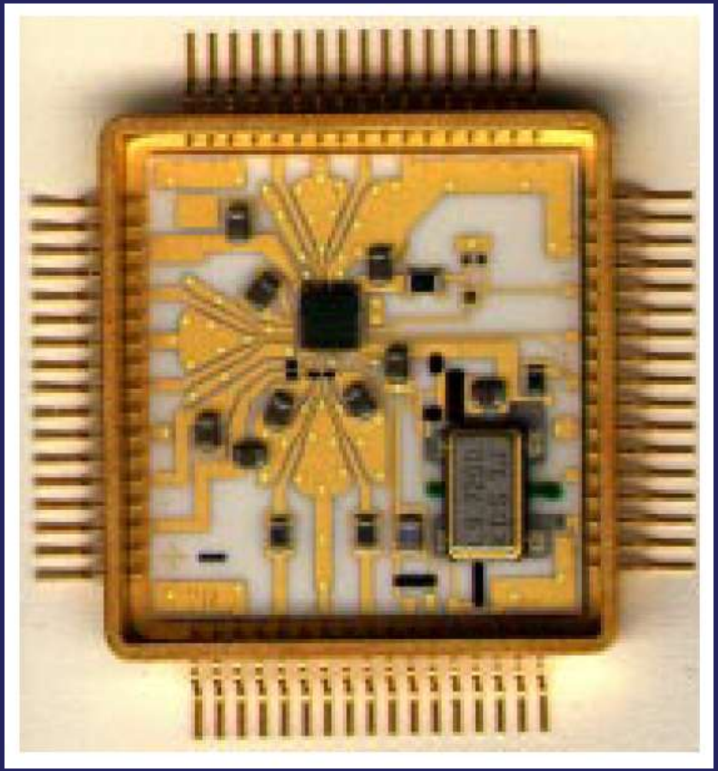
Predicting PLL Power Supply Jitter Using Nonlinear Oscillator Macromodels

Jaijeet Roychowdhury
University of Minnesota

(joint work with Xiaolue Lai)

PLLs: Commodity, High-Margin

Rick Walker, HP/Agilent



Rick Walker, HP/Agilent

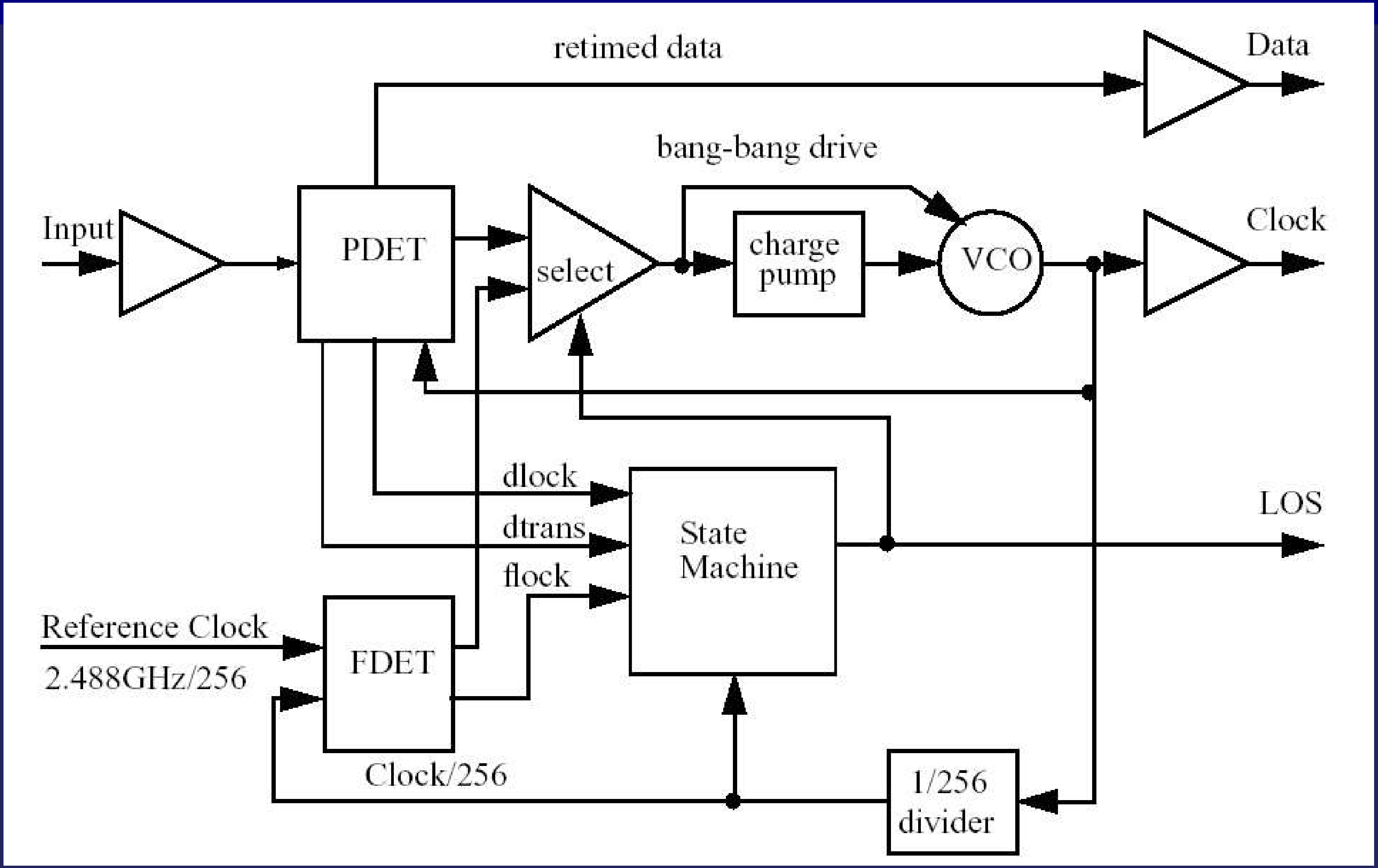
1.25Gbps Ethernet xcvr
<< $10

2.44Gbps SONET CDR
$500

PLL Applications

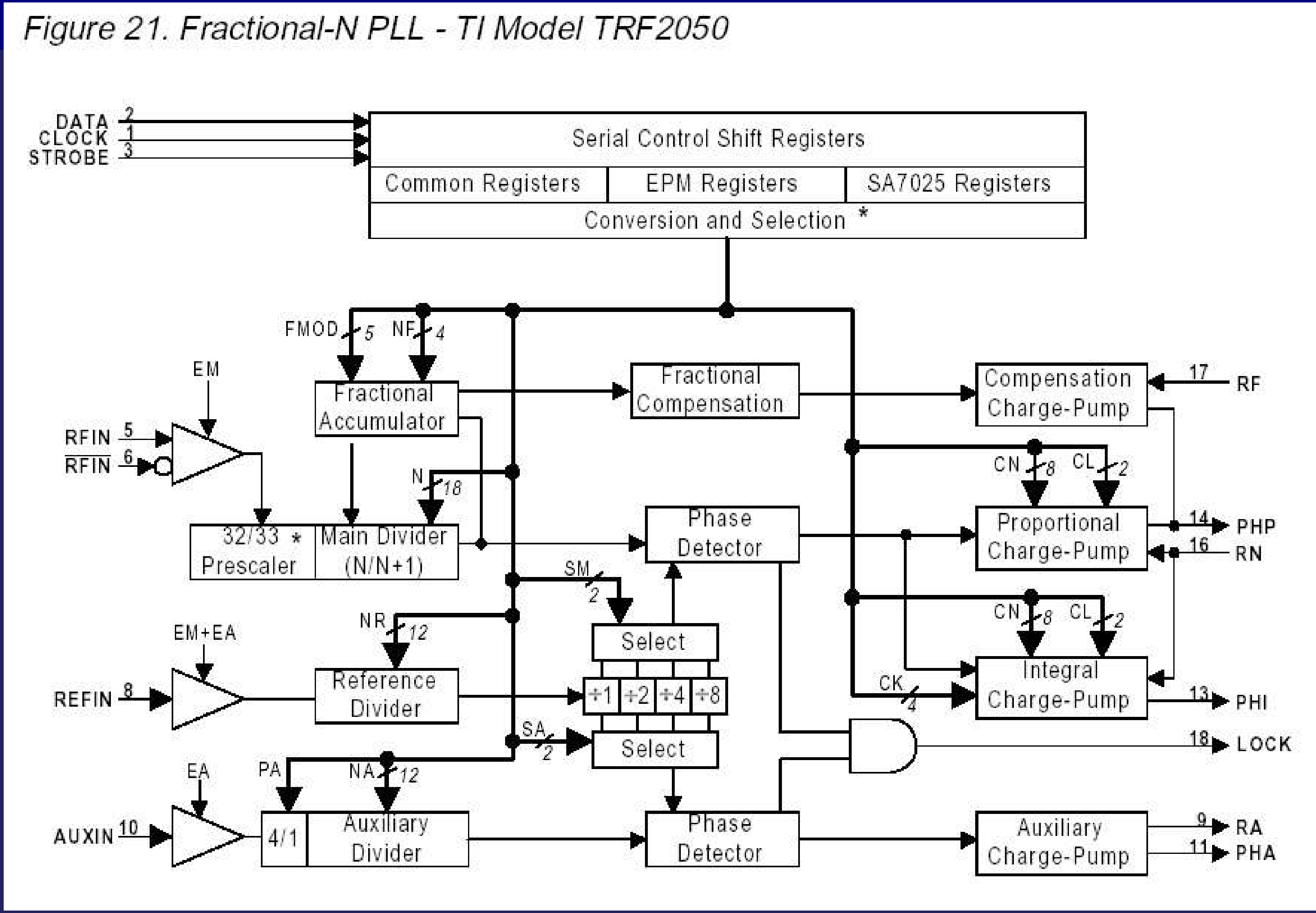
- Clock and data recovery (CDR)
  - synchronization: Costas Loop
  - applications: communication systems, disk-drive read-channels
- Frequency synthesizers
  - integer-N, fractional-N synthesizers
  - direct digital to GPSK modulation

Clock and Data Recovery (CDR) ckt



Mixed-signal PLL: very hard to simulate at ckt level

TI TRF-2050 Fractional-N PLL



Texas Instruments

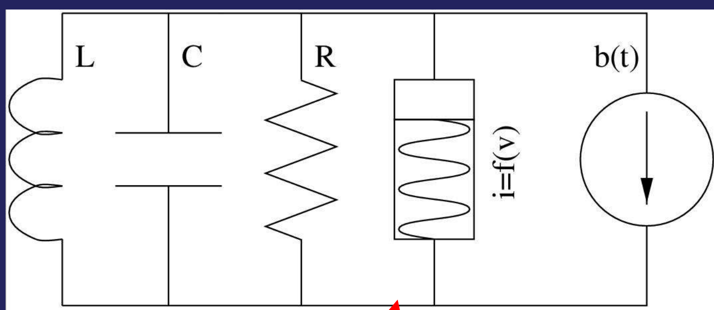
Large; complex; many interconnected functional blocks

