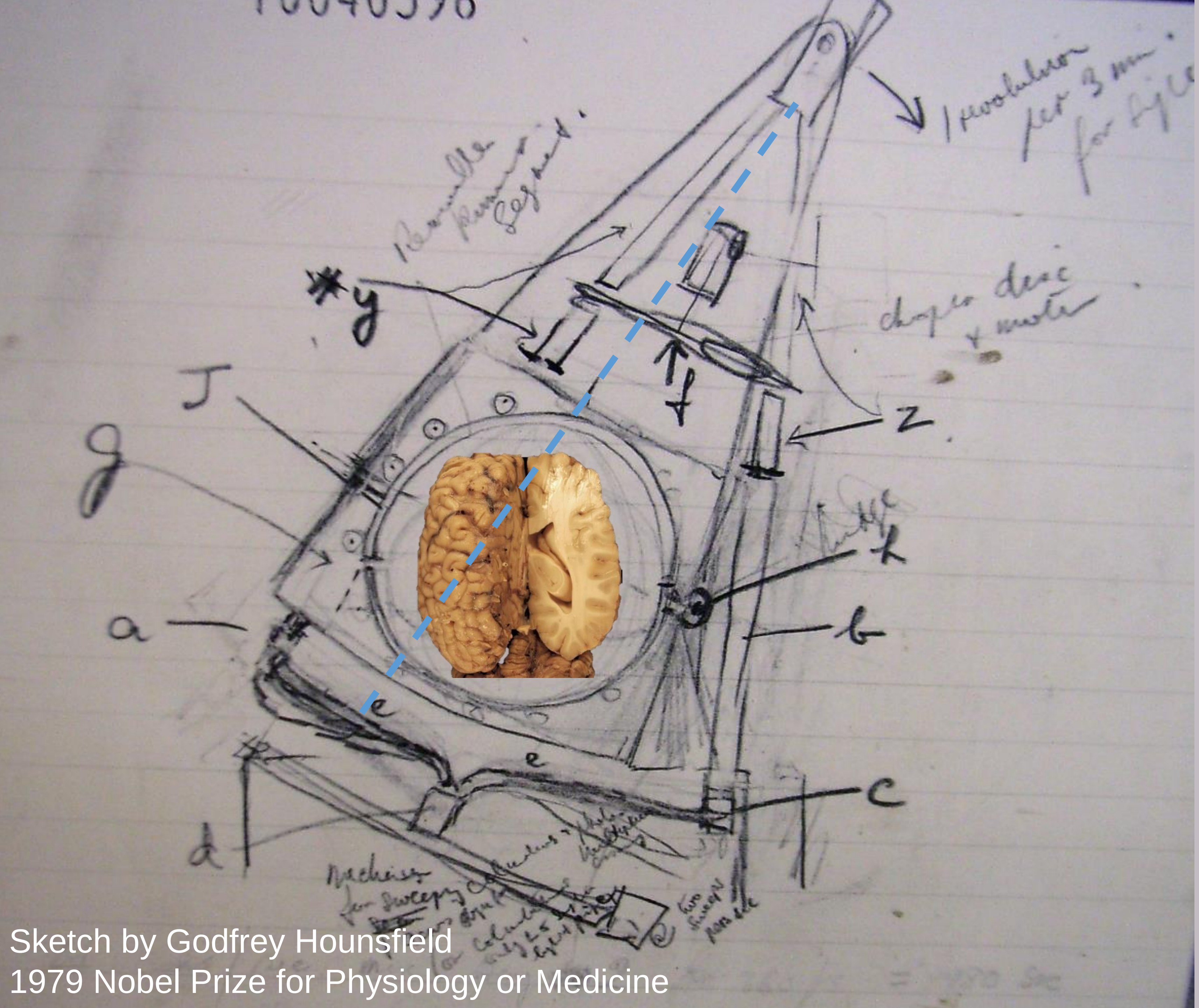
GHENT
UNIVERSITY

TOMOGRAPHY & RECONSTRUCTION

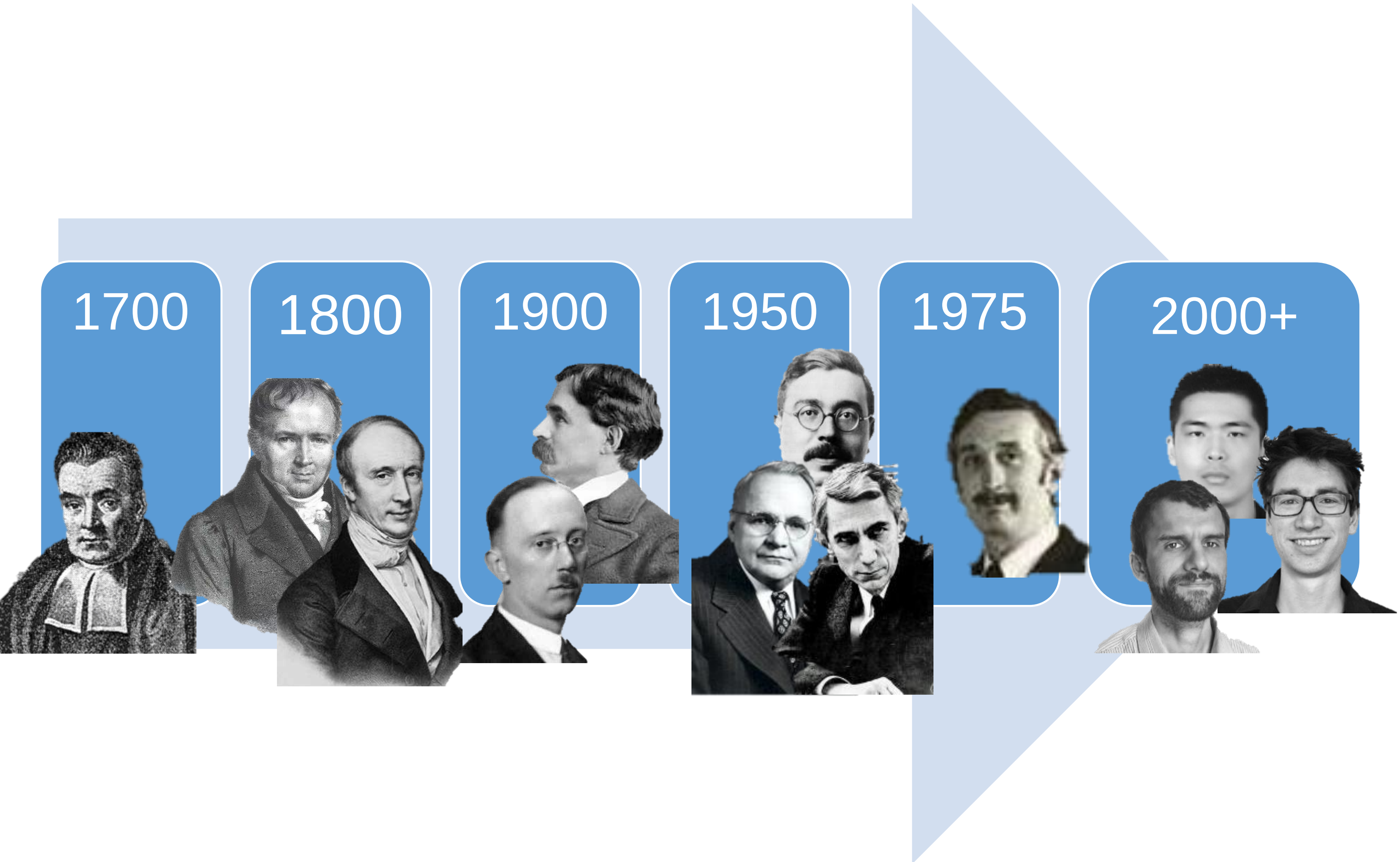
9:00-11:00

Jan Aelterman



Sketch by Godfrey Hounsfield
1979 Nobel Prize for Physiology or Medicine

A HISTORY OF TOMOGRAPHY



TOMOGRAPHY & RECONSTRUCTION

- What is it?
- How is it done?
  - Analytical
  - Iterative
- Link to estimation theory
  - Bayesian vs ML
  - Priors
- Link to machine learning

WHAT IS TOMOGRAPHY?

Etymology:
from the Greek tome (slice) and graphein (to write)

“imaging by sections”

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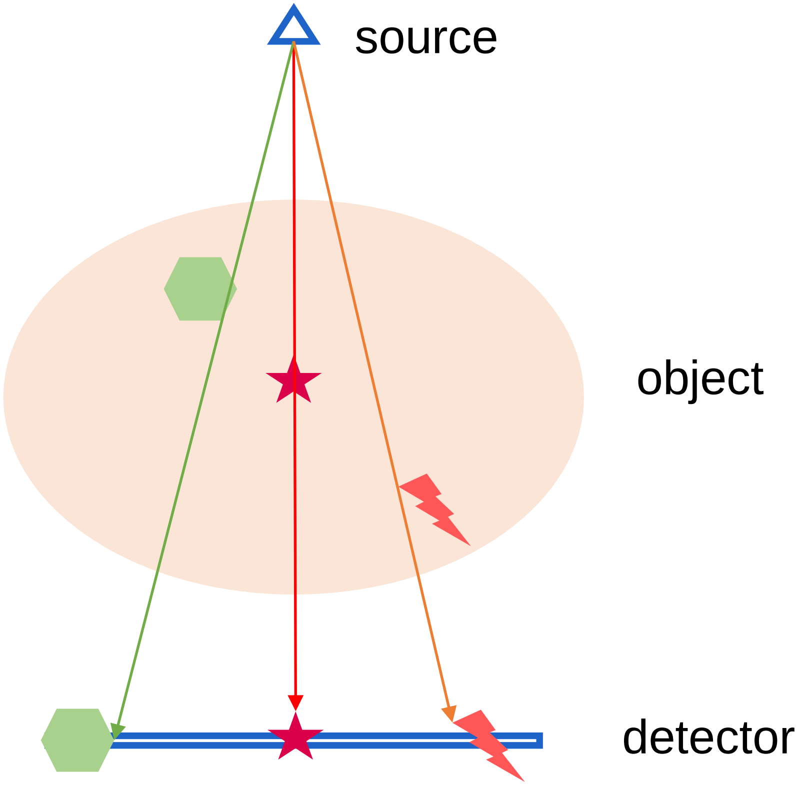

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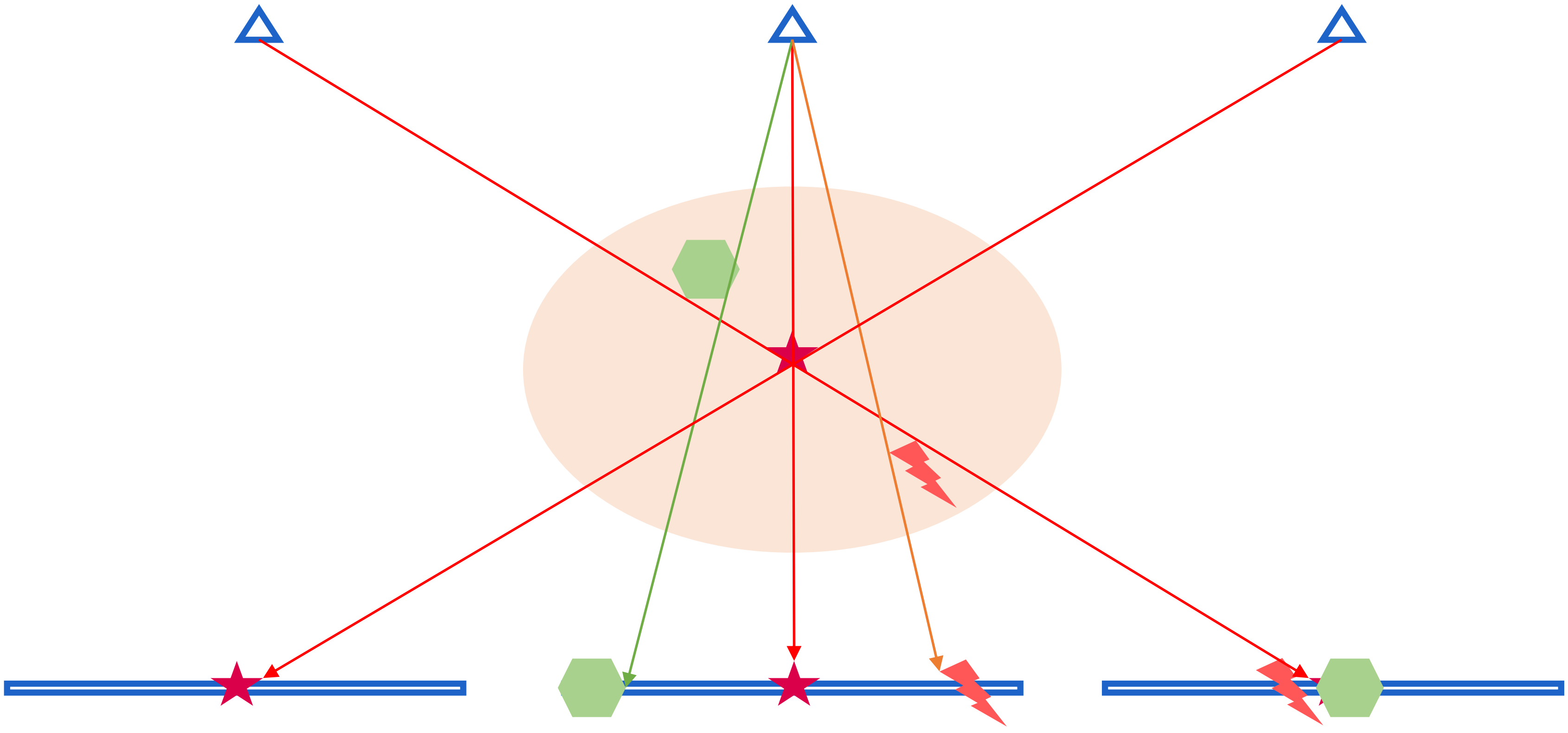

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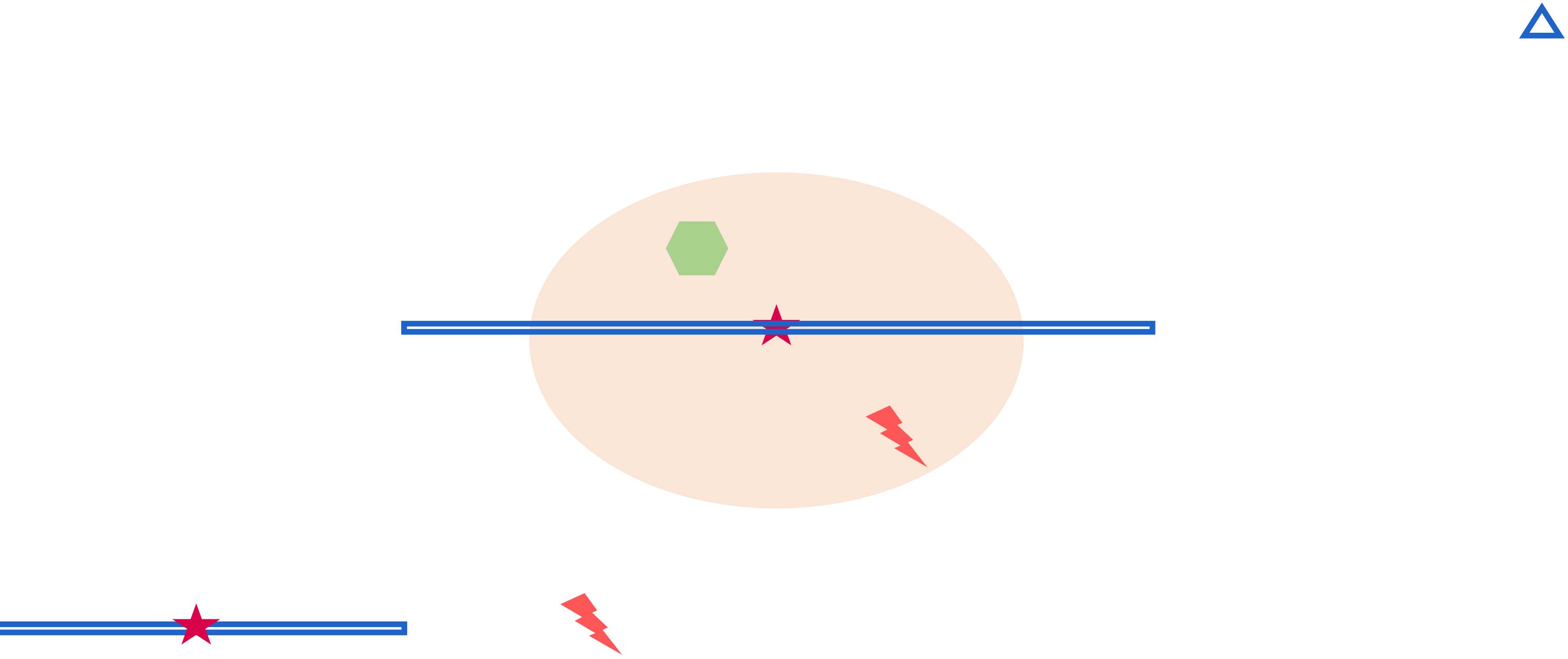

WHAT IS TOMOGRAPHY?

Etymology:
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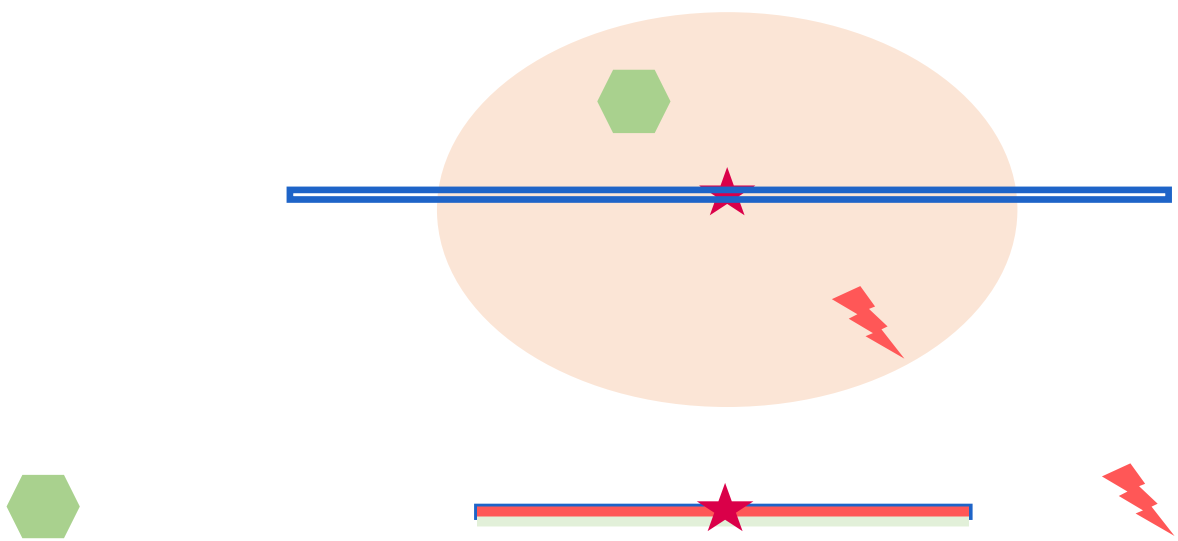
Focal Plane Tomography

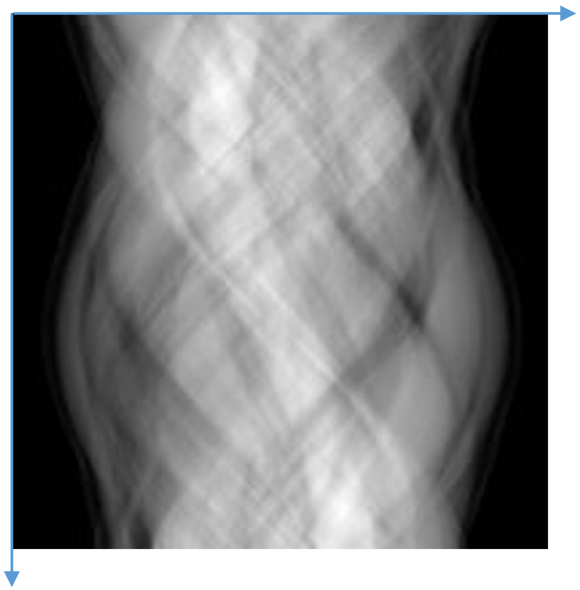


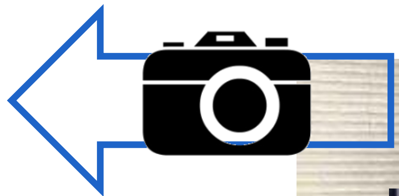
Only section of interest is sharp, the rest is blurred to the point it generates no contrast

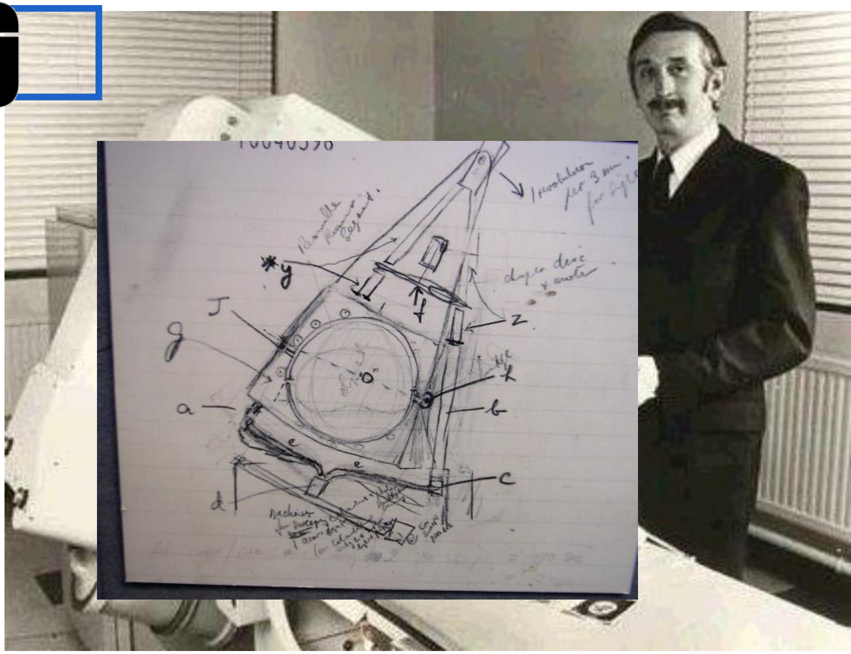
WHAT IS TOMOGRAPHY?

Etymology:
from the Greek tome (slice) and graphein (to write)

“imaging by sections”

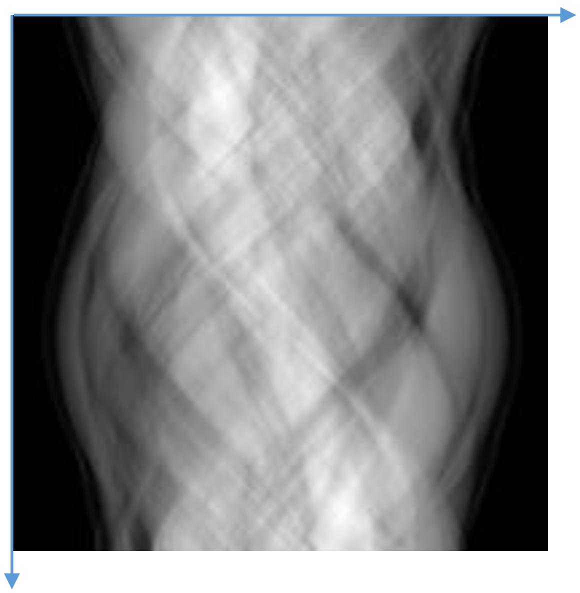






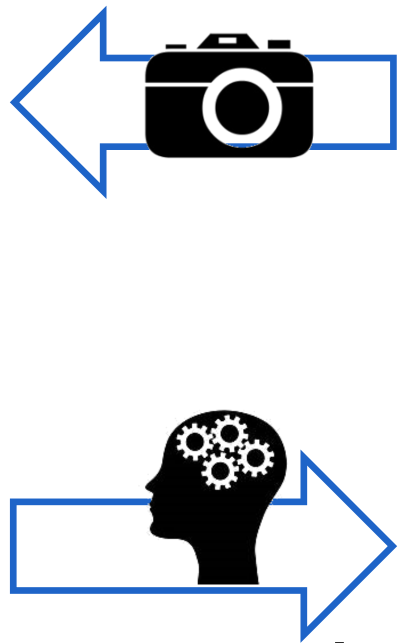
time

WHAT IS RECONSTRUCTION?

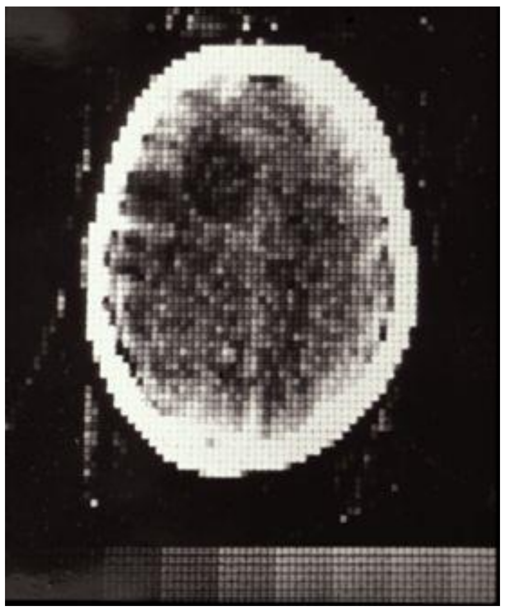


time

“acquisition”



“reconstruction”



There exist many types of tomography

From Wikipedia:

Name	Source of data	Abbreviation	Year of introduction
Atom probe tomography	Atom probe	APT	
Computed tomography imaging spectrometer[2]	Visible light spectral imaging	CTIS	
Confocal microscopy (Laser scanning confocal microscopy)	Laser scanning confocal microscopy	LSCM	
Cryogenic electron tomography	Cryogenic transmission electron microscopy	CryoET	
Electrical capacitance volume tomography	Electrical capacitance	ECVT	
Electrical resistivity tomography	Electrical resistivity	ERT	
Magnetic induction tomography	Magnetic induction	MIT	
Magnetic resonance imaging or nuclear magnetic resonance tomography	Nuclear magnetic moment	MRI or MRT	
Muon tomography	Muon		
Microwave tomography[10]	Microwave (1-10 GHz electromagnetic radiation)		
Neutron tomography	Neutron		
Ocean acoustic tomography	Sonar	OAT	
Optical coherence tomography	Interferometry	OCT	
Optical diffusion tomography	Absorption of light	ODT	
Optical projection tomography	Optical microscope	OPT	
Photoacoustic imaging in biomedicine	Photoacoustic spectroscopy	PAT	
Positron emission tomography	Positron emission	PET	
Positron emission tomography - computed tomography	Positron emission & X-ray	PET-CT	
Quantum tomography	Quantum state	QST	
Single-photon emission computed tomography	Gamma ray	SPECT	
Seismic tomography	Seismic waves		
Terahertz tomography	Terahertz radiation	THz-CT	
Thermoacoustic imaging	Photoacoustic spectroscopy	TAT	
Ultrasound-modulated optical tomography	Ultrasound	UOT	
Ultrasound computer tomography	Ultrasound	USCT	
Ultrasound transmission tomography	Ultrasound		
X-ray microtomography	X-ray	microCT	
Zeeman-Doppler imaging	Zeeman effect		
Focal plane tomography	X-ray		1930s
Electron tomography	Transmission electron microscopy	ET	1968[7][8]
X-ray computed tomography	X-ray	CT, CATScan	1971
Electrical impedance tomography	Electrical impedance	EIT	1984
Electrical capacitance tomography	Electrical capacitance	ECT	1988[6]
Functional magnetic resonance imaging	Magnetic resonance	fMRI	1992
Hydraulic tomography	fluid flow	HT	2000
Magnetic particle imaging	Superparamagnetism	MPI	2005
Computed tomography of chemiluminescence[3][4][5]	Chemiluminescence Flames	CTC	2009
Photoemission Orbital Tomography	Angle-resolved photoemission spectroscopy	POT	2009[11]
Infrared microtomographic imaging[9]	Mid-infrared		2013
Laser Ablation Tomography	Laser Ablation & Fluorescent Microscopy	LAT	2013
Aerial tomography	Electromagnetic radiation	AT	2020

