

OPPENHEIM & SCHAFER: 2.24, 2.47, 2.66, 2.73

2.24

FOR AN ARBITRARY LINEAR SYSTEM WITH INPUT $x[n]$ AND OUTPUT $y[n]$, IF $x[n] = 0$ FOR ALL n , THEN $y[n]$ MUST BE ZERO FOR ALL n .

AN ARBITRARY LINEAR SYSTEM IS DESCRIBED BY THE CONVOLUTION SUM

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

WHERE THE OUTPUT IS A LINEAR COMBINATION OF INPUT SAMPLES, IF ALL SAMPLES ARE ZERO, THE SUMMATION IS ZERO.

2.47

GIVEN A "WINDOWED COSINE SIGNAL"

$$x[n] = w[n] \cos(\omega_0 n)$$

a) THE FOURIER TRANSFORM IS

$$X(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathcal{F}\{\cos(\omega_0 n)\} W(e^{j(\omega-\theta)}) d\theta$$

BY THE WINDOWING THEOREM, BY TABLE 2.3 SEQUENCE 11,

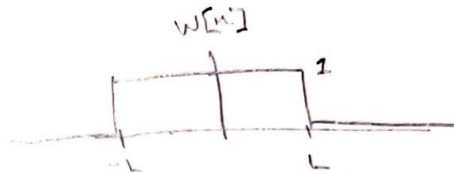
$$\mathcal{F}\{\cos(\omega_0 n)\} = \sum_{k=-\infty}^{\infty} [\pi \delta(\omega - \omega_0 + 2\pi k) + \pi \delta(\omega + \omega_0 + 2\pi k)]$$

b) FOR THE SEQUENCE

$$w[n] = \begin{cases} 1 & -L \leq n \leq L \\ 0 & \text{OTHERWISE} \end{cases}$$

THE SIGNAL CAN BE WRITTEN AS

$$W[n] = u[n+L] - u[n-L]$$



THE FOURIER TRANSFORM IS, BY TABLE 2.2 THEOREM 1,

$$W(e^{j\omega}) = F\{u[n+L]\} - F\{u[n-L]\}$$

WHERE, BY TABLE 2.3 SEQUENCE 5,

$$F\{u[n]\} = \frac{1}{1-e^{-j\omega}} + \sum_{K=-\infty}^{\infty} \pi \delta(\omega + 2\pi K)$$

AND, BY TABLE 2.2 THEOREM 2,

$$F\{u[n-L]\} = e^{-j\omega L} \left(\frac{1}{1-e^{-j\omega}} + \sum_{K=-\infty}^{\infty} \pi \delta(\omega + 2\pi K) \right)$$

SO,

$$W(e^{j\omega}) = (e^{j\omega L} - e^{-j\omega L}) \left(\frac{1}{1-e^{-j\omega}} + \sum_{K=-\infty}^{\infty} \pi \delta(\omega + 2\pi K) \right)$$

∴
THIS IDENTITY?



2.66

THE AUTO CORRELATION OF A SIGNAL IS

$$R_x[n] = \sum_{k=-\infty}^{\infty} x^*[k] x[n+k]$$

a) TO FIND A SIGNAL $g[n]$ SUCH THAT

$$R_x[n] = x[n] * g[n]$$

WE CAN EXPAND THE CONVOLUTION SUM AS

$$R_x[n] = \sum_{k=-\infty}^{\infty} x[k] g[n-k] = \sum_{k=-\infty}^{\infty} g[k] x[n-k]$$

$$\text{IF } g[n] = x^*[-n]$$

$$R_x[n] = \sum_{k=-\infty}^{\infty} x^*[-k] x[n-k]$$

BUT k IS ARBITRARY AND INFINITE SO THIS IS EQUIVALENT TO

$$R_x[n] = \sum_{k=-\infty}^{\infty} x^*[k] x[n+k]$$

b)