PLL Simulation: Difficulties

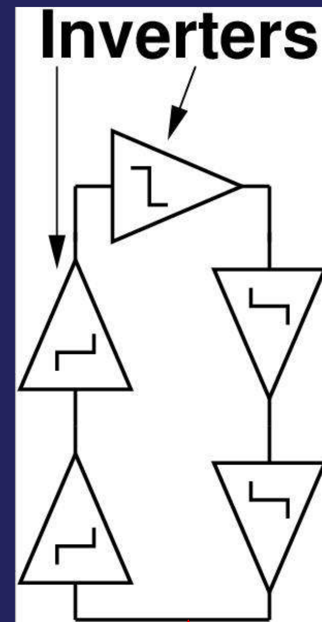
- SPICE: Inaccurate + extremely time-consuming
  - biggest single culprit: VCO
    - 100K cycles of VCO typical for PLL simulation
  - combination of digital (PFD, dividers) and complicated analog (charge pump, VCO)
  - noise characterization (jitter) critical and difficult
  - capture/lock-in/freq-hopping dynamic phenomena
  - Injection-locking: 1st order PLL
- Full system with PLLs: very difficult today
  - Current macromodel-based approaches ad-hoc, manual, cannot capture dynamics, nonlinearity well

Oscillators

- Voltage-controlled oscillators (VCOs) are the central component of PLLs:
  - LC oscillators
  - Ring oscillators



-ve resistance LC oscillator

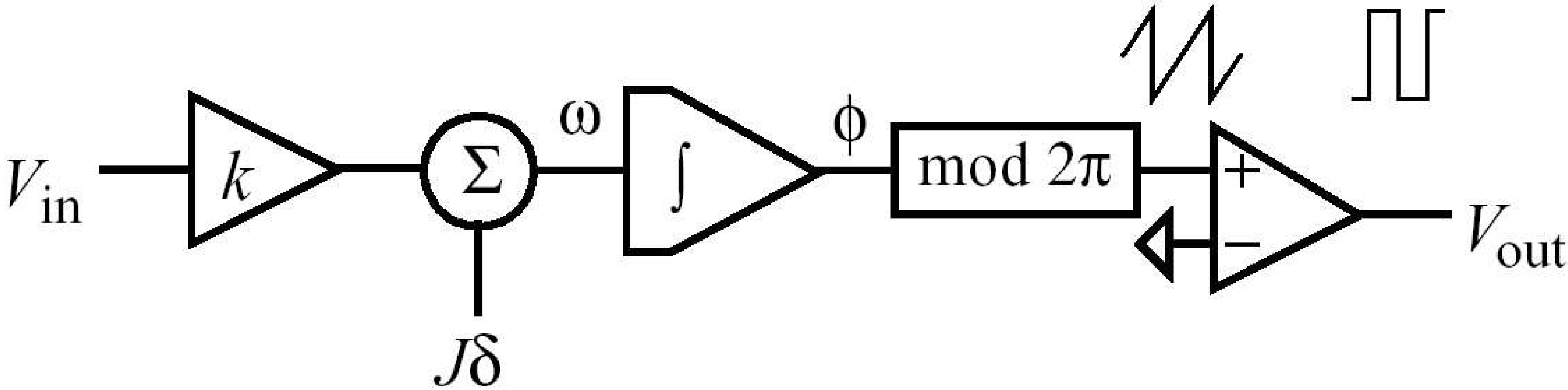


Ring oscillator

Why Oscillators are a Special Simulation Challenge

- Computation/size/accuracy: much greater than amps/mixers
- Even 1-transistor oscillators (eg, UHF oscs, >100GHz)
  - long startups, tiny timesteps needed
- On-chip RF: 100s to 1000s of transistors
  - VERY challenging to simulate
- Oscillators feature complex phenomena: injection locking
  - oscillator's frequency “locks” to frequency of external input
  - if frequencies close enough, even if input is very small
  - can take extremely long to simulate
  - universal phenomenon: grandfather clocks, fireflies flashing, etc
- Macromodelling (esp phase) offers dramatic speedup
  - Even for 1-transistor oscillator

Linear Macromodelling of VCOs/PLLs



Ken Kundert, Cadence

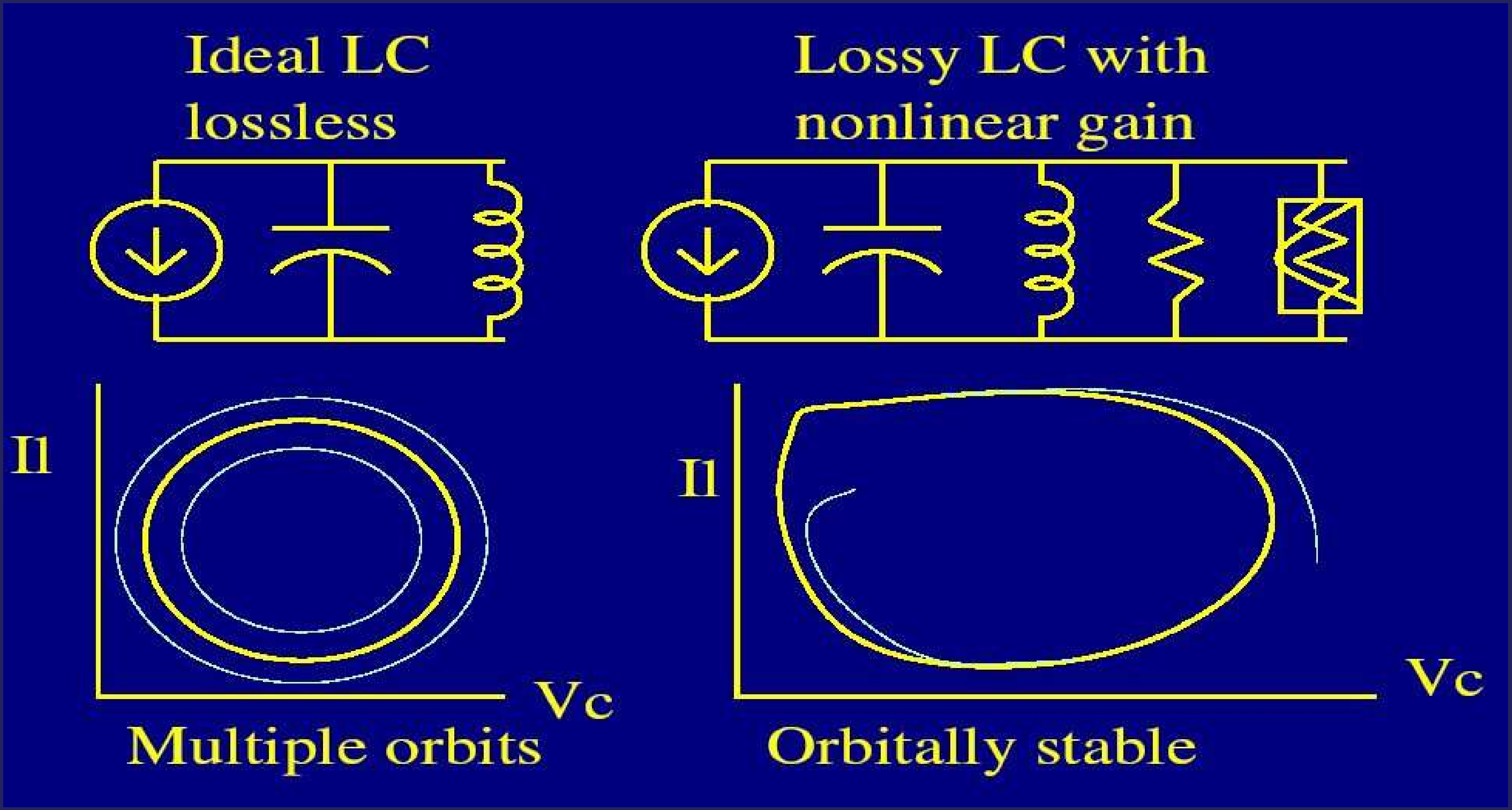
[Kundert 02]

- Manually generated VCO macromodel
- Output VCO phase = integral of input control
- Linear => cannot capture nonlinear phenomena (injection locking, jumps, cycle slipping, etc)
- Good for intuition, hand calculations, noise trends
- Can be grossly wrong for jitter caused by power grid/substrate

Automated Nonlinear Oscillator Macromodelling

- Nonlinear: **accurate** I/O capture
  - Injection locking, phase noise spectrum, ...
- Automated generation
  - **SPICE in, macromodel out (Verilog-A, MATLAB, SPICE, etc)**
- Small size
  - **Very fast to simulate compared to full SPICE oscillator circuit**

