Application of multi-variable synthesis using Matlab

Maneuvering a submarine

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1. Introduction

The goal of this is to be able to make the study of the system of a submarine, analyzing its stability and make the necessary changes in order to obtain a less sensitive system and decoupled.

To achieve this, we are going the use the software Matlab to simplify the calculations and see how the system is responding, verifying that the dynamics of the system and its decoupling is the ones we want

2. Analysis of the natural system:

2.1. State space model

This is the linearized state space model that we are going to use for the submarine system:

$$\dot{X} = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{41} \\ a_{21} & a_{22} & 0 & a_{42} \\ 1 & 0 & 0 & a_{43} \\ 0 & 1 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} b_{11} & b_{21} \\ b_{21} & b_{22} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} U \qquad Y = \begin{bmatrix} \delta y_1 \\ \delta y_2 \end{bmatrix} \qquad U = \begin{bmatrix} \delta u_1 \\ \delta u_1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X$$

Figure 2.1: State space model

We choosed the speed of 30 knots which correspond to the following values in the state space model:

$$\begin{array}{lll} a_{11} = -0.19003 & a_{12} = 4.4802 & a_{14} = 0.0014673 \\ a_{21} = 0.0085526 & a_{22} = -0.45988 & a_{24} = -0.0056095 \\ a_{34} = -15.433 & b_{11} = -0.1855 & b_{12} = -0.57149 \\ b_{21} = 0.043308 & b_{22} = -0.055543 & \end{array}$$

Figure 2.2: Values for speed of 30 knots

2.2. Observability and Controllability

Is good to remember that our matrix A has a rank of 4, this value is going to be use to determine the observability and the controllability of the system.

2.2.1 Observability

We need to calculate the matrix of observability which has the following form:

$$Q_0 = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Figure 2.3: Matrix of observability

For our case the matrix Q_o is:

$$Q_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -15.433 \\ 0 & 1 & 0 & 0 \\ -0.19 & -10.9528 & 0 & 0.0015 \\ 0.0086 & -0.4599 & 0 & -0.0056 \\ -0.0576 & 4.1871 & 0 & 0.0612 \\ -0.0056 & 0.2442 & 0 & 0.0026 \end{bmatrix}$$

Figure 2.4: Resulting matrix of observability

We can see that our observability matrix has a rank of 4 equal to the rank of our matrix A, with this conclude that our matrix is observable.

2.2.2 Controllability

The same procedure that we used to determine the observability, but in this case we need to calculate the matrix of controllability:

$$Q_c = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$

Figure 2.5: Matrix of controllability

After the calculations we obtain the following matrix

$$Q_c = \begin{bmatrix} -0.155 & -0.5715 & 0.2293 & -0.1402 & -0.1398 & 0.1191 & 0.0785 & -0.0691 \\ 0.0433 & -0.0555 & -0.0215 & 0.0207 & 0.0116 & -0.0104 & -0.0064 & 0.0057 \\ 0 & 0 & -0.1855 & -0.5715 & -0.4391 & 0.717 & 0.192 & -0.1997 \\ 0 & 0 & 0.0433 & -0.0555 & -0.0215 & 0.0207 & 0.0116 & -0.0104 \end{bmatrix}$$

Figure 2.6: Resulting matrix of controllability

We can see that our system is also controllable.

2.3. Modes in laplace plane

For the matrix A we obtain the following eigenvalues (modes):

$$\begin{aligned} P1 &= 0 \\ P2 &= -0.5548 \\ P3 &= -0.0667 \\ P4 &= -0.0285 \end{aligned}$$

Figure 2.7: Modes of the system

Since we have a mode in 0 and the others are negatives, we conclude that our system in open loop is stable.

This is the visualization of theses modes in the Laplace plane:

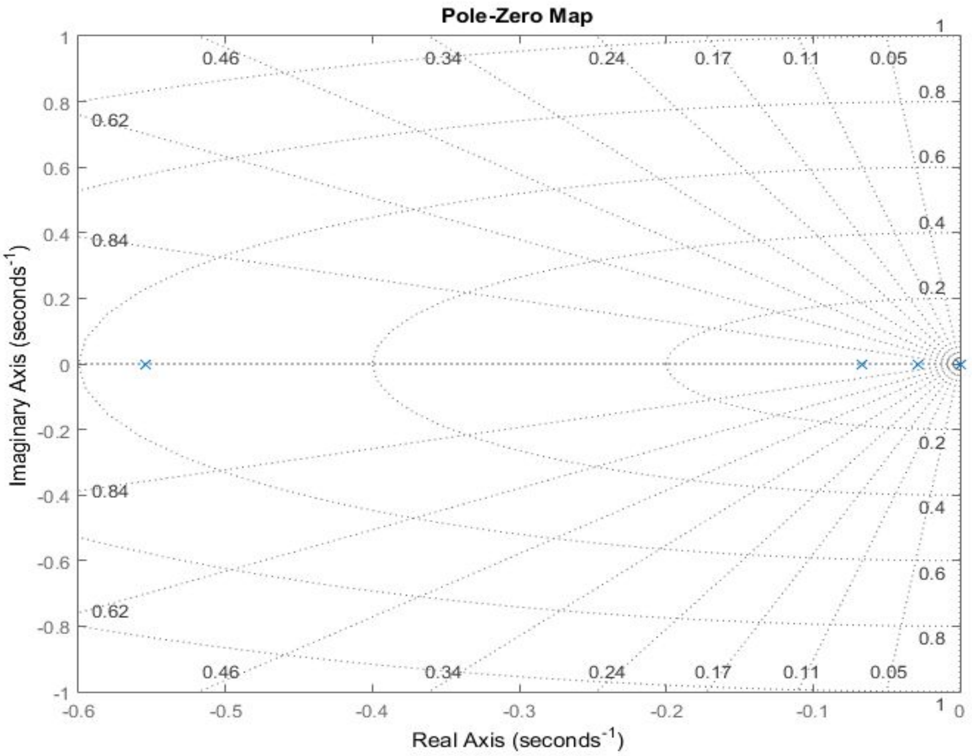

Figure 2.8: Laplace plane

With the help of the function damp() of matlab, we can know the damping, frequency and time constant of the poles.