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- What is it?
- How is it done?
    - Analytical
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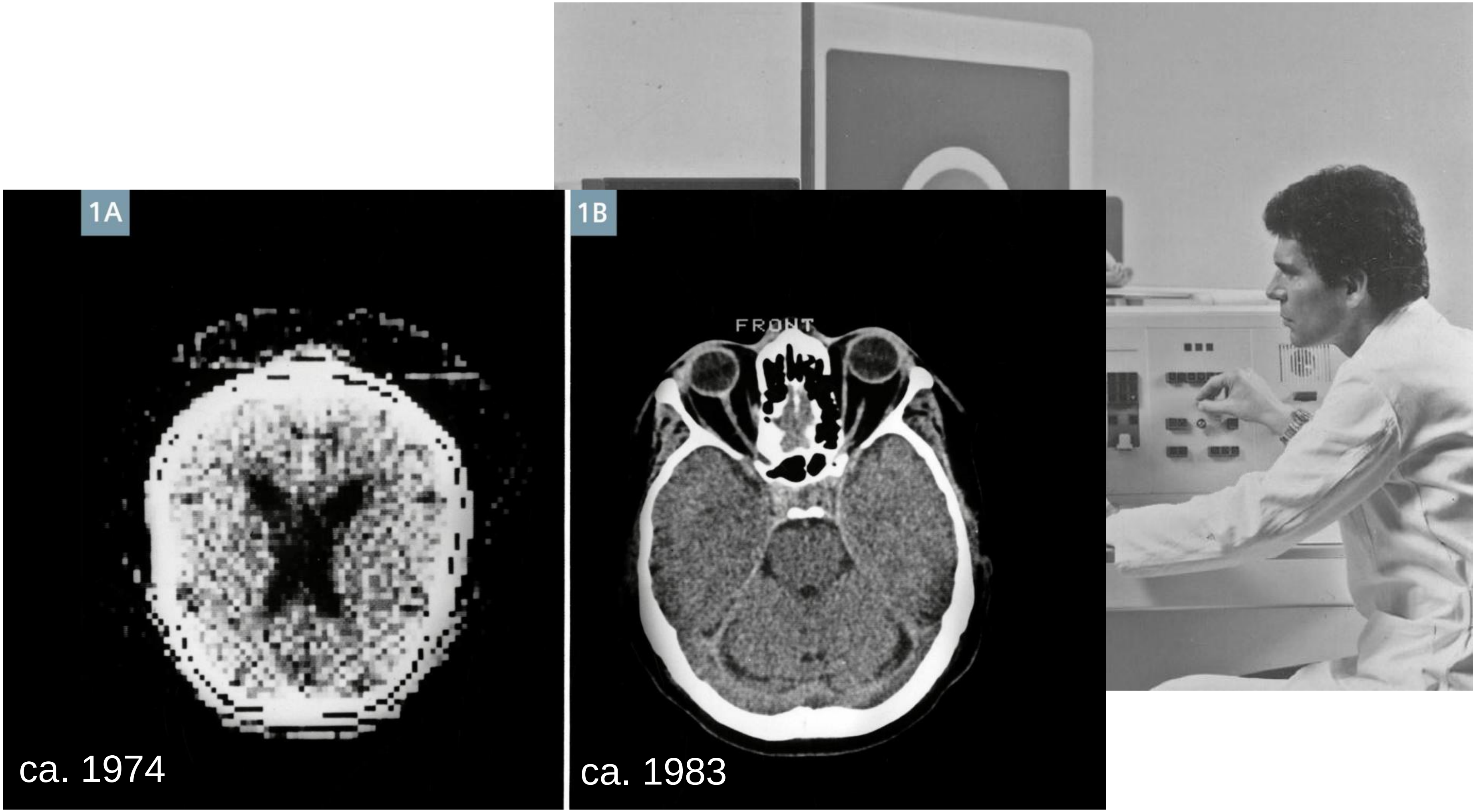
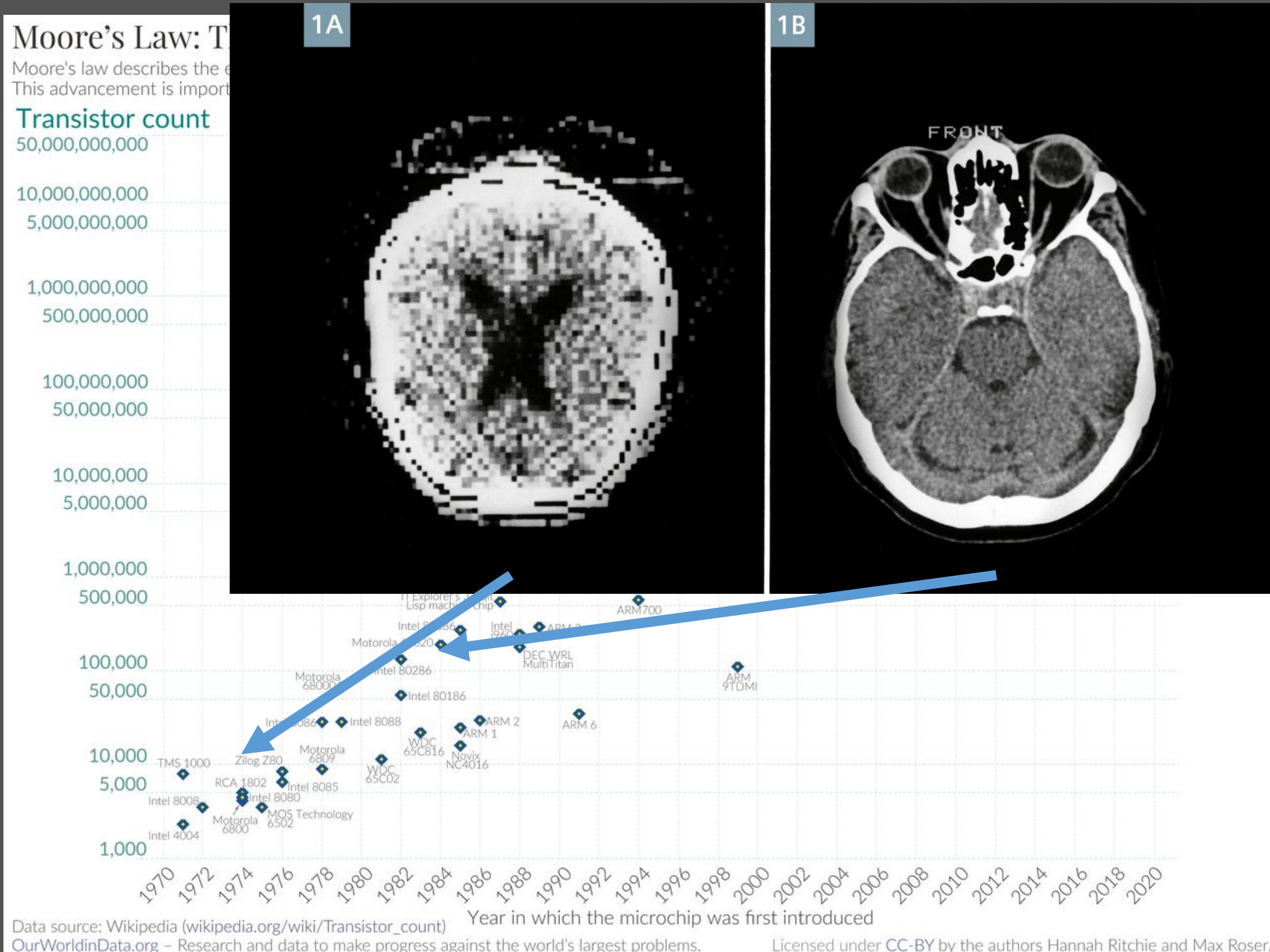
HOW IS IT DONE?

With computational imaging



https://www.siemens-healthineers.com/ar/computed-tomography/news/mso-the-history-of-ct-2.html

THE BACKDROP



HOW IS IT DONE?

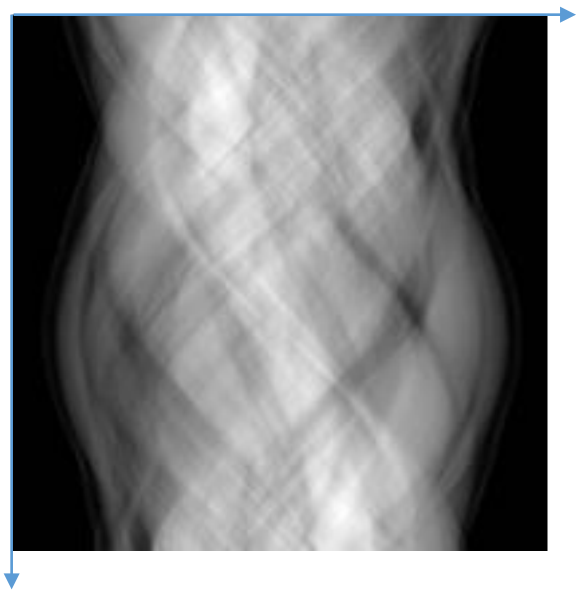
Foundation in mathematics

For CT: proven possible in 1917

Johann Radon ca 1920

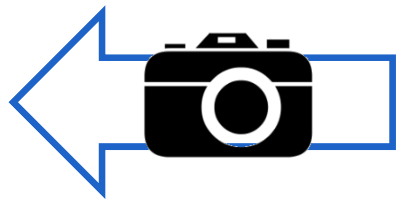
Radon J (1917). "Uber die Bestimmung von Funktionen durch ihre Integralwerte Langs Gewisser Mannigfaltigkeiten" [On the determination of functions from their **integrals** along certain manifolds]. *Ber. Saechsische Akad. Wiss*. 29: 262.

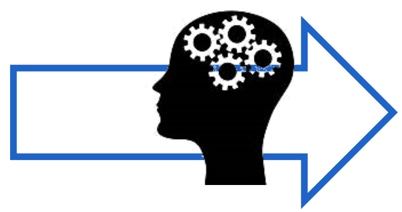
HOW IS RECONSTRUCTION PERFORMED?



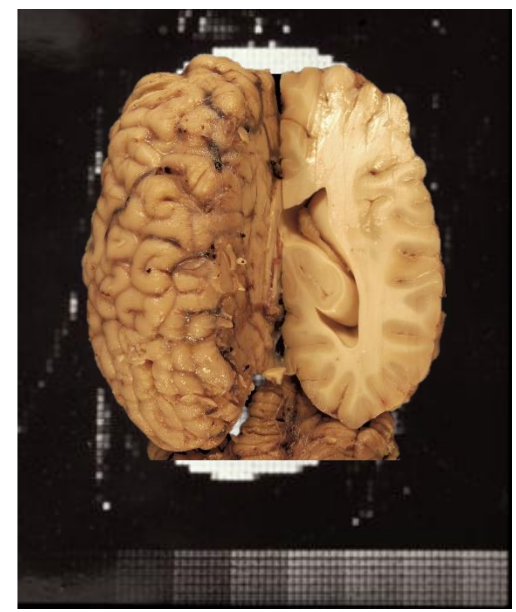
time

“acquisition”
“forward problem”





“reconstruction”
“inverse problem”

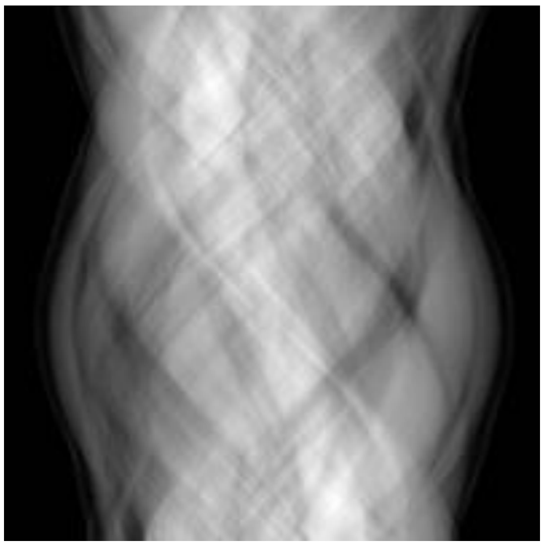


SIMPLIFIED CT FORWARD MODEL

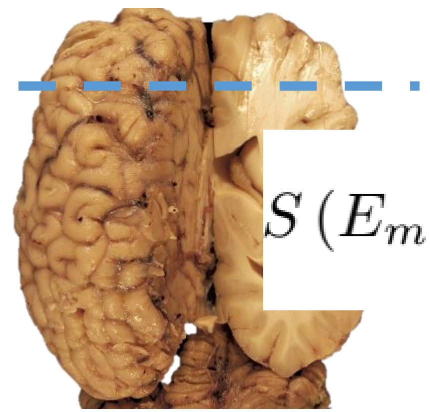
Let's formulate this **"forward problem"**

$$\exp\left(-\int_l \mu(r, E_m)\, \mathrm{d}r\right)$$

x



$D(E_m)$



$S(E_m)$

y

$$I(\;\ln\left(\frac{S(E_m)\, D(E_m)}{I(l)}\right)$$

$$p(y) = \int_{-\infty}^{\infty} \mu(x, y)\, \mathrm{d}x$$

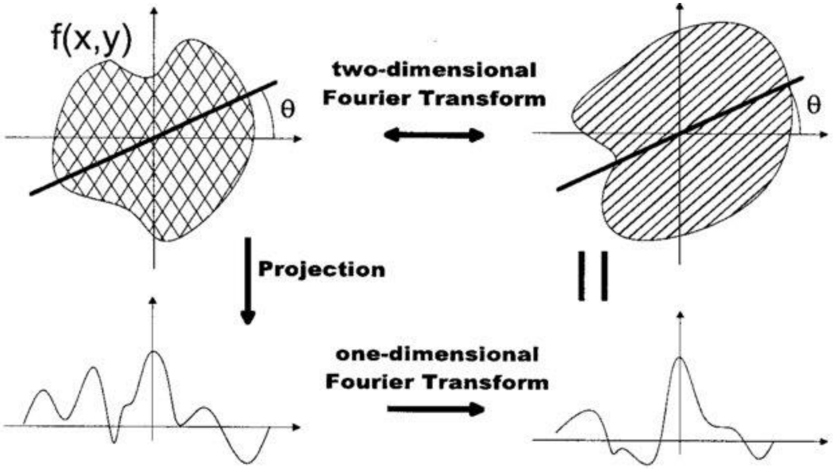
FOURIER SLICE THEOREM

$$p\,(y) = \int_{-\infty}^{\infty} \mu\,(x,y)\,\mathrm{d}x$$

$$F(k_x,k_y) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \mu\,(x,y)\exp\left(-2\pi i\left(xk_x + yk_y\right)\right)\mathrm{d}x\right]\mathrm{d}y$$

$$F(0,k_y) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \mu\,(x,y)\,\mathrm{d}x\right]\exp\left(-2\pi i y k_y\right)\mathrm{d}y$$

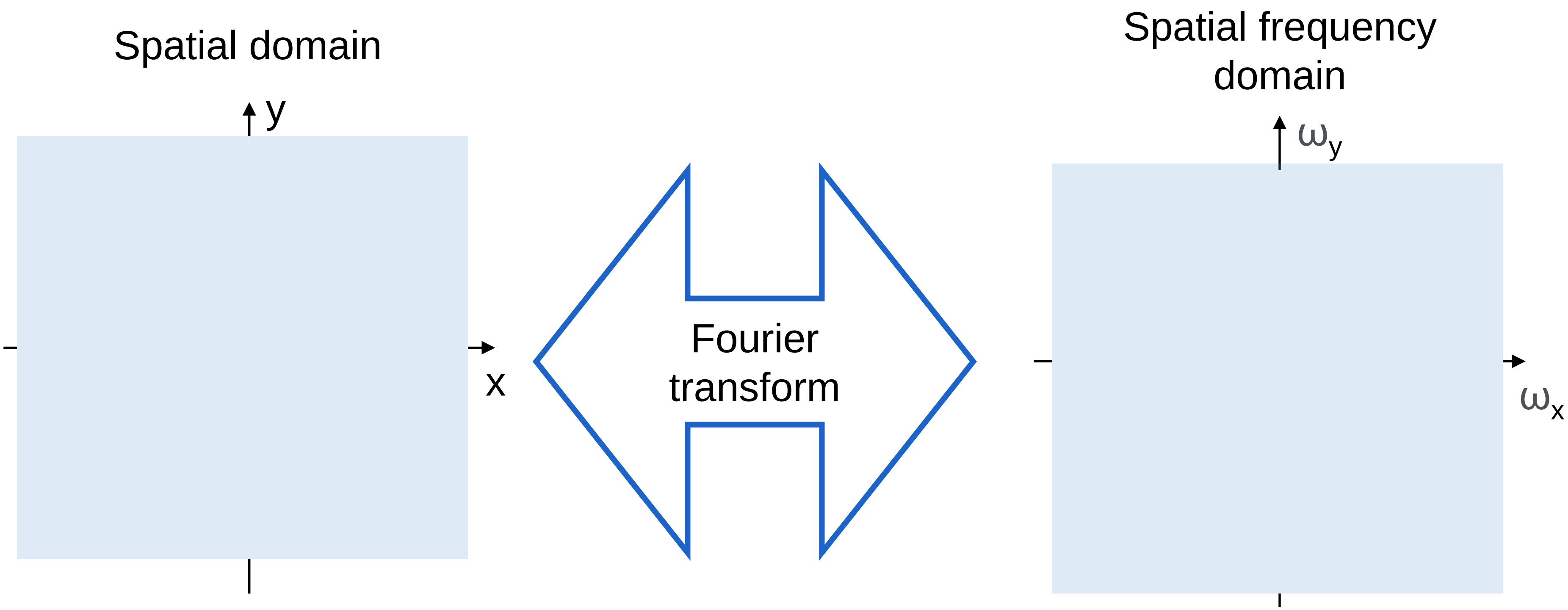
$$F(0,k_y) = \int_{-\infty}^{\infty} p(y)\exp\left(-2\pi i y k_y\right)\mathrm{d}y$$



Jean-Baptiste Joseph Fourier
Ca 1800

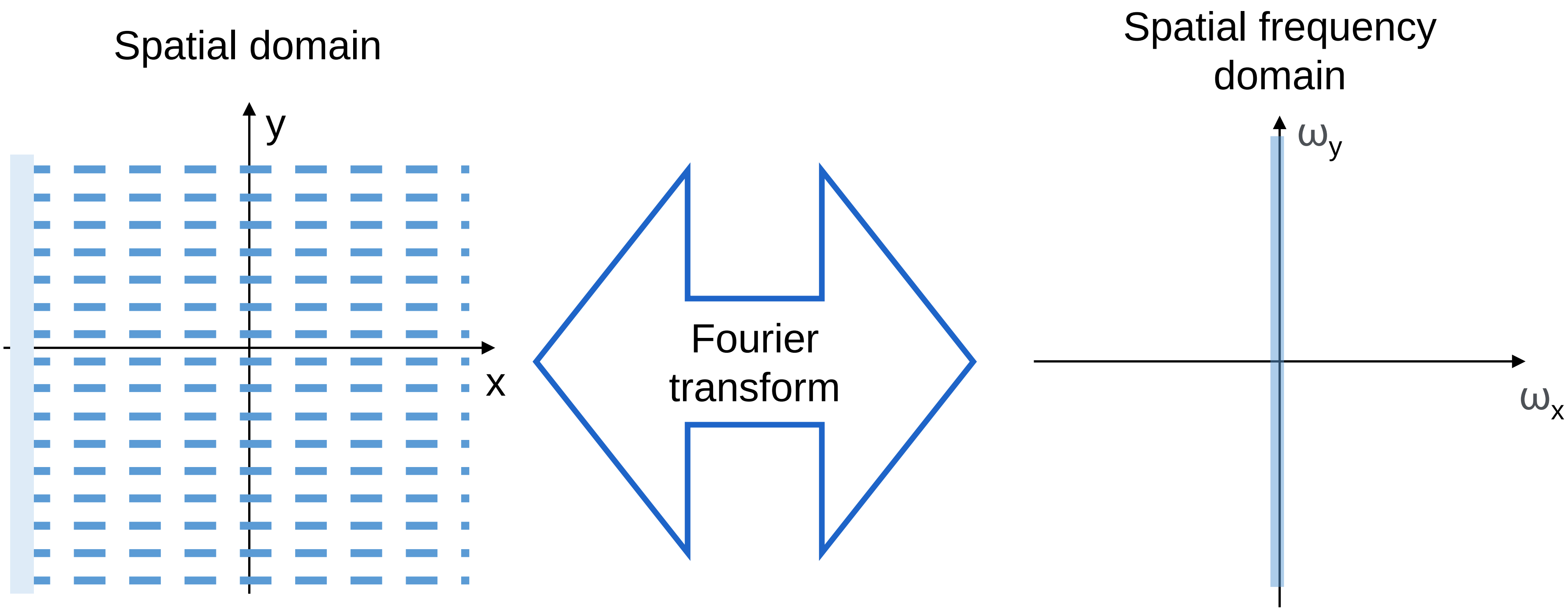
FOURIER RECONSTRUCTION

If we have this link with the Fourier transform, we could use the inverse Fourier transform to get ***analytical reconstruction***



FOURIER RECONSTRUCTION

If we have this link with the Fourier transform, we could use the inverse Fourier transform to get ***analytical reconstruction***



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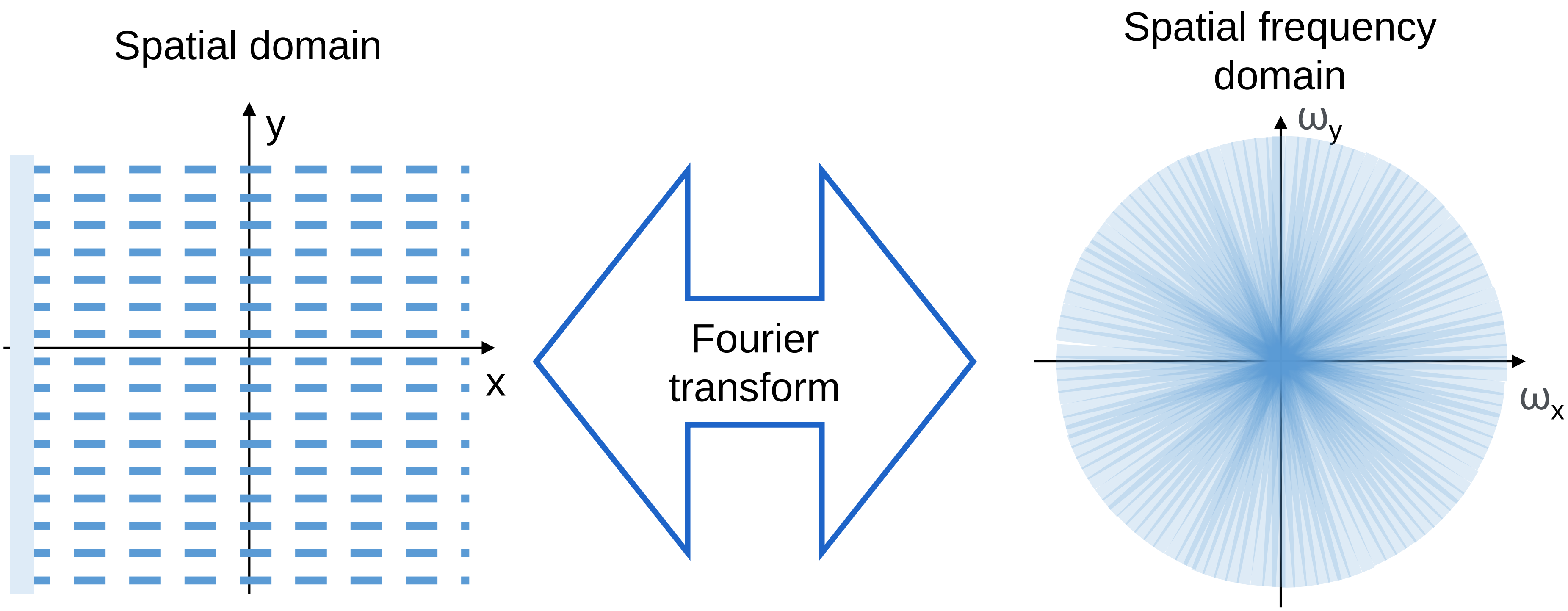
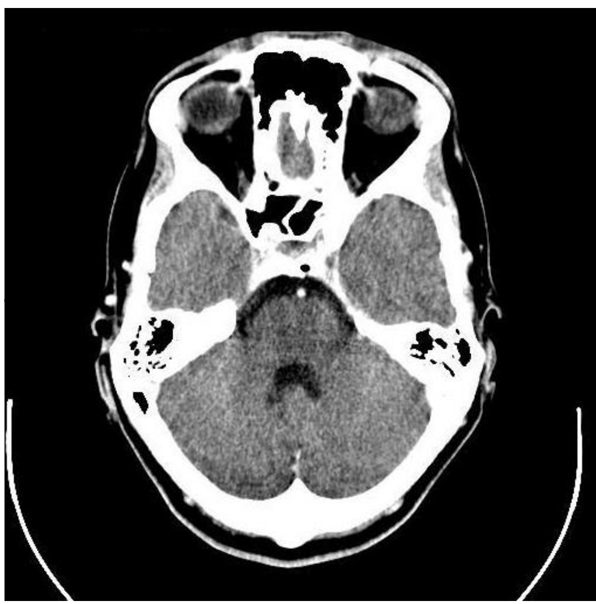


FOURIER RECONSTRUCTION

If we have this link with the Fourier transform, we could use the inverse Fourier transform to get ***analytical reconstruction***

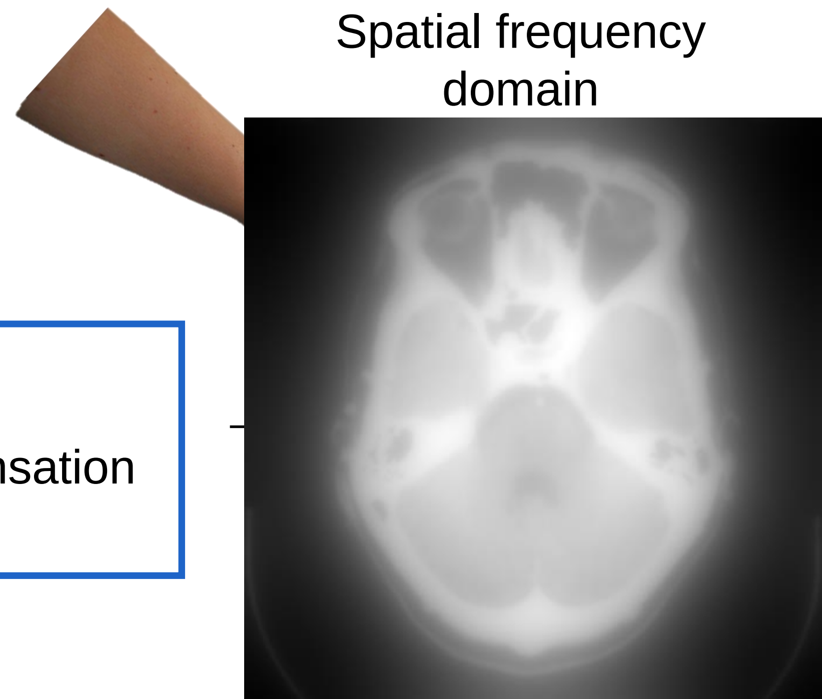
Density variation

Spatial frequency domain



Density Compensation

Spatial frequency domain

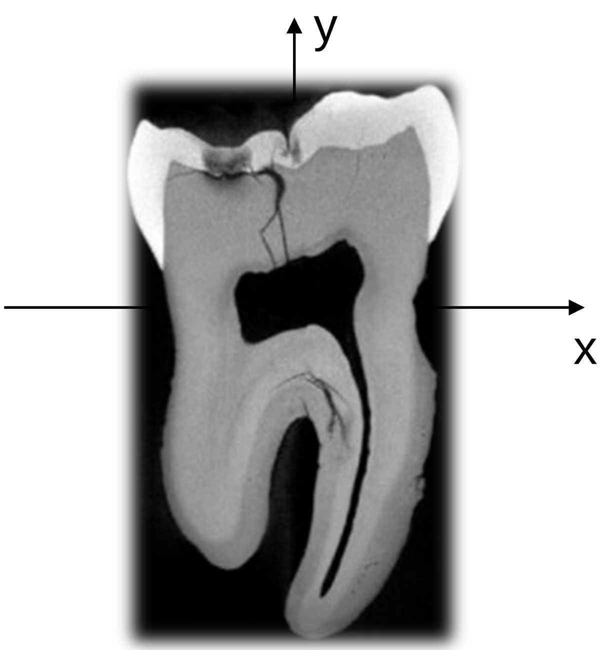


ON FOURIER TRANSFORMS

$$F(0,k_y) = \int_{-\infty}^{\infty} p(y)\exp\left(-2\pi i y k_y\right)\mathrm{d}y$$

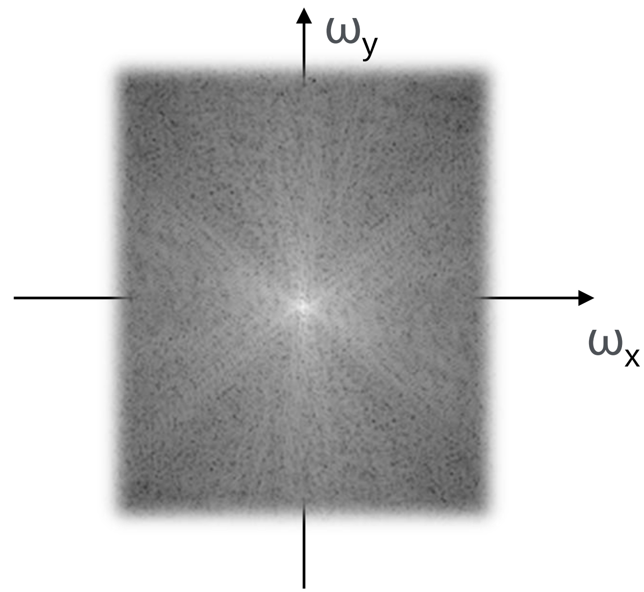
$$F_d(0,k_y) = \sum_{y=-\infty}^{\infty} p(y\Delta_y)\exp\left(-2\pi i y \Delta_y k_y\right)\Delta_y$$

Spatial domain



Continuous, infinite

Spatial frequency domain



Continuous, infinite

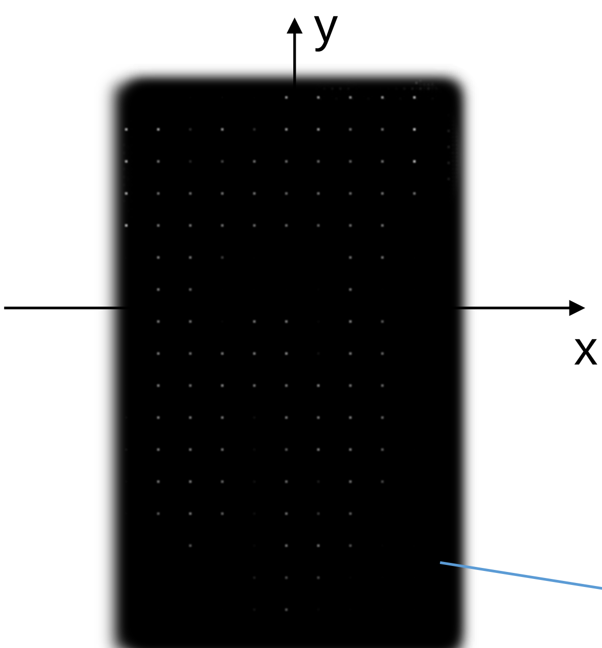
ON SAMPLING THEORY

$$F(0,k_y)=\int_{-\infty}^{\infty}p(y)\exp\left(-2\pi i y k_y\right)\mathrm{d}y$$

$$\Downarrow$$

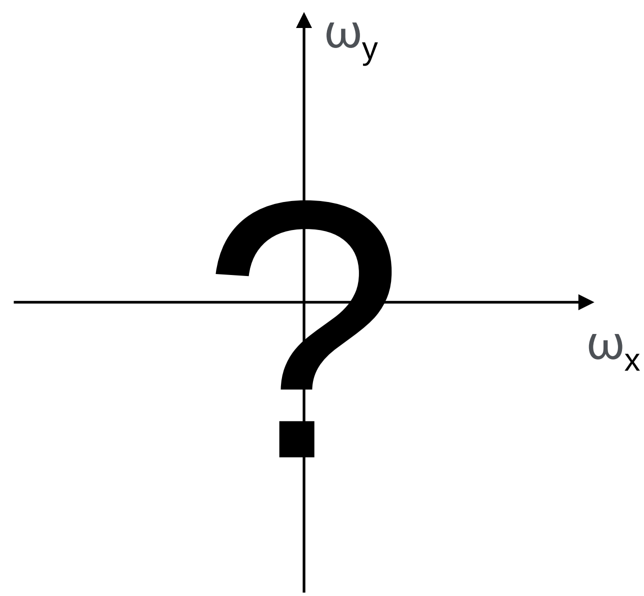
$$F_d(0,k_y)=\sum_{y=-\infty}^{\infty}p(y\Delta_y)\exp\left(-2\pi i y\Delta_y k_y\right)\Delta_y$$

Spatial domain



What's in the gaps?