"Ideal" vs Orbitally Stable Oscillators



Nonlinearity cannot be ignored – fundamental to oscillator operation

Quantifying Oscillator Response

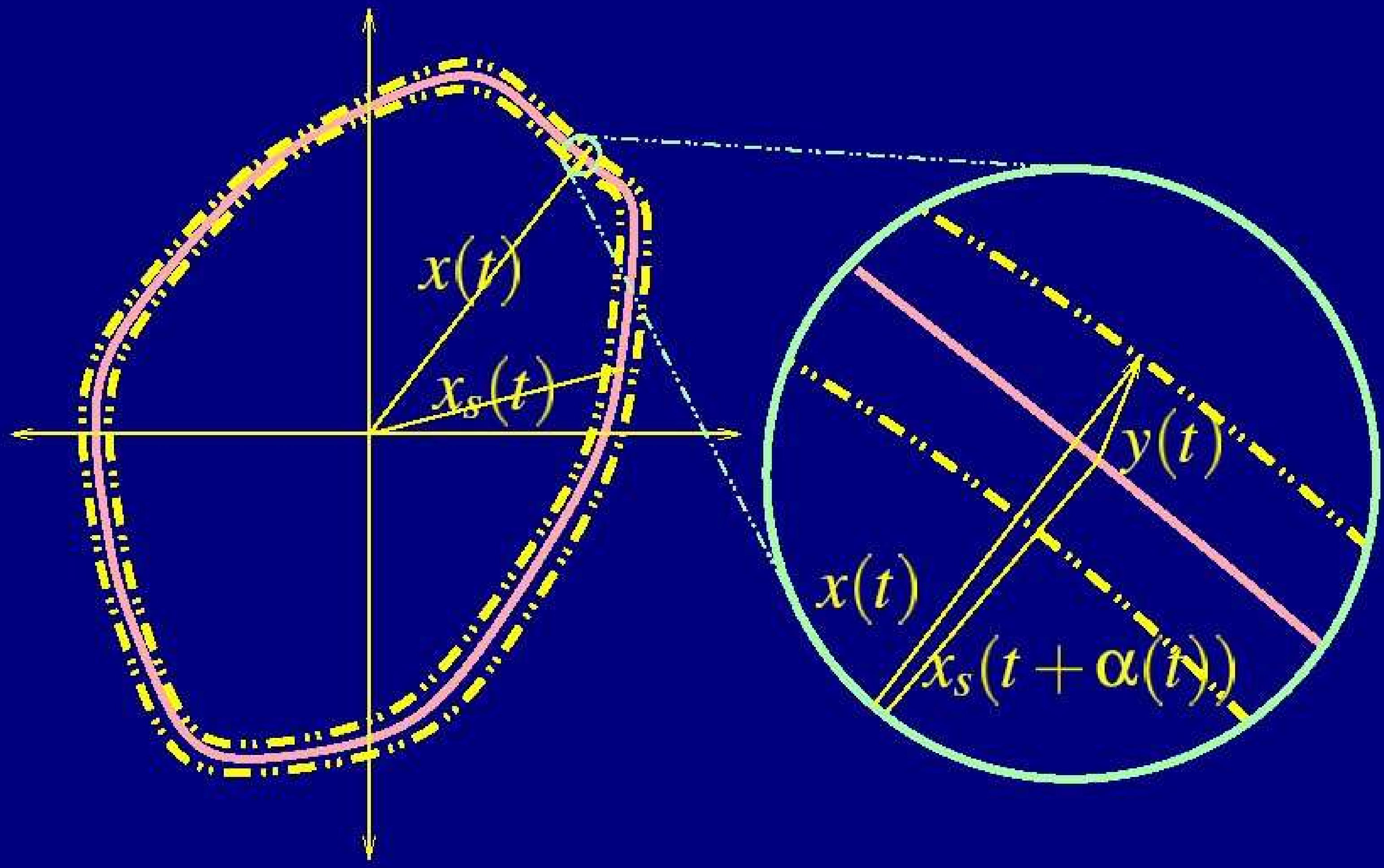
How does the oscillator (VCO) respond to “inputs”?

$$\dot{x}(t) = f(x) + \underbrace{b(t)}_{\text{input perturbation}}$$

- No perturbation $\Rightarrow$ perfect periodic solution $x_s(t)$
- Small $b(t)$ perturbation:

$$x(t) = x_s(t + \underbrace{\alpha(t)}_{\text{growing phase error}}) + \underbrace{y(t)}_{\text{small}}$$

Oscillator: Response to “Inputs”



- Phase error $\alpha(t)$ *shifts track increasingly along limit cycle*
- $y(t)$ creates deviations from limit cycle that *remain small*

Nonlinear Differential Equation for Phase

$$\boxed{\dot{\alpha}(t) = v_1(t + \alpha(t)) \cdot b(t)}$$

- *Scalar, nonlinear ODE* governs $\alpha(t)$
- $v_1(\cdot)$ is the *Perturbation Projection Vector (PPV)*
- *Projection* of noise perturbation onto PPV determines phase error growth
- PPV is *not obviously related* to anything!
  - periodic Floquet eigenvector of time-varying (linearized) adjoint system
  - PPV can be found from *purely LPTV analysis*
- But: periodicity of PPV makes α equation nonlinear

[Demir Mehrotra Roychowdhury 97, 01]

The Perturbation Projection Vector

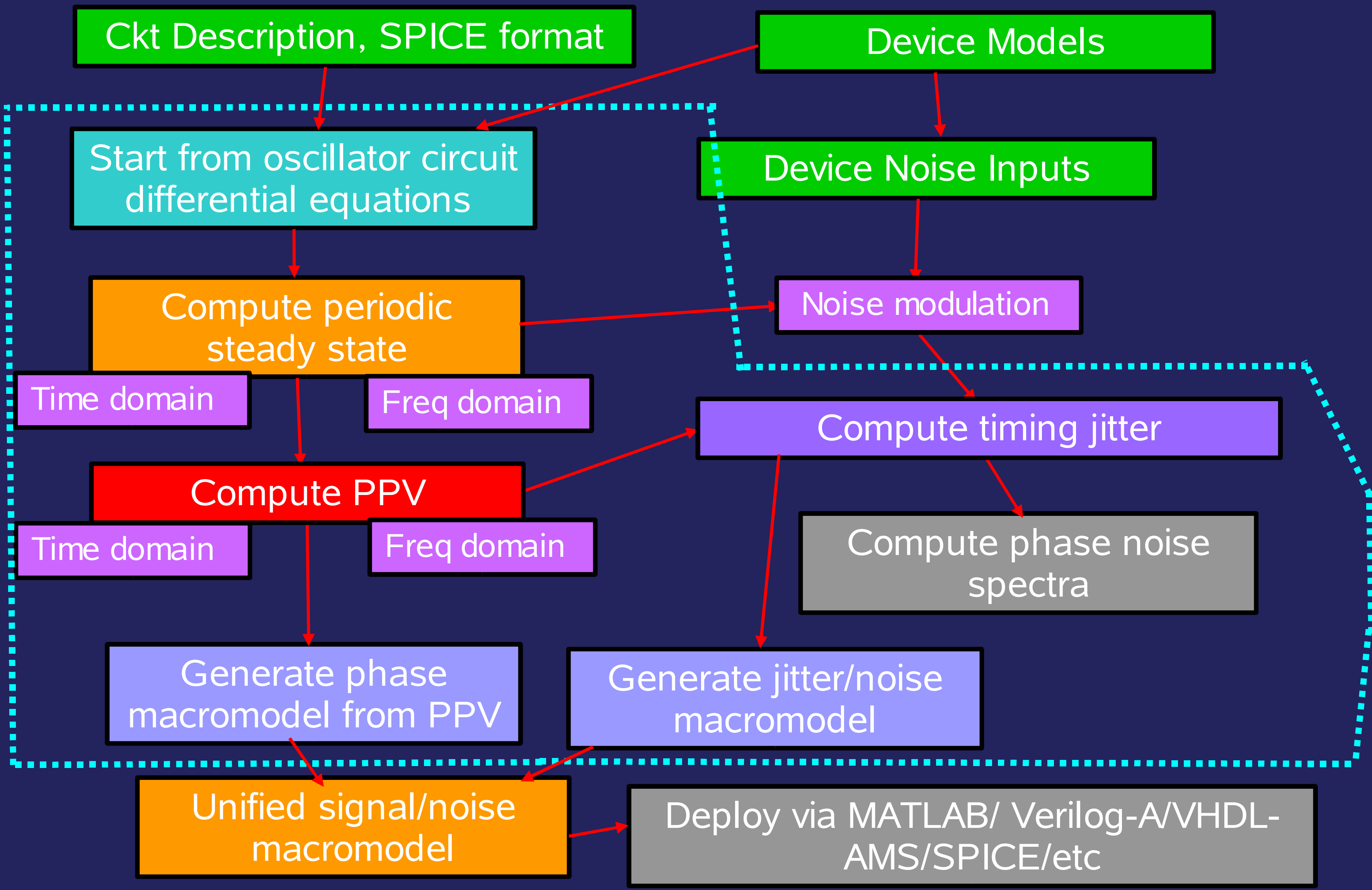
- $v_1(t)$: "transfer function" relating "input" to oscillator phase response
  - termed the PPV: Perturbation Projection Vector
  - procedure for calculating the PPV is not obvious
    - but computationally efficient
  - In general, PPV does NOT equal the tangent vector of the phase plane plot
    - ie, not equal to the "ISF" [Hajimiri 98]

Computing the PPV

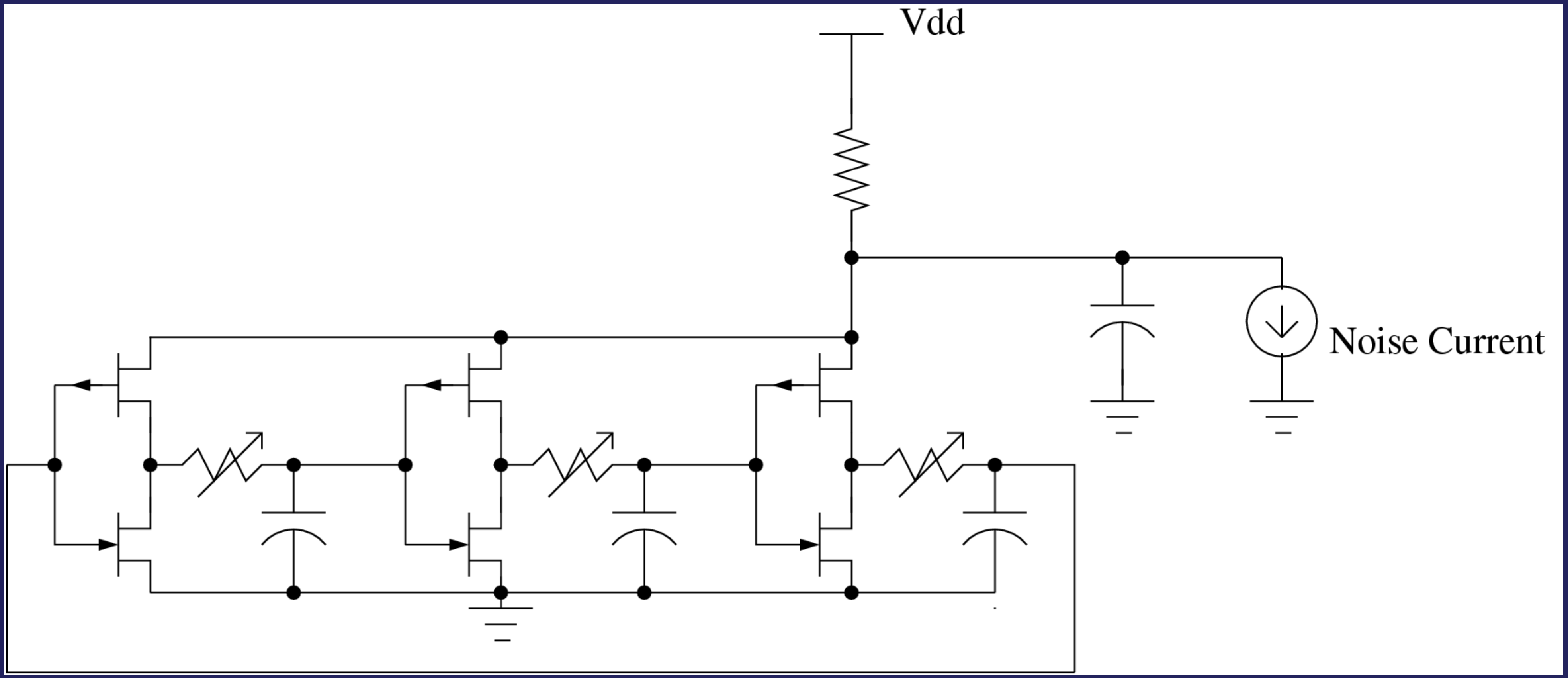
- PPV can be computed efficiently from oscillator steady-state quantities
  - first: find the periodic steady-state of oscillator
    - using, eg, HB, shooting, etc.
  - then obtain the same G(t) and C(t) used in LTV reduction
  - form a large block matrix A from "samples" of G(t) and C(t)
  - perform one single linear matrix solution with A
    - can be performed efficiently for large oscillators

