Primes and Primality Testing

Number Theory and Cryptography Workshop
Maths and Physics Club, IIT Bombay

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- Define a sequence by $a(1) = 1$, $a(2) = 11, a(3) = 111, a(4) = 1111$ and so on. Let p be a prime. Define $A_p = n, p|a(n)$. Prove that, if $p = 2$ or $p = 5$, then the set A_p is empty, and is an infinite set for all other primes. Bonus: For a given prime p, find the set A_p. (Difficulty level- easy)
- An integer k is an even perfect number if and only if it has the form $2^{n-1}(2^n - 1)$ and $2^n - 1$ is prime. (Difficulty level- moderate)
- For each integer $a \geq 2$, there are infinitely many Fermat pseudoprimes base a. (In other words, for a given a, there are infinitely many n such that n is composite and $a^{n-1} \equiv 1(modn)$. (Difficulty level- hard)