Modeling and Simulation of Noise in Analog/Mixed-Signal Communication Systems

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Abstract

Noise in analog and mixed-signal electronic systems is an undesired but unavoidable excitation on the circuit. Its analysis and modeling is relatively straightforward in linear analog circuits, such as amplifiers. For such circuits, the SPICE AC noise analysis is, most of the time, adequate for noise performance characterization. However, for communication circuits, where nonlinearities and frequency translation are inherent, SPICE AC noise analysis is not adequate. Recently, there has been a great deal of activity, both in the design and CAD communities, to develop noise analysis techniques and tools to treat problems with nonlinear and frequency translation effects. In this tutorial, we first present the basic concepts and fundamental techniques used in noise analysis and modeling both for linear and nonlinear circuits. Then, we concentrate on some specific noise modeling and analysis problems in mixed-signal communication system design, e.g., mixers, phase noise and timing jitter, digital switching interference, etc.

Bibliography on noise analysis

Random variables, stochastic processes, stochastic differential systems: [Pap91], [Gar90], [Gar83], [Ris89], [Arn74], [Sob91], [Hak83], [GS92], [KP92], [PS87].

Device noise source models: [Rob74], [Tsi87], [Amb82], [vdZ86], [PSNT85], [WC96], [Gup82], [Kes82], [GM84], [JJ65], [WHS94].

Linear time-invariant noise analysis: [RNMW71], [MNL73], [FF97].

Linear periodically time-varying (cyclostationary) noise analysis: [RLF98], [HM93], [OTIS93], [TKW96].

Time-domain linear time-varying noise analysis: [DLSV96, Dem97].

Monte Carlo noise simulation: [BP92].

Phase noise in oscillators: [Raz95], [WKG94], [AM83], [Kae89, Kae90], [Lee66], [VV83], [Haf66], [Rob91], [Fos88], [Roh83], [McN94], [RCMC94], [Lax67], [Kur68], [HL98], [DTS98], [DMR98, Dem98, DMR99, Dem99]

Digital switching interference analysis: [MCCSV96], [DF99].

1 Introduction

The increasing demand for low cost, high performance, highly integrated solutions for RF mobile communications products, and time-to-market pressures, made the currently required number of design iterations for RF ICs unacceptable. The analog front-end continues to be the bottleneck in the design of RF communications transceivers.

The basic functionality of the RF analog front-end of a communications receiver is to extract the low-bandwidth information signal from a high frequency carrier signal which rides on top of it by modulating its amplitude, phase or frequency. Even though there are many different radio architectures, all of them consist of several stages of amplification, filtering and mixing operations. All of these operations act on the information bearing signal, on top of a carrier or at baseband, and modify some of its characteristics. There are also internally generated signals, e.g., a clock signal for a switched-capacitor circuit or the output of a frequency synthesizer, which are required in one of the amplification, filtering or mixing operations.

Verifying the correctness of the basic ideal functionality of a given RF front-end is trivial, because of the extremely simple nature of the operation, unlike, for instance, the operation of a control logic circuitry. What is far from trivial is to verify that the basic functionality is corrupted below a specified acceptable level by the non-ideal operation of the system components caused by the *non-linearities* in the RF signal path and *noise*. Non-linearities in the signal path corrupt the information bearing signal by distorting it. Non-linearities also create corruption by translating other undesired information signals (which are in the spectrum received) in frequency and adding them to the desired signal. Noise, which is generated either within the components of the devices that make up the analog-front end (i.e., thermal, shot, flicker noise), or in other parts of the IC (e.g., digital switching noise coupled through the power supply lines, the chip interconnect and the IC substrate), corrupts by coupling to the signal path and adding on top of the information signal. Apart from directly adding on top of the information signal in the signal path, noise also causes distortion and interference from other signals in the spectrum. This happens through the local oscillator (LO) signal that is used in the mixing operation, which is not in the signal path. The LO signal, ideally, is a perfectly periodic signal, i.e., discrete tones in the spectrum, which is required in an ideal mixing (i.e., frequency translation) operation. Any noise in the device (i.e., frequency synthesizer) that generates the LO signal results in corruption of these discrete tones. The way a corrupted LO signals affects the information signal in the mixing operation is quite different than, for instance, the noise in an amplifier affects it.

The origin of the information signal is, most of the time, digital, i.e., a bit stream. The effect of the nonlinearities and noise in the RF analog front-end manifests itself as bit errors one detects by comparing the demodulated information signal bit stream at the output of the receiver with the ones that was used to modulate the carrier at the transmitter. The ultimate performance measure for an RF transceiver is the bit-error-rate (BER). One would like to *analyze* and *simulate* the whole RF transceiver system before fabricating the RFIC to make sure that it meets the BER specification in a variety of circumstances. If it does not meet the specification, one would like to identify the dominant sources of error so that the problem can be fixed by focusing on that part of the system and not by redesigning the whole thing. If it meets the specification with a margin larger than required, one would like to know, design of what parts of the system can be relaxed so that the specifications are still met with lower cost, lower power dissipation, etc. Thus, one would like to have ways of characterizing the individual performances of the components that make up the system and know how they contribute to the overall system performance. Moreover, one would like to use the analysis/simulation techniques not only for veri-

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fying the performance of an already designed system or subsystem but also in making design decisions.

