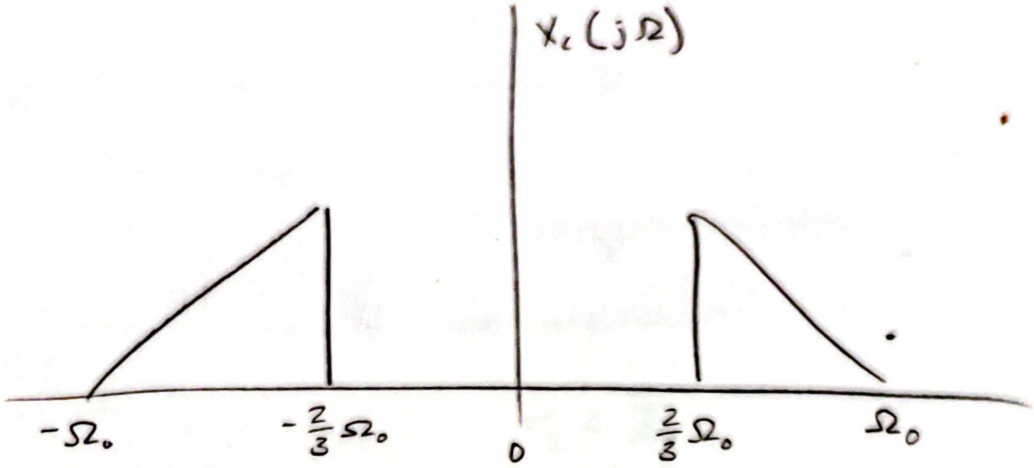
EE 518 HOMEWORK #3 JAIDON LYBBERT

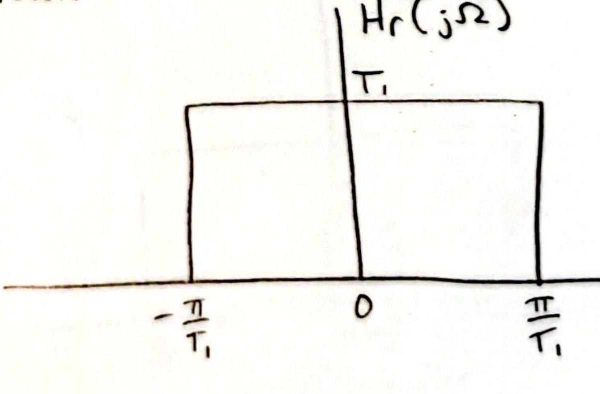
4.21) CONSIDER $x_c(t)$ WITH FT $X_c(j\Omega)$ SHOWN



a) A SIGNAL $x_r(t)$ IS OBTAINED BY SAMPLING $x_c(t)$

$$x_s(t) = \sum_{n=-\infty}^{+\infty} x_c(t)\,\delta(t-nT_1)$$

THEN PASSING $x_s(t)$ THROUGH A LOW-PASS WITH RESPONSE $H_r(j\Omega)$ SHOWN



FOR $x_r(t)$ TO BE EQUAL TO $x_c(t)$, THE SAMPLE RATE MUST BE HIGH ENOUGH TO RECONSTRUCT THE SIGNAL, AND THE LOW-PASS MUST ENCOMPASS THE FREQUENCIES OF THE SIGNAL. THIS RESULTS IN THE INEQUALITIES:

$$\frac{2\pi}{T_1} > 2\Omega_0 \qquad \frac{\pi}{T_1} > \Omega_0$$

(NO ALIASING) (SUFFICIENT BANDWIDTH)

BOTH INEQUALITIES REDUCE TO THE CONDITION

$$\boxed{T_1 < \frac{\pi}{\Omega_0}}$$

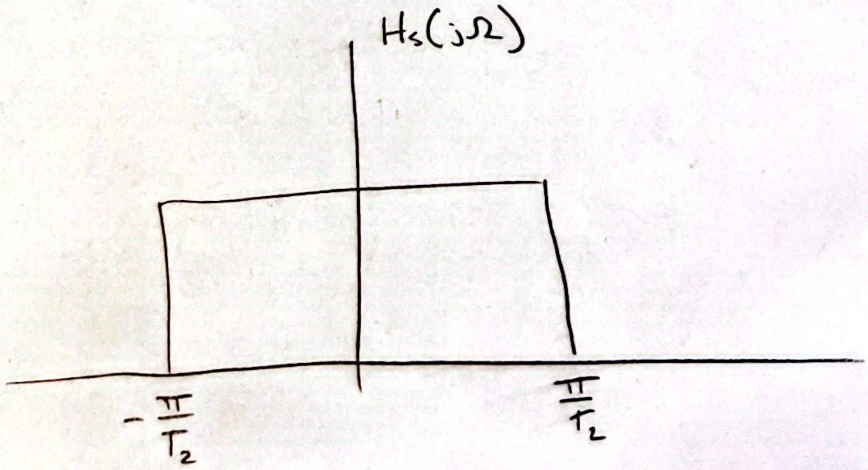
b) CONSIDER A SIMILAR SYSTEM, BUT WITH SAMPLING PERIOD T_2. FIND ALL VALUES OF T_2 FOR WHICH $x_0(t) = x_c(t)$ FOR ALL t. FOR THE LARGEST T_2 POSSIBLE, SKETCH $H_s(j\Omega)$.

