# Resolving Fundamental Issues of Slowness in Envelope Simulation Methods

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**ABSTRACT: Fourier-envelope algorithms are an important component of the mixed-signal/RF verification toolbox. In this article the unpredictability and lack of robustness that has been reported for these algorithms is addressed. It is shown that the problem stems from "slow" envelopes that fundamentally feature fast oscillations. We demonstrate that this is related to the fact that the envelope equations are always stiff, whether or not the underlying system is. We show that careful choice of the envelope initial conditions is necessary to obtain useful solutions, and propose two techniques for finding good initial conditions. By applying these techniques and solving the envelope equations with stiffly-stable numerical methods, we obtain a robust and reliable Fourier-envelope method. The new methods are illustrated by a direct-downconversion mixer circuit.**  Int J RF and Microwave CAE 15: 371–381, 2005.

**Keywords:** simulation; RF; multitone; multitime methods

## I. INTRODUCTION

A long-standing problem of considerable importance in mixed-signal/RF simulation and verification is that of predicting the slow envelopes of circuit responses. Envelopes are of interest, for example, when investigating startup transients of circuits that have rapid oscillations which change amplitude/phase only much more slowly. Another area of application is in communication circuits, where fast carrier waveforms are modulated by much slower information signals.

Finding these envelopes via initial-value simulation of differential equations (for example, using SPICE's transient simulation capability) is often not practical because of the widely-separated time scales of the waveforms involved. Initial value-solution techniques are constrained to taking timesteps that are much smaller than the time period of the fastest components of the signal. Hence, an excessive number of time points can be required to track the envelopes, which are often slower by orders of magnitude. As a result, numerical algorithms that alleviate this limitation have been sought.

Several such techniques have appeared in the literature. The earliest appears to be the time-domain envelope-following technique proposed in [7], and later adapted for circuit applications in [5]. So-called Fourier-envelope methods, which compute slowly-varying Fourier components of fast oscillations as they change with time, were proposed in 1996 [3, 6, 16]. Shortly after, multitime PDE techniques were used to place Fourier-envelope methods on a better mathematical footing, and also to propose novel time-domain envelope methods [14, 15]. Following their advent, Fourier-envelope techniques were implemented in commercial simulation tools, notably those from Agilent Technologies.

Despite this progress, practitioners have observed that Fourier-envelope techniques do not always succeed in finding slow envelopes in a predictable and

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robust manner. Even for some extremely simple linear circuits, it has been noted that Fourier-envelope simulation can produce (instead of the smooth envelope expected) large and seemingly random oscillations [9]. This behaviour is apparently also present in Agilent's ADS simulator [13]. Yet, at the same time, the Fourier envelope appears to produce reasonable results for many circuits. The resultant prevailing uncertainty about the correctness and reliability of Fourier-envelope methods appears to have hindered their adoption.

This article addresses the issue of incorrect and strange behaviour in Fourier-envelope methods, and proposes remedies to make them robust and reliable. We reproduce the undesirable phenomena, understand their causes, and apply this understanding to circumvent them. In determining the causes, we find that the root of the problem goes deeper than purely numerical issues. We show that contrary to intuition, the exact solution of the Fourier-envelope equations need not be slowly varying and, indeed, can be expected to have fast oscillations in the typical case. We prove another fact that relates to this phenomenon, namely, that the Fourier-envelope differential equations are very stiff even when the underlying circuit has no stiffness. However, we show that rapid oscillations can be reduced to insignificance via careful selection of envelope initial conditions.

Based on the above observations, we advocate the use of stiffly-stable integration techniques for Fourier-envelope equations. We propose two methods for the important task of finding "good" envelope initial conditions to eliminate rapid envelope oscillations. The methods are based on mapping initial conditions between orthogonal lines in the multitime domain and the use of artificial damping, respectively.

These enhancements have a significant positive impact on the reliability of Fourier-envelope techniques. We present an application of the enhanced method to a direct-downconversion mixer circuit. We are able to predict 10-kb/s bitstreams recovered from a 900-MHz carrier in the presence of slow gain modulation.

Throughout this article, multitime PDE concepts are used for both diagnosis and circumvention of Fourier-envelope problems. The remainder of the article is organized as follows. In section II, we demonstrate the failure of existing Fourier-envelope techniques. In section III, we develop an understanding of the underlying phenomena responsible for this failure. In section IV, we propose and discuss remedial enhancements, and in section V, we demonstrate their application.

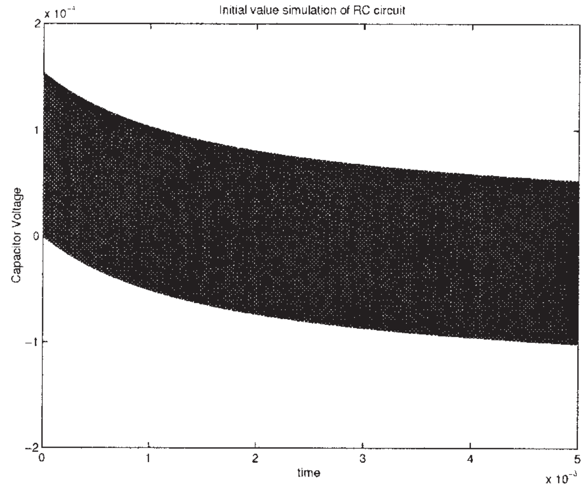



**Figure 1.** Standard test ODE: transient simulation.

## II. FAILURE IN FOURIER-ENVELOPE METHODS

Consider the standard test problem for investigating stability and accuracy properties of linear multistep integration methods [2, 11]:

$$\dot{x} = \lambda x + b(t). \tag{1}$$

In circuit terms, this corresponds to a simple RC network, with a capacitor voltage of $x(t)$, and driven by a voltage source with value $b(t)$. For illustration, we will use the values

$$\lambda = 10^3, \quad b(t) = \sin(2\pi 10^6 t), \tag{2}$$

corresponding to an RC time constant of 1 ms and a sinusoidal excitation of 1 MHz.