In this tutorial, we present an overview of the mathematical concepts from the theory of *dynamical systems* and *stochastic processes* that are used in developing an analysis/simulation methodology for the so-called *RF noise* problem. Even though we will touch upon the actual (numerical) algorithms used for analyzing RF noise, our emphasis will be on the concepts used in the mathematical representation of the signals and systems, and how input signal representations can be *passed through* the system representations for the output signal representations.

We will first concentrate on the signal representation problem. The modeling of noise-like signals using deterministic and stochastic representations will be discussed and compared against each other from the viewpoints of generality, efficiency and practicality. Both time and frequency domain concepts in stochastic modeling will be covered. We will introduce and illustrate the concepts of stationarity, non-stationarity, cyclo-stationarity, probability density functions, and *correlation* both in time and frequency domain, and discuss how they arise in dealing with the RF noise problem. The issues that arise in modeling the noise generated within the devices that make up the RF front-end due to the fundamentals of the IC device operation (thermal, shot, flicker noise, etc.), which is unavoidable, as well as the ones that are generated in other parts of the chip (digital switching noise coupled through the substrate, supply lines and the interconnect), which has a significant impact on the performance of the RF components, will be brought out. The importance of the Gaussian density, white noise (no correlation between time samples) and its dual, stationary noise (no correlation between frequency samples), will be explained.

Next, we consider the system representation problem, i.e., the modeling of the components that make up the front-end using partial differential equations (PDEs), ordinary differential equations (ODEs), transfer functions, etc. The implications of a system being *non-autonomous* (e.g., amplifiers, mixers), *autonomous* (e.g., oscillators), or *"semi"-autonomous* (e.g., phase-locked loops, frequency dividers) from the noise analysis viewpoint will be discussed. The distinction between how noise affects a non-autonomous system versus an autonomous one will be emphasized. In particular, the so-called *phase noise* problem that is associated with oscillators (an autonomous system) will be covered. The concepts of a system being linear, nonlinear, time-invariant and time-varying will be explained from the perspective of noise analysis.

Given the mathematical representations of the desired signals, noise and the system they are influencing, we would like to characterize the system output using a similar mathematical representation as the one used for the inputs. These characterizations should be such that various figures of merit (e.g., noise figure, single-sideband phase noise, timing jitter, etc.) RF circuit and system designers are used to describe the performance of their circuits can be derived/computed easily. There are two, seemingly contradictory, requirements on noise analysis techniques: They should be rigorous, but at the same time provide intuitive characterizations that can guide design. Most of the currently available rigorous noise analysis techniques are not widely used by designers because they fail to provide intuition.

RF systems are complex and large, in the sense of the number of variables, differential equations and the noise sources that are used to model the dynamics. This puts a restriction on the kinds of noise analysis algorithms that can be used for practical circuits. Usually, the more general the noise analysis algorithm is, the less efficient it is. To deal with the complexity and the large size of the noise analysis problem, one uses the same principle used in the design of these systems, i.e., exploit the hierarchy and handle the components that make up the system separately. However, in design, one still has to put the components together and make sure they work together. Same in noise analysis. One can design a mixer and verify that it meets a certain noise figure specification, and design an oscillator and verify its phase noise performance. Then they need to be put together, and verify that the corruption of the information signal (because of the non-ideal function of the mixer and the oscillator) that is frequency translated is at acceptable levels. We will present an overview of noise analysis algorithms that have been, or are being, developed. We will cover linear time-invariant analysis with stationary noise, linear (periodically) time-varying analysis with (cyclo-stationary) non-stationary noise, phase noise/timing jitter analysis and nonlinear noise analysis with large noise signals. Rather than the details of the formulation of these analysis algorithms, our emphasis will be on their general features and how they fit into a hierarchical methodology for analysis and design of RF front-ends. We will also discuss *reduced-order* and *black-box* modeling techniques, both for the noise signals and the systems, which are going to be indispensable tools in dealing with both the complexity and the modularity of RF circuit and system design. The concepts introduced in the tutorial will be illustrated with noise analysis examples for RF circuits and systems.

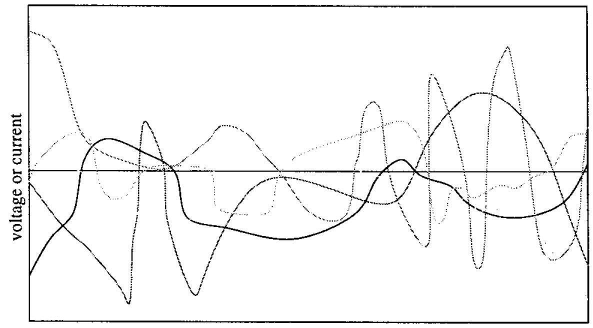

Figure 1: Noise as a collection of random waveforms

2 Representation of noise signals

Intuitively, noise is often visualised as an undesirable, fuzzy waveform corrupting a signal on, say, an oscilloscope screen or spectrum analyser, as illustrated in Figure 1. Implicit in this intuitive picture is the important notion that a noise waveform is not known exactly, but only in terms of average spread. The theory of stochastic processes enables us to concretize this notion and use it to analyse complex systems. A full review of stochastic processes is beyond the scope of this tutorial; here, we remind the reader of some the important concepts relevant to our further discussion.

