In[1]:=
```
P[p_, p50_, b_] = (1 / 2) ^ ((p / p50) ^ b)
```

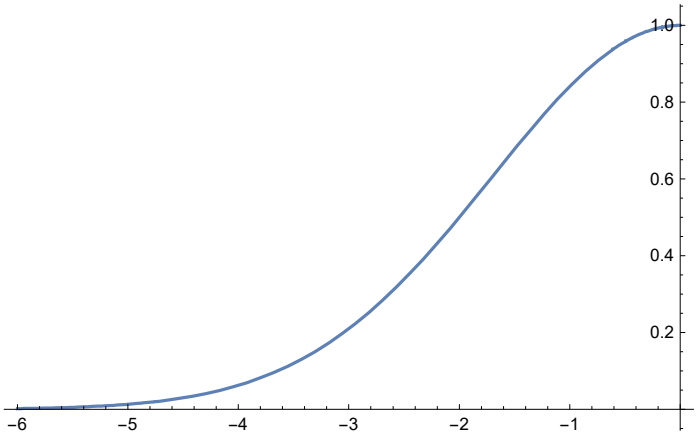
Out[1]=
$$2^{-\left(\frac{p}{p50}\right)^b}$$

In[2]:=
```
2^-(p/p50)^b
```

Out[2]=
$$2^{-\left(\frac{p}{p50}\right)^b}$$

In[3]:=
```
Plot[P[p, -2, 2], {p, -6, 0}]
```

Out[3]=



In[4]:=
```
Pp := Derivative[1, 0, 0][P]
```

In[5]:=
```
Pp[p, p50, b]
```

Out[5]=
$$-\frac{2^{-\left(\frac{p}{p50}\right)^b}\, b \left(\frac{p}{p50}\right)^{-1+b} \text{Log}[2]}{p50}$$

In[6]:=
```
Vc = (J / 4) (x ca + Km) / (x ca + 2 gstar)
```

Out[6]=
$$\frac{J\,(Km + ca\,x)}{4\,(2\,gstar + ca\,x)}$$

In[7]:=
```
Ac[x1_] := Vc (x1 ca - gstar) / (x1 ca + Km) - br Vc
```

In[8]:=
```
Aj[x1_] := (J / 4) (x1 ca - gstar) / (x1 ca + 2 gstar) - br Vc
```

In[9]:=
```
(*Solve[Ac[x]==Aj[x],Vc]*)
```

In[10]:=
```
Simplify[Aj[x]]
```

Out[10]=
$$-\frac{J\,(gstar - ca\,x + br\,(Km + ca\,x))}{8\,gstar + 4\,ca\,x}$$

In[11]:=
```
J[gs1_, x1_] := Solve[gs1 ca (1 - x1) == Aj[x1], J][[1, 1, 2]]
```

In[12]:=
```
(*J[gs1_,x1_]:= Solve[gs1 ca (1-x1)== Aj,J]
(*4gs1 ca (1-x1)(x1 ca+2gstar)/(x1 ca (1-br) - (gstar+br Km))*)*)
```

In[13]:=
```
J[gs, x]
```

Out[13]=
$$\frac{4\,\text{ca}\,\text{gs}\,(-1+\text{x})\,(2\,\text{gstar}+\text{ca}\,\text{x})}{\text{gstar}+\text{br}\,\text{Km}-\text{ca}\,\text{x}+\text{br}\,\text{ca}\,\text{x}}$$

In[14]:=
```
Jmax[j_] := 4 phi0 Iabs / Sqrt[(4 phi0 Iabs / j)^2 - 1]
```

In[15]:=
```
Jmax[J]
```

Out[15]=
$$\frac{4\,\text{Iabs}\,\text{phi0}}{\sqrt{-1+\frac{16\,\text{Iabs}^2\,\text{phi0}^2}{\text{J}^2}}}$$

In[16]:=
```
A[gs1_, x1_] := gs1 ca (1 - x1)
```

In[17]:=

In[18]:=
```
A[gs, x]
```

Out[18]=
$$\text{ca}\,\text{gs}\,(1-\text{x})$$

In[19]:=

In[20]:=
```
(* K here is K/eta*)
gs[dpsi_] = K * (Pox - Ppox * dpsi / 2) * dpsi
```

Out[20]=
$$\text{dpsi}\,\text{K}\left(\text{Pox}-\frac{\text{dpsi}\,\text{Ppox}}{2}\right)$$

