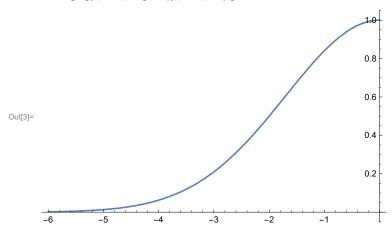
$$ln[1]:= P[p_, p50_, b_] = (1/2)^((p/p50)^b)$$

Out[1]= 
$$2^{-\left(\frac{p}{p50}\right)^b}$$

In[2]:= 
$$2^{-\left(\frac{p}{p50}\right)^b}$$

Out[2]= 
$$2^{-\left(\frac{p}{p50}\right)^b}$$



In[4]:= Pp := Derivative[1, 0, 0][P]

In[5]:= **Pp[p, p50, b]** 

Out[5]= 
$$-\frac{2^{-\left(\frac{p}{p5\theta}\right)^b}b\left(\frac{p}{p5\theta}\right)^{-1+b}Log[2]}{p50}$$

$$ln[6]:=$$
 Ajm[gs1\_, x1\_] := gs1 ca (1 - x1) (x1 ca + 2 gstar) / (x1 ca (1 - br) - (gstar + br Km))

In[7]:= Ajm[gs, x]

$$\label{eq:outprob} \mbox{Outproblem} \begin{array}{l} \mbox{Ca gs } (1-x) & (2 \mbox{ gstar} + \mbox{ca} \ x) \\ \mbox{-gstar} - \mbox{br Km} + (1-\mbox{br}) \mbox{ ca} \ x \end{array}$$

In[9]:= Jmax[Ajm]

$$ln[10] = A[gs1_, x1_] := gs1 ca (1 - x1)$$

In[11]:=

$$ln[12] = A[gs, x]$$

Out[12]= 
$$cags(1-x)$$

In[13]:=

$$\text{Out} [14] = \text{ dpsi K } \left( \text{Pox} - \frac{\text{dpsi Ppox}}{2} \right)$$

$$ln[15]:=$$
 gs[dpsi] /. Pox  $\rightarrow$  P[psoil, p50, b] /. Ppox  $\rightarrow$  Pp[psoil, p50, b]

$$\text{Out[15]= dpsik} \left( 2^{-\left(\frac{psoil}{p50}\right)^b} + \frac{2^{-1-\left(\frac{psoil}{p50}\right)^b} \text{ b dpsi} \left(\frac{psoil}{p50}\right)^{-1+b} \text{Log}\left[2\right]}{p50} \right)$$

$$ln[16]:= F[x_, dpsi_] := A[gs, x] - a Jmax[mj] - y dpsi^2$$

In[17]:= **F**[x, dpsi]

$$\text{Out[17]= } - \frac{\text{4 a labs phi0}}{\sqrt{-1 + \frac{\text{labs}^2 \, \text{phi0}^2}{\text{mj}^2}}} + \text{cags } (1-x) - \text{dpsi}^2 \, y$$

$$ln[18]:= F[x, dpsi] /. gs \rightarrow gs[dpsi]$$

$$\text{Out[18]= } - \frac{\text{4 a Iabs phi0}}{\sqrt{-1 + \frac{\text{Iabs}^2 \, phi0^2}{\text{mj}^2}}} + \text{ca dpsi K} \left( \text{Pox} - \frac{\text{dpsi Ppox}}{2} \right) \, \left( \text{1-x} \right) \, - \text{dpsi}^2 \, y$$

$$ln[19] = Fsubs[x_, dpsi_] := F[x, dpsi_] /. gs \rightarrow gs[dpsi_] /. mj \rightarrow Ajm[gs[dpsi_], x]$$

In[20]:=

Fsubs[x, dpsi]

$$\text{Out} [20] = \text{ ca dpsi K } \left( Pox - \frac{\text{dpsi Ppox}}{2} \right) \ (1-x) \ - \frac{\text{4 a Iabs phi0}}{\sqrt{-1 + \frac{\text{Iabs}^2 \, phi0^2 \, (-gstar-br \, Km + \, (1-br) \, ca \, x)^2}{\text{ca}^2 \, dpsi^2 \, K^2 \, \left( Pox - \frac{dpsi \, Ppox}{2} \right)^2 \, \left( 1-x \right)^2 \, \left( 2 \, gstar + ca \, x \right)^2}} \ - \, dpsi^2 \, y$$