A solution of eq. (1) with initial condition $x(t = 0) = 0$ is shown in Figure 1. As expected, the 1-MHz excitation results in a "fast response" that varies at the same rate; due to the slow RC time-constant, however, there is also a decaying transient "slow response" that eventually dies out in this case. This is the classic widely-separated time-scale issue, a challenge for transient simulation because of the large number of timepoints required.

The goal of envelope methods is to predict the waveform in Figure 1 more efficiently than lengthy initial-value simulations. Fourier envelope is particularly attractive for this system, since the response can be captured exactly in the form

$$\begin{aligned} x(t) &= X_0(t) + a(t)\sin(2\pi 10^6 t + \phi(t)) \\ &= X_0(t) + X_1(t)e^{j2\pi 10^6 t} + X_{-1}(t)e^{-j2\pi 10^6 t}, \end{aligned} \tag{3}$$

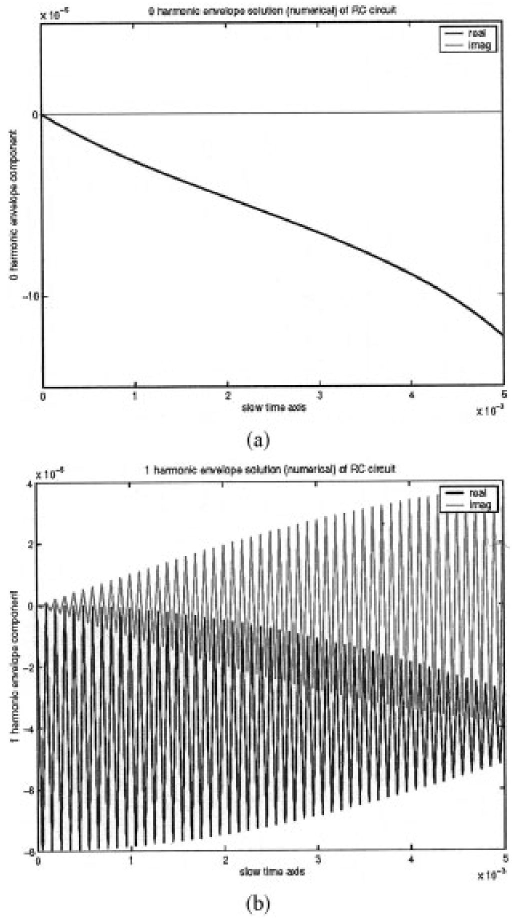



**Figure 2.** Test ODE: Fourier envelope simulation: (a) DC component; (b) 1st harmonic.

where $X_0(t)$ is called the DC envelope, while $a(t)$, $\phi(t)$, $X_1(t)$, and $X_{-1}(t)$ are various components of the first harmonic envelope. For envelope simulation to be useful, these must all be slowly-varying quantities. Simply from examination of Figure 1, it may be expected that the DC envelope for this system should be a slowly-decaying curve, while the first harmonic envelope (that is, the amplitude of the oscillations) should remain substantially constant.

Unfortunately, when existing Fourier-envelope techniques [3, 16] are applied to this problem, the waveforms obtained can be very different. The results of Fourier-envelope simulation [3] with zero initial condition are shown in Figure 2. Not only is the DC envelope substantially different from that expected, the 1st-harmonic envelope shows large oscillations that appear clearly unphysical. This phenomenon has been observed by practitioners [9, 13] in commercial tools implementing Fourier envelope methods, including Agilent's ADS simulator.

Such undesirable phenomena appear to have limited the application of envelope methods. To the author's knowledge, a clear understanding of the causes of this unpredictable envelope behaviour has not been arrived at to date. An important basic question is whether these undesirable phenomena are caused by purely numerical issues, or whether they have other causes as well.

In the remainder of this article, we first provide an answer to the abovementioned question, and then propose and evaluate techniques to circumvent the undesirable phenomena of Figure 2.

## III. NONSLOW ENVELOPES, AND STIFF SYSTEMS

To investigate the problem identified above, we use the mathematical framework of multitime partial-differential equations (MPDEs, see, for example, [1, 15]). The MPDE formulation is useful because it provides an underlying unifying basis from which a number of envelope simulation techniques, including Fourier envelope, can be derived [15]. An MPDE corresponding to eq. (1) is

$$\frac{d}{dt_1}\hat{x}(t_1, t_2) + \frac{d}{dt_2}\hat{x}(t_1, t_2) = \lambda\hat{x} + \sin(2\pi 10^6 t_2). \tag{4}$$

Recall that the usefulness of eq. (4) stems from that if $\hat{x}(t_1, t_2)$ is a solution, then $x(t) = \hat{x}(t, t)$ solves eq. (1). In the above MPDE, $t_2$ is the artificial time scale corresponding to the 1-MHz fast variations, while $t_1$ captures slow variations. The envelope should then be a slow function of $t_1$.