In[21]:=
```
gs[dpsi] /. Pox → P[psoil, p50, b] /. Ppox → Pp[psoil, p50, b]
```

Out[21]=
$$\text{dpsi}\,\text{K}\left(2^{-\left(\frac{\text{psoil}}{\text{p50}}\right)^{\text{b}}}+\frac{1}{\text{p50}}2^{-1-\left(\frac{\text{psoil}}{\text{p50}}\right)^{\text{b}}}\,\text{b}\,\text{dpsi}\left(\frac{\text{psoil}}{\text{p50}}\right)^{-1+\text{b}}\text{Log}[2]\right)$$

In[22]:=
```
F[x_, dpsi_] := A[gs, x] - a Jmax[J] - y dpsi^2
```

In[23]:=
```
F[x, dpsi]
```

Out[23]=
$$-\frac{4\,\text{a}\,\text{Iabs}\,\text{phi0}}{\sqrt{-1+\frac{16\,\text{Iabs}^2\,\text{phi0}^2}{\text{J}^2}}}+\text{ca}\,\text{gs}\,(1-\text{x})-\text{dpsi}^2\,\text{y}$$

In[24]:=
```
F[x, dpsi] /. gs → gs[dpsi]
```

Out[24]=
$$-\frac{4\,\text{a}\,\text{Iabs}\,\text{phi0}}{\sqrt{-1+\frac{16\,\text{Iabs}^2\,\text{phi0}^2}{\text{J}^2}}}+\text{ca}\,\text{dpsi}\,\text{K}\left(\text{Pox}-\frac{\text{dpsi}\,\text{Ppox}}{2}\right)(1-\text{x})-\text{dpsi}^2\,\text{y}$$

In[25]:=
```
Fsubs[x_, dpsi_] := F[x, dpsi] /. gs → gs[dpsi] /. J → J[gs[dpsi], x]
```

In[26]:=
```
Fsubs[x, dpsi]
```
Out[26]=
$$ca\ dpsi\ K\left(Pox-\frac{dpsi\ Ppox}{2}\right)(1-x)-(4\ a\ Iabs\ phi0)\Big/\left(\sqrt{\left(-1+\left(4\ Iabs^2\ phi0^2\ (gstar+br\ Km-ca\ x+br\ ca\ x)^2\right)\Big/\left(ca^2\ dpsi^2\ K^2\ (-2\ Pox+dpsi\ Ppox)^2\ (-1+x)^2\ (2\ gstar+ca\ x)^2\right)\right)}\right)-dpsi^2\ y$$

In[27]:=
```
(*F1=FullSimplify[Derivative[1,0][Fsubs][x,dpsi]]*)
```

In[28]:=
```
(*F2=FullSimplify[Derivative[0,1][Fsubs][x,dpsi]]*)
```

In[29]:=
```
dAdx = Derivative[0, 1][A][gs, x]
```
Out[29]=
$$-ca\ gs$$

In[30]:=
```
dgsddpsi = Simplify[D[gs[dpsi], dpsi]]
```
Out[30]=
$$K\ (Pox-dpsi\ Ppox)$$

In[31]:=
```
dAddpsi = D[A[Gs[dpsi], x], dpsi]
```
Out[31]=
$$ca\ (1-x)\ Gs'[dpsi]$$

In[32]:=
```
dAddpsi /. Gs'[dpsi] → dgsddpsi
```
Out[32]=
$$ca\ K\ (Pox-dpsi\ Ppox)\ (1-x)$$

In[33]:=
```
dJmaxdJ = Derivative[1][Jmax]
```
Out[33]=
$$\frac{64\ Iabs^3\ phi0^3}{\left(-1+\frac{16\ Iabs^2\ phi0^2}{\#1^2}\right)^{3/2}\#1^3}\&$$

In[34]:=
```
dJdx = Simplify[D[J[gs, x], x]]
```
Out[34]=
$$\frac{4\ ca\ gs\left(2\ gstar\ (gstar+br\ Km)+(-1+br)\ ca^2\ x^2+br\ ca\ Km\ (-1+2\ x)+ca\ gstar\ (-3+2\ br+2\ x)\right)}{(gstar-ca\ x+br\ (Km+ca\ x))^2}$$

In[35]:=
```
Collect[Numerator[dJdx], {gs ca, br, gstar}] / Denominator[dJdx]
```
Out[35]=
$$\left(ca\ gs\left(8\ gstar^2-4\ ca^2\ x^2+4\ gstar\ (-3\ ca+2\ ca\ x)+br\left(4\ gstar\ (2\ ca+2\ Km)+4\left(ca^2\ x^2+ca\ Km\ (-1+2\ x)\right)\right)\right)\right)\Big/(gstar-ca\ x+br\ (Km+ca\ x))^2$$

