Advanced Analysis Methods and Simulation tools for Noise in VCOs and PLLs

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Outline

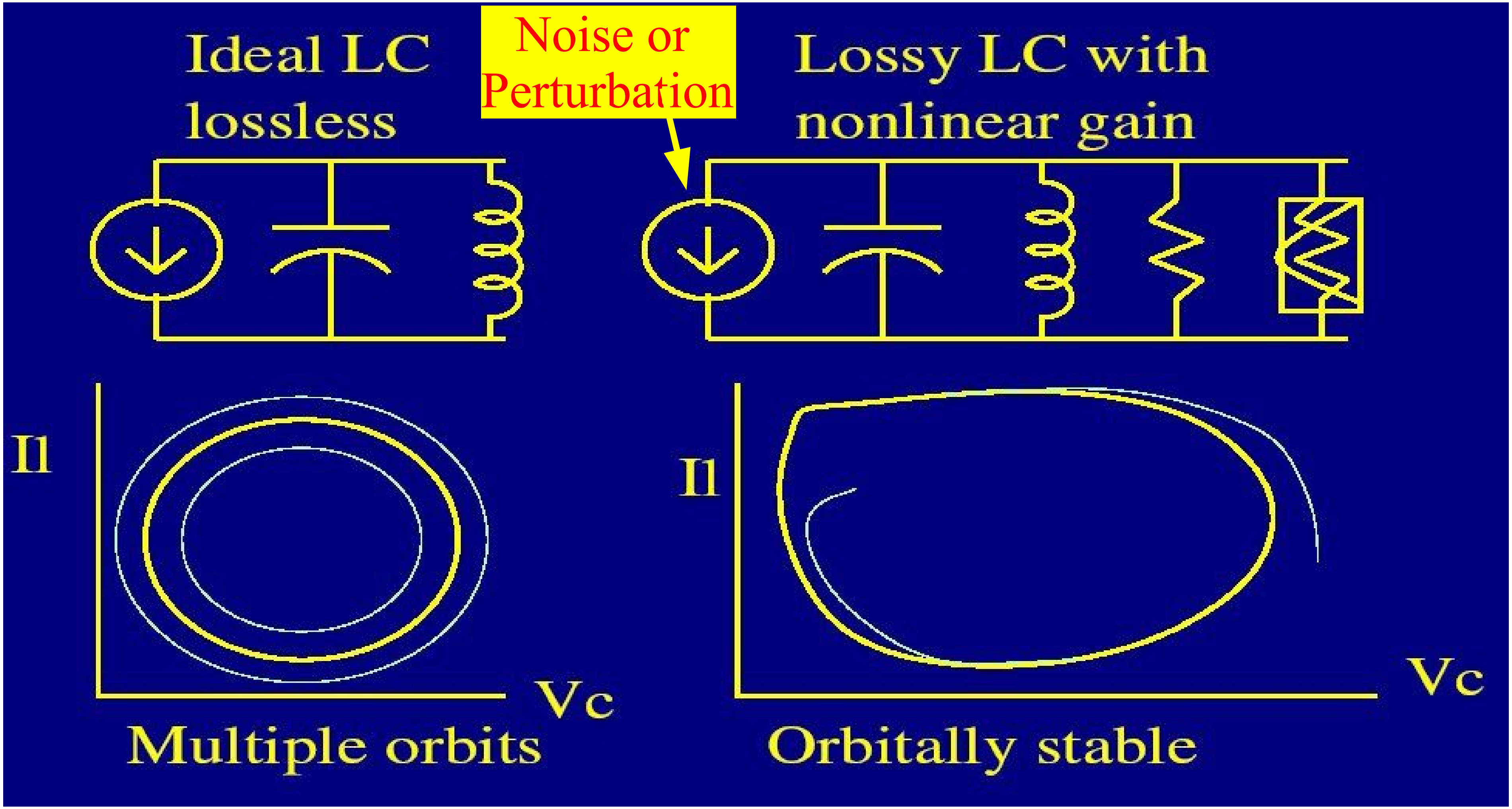
- Basics of oscillator operation
- Perturbation analysis of oscillators: the PPV/NISF
- Phase noise and jitter basics
- Injection locking
- Oscillator/PLL jitter due to interference noise
- Phase noise examples and design guidelines
- PLL noise analysis and examples

Basic Features of Oscillator Operation

Why Oscillators are a Special Simulation Challenge

- **Computation/size/accuracy**: much greater than for amps/mixers
  - long startups, tiny timesteps needed
  - inefficient for even 1-transistor oscillators
  - integrated RF: 100s to 1000s of transistors
  - errors dependent on size of timesteps, integration method, ...
  - fundamental cause: **marginal phase stability** of all oscillators
    - numerical errors integrate over time
- Using SPICE directly for oscillator phase simulation should be avoided if you have alternatives

Amplitude Stability and Nonlinearity



For **quantitative analysis**: **<u>nonlinearity cannot be ignored</u>** – fundamental to oscillator operation

Perturbation Analysis of Oscillators

Quantitative Perturbation Analysis

- Mathematically, circuits are **nonlinear differential equations**
  - Eg, for an RLC tank (using KCL and KVL):

$\mathbf{x}$

$$\frac{\mathbf{d}}{\mathbf{dt}}\begin{bmatrix} v(t) \\ i(t) \end{bmatrix} = -\begin{bmatrix} C & \\ & L \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{R} & 1 \\ 1 & \end{bmatrix} \begin{bmatrix} v(t) \\ i(t) \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

Quantifying Oscillator Response

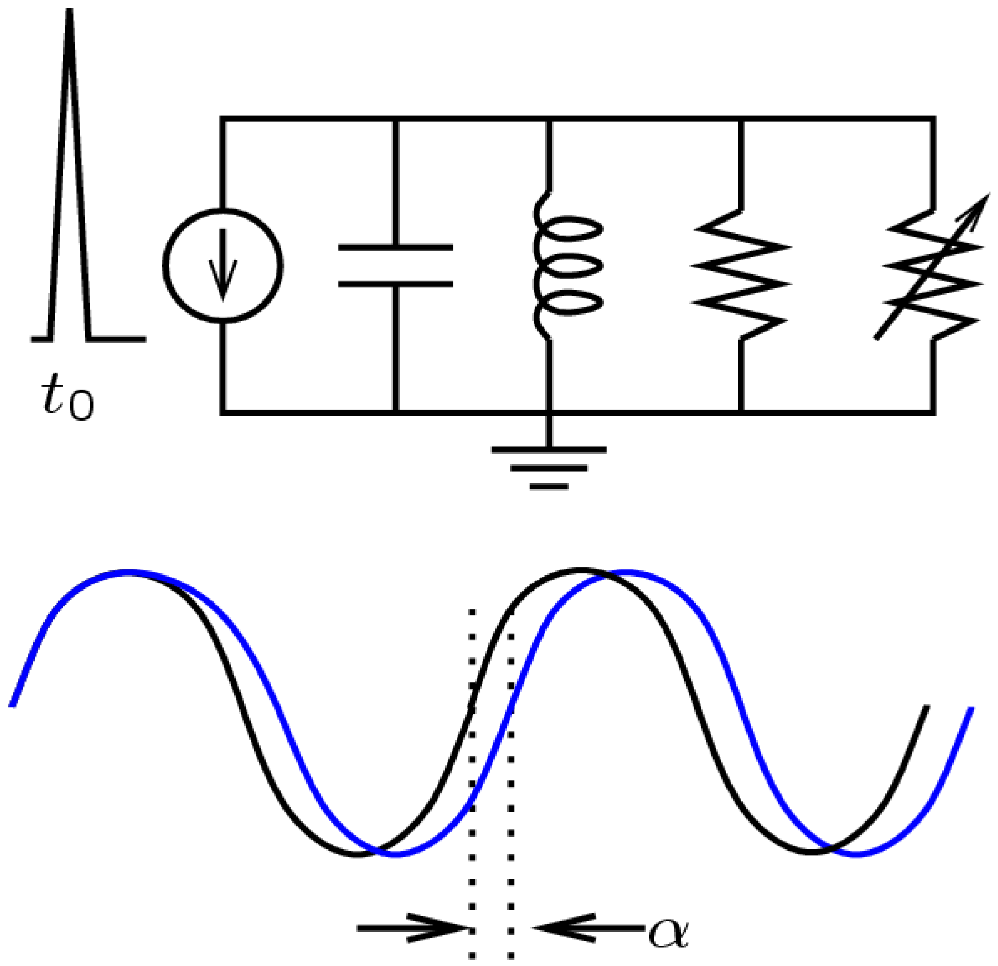
How does the oscillator (VCO) respond to “inputs”?