$x(t)$ is called a stochastic process or random process if for any fixed value of time t, $x(t)$ is a random variable. $x(t)$ therefore represents an entire collection or *ensemble* of waveforms. Like any random variable, $x(t)$ has a mean $\mu_x(t)$ and variance $\sigma_x^2(t)$. While it is meaningless to ask what the value of $x(t)$ is at a given time t (for $x(t)$ represents a collection of waveforms, which are not individually known), the mean and variance provide useful imformation about averages of the ensemble.

In communication systems, one is often interested in frequency-domain representations, *i.e.*, Fourier transforms of time-domain waveforms. It may be asked what the Fourier transform of a stochastic process $x(t)$ is. In principle, one may consider the Fourier transform of each member of the ensemble $x(t)$ (or its square for the power), to obtain a new ensemble $X(f)$, parametrized by the frequency f. Such a definition is not, however, strictly correct, simply because members of the $x(t)$ ensemble may not have finite energy and hence cannot be Fourier-transformed. Nevertheless, a technically correct definition that retains this intuition is possible. The transformed ensemble $X(f)$ is a stochastic process in frequency, with mean $\mu_X(f)$ and variance $\sigma_X^2(f)$ providing information about its averages. We will not follow the apparently intuitive course above to understand stochastic processes in the frequency domain[1], but we will return to it when necessary. Instead, we will continue by considering the *autocorrelation function* of $x(t)$, a concept of central importance in noise analysis, both from the time- and frequency-domain points of view.

The autocorrelation function $R_{xx}(t,\tau)$ is the expected value of $x(t)x(t+\tau)$, *i.e.*, the quantity $x(t)x(t+\tau)$ averaged over all members of the ensemble of $x(t)$. The function denotes the correlation between values of the stochastic process at different times. If $R_{xx}(t,\tau)$ depends only on τ, *i.e.*, only on the time difference between the two timepoints, $x(t)$ is called a *stationary stochastic process*; if there is a dependence on t as well, it is called *nonstationary*. Stationary processes are of great importance in circuits and systems. We will first consider stationary processes.

If $x(t)$ is stationary, the Fourier transform of its autocorrelation $R_{xx}(\tau)$ is called the *power spectral density* of $x(t)$, and denoted by $S_{xx}(f)$. The power spectral density (or PSD) is a useful characterization of a stochastic process, for it can be measured directly on a spectrum analyzer. This is so because there is a direct connection between the PSD and the Fourier transforms of the squares of the stochasic process ensemble denoted by $X(f)$ above. It can be shown that the mean value of the Fourier transforms of the squared process, at a given value of frequency f, is equal to $S_{xx}(f)$. Further, two random variables $X(f_1)$ and $X(f_2)$ can be shown to be uncorrelated if $f_1 \neq f_2$. Spectrum analyzers display Fourier transforms of the squared input, averaged over separated sections of time, thus approximating an ensemble average and measuring the power spectral density directly. Another property of the PSD is that its integral over frequency equals the variance of $x(t)$. This is the total noise power, an important figure of merit.

If the PSD is independent of frequency, *i.e.*, $S_{xx}(f)$ is a constant, the process is known as *white noise*. This corresponds to the autocorrelation function $R_{xx}(\tau)$ being a delta function in τ, implying that neighbouring points of the process are totally uncorreleated. Intuitively, this suggests underlying physical processes that are extremely rapid or with very short memory. Many important sources of noise in circuits can be modelled adequately as white noise. Thermal noise in resistors and shot noise in semiconductors are white, provided bias conditions are steady.

The concepts of stationarity suffices for noise calculations in systems in a small linear region about a DC operating point, such as linear amplifiers. However, some components in RF systems, such as mixers, do not operate in a small region around a quiescent point, but have large-signal swings that are crucial to their operation. Stationary concepts no longer suffice for describing noise in such systems. It is easy to appreciate why. Consider a switch that turns on and off periodically, either passing its input through or blocking it completely. This ideal switching mixer cannot be time-invariant, since the periodic switch control makes the output dependent on the time in the cycle that an input is applied. Furthermore, if the input is a stationary stochastic process, the output is no longer stationary, for its power is zero when the switch is off, whereas it is the same as that of the input when the switch is on. Since the output power varies with time, the output noise is not stationary.

If $x(t)$ is nonstationary, its autocorrelation function $R_{xx}(t,\tau)$ is a function of t. When the dependence on t is periodic or quasiperiodic, the process is called cyclostationary. Cyclostationary processes usually arise in systems such as mixers, that are in periodic or quasiperiodic steady state.