In[36]:=
```
dJddpsi = D[J[Gs[dpsi], x], dpsi]
```
Out[36]=
$$\frac{4\ ca\ (-1+x)\ (2\ gstar+ca\ x)\ Gs'[dpsi]}{gstar+br\ Km-ca\ x+br\ ca\ x}$$

In[37]:=
```
dFdx = dAdx - a DJmaxdJ dJdx
```

Out[37]=
$$-\text{ca gs}-\left(4\,\text{a ca DJmaxdJ gs}\left(2\,\text{gstar}\,(\text{gstar}+\text{br Km})+(-1+\text{br})\,\text{ca}^2\,\text{x}^2+\text{br ca Km}\,(-1+2\,\text{x})+\text{ca gstar}\,(-3+2\,\text{br}+2\,\text{x})\right)\right)\Big/\left(\text{gstar}-\text{ca x}+\text{br}\,(\text{Km}+\text{ca x})\right)^2$$

In[38]:=
```
X1 = Solve[dFdx == 0, DJmaxdJ][[All, 1, 2]][[1]]
```

Out[38]=
$$-\left((\text{gstar}+\text{br Km}-\text{ca x}+\text{br ca x})^2\Big/\left(4\,\text{a}\left(-3\,\text{ca gstar}+2\,\text{br ca gstar}+2\,\text{gstar}^2-\text{br ca Km}+2\,\text{br gstar Km}+2\,\text{ca gstar x}+2\,\text{br ca Km x}-\text{ca}^2\,\text{x}^2+\text{br ca}^2\,\text{x}^2\right)\right)\right)$$

In[39]:=
```
dFddpsi = dAddpsi - a DJmaxdJ dJddpsi - 2 y dpsi
```

Out[39]=
$$-2\,\text{dpsi y}+\text{ca}\,(1-\text{x})\,\text{Gs}'[\text{dpsi}]-\frac{4\,\text{a ca DJmaxdJ}\,(-1+\text{x})\,(2\,\text{gstar}+\text{ca x})\,\text{Gs}'[\text{dpsi}]}{\text{gstar}+\text{br Km}-\text{ca x}+\text{br ca x}}$$

In[40]:=
```
X2 = Solve[dFddpsi == 0, DJmaxdJ][[All, 1, 2]][[1]]
```

Out[40]=
$$-\frac{(\text{gstar}+\text{br Km}-\text{ca x}+\text{br ca x})\,(2\,\text{dpsi y}-\text{ca Gs}'[\text{dpsi}]+\text{ca x Gs}'[\text{dpsi}])}{4\,\text{a ca}\,(-1+\text{x})\,(2\,\text{gstar}+\text{ca x})\,\text{Gs}'[\text{dpsi}]}$$

In[41]:=
```
Collect[Numerator[Simplify[1 / X1 - 1 / X2]], x, Simplify]
```

Out[41]=
$$4\,\text{a ca}^2\,\text{x}^2\,(-2\,(-1+\text{br})\,\text{dpsi y}+((-3+2\,\text{br})\,\text{gstar}-\text{br Km})\,\text{Gs}'[\text{dpsi}])+4\,\text{a}\left(2\,\text{dpsi}\,((3-2\,\text{br})\,\text{ca gstar}+\text{br ca Km}-2\,\text{gstar}\,(\text{gstar}+\text{br Km}))\,\text{y}+\text{ca}^2\,((-3+2\,\text{br})\,\text{gstar}-\text{br Km})\,\text{Gs}'[\text{dpsi}]\right)+8\,\text{a ca x}\,(-2\,\text{dpsi}\,(\text{gstar}+\text{br Km})\,\text{y}+\text{ca}\,((3-2\,\text{br})\,\text{gstar}+\text{br Km})\,\text{Gs}'[\text{dpsi}])$$

In[42]:=
```
(*Expand X2 by approximate formula for Gs*)
X2approx = X2 /. Gs'[dpsi] → dgsddpsi
```

Out[42]=
$$-\frac{(\text{gstar}+\text{br Km}-\text{ca x}+\text{br ca x})\,(-\text{ca K}\,(\text{Pox}-\text{dpsi Ppox})+\text{ca K}\,(\text{Pox}-\text{dpsi Ppox})\,\text{x}+2\,\text{dpsi y})}{4\,\text{a ca K}\,(\text{Pox}-\text{dpsi Ppox})\,(-1+\text{x})\,(2\,\text{gstar}+\text{ca x})}$$

In[43]:=
```
(* Express dpsi in terms of x -->
 Final eqn to solve for x. Approximation of Gs required*)
dpsiasfx = Simplify[Solve[X1 == X2approx, dpsi][[All, 1, 2]][[1]]]
```

