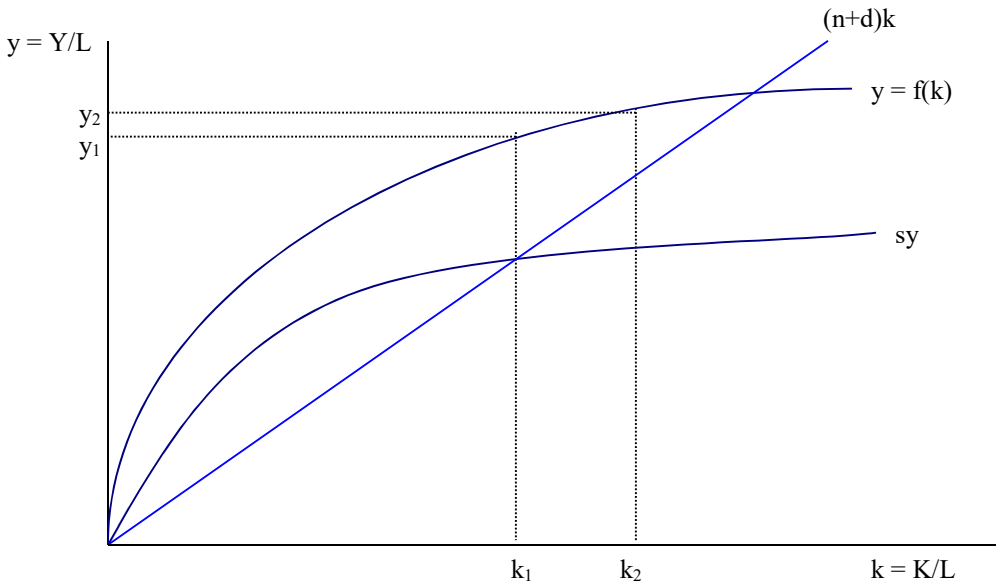
Problem Set # 3
Solutions

Chapter 8 #2

a) The production function in the Solow growth model is Y = f(K,L), or expressed in terms of output per worker, y = f(k). If a war reduces the labor force through casualties, the L falls but Capital-labor ratio k = K/L rises. The production function tells us that total output falls because there are fewer workers. Output per worker increases, however, since each worker has more capital.

b) The reduction in the labor force means that the capital stock per worker is higher after the war. Therefore, if the economy were in a steady state prior to the war, then after the war the economy has a capital stock that is higher than the steady-state level. This is shown in the figure below as an increase in capital per worker from k_1 to k_2. As the economy returns to the steady state, the capital stock per worker falls from k_2 back to k_1, so output per worker also falls.



Chapter 8 #4

Suppose the economy begins with an initial steady-state capital stock below the Golden Rule level. The immediate effect of devoting a larger share of national output to investment is that the economy devotes a smaller share to consumption; that is, "living standards" as measured by consumption fall. The higher investment rate means that the capital stock increases more quickly, so the growth rates of output and output per worker rise. The productivity of workers is the average amount produced by each worker – that is, output per worker. So productivity growth rises. Hence, the immediate effect is that living standards fall but productivity growth rises.

In the new steady state, output grows at rate n+g, while output per worker grows at rate g. This means that in the steady state, productivity growth is independent of the rate of investment. Since we begin with an initial steady-state capital stock below the Golden rule level, the higher investment rate means that the new steady state has a higher level of consumption, so living standards are higher

Thus, an increase in the investment rate increases the productivity growth rate in the short run but has no effect in the long run. Living standards, on the other hand, fall immediately and only rise over time. That is, the quotation emphasizes growth, but not the sacrifice required to achieve it.

Chapter 9, #3

To solve this problem, it is useful to establish what we know about the U.S. economy:

- A Cobb-Douglas production function has the form $y = k^{\alpha}$, where α is capital's share of income. The question tells us that $\alpha = 0.3$, so we k now that the production function is $y = k^{.3}$
- In the steady state, we know that the growth rate of output equals 3%, so we know that (n+g) = .03
- The depreciation rate $\delta = .04$
- The capital-output ratio K/Y = 2.5. Because k/y = [K/(LxE)]/[Y/(LxE)] = K/Y, we also know that k/y = 2.5. (That is, the capital-output ratio is the same in terms of effective workers as it is in levels.)

a) Begin with the steady-state condition, $sy = (\delta + n + g)k$. Rewriting this equation leads to a formula for saving in the steady state:

$s = (\delta + n + g)(k/y)$

Plugging in the values from above: $s = (0.04 + 0.03)(2.5) = .175$
The initial saving rate is 17.5%.

b) We know from Chapter 3 that with a Cobb-Douglas production function, capital's share of income $\alpha = MPK(K/Y)$. Rewriting, we have:

$MPK = \alpha/(K/Y)$

Plugging in the values from above: $MPK = 0.3/2.5 = .12$

c) We know that at the Golden Rule steady state:

$MPK = (n + g + d)$

Plugging in the values from above: $MPK = (.03 + .04) = .07$
At the Golden Rule steady state, the marginal product of capital is 7%, whereas it is 12% in the initial steady state. Hence, from the initial steady state we need to increase k to achieve the Golden Rule steady state.

d) We know from Chapter 3 that for a Cobb-Douglas production function,
$MPK = \alpha(Y/K)$. Solving this for the capital-output ratio, we find:

$K/Y = \alpha/MPK$

We can solve for the Golden Rule capital-output ratio using this equation. If we plug in the value 0.07 for the Golden Rule steady-state MPK, and the value 0.3 for a, we find:

$K/Y = 0.3/0.07 = 4.29$

In the Golden Rule steady state, the capital-output ratio equals 4.29, compared to the current capital-output ratio of 2.5.

e) We know from part (a) that in the steady state

$s = (n + g + \delta)(k/y)$

where k/y is the steady-state capital-output ratio. In the introduction to this answer, we showed that k/y = K/Y, and in part (d) we found that the Golden Rule K/Y = 4.29. Plugging in this value and those established above:

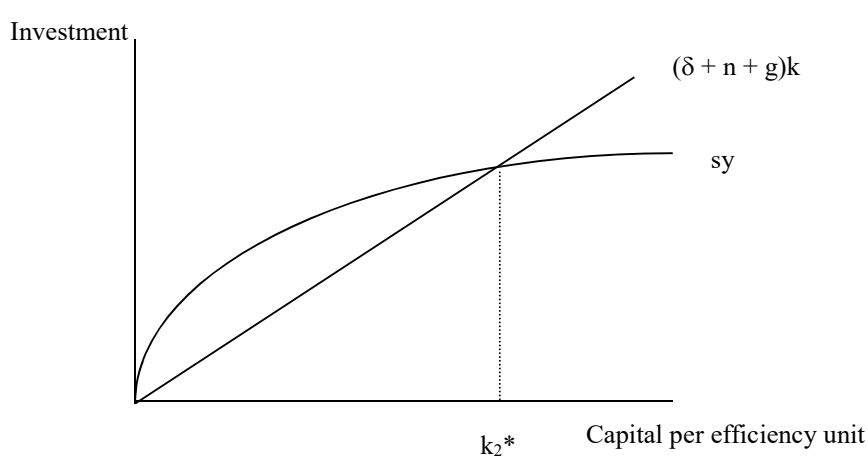
$s = (0.04 + 0.03)(4.29) = 0.30$

To reach the Golden Rule steady-state, the saving rate must rise from 17.5% to 30%.

Chapter 9, #6

How do differences in education across countries affect the Solow Growth Model? Education is one factor affecting the *Efficiency of labor*, which we denoted by E. (Other factors affecting the efficiency of labor include levels of health, skill and knowledge.) Since country 1 has a more highly educated labor force than country 2, , each worker in country 1 is more efficient. That is E1>E2. We will assume that both countries are in steady state.

a. In the Solow Growth model, the rate of growth of total income is equal to n+g, which is independent of the work force's *level* of education. The two countries will, thus, have the same rate of growth of total income because they have the same rate of population growth and the same rate of technological progress.
b. Because both countries have the same saving rate, the same population growth rate, and the same rate of technological progress, we know that the two countries will converge to the same steady-state level of capital per efficiency unit of labor k*. This is shown in the figure below.



Hence, output per efficiency unit of labor in the steady state, which is $y^* = f(k^*)$ is the same in both countries. But $y^* = Y/(L^*E)$ or $Y/L = y^*E$. We know that y^* will be the same in both countires, but that E1>E2. Therefore $y^*E_1 > y^*E_2$. This implies that $(Y/L)_1 > (Y/L)_2$. Thus, the level of income per worker will be higher in the country with the more educated labor force.

c. We know that the real rental price of capital r equals the marginal product of capital (MPK). But the MPK depends on the capital stock per efficiency unit of labor. In the steady state, both countries have $k^*_1 = k^*_2 = k^*$ because both countries have the same saving rate, the same population growth rate, and the same rate of technological progress. Therefore, it must be true that $r_1 = r_2 = MPK$. Thus, the real rental price of capital is identical in both countries.

d. Output is divided between capital income and labor income. Therefore, the wage per efficiency unit of labor can be expressed as:

$$w = f(k) - MPK * k$$

As discussed in parts (b) and (c), both countries have the same steady-state capital stock k and the same MPK. Therefore, the wage per efficiency unit in the two countries is equal.

Workers, however, care about the wage per unit of labor, not the wage per efficiency unit. Also, we can observe the wage per unit of labor but not the wage per efficiency unit. The wage per unit of labor is related to the wage per efficiency unit of labor by the equation:

$$\text{Wage per Unit of L} = wE$$

Thus, the wage per unit of labor is higher in the country with the more educated labor force.