

Due Wednesday, July 6, 2022 (11:59 pm)

Assigned problems

Problem 1: If 7 cards are dealt from an ordinary deck of 52 playing cards, what is the probability that

(a) exactly 2 of them will be face cards?

a. 0.3246

(b) at least 1 of them will be a queen?

a. 0.4496

Hypergeometric

$N = 52$

$R = 12$ for the 12 face cards (a), 4 for (b)

$n = 7$ (number of items sampled)

$X = 2$ (number of successes), 0 for no queens on b

$$X \sim H(52, 12, 7) = (\text{Comb}(12, 2) * \text{Comb}((52-12), (7-2))) / \text{Comb}(52, 7) = 66 * 658008 / 133784560 = 0.3246$$

$$X \sim H(52, 4, 7) = (\text{Comb}(4, 0) * \text{Comb}((52-4), (7-0))) / \text{Comb}(52, 7) = 1 * 73629072 / 133784560 = 0.55035$$

$$1 - (0.55035) = 0.4496 \text{ for at least 1 queen}$$

Problem 2: The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that a given student will pass the test

(a) on the third try;

a. 0.063

(b) before the fourth try.

a. 0.973

Negative Binomial

$P = 0.7$ (0.3 for non written part)

$X = 3$ (number up to and including the success)

$R = 1$ number of successes observed

$$X \sim NB(1, 0.7) = \text{Comb}((3-1), (1-1)) (0.7)^1 (1-0.7)^{(3-1)} = 1 * 0.7 * 0.09 = 0.063$$

(part b. pg 236 summary box)

$$P(X < 4) = p + (1-p)p + (1-p)^2(p) = 0.7 + 0.21 + 0.063 = 0.973$$

Problem 3: A certain type of thread is manufactured with a mean tensile strength of 78.3 kilograms and a standard deviation of 5.6 kilograms. How is the variance of the sample mean changed when the sample size is

(a) increased from 64 to 196?

a. **It decreases**

(b) decreased from 784 to 49?

a. **It increases**

Central Limit Theorem

(leads back to normal distribution, also be solved with $\text{variance}/\text{sq}(n)$)

a) it decreases

a. $5.6^2/64 = 0.49$

b. $5.6^2/196 = 0.16$

b) Increases

a. $5.6^2/784 = 0.04$

b. $5.6^2/49 = 0.64$

Problem 4: A random sample of size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3. Find the probability that the sample mean computed from the 25 measurements will exceed the sample mean computed from the 36 measurements by at least 3.4 but less than 5.9. Assume the difference of the means to be measured to the nearest tenth. **0.7146**

(central limit, ex 4.72 + normal distribution section)

$n = 25$

mean = 80

std. dev = 5

$n_2 = 36$

mean₂ = 75

std. dev₂ = 3

$P(3.4 \leq \text{mean difference} < 5.9) = (\text{use } z \text{ equation})$

(lower): $3.4 - (80 - 75) / \text{sq}((5^2/\text{sq}(25) + 3^2/\text{sq}(36))) = -1.6/1.118 = -1.431 \rightarrow 0.0764$

(upper): $5.9 - (5) / 1.118 = 0.9 / 1.118 = 0.805 \rightarrow 0.7910$

Mean difference = z between 3.4 and 5.9 which gives $0.7910 - 0.0764 = \mathbf{0.7146}$