To find a unique envelope solution of eq. (4), an envelope initial condition must be specified (akin to the initial condition for eq. (1)). We set the envelope initial condition at $t_1 = 0$ to be

$$\hat{x}(t_1 = 0, t_2) \equiv 0. \tag{5}$$

Note that this envelope initial condition is consistent with the zero initial condition used for the initial value solution of eq. (1).

### A. Rapid Undulations in Envelopes

For the simple MPDE (4) with initial condition (5), an exact analytical solution can easily be found; it is given by

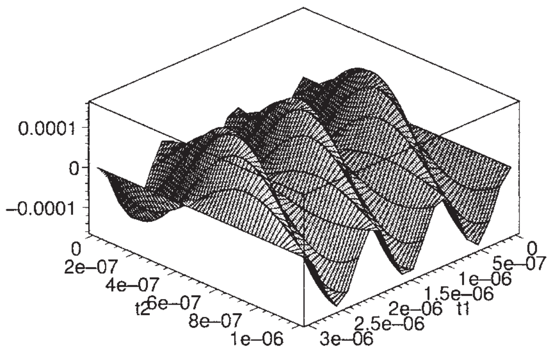



**Figure 3.** Standard test ODE: analytical multitime solution.

$$\hat{x}(t_1, t_2) = \lambda \frac{e^{-(\lambda + j2\pi 10^6)t_1} - 1}{4\pi 10^6 - 2j\lambda} e^{j2\pi 10^6 t_2} + \text{c.c. terms.} \tag{6}$$

Observe that the $t_2$-variation of eq. (6) is a pure sinusoid at 1 MHz, and periodic with period 1 $\mu$s. $\hat{x}(t_1, t_2)$ is plotted in Figure 3 for $t_1 = [0, 3\ \mu\text{s}]$.

Figure 3 reveals an unexpected phenomenon: the "slow" envelope, that is, a variation of $\hat{x}(t_1, t_2)$ with respect to $t_1$, contains fast oscillations of frequency about 1 MHz. This is contrary to the fundamental expectation for envelope techniques, that is, that the envelope will vary slowly. We remind the reader that Figure 3 is plotted from the exact analytical solution (6), and is therefore not affected by any possible numerical artifacts. This is borne out by the multitime envelope-simulation results shown in Figure 4.

We now turn to the manifestation of this phenomenon in Fourier-envelope methods. The Fourier-envelope equations corresponding to eq. (4) are given by

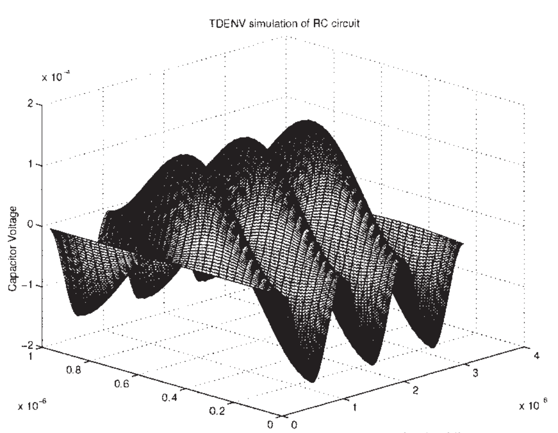



**Figure 4.** Standard test ODE: short multitime envelope simulation.

$$\frac{d}{dt} X_0(t) = -\lambda X_0(t),$$

$$\frac{d}{dt} X_1(t) = -(\lambda + j2\pi 10^6) X_1(t) - \frac{1}{2} j\lambda,$$

$$\frac{d}{dt} X_{-1}(t) = -(\lambda - j2\pi 10^6) X_{-1}(t) + \frac{1}{2} j\lambda. \tag{7}$$

The solution of these equations with initial condition $X_i(0) = 0$ corresponds to the multitime solution (6), from which it is apparent that the Fourier envelope solution is given by

$$X_0(t) \equiv 0, \quad X_1(t) = \bar{X}_{-1}(t) = \lambda \frac{e^{-(\lambda + j2\pi 10^6)t} - 1}{4\pi 10^6 - 2j\lambda}. \tag{8}$$

Observe that rapid undulations (of 1-MHz frequency) are present in the envelope waveform given by eq. (8); these are verified by a few fast cycles of Fourier-envelope simulation, as shown in Figure 5.

## B. Envelope Initial Conditions

The above observations begin to shed light on the undesired phenomena of Figure 2. Clearly, if the analytically-obtained envelope has fast oscillations that persist, it is only to be expected that numerical integration with time-steps much larger than the oscillation period will provide meaningless, artifact-ridden results.

