Assignment 17.1 : Problem Statement

A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution :

Let us assume that in this experiment done,

- ⇨ 'n' is representing the number of trials attempted, and that
- ⇨ 'k' is the count of successes that is to be attained in those 'n' trials.
- ⇨ This implies that number of failures clearly will be 'n - k'.

Here, n = 20, k = 20 - 5 = 15 , n - k = 5

Assuming, 's' to be the probability of succeeding in a trial, we get that the probability of failure is '1 - s'.

Here the probability of success = probability of giving a right answer = s = 1/4

Hence, the probability of failure = probability of giving a wrong answer = 1 - s = 1 – ¼ = ¾

P ('k' successes in 'n' trials) = C(n,k) s^k $(1-s)^{(n-k)}$

C (n, k) is called the coefficient for binomial distribution or binomial coefficient.

When we substitute these values in the formula for Binomial distribution we get,

P(15 out of 20 answers correct) = P (5 out of 20 answers incorrect)

= C(n,k) s^k $(1-s)^{(n-k)}$

➔ C (20, 15) * $(¼)^{15}$ * $(¾)^5$

⇨ C(20,15) = n!/(k! * (n-k)!) = 20! / (15! 5!)

⇨ C(20,15) = (20*19*18*17*16)/(5*4*3*2*1) = 15504
⇨ $(¼)^{15}$ = 0.00000000093132257462
⇨ $(¾)^{5}$ = 0.24
⇨ P (5 out of 20)
= 15504 * 0.00000000093132257462 * 0.24
= 0.000003465414

= 0.0000034 (approximately)

Thus the required probability that that a person undertaking that test has answered exactly 5 questions wrong is 0.0000034 approximately.