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
0.00e+00	-1.00e+00	0.00e+00	Inf
-2.85e-02	1.00e+00	2.85e-02	3.51e+01
-6.67e-02	1.00e+00	6.67e-02	1.50e+01
-5.55e-01	1.00e+00	5.55e-01	1.80e+00

Figure 2.9: Information about the modes

We can see that except for the mode in the origin which is underdamped, the others poles has a critically damped. Remembering was is the meaning of having a underdamped and critically damped modes:

- **Mode underdamped**

The oscillations tends to overshoot its starting position, and then return, overshooting again. With each overshoot, some energy in the system is dissipated, and the oscillations die towards zero.

- **Mode critically damped**

In this case the system will just fail to overshoot and will not make a single oscillation. In other word the system returns to equilibrium in the minimum amount of time.

2.4. Step and impulse response

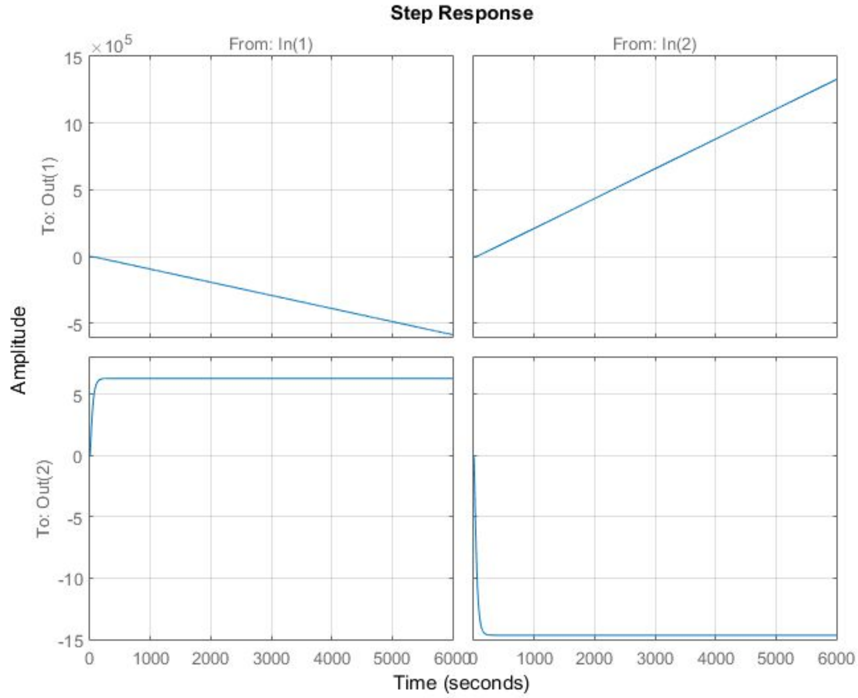

Figure 2.10: Step response of the system

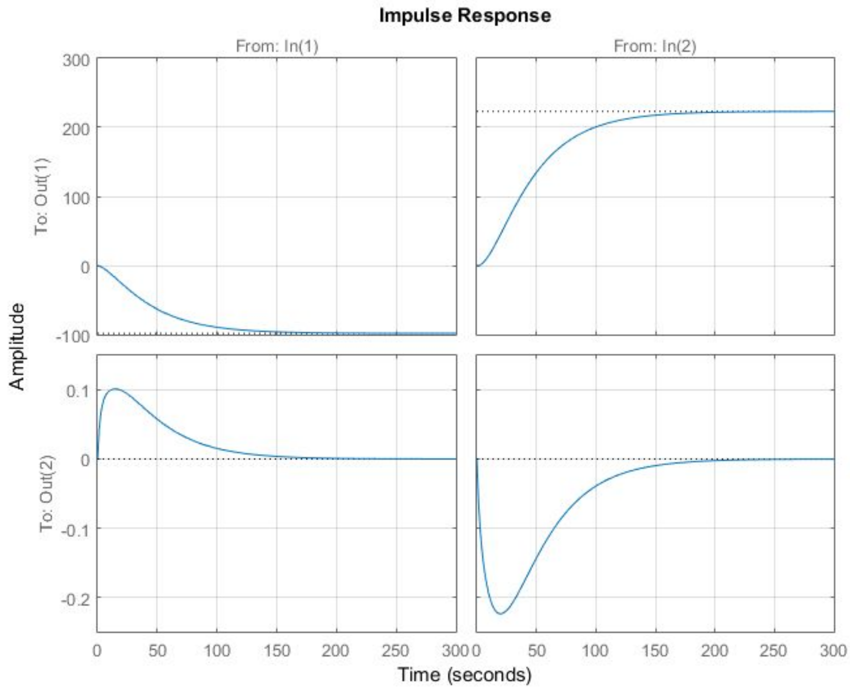

Figure 2.11: Impulse response of the system

We see from the step and impulse response, that the modes affecting the output 2 are stabilizing the output after a certain time. But in the case of the modes of the output 1, if the system has a continued perturbation like the step impulse response, the modes are not going to stabilize the output 1, but if it is a small perturbation like the impulse response the modes will stabilize the output 1.

2.5. Couplings

From the previous curves we see that the all the entries of our system have an influence in the outputs, but we also see that the entry 2 which correspond to stern angle has a great influence to the output 1 (depth) and the output 2 (pitch angle), the entry 1 (bow angle) only have influence in the output 1, it has a minimal influence in the other output.

3. Decoupling and control law:

In order to obtain a system in which the bow angle we will command the depth and the stern angle will command the pitch angle we need to do a decoupling with the following state feedback control law:

$$U = -KX + HY_c$$

Figure 2.12: State feedback control law

We are going to have two subsystems $\sum_1$ and $\sum_2$, there is going to be only one

output per subsystem and it will be stimulated by a single input. The subsystem one is going to be assigned the modes λ_1 and λ_2 and for the other subsystem the modes λ_3 and λ_4. We need to have a damping greater than 0.65 for the modes and a rising time of 80s for the depth and 100s for the pitch angle.