In[21]:=

In[22]:=

Out[23]= 
$$-cags$$

```
In[24]:= dgsddpsi = Simplify[D[gs[dpsi], dpsi]]
Out[24]= K (Pox - dpsi Ppox)
In[25]:= dAddpsi = D[A[Gs[dpsi], x], dpsi]
Out[25]= ca (1 - x) Gs'[dpsi]
In[26]:= dAddpsi /. Gs'[dpsi] → dgsddpsi
Out[26]= ca K (Pox - dpsi Ppox) (1 - x)
In[27]:= dJmaxdAjm = Derivative[1][Jmax]
\text{Out[27]=} \  \  \frac{ 4 \ \text{Iabs}^3 \ \text{phi0}^3 }{ \left( -1 + \frac{\text{Iabs}^2 \ \text{phi0}^2}{\text{m1}^2} \right)^{3/2} \ \text{m1}^3 } \ \& 
In[28]:= dAjmdx = Simplify[D[Ajm[gs, x], x]]
       Out[28]=
                                            (gstar - ca x + br (Km + ca x))^2
In[29]:= Collect[Numerator[dAjmdx], {gs ca, br, gstar}] / Denominator[dAjmdx]
       Out[29]=
                                              (gstar - ca x + br (Km + ca x))^2
In[30]:= dAjmddpsi = D[Ajm[Gs[dpsi], x], dpsi]
       \frac{\mathsf{ca}\ (1-\mathsf{x})\ (2\,\mathsf{gstar}+\mathsf{ca}\,\mathsf{x})\ \mathsf{Gs'}\,[\mathsf{dpsi}]}{-\mathsf{gstar}-\mathsf{br}\,\mathsf{Km}+\,(1-\mathsf{br})\ \mathsf{ca}\,\mathsf{x}}
Out[30]=
In[31]:= dFdx = dAdx - a DJmaxdAjm dAjmdx
Out[31]= - ca gs - (a ca DJmaxdAjm gs
            (2 gstar (gstar + br Km) + (-1 + br) ca^2 x^2 + br ca Km (-1 + 2 x) + ca gstar (-3 + 2 br + 2 x)))
          (gstar - ca x + br (Km + ca x))^2
ln[32] = X1 = Solve[dFdx == 0, DJmaxdAjm][[All, 1, 2]][[1]]
Out[32]= - ( (gstar + br Km - ca x + br ca x) ^2 /
           (a (-3 ca gstar + 2 br ca gstar + 2 gstar<sup>2</sup> - br ca Km + 2 br gstar Km +
                2 ca gstar x + 2 br ca Km x - ca^2 x^2 + br ca^2 x^2)))
In[33]:= dFddpsi = dAddpsi - a DJmaxdAjm dAjmddpsi - 2 y dpsi
Out[33]= -2 dpsiy + ca (1 - x) Gs'[dpsi] - 

a ca DJmaxdAjm (1 - x) (2 gstar + ca x) Gs'[dpsi]
                                                        -gstar - br Km + (1 - br) ca x
```

```
In[34]:= X2 = Solve[dFddpsi == 0, DJmaxdAjm][[All, 1, 2]][[1]]
                                             (gstar + br Km - ca x + br ca x) (2 dpsi y - ca Gs'[dpsi] + ca x Gs'[dpsi])
Out[34]= -
                                                                                                                                   a ca (-1 + x) (2 gstar + ca x) Gs' [dpsi]
    In[35]:= Collect[Numerator[Simplify[1/X1-1/X2]], x, Simplify]
Out[35]= a ca^2 x^2 (-2 (-1 + br) dpsi y + ((-3 + 2 br) gstar - br Km) Gs' [dpsi]) +
                                         a (2 dpsi ((3 - 2 br) ca gstar + br ca Km - 2 gstar (gstar + br Km)) y +
                                                            ca^2((-3+2 br) gstar - br Km) Gs'[dpsi]) +
                                         2 \text{ a ca x } (-2 \text{ dpsi } (\text{gstar} + \text{br Km}) \text{ y} + \text{ca} ((3-2 \text{ br}) \text{ gstar} + \text{br Km}) \text{ Gs}'[\text{dpsi}])
   ln[36] = Simplify[Solve[X1 = X2, x]]
Out[36]= \left\{ \left\{ x \rightarrow \frac{\text{gstar} + \text{br Km}}{\text{ca} - \text{br ca}} \right\} \right\}
                                      \left\{ x \rightarrow \left( -2 \; \text{ca dpsi (gstar + br Km)} \; \; y + \text{ca}^2 \; (\; (3-2 \; \text{br}) \; \; \text{gstar + br Km}) \; \; \text{Gs'} \left[ \; \text{dpsi} \right] \right. \\ \left. - \sqrt{2} \; \text{ca dpsi (gstar + br Km)} \; \; \text{dpsi} \left[ \; \text{dpsi} \right] \right\} \\ \left. - \sqrt{2} \; \text{ca dpsi (gstar + br Km)} \; \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{ca dpsi (gstar + br Km)} \; \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{ca dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{dpsi} \left[ \; \text{dpsi} \left[ \; \text{dpsi} \right] \right] \right] \\ \left. - \sqrt{2} \; \text{dpsi} \left[ \; \text{d