Out[43]=
$$\left(\text{ca}^2\,\text{K}\,((-3+2\,\text{br})\,\text{gstar}-\text{br Km})\,\text{Pox}\,(-1+\text{x})^2\right)\Big/\left(4\,\text{gstar}\,(\text{gstar}+\text{br Km})\,\text{y}+2\,\text{ca}\,(\text{br Km}\,(-1+2\,\text{x})+\text{gstar}\,(-3+2\,\text{br}+2\,\text{x}))\,\text{y}+\text{ca}^2\left((-3+2\,\text{br})\,\text{gstar K Ppox}\,(-1+\text{x})^2-2\,\text{x}^2\,\text{y}-\text{br}\left(\text{K Km Ppox}\,(-1+\text{x})^2-2\,\text{x}^2\,\text{y}\right)\right)\right)$$

In[44]:=
```
Collect[Numerator[dpsiasfx], {br, Pox, PPox, ca, K, x}] /
 Collect[Denominator[dpsiasfx], {br, Pox, Ppox, ca, K, x}]
```

Out[44]=
$$\left(ca^2\,K\,Pox\left(-3\,gstar+6\,gstar\,x-3\,gstar\,x^2\right)+br\,ca^2\,K\,Pox\left(2\,gstar-Km+(-4\,gstar+2\,Km)\,x+(2\,gstar-Km)\,x^2\right)\right)\Big/\left(ca^2\,K\,Ppox\left(-3\,gstar+6\,gstar\,x-3\,gstar\,x^2\right)+4\,gstar^2\,y-2\,ca^2\,x^2\,y+ca\,(-6\,gstar\,y+4\,gstar\,x\,y)+br\left(ca^2\,K\,Ppox\left(2\,gstar-Km+(-4\,gstar+2\,Km)\,x+(2\,gstar-Km)\,x^2\right)+4\,gstar\,Km\,y+2\,ca^2\,x^2\,y+ca\,(4\,gstar\,y-2\,Km\,y+4\,Km\,x\,y)\right)\right)$$

In[45]:=
```
(*Express x in terms of dpsi -->
 Final eqn to solve for dpsi. No approximation required,
but dpsi bound needs to be calc*)
xasfdpsiAll = Simplify[Solve[X1 == X2, x][[All, 1, 2]]]
```

Out[45]=
$$\left\{\frac{gstar+br\,Km}{ca-br\,ca},\ \left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]-\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/\left(ca^2\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi])\right),\ \left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/\left(ca^2\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi])\right)\right\}$$

In[46]:=
```
Qdelta =
 ca^2 * dpsi * ((-3 + 2 * br) * gstar - br * Km) * ((-1 + br) * ca + gstar + br * Km) * y *
   (-2 * dpsi * y + (ca + 2 * gstar) * Derivative[1][Gs][dpsi])
```

Out[46]=
$$ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])$$

In[47]:=
```
Factor[Qdelta]
```

Out[47]=
$$ca^2\,dpsi\,(-3\,gstar+2\,br\,gstar-br\,Km)\,(-ca+br\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+ca\,Gs'[dpsi]+2\,gstar\,Gs'[dpsi])$$

In[48]:=
```
(* When is determinant > 0? When dpsi < some value. Lets calculate that value*)
Collect[(-2 dpsi y + (ca + 2 gstar) Gs'[dpsi]) /. Gs'[dpsi] → dgsddpsi, dpsi]
```

Out[48]=
$$(ca+2\,gstar)\,K\,Pox+dpsi\,((-ca-2\,gstar)\,K\,Ppox-2\,y)$$

In[49]:=
```
(* dpsi bound is given as below *)
Solve[(Simplify[Qdelta] /. Gs'[dpsi] → dgsddpsi) == 0, dpsi]
```

Out[49]=
$$\left\{\{\text{dpsi} \to 0\}, \left\{\text{dpsi} \to \frac{\text{ca K Pox} + 2\,\text{gstar K Pox}}{\text{ca K Ppox} + 2\,\text{gstar K Ppox} + 2\,\text{y}}\right\}\right\}$$

In[50]:=
```
xasfdpsi = xasfdpsiAll[[3]]
```

Out[50]=
$$\Big(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar} + \text{br}\,\text{Km})\,\text{y} + \text{ca}^2\,((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}] + \sqrt{2}\,\sqrt{\Big(\text{ca}^2\,\text{dpsi}\,((-3 + 2\,\text{br})\,\text{gstar} - \text{br}\,\text{Km})\,((-1 + \text{br})\,\text{ca} + \text{gstar} + \text{br}\,\text{Km})\,\text{y}\,(-2\,\text{dpsi}\,\text{y} + (\text{ca} + 2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\Big)}\Big) \Big/ \Big(\text{ca}^2\,(2\,(-1 + \text{br})\,\text{dpsi}\,\text{y} + ((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}])\Big)$$