The autocorrelation function of a cyclostationary process can be expanded in a Fourier series in t:

$$R_{xx}(t,\tau) = \sum_i R_i(\tau) e^{ji2\pi f_0 t}, \tag{1}$$

where $R_i(\tau)$ are called harmonic autocorrelation functions. Typically, a finite number of harmonics are sufficient to describe the process. The Fourier transform of $R_i(\tau)$, denoted by $S_i(f)$, is called the harmonic power spectral density or HPSD. We observe that while a stationary process has a single autocorrelation function and PSD of one argument, a cyclostationary one has many of them.

If $x(t)$ is cyclostationary, it can further be shown that the Fourier transformed process $X(f)$, alluded to previously, is no longer uncorrelated at different values of f. In fact, $X(f)$ and $X(f+if_0)$ can be shown to be correlated with value $S_i(f)$. This phenomenon is known as frequency correlation, and is equivalent to that of having nontrivial HPSDs. The stationary part of the power spectrum (*i.e.*, the component that is independent of t) is given by $S_0(f)$ and denotes the average noise power. This is usually the only output quantity of interest. However, as explained in Section 4.2, it is very important to keep track of the other HPSDs during noise analysis of time-varying systems, for they can affect $S_0(f)$ at the outputs.

3 System representation for noise analysis

Electronic circuits as dynamical systems are modeled with partial and ordinary differential equations, transfer functions, finite-state machines, etc. For the RF noise analysis problem, a system of differential/algebraic equations and transfer functions are most appropriate. Transfer functions are especially useful, because they represent the system components in frequency domain, the domain of choice for RF design, and as the basis for input-output black-box and reduced-order models.

Let us define a system as a mapping $y = \mathcal{H}(x)$ from from an input $x(t)$ into an output $y(t)$. A system $\mathcal{H}$ is said to be *linear* if $\mathcal{H}(ax_1 + bx_2) = a\mathcal{H}(x_1) + b\mathcal{H}(x_2)$, and *time-invariant* if $\mathcal{H}(x(t+\tau)) = \mathcal{H}(x)(t+\tau)$. For a linear system, the *impulse response* is given by $h(t,u) = \mathcal{H}(\delta(t-u))$. For an arbitrary input, the system output is given by

$$\mathcal{H}(x)(t) = \int_{-\infty}^{\infty} x(u)h(t,u)du \tag{2}$$

If the system is time-invariant, then $h(t,u) = h(t-u)$. If the input to an LTI system is a complex exponential at frequency f, $x(t) = \exp(j2\pi ft)$, then the output is

$$\mathcal{H}(x)(t) = H(f)\exp(j2\pi ft) \tag{3}$$

where $H(f)$ is the Fourier transform of the impulse response $h(t)$,

$$H(f) = \int_{-\infty}^{\infty} h(t-u)\exp(-j2\pi f(t-u))du \tag{4}$$

and is called the *system transfer function*. For an arbitrary input with Fourier transform $X(f) = \mathrm{F}\{x(t)\}$, the output is

$$\mathcal{H}(x)(t) = \int_{-\infty}^{\infty} H(f)X(f)\exp(j2\pi ft)df \tag{5}$$

with the Fourier transform

$$Y(f) = \mathrm{F}\{\mathcal{H}(x)(t)\} = H(f)X(f). \tag{6}$$

[1]Though possible, this turns out to be considerably more involved than the alternative presented.

By analogy with (4), the system transfer function $H(t,f)$ for a linear time-varying (LTV) system is defined by

$$H(t,f) = \int_{-\infty}^{\infty} h(t,u)\exp(-j2\pi f(t-u))du \tag{7}$$

Note that, in contrast to $H(f)$ in (4), $H(t,f)$ in (7) is a function of both f and t. If the input to an LTV system $\mathcal{H}$ is $x(t) = \exp(j2\pi ft)$, then the output is

$$\mathcal{H}(x)(t) = H(t,f)\exp(j2\pi ft) \tag{8}$$

which is a generalization of (3) to LTV systems. For an arbitrary input with $X(f) = \mathrm{F}\{x(t)\}$, the output is

$$\mathcal{H}(x)(t) = \int_{-\infty}^{\infty} H(t,f)X(f)\exp(j2\pi ft)df \tag{9}$$

A linear system is (linear) periodically time-varying (LPTV), if the impulse response is periodic in t:

$$h(t+\tau,t) = h(t+T+\tau,t+T) \tag{10}$$

with Fourier series representations for the impulse response and the transfer function

$$h(t+\tau,t) = \sum_{n=-\infty}^{\infty} h_n(\tau)\exp(j2\pi nf_c t) \tag{11}$$

$$H(t,f) = \sum_{n=-\infty}^{\infty} H_n(f+nf_c)\exp(j2\pi nf_c t) \tag{12}$$

where $H_n(f) = \mathrm{F}\{h_n(\tau)\}$ are the harmonic transfer functions. If the input to an LPTV system $\mathcal{H}$ is $x(t) = \exp(j2\pi ft)$, then the output is

$$\mathcal{H}(x)(t) = \sum_{n=-\infty}^{\infty} H_n(f+nf_c)\exp(j2\pi(f+nf_c)t) \tag{13}$$

which is a special case of (8). The Fourier transform of the output of an LPTV system is $Y(f) = \sum_{n=-\infty}^{\infty} H_n(f)X(f+nf_c)$, where $X(f)$ is the Fourier transform of the input. This is a generalization of (6) to LPTV systems.