                                                                                 \sqrt{\left(\text{ca}^2\,\text{dpsi}\,\left(\left(-3+2\,\text{br}\right)\,\,\text{gstar}-\text{br}\,\text{Km}\right)\,\left(\left(-1+\text{br}\right)\,\,\text{ca}+\text{gstar}+\text{br}\,\text{Km}\right)}
                                                                                                   y (-2 dpsi y + (ca + 2 gstar) Gs'[dpsi])) /
                                                             \left( \text{ca}^{2} \ (\text{2 (-1+br) dpsi} \ y + \ ( \ (\text{3-2br}) \ \text{gstar+br} \ \text{Km}) \ \text{Gs'} [\text{dpsi}] \ ) \ \right) \right) \text{,}
                                       \left\{ x \rightarrow \left( -\, 2\; \text{ca}\; \text{dpsi}\; \left(\, \text{gstar} +\, \text{br}\; \text{Km}\right)\; y \,+\, \text{ca}^2\; \left(\, \left(\, 3\,-\, 2\; \text{br}\right)\; \text{gstar} \,+\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \right\} \right\} = \left\{ -\, 2\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; \text{Gs'}\left[\, \text{dpsi}\, \right]\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, \text{gstar}\, +\, \text{br}\; \text{Km}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Km}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Km}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \text{br}\; \text{Cm}\right)\; +\, \left(\, \left(\, 3\,-\, 2\, \text{br}\right)\, +\, \left(\, 3\,-\, 2\, \text{br}\right)\; +\, \left(\, 3
                                                                          \sqrt{2} \sqrt{\left(\text{ca}^2\,\text{dpsi}\,\left(\left(-3+2\,\text{br}\right)\,\text{gstar}-\text{br}\,\text{Km}\right)\,\left(\left(-1+\text{br}\right)\,\text{ca}+\text{gstar}+\text{br}\,\text{Km}\right)}
                                                                                                   y (-2 dpsi y + (ca + 2 gstar) Gs'[dpsi]))/
                                                             \left(ca^{2}\left(2\left(-1+br\right)dpsiy+\left((3-2br)gstar+brKm\right)Gs'[dpsi]\right)\right)\right\}
    In[37]:= (*Expand X2 by approximate formula for Gs*)
                                X2approx = X2 /. Gs'[dpsi] → dgsddpsi
                                             (gstar + br Km - ca x + br ca x) (-ca K (Pox - dpsi Ppox) + ca K (Pox - dpsi Ppox) x + 2 dpsi y)
Out[37]= -
                                                                                                                                                                a ca K (Pox - dpsi Ppox) (-1 + x) (2 gstar + ca x)
    In[38]:= (* Express dpsi in terms of x -->
                                        Final eqn to solve for x. Approximation of Gs required*)
                                 dpsiasfx = Simplify[Solve[X1 == X2approx, dpsi][[All, 1, 2]][[1]]]
Out[38]= (ca^2 K ((-3 + 2 br) gstar - br Km) Pox (-1 + x)^2)
                                         (4 gstar (gstar + br Km) y + 2 ca (br Km (-1 + 2 x) + gstar (-3 + 2 br + 2 x)) y +
                                                     ca^{2} ( (-3 + 2 br) gstar K Ppox (-1 + x)^{2} - 2 x^{2} y - br (K Km Ppox (-1 + x)^{2} - 2 x^{2} y)))
```

