IMAGE TRANSFORMATIONS - ROTATION

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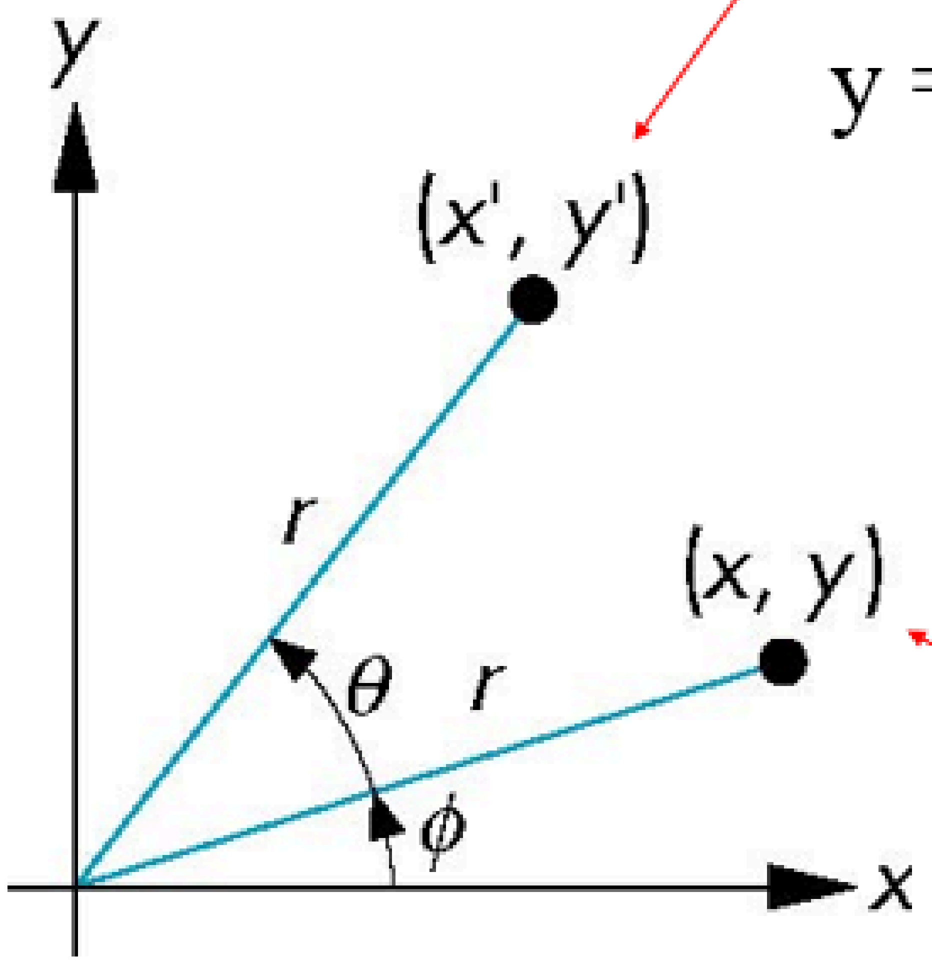
Rotation is a process in which a image is simply rotated around the origin or an image center by a given angle. This rotates the image or changes the orientation of an image depending on the angle it has been set to.

Its equation is:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Consider rotation about the origin by θ degrees

