EE 518 HOMEWORK #2 JAIDON LYBBERT

3.16) $x[n] = \left(\frac{1}{3}\right)^n u[n] + (2)^n u[-n-1]$

$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n]$

a)

TABLE 3.1 #5 & 6 R.O.C

$$X(z) = \frac{1}{1-\left(\frac{1}{3}\right)z^{-1}} - \frac{1}{1-2z^{-1}} \qquad |z| > \frac{1}{3} \quad |z| < 2$$

TABLE 3.1 #5 R.O.C

$$Y(z) = \frac{5}{1-\left(\frac{1}{3}\right)z^{-1}} - \frac{5}{1-\left(\frac{2}{3}\right)z^{-1}} \qquad |z| > \frac{1}{3} \quad |z| > \frac{2}{3}$$

$$X(z) = \frac{(1-2z^{-1}) - (1-\left(\frac{1}{3}\right)z^{-1})}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$$

$$Y(z) = \frac{5(1-\frac{2}{3}z^{-1}) - 5(1-\frac{1}{3}z^{-1})}{(1-\frac{2}{3}z^{-1})(1-\frac{1}{3}z^{-1})} = \frac{(-\frac{10}{3}+\frac{5}{3})\cdots}{\cdots} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{2}{3}z^{-1})(1-\frac{1}{3}z^{-1})}$$

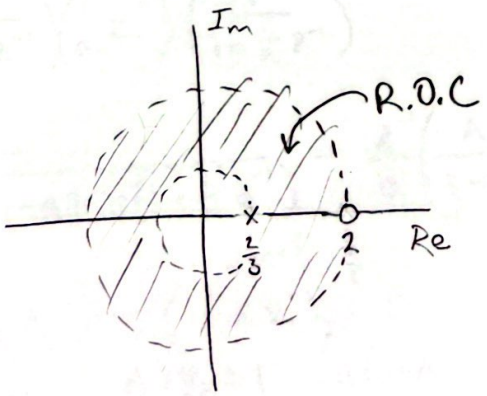
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\left(-\frac{5}{3}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)}{\left(1-\frac{2}{3}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)\left(-\frac{5}{3}z^{-1}\right)} = \frac{(1-2z^{-1})}{(1-\frac{2}{3}z^{-1})}$$

$$H(z) = \frac{(1-2z^{-1})}{(1-\frac{2}{3}z^{-1})}$$

POLES: $z = \frac{2}{3}$

ZEROS: $z = 2$

R.O.C: $\frac{2}{3} < |z| < 2$



b) TABLE 3.1 #5 TABLE 3.2 #2

$$h[n] = Z^{-1}\{H(z)\} = Z^{-1}\left\{\frac{1}{1-\frac{2}{3}z^{-1}} - \frac{2z^{-1}}{(1-\frac{2}{3}z^{-1})}\right\}$$

$$h[n] = \left(\frac{2}{3}\right)^n u[n] - 2\left(\frac{2}{3}\right)^{n-1} u[n-1] \qquad \text{R.O.C: } \frac{2}{3} < |z| < 2$$

c) $n-K-1 > 0$

$$y[n] = \sum_{K=-\infty}^{\infty} x[K]\,h[n-K]$$

$$y[n] = \sum_{K=-\infty}^{\infty} x[K]\left(\frac{2}{3}\right)^{n-K} u[n-K] - x[K]\,2\left(\frac{2}{3}\right)^{n-K-1} u[n-K-1]$$

d) BY SECTION 3.2 PROPERTY 5, THE SYSTEM IS TWO-SIDED AND THEREFORE NOT CAUSAL

3.58)

$$C_{xx}[n] = \sum_{K=-\infty}^{\infty} x[K]\,x[n+K]$$

a) $C_{xx}[n] = x[-n] * x[n]$ — TIME REVERSAL (Pg. 129) → $x[-n] \overset{Z}{\Longleftrightarrow} X\left(\frac{1}{z}\right)$ ROC = $\frac{1}{R_x}$ U ROC = R_x

$$C_{xx}[n] \overset{Z}{\Longleftrightarrow} X\left(\frac{1}{z}\right)X(z) = X(z)X(z^{-1})$$