[Demir Roychowdhury TCAD 03]

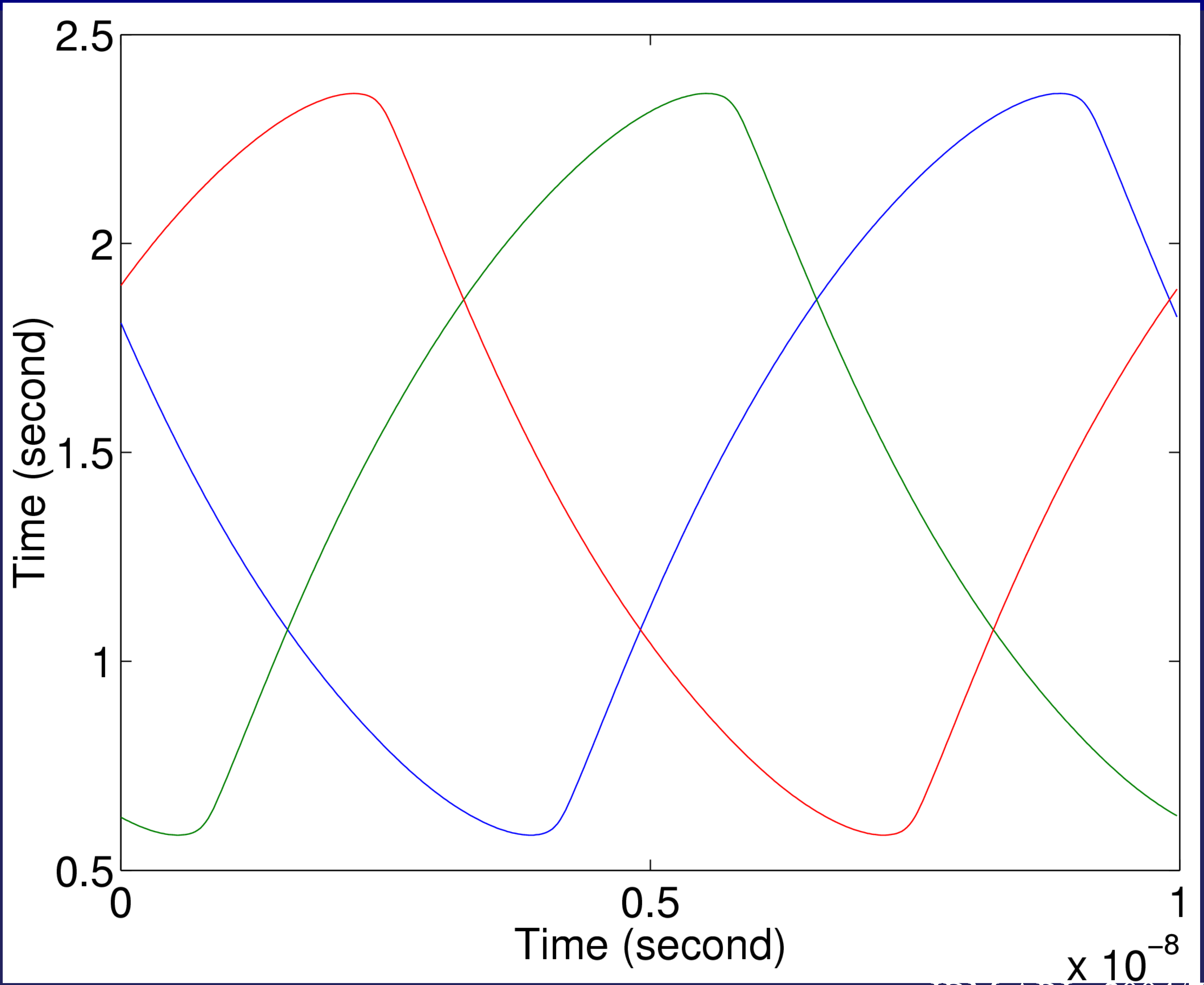
Oscillator Macromodelling Steps



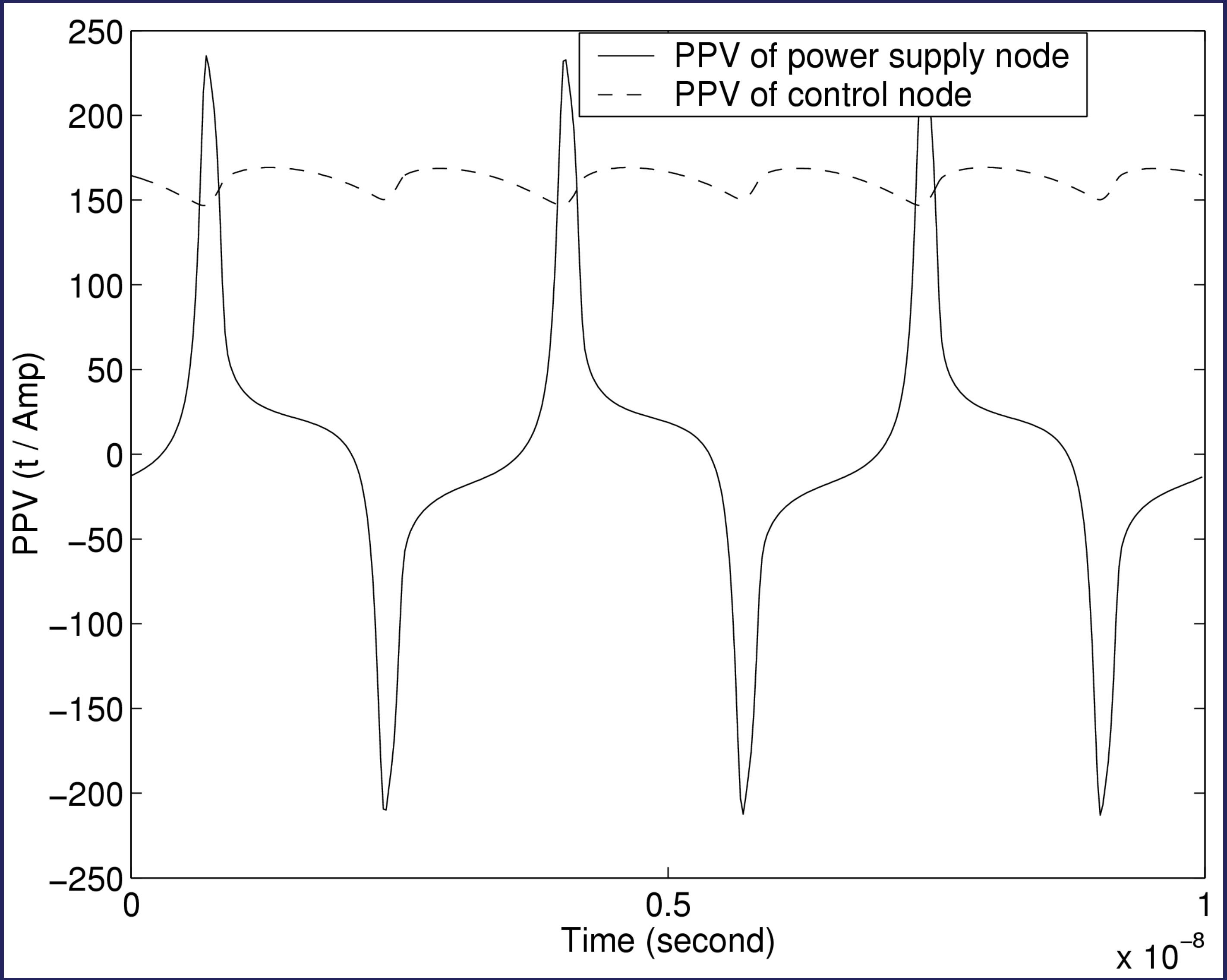
Example: Ring-Oscillator based VCO



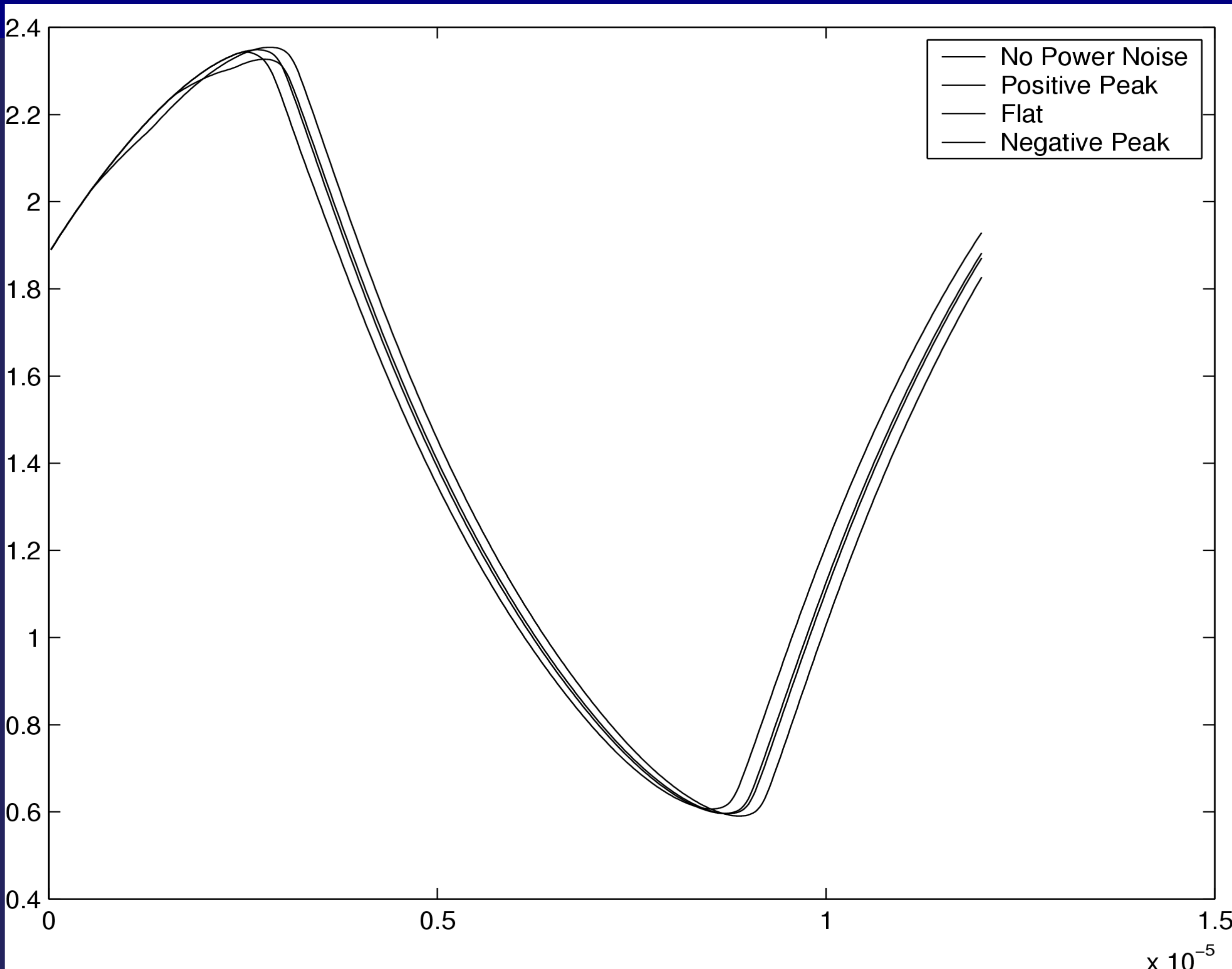
Ring Oscillator VCO: Steady State Oscillation



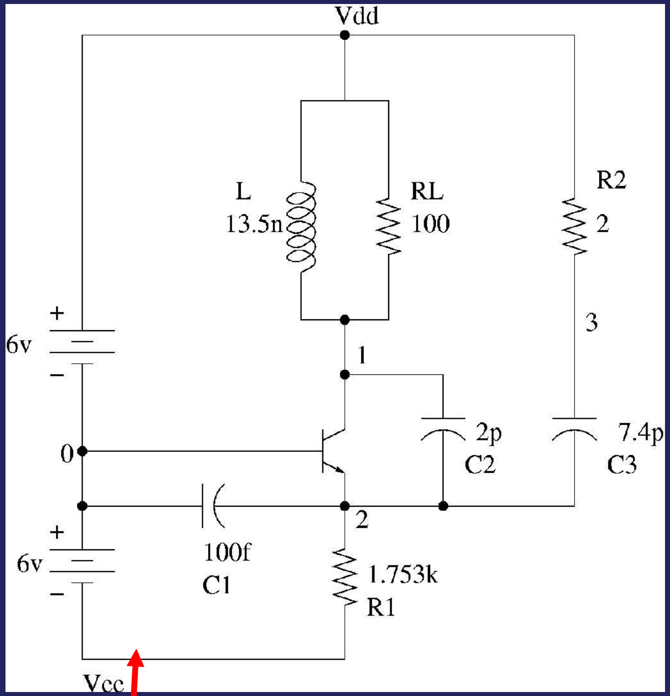
VCO Perturbation Projection Vectors: Control Node and Power Supply Components

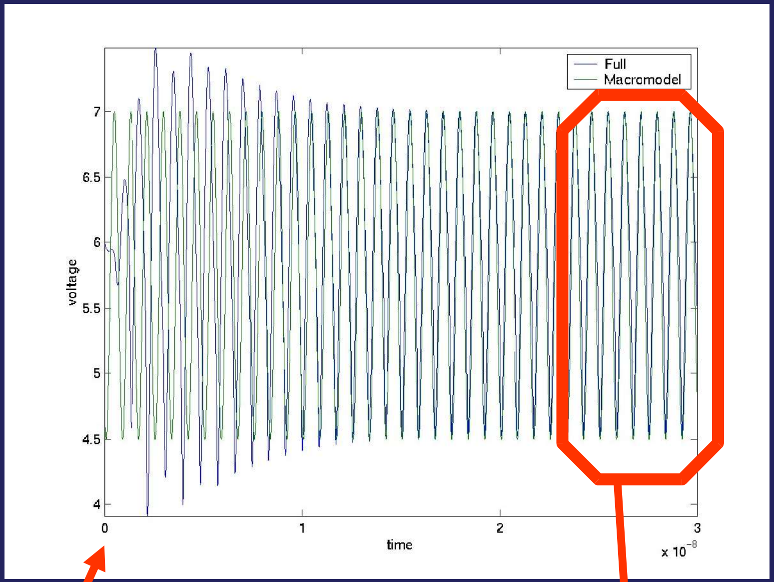


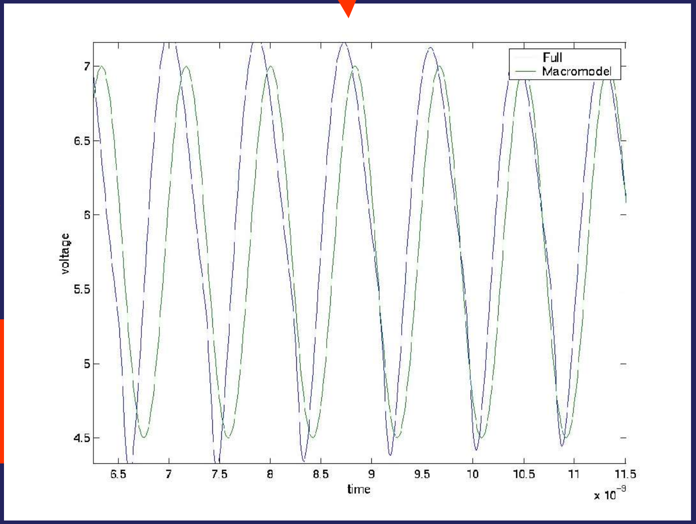
Ring Oscillator VCO: Shifts due to Supply Noise