$$\dot{x}(t) = f(x) + \underbrace{b(t)}_{\text{input perturbation}}$$

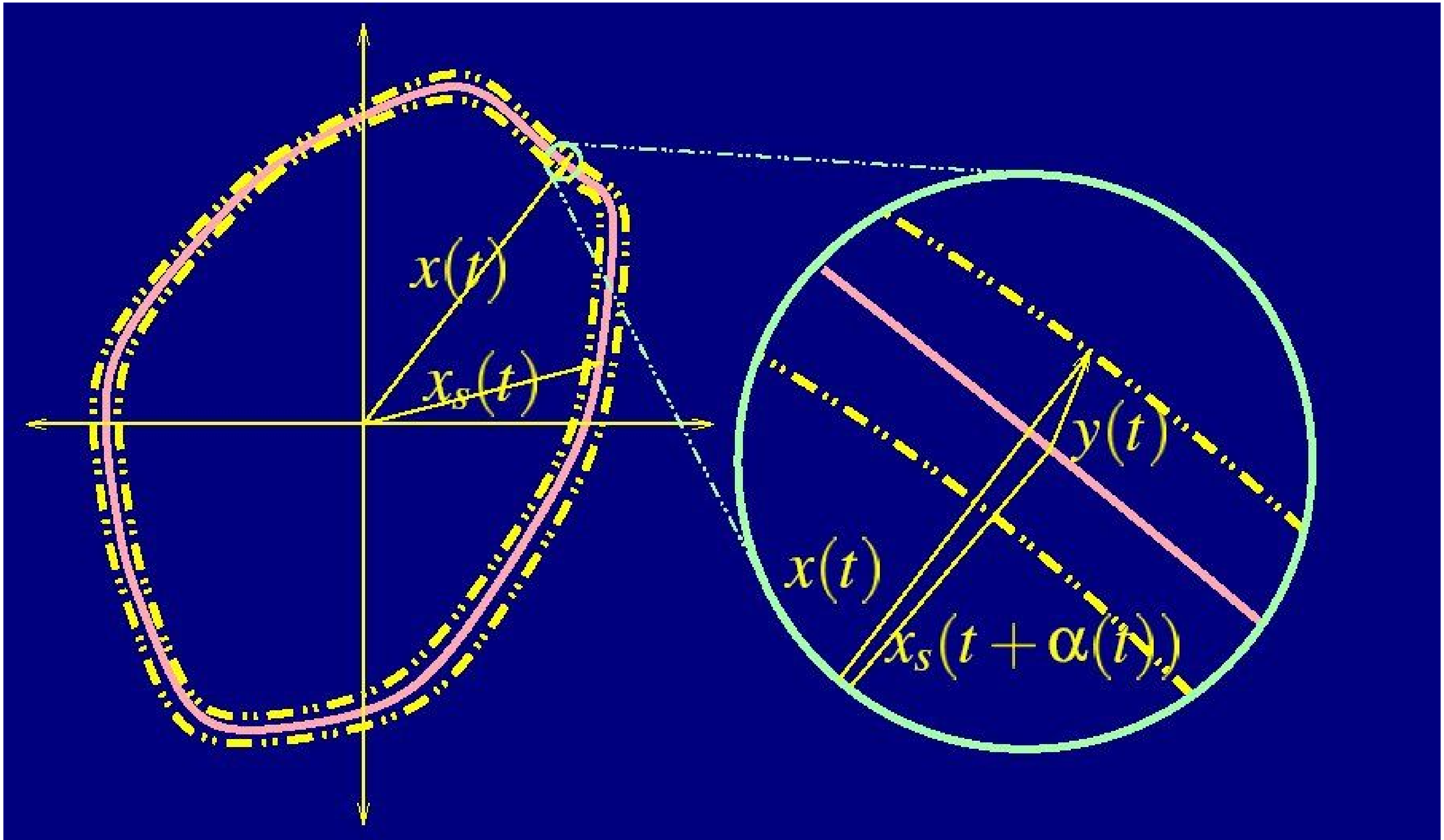
- No perturbation $\Rightarrow$ perfect periodic solution $x_s(t)$
- Small $b(t)$ perturbation:

$$x(t) = x_s(t + \underbrace{\alpha(t)}_{\text{growing phase error}}) + \underbrace{y(t)}_{\text{small}}$$

Phase Response to Delta Function



Oscillator: Response to “Inputs”



- Phase error $\alpha(t)$ *shifts track increasingly along limit cycle*
- $y(t)$ creates deviations from limit cycle that *remain small*

Nonlinear Differential Equation for Phase

$$\dot{\alpha} = \mathbf{v}_1^T(t + \alpha(t)) \cdot \mathbf{b}(t)$$

- Simple differential equation governs phase
  - **nonlinear (because PPV is periodic)**
  - **scalar (much smaller than oscillator equations)**

[Demir Mehrotra Roychowdhury 97, 98, 01]

The Perturbation Projection Vector (PPV): a *Nonlinear* ISF (NISF)

- $v_1(t)$: **"nonlinear transfer or sensitivity function"** relating "input" to oscillator phase response
  - (via nonlinear differential equation on previous slide)
  - $v_1(t)$: the PPV (Perturbation Projection Vector) or NISF (nonlinear ISF)
- In general, PPV/NISF does NOT equal the tangent vector of the phase plane plot
    - ie, not equal to the ISF [Hajimiri 98]
    - but **intuition is similar:** sensitivity of phase/jitter to external perturbations as a function of time

PPV/NISF vs the ISF

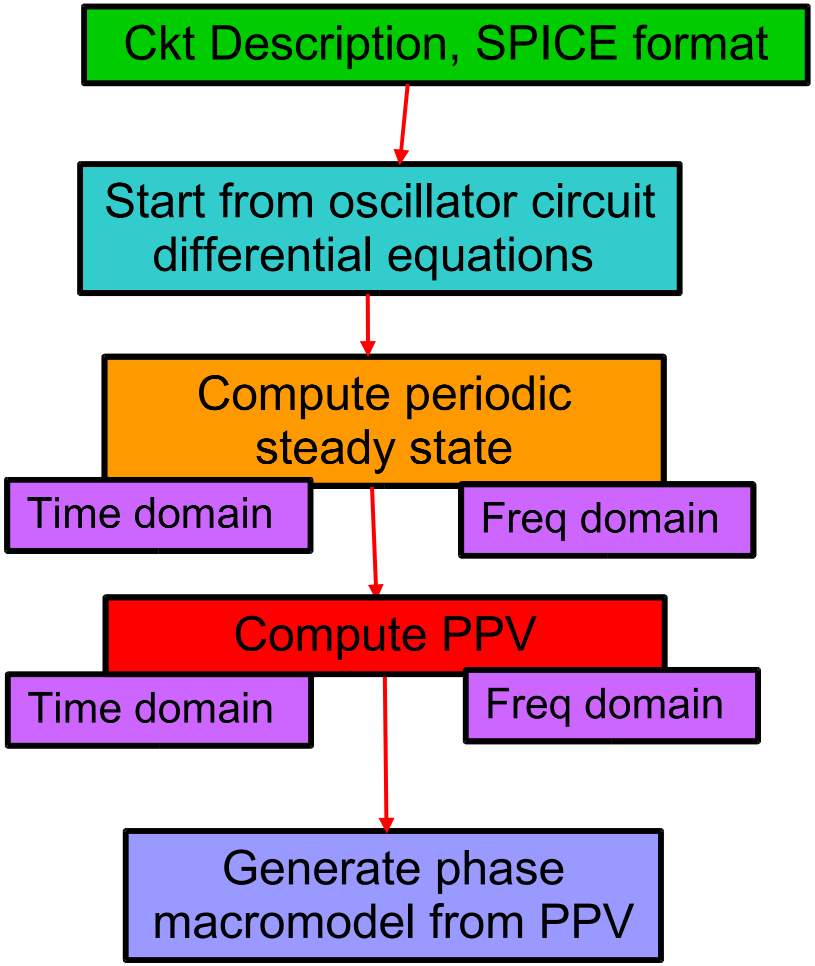
- Similarities
  - PPV/NISF and ISF identical for high-Q negative resistance oscillators
  - (still important to use the NONLINEAR phase equation)
- Differences
  - PPV/NISF and ISF are **NOT IDENTICAL** for most other oscillators, including **ring oscillators**
- Applications where the difference matters (greatly):
  - Phase noise/jitter due to random noise
  - Injection locking
  - Jitter from interference noise (power/ground noise)