3.1 Subsystem $\sum_1$

For the mode λ_1 who was given by the problem,we can redo how to obtain it:

$$\lambda_1 = -\frac{1}{\tau_1}$$
$$T_{r1} = 80s = 3\tau_1$$
$$\lambda_1 = -\frac{3}{80}$$

For the mode λ_2 we chose his value in order to its dynamic to be negligible,

3.2 Subsystem $\sum_2$

For the mode λ_3 who was given by the problem,we can redo how to obtain it:

$$\lambda_3 = -\frac{1}{\tau_3}$$
$$T_{r3} = 100s = 3\tau_3$$
$$\lambda_3 = -\frac{3}{100}$$

For the mode λ_4 we did the same as for λ_2,

$$\lambda_4 = -\frac{3.85}{100}$$

We after using the function place() in matlab we obtain the following step response for the closed-loop system:

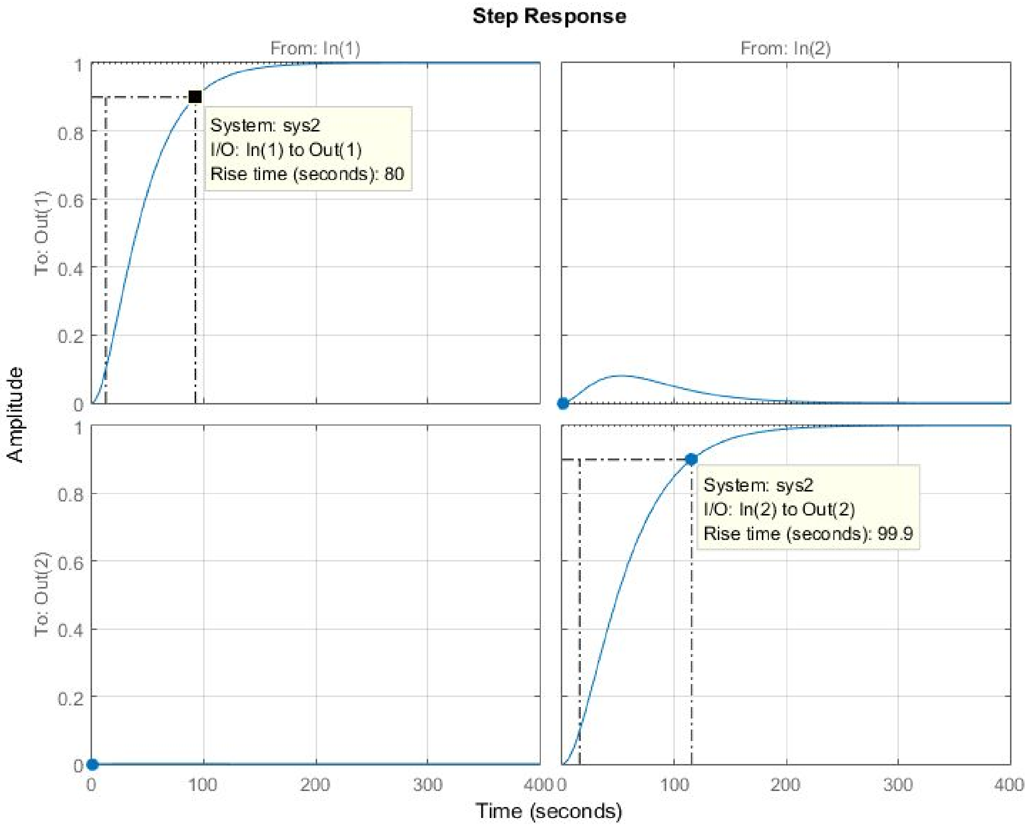

Figure 2.12: Step response of the closed-loop system

The step response tell us that for the propose modes, the rising times for the outputs are correct, but the system is not completely decoupled, since matlab doesn’t do the the correct decoupling constraints.

4. Insensitive pole placement:

4.1. Place, Cond and Sensible matlab functions

- **place.m**:

This function is used to find the state feedback matrix K. This matrix allows situate the poles of the closed-loop matrix, normally (A-BK). It is also possible to use this function for calculate the state feedback matrix L for a state observer (A-LC).

- **cond.m**:

This function returns the matrix condition value, that measure means the sensitivity of the solution to small errors in the modeled system related to the incertitude of the parameters. Condition near to 1 indicates a well-conditioned matrix.

- **sensible.m**:

This functions allows visualize the modes comportament when a random non-structure perturbation is induced in the system, this functions also return the maximum real value in the set of modes of the systems perturbed, as well as the maximum distance between two modes.

4.2. Comparing sensibilities

In order to find the best state feedback control that allows the smallest sensitivity possible, we made 4 iterations variating the pole placement and analysing the closed-loop matrix condition for each modes set.

Poles [λ_1 λ_2 λ_3 λ_4]	Condition
-3/100 -3.85/100 -3/80 -3.85/80	3.6714e+03
-3/100 -15/100 -3/80 -15/80	87.3839
-3/100 -15 -3/80 -15	37.9654
-3/100 -50 -3/80 -50	38.2246

Table 4.1. Matrix condition for differents poles placements

The matrix condition decrease with greater poles, but we have a big restriction in practice because choosing bigs poles we force the system to act fast, if the actuators do not have the response time sufficiently small this control will be impossible to set.

In order to know the system sensitivity, we observe le comportement of the modes when a perturbation is induced in the system, we simulate 4 differents perturbations to know the maximum value of perturbation accepted for the system.