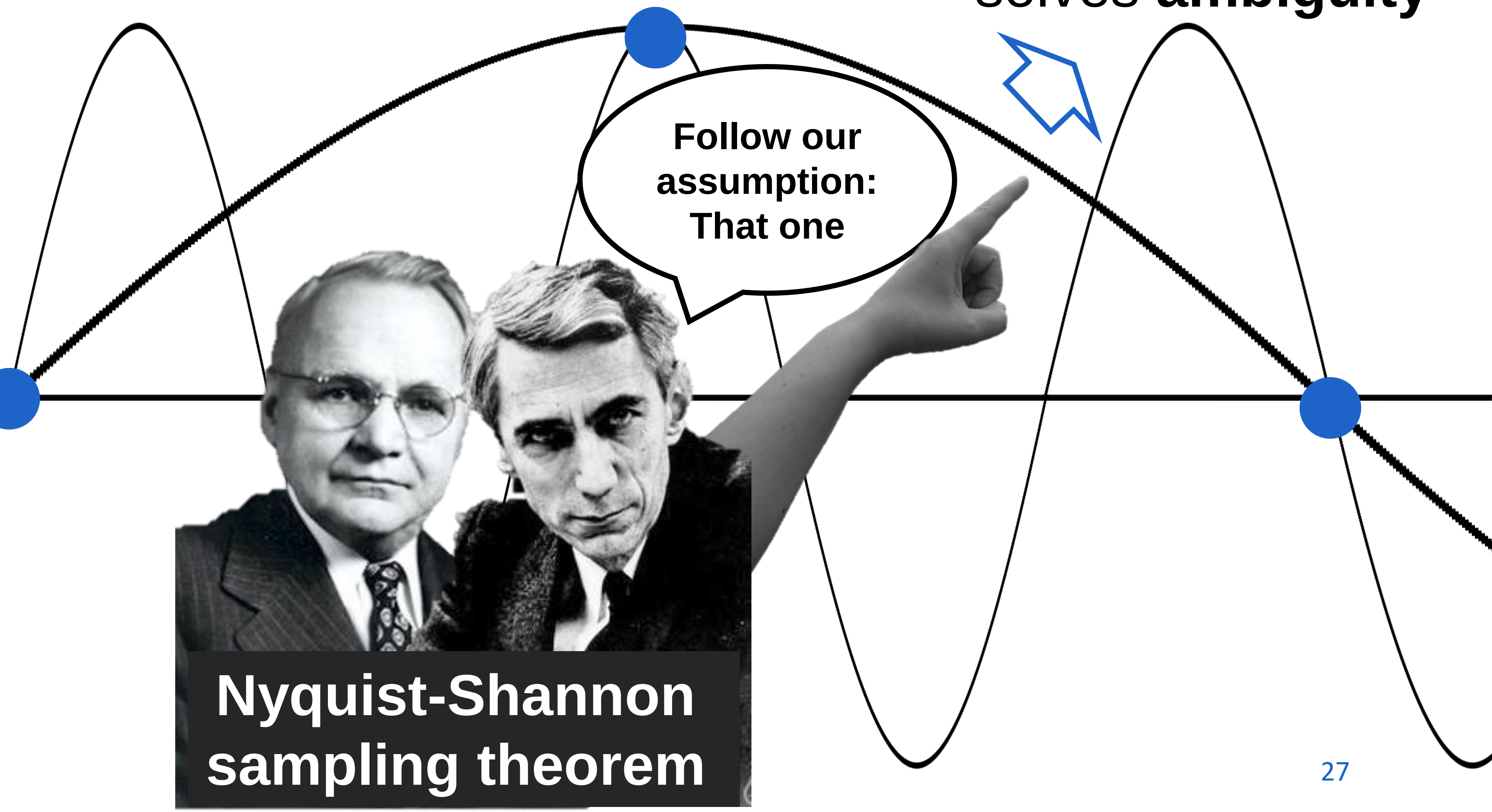
Spatial frequency domain



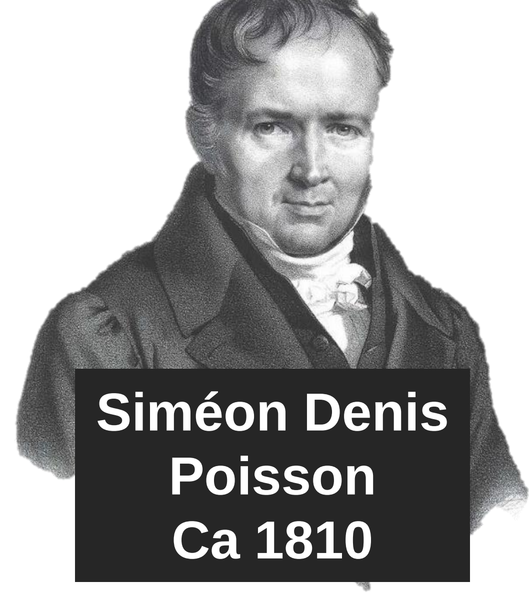
SAMPLING THEORY

Which sine gave rise to the following samples?

an **assumption**
solves **ambiguity**



ON FOURIER TRANSFORMS

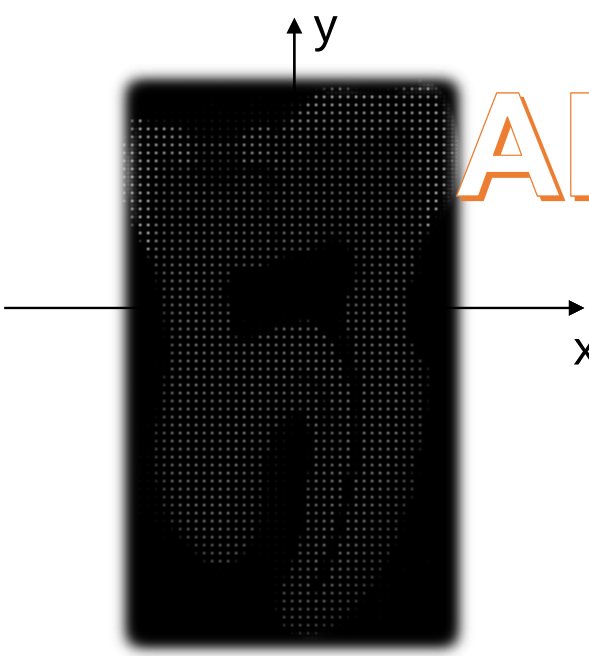


$$F(0,k_y) = \int_{-\infty}^{\infty} p(y) \exp\left(-2\pi i y k_y\right) \mathrm{d}y$$

$$F_d(0,k_y) = \sum_{y=-\infty}^{\infty} p(y\Delta_y) \exp\left(-2\pi i y \Delta_y k_y\right) \Delta_y$$

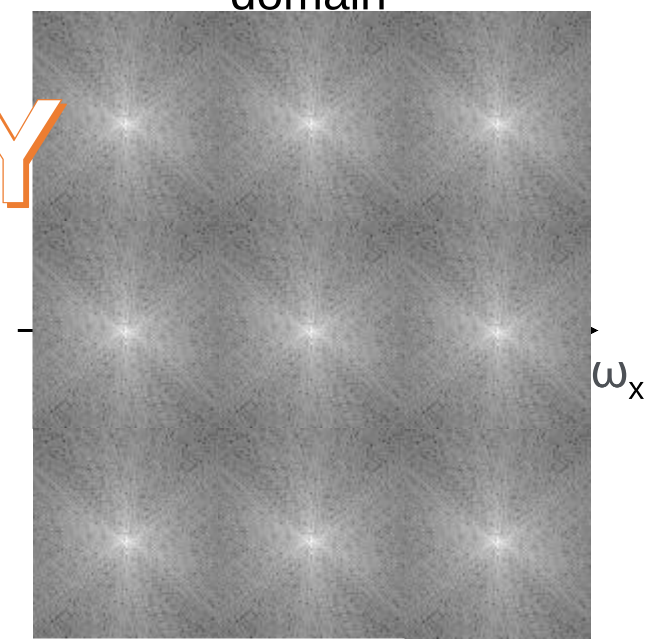
$$F_d(0,k_y) = \sum_{l=-\infty}^{\infty} F(0, k_y \pm \frac{l}{\Delta_y})$$

Spatial domain



AMBIGUITY

Spatial frequency domain



ON FOURIER TRANSFORMS

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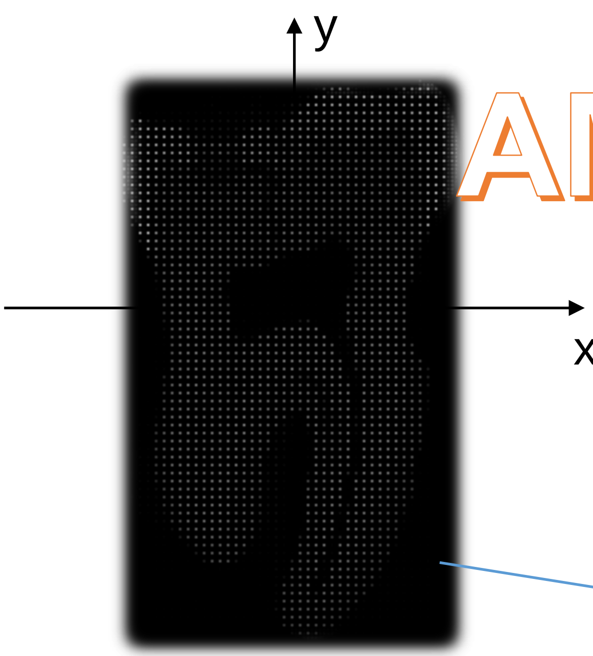
Spatial domain

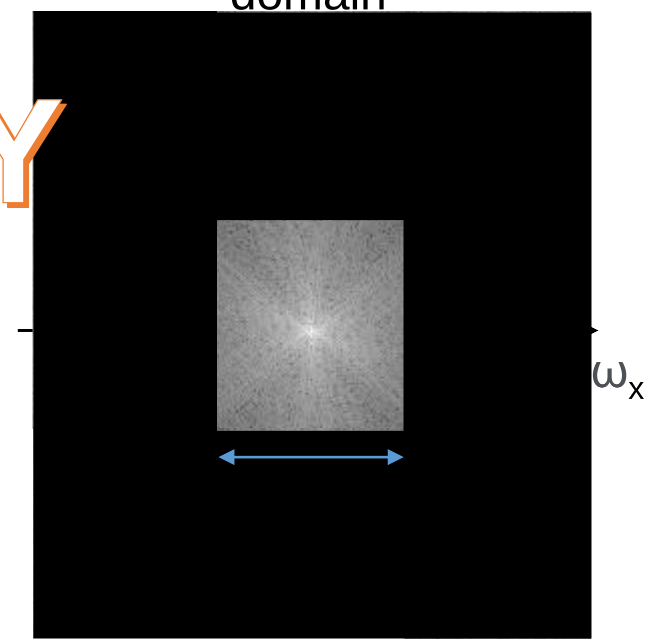


Sample distance: Δ_y

AMBIGUITY

What's in the gaps?

Spatial frequency domain



Bandwidth: $\frac{2}{\Delta_y}$

ON FOURIER TRANSFORMS

$$F(0,k_y) = \int_{-\infty}^{\infty} p(y) \exp\left(-2\pi i y k_y\right) \mathrm{d}y$$

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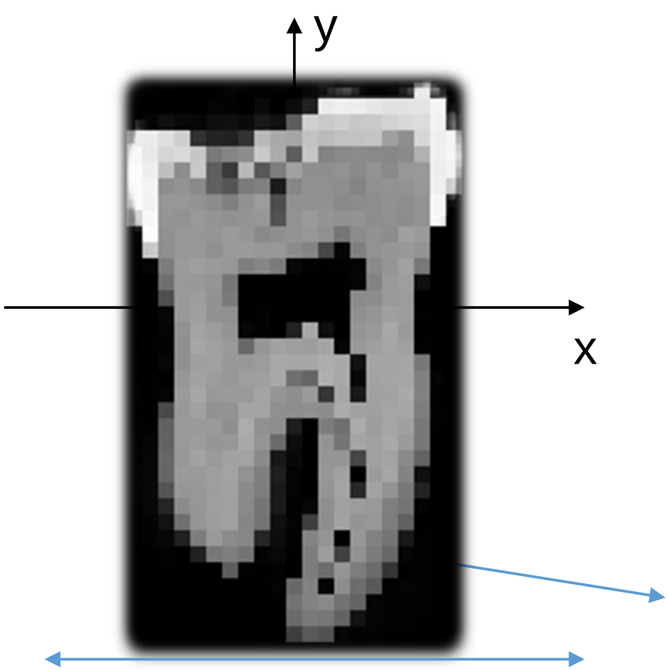
Spatial domain

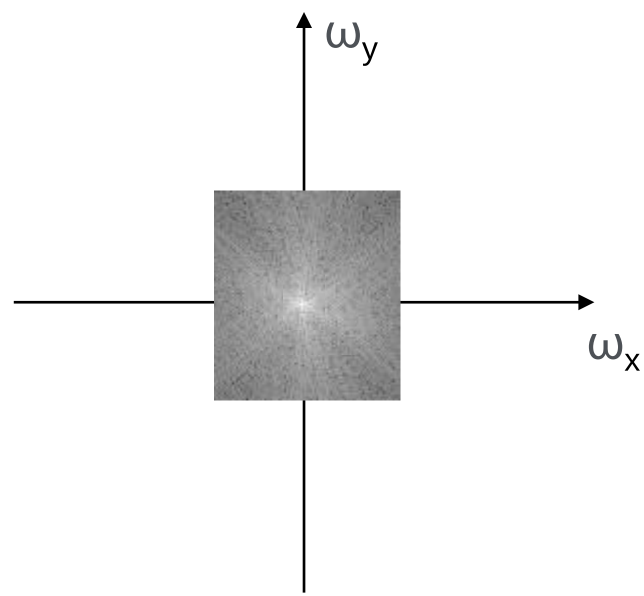


Sample distance: Δ_y

Discrete, infinite

What's in the gaps?

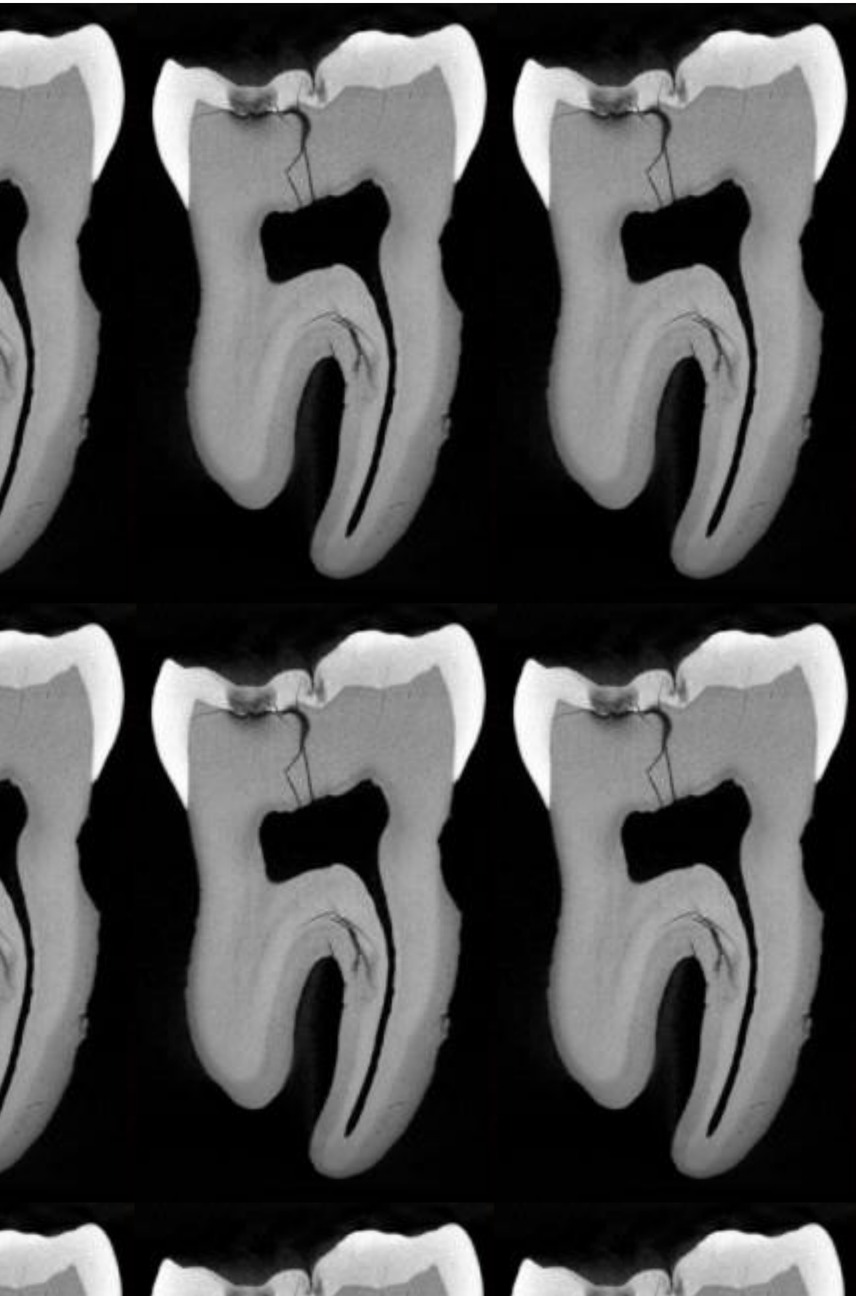
Spatial frequency domain



Bandwidth: $\frac{2}{\Delta_y}$

Continuous, finite

ON FOURIER TRANSFORMS

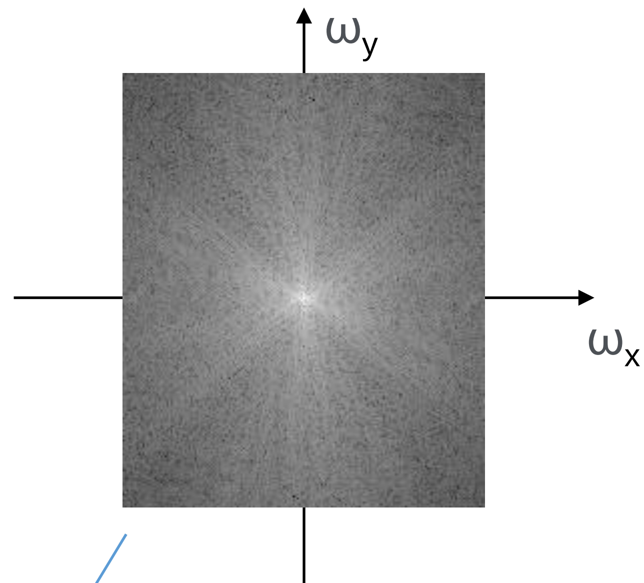


$\exp\left(-2\pi i y k_y\right) \mathrm{d}y$

$\ldots p\left(-2\pi i y \Delta_y k_y\right) \Delta_y$

$\ldots xp\left(-2\pi i y \Delta_y k_y\right) \Delta_y$

Spatial frequency domain

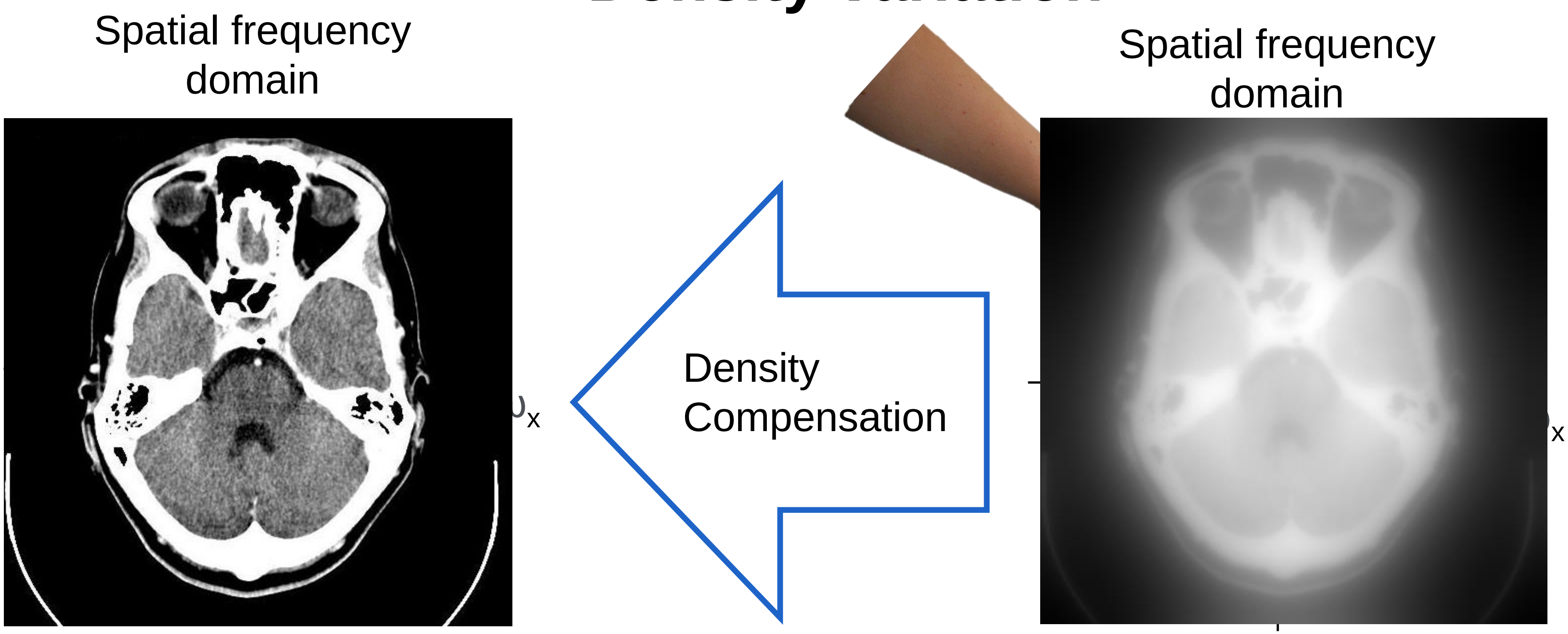


What's in the gaps?

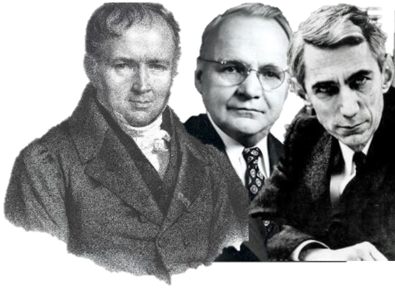
FOURIER RECONSTRUCTION

If we have this link with the **discrete** Fourier transform, we use the inverse **discrete** Fourier transform reconstruct, this is called ***analytical reconstruction***

Density variation



ON RESOLUTION



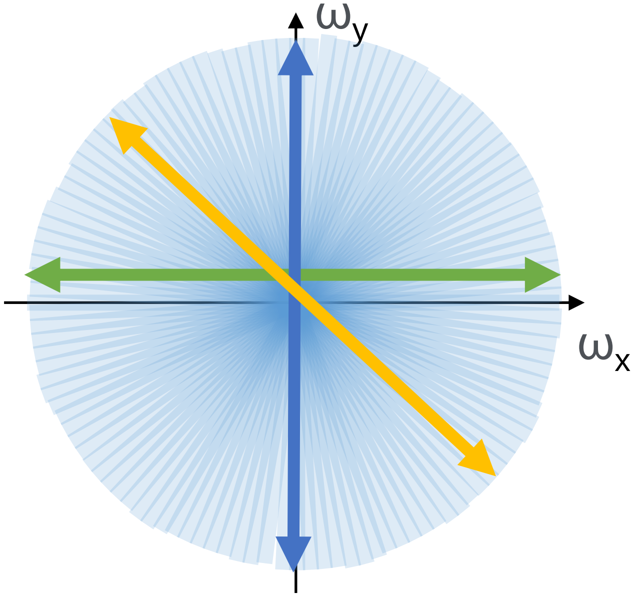
The relation

Sample distance: Δ_y

Bandwidth: $\frac{2}{\Delta_y}$

allows us a reasonable definition of spatial resolution

Spatial frequency domain

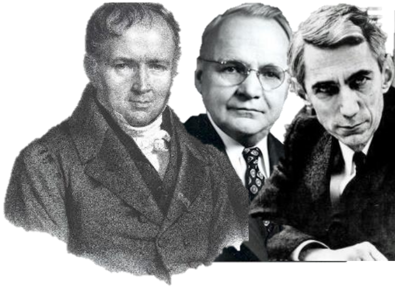


Horizontal bandwidth
~ horizontal sample distance
~ min vertical size of resolvable point

Vertical bandwidth
~ vertical sample distance
~ min horizontal size of resolvable point

Diagonal bandwidth
~ diagonal sample distance
~ min diagonal size of resolvable point

ON RESOLUTION



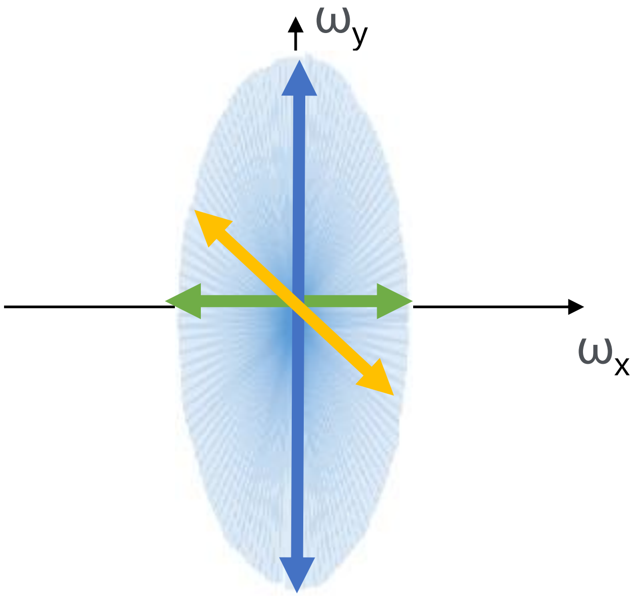
The relation Sample distance: Δ_y

Bandwidth: $\frac{2}{\Delta_y}$

allows us a reasonable definition of spatial resolution

Independent of voxel size

Spatial frequency domain



Horizontal bandwidth
~ horizontal sample distance
~ min vertical size of resolvable point

Vertical bandwidth
~ vertical sample distance
~ min horizontal size of resolvable point

Diagonal bandwidth
~ diagonal sample distance
~ min diagonal size of resolvable point

FILTERED BACKPROJECTION

- Inspired by analytical reconstruction
- Very efficient technique!
  - Can be done in terms of 1D signals
  - Filtering to compensate for density imbalance in Fourier space
  - Doesn’t need to be performed in Fourier space

Multiplication in Fourier space $\Longleftrightarrow$ equivalent $\Longleftrightarrow$ Filtering in Image space

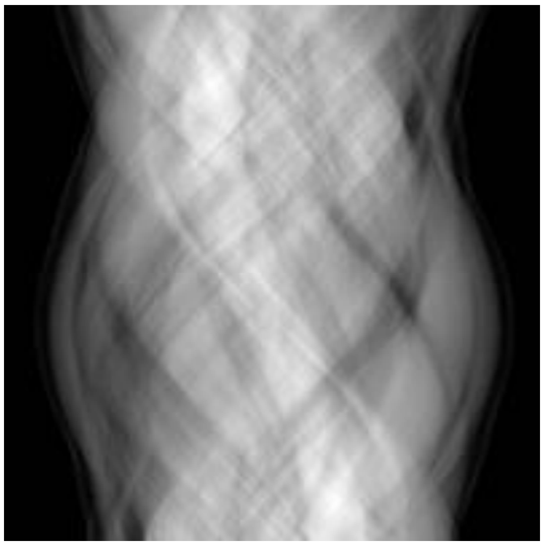
TOMOGRAPHY & RECONSTRUCTION

- What is it?
- How is it done?
  - Analytical
  - Iterative
- Link to estimation theory
  - Bayesian vs ML
  - Priors
- Link to machine learning