In[51]:=
```
Collect[Numerator[xasfdpsi], {br, ca, dpsi}] /
 Collect[Denominator[xasfdpsi], {br, ca, dpsi}]
```

Out[51]=
$$\Big(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,\text{y} + 3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}] + \sqrt{2}\,\sqrt{\Big(\text{ca}^2\,\text{dpsi}\,((-3 + 2\,\text{br})\,\text{gstar} - \text{br}\,\text{Km})\,((-1 + \text{br})\,\text{ca} + \text{gstar} + \text{br}\,\text{Km})\,\text{y}\,(-2\,\text{dpsi}\,\text{y} + (\text{ca} + 2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\Big)} + \text{br}\,\Big(-2\,\text{ca}\,\text{dpsi}\,\text{Km}\,\text{y} + \text{ca}^2\,(-2\,\text{gstar}\,\text{Gs}'[\text{dpsi}] + \text{Km}\,\text{Gs}'[\text{dpsi}])\Big)\Big) \Big/ \Big(\text{ca}^2\,(-2\,\text{dpsi}\,\text{y} + 3\,\text{gstar}\,\text{Gs}'[\text{dpsi}]) + \text{br}\,\text{ca}^2\,(2\,\text{dpsi}\,\text{y} - 2\,\text{gstar}\,\text{Gs}'[\text{dpsi}] + \text{Km}\,\text{Gs}'[\text{dpsi}])\Big)$$

In[52]:=
```
Rx = dFdx /. DJmaxdJ → dJmaxdJ[J[gs, x]] /. gs → gs[dpsi] /. x → xasfdpsi
```

Out[52]=
$$-\text{ca}\,\text{dpsi}\,\text{K}\left(\text{Pox} - \frac{\text{dpsi}\,\text{Ppox}}{2}\right) - \Big(4\,\text{a}\,\text{Iabs}^3\,\text{phi0}^3\,\Big(\text{gstar} + \text{br}\,\text{Km} - \Big(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar} + \text{br}\,\text{Km})\,\text{y} + \text{ca}^2\,((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}] + \sqrt{2}\,\sqrt{\Big(\text{ca}^2\,\text{dpsi}\,((-3 + 2\,\text{br})\,\text{gstar} - \text{br}\,\text{Km})\,((-1 + \text{br})\,\text{ca} + \text{gstar} + \text{br}\,\text{Km})\,\text{y}\,(-2\,\text{dpsi}\,\text{y} + (\text{ca} + 2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\Big)}\Big) \Big/ (\text{ca}\,(2\,(-1 + \text{br})\,\text{dpsi}\,\text{y} + ((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}])) + \Big(\text{br}\,\Big(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar} + \text{br}\,\text{Km})\,\text{y} + \text{ca}^2\,((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}] + \sqrt{2}\,\sqrt{\Big(\text{ca}^2\,\text{dpsi}\,((-3 + 2\,\text{br})\,\text{gstar} - \text{br}\,\text{Km})\,((-1 + \text{br})\,\text{ca} + \text{gstar} + \text{br}\,\text{Km})\,\text{y}\,(-2\,\text{dpsi}\,\text{y} + (\text{ca} + 2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\Big)}\Big)\Big) \Big/ (\text{ca}\,(2\,(-1 + \text{br})\,\text{dpsi}\,\text{y} + ((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]))\Big)^3 \Big(2\,\text{gstar}\,(\text{gstar} + \text{br}\,\text{Km}) + \Big((-1 + \text{br})\,\Big(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar} + \text{br}\,\text{Km})\,\text{y} + \text{ca}^2\,((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}] + \sqrt{2}\,\sqrt{\Big(\text{ca}^2\,\text{dpsi}\,((-3 + 2\,\text{br})\,\text{gstar} - \text{br}\,\text{Km})\,((-1 + \text{br})\,\text{ca} + \text{gstar} + \text{br}\,\text{Km})\,\text{y}\,(-2\,\text{dpsi}\,\text{y} + (\text{ca} + 2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\Big)}\Big)^2\Big) \Big/ \Big(\text{ca}^2\,(2\,(-1 + \text{br})\,\text{dpsi}\,\text{y} + ((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}])^2\Big) + \text{br}\,\text{ca}\,\text{Km}\,\Big(-1 + \Big(2\,\Big(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar} + \text{br}\,\text{Km})\,\text{y} + \text{ca}^2\,((3 - 2\,\text{br})\,\text{gstar} + \text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}] +$$