THE FOURIER TRANSFORM OF

$$R_x[n] = x[n] * x^*[-n]$$

IS

$$R_x(e^{j\omega}) = X(e^{j\omega}) X^*(e^{j\omega})$$

BY, TABLE 2.1 SEQUENCE 2

SO, BY COMPLEX MULTIPLICATION

$$R_x(e^{j\omega}) = |X(e^{j\omega})|^2$$

2.73

a) $E(e^{j\omega}) = H_1(e^{j\omega})X(e^{j\omega})$

$$F(e^{j\omega}) = H_1(e^{-j\omega})X(e^{j\omega})$$

$$G(e^{j\omega}) = H_1(e^{j\omega})H_1(e^{-j\omega})X(e^{-j\omega})$$

$$Y(e^{j\omega}) = H_1(e^{-j\omega})H_1(e^{j\omega})X(e^{j\omega})$$

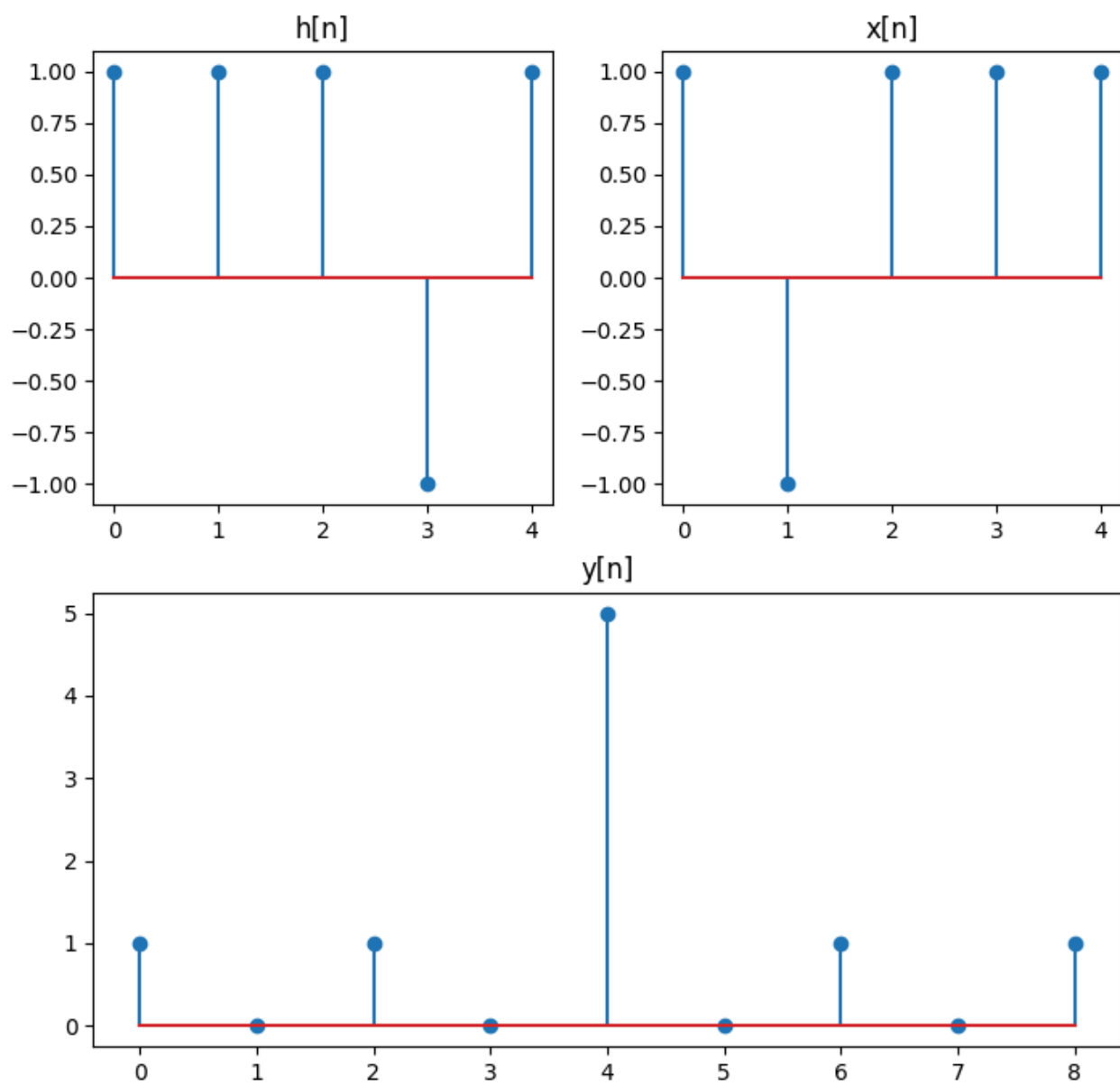
b)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = H_1(e^{-j\omega})H_1(e^{j\omega})$$

c)

$$h[n] = h_1[n] * h_1[-n]$$

Part 1: Convolution



Part 2: Difference Equation