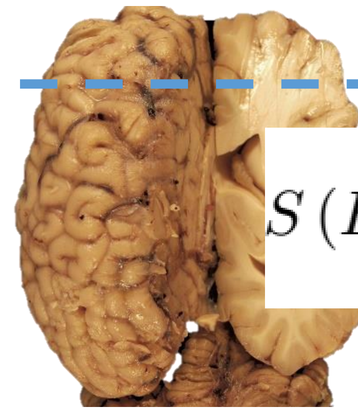
ANOTHER VIEWPOINT

Let's formulate this **"forward problem"**

$$\exp\left(-\int_l \mu\left(r, E_m\right) \mathrm{d}r\right)$$



$$D\left(E_m\right)$$

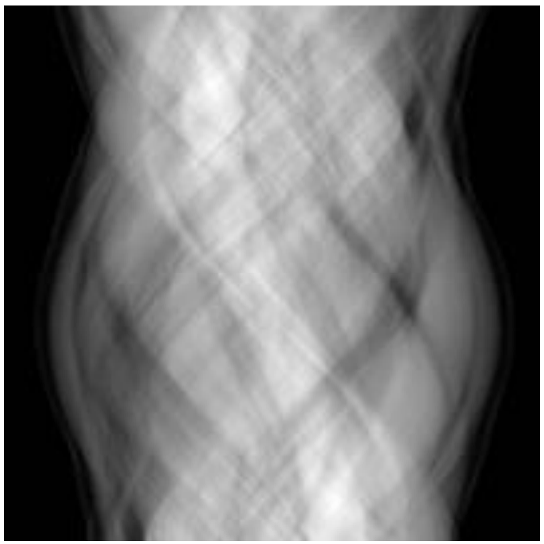


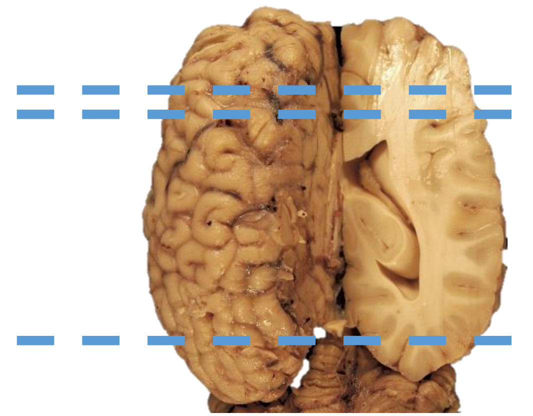
$$S\left(E_m\right)$$

$$\ln\left(\frac{S\left(E_m\right) D\left(E_m\right)}{I\left(l\right)} y = \sum_{r=0}^{L} x_r h_{r\ m}\right) \mathrm{d}r$$

SIMPLIFIED EXAMPLE: CT FORWARD MODEL

Let's formulate a **"forward problem"**





$$\mathbf{y} = H\mathbf{x}$$

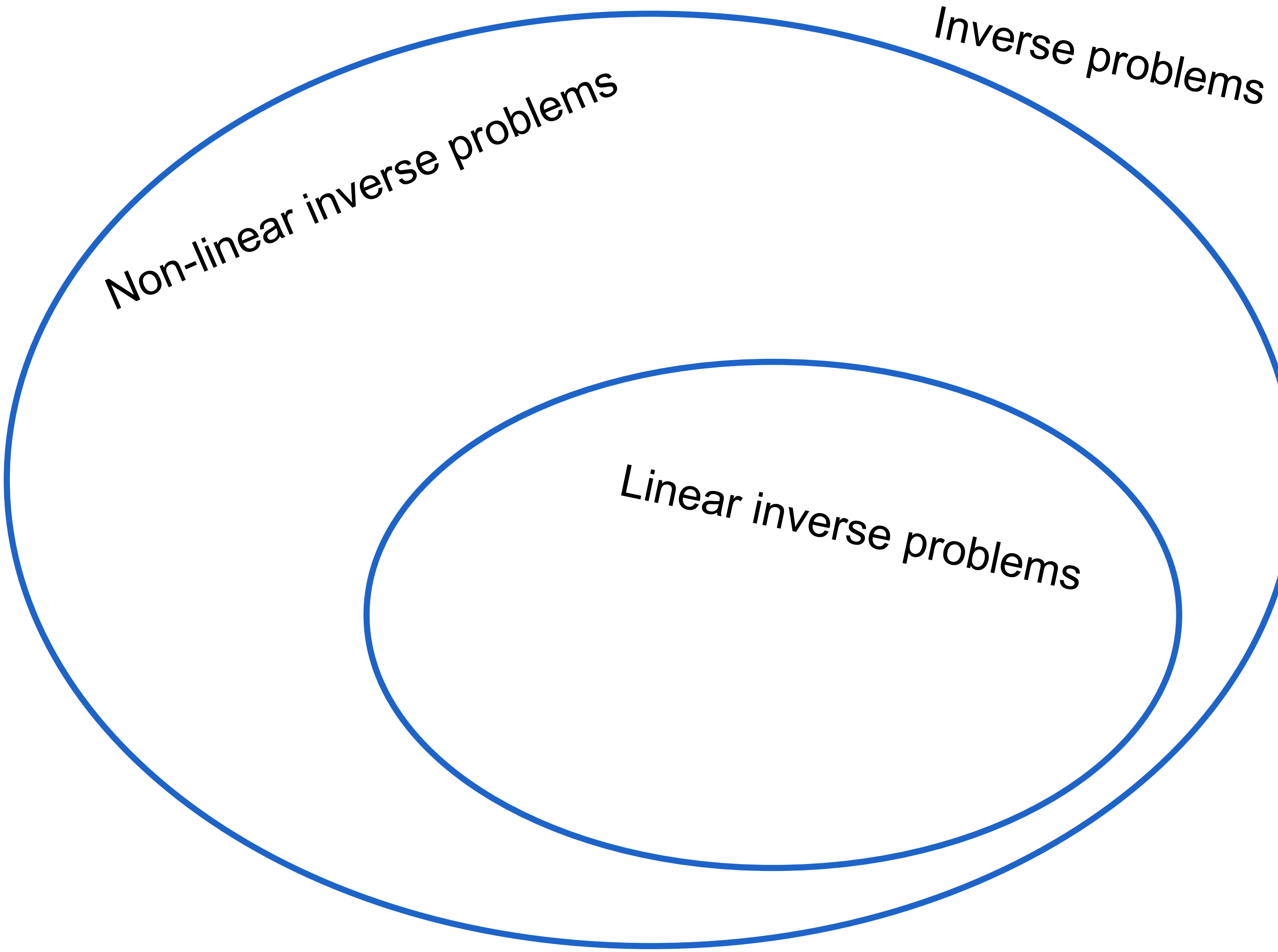
$$\begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_{k-1} \\ y_k \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 0 \\ & & & 0 \\ & & \vdots & \\ 0 & \cdots & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

SIMPLIFIED CT RECONSTRUCTION

a large system of
linear algebraic equations

$$\mathbf{y} = H\mathbf{x}$$

LINEAR VS NON-LINEAR TOMOGRAPHY



MOORE-PENROSE PSEUDO-INVERSE

a large system of
linear algebraic equations

"inverse problem"

$$\boxed{\mathbf{x} = \left(H^{\mathrm{T}} H\right)^{-1} H^{\mathrm{T}} \mathbf{y}}$$

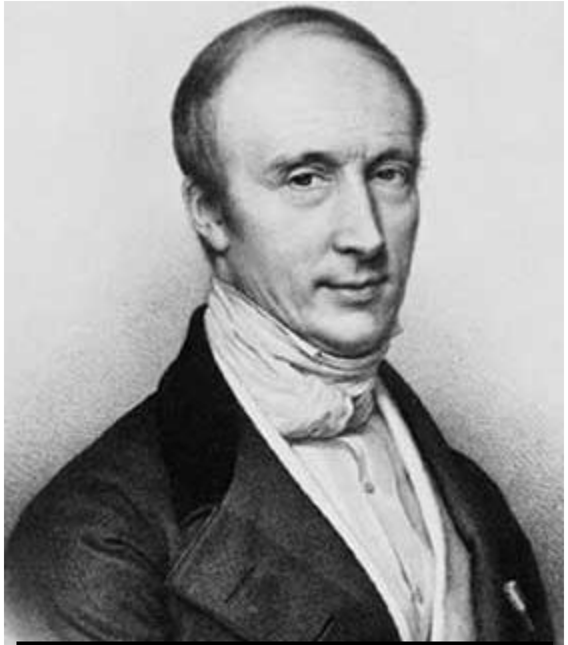
$$\mathbf{y} = H\mathbf{x}$$

Eliakim Hastings Moore
Ca 1915

Complexity
$O(n^{2.3 \to 3})$
For n = number of pixels

GRADIENT DESCENT

a large system of
linear algebraic equations



Augustin-Louis Cauchy
Ca 1840

$$x_{i+1} = x_i + \epsilon H^{\mathrm{H}} (y - H x_i)$$

Complexity
O(n^2)
For n = number of pixels

$$\mathbf{y} = H\mathbf{x}$$

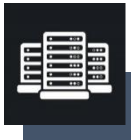
KACZMARZ ITERATIONS

method to find an **approximate solution** to a large system of linear algebraic equations in 1937

Aka **algebraic reconstruction technique (ART)**


Stefan Kaczmarz
Ca 1935

$$x_{i+1} = x_i + \frac{y_j - x_i^{\mathrm{T}} h_j}{\|h_j\|^2} \overline{h_j}$$


Complexity
O(n)
For n = number of pixels

$$\mathbf{y} = H\mathbf{x}$$

NOT ALL ITERATIONS ARE CREATED EQUALLY

method to find an **approximate solution**

algebraic reconstruction technique (ART)

Simultaneous Iterative Reconstruction Technique (SIRT)

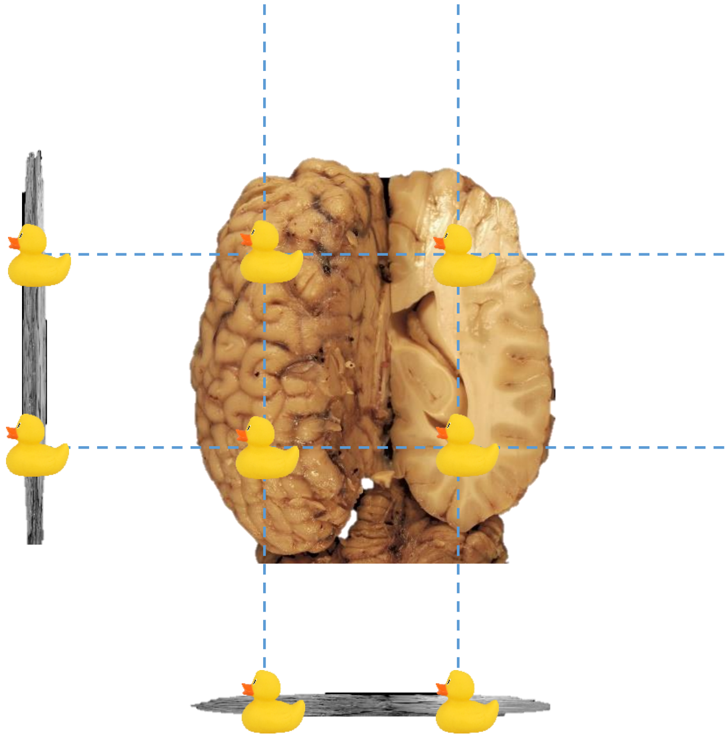
Simultaneous algebraic reconstruction technique (SART)

…

WELL-POSED VS ILL-POSED

The ducks are not located **unambiguously** if is $\mathbf{y} = H\mathbf{x}$ satisfied

"the inverse problem is **ill-posed**"

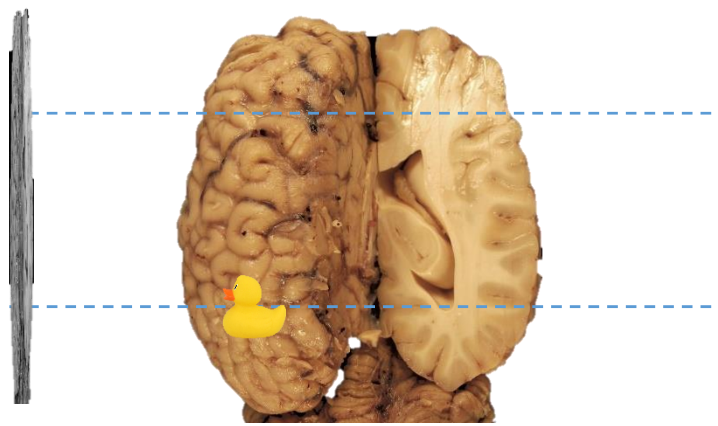


AMBIGUITY

WELL-POSED VS ILL-POSED

If satisfying $\mathbf{y} = H\mathbf{x}$ locates ducks unambiguously

"the inverse problem is **well-posed**"



WELL-POSED VS ILL-POSED

If satisfying $\mathbf{y} = H\mathbf{x}$ locates ducks unambiguously

"the inverse problem is **well-posed**"

Conditions for **well-posed** problem:

1. A solution **exists**
2. The solution is **unique**
3. The solution changes **continuously** in function of the initial conditions.

TOMOGRAPHY & RECONSTRUCTION

- What is it?
- How is it done?
  - Analytical
  - Iterative
- Link to estimation theory
  - Bayesian vs ML
  - Priors
- Link to machine learning

ESTIMATION THEORY

Equality physically impossible!

$$\mathbf{y} = H\mathbf{x}$$

Why? Because of statistical deviations (e.g. noise)
So being accurate means defining the relation between x and y in a **statistical** sense:

For example: $\mathrm{p}\left(\mathbf{y}|\mathbf{x}\right) = \mathcal{N}\left(H\mathbf{x}, \Sigma\right)$

Thus, the inverse problem becomes an **estimation theory** problem.
For example, **maximum likelihood**:

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \mathrm{p}\left(\mathbf{y}|\mathbf{x}\right)$$

Looks familiar?

$$\boxed{\mathbf{x} = \left(H^{\mathrm{T}}H\right)^{-1} H^{\mathrm{T}}\mathbf{y}}$$

a more accurate model

$$\hat{\mathbf{x}} = \left(H^{\mathrm{T}}\Sigma^{-1}H\right)^{-1} H^{\mathrm{T}}\Sigma^{-1}\mathbf{y}$$

is this invertible?

if not: AMBIGUITY!

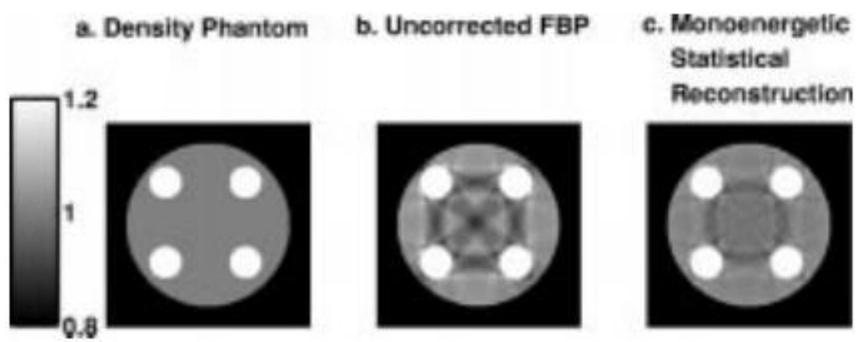
ESTIMATION THEORY

But wait, why a Normal distribution assumption??

$$\cancel{\mathrm{p}\left(\mathbf{y}|\mathbf{x}\right) \sim \mathcal{N}\left(H\mathbf{x}, \Sigma\right)}$$

$$I_i = I_0 \exp\left(-\left[H\mathbf{x}\right]_i\right) \sim \text{Poisson}\left(\overline{I_i}\right)$$

$\hat{\mathbf{x}} = \arg \mathrm{m}$ … $))$



…pproximation

$\hat{\mathbf{x}} = \arg$ … $\mathbf{x}$ … $\sum_{i=1}$ … $\left(\overline{I_i}\right)\Big)$

Fig. 3. Bone/water density phantom results.

[Elbakri2002] Elbakri, Idris A., and Jeffrey A. Fessler. "Statistical image reconstruction for polyenergetic X-ray computed tomography." *IEEE transactions on medical imaging* 21, no. 2 (2002): 89-99.

TOMOGRAPHY & RECONSTRUCTION

- What is it?
- How is it done?
    - Analytical
    - Iterative
- Link to estimation theory
    - Bayesian vs ML
    - Priors
- Link to machine learning

ESTIMATION THEORY: PRIOR KNOWLEDGE

Solve **ambiguity** by imposing **assumptions**.

For example by **maximum a posteriori** estimation:

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \mathrm{p}\left(\mathbf{x}|\mathbf{y}\right)$$

$$\mathrm{p}\left(\mathbf{x}|\mathbf{y}\right) = \frac{\mathrm{p}\left(\mathbf{y}|\mathbf{x}\right)\mathrm{p}\left(\mathbf{x}\right)}{\mathrm{p}\left(\mathbf{y}\right)}$$

Thomas Bayes

ESTIMATION THEORY: PRIOR KNOWLEDGE

Solve **ambiguity** by imposing **assumptions**.

For example by **maximum a posteriori** estimation:

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \mathrm{p}\left(\mathbf{x}|\mathbf{y}\right)$$

$$\Downarrow$$

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \underbrace{\mathrm{p}\left(\mathbf{y}|\mathbf{x}\right)}_{\text{Forward model}} \underbrace{\mathrm{p}\left(\mathbf{x}\right)}_{\textbf{prior}}$$

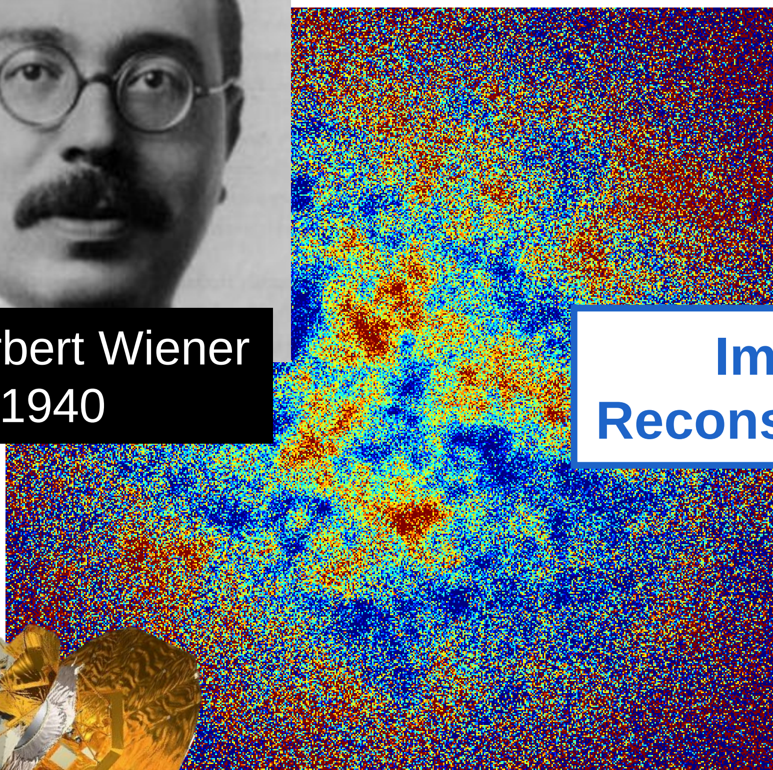
PRIOR EXAMPLES

Wiener filter

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \mathrm{p}(\mathbf{y}|\mathbf{x})\,\mathrm{p}(\mathbf{x})$$

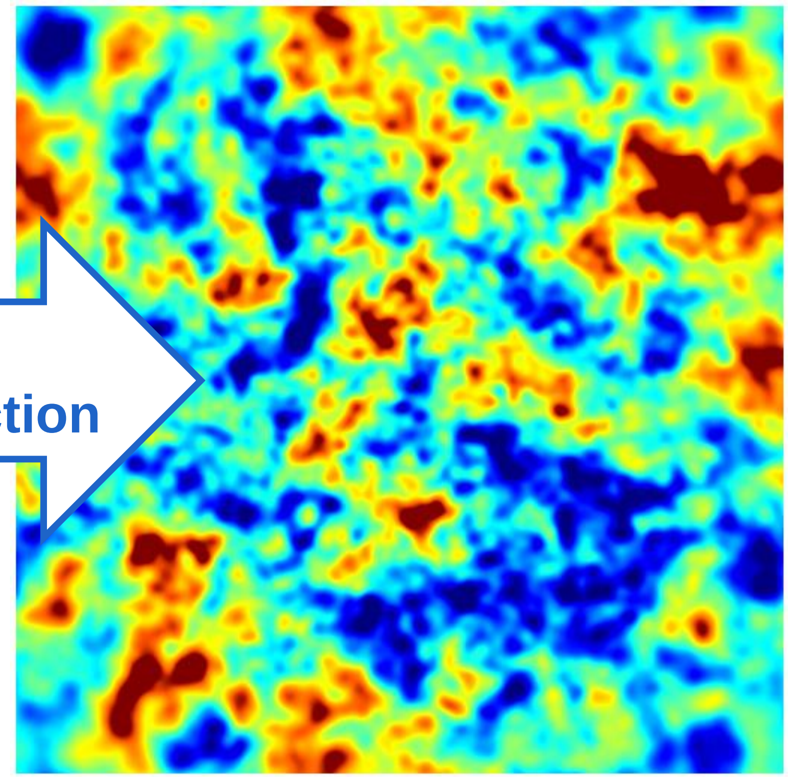
$$\mathrm{p}(\mathbf{y}|\mathbf{x}) = \mathcal{N}(H\mathbf{x}, \Sigma) \qquad \mathrm{p}(\mathbf{x}) = \mathcal{N}(\mathbf{0}, \Sigma_x)$$

Norbert Wiener
Ca 1940

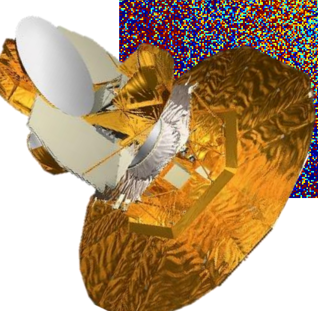


Noisy data

Image Reconstruction



Wiener filtered



Denoising the cosmic microwave background radiation signal

PRIOR EXAMPLES

Filtered backprojection?

It’s an analytical Fourier-based method to solve

$$\mathbf{y} = H\mathbf{x}$$

Actually by minimizing (maximum likelihood estimator)

$$\left\| F\mathbf{x} - WFH^{\mathrm{T}}\mathbf{y} \right\|^2$$

We get

2D Discrete Fourier transform

$$\hat{\mathbf{x}} = F^{\mathrm{H}} W F H^{\mathrm{T}} \mathbf{y}$$

Ramp filter

$$\hat{\mathbf{x}} = F^{\mathrm{H}} \left(1 + \Lambda^2\right)^{-1} W F H^{\mathrm{T}} \mathbf{y}$$

PRIOR EXAMPLES

– Filtered backprojection?

Now let's assume we know some image statistics

Let's minimize

$$\left\|F\mathbf{x} - WFH^{\mathrm{T}}\mathbf{y}\right\|^2 + \left\|\Lambda F\mathbf{x}\right\|^2$$

$\mathrm{p}\left(\mathbf{y}|\mathbf{x}\right)$

$\mathrm{p}\left(\mathbf{x}\right)$

Diagonal matrix
allows you to encode image PSD statistics

we get

$$\hat{\mathbf{x}} = F^{\mathrm{H}}\left(1 + \Lambda^2\right)^{-1} WFH^{\mathrm{T}}\mathbf{y}$$

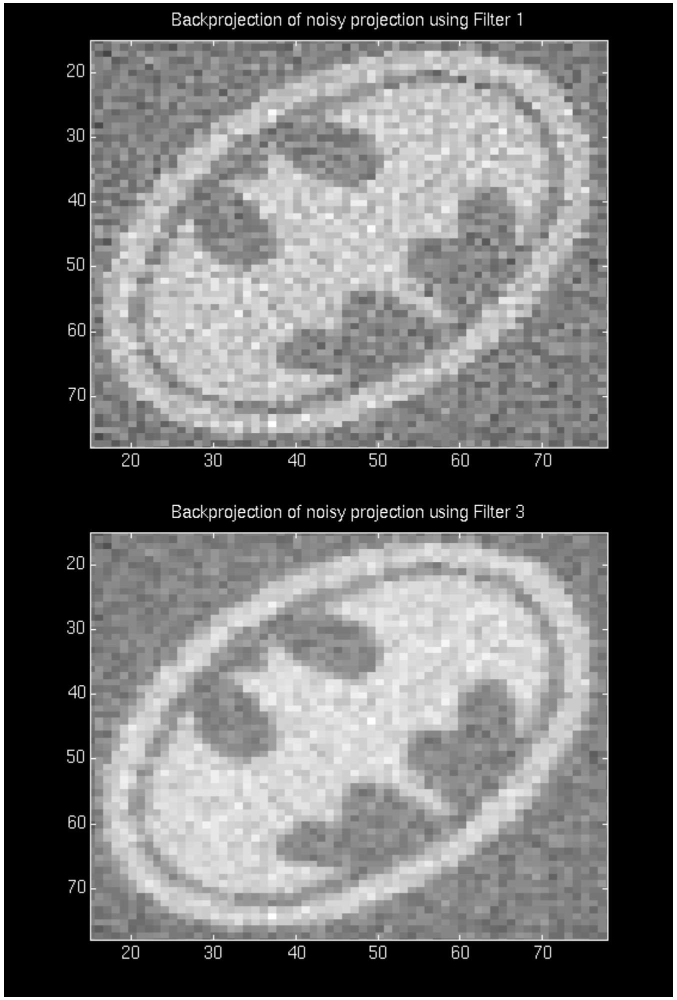
Frequency domain modification to Ramp filter

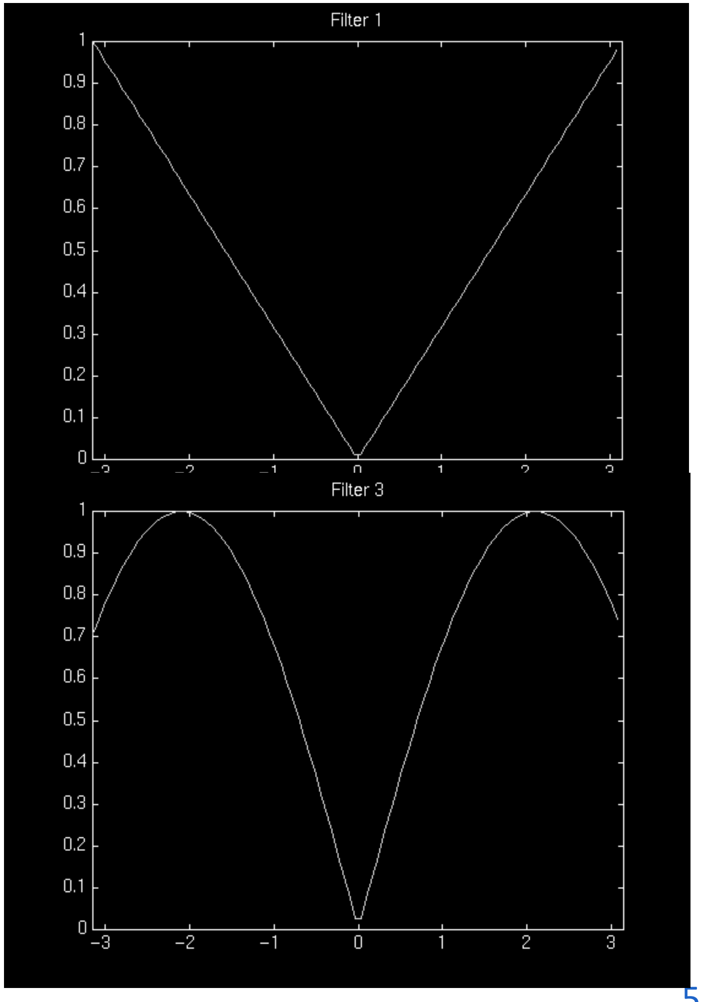
PRIOR EXAMPLES

– Filtered backprojection?