$$\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\,\left(\left(-1+br\right)\,ca+gstar+br\,Km\right)\,y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)}\Big)\Big)\Big/$$
$$\left(ca^2\,\left(2\,\left(-1+br\right)\,dpsi\,y+\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]\right)\right)\Big)+$$
$$ca\,gstar\left(-3+2\,br+\left(2\left(-2\,ca\,dpsi\,\left(gstar+br\,Km\right)\,y+ca^2\,\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\right.\right.\right.$$
$$Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\right.}$$
$$\left.\left.\left.\left.\left(\left(-1+br\right)\,ca+gstar+br\,Km\right)\,y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)\right)\right)\Big/$$
$$\left(ca^2\,\left(2\,\left(-1+br\right)\,dpsi\,y+\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]\right)\right)\Big)\Big)\Big)\Big/$$
$$\left(ca^2\,dpsi^2\,K^2\left(Pox-\frac{dpsi\,Ppox}{2}\right)^2\left(-1+\left(-2\,ca\,dpsi\,\left(gstar+br\,Km\right)\,y+\right.\right.\right.$$
$$ca^2\,\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\right.}$$
$$\left.\left.\left.\left(\left(-1+br\right)\,ca+gstar+br\,Km\right)\,y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)\right)\Big/$$
$$\left.\left(ca^2\,\left(2\,\left(-1+br\right)\,dpsi\,y+\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]\right)\right)\right)^3$$
$$\left(2\,gstar+\left(-2\,ca\,dpsi\,\left(gstar+br\,Km\right)\,y+ca^2\,\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]+\right.\right.$$
$$\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\,\left(\left(-1+br\right)\,ca+gstar+br\,Km\right)\right.}$$
$$\left.\left.\left.y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)\right)\Big/$$
$$\left.\left(ca\,\left(2\,\left(-1+br\right)\,dpsi\,y+\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]\right)\right)\right)^3$$
$$\left(gstar-\left(-2\,ca\,dpsi\,\left(gstar+br\,Km\right)\,y+ca^2\,\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]+\right.\right.$$
$$\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\,\left(\left(-1+br\right)\,ca+gstar+br\,Km\right)\right.}$$
$$\left.\left.\left.y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)\right)\Big/$$
$$\left(ca\,\left(2\,\left(-1+br\right)\,dpsi\,y+\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]\right)\right)+$$
$$br\left(Km+\left(-2\,ca\,dpsi\,\left(gstar+br\,Km\right)\,y+ca^2\,\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]+\right.\right.$$
$$\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\,\left(\left(-1+br\right)\,ca+gstar+br\,Km\right)\right.}$$
$$\left.\left.\left.y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)\right)\Big/$$
$$\left.\left.\left(ca\,\left(2\,\left(-1+br\right)\,dpsi\,y+\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]\right)\right)\right)^2$$
$$\left(-1+\left(Iabs^2\,phi0^2\left(gstar+br\,Km-\left(-2\,ca\,dpsi\,\left(gstar+br\,Km\right)\,y+ca^2\,\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\right.\right.\right.\right.$$
$$Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\,\left(\left(-1+br\right)\,ca+\right.\right.}$$
$$\left.\left.\left.\left.\left.gstar+br\,Km\right)\,y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)\right)\Big/$$
$$\left(ca\,\left(2\,\left(-1+br\right)\,dpsi\,y+\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]\right)\right)+$$
$$\left(br\left(-2\,ca\,dpsi\,\left(gstar+br\,Km\right)\,y+ca^2\,\left(\left(3-2\,br\right)\,gstar+br\,Km\right)\,Gs'[dpsi]+\right.\right.$$
$$\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,\left(\left(-3+2\,br\right)\,gstar-br\,Km\right)\,\left(\left(-1+br\right)\,ca+gstar+br\,Km\right)\right.}$$
$$\left.\left.\left.y\,\left(-2\,dpsi\,y+\left(ca+2\,gstar\right)\,Gs'[dpsi]\right)\right)\right)\Big/$$

$$\left(\text{ca}\,(2\,(-1+\text{br})\,\text{dpsi}\,\text{y}+((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}])\right)^2\Big)\Big/\Big(\text{ca}^2$$