$$\text{ROC: } \frac{1}{R_x} < |z| < R_x$$

b) $x[n] = a^n u[n]$ $\quad X(z) = \frac{1}{1-az^{-1}}$ $\quad |z| > a$ $\quad X(z^{-1}) = \frac{1}{1-az} = \frac{z^{-1}}{(z^{-1}-a)}$

$$X(z^{-1}) = \frac{1}{a}z^{-1}\frac{1}{1-\frac{1}{a}z^{-1}} \qquad |z| > \frac{1}{a}$$

$$C_{xx}(z) = \left(\frac{1}{1-az^{-1}}\right)\left(\frac{1}{a}z^{-1}\right)\left(\frac{1}{1-\frac{1}{a}z^{-1}}\right)$$

$$C_{xx}(z) = \frac{z^{-1}}{a}\,\frac{1}{(1-az^{-1})(1-\frac{1}{a}z^{-1})} = \frac{z^{-1}}{a}\left(\frac{A}{1-az^{-1}} + \frac{B}{1-\frac{1}{a}z^{-1}}\right)$$

$$A(1-\tfrac{1}{a}z^{-1}) + B(1-az^{-1}) = 1$$

$$A + B = 1 \qquad B = 1-A$$

$$-\frac{A}{a} - Ba = 0$$

$$-\frac{A}{a} - (1-A)a = 0$$

$$A(a-\tfrac{1}{a}) - a = 0$$

$$\rightarrow A = \frac{a}{a-\frac{1}{a}}$$

$$B = 1 - \frac{1}{1-a^2}$$

$$C_{xx}[z] = \left(\frac{1}{a-\frac{1}{a}}\right) z^{-1} \frac{1}{1-az^{-1}} + \left(a+\frac{a}{1-a^2}\right) z^{-1} \frac{1}{1-\frac{1}{a}z^{-1}} \qquad a<|z|$$

$$C_{xx}[n] = \left(\frac{1}{a-\frac{1}{a}}\right) a^{n-1} u[n-1] + \left(a+\frac{a}{1-a^2}\right)\left(\frac{1}{a}\right)^{n-1} u[n-1]$$

c) A TIME REVERSED INPUT $x[n]$ WOULD HAVE THE SAME $C_{xx}[n]$

$$x[n] = x[-n] = a^{-n} u[-n]$$

d) A TIME SHIFTED INPUT $x_1[n]$ WOULD HAVE THE SAME $C_{xx}[n]$

$$x_1[n] = x[n-k] = a^{n-k} u[n-k]$$

M1)

PROBLEM 3.32(a) Pg. 145 DO PFD

$$X(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-2z^{-1}\right)\left(1-3z^{-1}\right)} = \frac{A}{\left(1+\frac{1}{2}z^{-1}\right)^2} + \frac{B}{\left(1-2z^{-1}\right)} + \frac{C}{\left(1-3z^{-1}\right)} + \frac{D}{1+\frac{1}{2}z^{-1}}$$

$1-\frac{3}{2}-\frac{1}{2}$

$$A\left(1-2z^{-1}\right)\left(1-3z^{-1}\right) + B\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-3z^{-1}\right) + C\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-2z^{-1}\right) + D\left(1+\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)\left(1-3z^{-1}\right) = 1$$

$z=2$

$$\left(\frac{5}{4}\right)^2\left(-\frac{1}{2}\right)B = 1$$

$$\left(\frac{5}{4}\right)^2\left(-\frac{1}{2}\right)B = 1$$

$$-\left(\frac{25}{16}\right)\left(\frac{1}{2}\right)B = 1$$

$$-\frac{25}{32}B = 1$$

$$B = \frac{32}{-25}$$

$$B = -1.28$$

$z=3$

$$\left(\frac{7}{6}\right)^2\left(\frac{1}{3}\right)C = 1$$

$$\left(\frac{49}{36}\right)\left(\frac{1}{3}\right)C = 1$$

$$\frac{49}{108}C = 1$$

$$C = \frac{108}{49}$$

$$C = 2.2$$

$z=-\frac{1}{2}$

$$(1+4)(1+6)A = 1$$

$$(5)(7)A = 1$$

$$A = \frac{1}{35}$$

$$A = 0.028$$

$$A+B+C+D = 1$$

$$D = 1-A-B-C$$

$$D = 0.047$$

$$x(z) = \frac{0.028}{\left(1+\frac{1}{2}z^{-1}\right)^2} - \frac{1.28}{\left(1-2z^{-1}\right)} + \frac{2.2}{\left(1-3z^{-1}\right)} + \frac{0.047}{\left(1+\frac{1}{2}z^{-1}\right)}$$