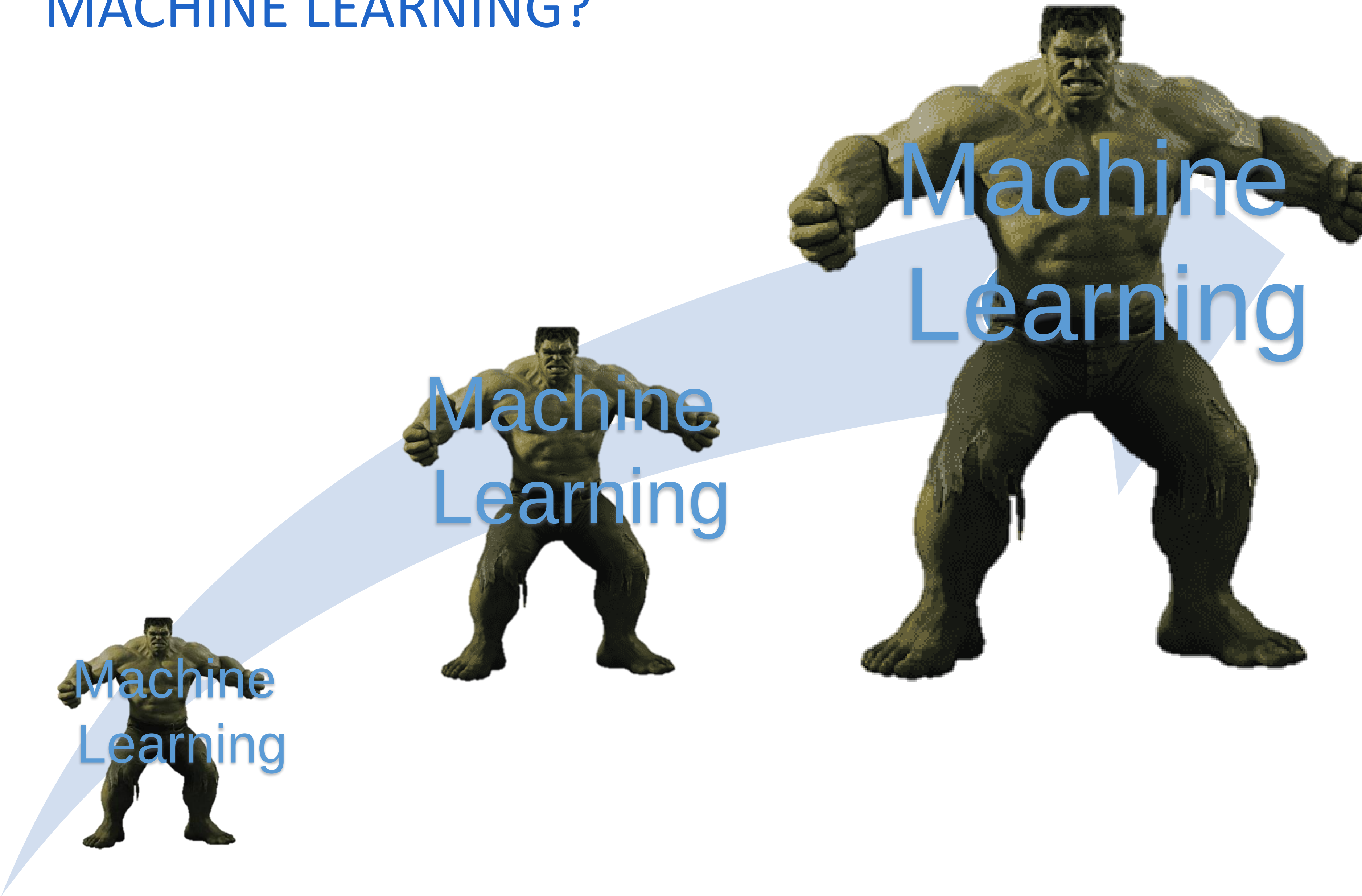
Lots of heuristic filters

w

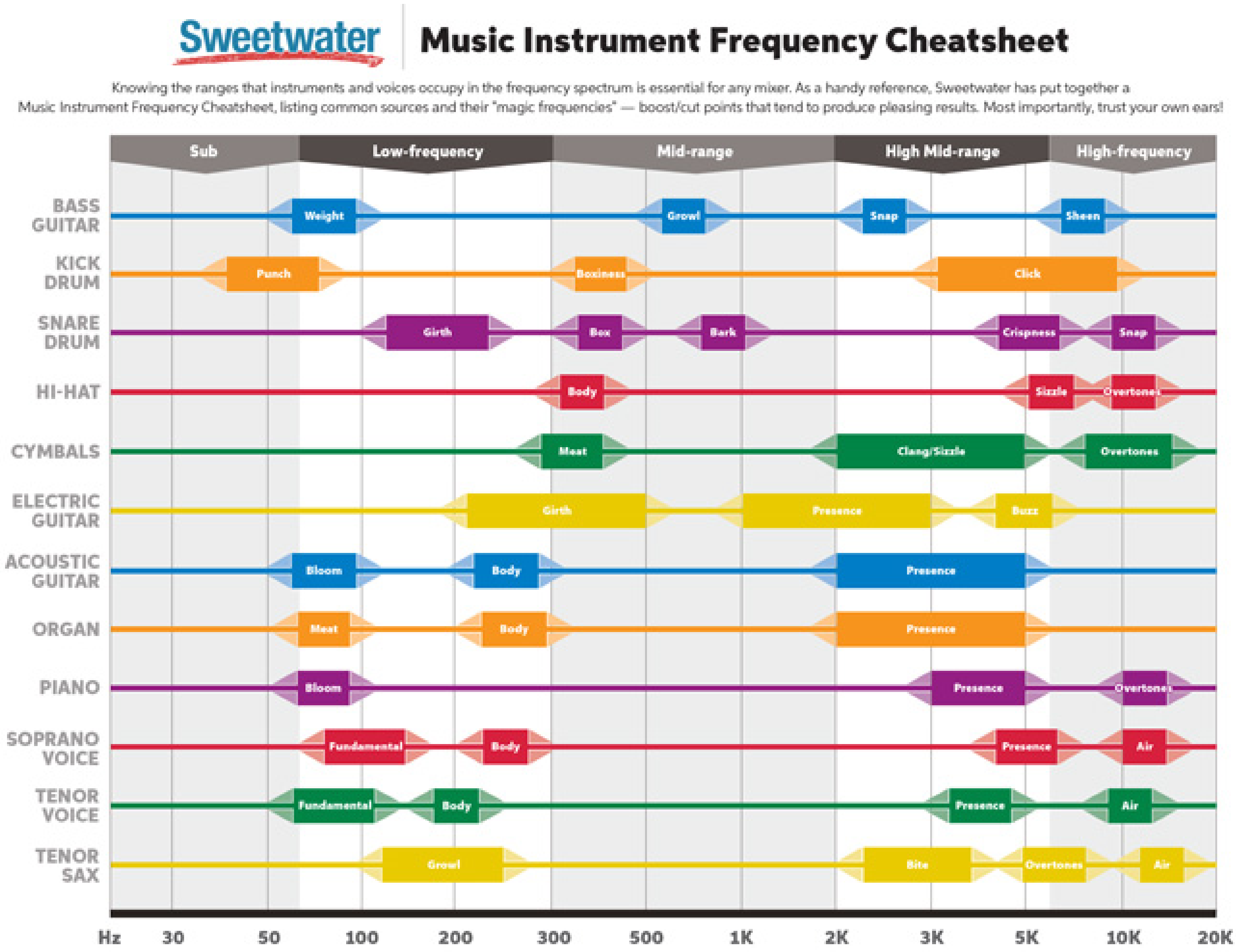




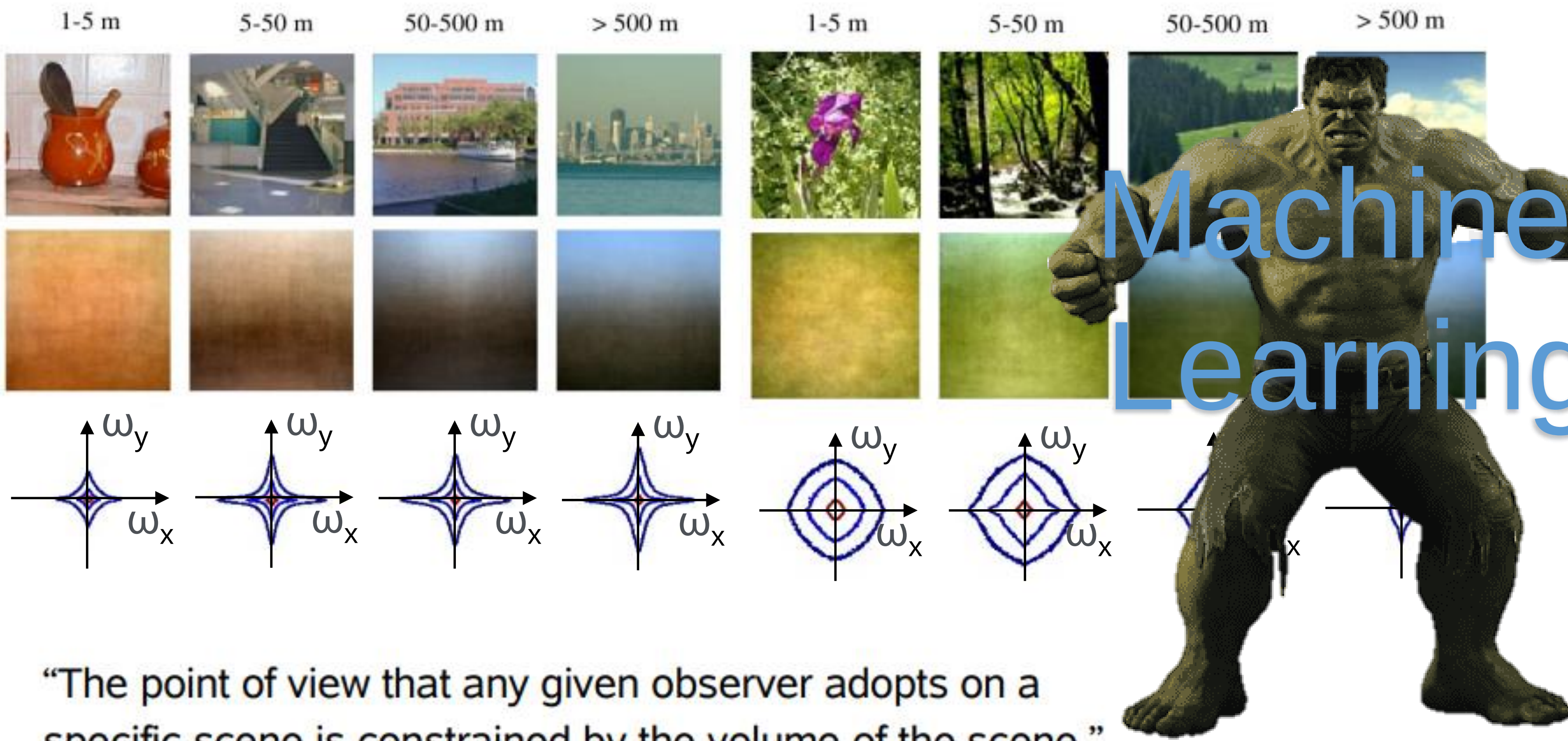
MACHINE LEARNING?



TYPICAL IMAGE POWER SPECTRAL DENSITY



TYPICAL IMAGE POWER SPECTRAL DENSITY



“The point of view that any given observer adopts on a specific scene is constrained by the volume of the scene.”

[Torralba2003] Torralba, Antonio, and Aude Oliva. "Statistics of natural image categories." *Network: computation in neural systems* 14, no. 3 (2003): 391.

ALTERNATIVE IMAGE REPRESENTATIONS

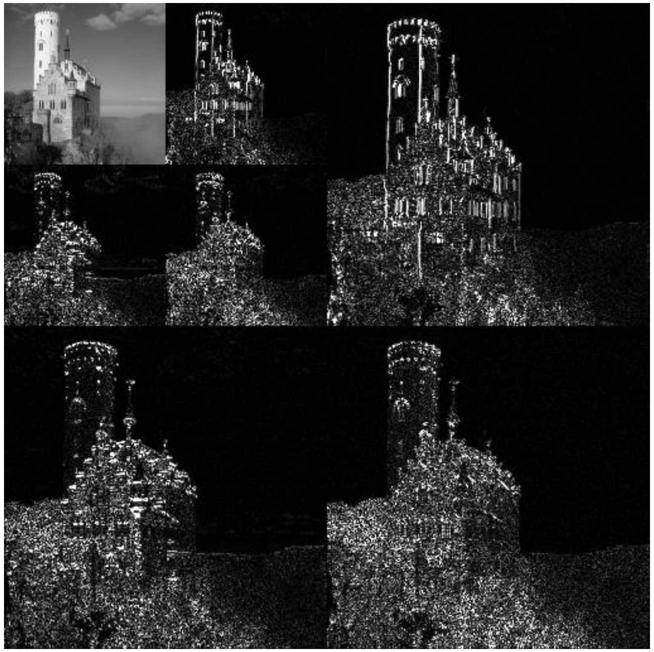
image representations facilitate assumption modeling

$\mathbf{x}$

"an image is a vector of pixel values"

$\mathbf{w}$

"an image is a vector of wavelet basis functions coefficients"



PRIOR EXAMPLES

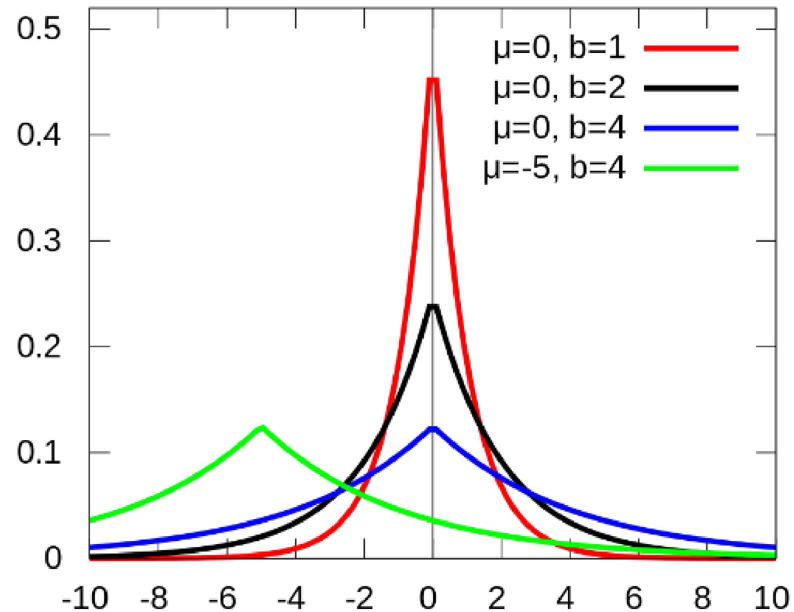
Wavelet ‘sparsity’

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}} \mathrm{p}\left(\mathbf{y}|\mathbf{w}\right) \mathrm{p}\left(\mathbf{w}\right)$$

$$\mathrm{p}\left(\mathbf{y}|\mathbf{w}\right) = \mathcal{N}\left(H\mathbf{w}, \Sigma\right)$$

$$\mathrm{p}\left(\mathbf{w}\right) = \frac{1}{2b} \exp\left(-\frac{|\mathbf{w}-\mu|}{b}\right)$$

“Laplace Distribution”



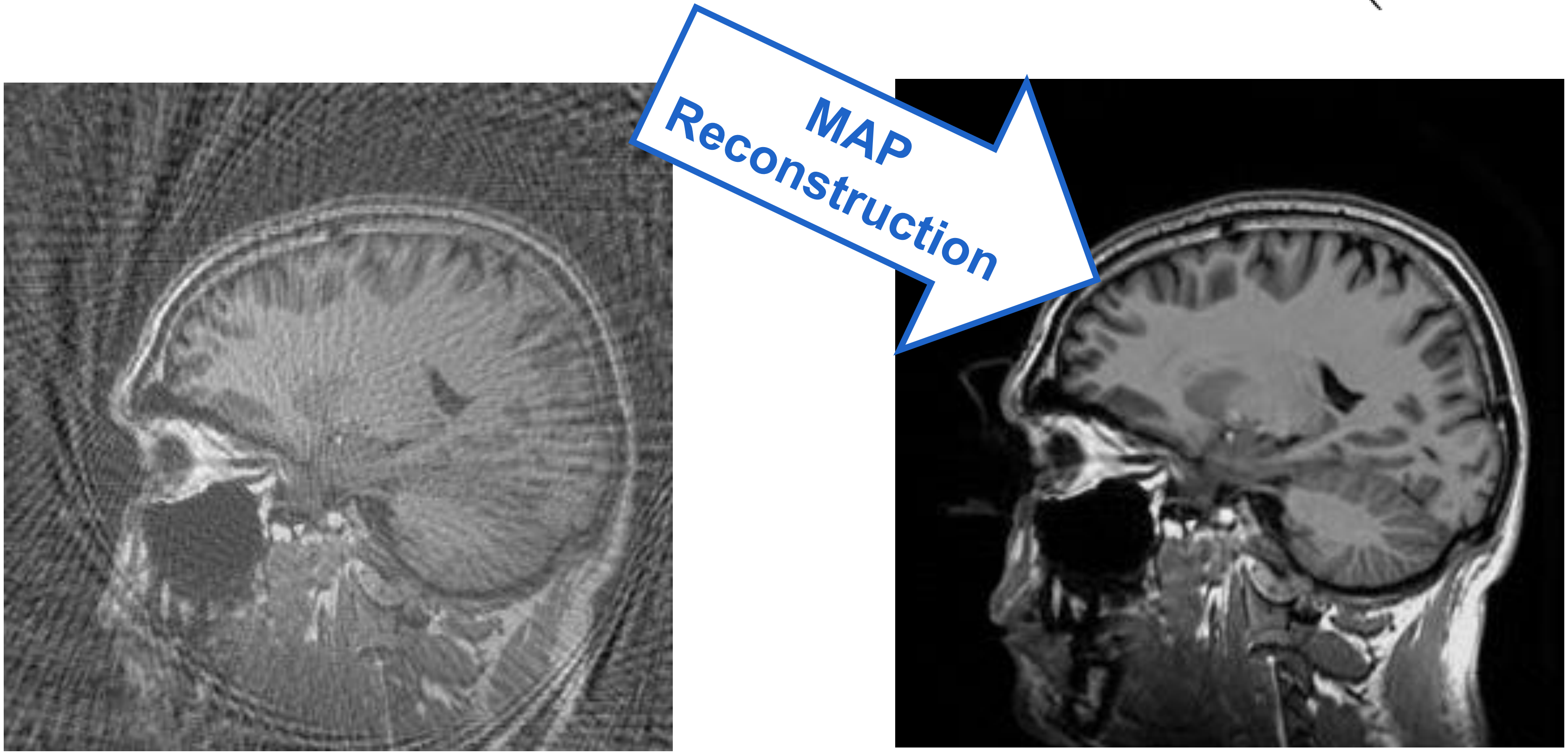
Reconstructing fast MRI acquisition

PRIOR EXAMPLES

Wavelet 'sparsity'

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}} \mathrm{p}(\mathbf{y}|\mathbf{w})\, \mathrm{p}(\mathbf{w})$$

$$\mathrm{p}(\mathbf{y}|\mathbf{w}) = \mathcal{N}(H\mathbf{w}, \Sigma) \qquad \mathrm{p}(\mathbf{w}) = \frac{1}{2b} \exp\left(-\frac{|\mathbf{w}-\mu|}{b}\right)$$



'Classic reconstruction'

Wavelet-regularized reconstruction

Reconstructing fast MRI acquisition

KINDS OF PRIORS

Some models the **spatial** distribution of pixel values

Spatial upscaling of images, denoising, …

Others model the **temporal** distribution of pixel values

Temporal upscaling of images, denoising, …

Or the **spectral** distribution of pixel values

Spectral upscaling of images, denoising, …

Or the relation to **other images** (other modalities)

Multi-modal image restoration…

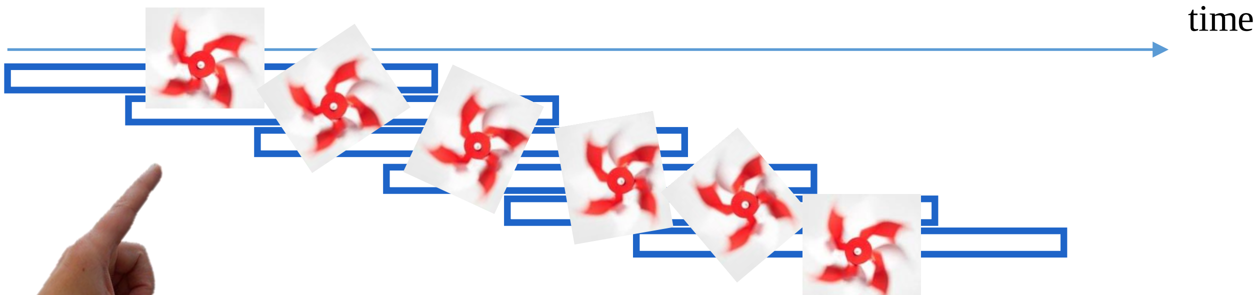
PRIOR EXAMPLES

Temporal upscaling?

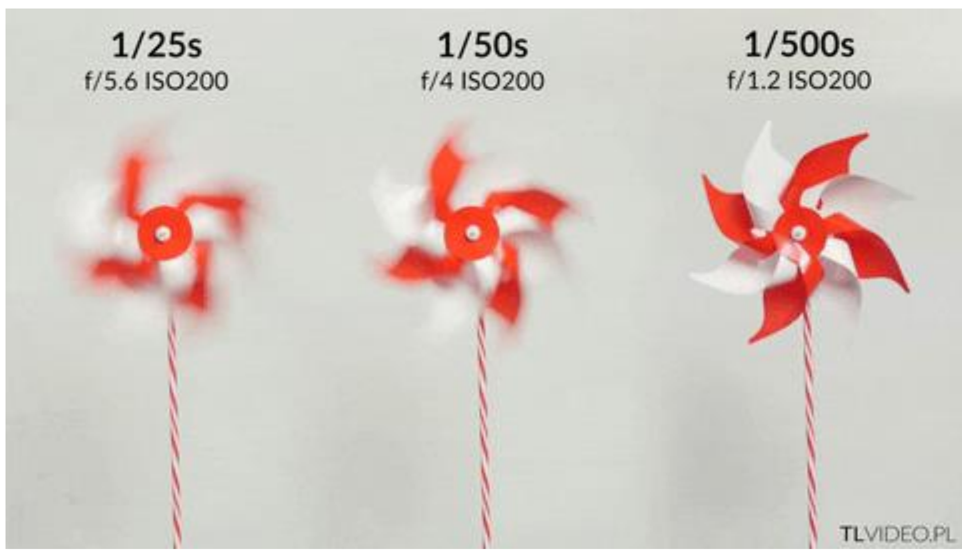
What is temporal Resolution?

– Not a synonym for Framerate

1/25s



The object’s blurriness is not only a function of framerate

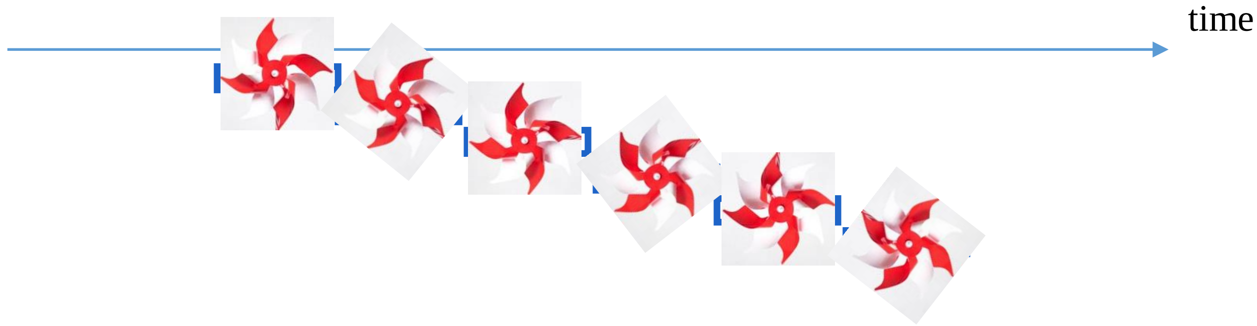


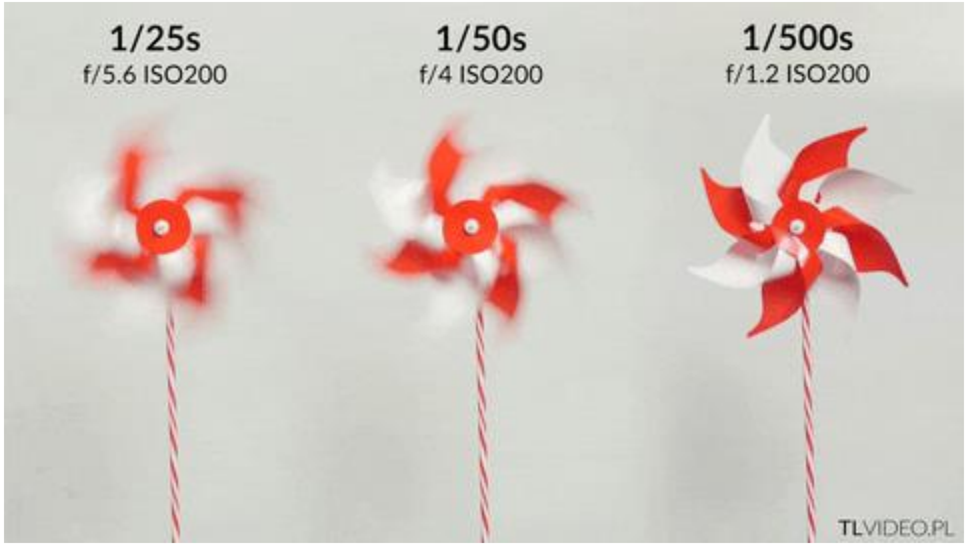
PRIOR EXAMPLES

What is temporal Resolution?

– Not a synonym for Framerate

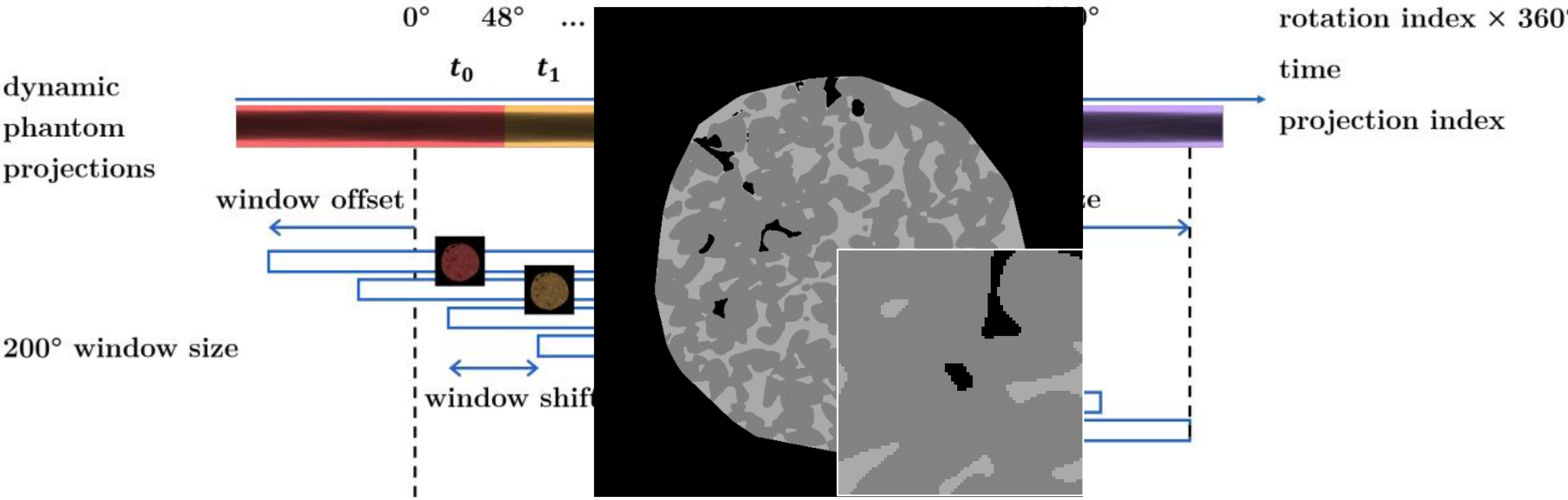
1/500s

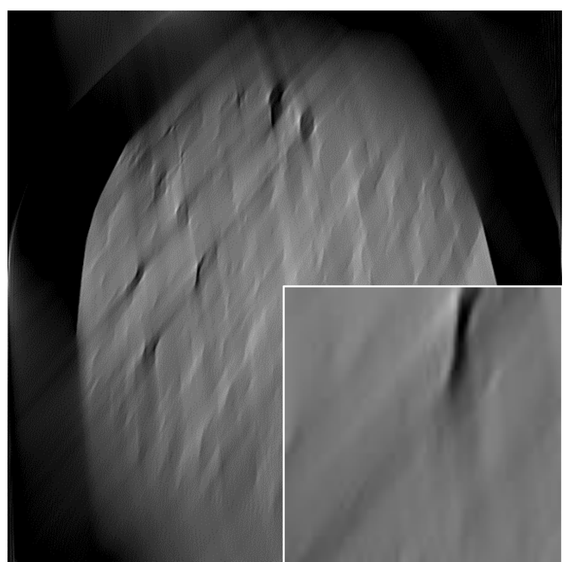




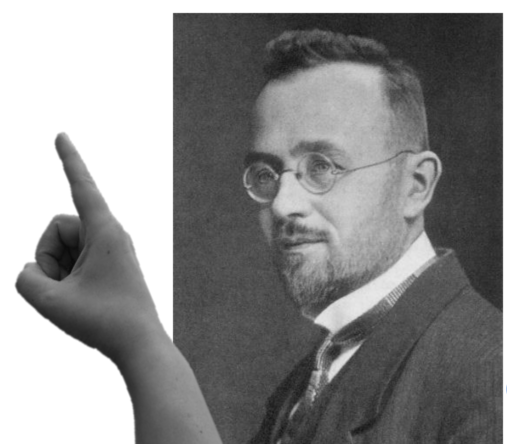
PRIOR EXAMPLES

The same happens in CT





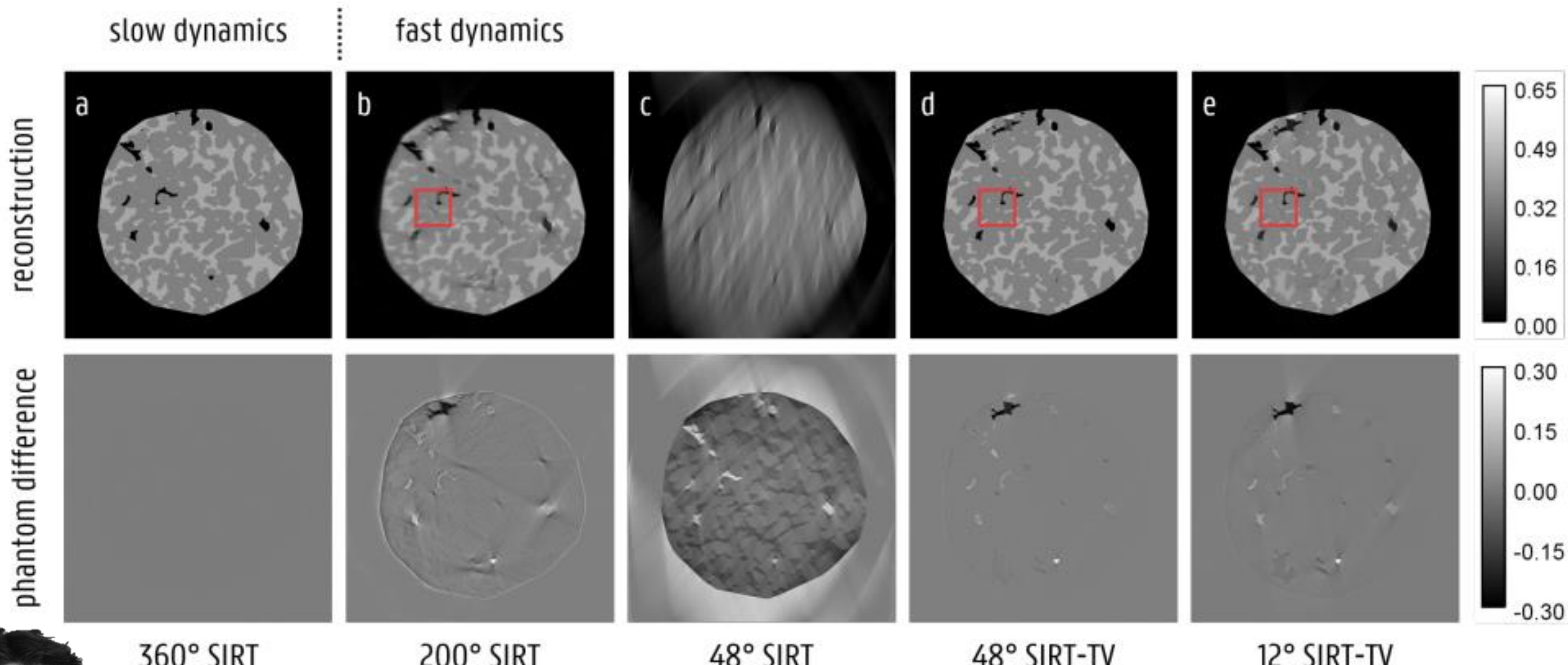
A narrower reconstruction window violates reconstruction conditions: **Limited angle artifact**