$$\text{dpsi}^2\,\text{K}^2\left(\text{Pox}-\frac{\text{dpsi}\,\text{Ppox}}{2}\right)^2\left(-1+\left(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar}+\text{br}\,\text{Km})\,\text{y}+\text{ca}^2\,((3-2\,\text{br})\,\text{gstar}+\right.\right.$$

$$\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]+\sqrt{2}\,\sqrt{\left(\text{ca}^2\,\text{dpsi}\,((-3+2\,\text{br})\,\text{gstar}-\text{br}\,\text{Km})\,((-1+\text{br})\right.}$$

$$\left.\left.\text{ca}+\text{gstar}+\text{br}\,\text{Km})\,\text{y}\,(-2\,\text{dpsi}\,\text{y}+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\right)\right)\Big/$$

$$\left(\text{ca}^2\,(2\,(-1+\text{br})\,\text{dpsi}\,\text{y}+((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}])\right)^2$$

$$\left(2\,\text{gstar}+\left(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar}+\text{br}\,\text{Km})\,\text{y}+\text{ca}^2\,((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]+\right.\right.$$

$$\sqrt{2}\,\sqrt{\left(\text{ca}^2\,\text{dpsi}\,((-3+2\,\text{br})\,\text{gstar}-\text{br}\,\text{Km})\,((-1+\text{br})\,\text{ca}+\text{gstar}+\text{br}\,\text{Km})\right.}$$

$$\left.\left.\text{y}\,(-2\,\text{dpsi}\,\text{y}+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\right)\right)\Big/$$

$$\left.\left.\left(\text{ca}\,(2\,(-1+\text{br})\,\text{dpsi}\,\text{y}+((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}])\right)^2\right)\right)^{3/2}\Bigg)$$

In[53]:= **Rx /. br → 0**

Out[53]=

$$-\text{ca}\,\text{dpsi}\,\text{K}\left(\text{Pox}-\frac{\text{dpsi}\,\text{Ppox}}{2}\right)-$$

$$\left(4\,\text{a}\,\text{Iabs}^3\,\text{phi0}^3\left(\text{gstar}-\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,\text{y}+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\sqrt{6}\right.\right.\right.$$

$$\left.\left.\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,\text{y}\,(-2\,\text{dpsi}\,\text{y}+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)\Big/$$

$$\left.(\text{ca}\,(-2\,\text{dpsi}\,\text{y}+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}]))\right)$$

$$\left(2\,\text{gstar}^2-\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,\text{y}+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\right.\right.$$

$$\left.\sqrt{6}\,\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,\text{y}\,(-2\,\text{dpsi}\,\text{y}+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)^2\Big/$$

$$\left(\text{ca}^2\,(-2\,\text{dpsi}\,\text{y}+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}])^2\right)+\text{ca}\,\text{gstar}$$

$$\left(-3+\left(2\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,\text{y}+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\right.\right.\right.$$

$$\left.\left.\sqrt{6}\,\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,\text{y}\,(-2\,\text{dpsi}\,\text{y}+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)\Big/\right.$$

$$\left.\left.\left.\left(\text{ca}^2\,(-2\,\text{dpsi}\,\text{y}+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}])\right)\right)\right)\Big/$$

$$\left(\text{ca}^2\,\text{dpsi}^2\,\text{K}^2\left(\text{Pox}-\frac{\text{dpsi}\,\text{Ppox}}{2}\right)^2\left(-1+\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,\text{y}+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\right.\right.\right.$$

$$\left.\left.\sqrt{6}\,\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,\text{y}\,(-2\,\text{dpsi}\,\text{y}+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)\Big/\right.$$

$$\left.\left.\left.\left(\text{ca}^2\,(-2\,\text{dpsi}\,\text{y}+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}])\right)\right)\right)^3$$

$$\left(2\,\text{gstar}+\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,y+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\sqrt{6}\,\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,y\,(-2\,\text{dpsi}\,y+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)\Big/(\text{ca}\,(-2\,\text{dpsi}\,y+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}]))\right)^3$$

$$\left(-1+\left(\text{Iabs}^2\,\text{phi0}^2\left(\text{gstar}-\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,y+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\sqrt{6}\,\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,y\,(-2\,\text{dpsi}\,y+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)\Big/(\text{ca}\,(-2\,\text{dpsi}\,y+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}]))\right)^2\right)\Big/\left(\text{ca}^2\,\text{dpsi}^2\,\text{K}^2\left(\text{Pox}-\frac{\text{dpsi}\,\text{Ppox}}{2}\right)^2\left(-1+\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,y+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\sqrt{6}\,\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,y\,(-2\,\text{dpsi}\,y+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)\Big/(\text{ca}^2\,(-2\,\text{dpsi}\,y+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}]))\right)^2\left(2\,\text{gstar}+\left(-2\,\text{ca}\,\text{dpsi}\,\text{gstar}\,y+3\,\text{ca}^2\,\text{gstar}\,\text{Gs}'[\text{dpsi}]+\sqrt{6}\,\sqrt{-\text{ca}^2\,\text{dpsi}\,\text{gstar}\,(-\text{ca}+\text{gstar})\,y\,(-2\,\text{dpsi}\,y+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])}\right)\Big/(\text{ca}\,(-2\,\text{dpsi}\,y+3\,\text{gstar}\,\text{Gs}'[\text{dpsi}]))\right)^2\right)^{3/2}\right)$$