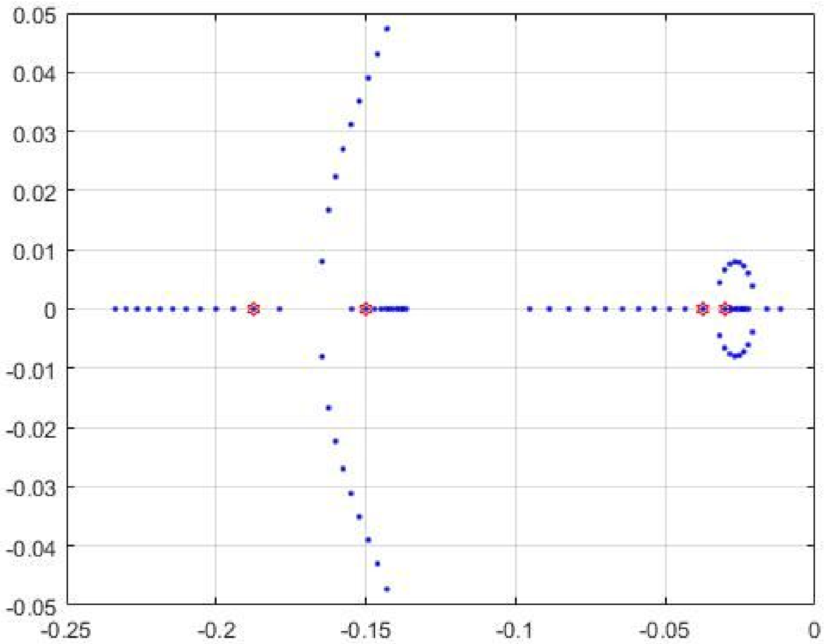

Figure 4.1. Poles sensitivity for Pmax= 0.001

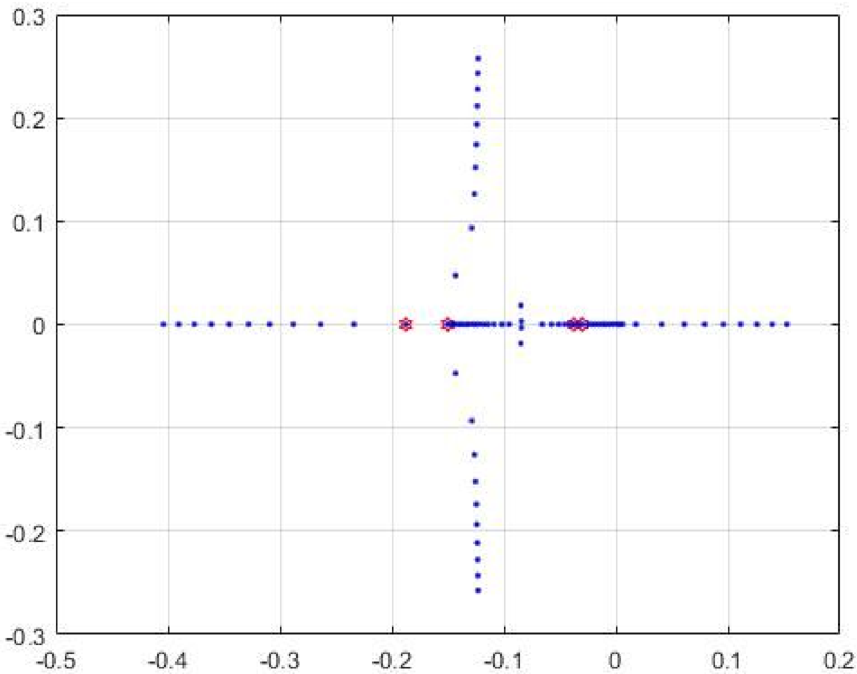

Figure 4.2. Poles sensitivity for Pmax= 0.01

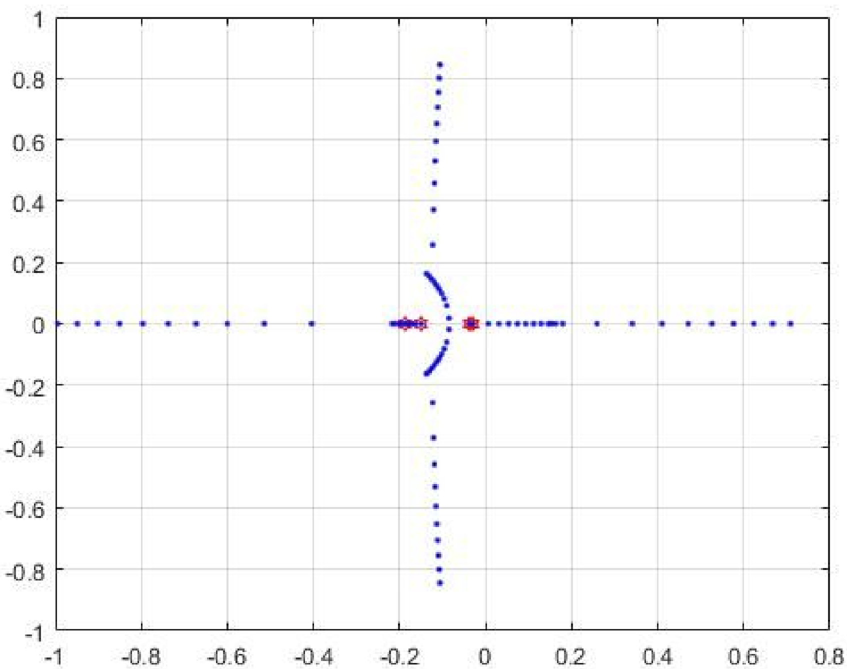

Figure 4.3. Poles sensitivity for Pmax= 0.1

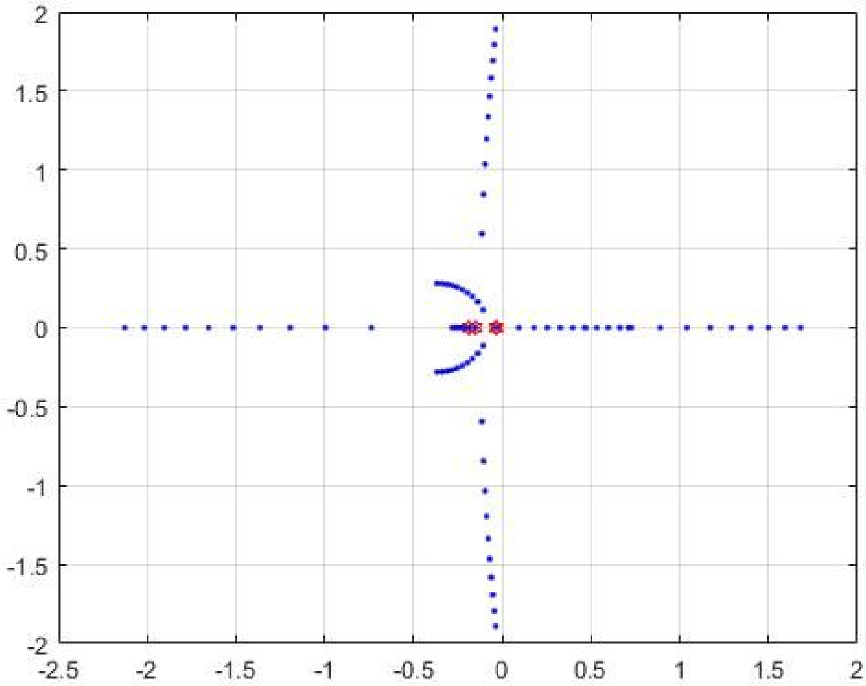

Figure 4.4. Poles sensitivity for Pmax= 0.5

The bigger the perturbation is, the greater is the poles variations. Firstly, we choose a perturbation of same order that the matrix coefficients. After the first iteration, the poles variation includes some values inside the semiplane positive, that means our system would be inestable for this range of perturbation. In the same way for pmax equal to 0,1 and 0,01.