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In[39]:= Collect[Numerator[dpsiasfx], {br, Pox, PPox, ca, K, x}] /
                        Collect[Denominator[dpsiasfx], {br, Pox, Ppox, ca, K, x}]
 Out[39]= (ca^2 \text{ K Pox } (-3 \text{ gstar} + 6 \text{ gstar } x - 3 \text{ gstar } x^2) +
                               br\;ca^2\;K\;Pox\;\left(\,2\;gstar\,-\,Km\,+\,\left(\,-\,4\;gstar\,+\,2\,Km\right)\;x\,+\,\left(\,2\;gstar\,-\,Km\right)\;x^2\,\right)\,\left/\,\right.
                         (ca^2 K Ppox (-3 gstar + 6 gstar x - 3 gstar x^2) + 4 gstar^2 y -
                                2 ca^2 x^2 y + ca (-6 gstar y + 4 gstar x y) +
                                br (ca^2 \text{ K Ppox } (2 \text{ gstar} - \text{Km} + (-4 \text{ gstar} + 2 \text{ Km}) \text{ x} + (2 \text{ gstar} - \text{Km}) \text{ x}^2) +
                                            4 gstar Km y + 2 ca<sup>2</sup> x<sup>2</sup> y + ca (4 gstar y - 2 Km y + 4 Km x y) ) )
   In[40]:= (*Express x in terms of dpsi -->
                        Final eqn to solve for dpsi. No approximation required,
                     but dpsi bound needs to be calc*)
                    xasfdpsiAll = Simplify[Solve[X1 == X2, x][[All, 1, 2]]]
Out[40]= \left\{ \frac{\text{gstar} + \text{br Km}}{\text{ca} - \text{br ca}}, \left( -2 \text{ ca dpsi } (\text{gstar} + \text{br Km}) \text{ y} + \right) \right\}
                                    \mathsf{ca^2} \ (\ (3-2\ \mathsf{br})\ \mathsf{gstar} + \mathsf{br}\ \mathsf{Km})\ \mathsf{Gs'}\ [\mathsf{dpsi}]\ -\sqrt{2}\ \sqrt{\left(\mathsf{ca^2}\ \mathsf{dpsi}\ (\ (-3+2\ \mathsf{br})\ \mathsf{gstar} - \mathsf{br}\ \mathsf{Km}\right)}
                                                    ((-1 + br) ca + gstar + br Km) y (-2 dpsi y + (ca + 2 gstar) Gs'[dpsi]))
                             (ca^{2}(2(-1+br)dpsiy+((3-2br)gstar+brKm)Gs'[dpsi])),
                         \left(-\,2\;ca\;dpsi\;\left(\,gstar\,+\,br\;Km\right)\;y\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\;gstar\,+\,br\;Km\right)\;Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,\left(\,3\,-\,2\;br\right)\,gstar\,+\,br\;Km\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left[\,dpsi\,\right]\,+\,ca^{2}\,\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{\prime}\left(\,a\,-\,a\,b\,\right)\,Gs^{
                                   \sqrt{2} \sqrt{(ca^2 dpsi ((-3 + 2 br) gstar - br Km))}
                                                    ((-1 + br) ca + gstar + br Km) y (-2 dpsi y + (ca + 2 gstar) Gs'[dpsi]))
                             (ca^{2}(2(-1+br)dpsiy+((3-2br)gstar+brKm)Gs'[dpsi]))
   In[41]:= Qdelta =
                        ca^2 * dpsi * ((-3 + 2 * br) * gstar - br * Km) * ((-1 + br) * ca + gstar + br * Km) * y *
                                (-2*dpsi*y + (ca + 2*gstar)*Derivative[1][Gs][dpsi])
 Out[41]= ca^2 dpsi ((-3 + 2 br) gstar - br Km)
                         ((-1+br) ca + gstar + br Km) y (-2 dpsi y + (ca + 2 gstar) Gs' [dpsi])
   In[42]:= Factor[Qdelta]
 Out[42] = ca^2 dpsi (-3 gstar + 2 br gstar - br Km)
                         (-ca+br\;ca+gstar+br\;Km)\;\;y\;\;(-2\;dpsi\;y+ca\;Gs'\;[dpsi]\;+2\;gstar\;Gs'\;[dpsi]\;)
   ln[43]≔ (* When is determinant > 0? When dpsi < some value. Lets calculate that value*)