Computing the PPV/NISF

- PPV/NISF can be computed efficiently from oscillator steady-state quantities
  - Find the periodic steady-state of oscillator
    - using, eg, HB, shooting, etc.
  - Various matrix computations
    - can be performed efficiently for large oscillators

[Demir Roychowdhury TCAD 03]

PPV/NISF: Computation Techniques



Phase Noise and Jitter

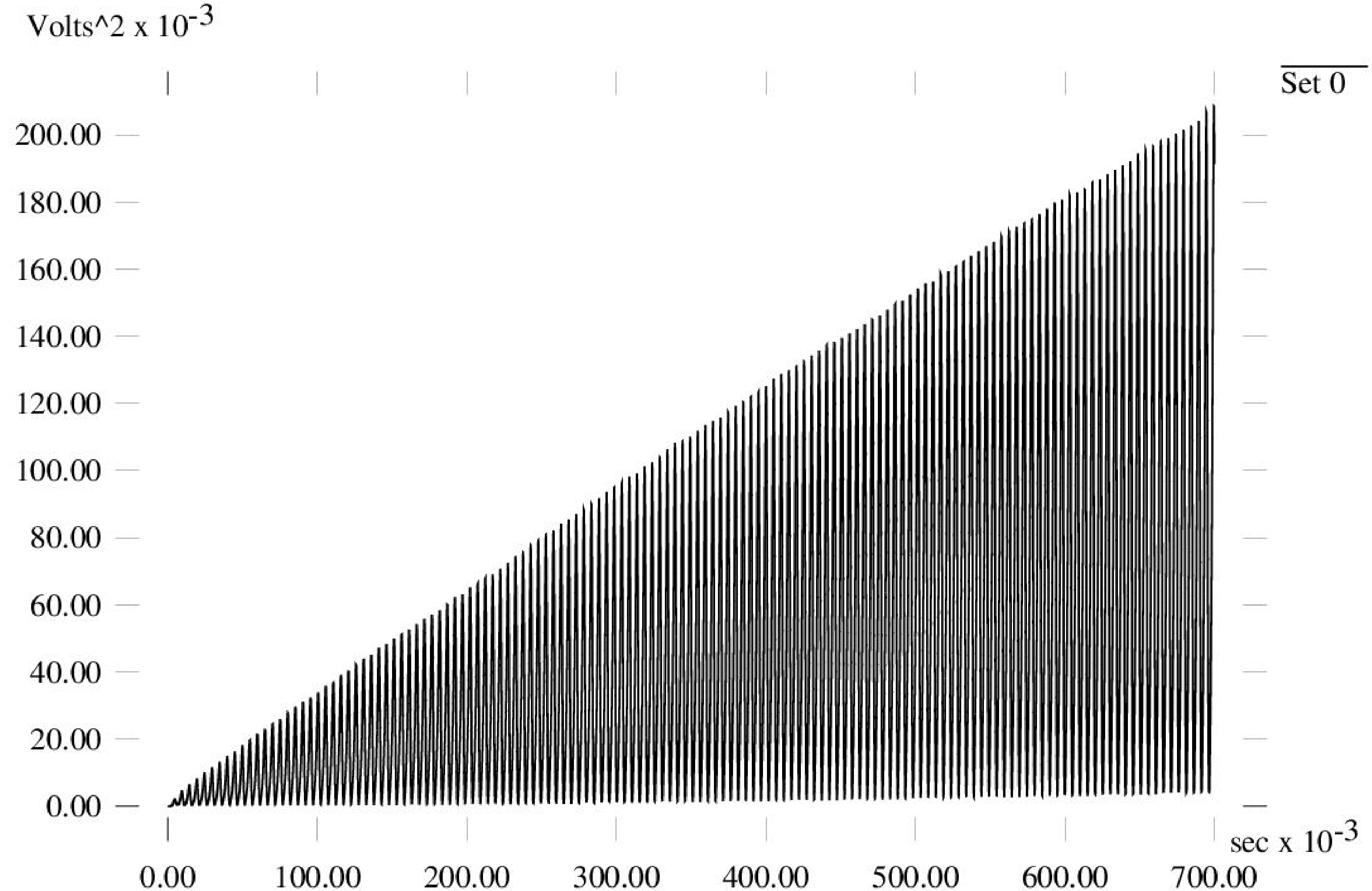
Using the PPV/NISF to find jitter and phase noise spectrum

- Noise: perturbation to oscillator comes from thermal, shot, flicker noise sources
  - b(t) is **random** (eg, white Gaussian)
  - Now concerned with **noise power** (a.k.a noise variance)
- Phase deviation $\alpha(t)$ now also random
  - Variance = $E[\alpha^2(t)]$ is **mean-squared jitter**

Mean-squared Jitter

- Key result: **mean-squared jitter grows linearly with time**
  - $E[\alpha^2(t)] = \mathbf{c}t$
  - (technically: random walk or Brownian motion)
  - **c**: **growth rate of mean-squared jitter**
    - Crucial scalar parameter
    - Depends on PPV/NISF and device (thermal/shot/etc) noises
    - Expression available for c [Demir et al TCAS-1 2000]

Linear rise of mean-square jitter: Monte-Carlo simulation

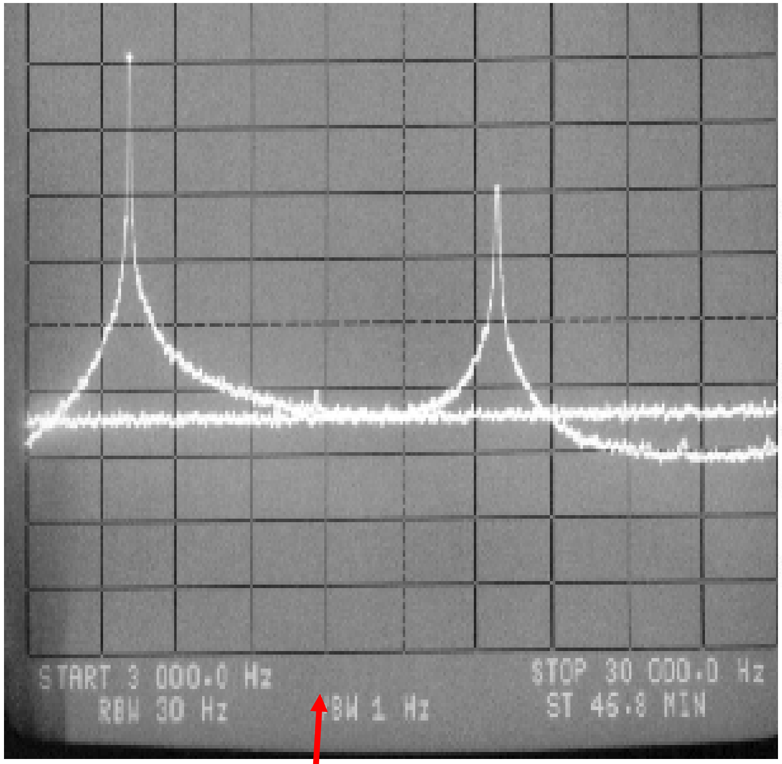


Phase Noise Spectrum

- Measured spectrum is of voltage/current
  - of, eg, $\sin(wt + \alpha(t))$; **<u>not</u>** of the phase $\alpha(t)$ itself!
- Spectrum is directly related to jitter growth-rate c
- Shape is **<u>Lorenzian</u>** – broadens as c increases
  - No explosion at center frequency