Note, however, that eqs. (6) and (8) are not the only possible envelope solutions to this problem. As noted previously, the envelope is determined as much by the initial condition (5) as by the system itself. The only requirement on the envelope initial condition $\hat{x}(t_1 = 0, t_2)$ is that it be consistent with the original problem's initial condition at $t = 0$, that is, $\hat{x}(t_1 = 0, 0) = x(t = 0)$. This leaves open the possibility that with appropriate choice of envelope initial condition, the envelope solution can be made free of fast undulations.

To illustrate this, we return to the Fourier-envelope equations given by eq. (7). Instead of the previous initial condition $X_0(0) = X_1(0) = X_{-1}(0) \equiv 0$, consider the alternative initial condition:

$$X_1(0) = \frac{\lambda}{2j\lambda - 4\pi 10^6}, \quad X_{-1}(0) = \bar{X}_1(0),$$

$$X_0(0) = -2\Re(X_1(0)). \tag{9}$$

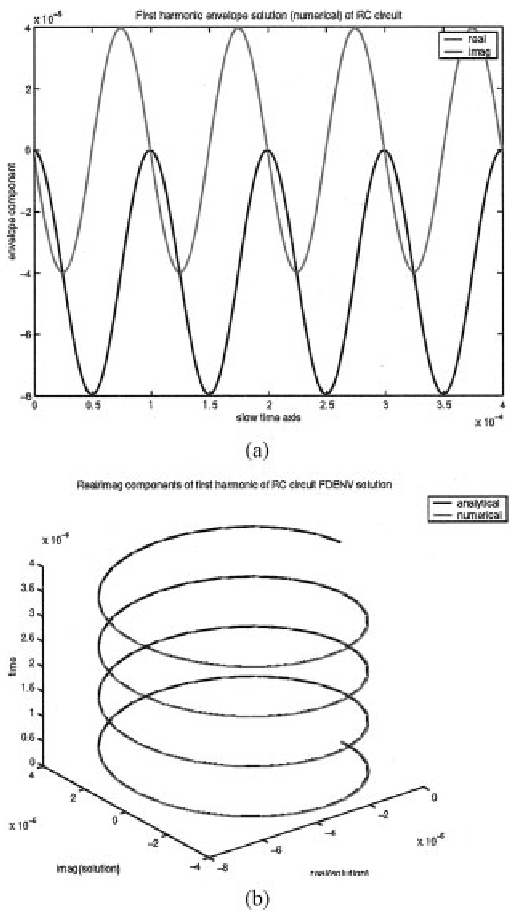



**Figure 5.** Short Fourier-envelope simulation of test ODE: (a) 1st harmonic envelope; (b) numerical vs. analytical.

It may be verified that this new envelope initial condition leads to the envelope solution

$$X_1(t) \equiv X_1(0) \;\; \forall\, t, \quad X_{-1}(t) = \bar{X}_1(t), \quad X_0(t) = X_0(0)e^{-\lambda t}. \quad (10)$$

Unlike (8), the envelopes (10) do not contain fast variations, and it is thus meaningful to use numerical methods to find them.

## C. Envelope Equations Are Stiff

Having established that proper initial conditions are critical for obtaining meaningful and slowly-varying envelopes, we now turn our attention to properties of the Fourier-envelope equations that may impact their numerical solution.

We first note that the test problem (1) has only a single intrinsic time-constant $1/\lambda$, which in our illustrative example is 1 ms. The problem is therefore not stiff.* It is the fact that the external input is far more rapid than the system's intrinsic relaxation rates that creates a widely-separated time-scale problem requiring envelope methods for efficient solution.

Now consider the corresponding Fourier-envelope equations given by eq. (7). Note that the input to each of these equations has no fast variations (being constants); this is the feature that enables the envelope equations, potentially, to have slowly-varying solutions. However, note that the intrinsic time-constants of the envelope equations are now widely separated. For example, in the equation for $X_1(t)$, the real part of the time constant is $\lambda = 10^3$, while the imaginary part is $2\pi 10^6$. The fast-time variation of the input to (1) has been transformed into an intrinsic time-constant in the envelope equations.

In other words, even though the original circuit or DAE problem was not stiff, the Fourier-envelope equations (7) are. It is the stiffness that manifests itself as fast oscillations that decay only as slowly as (potentially) the slowest time-constant of the original system. It is also apparent that the larger the number of harmonics considered in the Fourier-envelope method, the stiffer the Fourier-envelope equations become.

## D. The Main Issues

The main insights obtained in this section are as follows.

- The exact solution of envelope equations can have rapid variations in addition to slow ones, thereby undermining the purpose of envelope simulation. Without first addressing this basic issue, it is pointless to seek or apply 'better' numerical techniques.
- The nature of the envelope solution, in particular its fast variations, depends strongly on the envelope initial condition applied. The possibility remains that with appropriate initial conditions, the envelope solution will have *only* slow variations.
- Even when using initial conditions that eliminate fast variations, care must be taken to use numerical integration methods appropriate for stiff problems, to avoid numerical artifacts stemming from stiffness of the Fourier-envelope equations.

In the next section, we propose techniques to address these issues and devise a robust Fourier-envelope method.

* Recall that differential equations are stiff if they have widely-separated intrinsic time-constants or eigenvalues [2, 4].

## IV. ROBUST FOURIER ENVELOPE

As pointed out in the previous section, the two main tasks in devising a robust Fourier-envelope method are (i) to find envelope initial conditions that result in slowly-varying solutions, and (ii) to use stiffly-stable integration techniques for solving the envelope equations numerically.