If a single complex exponential at frequency f is input to an LTI system, the output is also a single complex exponential at frequency f with a scaled "amplitude", where the scaling is set by the transfer function $H(f)$. For an LTV system, the output for a single complex exponential input, in general, contains a *continuum* of frequencies. For LPTV systems, from (13), we observe that the output corresponding to a single complex exponential at frequency f is a summation of complex exponentials at frequencies $f+nf_c$, where f_c is the fundamental frequency. An LPTV system can be used to model a mixer as a single-input (RF) single-output (IF) system considering the LO as part of the mixer itself. Such a model captures frequency translation which is the basic functionality of a mixer. On the other hand, an LPTV model can not capture nonlinearities of the signal path. The system transfer function concept can be generalized to *nonlinear* time-invariant or (periodically) time-varying systems to capture signal path nonlinearities. For a nonlinear periodically time-varying system, the output corresponding to a single complex exponential at frequency f is a summation of complex exponentials at frequencies $kf+nf_c$, $k = 1,\ldots,\infty$, $n = -\infty,\ldots,\infty$, where f_c is the fundamental frequency.

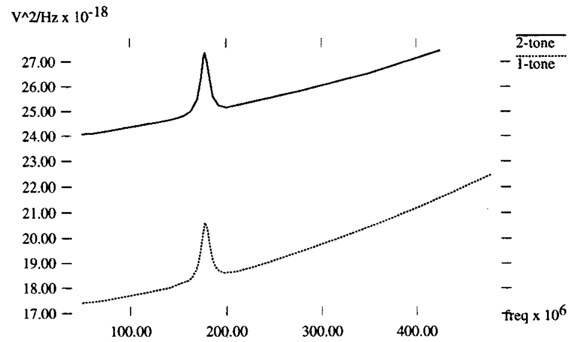

Figure 3: Stationary PSDs for mixer

4 Noise analysis

4.1 Linear time-invariant noise analysis with stationary noise

The propagation of stationary noise in LTI systems is relatively simple to analyse and to understand intuitively. This is because the PSD of a stationary signal behaves similarly to a deterministic signal when it passes through a LTI transfer function. The power spectral densities of the input and output processes $x(t)$ and $y(t)$ of a LTI system with transfer function $H(f)$ are related by:

$$S_{yy}(f) = |H(f)|^2 S_{xx}(f) \tag{14}$$

LTI noise computations all essentially implement the above relation. For a large dynamic system with many outputs and inputs, a matrix version of (14) is usually computed efficiently using adjoint techniques. The LTI noise computations can be made efficient enough to handle the huge and complicated linear network that models the power-ground grid/substrate/package/board ensemble in mixed-signal chips. Thus, LTI noise analysis can be used to analyze the interference produced by the digital switching activity of the entire chip in certain critical points which may host sensitive analog circuitry. Advanced stochastic models are used to compute the spectral characterization of the interference source signals. Figure 2 shows a comparison of spectral densities obtained by this direct noise analysis approach and a Monte Carlo experiment for a digital switching interference analysis.

4.2 Linear (periodically) time-varying noise analysis with (cyclostationary) non-stationary noise

Large signal swings in a RF system caused by, *e.g.*, an LO signal, can make significant changes to noise performance. Even the presence of a blocking signal can change the noise performance of a receiver, due to a nonlinear effect known as *noise folding*. Figure 3 illustrates how the noise level in an RFIC mixer increases when a large blocking signal is present, in addition to the local oscillator signal. Note that the noise peaks at the local oscillator frequency, indicating frequency translation of noise from the baseband – an effect that requires the machinery of cyclostationary processes and LPTV systems for correct prediction.

Noise in a LPTV system with cyclostationary processes is captured by a relation similar to (14). The HPSDs of the input process $x(t)$ and the output process $y(t)$ are related by

$$\mathbb{S}_{yy}(f) = \mathbb{H}(f)\mathbb{S}_{xx}(f)\mathbb{H}^*(f), \tag{15}$$

where $\mathbb{S}(f)$ is now a *matrix* of HPSDs arranged in a Toeplitz-like structure, and $\mathbb{H}(f)$ is a similar matrix of harmonic transfer functions of the LPTV system. The computation of (15) can be achieved efficiently, even for large systems.