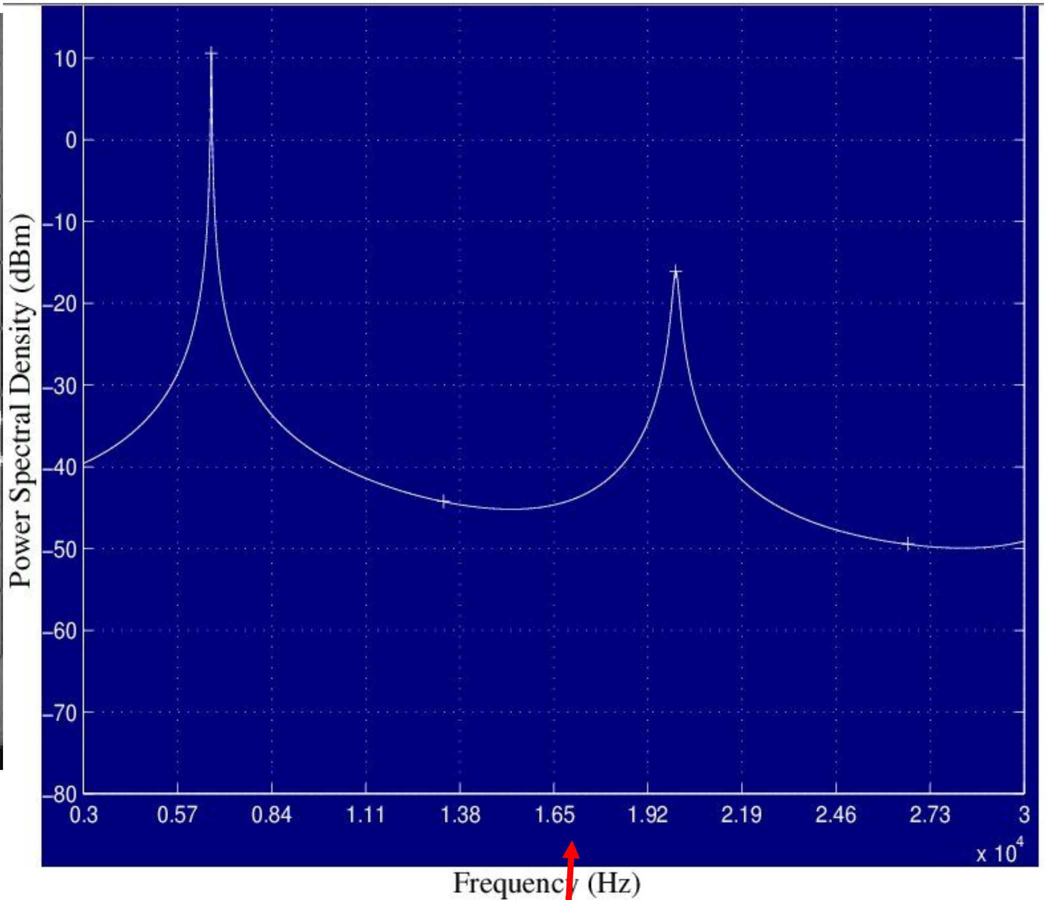
Spectrum Does Not Explode at Carrier

PPV/NISF-based nonlinear theory resolves contradiction in Leeson's model



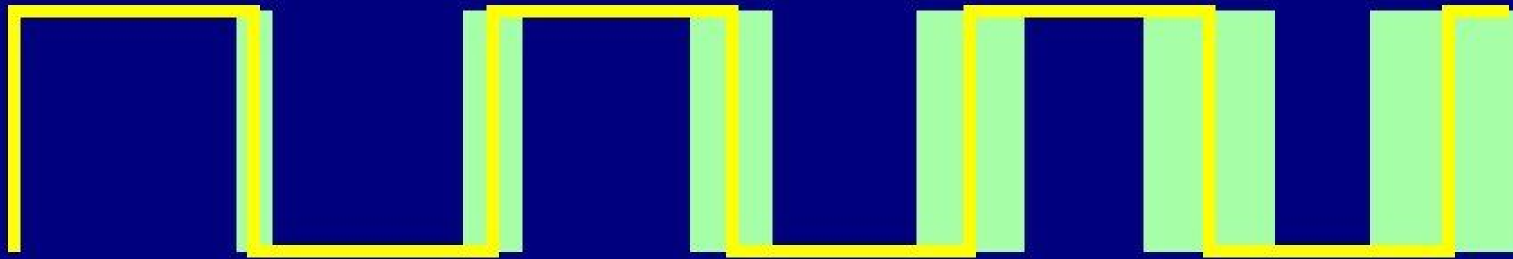
Measured

[Toth TCAS-1 1992]

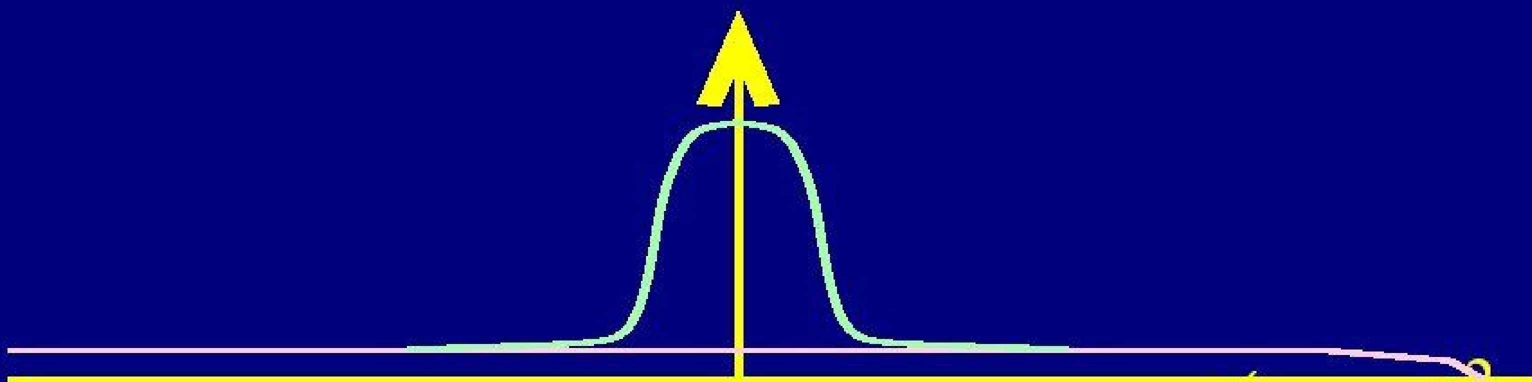


Computed from PPV

Timing Jitter and Phase Noise Spectrum



- Timing jitter variance (per cycle) = cT



- *Lorenzian spectrum:* $\mathcal{L}(f_m) = 10\log_{10}\left(\dfrac{f_0^2 c}{\pi^2 f_0^4 c^2 + f_m^2}\right)$