The only range of perturbation that does not make the system inestable is with values around 0.001. We can conclude that our system can reject only the little perturbations. This is a big problem because we have a very sensitive system to the perturbations.

Pmax	Maximum real part of the poles	Biggest difference with the system nominal poles
0.5	1.6807	69.8371
0.1	0.7088	32.1455
0.01	0.1528	12.4606
0.001	-0.0115	6.5484

Tabla 4.2. Sensitivity for poles λ_1 =-3/100 λ_2 =-15/100 λ_3 =-3/80 λ_4 =-15/80

In the same way, simulating with the other set of poles (See Table 4.1.) we obtain the next result:

Pmax	Maximum real part of the poles	Biggest difference with the system nominal poles
0.5	1.0323	50.3777
0.1	0.307	33.8433
0.01	0.0093	22.5451
0.001	-0.218	20.2436

Tabla 4.3. Sensitivity for poles λ_1 =-3/100 λ_2 =-50/100 λ_3 =-3/80 λ_4 =-50/80

Pmax	Maximum real part of the poles	Biggest difference with the system nominal poles
0.5	4.3550	504.1386
0.1	1.0023	34.4100
0.01	0.0786	3.6189
0.001	-0.0192	0.3608

Tabla 4.4. Sensitivity for poles λ_1 =-3/100 λ_2 =-15/100 λ_3 =-3/80 λ_4 =-15/80

Pmax	Maximum real part of the poles	Biggest difference with the system nominal poles
0.5	5.1626	173.0876
0.1	1.0805	1665
0.01	0.0826	3.7528
0.001	-0.0190	0.3674

Tabla 4.5. Sensitivity for poles λ_1 =-3/100 λ_2 =-50 λ_3 =-3/80 λ_4 =-50

Analysing the previous results, it is clear that the bigs poles do not make less sensitive the system. Although the bigs changes in the poles placements, the only one perturbation rejected is pmax=0.001, obtaining a slightly less sensitivity with the second set of poles.

Now, we obtain the maximum perturbation rejected for the system through an iteration process. This perturbation is the maximum value that do not make the system inestable, that means, every poles have real part negative.

Pmax=0.0069

4.3. Degraded mode:

In order to realize a deeper analysis, we inactive the input one and doing the same previous study, we can know the sensitivity for the input one. In like manner for the input two.

4.3.1. Study first Input (U1):

Taking the first column of the matrix B and the second row of the matrix C, we rebuild our system. We compute the state feedback control doing the same first pole placement.

$$K = [0.1950 \quad -4.8198 \quad -0.0003 \quad 0.1873]$$

Closing the loop, we obtain a condition value of 39903, thus the single input system is ill-conditioned.

Pmax	Maximum real part of the poles	Biggest difference with the system nominal poles
0.5	2.0271	73.9581
0.1	0.9713	33.6611
0.01	0.3202	11.6728
0.001	0.0915	5.5506

Tabla 4.6. Sensitivity U1 for poles $\lambda_1 =$-3/100 $\lambda_2 =$-15/100 $\lambda_3 =$-3/80 $\lambda_4 =$-15/80

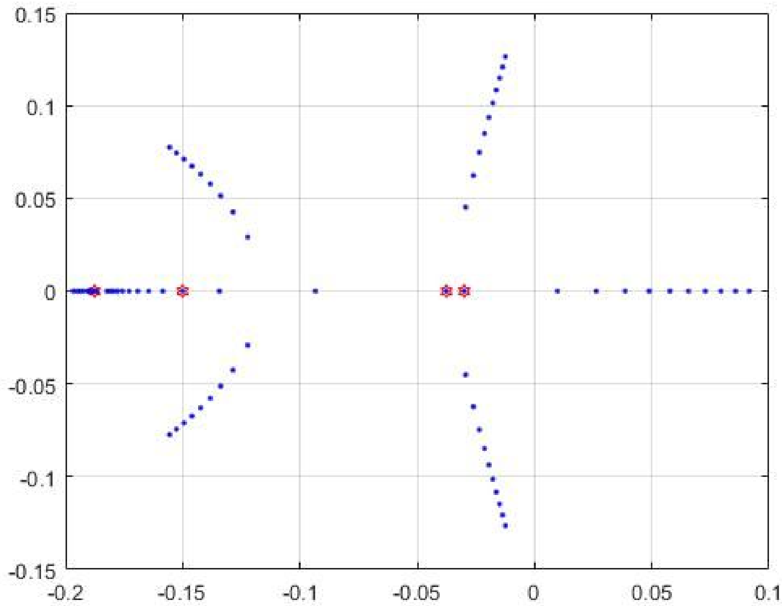

Figure 4.5. Poles sensitivity of the degraded system U1 pmax= 0.001

4.3.2. Study second Input (U2):

Taking the second column of the matrix B and the first row of the matrix C, We compute the state feedback control using the same previous poles.

$$K = [-0.1797 \quad 6.2587 \quad 0.0001 \quad -0.0380]$$

Closing the loop, we obtain a condition value of 92765, thus the single system for the second input is also ill-conditioned.