TO PREVENT ALIASING, THE INEQUALITY

$$\frac{2\pi}{T_2} > 2\Omega_0 \qquad T_2 < \frac{\pi}{\Omega_0}$$

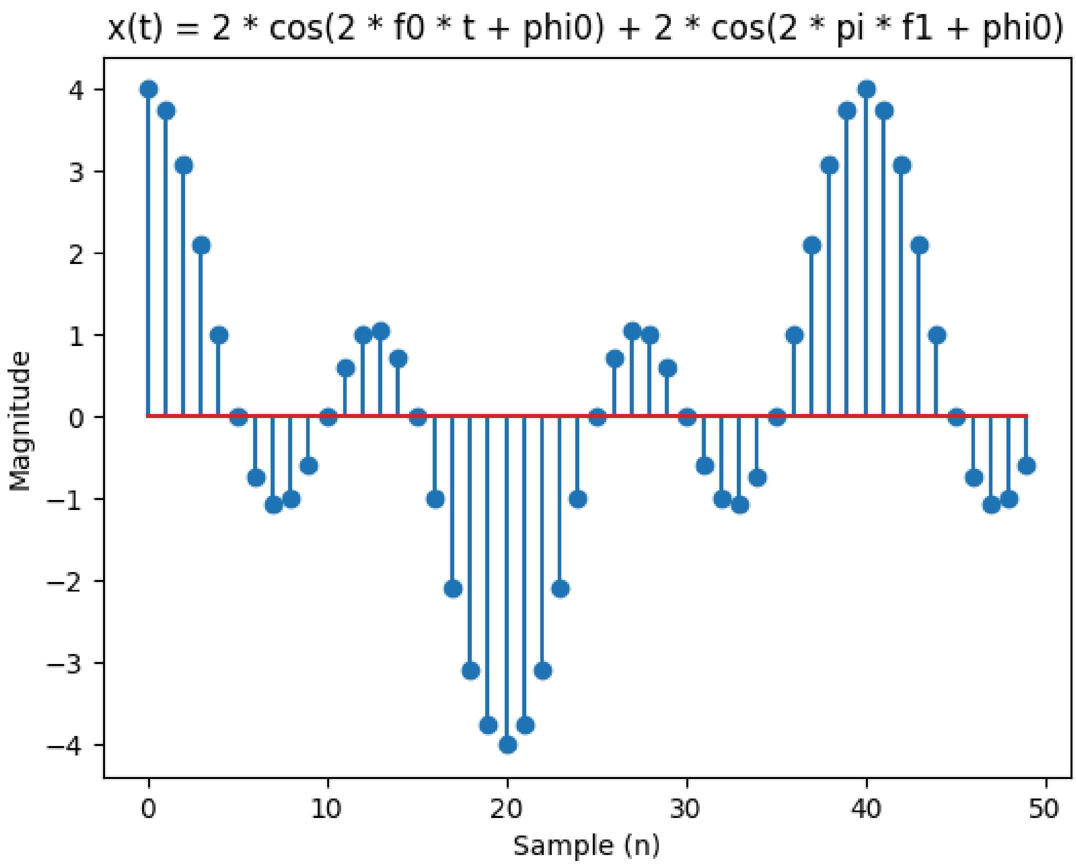
MUST BE TRUE. THE LARGEST VALUE OF T_2 IS THEN $\frac{\pi}{\Omega_0}$.