Capturing Injection Locking in a Colpitts (LC) Oscillator







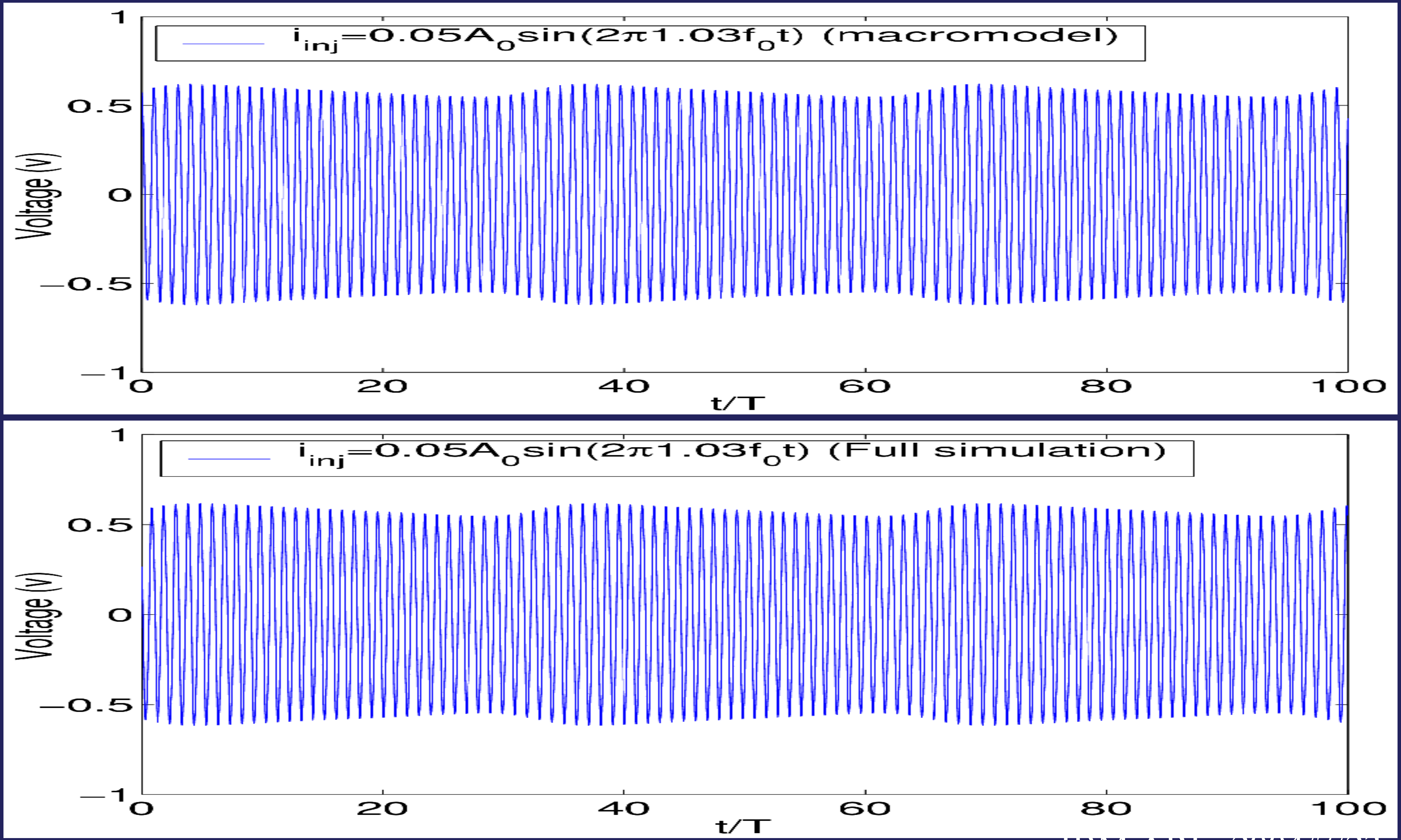
Colpitts oscillator

Full simulation vs macromodel

- ***89x speedup over original***

Capturing Amplitude Changes

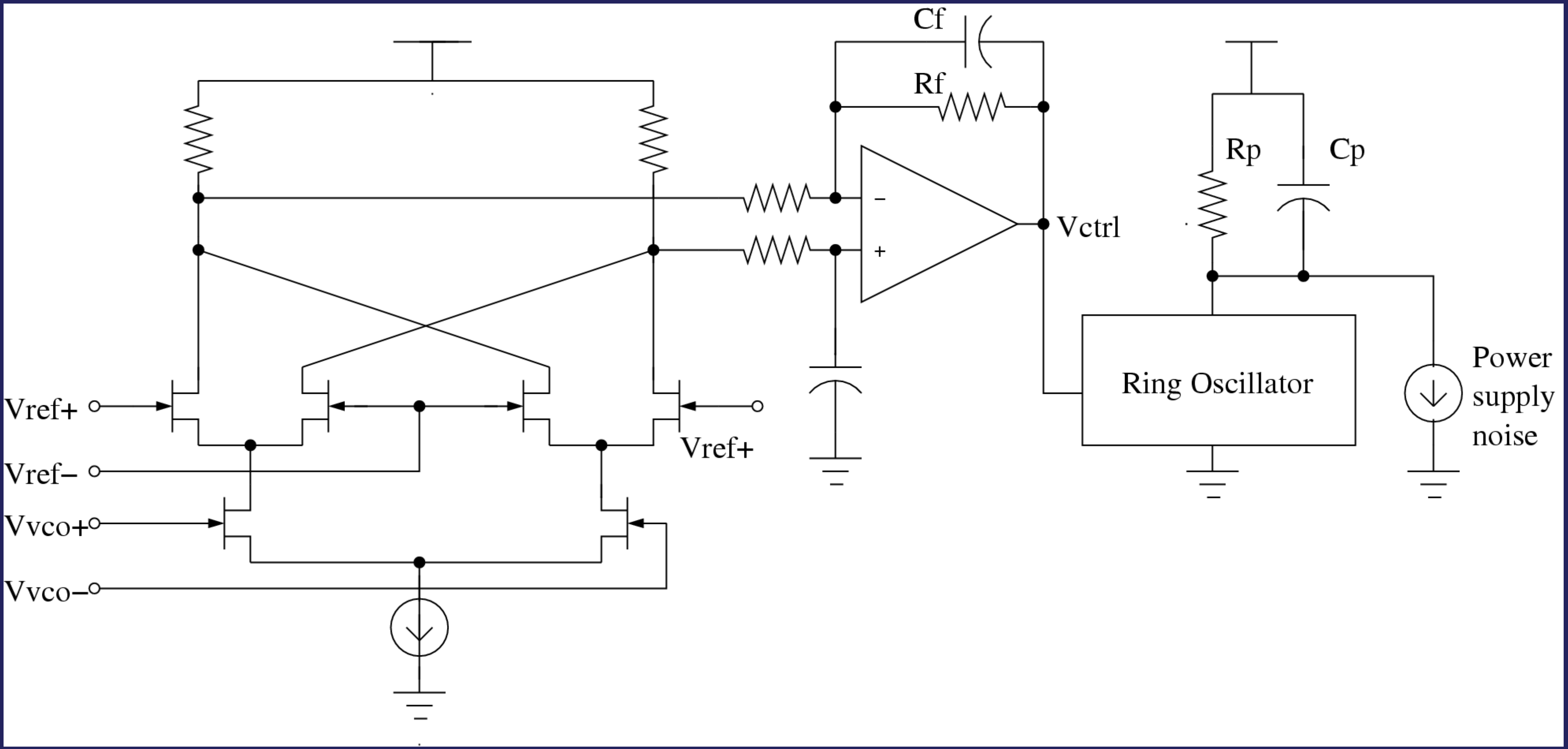
- ***nonlinear phase + amplitude components (via LTV reduction)***



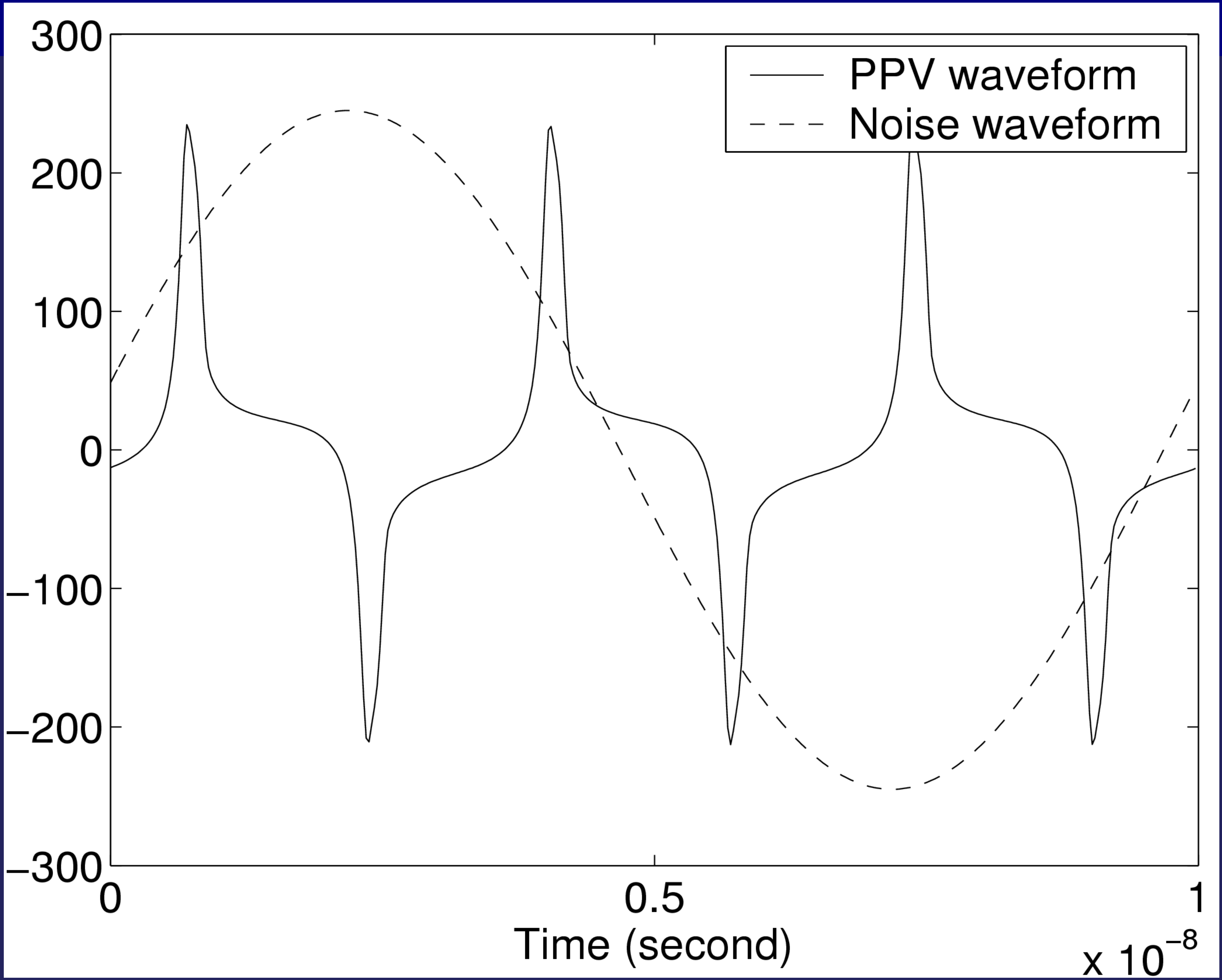
System Simulation with VCO Macromodels: PLLs

