EE 518 HOMEWORK #2 JAIDON LYBBERT

3.16) $x[n] = (\frac{1}{3})^n u[n] + (2)^n u[-n-1]$

$y[n] = 5(\frac{1}{3})^n u[n] - 5(\frac{2}{3})^n u[n]$

a)

TABLE 3.1 #5 & 6 — R.O.C

$$X(z) = \frac{1}{1-(\frac{1}{3})z^{-1}} - \frac{1}{1-2z^{-1}} \qquad |z| > \frac{1}{3},\ |z| < 2$$

TABLE 3.1 #5 — R.O.C

$$Y(z) = \frac{5}{1-(\frac{1}{3})z^{-1}} - \frac{5}{1-(\frac{2}{3})z^{-1}} \qquad |z| > \frac{1}{3},\ |z| > \frac{2}{3}$$

$$X(z) = \frac{(1-2z^{-1})-(1-(\frac{1}{3})z^{-1})}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$$

$$Y(z) = \frac{5(1-\frac{2}{3}z^{-1})-5(1-\frac{1}{3}z^{-1})}{(1-\frac{2}{3}z^{-1})(1-\frac{1}{3}z^{-1})} = \frac{(-\frac{10}{3}+\frac{5}{3})\cdots}{\cdots} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{2}{3}z^{-1})(1-\frac{1}{3}z^{-1})}$$

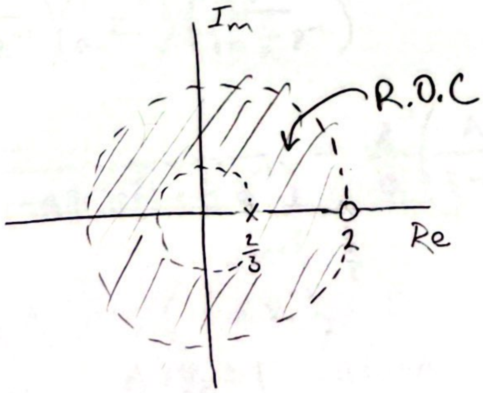
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\cancel{(-\frac{5}{3}z^{-1})}\cancel{(1-\frac{1}{3}z^{-1})}(1-2z^{-1})}{(1-\frac{2}{3}z^{-1})\cancel{(1-\frac{1}{3}z^{-1})}\cancel{(-\frac{5}{3}z^{-1})}} = \frac{(1-2z^{-1})}{(1-\frac{2}{3}z^{-1})}$$

$$H(z) = \frac{(1-2z^{-1})}{(1-\frac{2}{3}z^{-1})}$$

POLES: $z = \frac{2}{3}$

ZEROS: $z = 2$

R.O.C: $\frac{2}{3} < |z| < 2$



b)

TABLE 3.1 #5 TABLE 3.2 #2

$$h[n] = Z^{-1}\{H(z)\} = Z^{-1}\left\{ \frac{1}{1-\frac{2}{3}z^{-1}} - \frac{2z^{-1}}{(1-\frac{2}{3}z^{-1})} \right\}$$

$$h[n] = \left(\frac{2}{3}\right)^n u[n] - 2\left(\frac{2}{3}\right)^{n-1} u[n-1]$$ R.O.C: $\frac{2}{3} < |z| < 2$

$n-k-1 > 0$

c) $$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\left(\frac{2}{3}\right)^{n-k} u[n-k] - x[k]2\left(\frac{2}{3}\right)^{n-k-1} u[n-k-1]$$

d) BY SECTION 3.2 PROPERTY 5, THE SYSTEM IS TWO-SIDED AND THEREFORE NOT CAUSAL

3.58)

$$C_{xx}[n] = \sum_{k=-\infty}^{\infty} x[k]x[n+k]$$

a) $$C_{xx}[n] = x[-n] * x[n] \xrightarrow[\text{TIME REVERSAL}]{\text{Pg.129}} x[-n] \overset{z}{\longleftrightarrow} X\left(\frac{1}{z}\right)$$

$ROC = \frac{1}{R_x}$

$\cup$

$ROC = R_x$

$$C_{xx}[n] \overset{z}{\longleftrightarrow} X\left(\frac{1}{z}\right)X(z) = X(z)X(z^{-1})$$

$$ROC: \frac{1}{R_x} < |z| < R_x$$

b) $x[n] = a^n u[n]$ $\quad X(z) = \frac{1}{1-az^{-1}}$ $\quad |z| > a$ $\quad X(z^{-1}) = \frac{1}{1-az} = \frac{z^{-1}}{(z^{-1}-a)}$

$$X(z^{-1}) = \frac{1}{a}z^{-1}\frac{1}{1-\frac{1}{a}z^{-1}} \quad |z| > \frac{1}{a}$$

$$C_{xx}(z) = \left(\frac{1}{1-az^{-1}}\right)\left(\frac{1}{a}z^{-1}\right)\left(\frac{1}{1-\frac{1}{a}z^{-1}}\right)$$

$$C_{xx}(z) = \frac{z^{-1}}{a}\frac{1}{(1-az^{-1})(1-\frac{1}{a}z^{-1})} = \frac{z^{-1}}{a}\left(\frac{A}{1-az^{-1}} + \frac{B}{1-\frac{1}{a}z^{-1}}\right)$$