We have already observed that the PSD of stationary noise is acted upon by a LTI system much as a deterministic signal would be. The

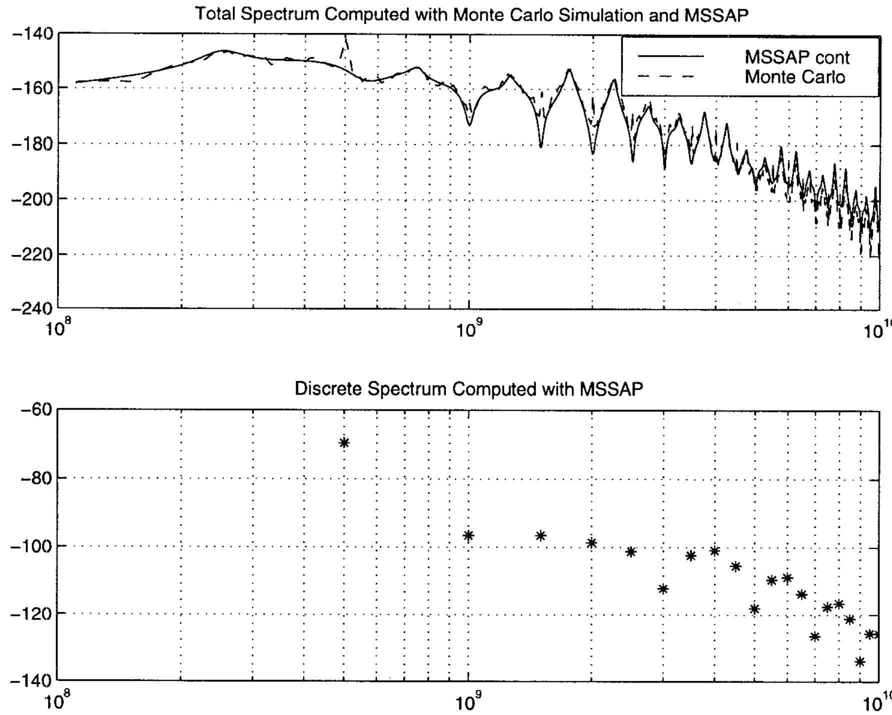

Figure 2: Interference noise analysis: Monte Carlo vs LTI

situation is similar for cyclostationary noise through an LPTV system. Just as the harmonics of a deterministic signal get mixed or frequency-translated by passage through a LPTV system, so do the HPSDs of cyclostationary noise. And just as in the deterministic case, the DC value of the output can depend on the harmonics at the input, so can the stationary HPSD at the output depend on all the HPSDs at the input. This is why the HPSDs need to be kept track of during analysis, even if it is only the stationary component at the output that is of interest.

4.3 Phase noise and timing jitter analysis

Oscillators constitute a special class among noisy physical systems: their *autonomous* nature makes them unique in their response to perturbations. Introducing even small noise into an oscillator leads to dramatic changes in its frequency spectrum and timing properties. In contrast to nonautonomous/forced systems such as mixers and amplifiers, oscillators exhibit *time instability* and *spectral dispersion.* This phenomenon, peculiar to oscillators, is known as *phase noise* or *timing jitter.* A perfect oscillator would have localized tones at discrete frequencies (i.e., harmonics), but any corrupting noise spreads these perfect tones, resulting in high power levels at neighboring frequencies.

The *linear* noise analysis techniques we described in Section 4.1 and Section 4.2 which can be used for amplifiers, filters and mixers are not directly applicable to oscillators. In LTI or LPTV noise analysis, it is assumed that the resultant deviation in the response of the circuit due to the noise sources is small. This assumption is not valid for oscillators or autonomous circuits in general. Oscillators are handled with a nonlinear perturbation analysis that is valid for autonomous systems, which is summarized next.

An oscillator without noise, i.e., a perfect oscillator, produces a periodic signal $x_s(t)$. When there are small disturbances, or noise, in an oscillator, the perfectly periodic response $x_s(t)$ is modified to $x_s(t+\alpha(t)) + z(t)$, where $\alpha(t)$ is a changing time shift, or *phase deviation,* in the periodic output of the unperturbed oscillator, $z(t)$ is an additive component, which we term the *orbital deviation,* to the phase-shifted oscillator waveform. $\alpha(t)$ and $z(t)$ are such that $\alpha(t)$ will, in general, keep increasing with time even if the noise is always small, and if the noise sources are removed, $\alpha(t)$ will settle to a constant value, and not decay to zero. The orbital deviation $z(t)$, on the other hand, will always remain small (within a factor of the noise), and if the noise sources are removed, $z(t)$ will decay to zero. If the circuit is not autonomous, i.e., it is not an oscillator, the phase deviation $\alpha(t)$ will not increase without bound, and will decay to zero if the disturbances are removed. In this case, representing the response without a phase deviation $x_s(t) + z(t)$ and using regular linear noise analysis to characterize the deviation $z(t)$ is appropriate.

