Homework #3 , due Tuesday 4/16 at 8pm (before discussion section)

Text Problems:
Oppenheim and Schafer, 3rd edition, published 2010: Problems - 4.21

Computer Exercise
Consider the signal *x*(*t*) that consists of two cosinusoids

$$x(t) = 2\,cos\ (2\pi f_0 t + \phi_0) + 2\,cos\ (2\pi f_1 t + \phi_1)$$

We will study the effects of sampling, downsampling and upsampling on such signals and their spectra for several values of f_0 and f_1. In this exercise, choose the sampling frequency $f_s = 2\ kHz$. Since you will only be looking at magnitude spectra, the phase terms ϕ_0 and ϕ_1 don't matter and you can set them both to 0 for this exercise.

a) Using $f_s = 2\ kHz$, generate a sampled sinusoid $x_1[n]$ which comes from sampling the continuous time signal in equation (1) with $f_0 = 50\ Hz$ and $f_1 = 150\ Hz$. Take exactly 2000 samples and let the time index n of $x_1[n]$ range from 0 to 1999. Plot the first about 50-100 samples using the appropriate Python plotting function.

b) Use `pydsm.ft.dtft( )` to estimate the Fourier transform $X_1(e^{j\omega})$ of this 2000 sample file $x_1[n]$. Choose the parameter *N* of of this DTFT function to equal to the length of this input sequence. This choice luckily ensures that the cosinusoids in $x_1[n]$ resolve nicely into individual lines in the spectrum $X_1(e^{j\omega})$. (A rare event for natural signals.) Plot the magnitude spectrum $|X_1(e^{j\omega})|$ for $-\pi \leq \omega < \pi$.
Give a short analysis and explanation of all spectra above. Pay attention to the location of the cosinusoid lines in the spectrum, as well as their magnitude (height).

c) Determine the signal $x_{1,s}[n]$ that is the *sampled* version of $x_1[n]$ as follows

$$x_{1,s}[n] = \{x_1[n], n = 4k, for\ k = \{0,1,2,\ldots,499\}\ 0, \qquad elsewhere$$

Follow the directions given in the above (a) and (b) to plot the signal $x_{1,s}[n]$ and its magnitude spectrum $|X_{1,s}(e^{j\omega})|$. While you are welcome to consider what you see, no need to analyze and explain spectra, yet. Our lectures will motivate the answer and that explanation might be part of HW 4, assigned next week.

Please Note: Power spectral densities are statistical quantities defined, for example, in (not required reading) sec. 2.10 of Oppenheim and Schafer. The above magnitude-squared spectrum $|X_1(e^{j\omega})|^2, -\pi \leq \omega < \pi$ provides only an *estimate*, often called a "periodogram," of the true power density spectrum of the signal *x*(*t*).