$$A(1-\tfrac{1}{a}z^{-1}) + B(1-az^{-1}) = 1$$

$A + B = 1$ $\quad B = 1 - A$

$-\frac{A}{a} - Ba = 0$

$-\frac{A}{a} - (1-A)a = 0$

$A(a - \frac{1}{a}) - a = 0$

$\rightarrow A = \frac{a}{a - \frac{1}{a}}$

$B = 1 - \frac{1}{1-a^{-2}}$

$a<|z|$

$$C_{xx}[z]=\left(\frac{1}{a-\frac{1}{a}}\right)z^{-1}\frac{1}{1-az^{-1}}+\left(a+\frac{a}{1-a^2}\right)z^{-1}\frac{1}{1-\frac{1}{a}z^{-1}}$$

$$C_{xx}[n]=\left(\frac{1}{a-\frac{1}{a}}\right)a^{n-1}u[n-1]+\left(a+\frac{a}{1-a^2}\right)\left(\frac{1}{a}\right)^{n-1}u[n-1]$$

c) A TIME REVERSED INPUT $x[n]$ WOULD HAVE THE SAME $C_{xx}[n]$

$$x[n]=x[-n]=a^{-n}u[-n]$$

d) A TIME SHIFTED INPUT $x_1[n]$ WOULD HAVE THE SAME $C_{xx}[n]$

$$x_1[n]=x[n-k]=a^{n-k}u[n-k]$$

M1)

PROBLEM 3.32(a) Pg. 145 DO PFD

$$x(z)=\frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})}=\frac{A}{(1+\frac{1}{2}z^{-1})^2}+\frac{B}{(1-2z^{-1})}+\frac{C}{(1-3z^{-1})}+\frac{D}{1+\frac{1}{2}z^{-1}}$$

$1-\frac{3}{2}\quad -\frac{1}{2}$

$$A(1-2z^{-1})(1-3z^{-1})+B(1+\tfrac{1}{2}z^{-1})^2(1-3z^{-1})$$
$$+\ C(1+\tfrac{1}{2}z^{-1})^2(1-2z^{-1})+D(1+\tfrac{1}{2}z^{-1})(1-2z^{-1})(1-3z^{-1})=1$$

$z=2$

$\left(\frac{5}{4}\right)^2\left(-\frac{1}{2}\right)B=1$

$\left(\frac{5}{4}\right)^2\left(\frac{1}{2}\right)B=$

$-\left(\frac{25}{16}\right)\left(\frac{1}{2}\right)B=1$

$-\frac{25}{32}B=1$

$B=\frac{32}{-25}$

$B=-1.28$

$z=3$

$\left(\frac{7}{6}\right)^2\left(\frac{1}{3}\right)C=1$

$\left(\frac{49}{36}\right)\left(\frac{1}{3}\right)C=1$

$\frac{49}{108}C=1$

$C=\frac{108}{49}$

$C=2.2$

$z=-\frac{1}{2}$

$(1+4)(1+6)A=1$

$(5)(7)A=1$

$A=\frac{1}{35}$

$A=0.028$

$A+B+C+D=1$

$D=1-A-B-C$

$D=0.047$

$$x(z)=\frac{0.028}{(1+\frac{1}{2}z^{-1})^2}-\frac{1.28}{(1-2z^{-1})}+\frac{2.2}{(1-3z^{-1})}+\frac{0.047}{(1+\frac{1}{2}z^{-1})}$$

$$X(z) = \frac{1}{(1+z^{-1}+\frac{1}{4}z^{-2})(1-5z^{-1}+6z^{-2})} = \frac{1}{1-5z^{-1}+6z^{-2}+z^{-1}-5z^{-2}+6z^{-3}+\frac{1}{4}z^{-2}-\frac{5}{4}z^{-3}+\frac{3}{2}z^{-4}}$$

$$X(z) = \frac{1}{1+(-5+1)z^{-1}+(6-5+\frac{1}{4})z^{-2}+(6-\frac{5}{4})z^{-3}+\frac{3}{2}z^{-4}}$$

SCIPY.SIGNAL.RESIDUEZ

↓

$$X(z) = \frac{0.047}{1+0.5z^{-1}} + \frac{0.0286}{(1+0.5z^{-1})^2} - \frac{1.28}{1-2z^{-1}} + \frac{2.20}{1-3z^{-1}}$$

M2)

a) $y[n] -1.85\cos(\frac{\pi}{18})y[n-1] + 0.83\,y[n-2] = x[n] + \frac{1}{3}x[n-1]$

$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1]$$

$$a_0 Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) = b_0 X(z) + b_1 z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \qquad (a_0 + a_1 z^{-1} + a_2 z^{-2})Y(z) = (b_0 + b_1 z^{-1})X(z)$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

$$H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-1.85\cos(\frac{\pi}{18})z^{-1}+0.83z^{-2}}$$

b) SEE PLOTS.
FILTER APPEARS TO BE LOW-PASS.

c) SEE PLOTS.