FOR THIS VALUE OF T_2, $H_s(j\Omega)$

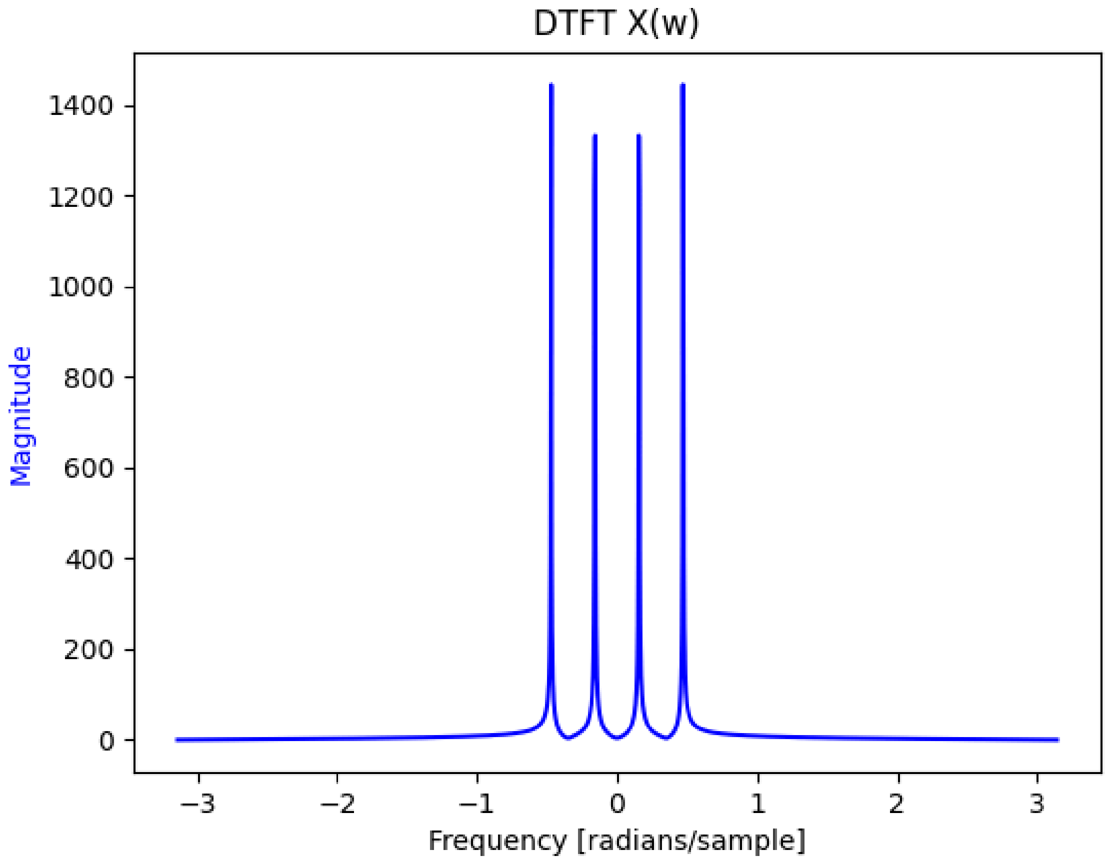


EE518 Sp24
Jaidon Lybbert
Homework 3

Part A:



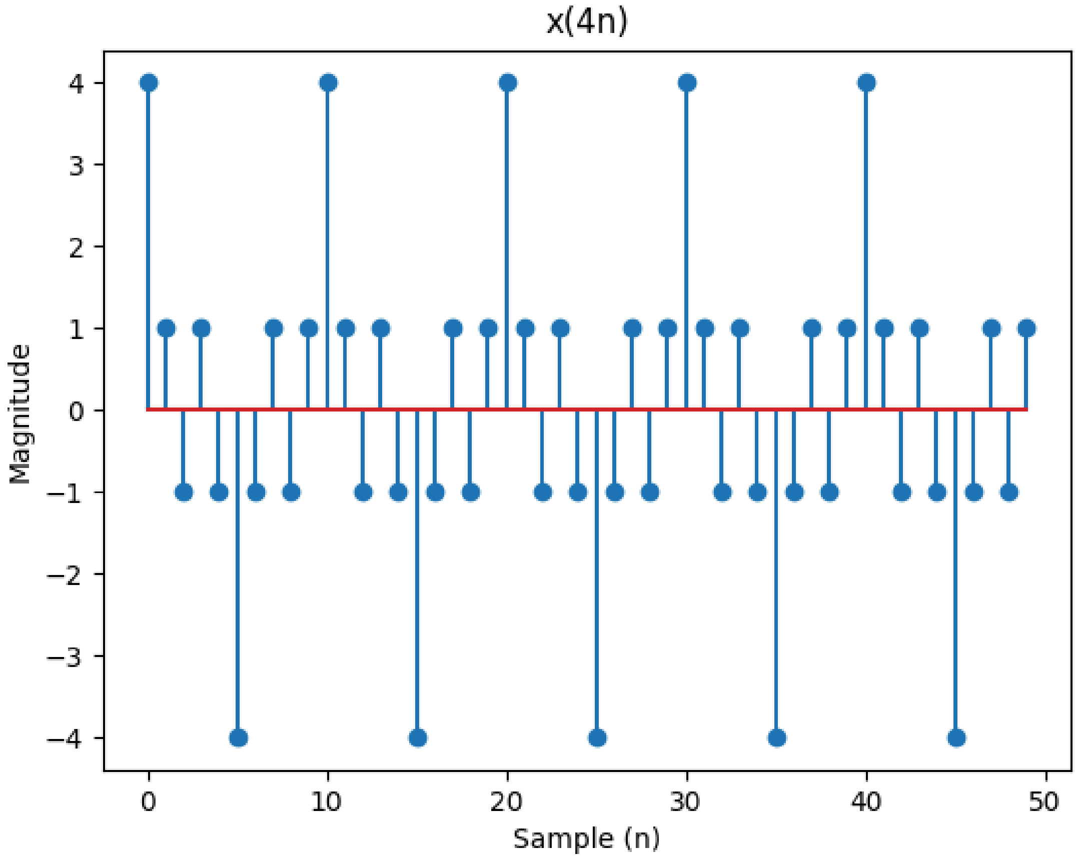
Part B:



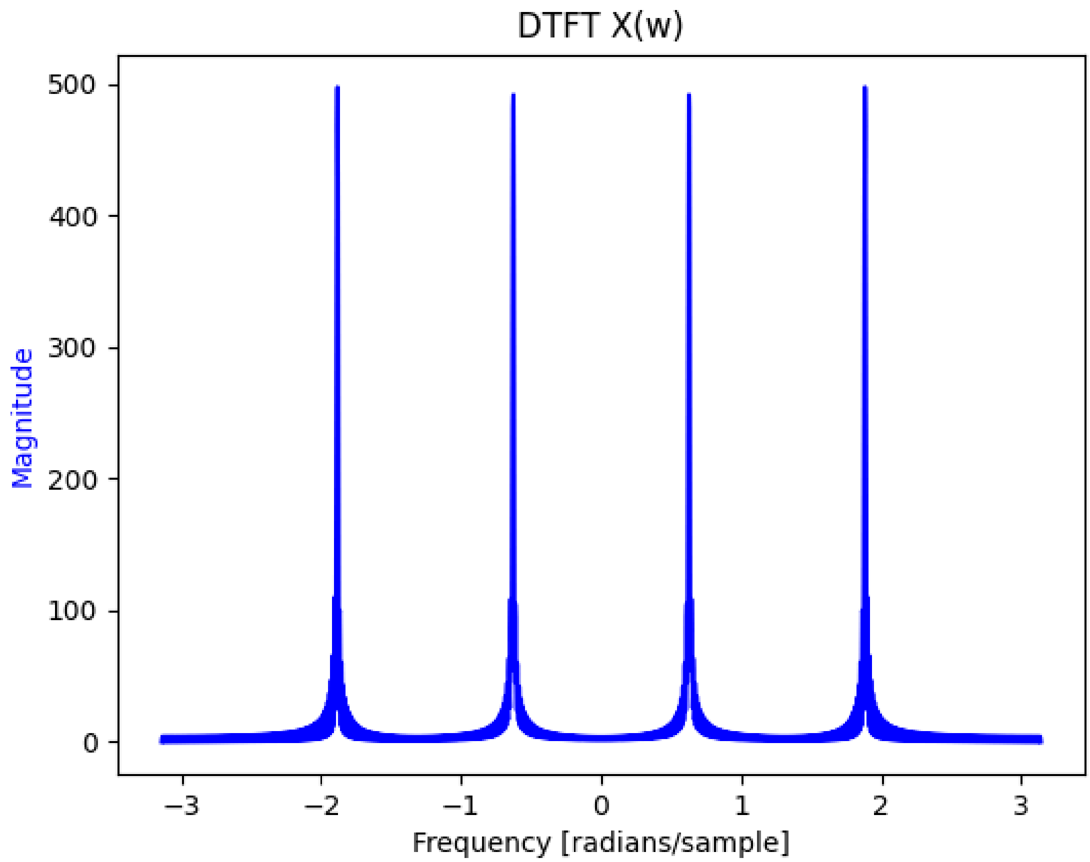
The peaks of the fourier transform correspond to the frequencies of the two cosine functions. At the frequencies which match the cosine functions, the complex exponential is in phase with the corresponding cosine. Since the magnitude of the complex exponential is always 1, the effect is that the summation accumulates all samples of the matching cosine as positive complex numbers of the same magnitude as the original sampled cosine. In general, when the frequency does not match, the summation adds positive and negative complex numbers which tend to cancel each other, and the magnitude of the DTFT is relatively low.

The specific number attached to the magnitude of the peaks is not very useful in analysis, since it is largely dependent on sample rate and sampling window. By increasing sample rate, or extending the duration of the sampled window, there will be more samples to accumulate and increase the value to an arbitrarily large value, without any significant implication about the underlying signal.

Part C:



x(4n)
Magnitude
Sample (n)
4
3
2
1
0
−1
−2
−3
−4
0
10
20
30
40
50


DTFT X(w)
Magnitude
500
400
300
200
100
0
−3
−2
−1
0
1
2
3
Frequency [radians/sample]