- Use nonlinear phase macromodel of VCO in phase macromodel for PLL [Lai/Roychowdhury 04]
  - PPV “extracted” by algorithm
  - amplitude components could also be used
- Replace other components (PFD, LPF, divider) also with macromodels if necessay
  - often, PFD and LPF small: keep full SPICE ckt
  - divider macromodel => simple scaling of phase
- Single VCO macromodel to capture desired and undesired influences

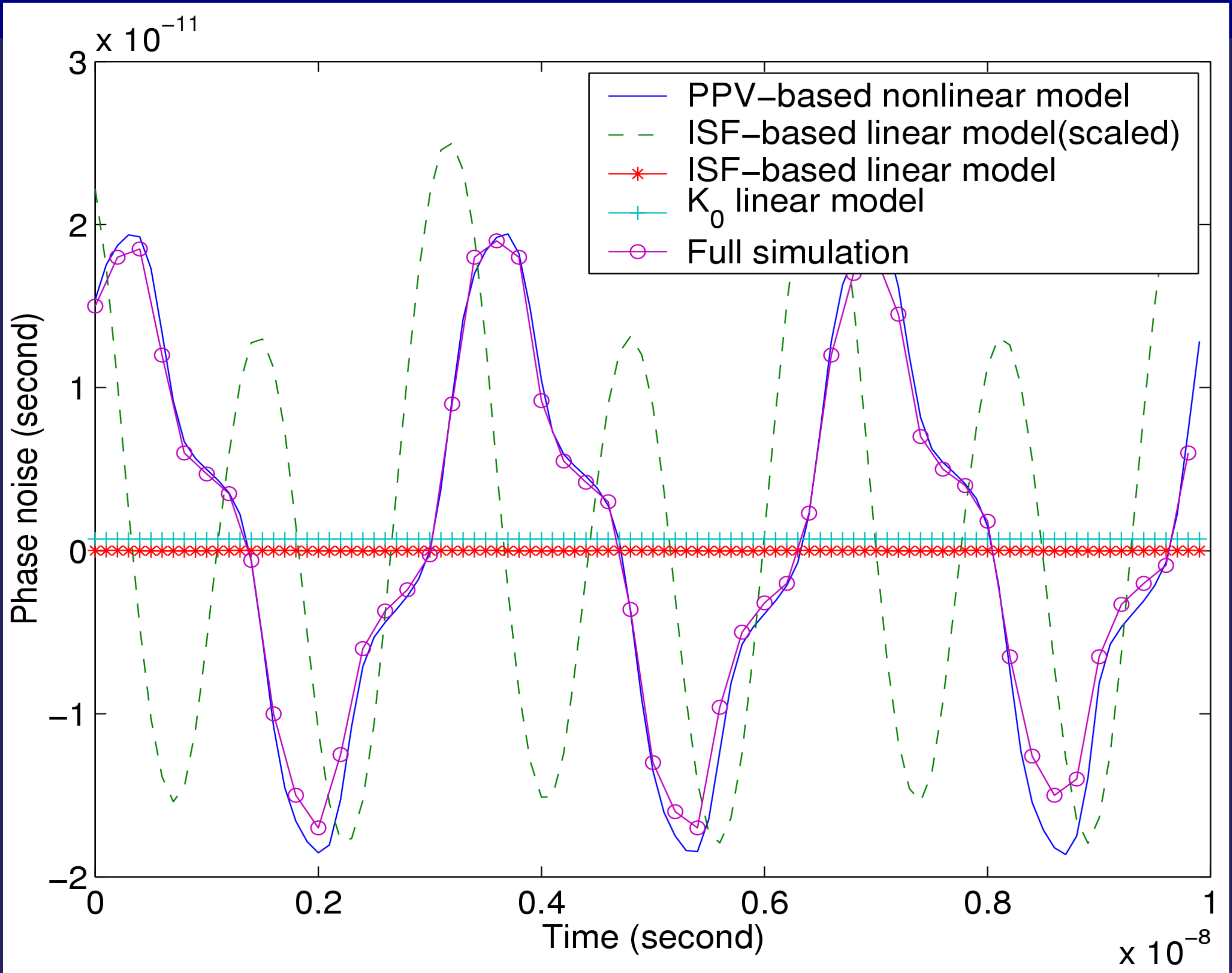
Simple Ring-Oscillator-based Phase-Locked Loop