The discussion above can be formalized for an oscillator described by a system of differential equations, and differential equations for the phase deviation $\alpha(t)$ and the orbital deviation $z(t)$ can be derived. Then, given the models of the noise sources in the oscillator as stochastic processes, one is interested in computing a stochastic characterization of the noisy oscillator output $x_s(t+\alpha(t)) + z(t)$, i.e., its spectrum. It turns out that, for the range of frequencies that are of practical interest, the dominant contribution comes from the phase deviation term $x_s(t+\alpha(t))$. Moreover, the orbital deviation term $z(t)$ can be "cleaned up" using a limiter at the output of the oscillator, leaving the phase deviation term unchanged, which can only be modified through the use of a phase-locked loop. Hence, noise analysis for oscillators involves characterizing the phase deviation $\alpha(t)$ as a stochastic process, which is the *timing jitter* itself, and also the resulting spectrum of $x_s(t+\alpha(t))$. The phase deviation $\alpha(t)$ does *not* change the total power in the periodic signal $x_s(t)$, but it alters the power density in frequency, i.e., the power spectral density. For the perfect periodic signal $x_s(t)$, the power

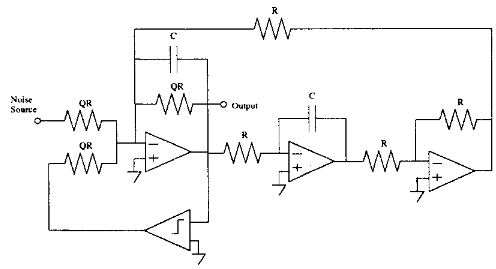

Figure 4: Oscillator with a band-pass filter and a comparator [DTS98]

spectral density has δ functions located at discrete frequencies (i.e., the harmonics). The phase deviation $\alpha(t)$ *spreads* the power in these δ functions in some specific forms depending on the stochastic properties of the noise sources. The exact shape and level of spectral spreading for an LO signal is extremely important from a practical point of view, since it is the direct cause of undesired phenomena such as interchannel interference in the mixing operation.

For example, Figure 4 shows an oscillator which consists of a Tow-Thomas second-order bandpass filter and a comparator. Figure 5(a) shows the computed PSD of the oscillator output, and Figure 5(b) shows the spectrum analyzer measurement. Figure 5(c) shows a blown up version of the PSD around the first harmonic. The single-sideband phase noise spectrum is in Figure 5(d).

4.4 Nonlinear noise analysis with large noise signals

The linear noise analysis techniques we outlined in Section 4.1 and Section 4.2 are based on a perturbation or small-signal analysis assuming that both the noise signals and the deviation of the response of the circuit due to the noise signals are *small*. In Section 4.3, we argued that, for oscillators, even when the noise signals are assumed to be small, the response deviation is not. However, by exploiting the unique nature of oscillators, we were able to *go around* this difficulty and devise a rigorous and efficient noise analysis technique. In other applications, the "small noise" or "small response deviation" may not be justified. For instance, the information signal itself in CDMA wireless communications systems is noise-like, and best modeled as a stochastic process.

One approach in dealing with the full nonlinear noise analysis problem is the so-called *Monte Carlo* method. With the Monte Carlo method, system of differential equations that model the circuit are numerically integrated *directly* in time domain to generate a number of *sample paths* for the stochastic processes that model the circuit variables. Thus, an *ensemble* of sample paths is created. Then, by calculating various expectations over this ensemble, one can evaluate various probabilistic characteristics, including correlation functions and spectral densities. If one can prove that the vector of stochastic processes satisfies some ergodicity properties, as is usually the case in practice, it may be possible to calculate time averages over a *single* sample path to evaluate some time-averaged probabilistic characteristics which provide adequate information in some applications. This method is referred to as a *Monte Carlo* method, because in generating the sample paths using numerical integration, one has to realize or simulate the noise sources or noise-like desired signals using a random number generator. Figure 6 illustrates the results of 60000 Monte-Carlo simulations of a mixer circuit.

Even though Monte-Carlo may prove to be useful for very specific problems where one can model the circuit with few differential equations, it is in general very inefficient and inaccurate. The efficiency can be increased by using specialized techniques (e.g., envelope following implemented on top of harmonic balance for simulation of nonlinear

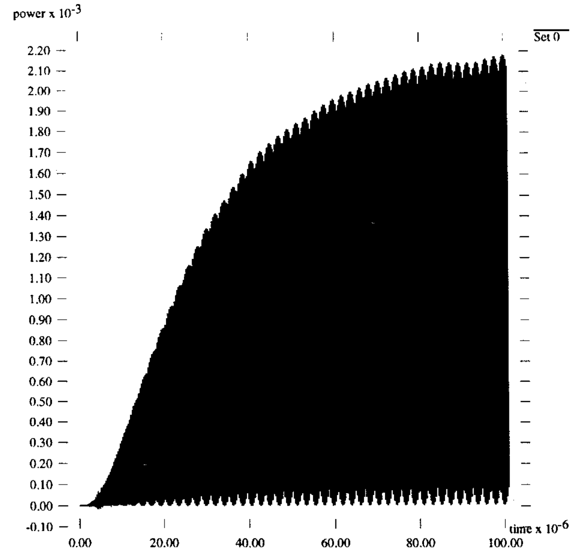

Figure 6: Time-varying filtered noise power, Monte-Carlo simulation

circuits with high-frequency narrowband signals) for the numerical solution of the differential equations that model the circuit, it is still not efficient and practical enough for realistic performance evaluation.