                    Collect[(-2 dpsi y + (ca + 2 gstar) Gs'[dpsi]) /. Gs'[dpsi] → dgsddpsi, dpsi]
 Out[43]= (ca + 2 gstar) K Pox + dpsi ((-ca - 2 gstar) K Ppox - 2 y)
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log[47] = Rx = dFdx /. DJmaxdAjm \rightarrow dJmaxdAjm[Ajm[gs, x]] /. gs \rightarrow gs[dpsi] /. dpsi \rightarrow dpsiasfx
Out[47]= -\left(\left(ca^{3}K^{2}\left(\left(-3+2br\right)gstar-brKm\right)Pox\right)\right)
                                                                                         (-1 + x)^{2} (Pox - (ca^{2} K ((-3 + 2 br) gstar - br Km) Pox Ppox (-1 + x)^{2}) /
                                                                                                                  \left(2\,\left(4\,\text{gstar}\,\left(\,\text{gstar}\,+\,\text{br}\,\text{Km}\right)\,\,y\,+\,2\,\,\text{ca}\,\left(\,\text{br}\,\text{Km}\,\left(\,-\,1\,+\,2\,\,x\right)\,\,+\,\text{gstar}\,\left(\,-\,3\,+\,2\,\,\text{br}\,+\,2\,\,x\right)\,\,\right)\,\,y\,+\,\text{ca}^{2}\,\right)
                                                                                                                                                             ((-3 + 2 br) gstar K Ppox (-1 + x)^2 - 2 x^2 y - br (K Km Ppox (-1 + x)^2 - 2 x^2 y)))))))
                                                                          \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \ y + \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \ y + \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{x)} \ )} \right) + \left( \text{4 gstar (gstar + br Km) } \ y + 2 \ \text{ca (br Km (-1 + 2 \ x) + gstar (-3 + 2 \ \text{br} + 2 \ \text{km}) \ )} \right)
                                                                                       ca^{2} ( (-3 + 2 br) gstar K Ppox (-1 + x) ^{2} - 2 x^{2} y - br (K Km Ppox (-1 + x) ^{2} - 2 x^{2} y))) -
                                                  \left(4 \text{ a Iabs}^3 \text{ phi0}^3 \left(-\text{gstar} - \text{br Km} + \left(1 - \text{br}\right) \text{ ca x}\right)^3\right)
                                                                           (2 gstar (gstar + br Km) + (-1 + br) ca^{2} x^{2} + br ca Km (-1 + 2x) + ca gstar (-3 + 2 br + 2x))
                                                                           (4 gstar (gstar + br Km) y + 2 ca (br Km (-1 + 2 x) + gstar (-3 + 2 br + 2 x)) y + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km) (-1 + 2 x) + 2 ca (br Km
                                                                                                 ca^{2} \, \left( \, \left( \, -3 \, + \, 2 \, \, br \right) \, \, gstar \, K \, Ppox \, \left( \, - \, 1 \, + \, x \, \right) \, ^{2} \, - \, 2 \, \, x^{2} \, \, y \, - \, br \, \, \left( \, K \, K \, m \, Ppox \, \, \left( \, - \, 1 \, + \, x \, \right) \, ^{2} \, - \, 2 \, \, x^{2} \, \, y \right) \, \right) \, \right) \, ^{2} \right) \, / \, \, ca^{2} \, \left( \, \left( \, - \, 3 \, + \, 2 \, \, br \, \right) \, \, gstar \, K \, Ppox \, \, \left( \, - \, 1 \, + \, x \, \right) \, ^{2} \, - \, 2 \, \, x^{2} \, \, y \, \right) \, \right) \, ^{2} \, \right) \, / \, \, ca^{2} \, \left( \, \left( \, - \, 3 \, + \, 2 \, \, br \, \right) \, \, gstar \, K \, Ppox \, \, \left( \, - \, 1 \, + \, x \, \right) \, ^{2} \, - \, 2 \, \, x^{2} \, \, y \, \right) \, \right) \, ^{2} \, \right) \, / \, \, ca^{2} \, \, (1 \, + \, x) \, ^{2} \, - \, 2 \, \, x^{2} \, \, y \, \right) \, ^{2} \, \, d^{2} \, \, d^{2} \, \, d^{2} \, \, d^{2} \,
