Proof of the Twin Primes Conjecture

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Abstract. In this paper, a proof of the Prime Number Conjecture is presented, using set theory.

1. Introduction

The Twin Primes Conjecture says: There are infinitely many pairs of twin primes. Twin primes are those that are separated by two units. Namely,
{ (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), ...}

Proof

Let W be a set, defined as

$$W = \{2n | n \in \mathbb{N} \ \ and \ \ n \geq 4\} \tag{1}$$

To have the entire set W, 5 cases are presented in the interval [n -1, n + 1] with n $\geq$ 4

i) (p, p + 2) where p is a prime number

ii) (p, t) where t is an odd number not prime

iii) (t, p)

iv) (s, s + 2) where s is an even number

v) (t, t + 2)

Derived from the previous cases, let us consider the following sets

$$\begin{aligned} A_1 &= \{(p, p+2) | p + (p+2) \in W\} && (2) \\ A_2 &= \{(p, t) | p + t \in W\} && (3) \\ A_3 &= \{(t, p) | t + p \in W\} && (4) \\ A_4 &= \{(s, s+2) | s + (s+2) \in W\} && (5) \\ A_5 &= \{(t, t+2) | t + (t+2) \in W\} && (6) \end{aligned}$$

From the above, we have

$$W = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) \tag{7}$$

But as the A_4 set

$$\begin{aligned} A_4 &= \{10, 14, 18, 22, 26, 30, ...\} = \{4(n-1)+10 | n \in \mathbb{N} \;\; and \;\; n \geq 1\} && (8) \\ \Rightarrow |A_4| &= \infty && (9) \end{aligned}$$

So we will only take

$$(A_1 \cup A_2 \cup A_3 \cup A_5) \tag{10}$$

and, we must prove

$$\begin{aligned} |A_1| &= \infty && (11) \\ |A_2| &= \infty && (12) \\ |A_3| &= \infty && (13) \\ |A_5| &= \infty && (14) \end{aligned}$$

Let B be the following set

$$\begin{aligned} B &= \{(2n+1, 2n+3) | (2n+1)+(2n+3) \in W \;\; with \;\; n \in \mathbb{N} \;\; and \;\; n \geq 1\} && (15) \\ \Rightarrow |B| &= \infty && (16) \end{aligned}$$

Also, note that

$$B = (A_1 \cup A_2 \cup A_3 \cup A_5) \tag{17}$$

Let us take the following differences between sets

$$\begin{aligned} B - (A_2 \cup A_3 \cup A_5) &= A_1 && (18) \\ B - (A_1 \cup A_3 \cup A_5) &= A_2 && (19) \\ B - (A_1 \cup A_2 \cup A_5) &= A_3 && (20) \\ B - (A_1 \cup A_2 \cup A_3) &= A_5 && (21) \end{aligned}$$

later

$$
\begin{aligned}
|B-(A_2\cup A_3\cup A_5)| &= |A_1| && (22)\\
|B-(A_1\cup A_3\cup A_5)| &= |A_2| && (23)\\
|B-(A_1\cup A_2\cup A_5)| &= |A_3| && (24)\\
|B-(A_1\cup A_2\cup A_3)| &= |A_5| && (25)
\end{aligned}
$$

now, suppose that

$$
\begin{aligned}
|B-(A_2\cup A_3\cup A_5)| &= |A_1|<\infty && (26)\\
\Rightarrow |B| &< \infty && (27)\\
\textit{and} &\\
|(A_2\cup A_3\cup A_5)| &< \infty && (28)
\end{aligned}
$$

But by equation (16)

$$
\begin{aligned}
|B| &= \infty && (29)\\
\Rightarrow\ \textit{than equation } (26)\ \ |A_1| &= \infty && (30)
\end{aligned}
$$

$$
\begin{aligned}
|B-(A_1\cup A_3\cup A_5)| &= |A_2|<\infty && (31)\\
\Rightarrow |B| &< \infty && (32)\\
\textit{and} &\\
|(A_1\cup A_3\cup A_5)| &< \infty && (33)
\end{aligned}
$$

But by equation (16)

$$
\begin{aligned}
|B| &= \infty && (34)\\
\Rightarrow\ \textit{than equation } (31)\ \ |A_2| &= \infty && (35)\\
\textit{and by equation } (30),\ \textit{equation } (33)\ \textit{leaves us} &\\
|(A_1\cup A_3\cup A_5)| &= \infty && (36)
\end{aligned}
$$

$$
\begin{aligned}
|B-(A_1\cup A_2\cup A_5)| &= |A_3|<\infty && (37)\\
\Rightarrow |B| &< \infty && (38)\\
\textit{and} &\\
|(A_1\cup A_2\cup A_5)| &< \infty && (39)
\end{aligned}
$$

But by equation (16)

$$
\begin{aligned}
|B| &= \infty && (40)\\
\Rightarrow \ \textit{than equation } (37) \ \ |A_3| &= \infty && (41)\\
\textit{and by equations } (30) \textit{ and } (35), \textit{ equation } (39) \textit{ leaves us} & && \\
|(A_1 \cup A_2 \cup A_5)| &= \infty && (42)
\end{aligned}
$$

$$
\begin{aligned}
|B - (A_1 \cup A_2 \cup A_3)| &= |A_5| < \infty && (43)\\
\Rightarrow |B| &< \infty && (44)\\
\textit{and} & && \\
|(A_1 \cup A_2 \cup A_3)| &< \infty && (45)
\end{aligned}
$$

But by equation (16)

$$
\begin{aligned}
|B| &= \infty && (46)\\
\Rightarrow \ \textit{than equation } (43) \ \ |A_5| &= \infty && (47)\\
\textit{and by equations } (30), \ (35), \textit{ and}(41), \textit{ equation } (45) \textit{ leaves us} & && \\
|(A_1 \cup A_2 \cup A_3)| &= \infty && (48)
\end{aligned}
$$

Then by the equations (35), (41) and (47), we are left with the equation (28)

$$
|(A_2 \cup A_3 \cup A_5)| = \infty \qquad (49)
$$

then, from equations (30) and (49) we conclude

$$
|A_1 \cup A_2 \cup A_3 \cup A_5| = \infty \qquad (50)
$$

of equations (16), (17) and (50)

$$
|B| = \infty = |A_1 \cup A_2 \cup A_3 \cup A_5| \qquad (51)
$$

<u>and from equations (2) and (30), we have infinite pairs of twin primes</u>

References

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