$$X(z) = \frac{1}{(1+z^{-1}+\frac{1}{4}z^{-2})(1-5z^{-1}+6z^{-2})} = \frac{1}{1-5z^{-1}+6z^{-2}+z^{-1}-5z^{-2}+6z^{-3}+\frac{1}{4}z^{-2}-\frac{5}{4}z^{-3}+\frac{3}{2}z^{-4}}$$

$$X(z) = \frac{1}{1+(-5+1)z^{-1}+(6-5+\frac{1}{4})z^{-2}+(6-\frac{5}{4})z^{-3}+\frac{3}{2}z^{-4}}$$

SCIPY.SIGNAL.RESIDUEZ

$$X(z) = \frac{0.047}{1+0.5z^{-1}} + \frac{0.0286}{(1+0.5z^{-1})^2} - \frac{1.28}{1-2z^{-1}} + \frac{2.20}{1-3z^{-1}}$$

M2)

a) $y[n] = -1.85\cos(\frac{\pi}{18})y[n-1] + 0.83y[n-2] = x[n] + \frac{1}{3}x[n-1]$

$$a_0y[n] + a_1y[n-1] + a_2y[n-2] = b_0x[n] + b_1x[n-1]$$

$$a_0Y(z) + a_1z^{-1}Y(z) + a_2z^{-2}Y(z) = b_0X(z) + b_1z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = (a_0+a_1z^{-1}+a_2z^{-2})Y(z) = (b_0+b_1z^{-1})X(z)$$

$$H(z) = \frac{b_0+b_1z^{-1}}{a_0+a_1z^{-1}+a_2z^{-2}}$$

$$H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-1.85\cos(\frac{\pi}{18})z^{-1}+0.83z^{-2}}$$

b) SEE PLOTS.
FILTER APPEARS TO BE LOW-PASS.

c) SEE PLOTS.