                                                           \left( ca^{6}K^{4} \left( (-3+2 br) gstar - brKm \right)^{2}Pox^{2} (1-x)^{3} (-1+x)^{4} (2 gstar + ca x)^{3} \right)
                                                                          (gstar - ca x + br (Km + ca x))^2
                                                                          \left( \text{Pox} - \left( \text{ca}^2 \text{ K} \left( \left( -3 + 2 \text{ br} \right) \text{ gstar} - \text{br Km} \right) \text{ Pox Ppox } \left( -1 + x \right)^2 \right) \right)
                                                                                                            (2 (4 gstar (gstar + br Km) y + 2 ca (br Km (-1 + 2 x) + gstar (-3 + 2 br + 2 x)) y + ca^{2})
                                                                                                                                                   ((-3 + 2 \text{ br}) \text{ gstar K Ppox } (-1 + x)^2 - 2 x^2 y - \text{br } (\text{K Km Ppox } (-1 + x)^2 - 2 x^2 y)))))^2
                                                                         \left(-1 + \left(\text{Iabs}^2 \text{ phi0}^2 \left(-\text{gstar} - \text{br} \text{Km} + \left(1 - \text{br}\right) \text{ ca x}\right)^2\right)^2 \left(4 \text{ gstar} \left(\text{gstar} + \text{br} \text{Km}\right) \text{ y} + 2 \right)^2 \left(4 \text{ gstar} \left(-\text{gstar} + \text{br} \text{Km}\right) \right)^2 \left(-1 + \left(-1 + \frac{1}{2} \right)^2 \right)^2 \left(-1 + \frac{1}{2} \right)^2 
                                                                                                                                                         ca (br Km (-1+2x) + gstar (-3+2br+2x)) y + ca^{2} ((-3+2br) gstar
                                                                                                                                                                                 K Ppox (-1 + x)^2 - 2x^2y - br(K Km Ppox (-1 + x)^2 - 2x^2y)))^2
                                                                                                            (ca^{6} K^{4} ((-3+2 br) gstar - br Km)^{2} Pox^{2} (1-x)^{2} (-1+x)^{4} (2 gstar + ca x)^{2})
                                                                                                                            (Pox - (ca^2 K ((-3 + 2 br) gstar - br Km) Pox Ppox (-1 + x)^2)
                                                                                                                                                             (2 (4 gstar (gstar + br Km) y + 2 ca (br Km (-1 + 2 x) + gstar (-3 + 2 br + 2 x)) y +
                                                                                                                                                                                            ca^{2} ( (-3 + 2 br) gstar K Ppox (-1 + x) ^{2} - 2 x^{2} y -
                                                                                                                                                                                                                   br (K Km Ppox (-1 + x)^2 - 2 x^2 y))))))))))))))