The ultimate characterization for a stochastic process is its *finite-dimensional* probability distributions, i.e., the joint probability density function for a number of its time samples. Note that, in a nonlinear noise analysis problem, the response of the circuit is not necessarily Gaussian even when the noise sources are Gaussian. This is one of the basic reasons why nonlinear noise analysis is (much) more difficult than linear(ized) noise analysis. Given a system of differential equations model with noise sources for a system, one can derive partial differential equations, known as Fokker-Planck equations, for the probability density of the circuit response. Solving Fokker-Planck-type equations *analytically* or *numerically* for probability densities for a general nonlinear noise analysis problem is out of the question. It is sometimes possible to obtain useful practical results using Fokker-Planck techniques for problems with very specific nature (e.g. cycle-slipping in PLLs, phase noise in oscillators).

In dealing with the nonlinear noise analysis problem for RF systems, the following observation is quite useful: The analog front-end of RF systems are generally designed to be as linear as possible from the input to the output to prevent distortion of the information signal, considering that the timing and synchronization signals such as the LO are part of the system and not inputs to it. The signal path is designed to be *nearly* linear though periodically time-varying as a result of timing and synchronization signals. Even though the signal path is designed to be as linear as possible, the low level nonlinearity can still have significant impact on the performance and needs to be taken into account for large noise-like information signals.

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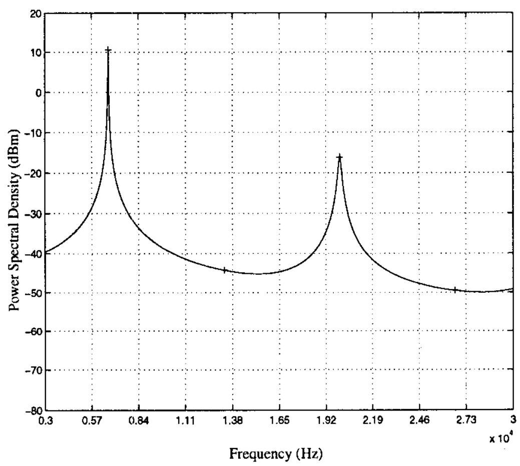
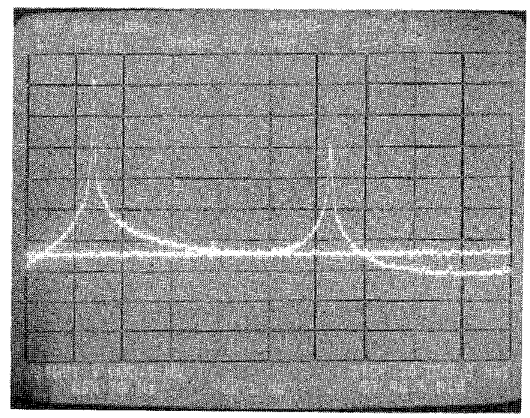
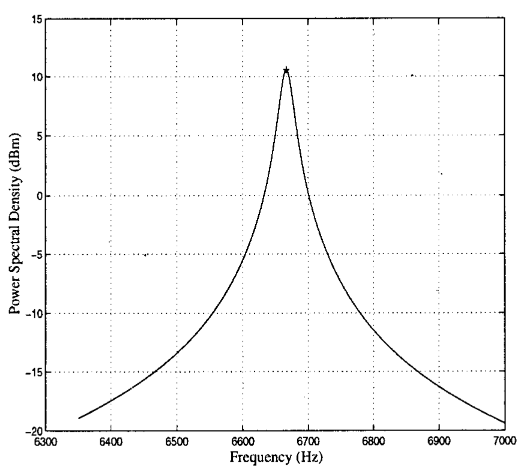


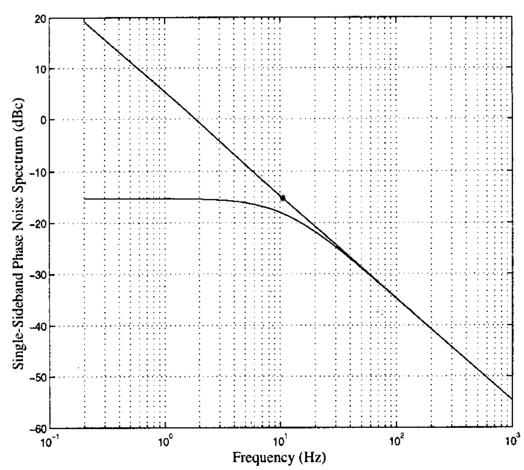
(a) Computed PSD (4 harmonics)



(b) Spectrum analyzer measured PSD [DTS98]



(c) Computed PSD (first harmonic)



(d) Computed $\mathcal{L}(f_m)$

Figure 5: Phase noise characterisation for the oscillator in Figure 4

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