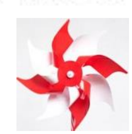


PRIOR EXAMPLES

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \mathrm{p}(\mathbf{y}|\mathbf{x})\,\mathrm{p}(\mathbf{x}) \longrightarrow \mathrm{p}(\mathbf{x}_{0..T}) \sim \prod_{i=1}^{T} \exp\big(\underbrace{-|\mathbf{x}_i - \mathbf{x}_{i-1}|}\big)$$

Differences between successive time frames (at time i and i-1) are small





PRIOR EXAMPLES

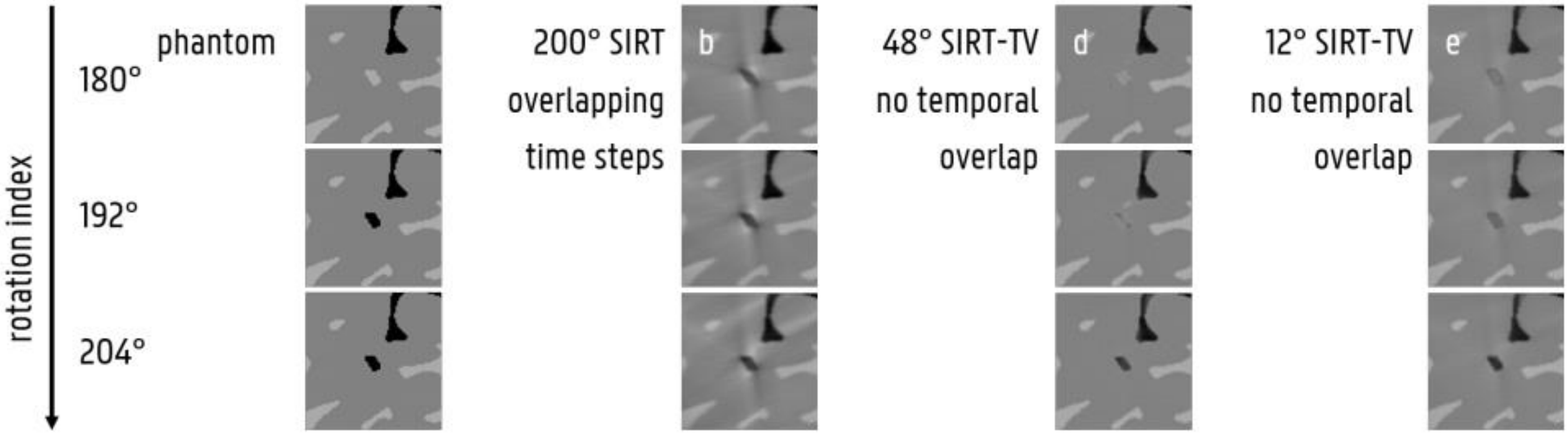


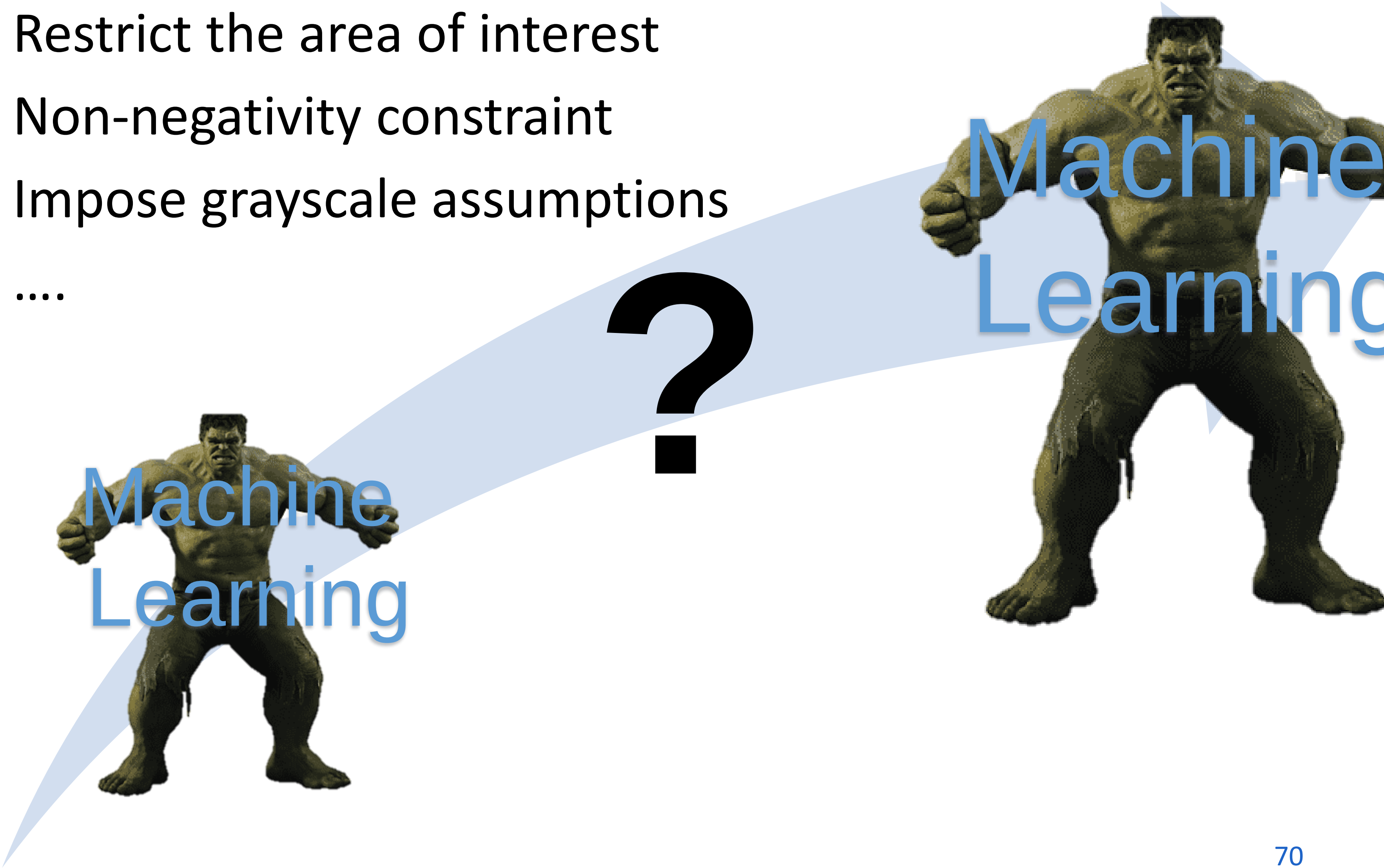
Figure 6. A selection of close-ups of the ROI before, during and after a jump in pore p2. In the right column, the phantom jumps at rotation index 192°. The 200° SIRT reconstruction in (b) contains streaks around the pore. The SIRT-TV reconstructions with 48° (d) and 12° (e) have clear transitions. The central time steps, during the jump, contain 50% projections before the jump and 50% projections after the jump.

[Goethals2021] Dynamic CT reconstruction with improved temporal resolution for scanning of fluid flow in porous media

PRIOR EXAMPLES: BE CREATIVE

What do I know about the signal-of-interest?

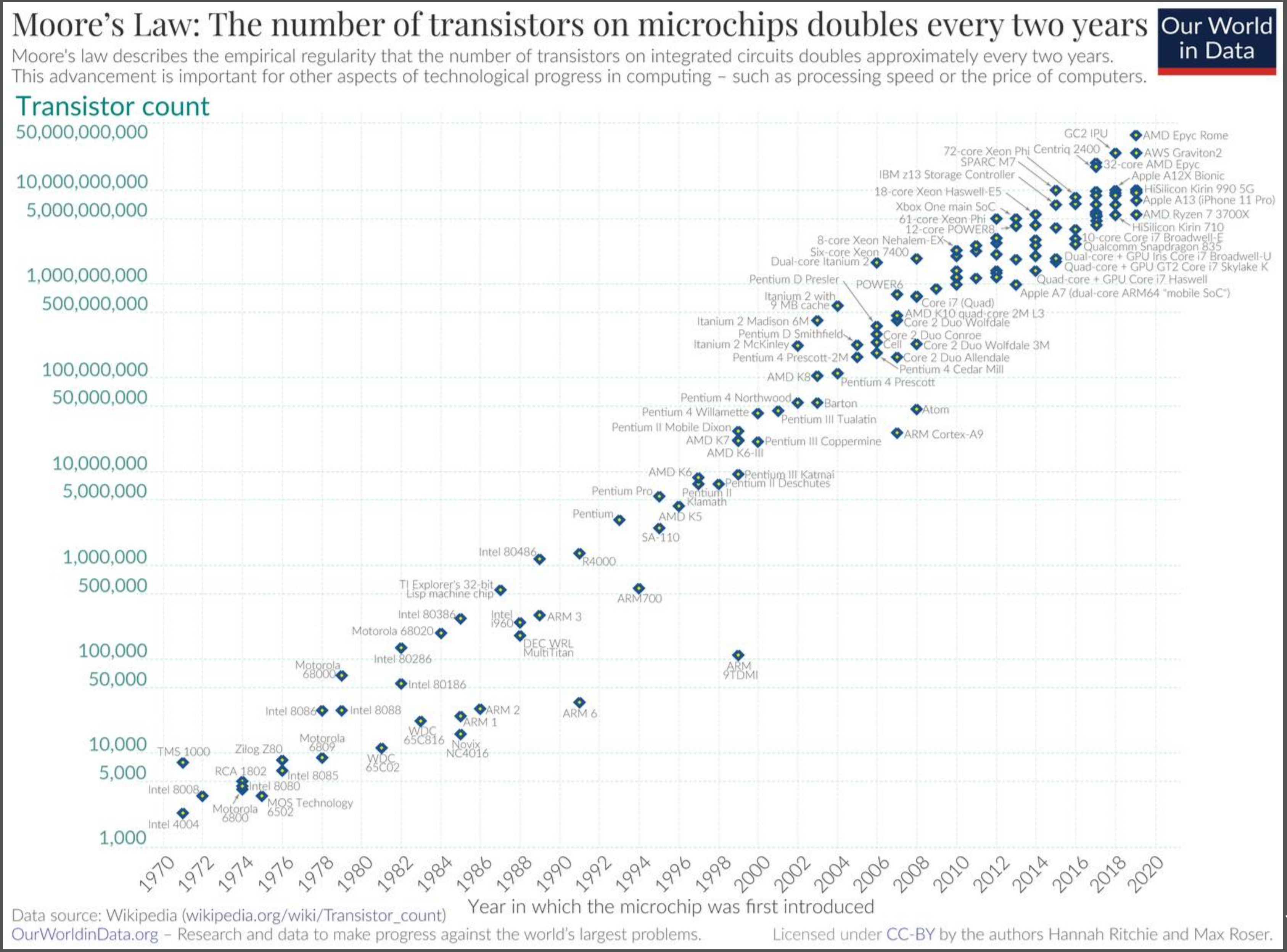
- Restrict the area of interest
- Non-negativity constraint
- Impose grayscale assumptions
- ….



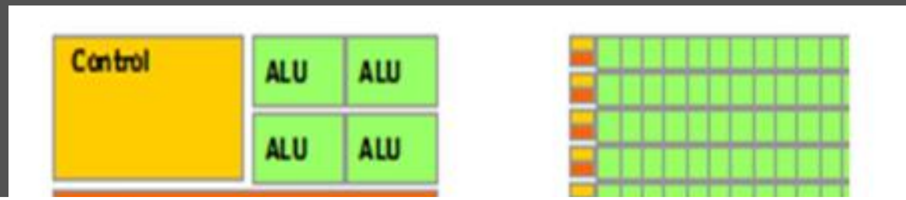
TOMOGRAPHY & RECONSTRUCTION

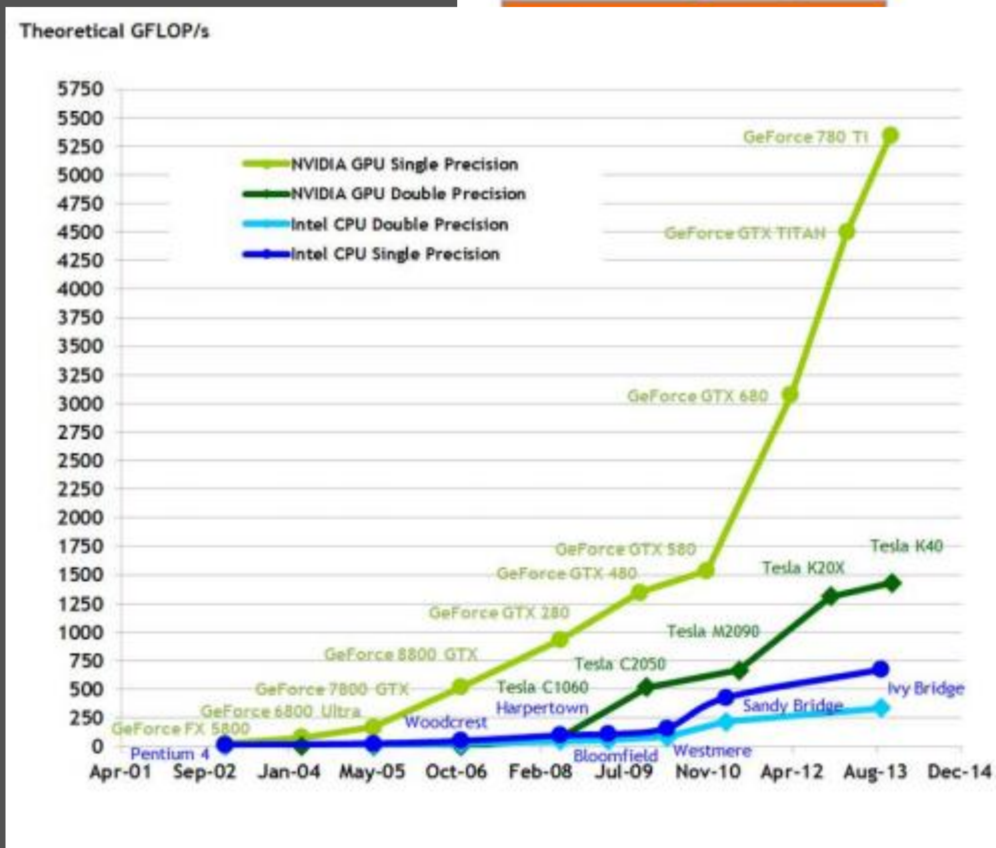
- What is it?
- How is it done?
  - Analytical
  - Iterative
- Link to estimation theory
  - Bayesian vs ML
  - Priors
- Link to machine learning

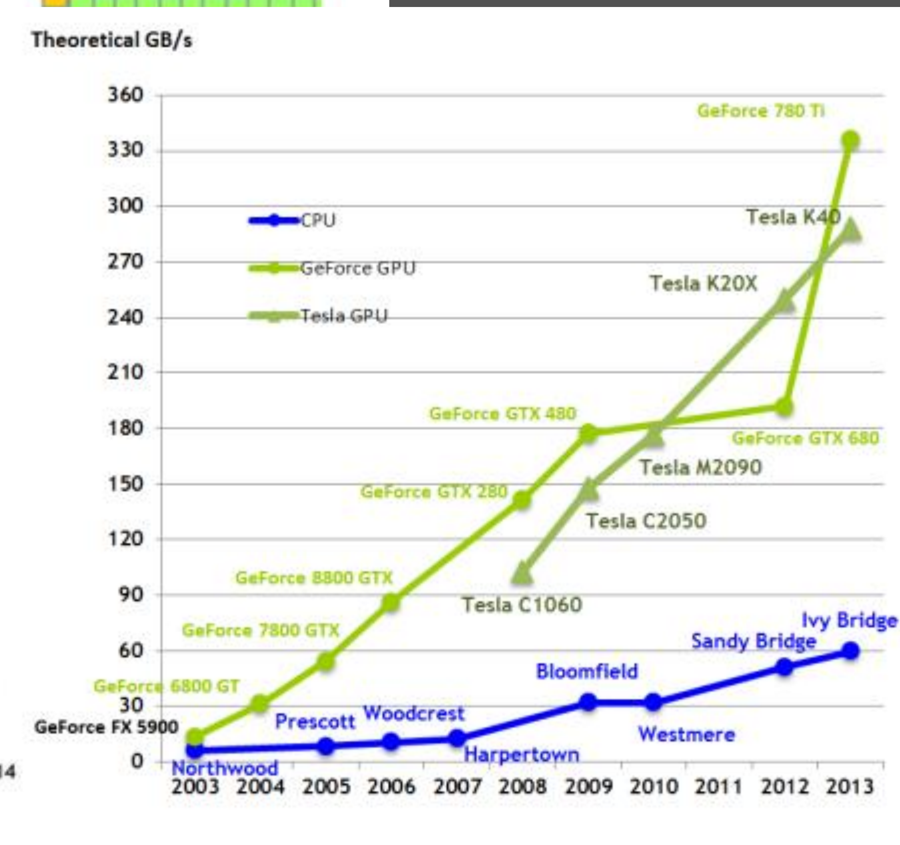
THE BACKDROP



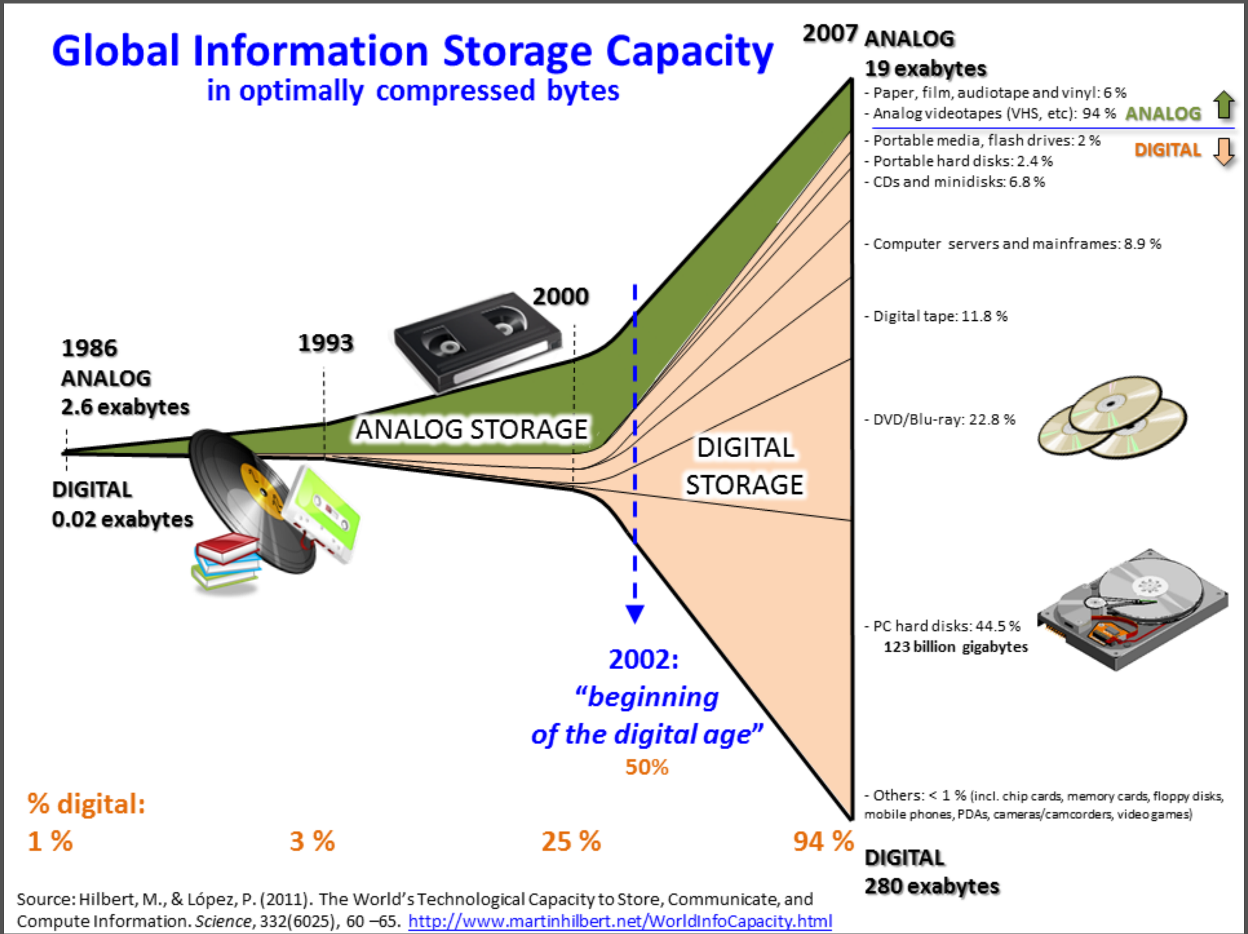
THE BACKDROP







THE BACKDROP



<1986: 10^{18} bytes = 1 000 000 000 000 000 000 bytes
2020 alone: 10^{22} bytes = 10 000 000 000 000 000 000 000 bytes
= 10 zettabytes
= 10 billion terabytes

PRIOR EXAMPLES: MACHINE LEARNING

What can you do with lots of data?

Thomas Bayes

$$\mathrm{p}\left(\mathbf{x}|\mathbf{y}\right) = \frac{\mathrm{p}\left(\mathbf{y}|\mathbf{x}\right)\mathrm{p}\left(\mathbf{x}\right)}{\mathrm{p}\left(\mathbf{y}\right)}$$

Learn these distributions (implicitly)
Learn estimators **(= a function)**
…
From EXAMPLES

PRIOR EXAMPLES: MACHINE LEARNING

What can you do with lots of data?

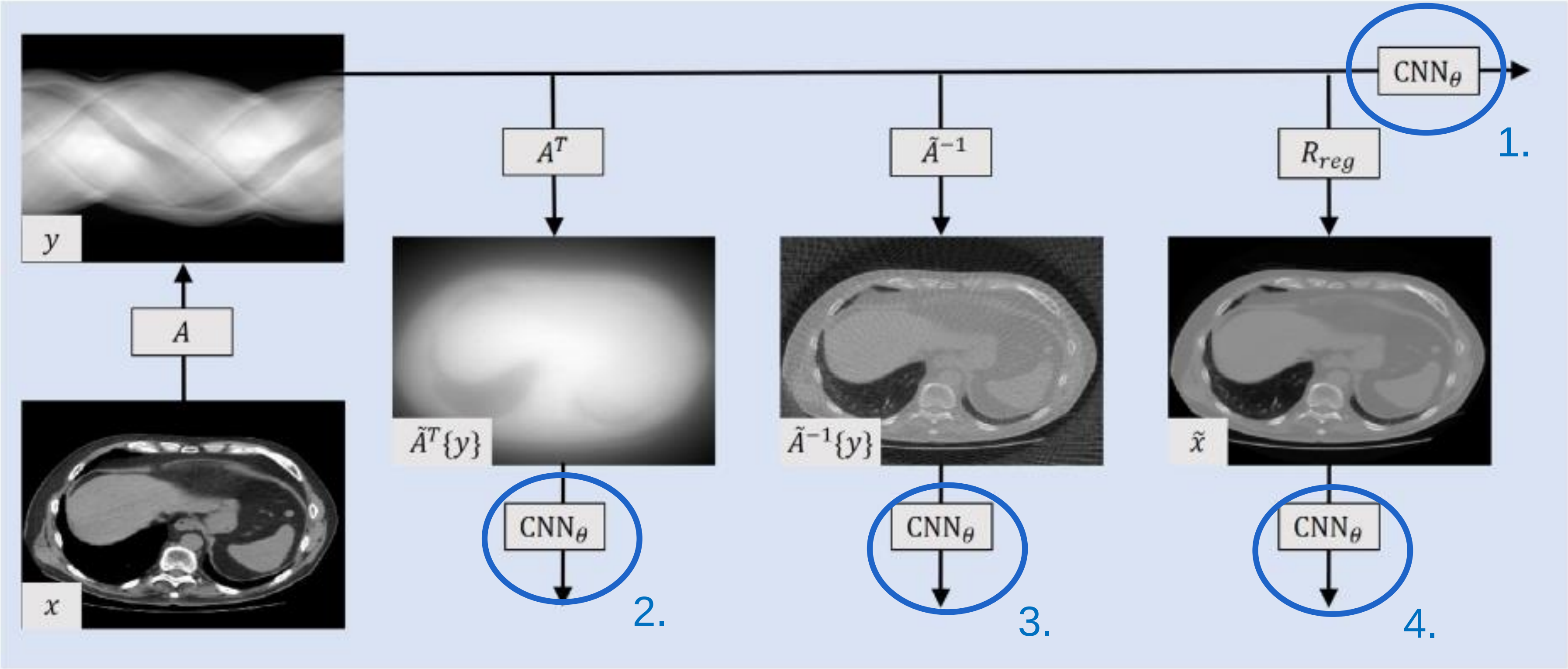
Universal approximation theorem:

A neural network can be made to approximate **ANY FUNCTION** with arbitrary precision

Learn these distributions (implicitly)
Learn estimators **(= a function)**
…
From EXAMPLES

A FUTURE OF TOMOGRAPHY

A Neural network may be applied in different ways

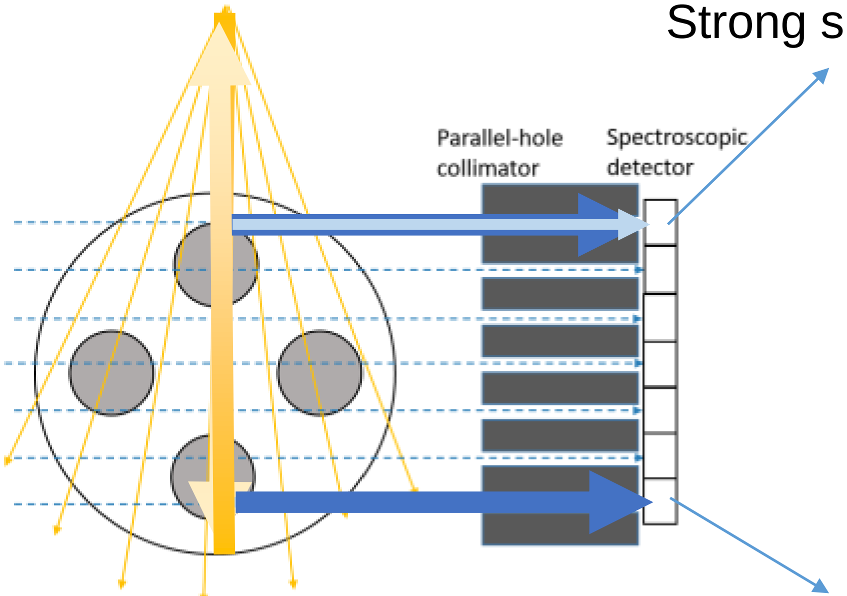


1. **To improve Sinograms**
2. To improve Backprojections
3. To improve Reconstructions as post-processing
4. To improve Reconstructions as regularization

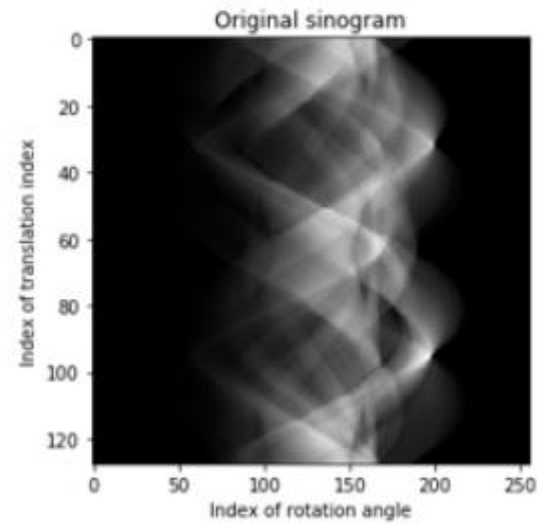
XFCT

Fluorescence tomography: a natural candidate for sinogram processing

Strong signal



Weakened signal

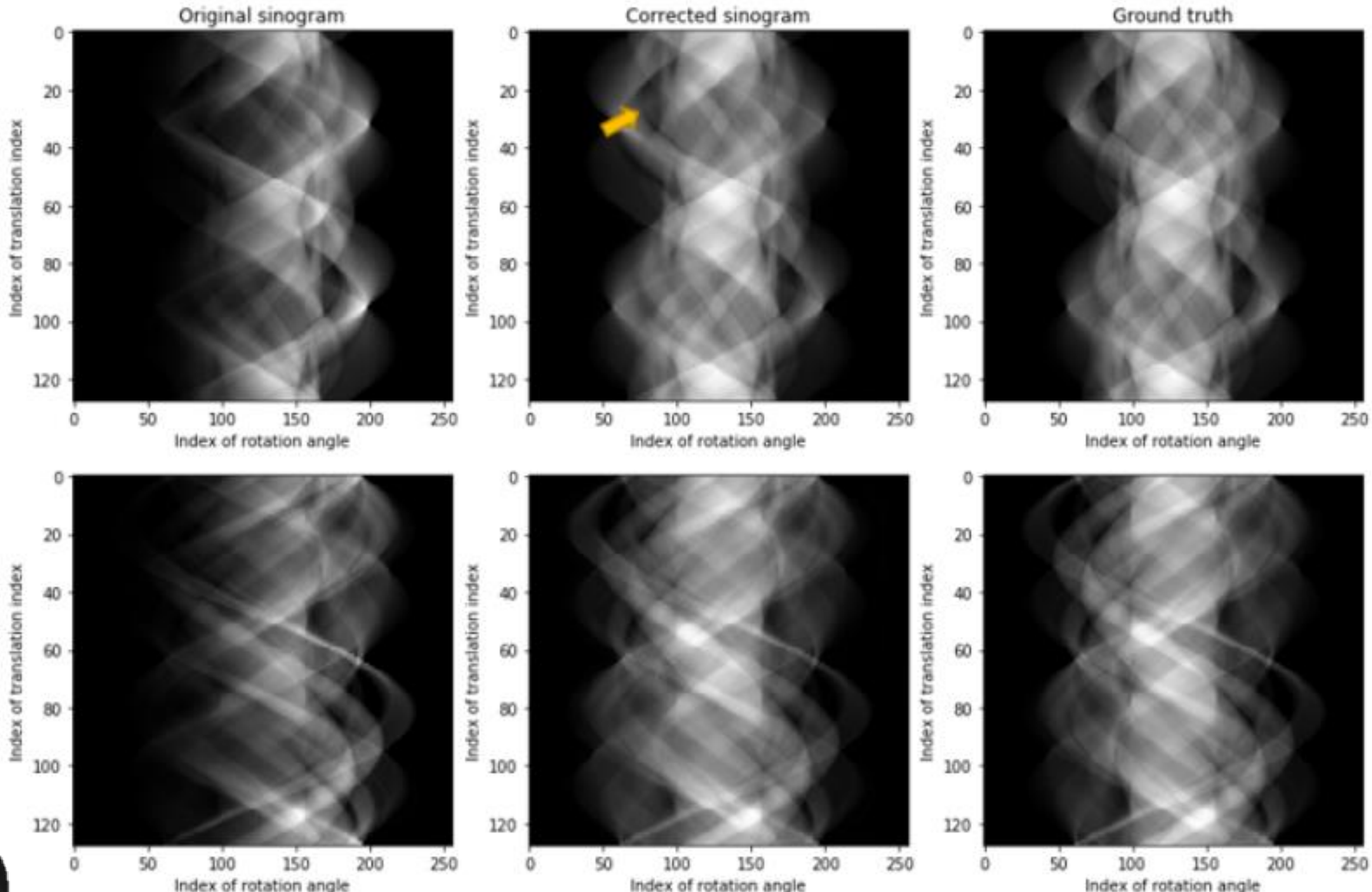


“Self-absorption”

Sinogram
Not symmetric!