- radius stays the same, angle increases by θ

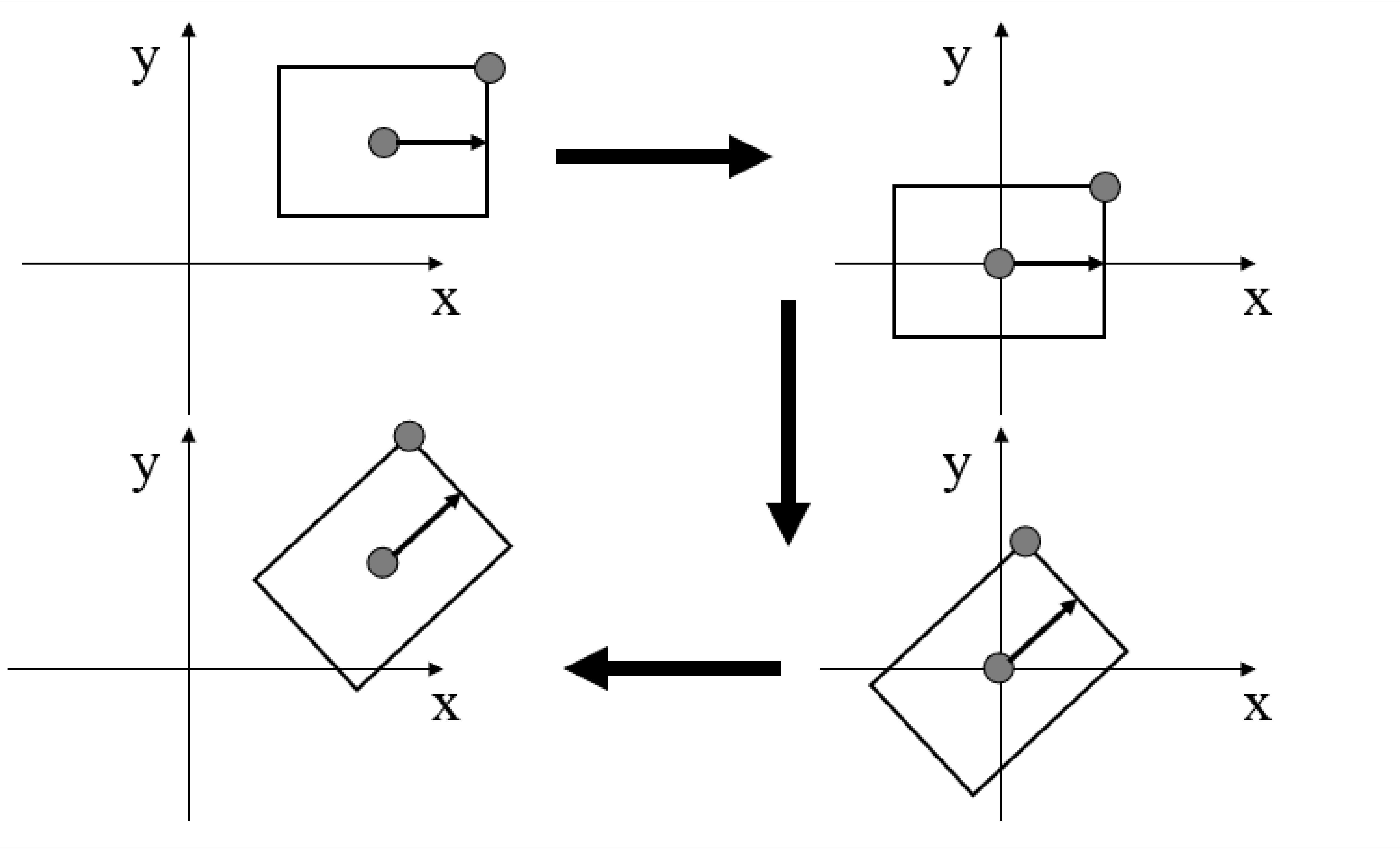


$$x = r \cos(\phi + \theta)$$
$$y = r \sin(\phi + \theta)$$

$$\boxed{\begin{aligned} x' &= x \cos\theta - y \sin\theta \\ y' &= x \sin\theta + y \cos\theta \end{aligned}}$$

$$x = r \cos\phi$$
$$y = r \sin\phi$$

Rotation about arbitrary point



Translation:

$x' = x - p$

Rotation:

$x'' = R(x') = Rx - Rp$

Translation Back:

$x''' = x'' + p = Rx - Rp + p$

Translation:

$x' = x - p$

Translate the center of rotation to the origin

Rotation:

$x'' = R(x') = Rx - Rp$

Rotate the object

Translation Back:

$x''' = x'' + p = Rx - Rp + p$

Translate back to original location

Rotation about x axis:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation about y axis:

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For about the axis, axis = 1

Rotation about z axis:

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ROTATION AND OTHER TRANFORMATIONS

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$T = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

->

$$T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & t_x \\ \sin\theta & \cos\theta & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate a Point in 2D Using a Rotation Matrix

Rotate the point

$$P = (2, 1)$$

by **90° counter-clockwise** around the **origin (0,0)**.

Step 1: Write the Rotation Matrix

For rotation by angle $\theta = 90^\circ = \frac{\pi}{2}$, the rotation matrix is:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Step 2: Multiply Matrix with Point

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} (0 \cdot 2 + -1 \cdot 1) \\ (1 \cdot 2 + 0 \cdot 1) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Final Answer:

$$P' = (-1, 2)$$

Rotate a Point in 3D Using a Rotation Matrix

Rotate the point

$$P = (2, 1, 0)$$

by **90° counter-clockwise** around the **Z-axis.**

Step 1: Write the 3×3 Z-axis Rotation Matrix

The rotation matrix for rotating around the **Z-axis** by an angle $\theta = 90^\circ = \frac{\pi}{2}$ is:

$$R_z(90^\circ) = \begin{bmatrix} \cos(90^\circ) & -\sin(90^\circ) & 0 \\ \sin(90^\circ) & \cos(90^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2: Multiply the Matrix by the Point