Pmax	Maximum real part of the poles	Biggest difference with the system nominal poles
0.5	2.0365	78.9012
0.1	0.9709	32.9558
0.01	0.3260	11.2486
0.001	0.1000	7.0451

Tabla 4.7. Sensitivity U2 for poles λ_1 =-3/100 λ_2 =-15/100 λ_3 =-3/80 λ_4 =-15/80

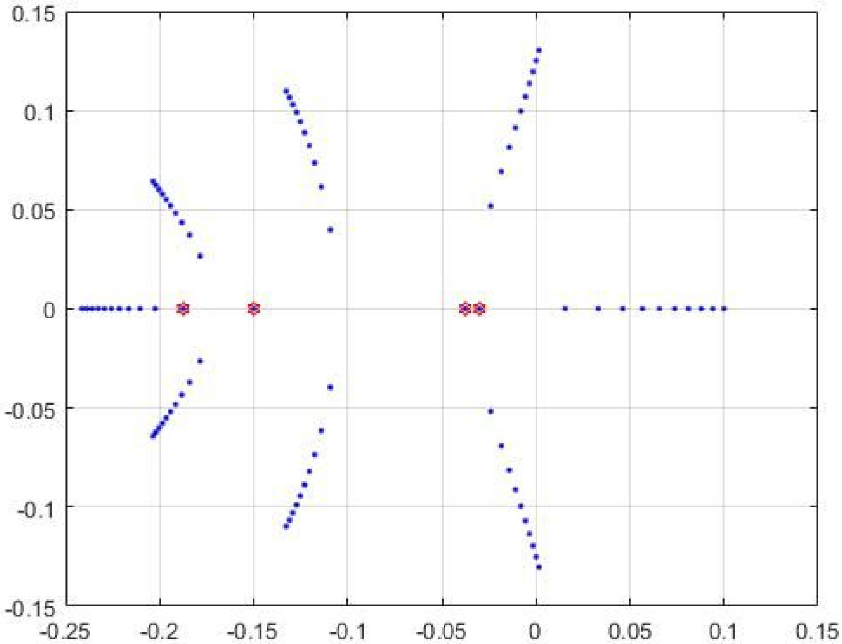

Figure 4.6. Poles sensitivity of the degraded system U2 pmax= 0.001

In conclusion the two systems degraded are more sensitive than the complete system, we observe that for the same perturbations used previously, the systems degraded are not capable to refuse it, making always inestable the system.

In the following table we can observe the maximum perturbation admissible for every system.

System	Maximum perturbation
With U1 and U2	0.0069
Only U1	0.00005983
Only U2	0.00005428

Tabla 4.8. Maximum perturbation admissible

We did not obtain results that meet the requirements, hence a state feedback control with eigenstructure assignment and decoupling will be considered.

5. Eigenstructure assignment and decoupling:

5.1. Admissible subspace associated with the modes:

We can find the admissible subspace thanks to the following equation:

$$N(\lambda_i) = -(I\lambda - A)^{-1}B$$

In order to simplify the calculations we select $M(\lambda) = I$.

Evaluating $N(\lambda_i)$ for every desired mode we obtain the following matrix.

$$N(\lambda_1) = \begin{pmatrix} -39 & 111 \\ -10 & 4 \\ 16232 & -43390 \\ 37 & -98 \end{pmatrix} N(\lambda_2) = \begin{pmatrix} -36 & 108 \\ -1 & 3 \\ 8674 & -24455 \\ 25 & -69 \end{pmatrix}$$

$$N(\lambda_3) = \begin{pmatrix} -150 & 400 \\ -10 & 10 \\ 99350 & -256320 \\ 18 & -470 \end{pmatrix} N(\lambda_4) = \begin{pmatrix} -37 & 107 \\ -1 & 4 \\ 14668 & -39401 \\ 34 & -91 \end{pmatrix}$$

From the decoupling constraints, we can determine the structural conditions the eigenvectors have to satisfy for guaranteeing all requirements.

$$CV = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_{11} & v_{21} & v_{31} & v_{41} \\ v_{12} & v_{22} & v_{32} & v_{42} \\ v_{13} & v_{23} & v_{33} & v_{43} \\ v_{14} & v_{24} & v_{34} & v_{44} \end{pmatrix} = \begin{pmatrix} * & * & 0 & 0 \\ 0 & 0 & * & * \end{pmatrix}$$

Solving the previous equation we obtain the following conditions in the eigenvectors for satisfy the right decoupling.

$$v_{14} = 0 \quad v_{24} = 0 \quad v_{33} = 0 \quad v_{43} = 0$$

Hence the eigenvectors will have the following structure:

$$V = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & 0 & 0 \\ 0 & 0 & k & l \end{pmatrix}$$

Next we can determine the values of the eigenvector and the values of the vector Z_i thanks to the following relation.

$$V_i = N(\lambda_i)\, Z_i$$

The easier way to determine these values is fixing the i, j, l *and* k, so the third and fourth rows of V are completely defined, rewriting the expression only for the two last rows we can calculate the values for Z being as N will be a square and invertible matrix

$$Z_i = N(\lambda_i)_{3,4}^{\;-1}\, V_{i\ 3,4}$$

After that all the remaining values of V will be placed automatically, so for $i = j = l = k = 2$ the following values are obtained.

$$Z_1 = \begin{pmatrix} -0.0286 \\ -0.0107 \end{pmatrix} Z_2 = \begin{pmatrix} -0.0351 \\ -0.0125 \end{pmatrix} Z_3 = \begin{pmatrix} 12.7895 \\ 4.9571 \end{pmatrix} Z_4 = \begin{pmatrix} 12.6030 \\ 4.6918 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.075 & -0.0962 & 30.866 & 30.866 \\ 0 & 0 & -0.06 & -0.077 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

The W matrix will be equal to Z because $M(\lambda) = I$. Next we can determine the gain K which make decouple the system.

$$K = WV^{-1}$$

$$K = \begin{pmatrix} 0.3049 & 10.9742 & -0.0029 & 2.0189 \\ 0.0837 & 15.6032 & -0.0022 & 1.6544 \end{pmatrix}$$

Now, we want to determine the gain H through the left decoupling. In the same manner, we know the form for the matrix UBH, so it is possible to find H.

$$UBH = \begin{pmatrix} * & 0 \\ * & 0 \\ 0 & * \\ 0 & * \end{pmatrix} = B^*$$

We have to fix some values to determine the matrix H, we search define two rows and in this manner make the system equation soluble, so we choose the following values.

$$UBH = \begin{pmatrix} 1 & 0 \\ * & 0 \\ 0 & 1 \\ 0 & * \end{pmatrix}$$

If we isolate the first and the third row we can determine H with the following expression.

$$H = UB_{1,3}^{-1} B_{1,3}^*$$

$$H = \begin{pmatrix} -0.0337 & 0.2772 \\ -0.0263 & -0.090 \end{pmatrix}$$

Simulating the closed-loop system we can verify if the decoupling conditions are satisfied, using a step like input we obtain the following plot.

