Homework 3

1. Answers
    a. 2.5
    b. $\frac{x^5+1}{2}$
    c. 0.502
    d. 0.83

2. Answers

	x						
y	0	1	2	3	4	5	$P_y(y)$
0	0.040	0.010	0.025	0	0.025	0	**0.1**
1	0.200	0	0.050	0.300	0	0	**0.55**
2	0.100	0.100	0	0	0	0.150	**0.35**
$P_x(x)$	**0.34**	**0.11**	**0.075**	**0.3**	**0.025**	**0.15**	1

    a. Because $P_x(x)$ and $P_y(y)$ =1 that means this is a valid joint probability distribution
    b. See bottom row of table (bolded)
    c. See last column of table (bolded)

	0	1	2	3	4	5
F(x|y=0)	0.4	0.1	0.25	0	0.25	0
F(x|y=1)	0.36	0	0.09	0.5454	0	0
F(x|y=2)	0.285	0.285	0	0	0	0.428

    d. See table above
    e. 0.6
    f. They are dependent, see work in the how I solved section

3. Answers
    a. 4
    b. 0.04
    c. 2x, $0 \le x \le 1$ (marginal density of x)
    d. 2y, $0 \le y \le 1$ (marginal density of y)
    e. 2x, , $0 \le x \le 1$ (f(x|y))
    f. 2y, , $0 \le y \le 1$ (f(y|x)
    g. 2x, , $0 \le x \le 1$ (f(x|y=0.5))
    h. Independent

How I solved:

1. Density Function (Continuous Random Variables)
    a. We know that the density function integrated with infinity and negative infinity as the bounds should equal 1. So we integrate cx^4 and set it equal to 1 with the bounds given to us (-1 to 1)
        i. $\int_{-1}^{1} cx^4 = 1$. Which gives us 5/2 or 2.5
    b. $\int_{-1}^{x} \frac{5}{2}x^4 = \frac{x^5+1}{2}$
        i. So now we have F(x) = 0 if x ≤ -1, $\frac{x^5+1}{2}$ if -1 < x < 1, and 1 if x ≥ 1.
    c. To solve F(1/3) plug 1/3 in for x which gives you 0.502
    d. |x| means we would have to integrate f(x) from -1 to -0.7 added to the integration of f(x) from 0.7 to 1, however, P(|x|>0.7) is equivalent to F(0.7)*2 which is 0.83
2. Joint Probability Distribution

	x						
y	0	1	2	3	4	5	$P_y(y)$
0	0.040	0.010	0.025	0	0.025	0	0.1
1	0.200	0	0.050	0.300	0	0	0.55
2	0.100	0.100	0	0	0	0.150	0.35
$P_x(x)$	0.34	0.11	0.075	0.3	0.025	0.15	1

   a. Looked at table
   b. Add whats in the columns together
   c. Add whats in the rows together
   d. F(x|y) = f(x,y)/f(y), plug and chug
   e. P(X > 0 | Y =0) i.e for all values of X greater than 0 given Y = 0
      a. P(X=1,2,3,4,5, Y = 0)/P(Y=0)
      b. 0.01+0.025+0.025/0.1=0.6
   f. Independence defined as p(x,y) = $p_x(x)p_y(y)$
      a. So we'll take P(2,2) = 0.075*0.35 = 0.026 but it should be 0 to be independent, so the variables are dependent
      b. As a sanity check we'll look at P(1,0) = 0.11*0.1= 0.011 but should be 0.010
3. Joint Density
    a. Take double integral with limits being 0 to 1 for both x and y
        i. Gives you 1/4. So we have c1/4 =1 which makes c = 4
    b. P(x < 0.4, Y < 0.5)
        i. Take the double integral of 4xy with the limits 0 to 0.5 for y and 0 to 0.4 for x
    c. Take the integral of f(x,y) with the limits of 0 to 1 with respect to y
    d. Do the same as did to find marginal density of X but with respect to x

e. $F(y|x) = f(x,y)/f(x)$
f. $F(x|y) = f(x,y)/f(y)$
g. $F(x|y=0.5) = f(x, 0.5)/f(y) = 4x(0.5)/2(0.5)$
h. $f(x,y) = f_x(x)f_y(y)$
    i. $4xy = 2x*2y = 4xy$, which means independent