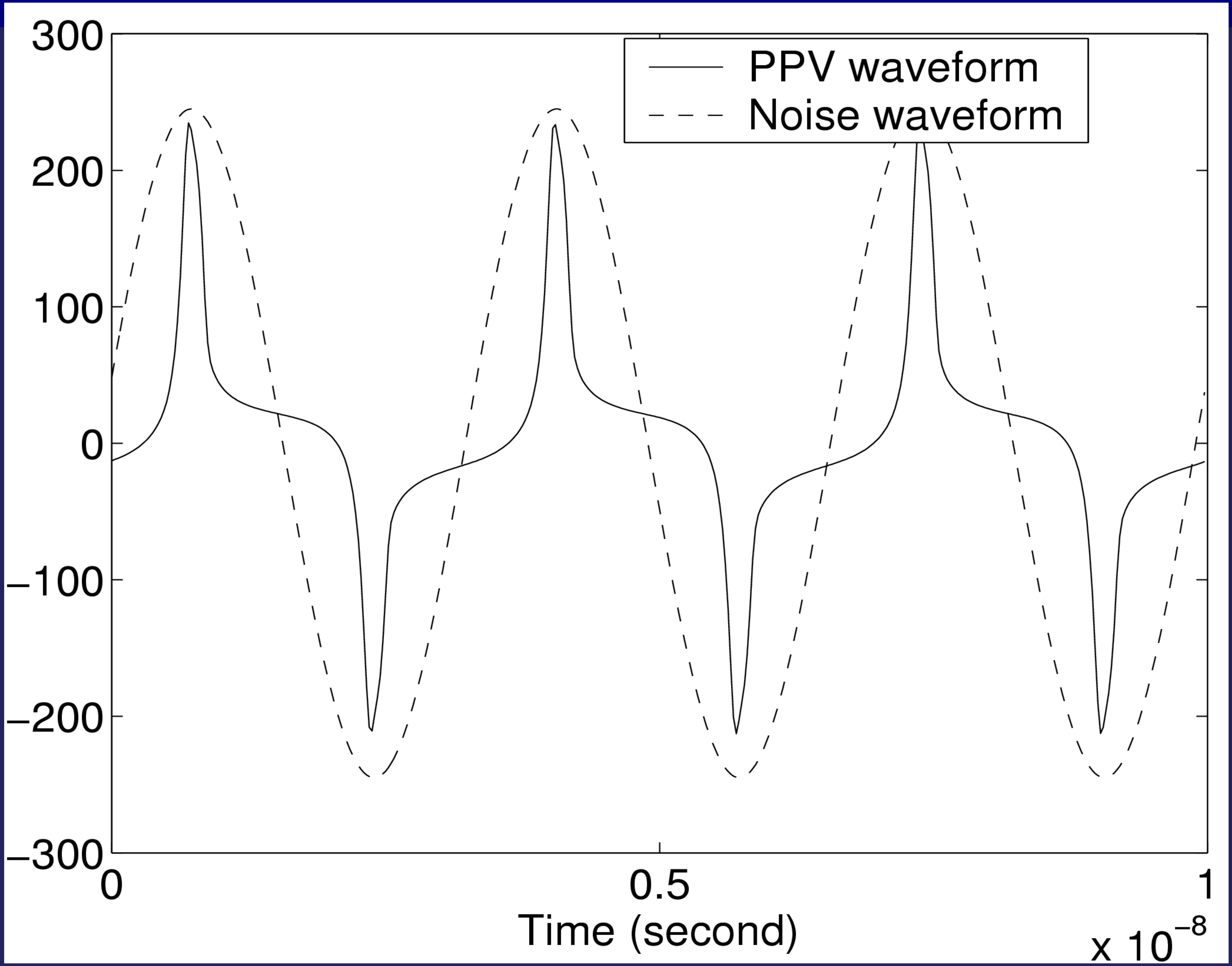
Expected Impact of 1st harmonic Supply Noise



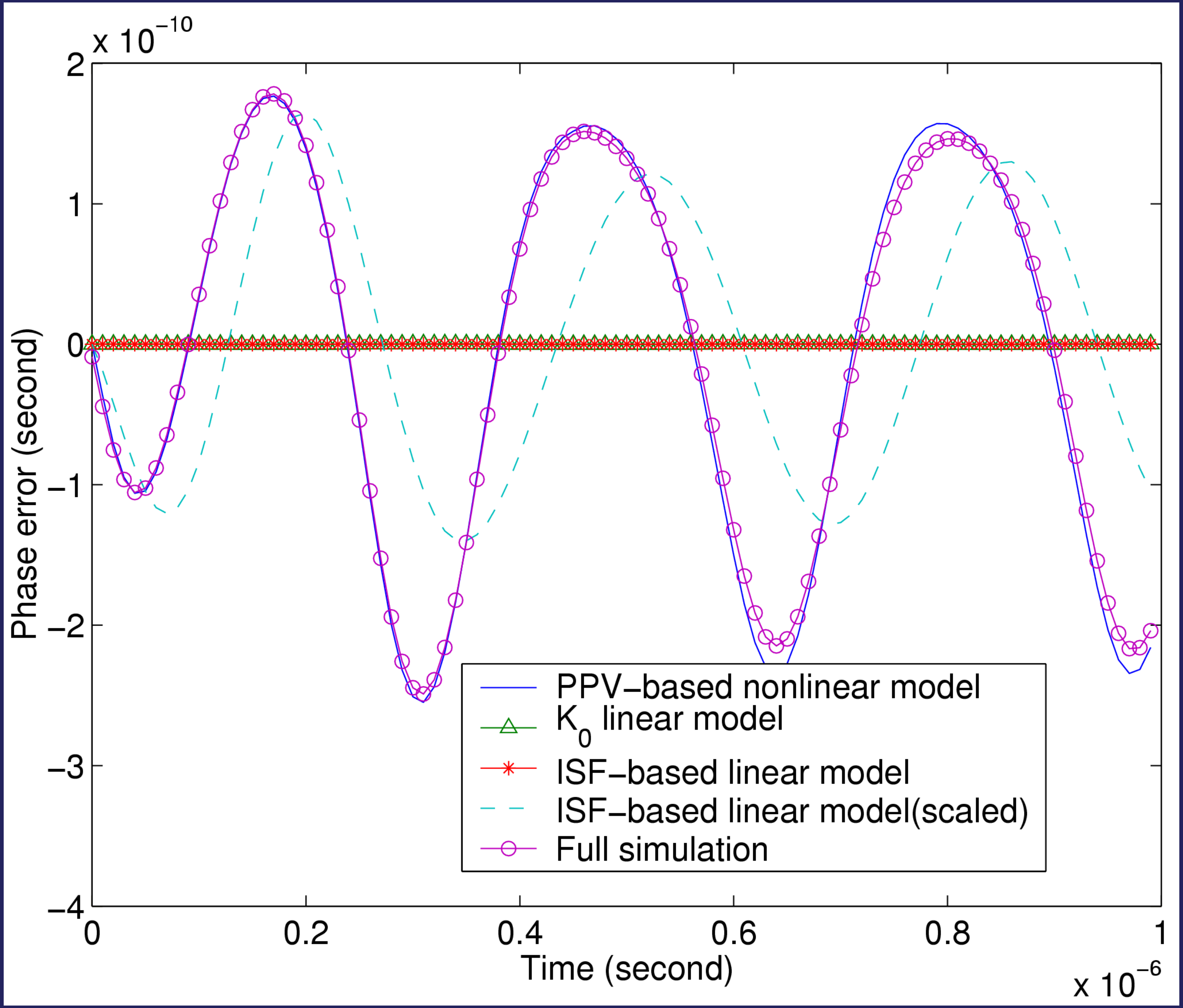
PLL Phase Reponse to Periodic Supply Noise