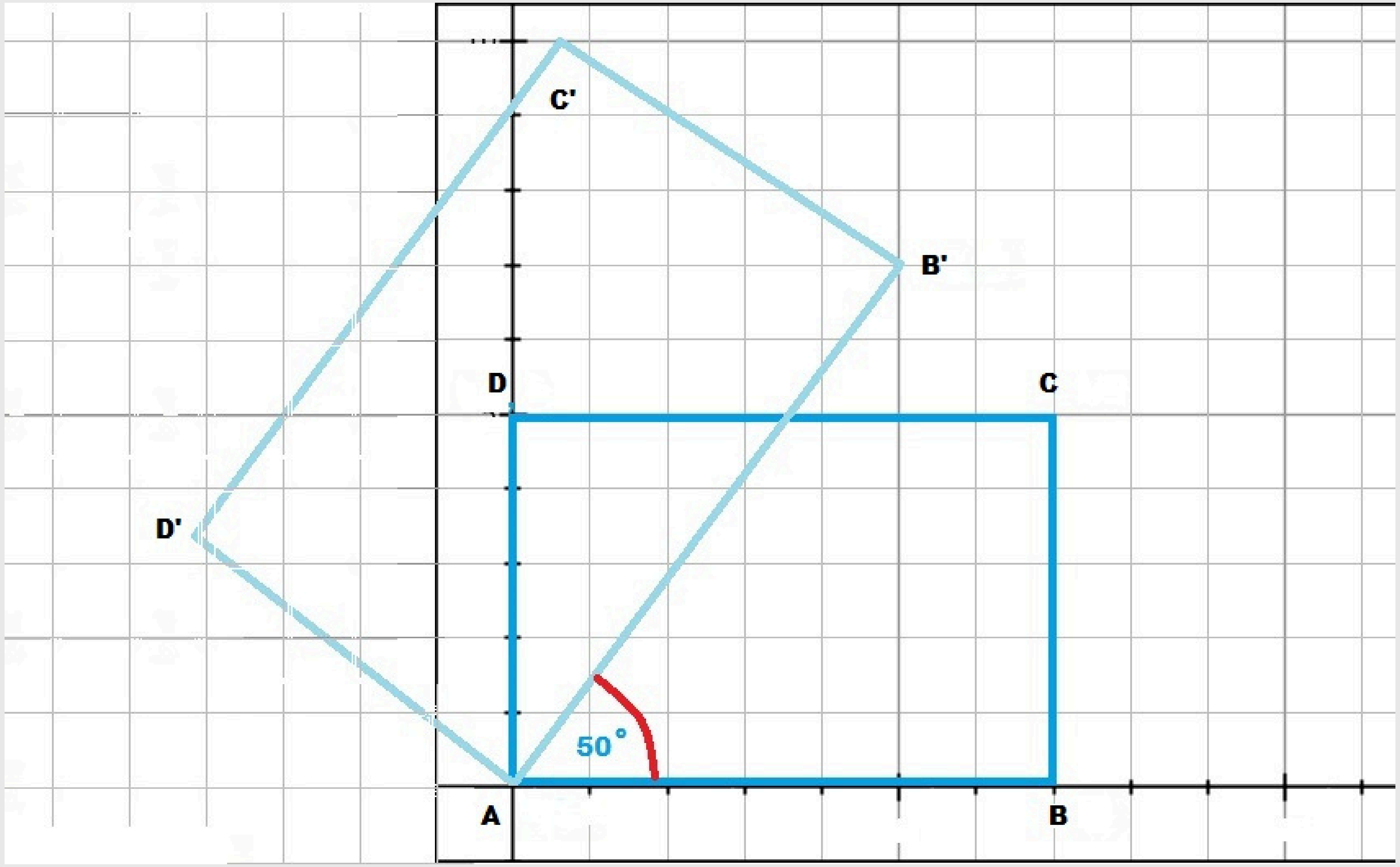
Now perform the matrix multiplication:

$$R_z \cdot P = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (0 \cdot 2 + -1 \cdot 1 + 0 \cdot 0) \\ (1 \cdot 2 + 0 \cdot 1 + 0 \cdot 0) \\ (0 \cdot 2 + 0 \cdot 1 + 1 \cdot 0) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

Final Answer:

The rotated point is

$$P' = (-1, 2, 0)$$


C'
B'
D
C
D'
50°
A
B

Thank you!