Injection Locking

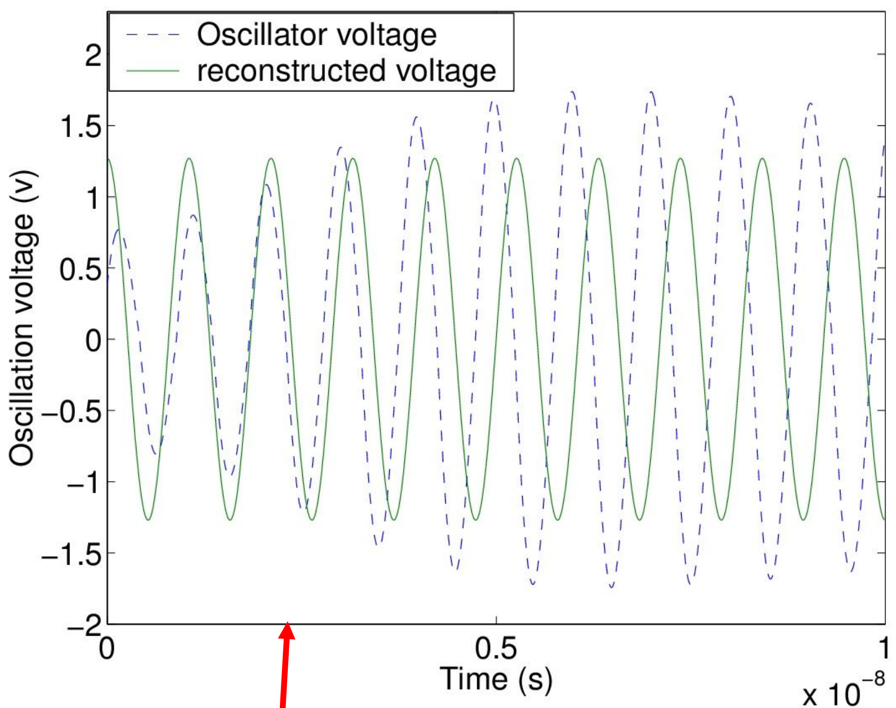
Injection Locking in Oscillators

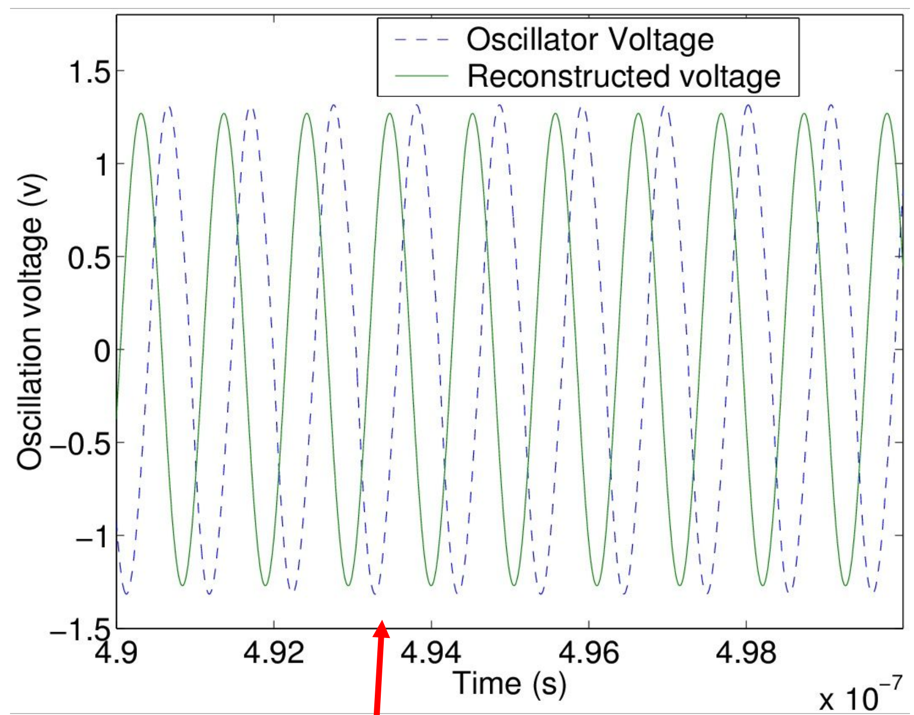
- Oscillator's frequency "**locks**" to frequency of external input
  - external frequency "**over-rides**" oscillator natural frequency
  - if frequencies close enough, even if input is very small
- Universal phenomenon: grandfather clocks, fireflies flashing, lasers, etc
- Impact: can be both good and bad
  - exploited in RF/mixed-signal design
  - undesired locking due to parasitic coupling a concern
- Can take extremely long to simulate in SPICE

Predicting Injection Locking

- Find nonlinear phase error $\alpha(t)$ due to b(t)
  - b(t) is now the deterministic injection signal
  - Use the nonlinear differential equation based on PPV/NISF to calculate $\alpha(t)$
    - Linear equations don't work: injection locking is a fundamentally nonlinear phenomenon
  - If $\alpha(t) = (w_1 - w_0)t$, the oscillator is injection locked
- Much faster/more accurate than SPICE simulation

Colpitts Oscillator: Injection Locking





Initially unlocked

Eventually Locked

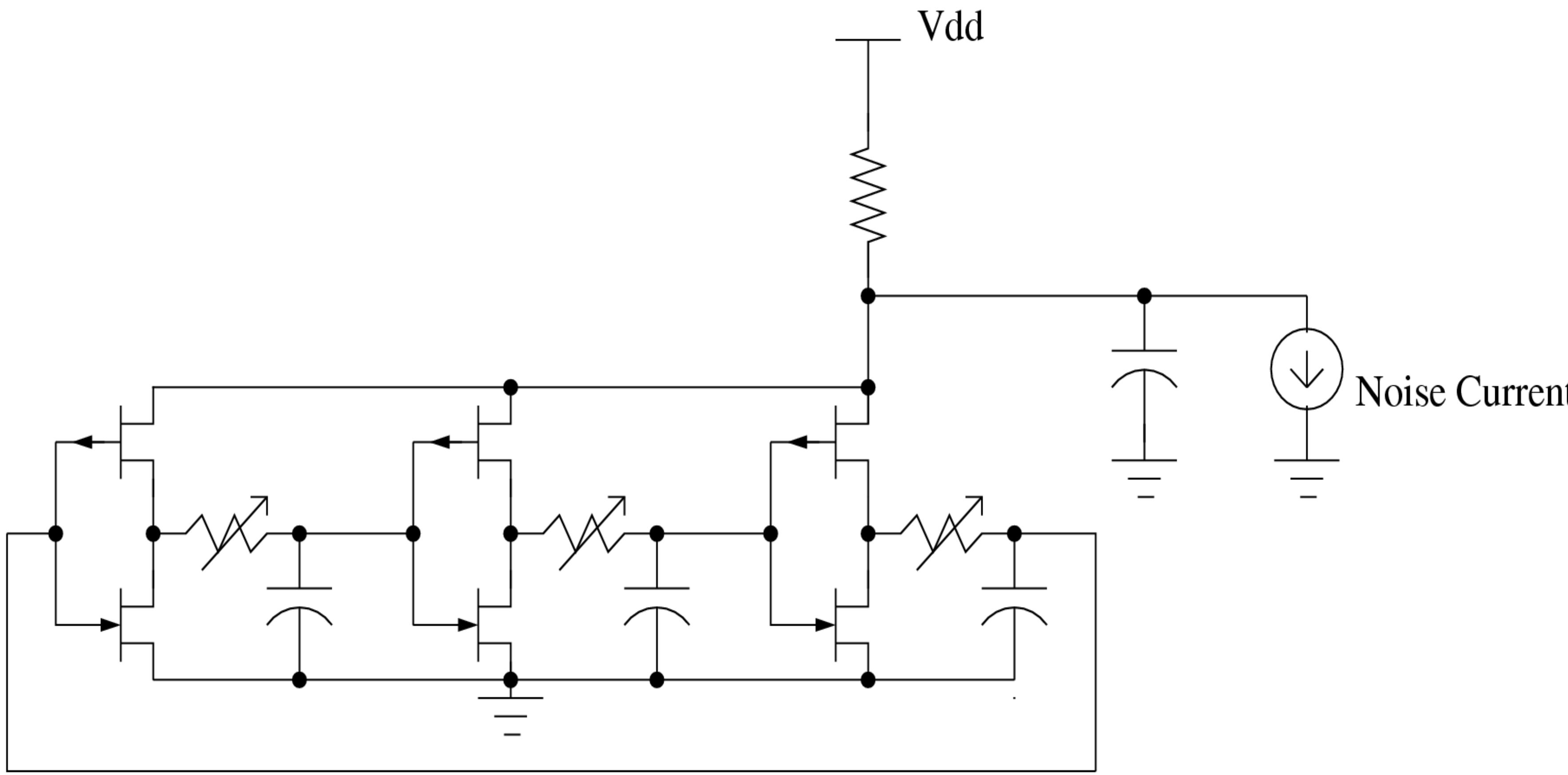
Full (SPICE) simulation, vs waveform reconstructed from $\alpha(t)$
(2 orders of magnitude speedup)