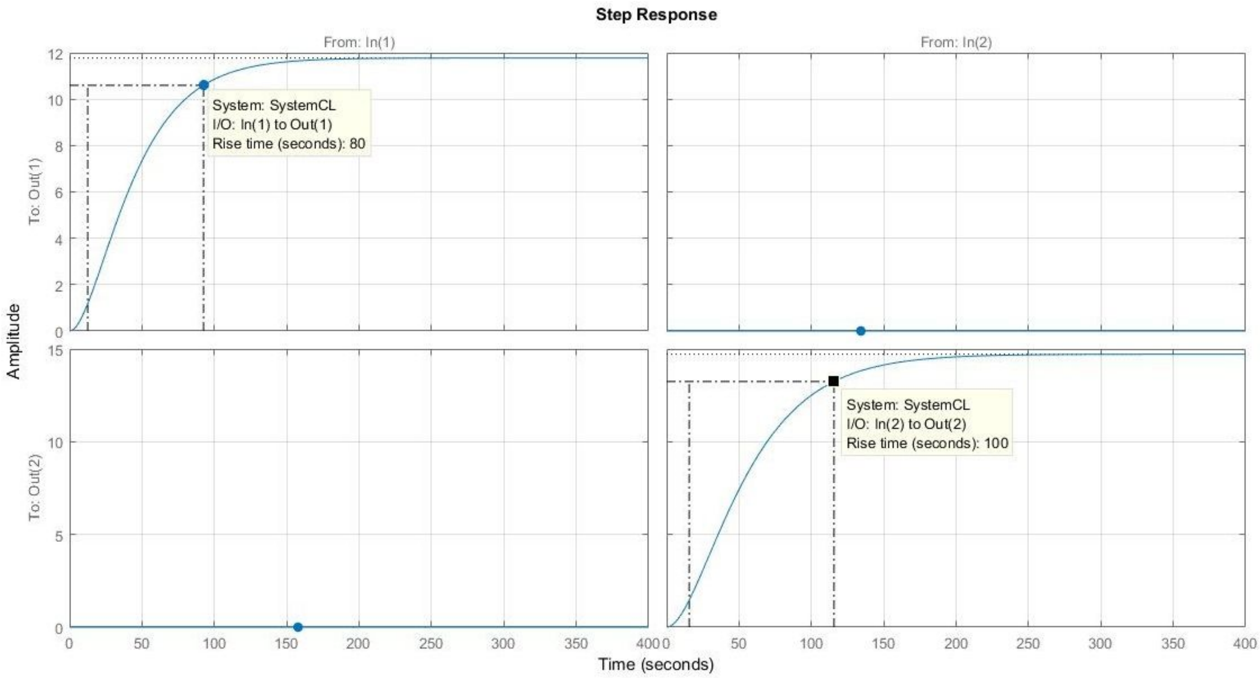

Figura 5.1 Step response system decoupled

In the previous figure we can constate the decoupling are satisfied, the first input does not have influence on the second output, in the same manner the second input does not affect the first output. Moreover the rise time for the two subsystem is reached taking a rise time equal to 80s for the deep (h) and 100s for the angle pitch (δ) as required.

An important question would be, is the decoupled system more or less sensitive than the previous system. In order to know the system sensitivity we apply the same procedure of the previous section.

Firstly, we calculate the condition for the closed-loop matrix.

$$Condition = 1.000$$

Obtaining a perfect condition equal to 1, that means the decoupled system is well-conditioned.

Secondly, we simulate the modes comportement when a perturbation equal to 0.001 is induced.

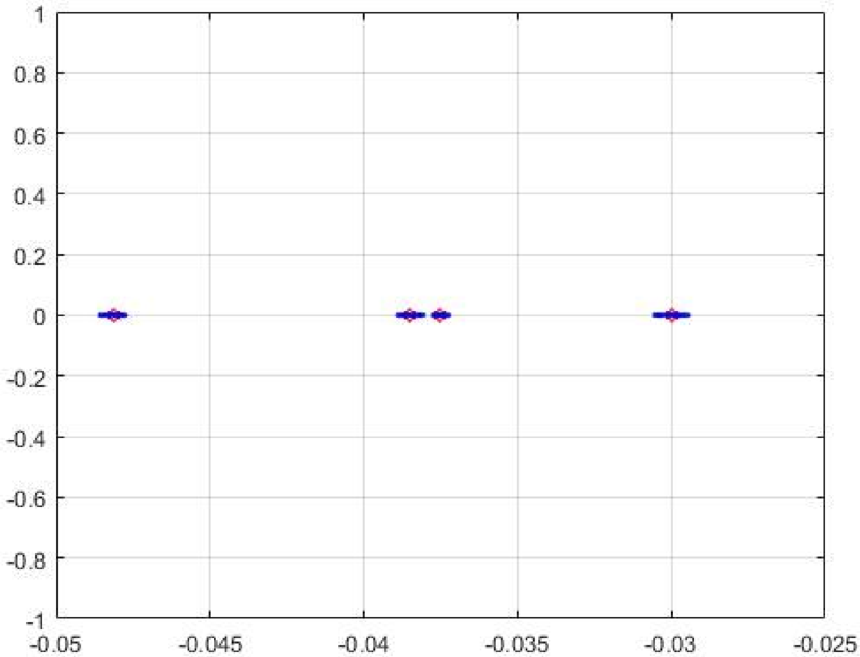

Figure 5.2. Poles sensitivity of the decoupled system Pmax= 0.001

Finally, we obtain the maximum admissible perturbation.

Pmax= 0.0275

In conclusion the decoupled system is much less sensitive than the system obtained through a simple state feedback control. Applying the same perturbation Pmax=0.001 the poles comportament for the decoupled system is almost not affected, the poles move only a little while the poles for the previous system are clearly disturbed. Another important difference is the maximum admissible perturbation, being for the second system almost 28 times bigger.