The latter task is the easier to address. A wide body of literature on effective numerical methods is available for stably integrating stiff differential equations (for example, see [2, 4, 8, 11]). In this work, we use the 2nd-order Backward Differentiation Formulae (BDF), better known as Gear's method.

The task of finding good envelope initial conditions for a general system is more challenging. In the remainder of this section, we present two approaches towards this end. The first uses several short transient simulations of the underlying DAE to find an initial condition for the envelope equations. The second exploits numerical damping properties of "overstable" integration methods, specifically the Backward Euler (BE) method in this work, to arrive at an envelope initial condition. We apply both methods, in succession, for the best results.

To facilitate exposition, the remainder of this section uses the simple illustrative example already encountered in sections II and III. It should be kept in mind, however, that the techniques proposed are motivated by, and applicable to, the general problem posed by the potentially large system of DAEs:

$$\dot{q}(x) + f(x) + b(t) = 0, \tag{11}$$

and its corresponding MPDE:

$$\left(\frac{d}{dt_1} + \frac{d}{dt_2}\right) q(\hat{x}) + f(\hat{x}) + \hat{b}(t_1, t_2) = 0, \tag{12}$$

descriptions adequate not only for circuit applications, but for a variety of optical, mechanical, and mixed-domain problems.

### A. Mapping Slow-Timescale Initial Conditions

The insight behind the first approach is best grasped using multiple time scales. Consider Figure 3. The envelope initial condition is specified along the entire $t_2$ line, at $t_1 = 0$; in the figure, this condition is identically zero, which as noted in the previous section, leads to the fast variations along $t_1$.

The only real constraint on the envelope initial condition is that it be consistent with whatever initial condition is specified for the underlying DAE problem (1)—in this case, that the value at $(t_1 = 0, t_2 = 0)$ is zero.

Furthermore, there is no real necessity that the envelope initial condition be specified at exactly $t_1 = 0$, so long as consistency with the DAE initial condition is retained. It could, for instance, be specified along the $t_2$ line at $t_1 = T_2$, where $T_2$ is the period of the fast time scale—1 $\mu$s in our example.

The key idea behind finding a "good" envelope initial condition is the following: instead of directly specifying a (hard-to-guess) envelope initial condition along the $t_2$ line at $t_1 = 0$, (i) specify instead a slowly-varying initial condition along a short segment of the $t_1$ line, specifically $t_1 = [0, T_2]$ at $t_2 = 0$, and then (ii) map this slow $t_1$ initial condition to the $t_2$ line at $t_1 = T_2$. The envelope simulation is then started from $t_1 = T_2$ instead of $t_1 = 0$.

The mapping between the two lines is carried out simply through transient simulations (that is, initial-value solutions of the underlying DAE). This is possible because diagonal lines along the $(t_1, t_2)$ plane of the MPDE correspond to initial value solutions of the underlying DAE with appropriate time-shifts to the excitation term [15].

The idea is illustrated in Figure 6. Using the underlying DAE initial condition at $(t_1, t_2) = (0, 0)$, a transient simulation (that is, initial value DAE solution) is first carried out to $(t_1, t_2) = (T_2, T_2)$. This is a short transient simulation, corresponding to one fast cycle. Periodicity with respect to $t_2$ implies that the solution at $(t_1, t_2) = (T_2, 0)$ has been obtained. A smooth curve is now fitted between the end-points $(t_1, t_2) = (0, 0)$ and $(t_1, t_2) = (T_2, 0)$ to obtain the slowly-varying initial condition along $t_1 = [0, T_2]$, $t_2 = 0$.

Starting with these initial conditions, a number of transient simulations are now carried out for periods less than $T_2$, as indicated by the diagonal arrows in Figure 6. The initial condition at $t_1 = T_2$, $t_2 = [0, T_2]$ is thus obtained, in which the envelope simulation along $t_1$, employing large timesteps, is initiated.

Figure 7 shows the initial condition obtained by this procedure for the test problem (1), as both a time-domain waveform and its Fourier components. It can be seen that the Fourier components are virtually identical to those in (9), even though they were obtained by a general procedure that does not rely on direct knowledge of the specific system. The Fourier-envelope simulation started with this initial condition (using a stiffly-stable integration method) are shown in Figure 8; observe that the slowly-varying solutions

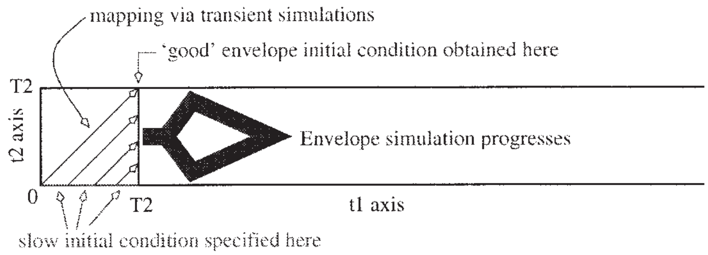


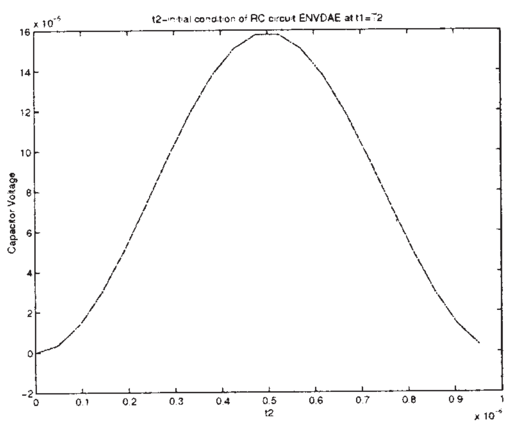

**Figure 6.** Slow ICs along $t_1$, mapped to 'good' $t_2$ ICs at $t_1 = T_2$.

are consistent both qualitatively and quantitatively with Figure 1 and eq. (10).