Jitter Induced by Power/Ground Noise

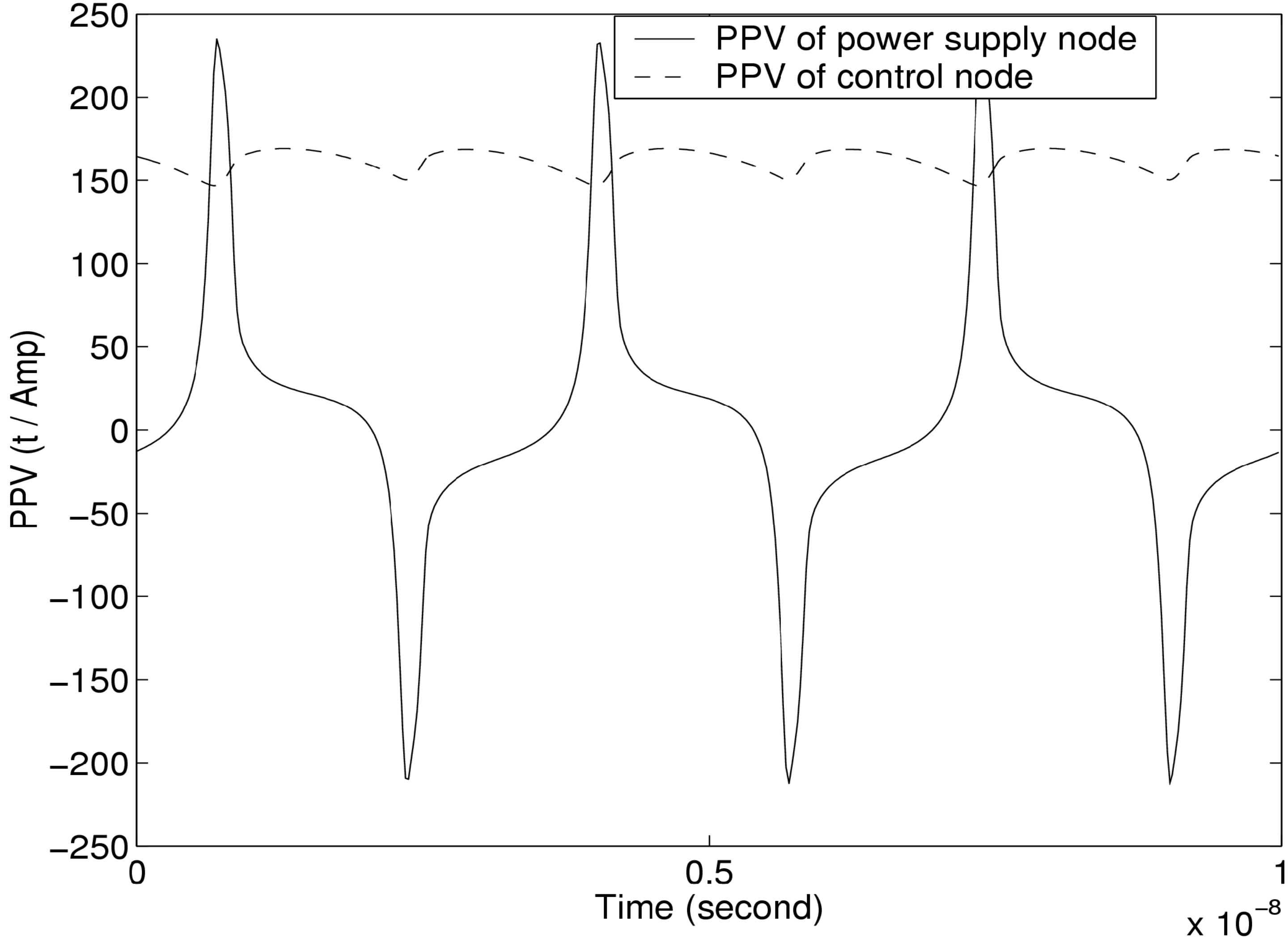
Interference: “Deterministic Noise”

- Power/ground noise: not “truly” random
  - caused by interference from other circuits
  - not Gaussian, not stationary: “spikes” of noise
  - can be relatively large (15-20% of VDD)
- Use nonlinear phase equation (again)
  - deterministic input
  - PPV component supplies design insight