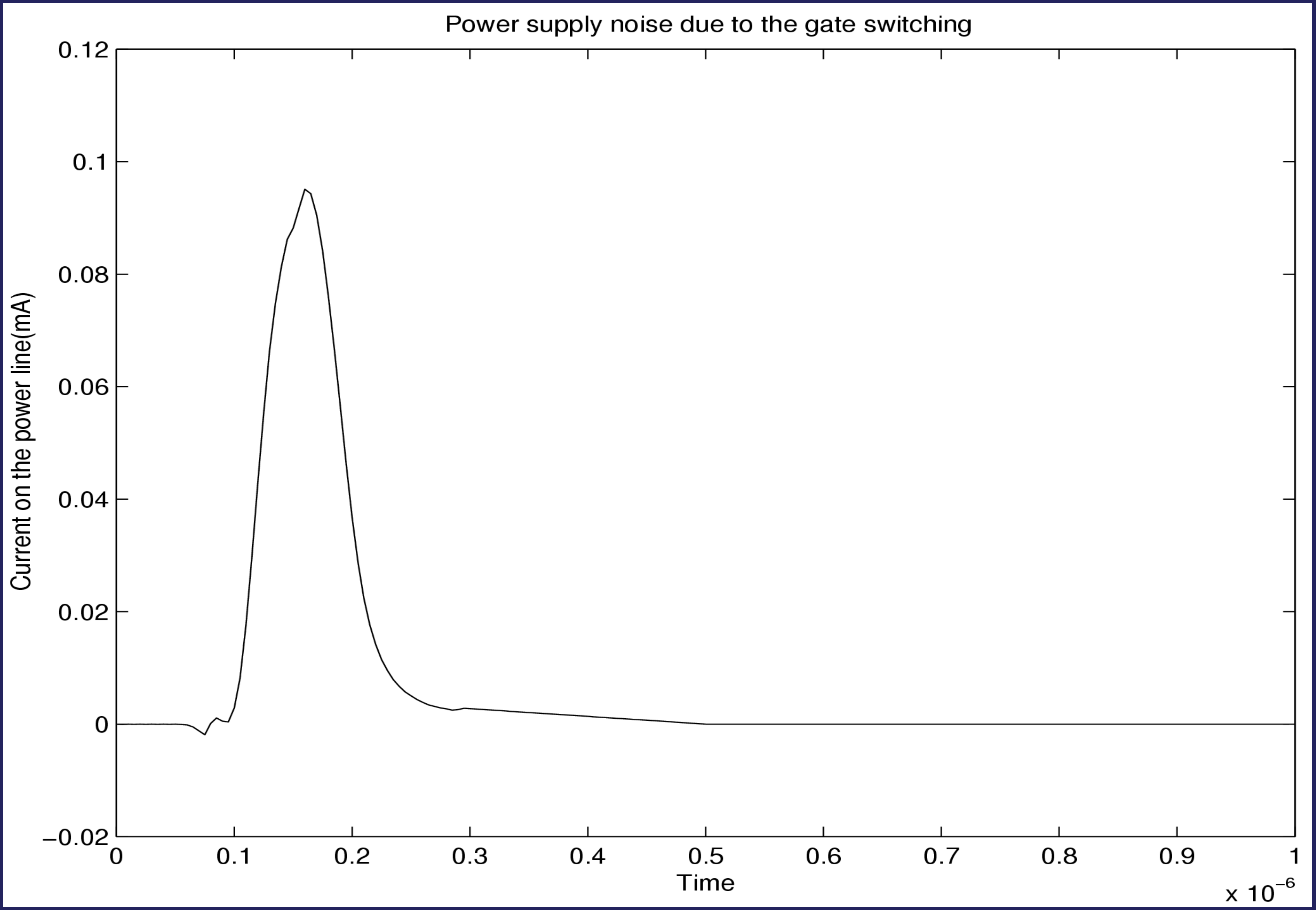
Expected Impact of 3rd Harmonic Supply Noise



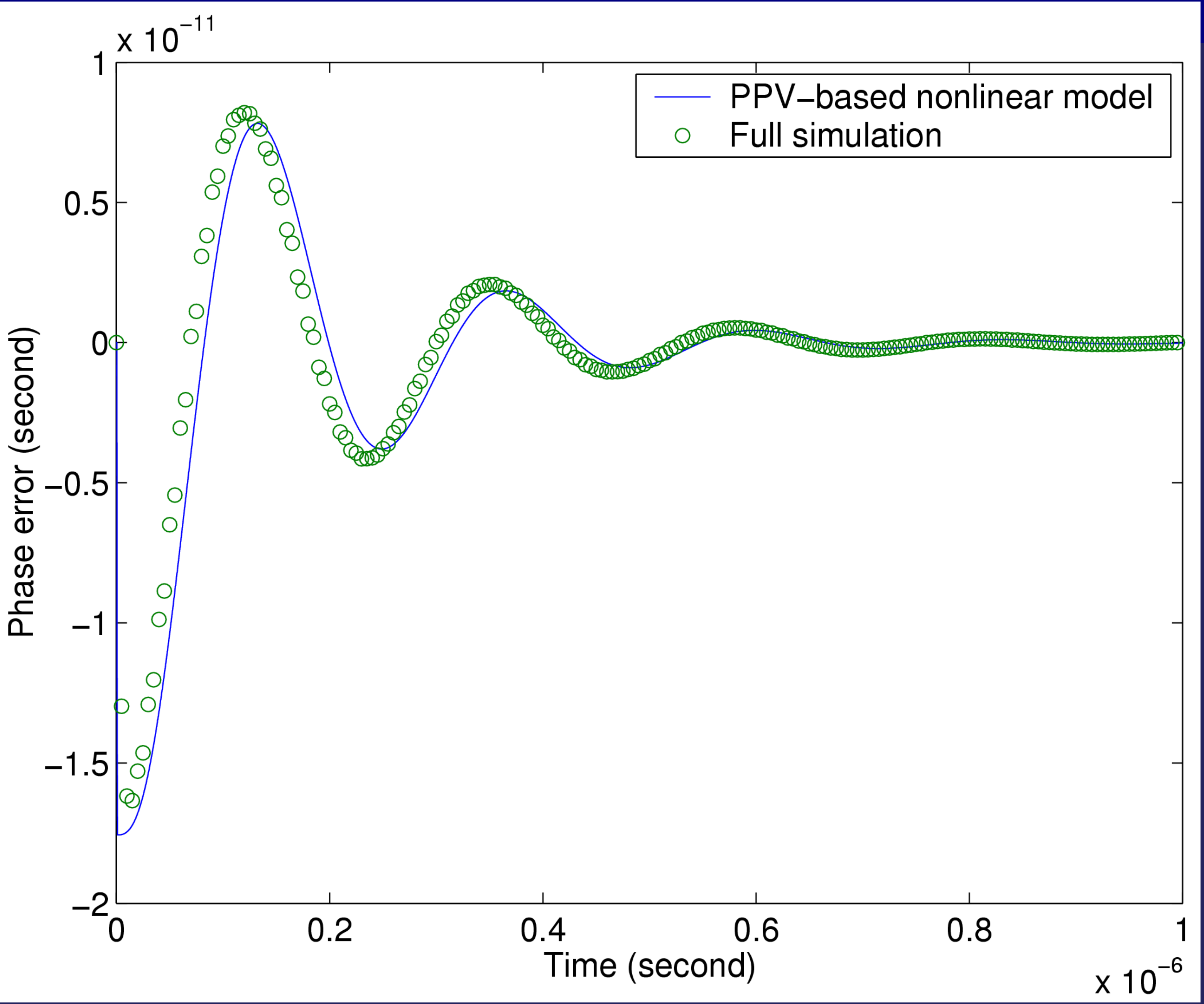
3^{rd} Harmonic Supply Noise: Phase Macromodel vs Full PLL simulation



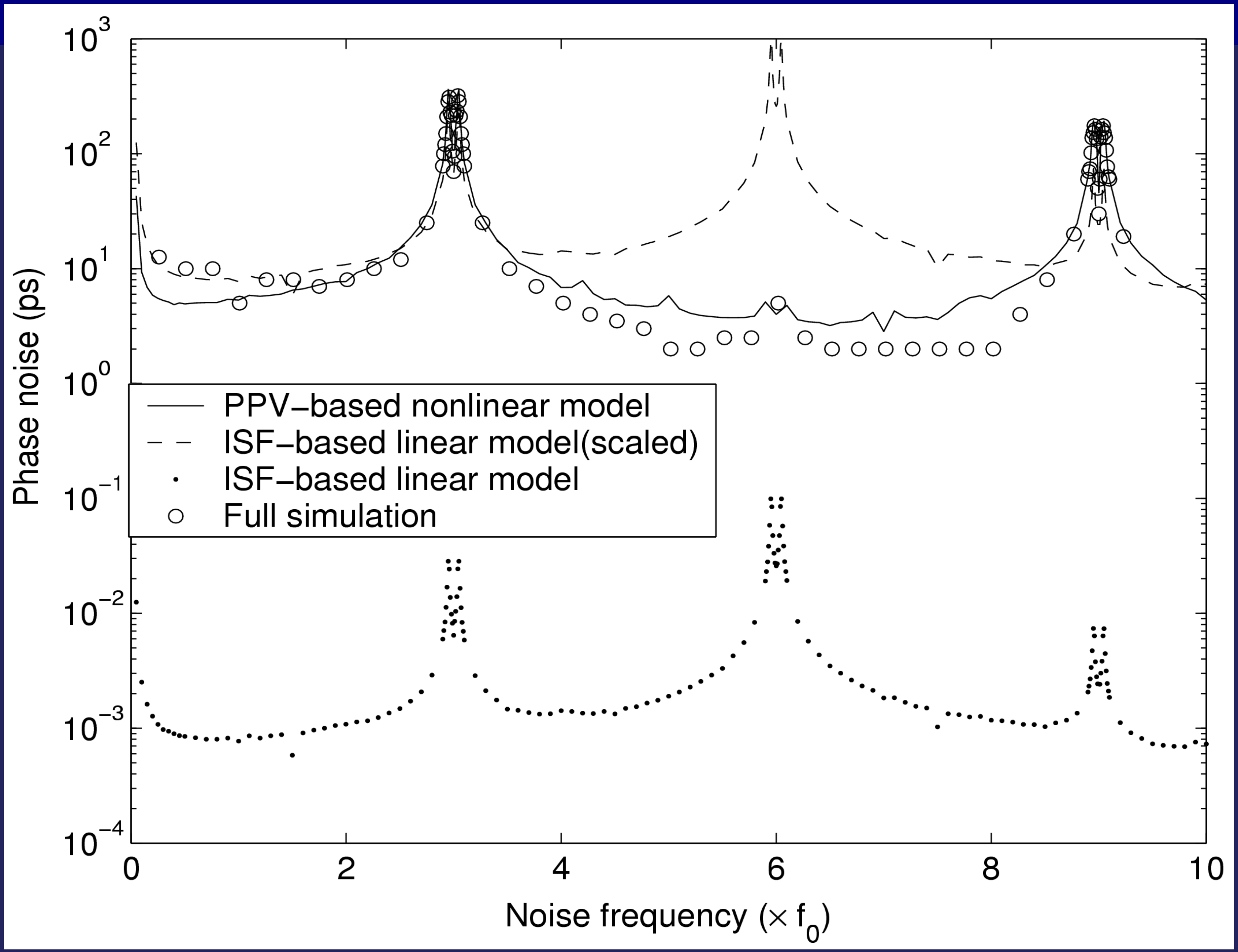
Power Supply Spike due to Single Switching



Phase Response of PLL to Supply Noise Spike



Sinusoidal Supply Noise: Effect on PLL Phase



Conclusion

- Nonlinear macromodel based PLL simulation
  - any type of VCO: LC/ring/relaxation/etc (and "multi-physics": lasers, etc)
  - Fast: orders of magnitude speedup over original
  - Capable of capturing: supply noise induced jitter, injection locking like effects, unlocked sidebands, phase jumps, cycle slipping, phase noise, etc..
- Purely analytical uses
  - ✳ Early design formulae
  - ✳ Design insight directly from PPV
  - ✳ Analytical formulae for locking, noise, ...