In[54]:=
```
Rdpsi = dFddpsi /. DJmaxdJ → dJmaxdJ[J[gs, x]] /. gs → gs[dpsi] /. x → xasfdpsi
```

Out[54]=

$$-2\,\text{dpsi}\,y+\text{ca}\,\text{Gs}'[\text{dpsi}]\left(1-\left(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar}+\text{br}\,\text{Km})\,y+\text{ca}^2\,((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]+\sqrt{2}\,\sqrt{\left(\text{ca}^2\,\text{dpsi}\,((-3+2\,\text{br})\,\text{gstar}-\text{br}\,\text{Km})\,((-1+\text{br})\,\text{ca}+\text{gstar}+\text{br}\,\text{Km})\,y\,(-2\,\text{dpsi}\,y+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\right)}\right)\Big/\left(\text{ca}^2\,(2\,(-1+\text{br})\,\text{dpsi}\,y+((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}])\right)\right)-\left(4\,\text{a}\,\text{Iabs}^3\,\text{phi0}^3\,\text{Gs}'[\text{dpsi}]\left(\text{gstar}+\text{br}\,\text{Km}-\left(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar}+\text{br}\,\text{Km})\,y+\text{ca}^2\,((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]+\sqrt{2}\,\sqrt{\left(\text{ca}^2\,\text{dpsi}\,((-3+2\,\text{br})\,\text{gstar}-\text{br}\,\text{Km})\,((-1+\text{br})\,\text{ca}+\text{gstar}+\text{br}\,\text{Km})\,y\,(-2\,\text{dpsi}\,y+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\right)}\right)\Big/(\text{ca}\,(2\,(-1+\text{br})\,\text{dpsi}\,y+((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]))+\left(\text{br}\left(-2\,\text{ca}\,\text{dpsi}\,(\text{gstar}+\text{br}\,\text{Km})\,y+\text{ca}^2\,((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]+\sqrt{2}\,\sqrt{\left(\text{ca}^2\,\text{dpsi}\,((-3+2\,\text{br})\,\text{gstar}-\text{br}\,\text{Km})\,((-1+\text{br})\,\text{ca}+\text{gstar}+\text{br}\,\text{Km})\,y\,(-2\,\text{dpsi}\,y+(\text{ca}+2\,\text{gstar})\,\text{Gs}'[\text{dpsi}])\right)}\right)\Big/(\text{ca}\,(2\,(-1+\text{br})\,\text{dpsi}\,y+((3-2\,\text{br})\,\text{gstar}+\text{br}\,\text{Km})\,\text{Gs}'[\text{dpsi}]))^2\right)\Big/$$

$$\left(ca^2\,dpsi^3\,K^3\left(Pox-\frac{dpsi\,Ppox}{2}\right)^3\left(-1+\left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/\left(ca^2\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi])\right)\right)^2\left(2\,gstar+\left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/(ca\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]))\right)^2\left(-1+\left(Iabs^2\,phi0^2\left(gstar+br\,Km-\left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/(ca\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]))+\left(br\left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/(ca\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]))\right)^2\right)\Big/\left(ca^2\,dpsi^2\,K^2\left(Pox-\frac{dpsi\,Ppox}{2}\right)^2\left(-1+\left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/\left(ca^2\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi])\right)\right)^2\left(2\,gstar+\left(-2\,ca\,dpsi\,(gstar+br\,Km)\,y+ca^2\,((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]+\sqrt{2}\,\sqrt{\left(ca^2\,dpsi\,((-3+2\,br)\,gstar-br\,Km)\,((-1+br)\,ca+gstar+br\,Km)\,y\,(-2\,dpsi\,y+(ca+2\,gstar)\,Gs'[dpsi])\right)}\right)\Big/(ca\,(2\,(-1+br)\,dpsi\,y+((3-2\,br)\,gstar+br\,Km)\,Gs'[dpsi]))\right)^2\right)^{3/2}\right)$$

In[55]:=