6. Decoupling in static mode

In static or steady-state mode we have that

$\dot{x} = 0$

thus we have that

$$(A - BK) + BHC = 0 \Leftrightarrow BHC = BK - A$$

And if we consider that Ω is equal to

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1}$$

Then we may transform this equation into the next two

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \times \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \times \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Leaving us with the next two systems

$$\begin{cases} \Omega_{11}A + \Omega_{12}C = 1 \\ \Omega_{11}B = 0 \\ \Omega_{21}A + \Omega_{22}C = 0 \\ \Omega_{21}A = 1 \end{cases} \qquad \begin{cases} A\Omega_{11} + B\Omega_{21} = 1 \\ C\Omega_{11} = 0 \\ A\Omega_{12} + B\Omega_{22} = 0 \\ C\Omega_{12} = 1 \end{cases}$$

Now we multiply $BHC = BK - A$ by Ω_{12}

$$BHC\,\Omega_{12} = BK\,\Omega_{12} - A\Omega_{12}$$

And replacing every term by the eight equations previously found we have that

$$BH = BK\Omega_{12} + B\Omega_{22}$$

In the same manner if we do it m=now for Ω_{21}

$$\Omega_{21}BH = \Omega_{21}BK\Omega_{12} + \Omega_{21}B\Omega_{22}$$

The we can conclude that

$$H = \Omega_{22} + K\Omega_{12}$$

6.1. Step and impulse response

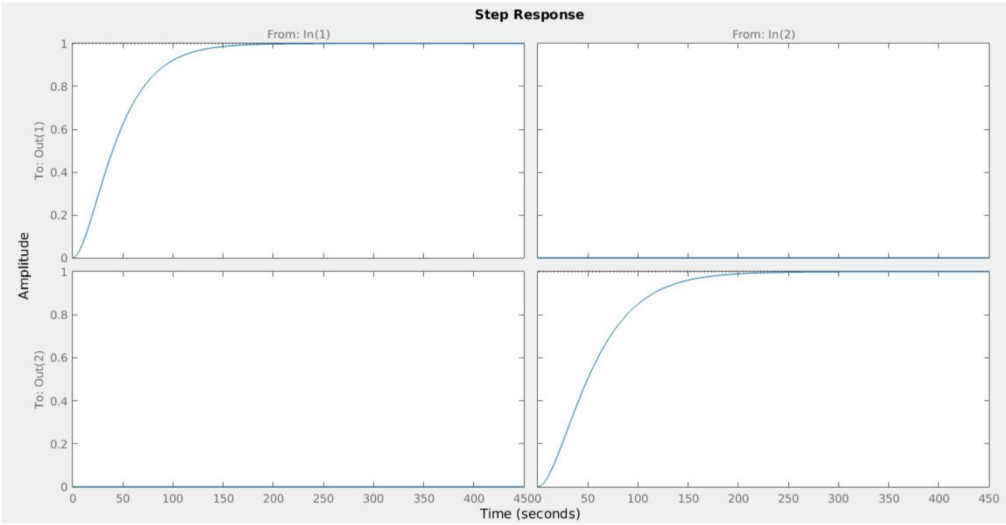

Figure 6.1: Step response of the system

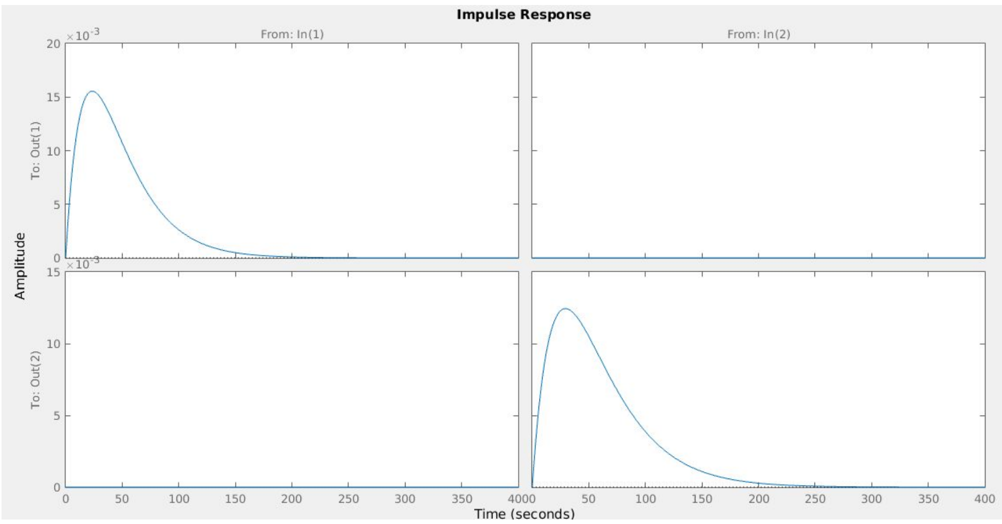

Figure 6.2: Impulse response of the system

We can conclude from this step and impulse responses that we have an almost perfect decoupled system and that in the step response we have an amplitude equal to 1 which means that our system has no intrinsic gain.

7. Conclusion

In this project we have been able to experience the process of analysing a real model of a multi input-multi output system and utilize tools and techniques to come up with the characteristics of the system.

The analyze of the modes made us realize how they were affecting the system and by a careful choose we accomplished to place new modes in order to have the required parameters (rising time and two poles with no much dynamique).

We obtained the observability and the controllability of our system and we proceeded to analyze the damping characteristics and then the sensitivity of the system by placing poles in different iterations and analyzing the sensitivity afterwards. In this point we face some of the constraints in the poles placing procedure and how it may affect the response of the actuators.

Making a feedback control law, does not guarantee that our system is going to be completely decouple, that's what happen when we did this in matlab, we saw that our system was no decouple has we wanted and did by ourselves the decouple, but another problem came up, we saw that we didn't have the gain wanted in the stationary state, this was caused by the matrix H of the system in closed-loop, another matrix H was calculated with a stationary gain.

We finally obtained a system that satisfies the decoupling and sensitivity constraints. In general this project help us to visualize and utilize concepts and techniques that otherwise may seems quite abstract.

This study was really interesting because we applied what we learned in classes and have the satisfaction that what we were taught was indeed useful to treat a problem that we may have in the future. With matlab is much easy and quicker to make the calculations, but if we didn't understood the theory, we would not be able to use this tool correctly and do a lot of mistakes, with the theory you know what you have to do and see the problems that you system has, correcting them in an efficient way.