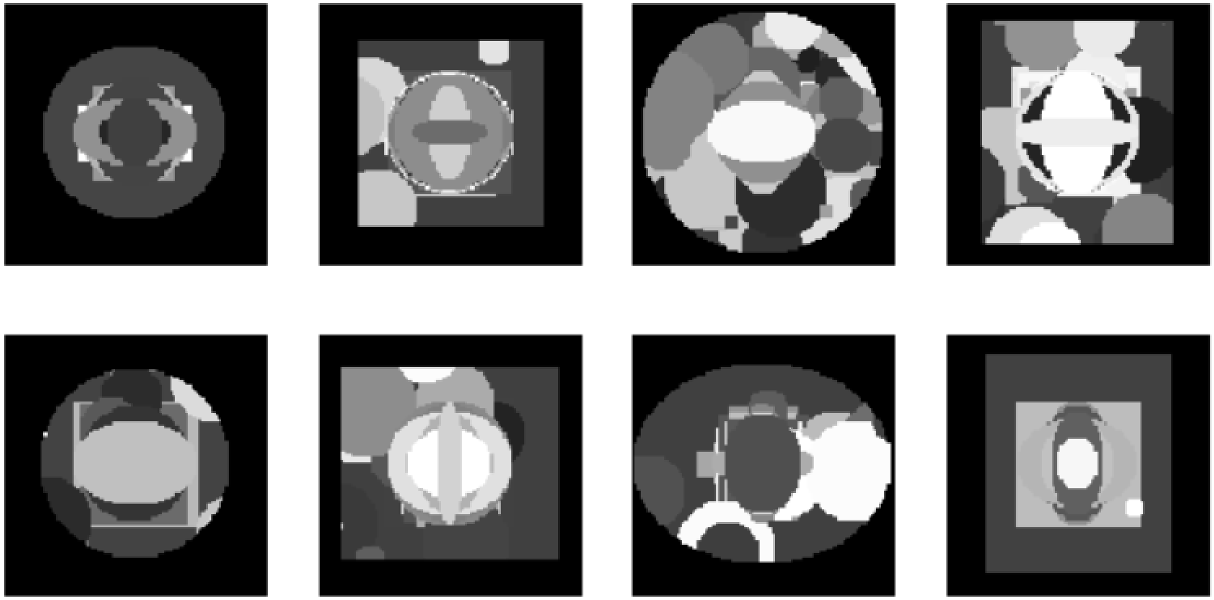
XFCT

– A Neural network can correct self-absorption



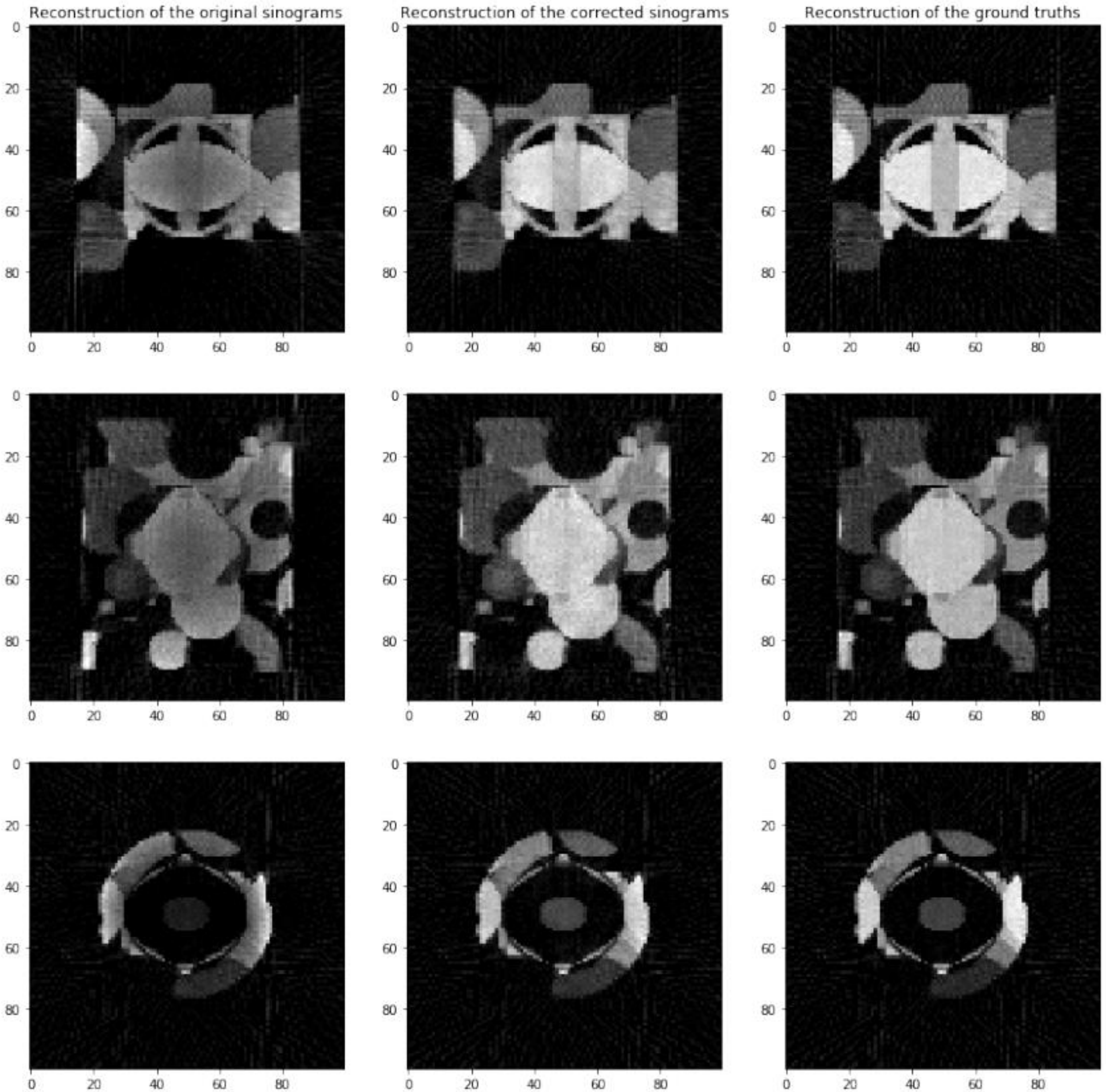
XFCT: CREATE YOUR OWN BIG DATA

Based on simulation? Yes



[Dhaene2015]Dhaene, Jelle, Elin Pauwels, Thomas De Schryver, Amélie De Muynck, Manuel Dierick, and Luc Van Hoorebeke. 2015. “Arion: a Realistic Projection Simulator for Optimizing Laboratory and Industrial micro-CT.” In Tomography of Materials and Structures, 2nd International Conference, Proceedings, ed. Bernard Long, 60–64.

XFCT: SELF-ABSORPTION CORRECTION RESULTS



HOW TO VALIDATE?

(a) Image for the generation of fluorescence X-rays

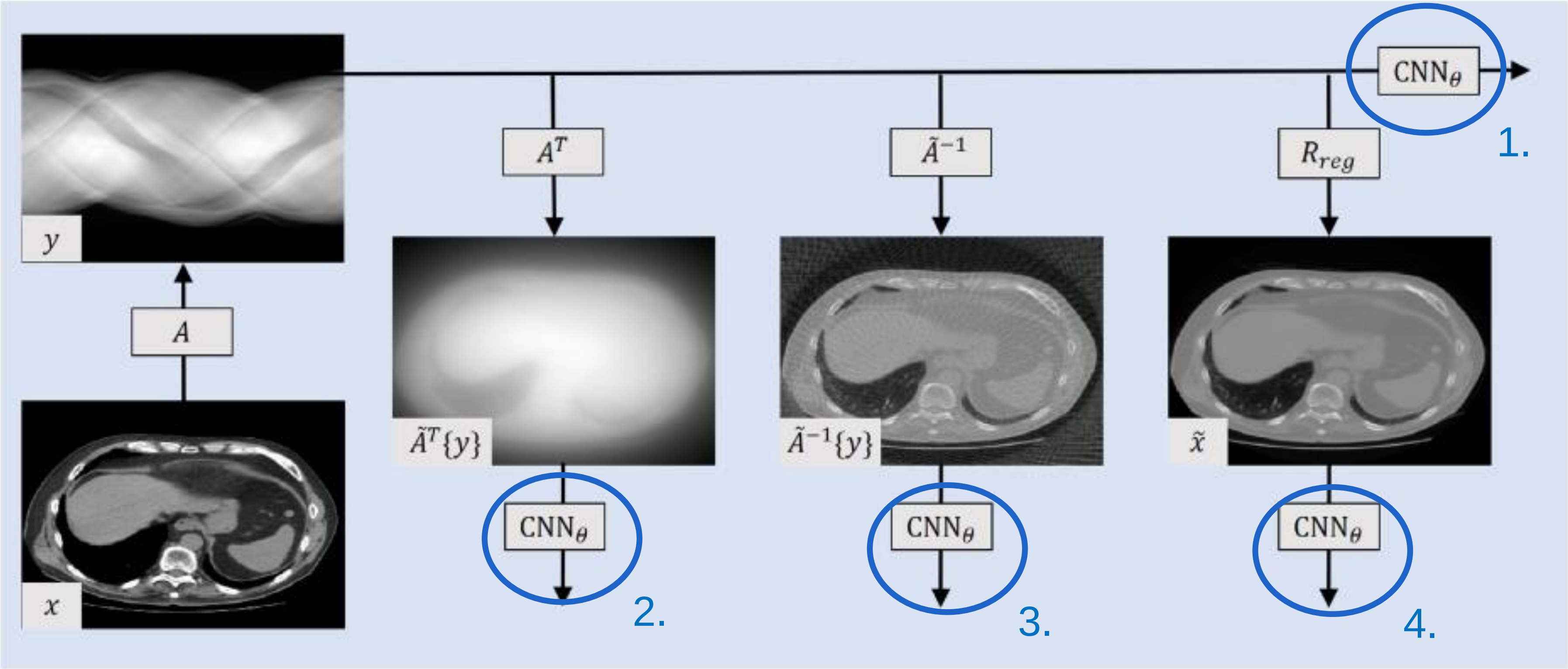
(b) Image for the generation of self-absorption effect

Figure 6.10: Numerical phantom for the validation of the training strategy, where the left image represents the relatively distribution of elements of interest and the image on the right denotes the attenuation map of the proposed phantom

Figure 6.12: Comparison among the FBP reconstructions of the fluorescence sinograms emitted from the validation phantom (left column), their corrections by DC-Unet (middle column) and the ground truths (right column). Similar to figure 6.11, the first row presents the reconstructions of phantom 1, the second row displays the reconstructions of phantom 2 while the bottom row shows the reconstructions of phantom 3. Moreover, all reconstructions are presented in the intensity range of the FBP reconstructions of the ground truth.

A FUTURE OF TOMOGRAPHY

A Neural network may be applied in different ways



1. To improve Sinograms
2. To improve Backprojections
3. To improve Reconstructions as post-processing
4. **To improve Reconstructions as regularization**

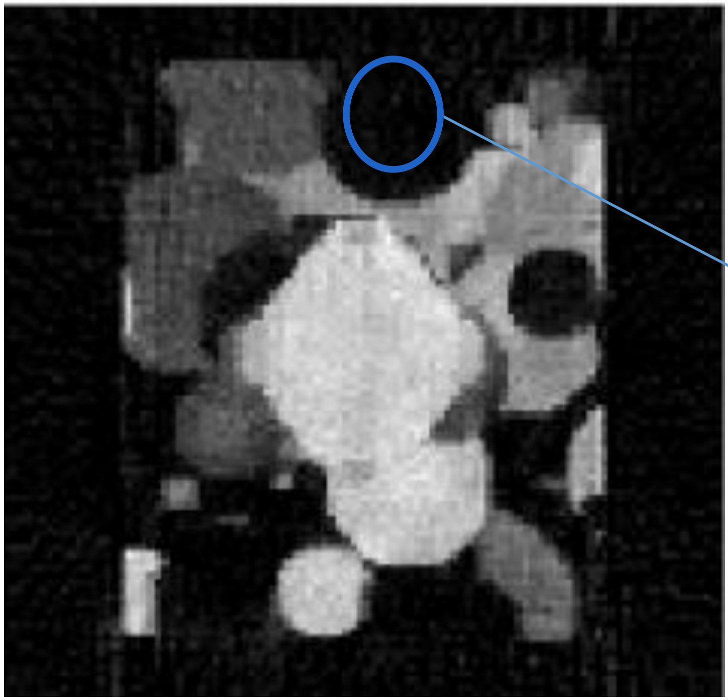
A WORD ON GANS

Neural networks rely on many examples to learn priors

Thomas Bayes

$$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{p(\mathbf{y})}$$

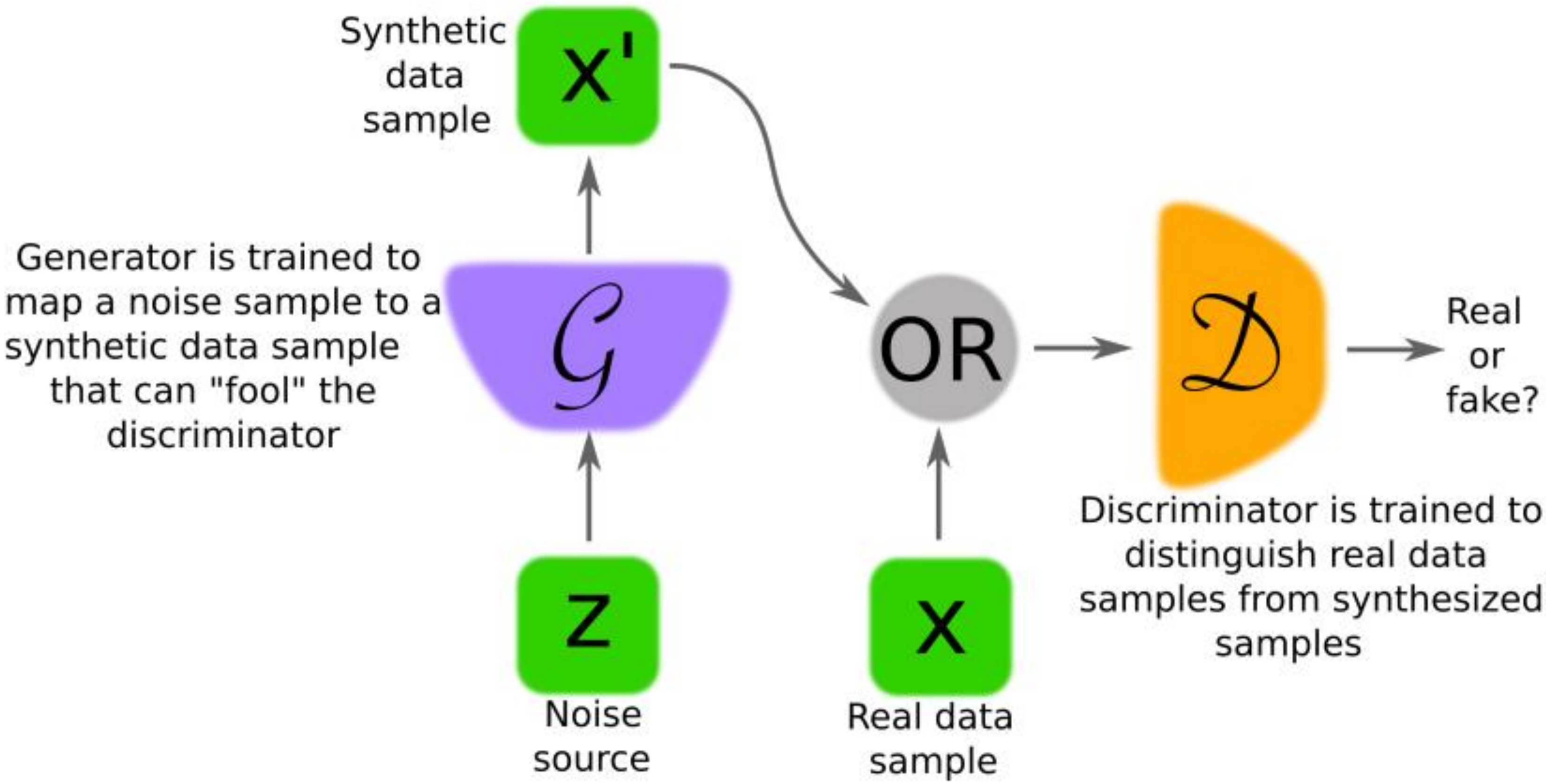
Examples show what is probable to occur
Not directly what is improbable to occur



NOT like this

A WORD ON GANS

A generative adversarial network dynamically changes optimization goal



Creswell, Antonia, Tom White, Vincent Dumoulin, Kai Arulkumaran, Biswa Sengupta, and Anil A. Bharath. "Generative adversarial networks: An overview." *IEEE Signal Processing Magazine* 35, no. 1 (2018): 53-65.

A WORD ON GANS

$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathrm{x} \sim p_{data}(\mathrm{x})} [\log D(\mathrm{x})] + \mathbb{E}_{\mathrm{z} \sim p_{\mathrm{z}}(\mathrm{z})} [\log(1 - D(G(\mathrm{z})))], \quad (1)$$

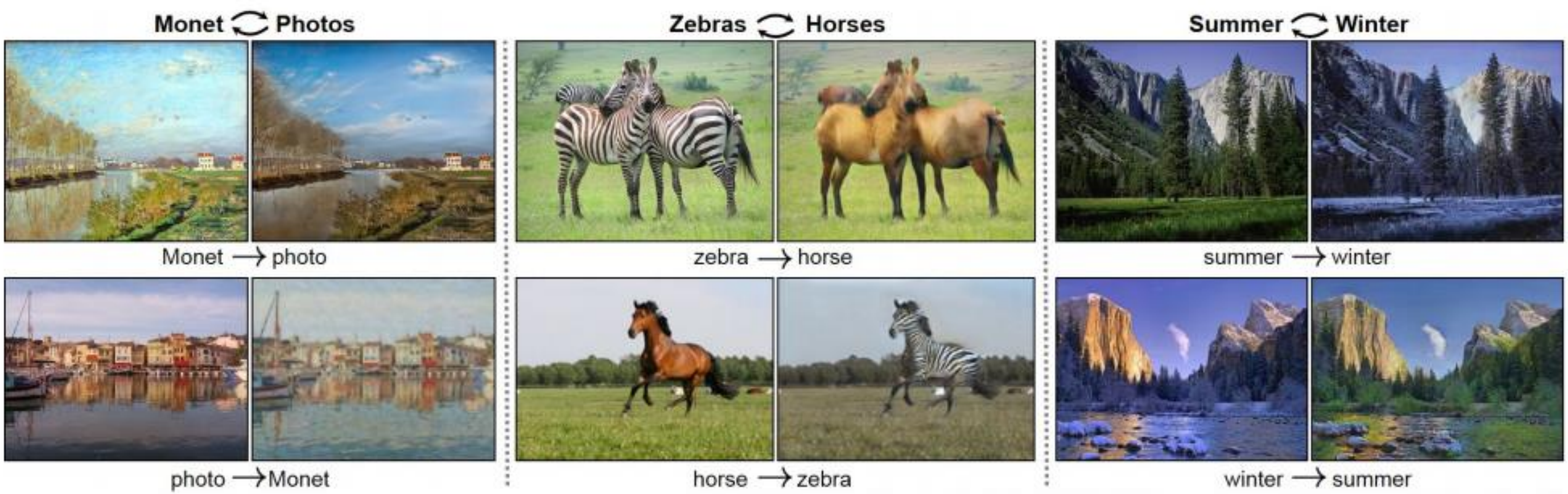
where x is the sample from the p_{data} distribution; z is a random encoding on the latent space. With some user interaction, GANs have been applied in interactive image editing [34]. However, GANs can not be applied to the the problem of inpainting directly, because they produce an entirely unrelated image with high probability, unless constrained by previously given corrupted image.

Creswell, Antonia, Tom White, Vincent Dumoulin, Kai Arulkumaran, Biswa Sengupta, and Anil A. Bharath. "Generative adversarial networks: An overview." *IEEE Signal Processing Magazine* 35, no. 1 (2018): 53-65.

A WORD ON GANS

Allows to generate samples that are indistinguishable from real data

And to focus the generator training on noticeable errors

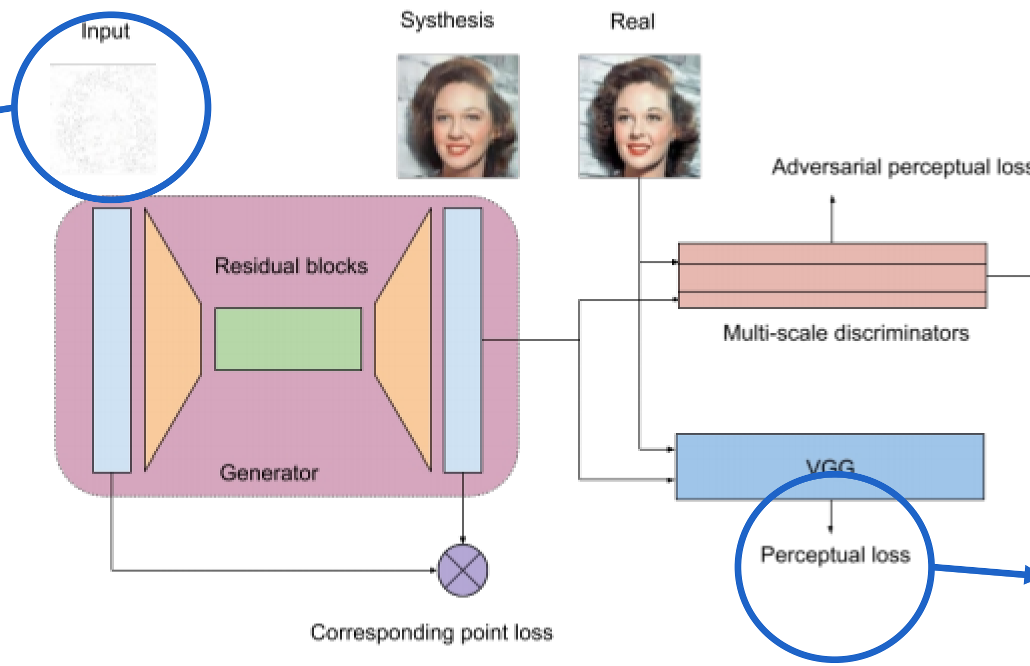


Creswell, Antonia, Tom White, Vincent Dumoulin, Kai Arulkumaran, Biswa Sengupta, and Anil A. Bharath. "Generative adversarial networks: An overview." *IEEE Signal Processing Magazine* 35, no. 1 (2018): 53-65.

A WORD ON GANS

GANS in tomography? The generator needs to ‘generate’ plausible solutions of reconstruction problem

Not a random vector



Limit to data-accurate solutions

Figure 2: Framework of the our network using GANs. We use a multi-dimensional loss functions architecture to help improve the quality of image reconstruction.

Liu, Longfei, Sheng Li, Yisong Chen, and Guoping Wang. "X-gans: Image reconstruction made easy for extreme cases." arXiv preprint arXiv:1808.04432 (2018).