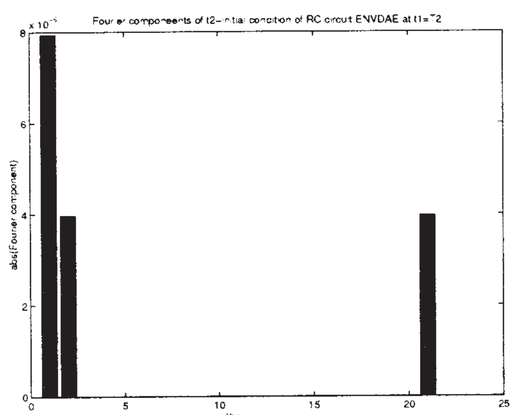
By construction, this mapping technique ensures that the envelope solution obtained is consistent with the specified initial condition of the underlying DAE problem. On the other hand, it does not guarantee an envelope initial condition that completely eliminates fast ringing, although these are substantially reduced in practice. Extensions of the mapping, using transient simulations for a few multiples of $T_2$, can produce even better initial conditions. However, when exact consistency with the underlying DAE's initial condition is not mandatory and approximate consistency suffices (as is often the case in applications), using artificial numerical damping to augment mapping, as described below, is simple to implement and proves beneficial in practice.

## B. Exploiting Artificial Numerical Damping

It is a well-known fact [2, 11] that certain numerical integration methods, like Backward Euler (BE), are

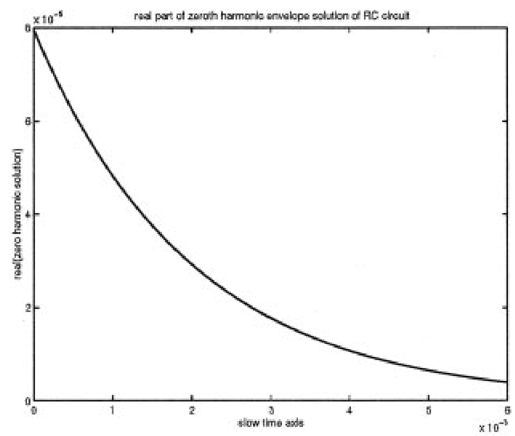



(a)

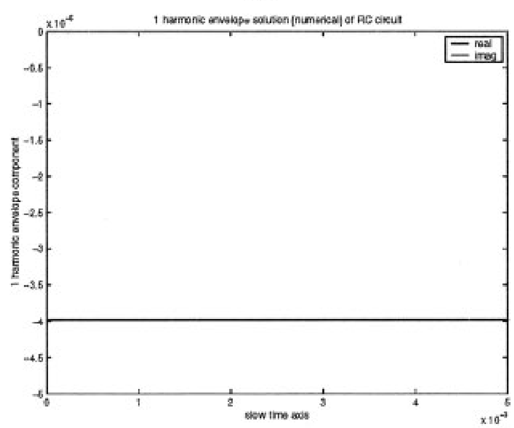



(b)

**Figure 7.** Test ODE: mapped envelope initial condition at $t_1 = 1$ $\mu$s: (a) time domain; (b) frequency domain.




(a)




(b)

**Figure 8.** Test ODE: Fourier envelope components: (a) DC component; (b) 1$^{st}$ harmonic.

"overly stable," that is, they can artificially damp out growing or persistent oscillatory solutions. Since the resulting solutions are wrong, these artifacts are not desired in most situations. Particularly when simulating oscillators, for instance, BE is notorious for requiring extremely small timesteps to achieve any semblance of accuracy, and is therefore invariably to be avoided.

It is instructive, however, to examine the mechanism and features of this artificial damping. In essence, the damping is caused by the introduction of spurious real (decaying) time-constants, of approximately the same order as the timesteps taken. If the timesteps taken are of the same order as the fast oscillatory time-constants, these are therefore rapidly eliminated within a few cycles. However, these spurious decaying time-constants have negligible impact on waveforms with much slower time-constants.

This insight suggests a way to use artificial damping to advantage for finding good envelope initial conditions. A few fast cycles of simulation using a numerical technique such as BE can be used to eliminate fast variations in the envelope. After the fast oscillations have (quickly) died out, the remaining solution can be used as an initial condition to restart the envelope simulation with accurate numerical techniques and much larger timesteps.

The new initial condition obtained via artificial damping is not consistent with the originally specified DAE initial condition. If, however, the artificial damping procedure is itself started with the mapped envelope initial condition previously described, the oscillations that are damped out are already relatively small, hence the damped initial condition is approximately consistent with the specified DAE initial condition, to within the amplitude of the small oscillations that are damped out. This translates to a small change in the originally specified DAE initial conditions, which is often acceptable in applications requiring envelope simulation.