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 \begin{aligned} & \text{Pe(48)} = \text{ RX /- br } \rightarrow \theta \\ & \text{ } 3 \text{ ca}^3 \text{ gstar } \text{K}^2 \text{ Pox } (-1 + \text{x})^2 \left( \text{Pox} + \frac{3 \text{ ca}^2 \text{ gstar } \text{K Pox Ppox } (-1 + \text{x})^2}{2 \left( 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } \text{K Ppox } (-1 + \text{x})^2 - 2 \text{ x}^2 \text{ y} \right)} \right) \\ & \text{ } 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } (-3 + 2 \text{ x}) \text{ y} + \text{ ca}^2 \left( -3 \text{ gstar } \text{K Ppox } (-1 + \text{x})^2 - 2 \text{ x}^2 \text{ y} \right)} \right) \\ & \text{ } 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } (-3 + 2 \text{ x}) \text{ y} + \text{ ca}^2 \left( -3 \text{ gstar } \text{K Ppox } (-1 + \text{x})^2 - 2 \text{ x}^2 \text{ y} \right) \right)^2 \right) \\ & \text{ } \left( 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } (-3 + 2 \text{ x}) \text{ y} + \text{ ca}^2 \left( -3 \text{ gstar } \text{K Ppox } (-1 + \text{x})^2 - 2 \text{ x}^2 \text{ y} \right) \right)^2 \right) \\ & \text{ } \left( 9 \text{ ca}^6 \text{ gstar}^2 \text{ K}^4 \text{ Pox}^2 \left( 1 - \text{x} \right)^3 \left( -1 + \text{x} \right)^4 \left( \text{ gstar } - \text{ ca} \text{ x} \right)^2 \left( 2 \text{ gstar } + \text{ ca} \text{ x} \right)^3 \right) \\ & \text{ } \left( 9 \text{ ca}^6 \text{ gstar}^2 \text{ K}^4 \text{ Pox}^2 \left( 1 - \text{x} \right)^3 \left( -1 + \text{x} \right)^4 \left( \text{ gstar } - \text{ ca} \text{ x} \right)^2 \left( 2 \text{ gstar } \text{ K Ppox } \left( -1 + \text{x} \right)^2 - 2 \text{ x}^2 \text{ y} \right) \right)^2 \right) \\ & \text{ } \left( -1 + \left( \text{Iabs}^2 \text{ phi}\theta^2 \left( -\text{gstar} + \text{ ca} \text{ x} \right)^2 \right) \\ & \text{ } \left( 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } \left( -3 + 2 \text{ x} \right) \text{ y} + \text{ ca}^2 \left( -3 \text{ gstar } \text{ K Ppox } \left( -1 + \text{x} \right)^2 - 2 \text{ x}^2 \text{ y} \right) \right)^2 \right) \right) \\ & \text{ } \left( 9 \text{ ca}^6 \text{ gstar}^2 \text{ K}^4 \text{ Pox}^2 \left( 1 - \text{x} \right)^2 \left( -1 + \text{x} \right)^4 \left( 2 \text{ gstar } + \text{ ca} \text{ x} \right)^2 \left( \text{Pox} + \frac{3 \text{ ca}^2 \text{ g}}{2 \left( 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } \left( -3 + 2 \text{ x} \right) \right) \right) \right) \right) \\ & \text{ } \left( 9 \text{ ca}^6 \text{ gstar}^2 \text{ K}^4 \text{ Pox}^2 \left( 1 - \text{x} \right)^2 \left( -1 + \text{x} \right)^4 \left( 2 \text{ gstar } + \text{ ca} \text{ x} \right)^2 \left( \text{Pox} + \frac{3 \text{ ca}^2 \text{ g}}{2 \left( 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } \left( -3 + 2 \text{ x} \right) \right) \right) \right) \right) \right) \\ & \text{ } \left( 9 \text{ ca}^6 \text{ gstar}^2 \text{ K}^4 \text{ Pox}^2 \left( 1 - \text{x} \right)^2 \left( -1 + \text{x} \right)^4 \left( 2 \text{ gstar } + \text{ ca} \text{ x} \right)^2 \left( \text{Pox} + \frac{3 \text{ ca}^2 \text{ g}}{2 \left( 4 \text{ gstar}^2 \text{ y} + 2 \text{ ca} \text{ gstar } \left( -3 + 2 \text{ x} \right) \right) \right) \right) \right) \right) \right) \\ & \text{ }
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