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PS C:\Users\Jaidon Lybbert\source\DSP-UW-SP24\homework> python .\hw2.py
a:  [1, -4, 1.25, 4.75, 1.5]
b:  [1, 0, 0, 0, 0]
r:  [ 0.04734694  0.02857143 -1.28        2.20408163]
p:  [-0.5 -0.5  2.   3. ]
k:  []
```

Fig. 1 Problem M1 - partial fraction decomposition with scipy.signal.residuez().

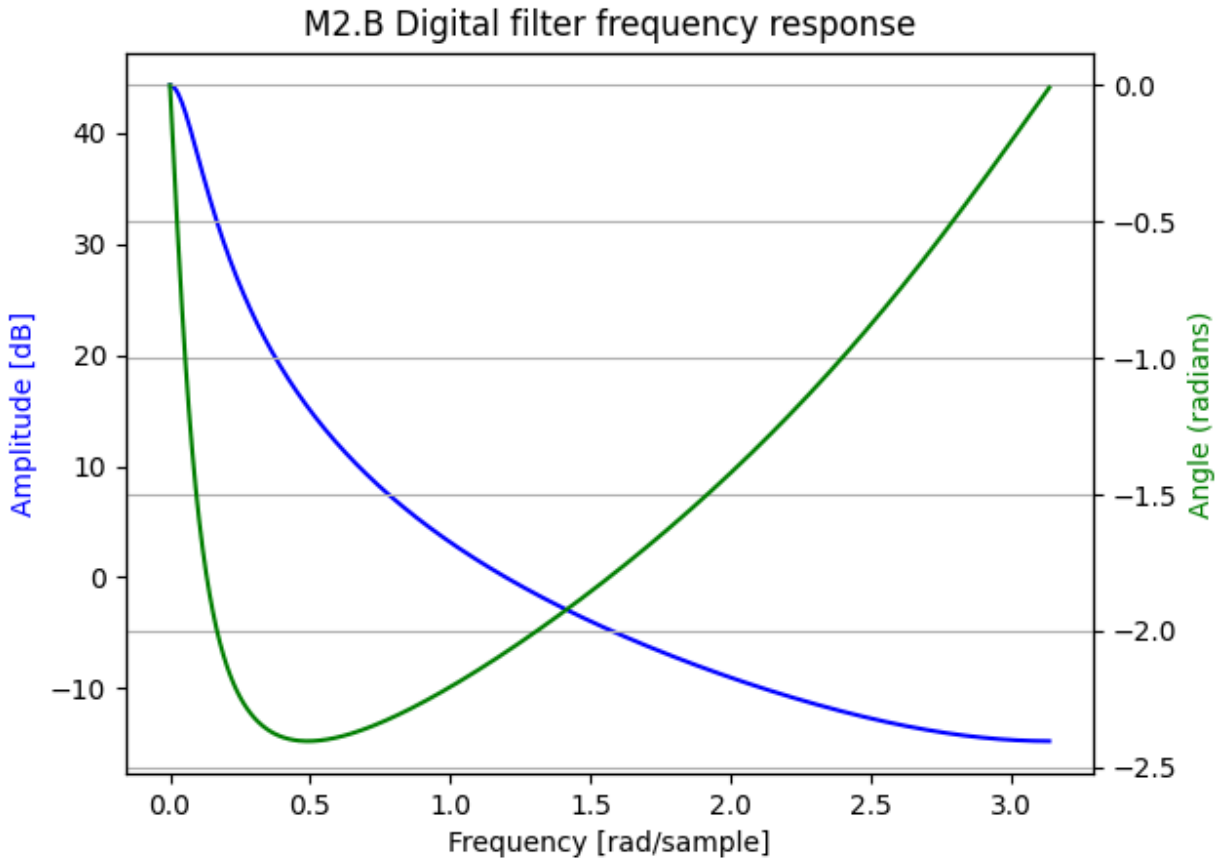

Fig. 2 Problem M2 part B, magnitude and frequency response.

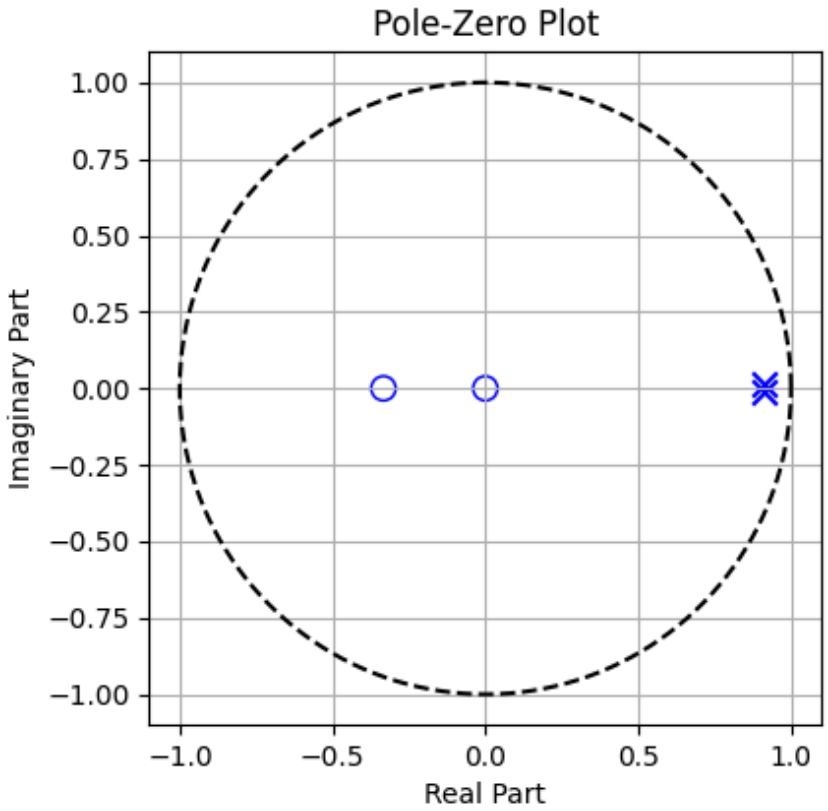

Fig. 3 Problem M2 part C, pole-zero plot.