Figure 9(a) shows the effect of using BE with 20 timesteps per fast cycle on the MPDE (4) with a zero-envelope initial condition. The rapid elimination of $t_1$ oscillations can be seen. The damping solution is compared with the correct analytical solution in Figure 9(b), which depicts a slice of the multitime solution, as it varies with $t_1$, at $t_2 \sim 0.5T_2$.

For the test ODE problem, the mapped initial condition described previously is excellent; hence, there is little extra benefit to applying the damping procedure in addition. The damping procedure is valuable in more complex circuits, however, as shown in section V.

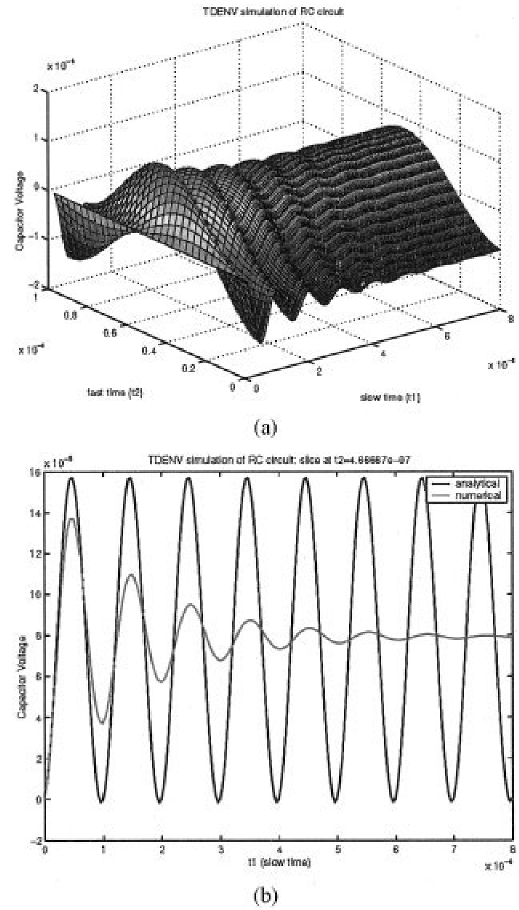
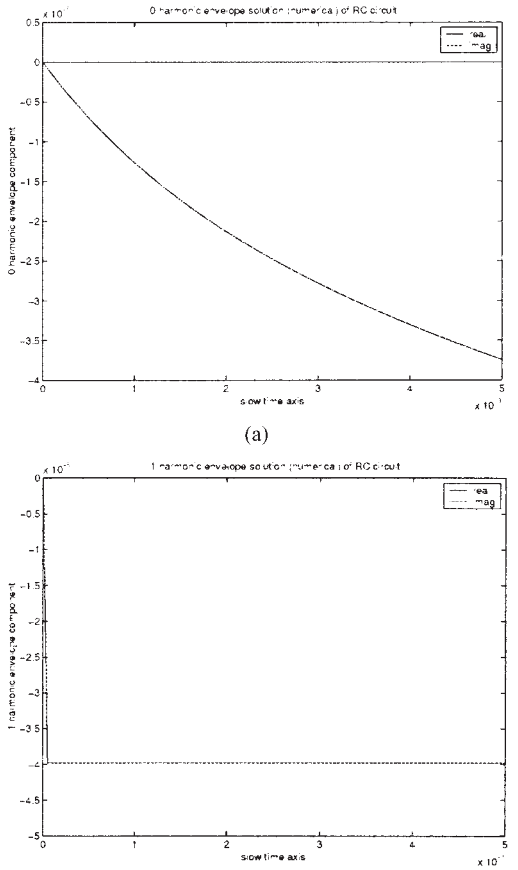



**Figure 9.** Test ODE: artificial damping by Backward Euler: (a) multitime solution; (b) slice along $t_2 \sim 0.5T_2$.

It is also interesting to note what happens when BE is applied directly to the Fourier-envelope equations with large timesteps, without first selecting good envelope initial conditions. The artificial damping property of BE eliminates fast oscillations within a single timestep, as seen in Figure 10(b). The DC envelope obtained in Figure 10(a), though slowly-varying, is however very different from the correct solution shown in Figure 8. This underscores the importance of paying careful attention to envelope initial conditions, even when the envelope solution is slowly varying and appears reasonable at first sight.