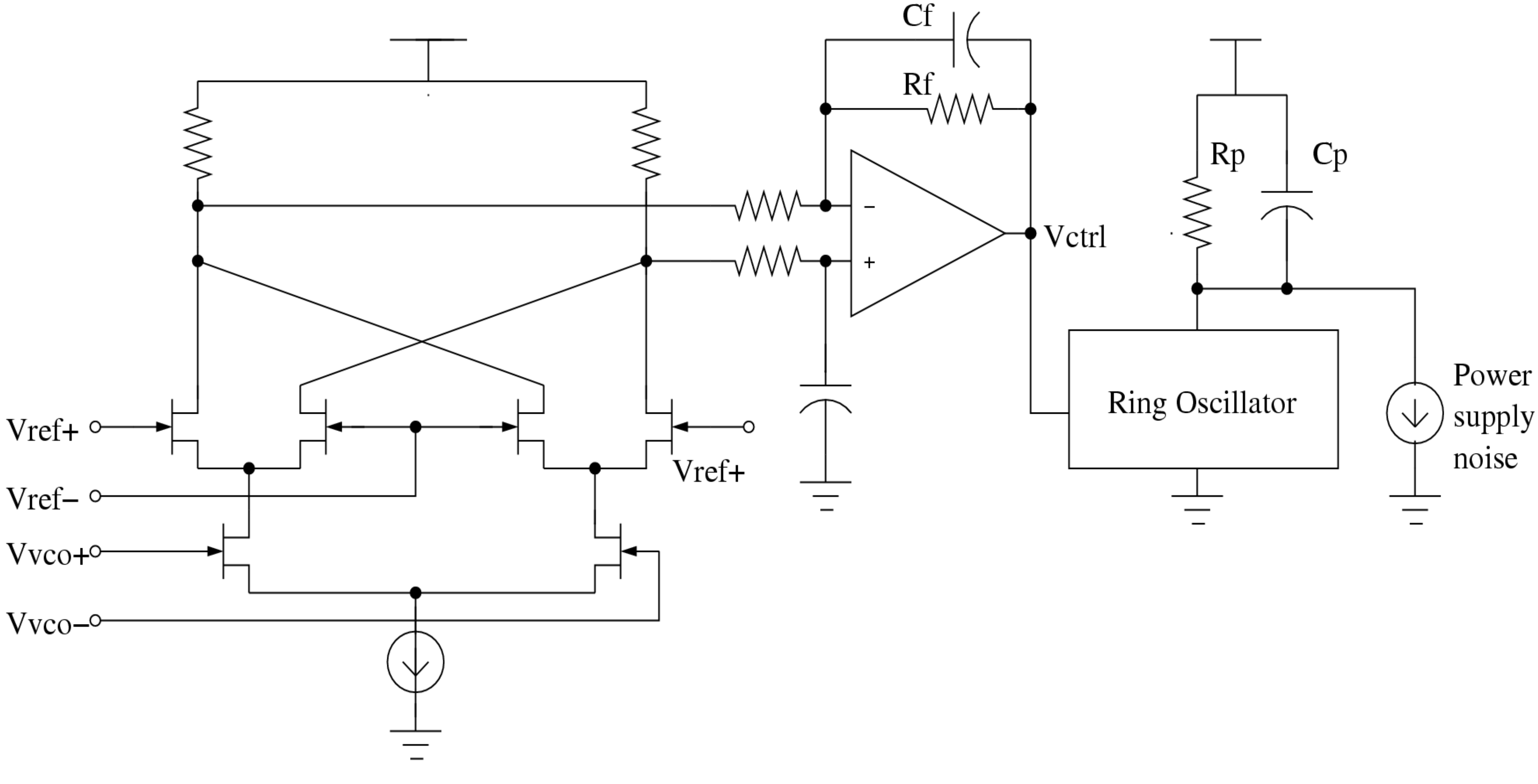
Example: Simple Ring-Oscillator VCO



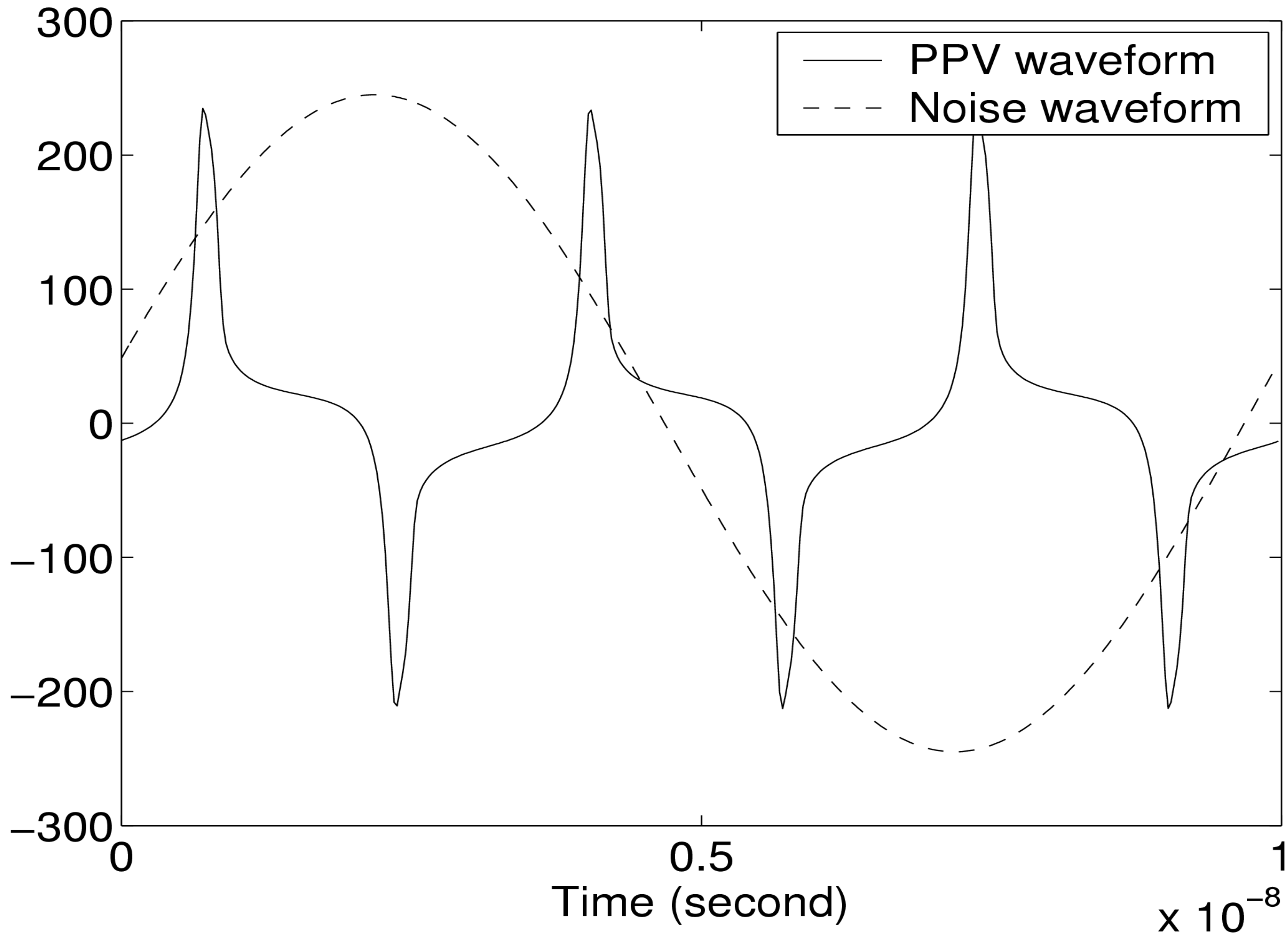
VCO Perturbation Projection Vectors: Control Node and Power Supply Components



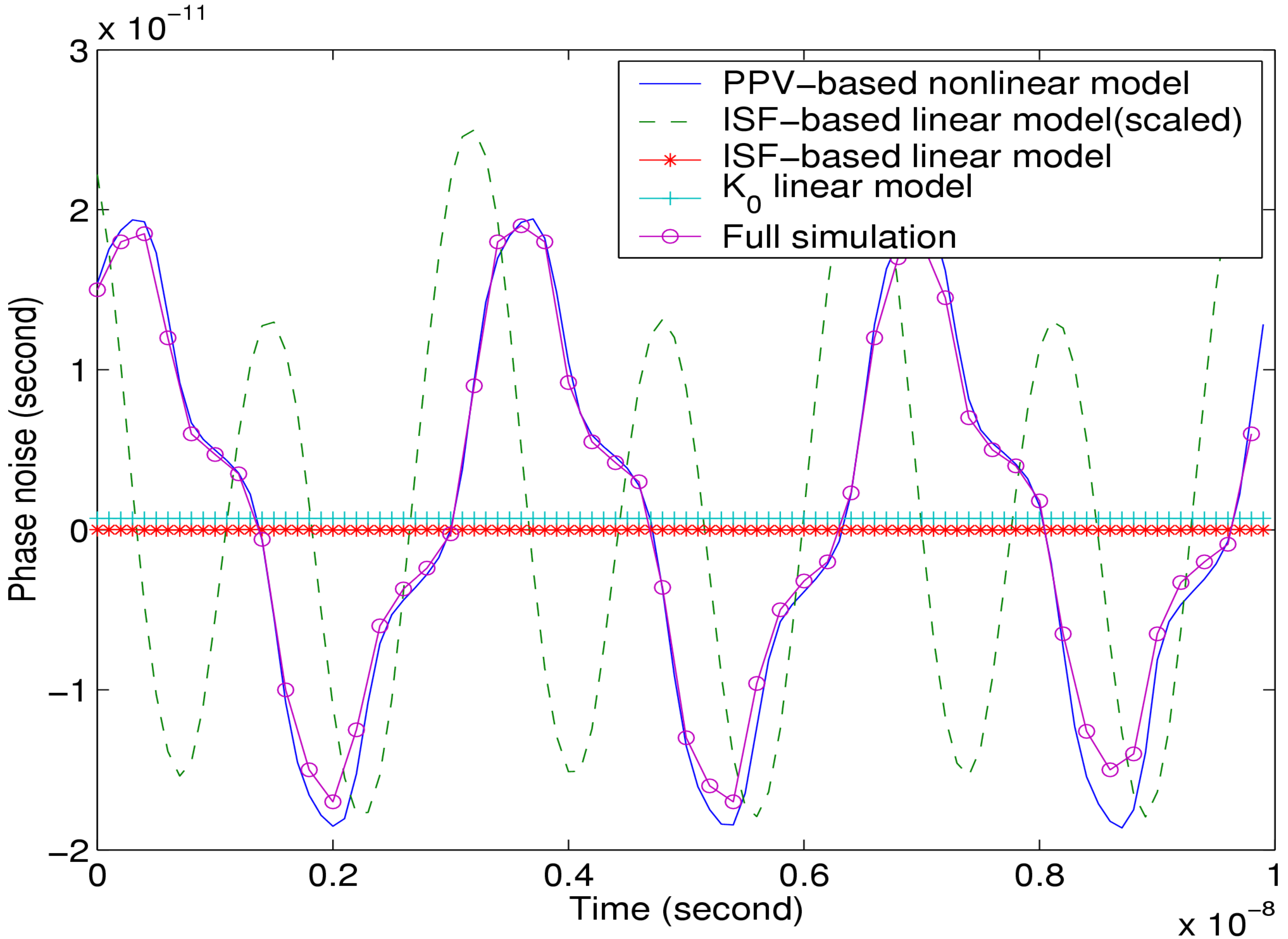
Simple Ring-Oscillator-based Phase-Locked Loop