GANS IN USE

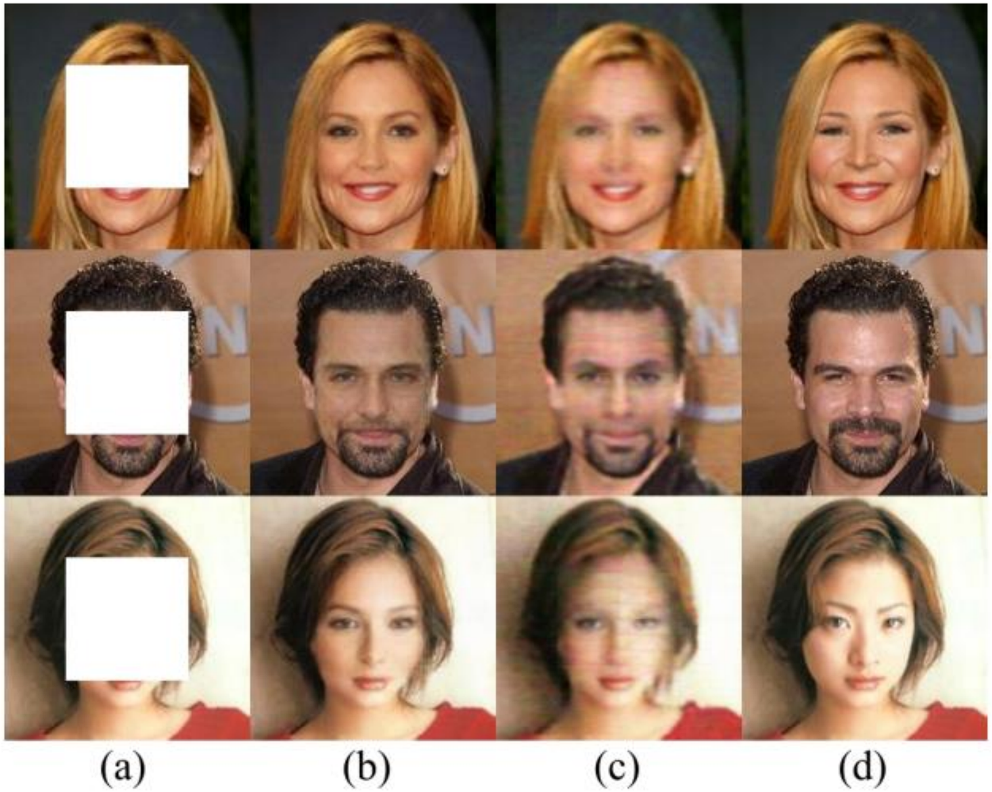
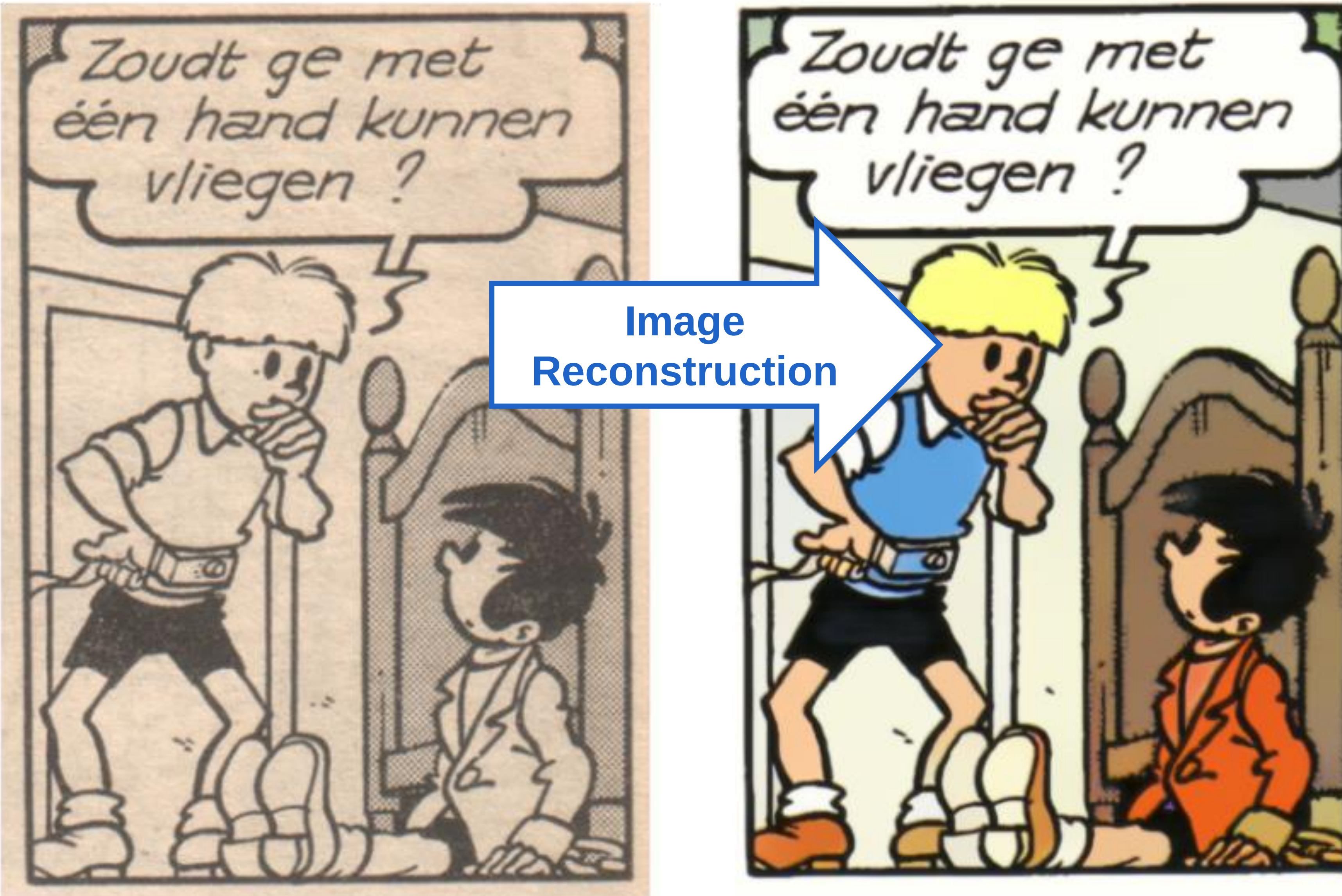


Figure 7: Reconstruction from missing block. Column (a) shows the missing block as the source image, column (b) is generated by our method, column (c) is generated by [6], and column (d) is the ground truth. Our method can synthesize better results.

Beware fake news!

GANS IN USE

Learning models using machine learning



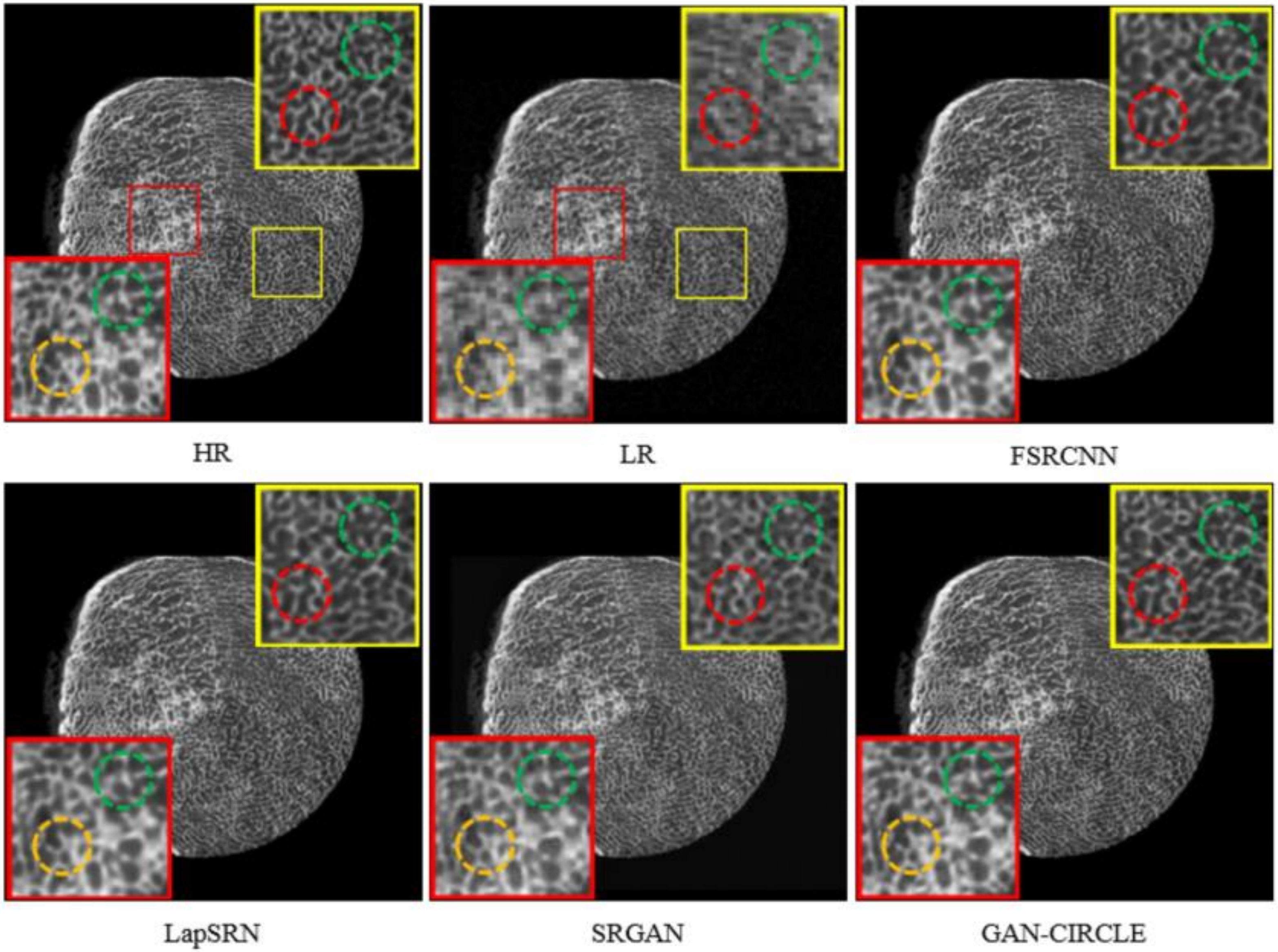
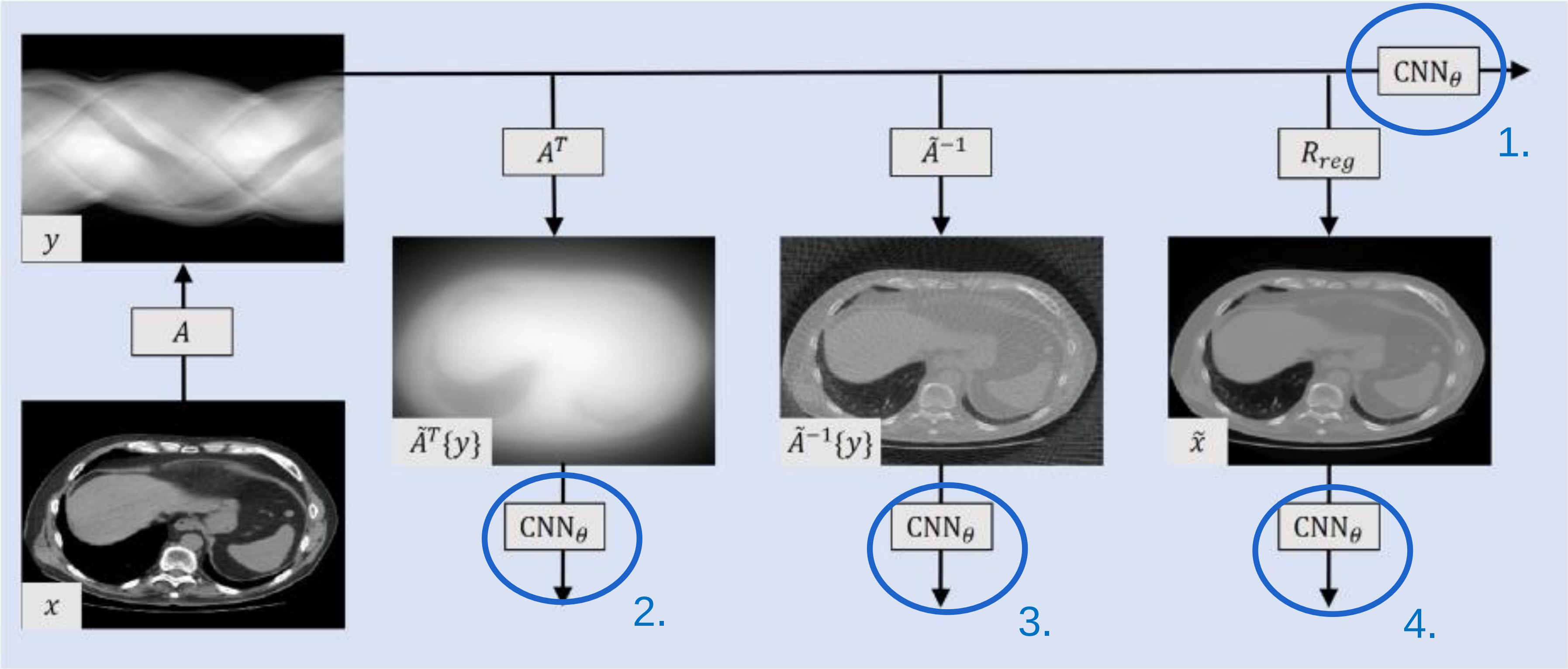


Figure 5.15. CT super-resolution results with different methods, including FSRCNN (Dong *et al* 2016), LapSRN (Lai *et al* 2017), SRGAN (Ledig *et al* 2017), and GAN-CIRCLE (You *et al* 2019), respectively.

A FUTURE OF TOMOGRAPHY

A Neural network may be applied in different ways



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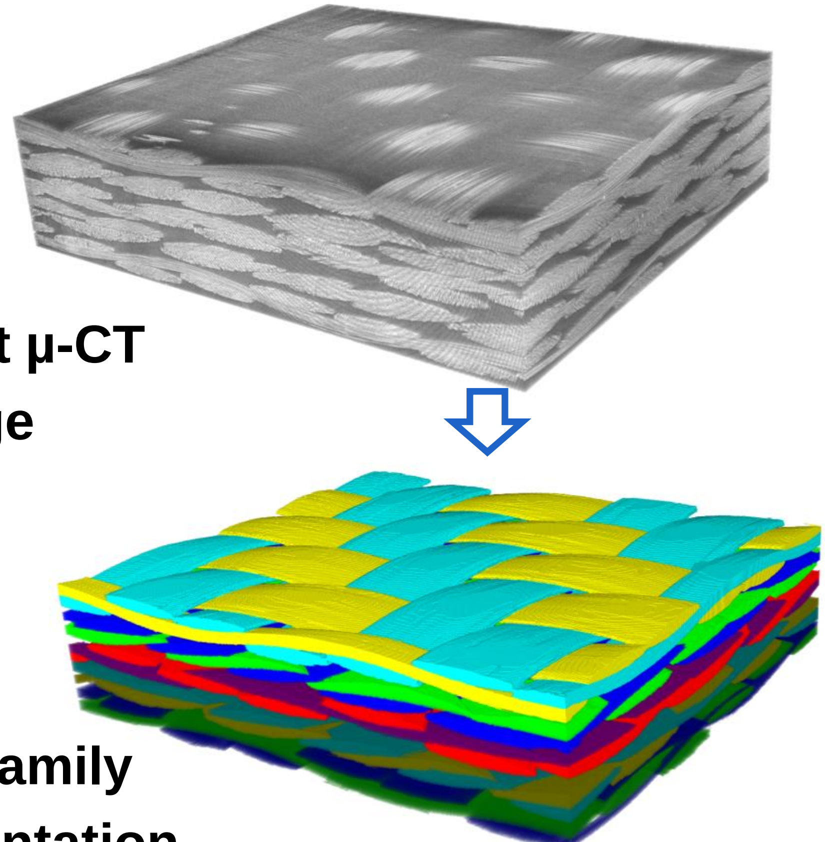
DATA ANALYSIS: POST-PROCESSING

Learn segmentation from examples



Input μ-CT image

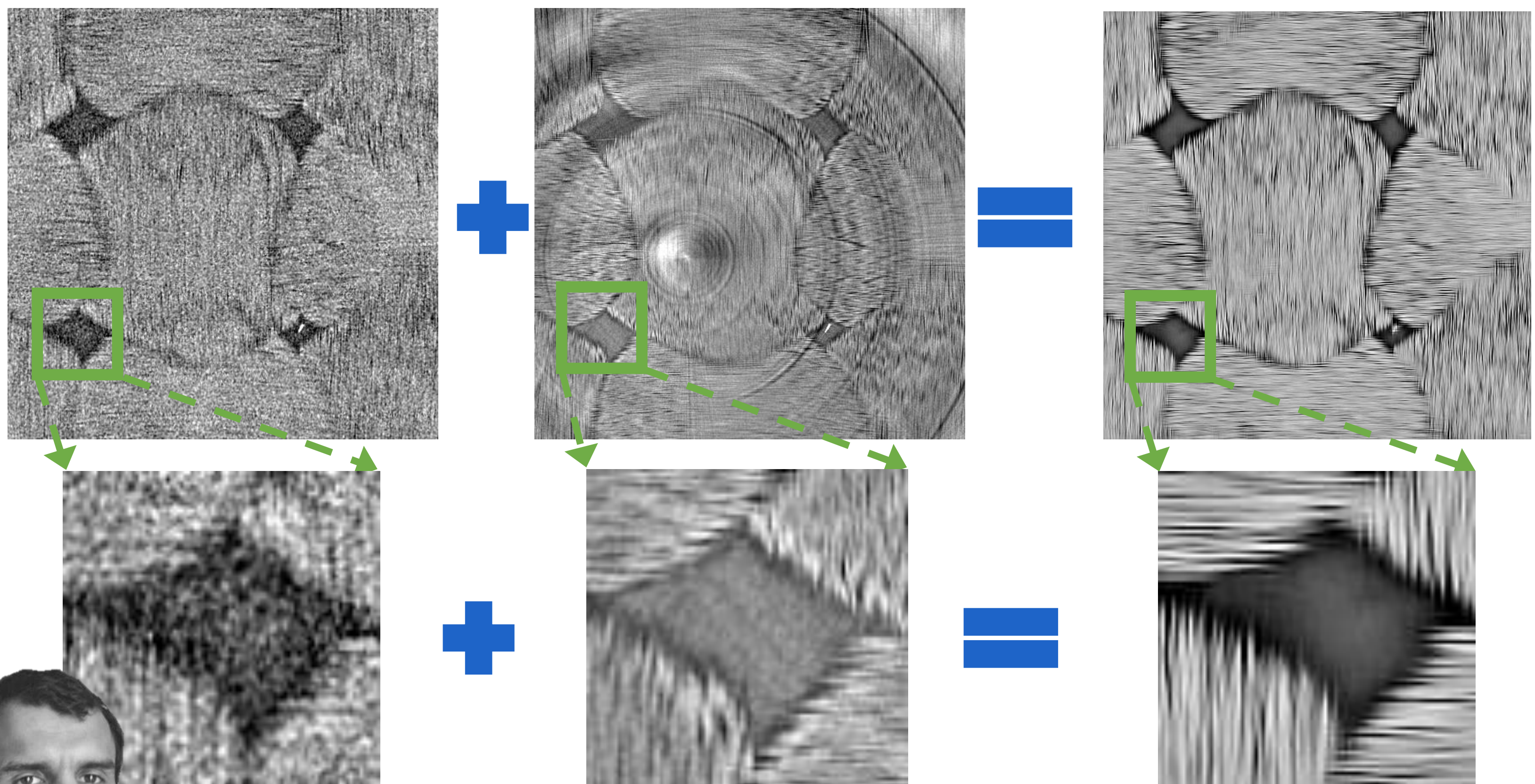
Tows family segmentation

Sinchuk, Yuriy, Pierre Kibleur, Jan Aelterman, Matthieu Boone, and Wim Van Paepegem. 2021. “Geometrical and Deep Learning Approaches for Instance Segmentation of CFRP Fiber Bundles in Textile Composites.” COMPOSITE STRUCTURES. https://doi.org/10.1016/j.compstruct.2021.114626.

BLURRED LINES

Fusion of reconstructed volumes

8µm CT slice | **TOMCAT slice (registered)** | **8µm CT slice (fused)**



BLURRED LINES

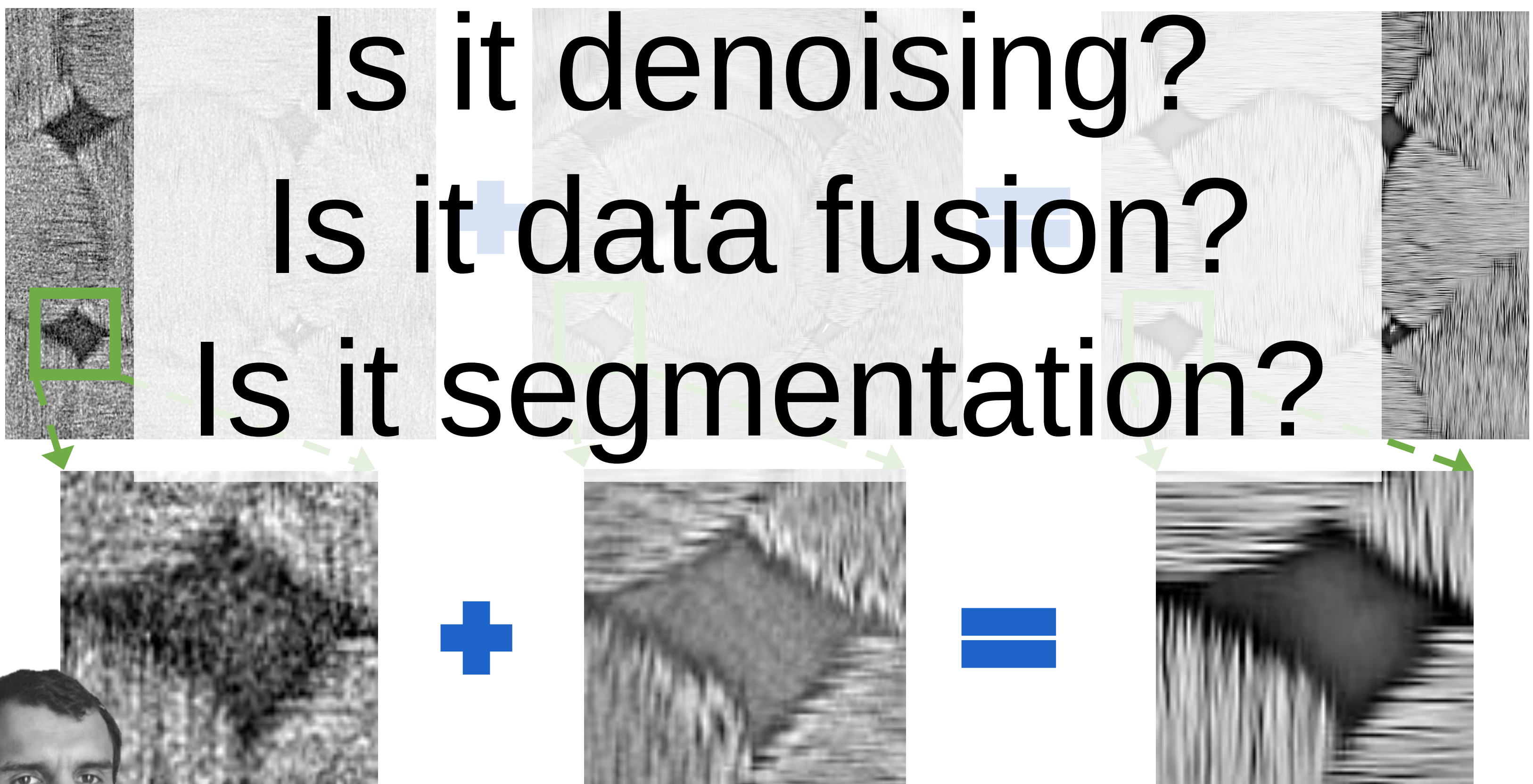
Fusion of reconstructed volumes

Is it reconstruction?

Is it denoising?

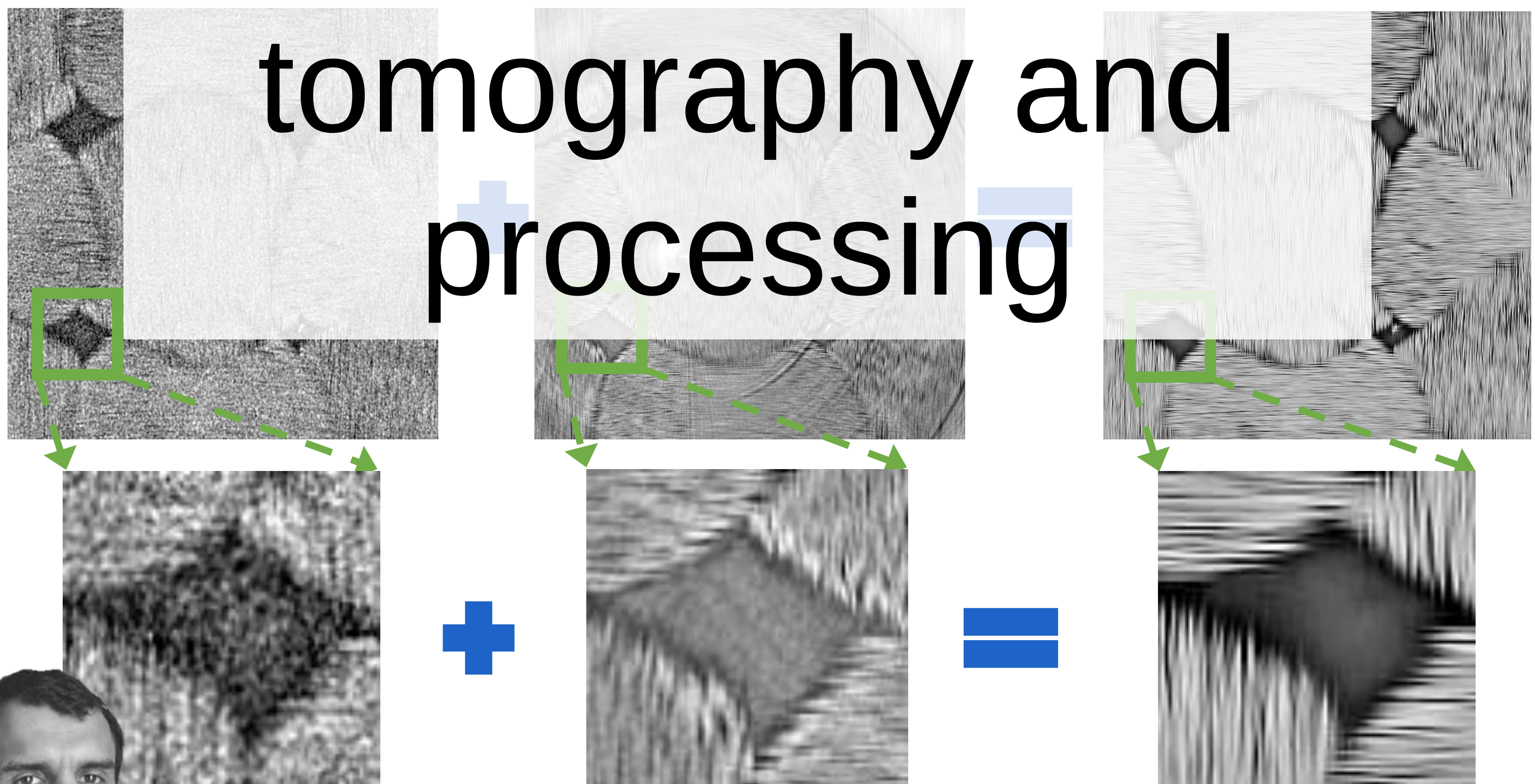
Is it data fusion?

Is it segmentation?

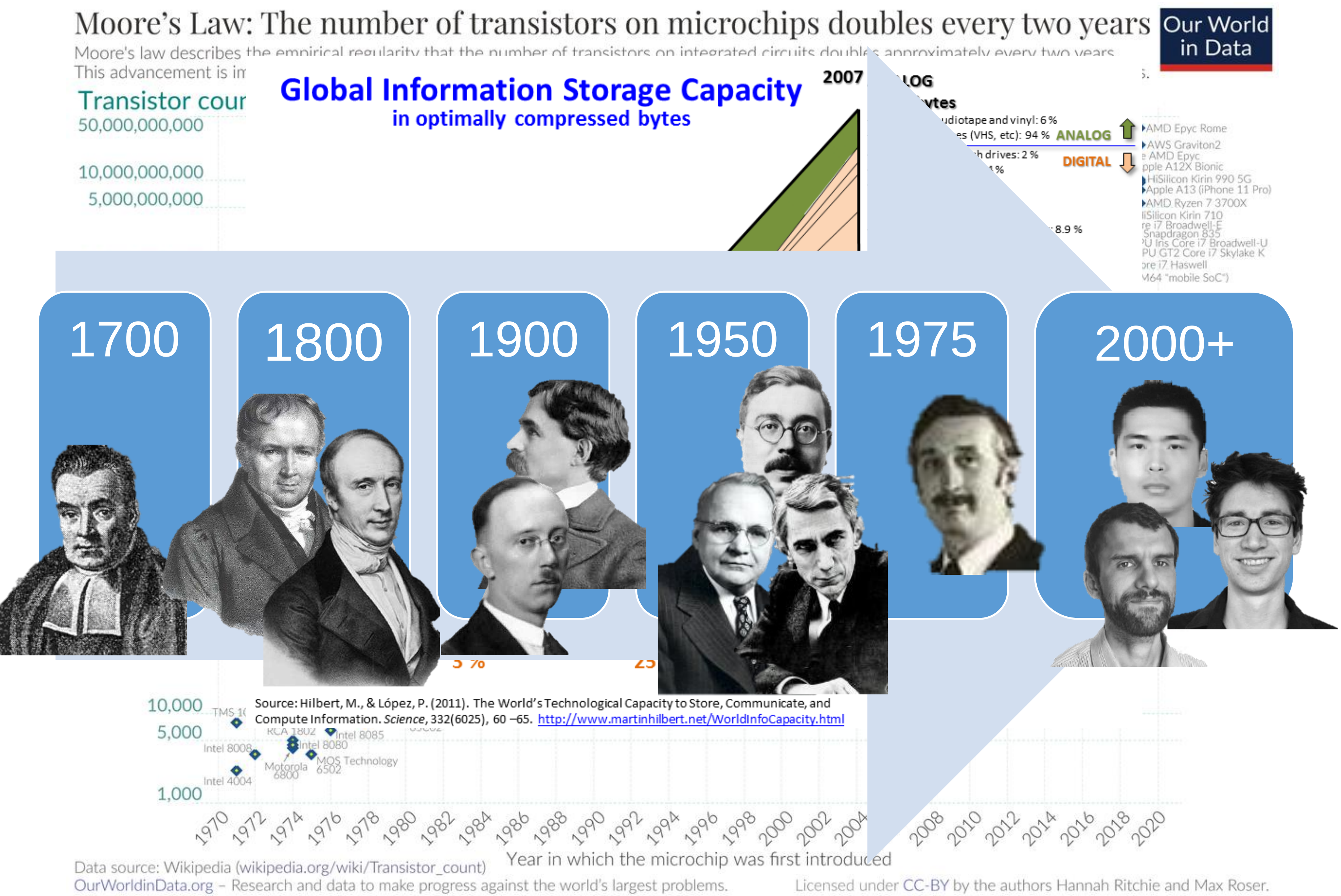


Fusion of reconstructed volumes

It's **joint** tomography and processing



A HISTORY OF TOMOGRAPHY



TOMOGRAPHY & RECONSTRUCTION

- What is it?
- How is it done?
  - Analytical
  - Iterative
- Link to estimation theory
  - Bayesian vs ML
  - Priors
- Link to machine learning

Questions

Jan.Aelterman@ugent.be