## V. EXPERIMENTAL RESULTS

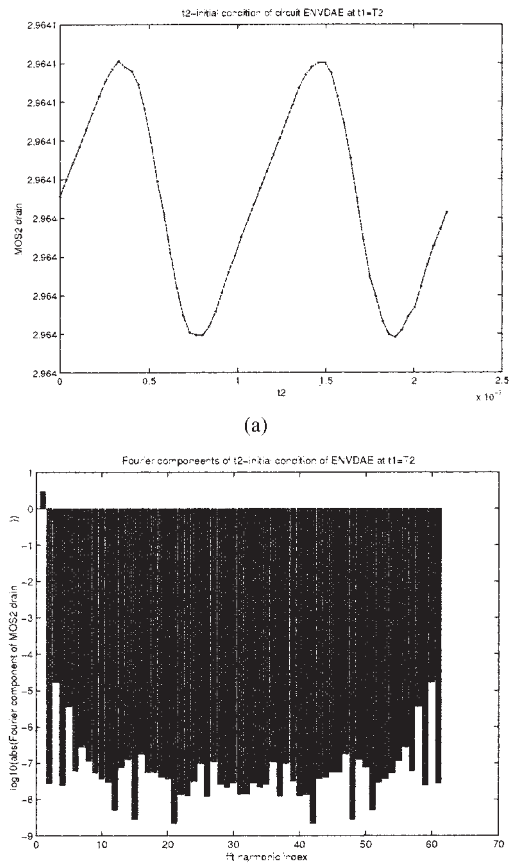
The circuit shown in Figure 11 is a balanced direct-downconversion mixer adapted from [17]. An important feature of this circuit is that the lower pair of MOSFET's constitutes a frequency doubler, generating a current at twice the LO frequency. This current



(a)

(b)

**Figure 10.** Test ODE: blind application of large-timestep BE: (a) DC component; (b) 1st harmonic.



(a)

(b)

**Figure 12.** Mapped envelope initial condition: (a) time domain; (b) frequency domain.

feeds the differential pair formed by the upper two MOSFETs, thus resulting in mixing and down-conversion. The LO signal is a sinusoid at 450 MHz, while the RF signal consists of a 900-MHz carrier modulated by a 10-kb/s bitstream. Furthermore, the

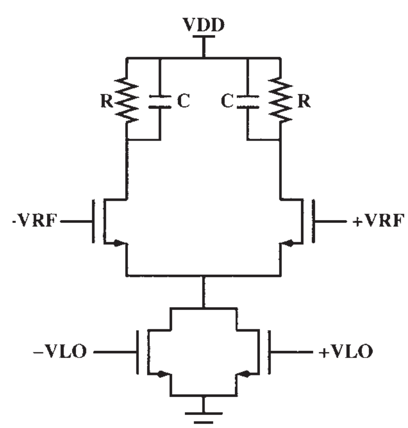



**Figure 11.** Balanced CMOS down-conversion mixer [10].

gain of the downconversion is changed over the bitstream by slowly changing the LO signal amplitude.

A Fourier-envelope simulation was started from the DC initial condition of the circuit. The DC initial condition was first mapped, using the procedure described in section IV.A., to an envelope initial condition at $t_1 = T_2 = 2.2$ ns, the time-period corresponding to 450 Mhz. Eighty short transient simulations were employed in the mapping. The mapped envelope initial condition at the drain of one of the upper MOSFETs is shown in Figure 12. The frequency-doubling action of the lower pair of MOSFETs is apparent. The initial condition was reduced to 16 Fourier components for the envelope simulation.

Figure 13 shows the DC and 2nd-harmonic Fourier components at the same output node, as they vary slowly with time. The initial startup transient of the down-converted bitstream is apparent in the DC component. The effect of slowly varying the downconversion gain can also be seen from the changing amplitude of the bits. The 900-MHz 2nd-harmonic component is largely filtered away at the outputs, but

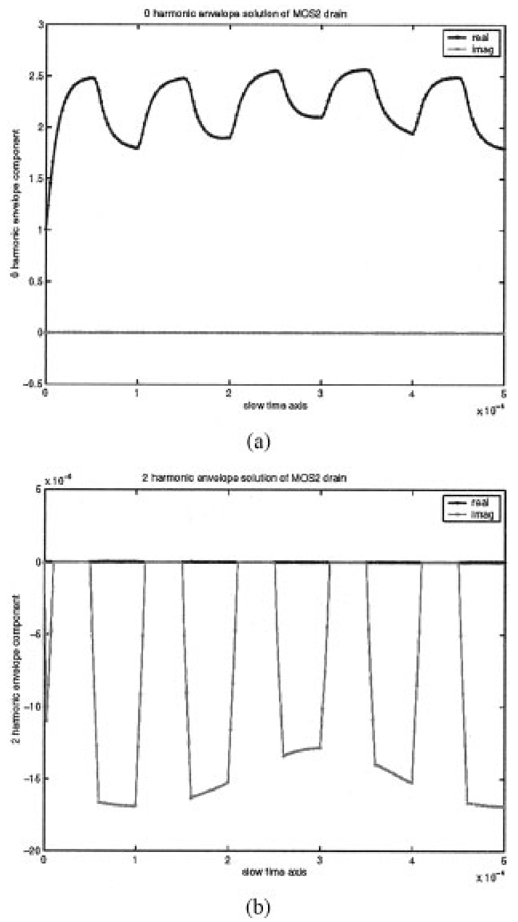



**Figure 13.** Fourier envelope simulation of mixer: (a) DC component; (b) $2^{nd}$ harmonic.

the small residual values can be seen to follow the same pattern as the down-converted bitstream.

## VI. CONCLUSION

Full exploitation of Fourier-envelope techniques has been hindered by robustness issues to date. In this article, we have clarified the mechanisms responsible for problems in these algorithms and proposed effective remedies. The techniques presented in this article, it is expected, will substantially resolve "numerical" robustness problems in envelope algorithms, thereby enabling their wider application.

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