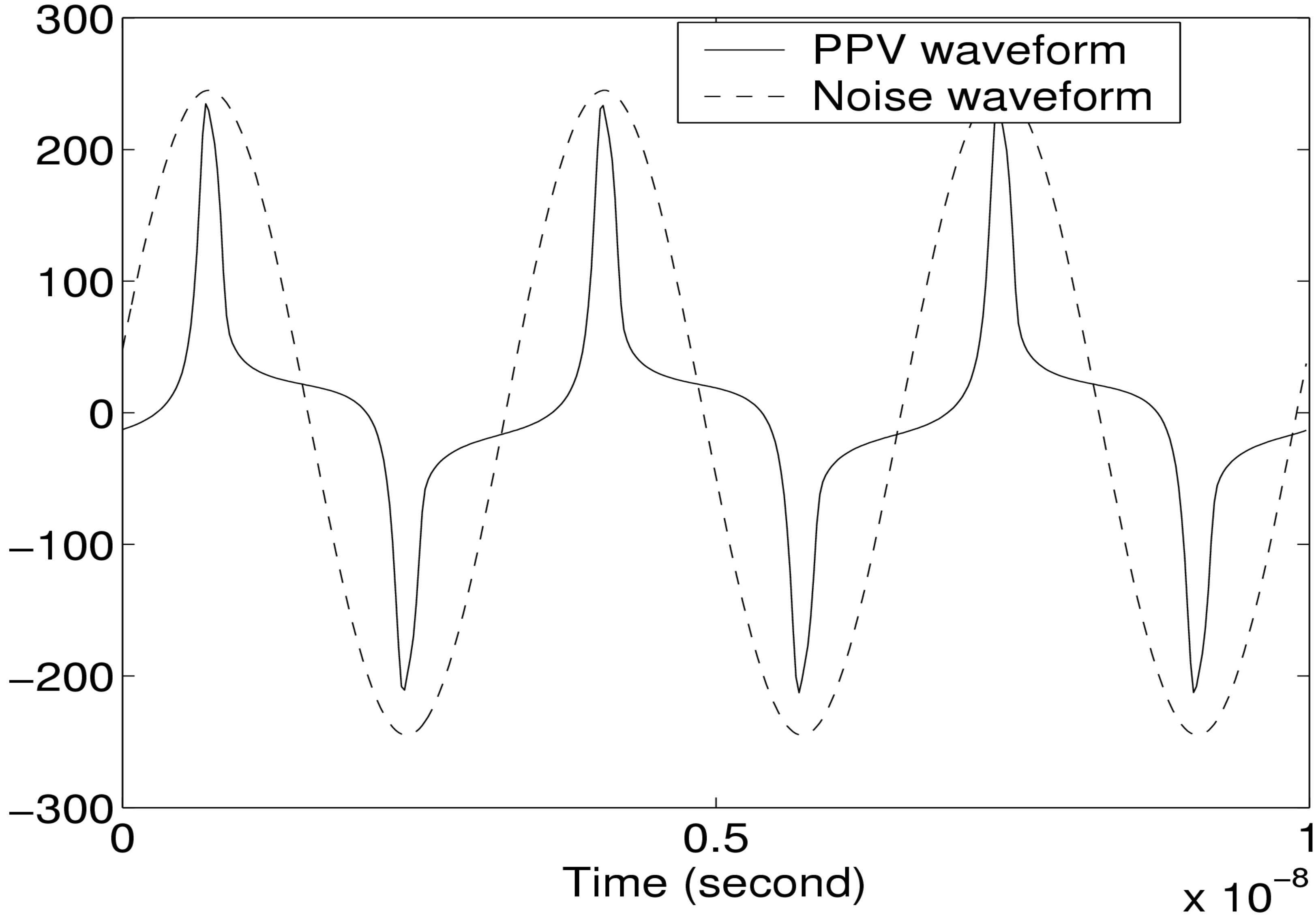
Expected Impact of 1st harmonic Supply Noise



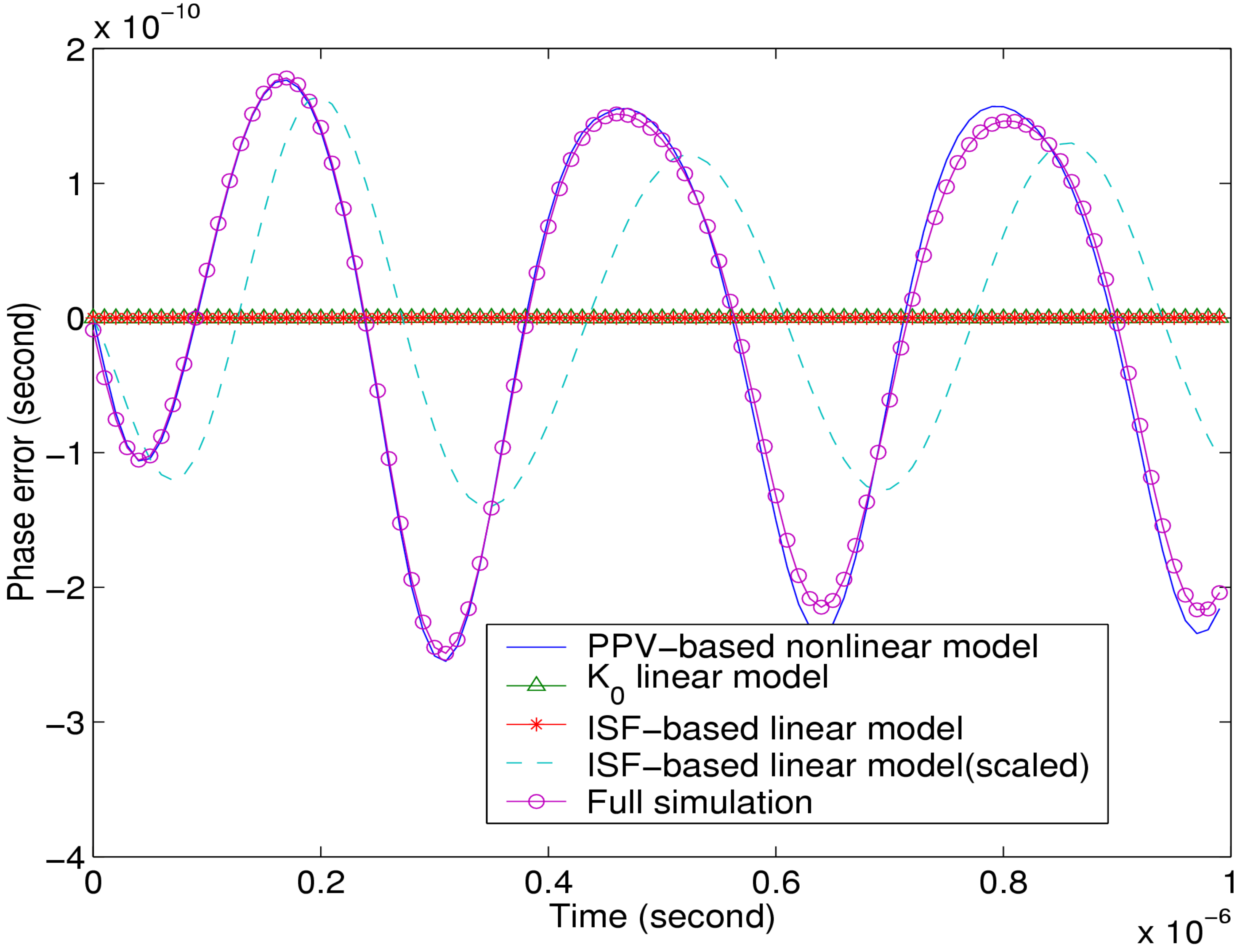
PLL Phase Reponse to Periodic Supply Noise



Expected Impact of 3rd Harmonic Supply Noise



3rd Harmonic Supply Noise: Phase Macromodel vs Full PLL simulation



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