EEP518 HOMEWORK 1 APRIL 2, 2024

OPPENHEIM $ SCHAFER: 2.24, 2.47, 2.66, 2.73

2.24

FOR AN ARBITRARY LINEAR SYSTEM WITH INPUT $x[n]$ AND OUTPUT $y[n]$, IF $x[n]=0$ FOR ALL n, THEN $y[n]$ MUST BE ZERO FOR ALL n.

AN ARBITRARY LINEAR SYSTEM IS DECRIBED BY THE CONVOLUTION SUM

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

WHERE THE OUTPUT IS A LINEAR COMBINATION OF INPUT SAMPLES, IF ALL SAMPLES ARE ZERO, THE SUMMATION IS ZERO.

2.47

GIVEN A "WINDOWED COSINE SIGNAL"

$$x[n] = w[n]\cos(\omega_0 n)$$

a) THE FOURIER TRANSFORM IS

$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathcal{F}\{\cos(\omega_0 n)\}\, W(e^{j(\omega-\theta)})\, d\theta$$

BY THE WINDOWING THEOREM, BY TABLE 2.3 SEQUENCE 11,

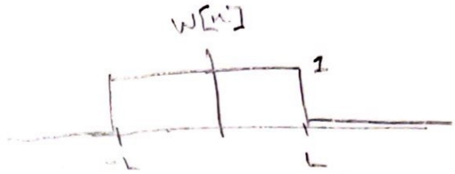
$$\mathcal{F}\{\cos(\omega_0 n)\} = \sum_{k=-\infty}^{\infty} \left[\pi\delta(\omega-\omega_0+2\pi k) + \pi\delta(\omega+\omega_0+2\pi k)\right]$$

b) FOR THE SEQUENCE

$$w[n] = \begin{cases} 1 & -L \le n \le L \\ 0 & \text{OTHERWISE} \end{cases}$$

THE SIGNAL CAN BE WRITTEN AS

$$w[n] = u[n+L] - u[n-L]$$



THE FOURIER TRANSFORM IS, BY TABLE 2.2 THEOREM 1,

$$W(e^{j\omega}) = \mathcal{F}\{u[n+L]\} - \mathcal{F}\{u[n-L]\}$$

WHERE, BY TABLE 2.3 SEQUENCE 5,

$$\mathcal{F}\{u[n]\} = \frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$$

AND, BY TABLE 2.2 THEOREM 2,

$$\mathcal{F}\{u[n-L]\} = e^{-j\omega L}\left(\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)\right)$$

SO,

$$W(e^{j\omega}) = (e^{j\omega L} - e^{-j\omega L})\left(\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)\right)$$

⋮

TRIG IDENTITY?

↓

2.66

THE AUTOCORRELATION OF A SIGNAL IS

$$R_x[n] = \sum_{K=-\infty}^{\infty} x^*[K]x[n+K]$$

a) TO FIND A SIGNAL $g[n]$ SUCH THAT

$$R_x[n] = x[n] * g[n]$$

WE CAN EXPAND THE CONVOLUTION SUM AS

$$R_x[n] = \sum_{K=-\infty}^{\infty} x[K]\,g[n-K] = \sum_{K=-\infty}^{\infty} g[K]\,x[n-K]$$

IF $g[n] = x^*[-n]$

$$R_x[n] = \sum_{K=-\infty}^{K=\infty} x^*[-K]\,x[n-K]$$

BUT K IS ARBITRARY AND INFINITE SO THIS IS EQUIVALENT TO

$$R_x[n] = \sum_{K=-\infty}^{\infty} x^*[K]\,x[n+K]$$

b)

THE FOURIER TRANSFORM OF

$$R_x[n] = x[n] * x^*[-n]$$

IS

$$R_x(e^{j\omega}) = X(e^{j\omega})X^*(e^{j\omega})$$

BY, TABLE 2.1 SEQUENCE 2

SO, BY COMPLEX MULTIPLICATION

$$R_x(e^{j\omega}) = |X(e^{j\omega})|^2$$

2.73

a) $E(e^{j\omega}) = H_1(e^{j\omega})X(e^{j\omega})$

$F(e^{j\omega}) = H_1(e^{-j\omega})X(e^{j\omega})$

$G(e^{j\omega}) = H_1(e^{j\omega})H_1(e^{-j\omega})X(e^{-j\omega})$

$Y(e^{j\omega}) = H_1(e^{-j\omega})H_1(e^{j\omega})X(e^{j\omega})$

b)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = H_1(e^{-j\omega})H_1(e^{j\omega})$$

c) $h[n] = h_1[n] * h_1[-n]$