

Computing Phase Noise Eigenfunctions Directly from Harmonic Balance/Shooting Matrices

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Abstract

A prerequisite for computing phase noise is finding the Perturbation Projection Vector (PPV), a periodic vector function that encapsulates the “transfer function” between individual noise sources and the phase noise of the oscillator. In this paper, we illustrate a novel technique for calculating the PPV that has several advantages over the monodromy matrix method currently used. The new method can be applied in the context of both frequency- and time-domain steady-state computations, and involves only a single linear solution of the appropriate Jacobian matrix. It also removes the need for applying heuristics in choosing the correct PPV from a potentially large set of choices, a feature particularly useful for high-Q oscillators. We compare PPVs obtained with the new method with those from the monodromy matrix method.

1 Introduction

Phase noise is a critical consideration in RF system design, since it corrupts spectral purity and generates large power content in a continuous spread of frequencies around the desired oscillator tone, thus contributing to adjacent channel interference. In digital circuits, the same phenomenon manifests itself as timing jitter. Recently, a rigorous theory has been developed for phase noise [3, 4, 2] that is uniformly applicable to any oscillatory system described by differential-algebraic equations. A key outcome of the theory is that a periodic vector function $v_1(t)$, termed the *Perturbation Projection Vector* or *PPV*, can always be found that represents a “transfer function” between the noise perturbations to the circuit, and the phase noise manifested in the oscillator. Existing numerical techniques for finding the PPV are based on explicit eigendecomposition of the monodromy matrix of the adjoint system. Full eigendecompositions are expensive for large systems; however, iterative linear methods can be used to find only a few eigenpairs with relatively little computation, to the point where the cost for finding the PPV becomes insignificant compared to, *e.g.*, that for finding a steady-state solution of the oscillator (a prerequisite).

For high-Q oscillators, however, monodromy matrices often have many eigenvalues close to 1 that are numerically indistinguishable from the oscillatory mode. In such situations, explicit eigendecomposition methods need to find a potentially large number of candidate PPVs and choose one from amongst them using heuristics. Finding many candidate eigenpairs can raise the computation to the point where it becomes dominant. Using heuristics to find the correct PPV is also often unreliable in such cases. Furthermore, monodromy matrix calculations have so far been limited to the time domain, where numerical accuracy is inherently poor compared to carefully implemented frequency-domain techniques.

In this paper, we provide a new computational procedure for the PPV that does not require the monodromy matrix. Instead, the method uses only a *single* linear solution of the steady-state Jacobian matrix of the oscillator. Heuristics are not required – the linear solution directly produces the correct PPV, with an accuracy limited only by the intrinsic numerical conditioning of the steady-state equations. Furthermore, the Jacobian matrix can be either a frequency-domain one (*e.g.*, from harmonic balance) or a time-domain one (*e.g.*, from shooting), as appropriate for the circuit in question. Hence the new technique is a) reliable and low cost for large circuits regardless of their nature, b) able to fully exploit the inherent accuracy of harmonic balance, and c) easy to implement in existing steady-state codes, since the same linear Jacobian solution required at each Newton step for finding the steady-state is simply invoked one extra time.

In Section 2, basic formulae enabling the new technique are first established at the DAE level, then specialised for harmonic balance and time-domain techniques to show that the PPV results from a single solution of the steady-state Jacobian matrix. In Section 3, the new technique is compared with monodromy-matrix eigendecomposition.

2 Relationship of the PPV $v_1(t)$ to the oscillator’s steady-state Jacobian

We consider an orbitally stable oscillator with a single oscillation mode, described by the DAE system:

$$\frac{\partial q(x)}{\partial t} + f(x) = 0. \quad (1)$$

We assume that this system has a known periodic solution $x_s(t)$. The linearization of (1) around the solution $x_s(t)$ is (*e.g.*, [4, 2]):

$$\frac{d}{dt}(C(t)y(t)) + G(t)y(t) = 0. \quad (2)$$

$C(t)$ and $G(t)$ are periodic matrices. The rank m of $C(t)$ can be less than the system size n ; m is assumed independent of t .

It can be shown (*e.g.*, [2]) that, under reasonable assumptions, the state-transition matrix

$$\Phi(t, s) = U(t)D(t-s)V^T(s)C(s), \quad (3)$$

with $D(t) = \text{diag}(e^{\mu_1 t}, \dots, e^{\mu_m t}, \underbrace{0, \dots, 0}_{k=n-m})$

satisfies (2). $U(t)$ and $V(t)$ are periodic matrices of full rank satisfying the biorthogonality condition

$$V^T(t)C(t)U(t) = I_n = \begin{pmatrix} I_m & 0 \\ 0 & 0_k \end{pmatrix}. \quad (4)$$

μ_1, \dots, μ_m are the Floquet eigenvalues. Since the system has an oscillatory mode, one of these is zero, say $\mu_1 = 0$. The Floquet eigenvectors corresponding to this mode are the first columns of $U(t)$ and $V(t)$, denoted by $u_1(t)$ and $v_1(t)$, respectively. It can be shown that $u_1(t)$ can be taken equal to $\tilde{x}_s(t)$ without loss of generality [] and computed easily from the known large-signal periodic solution. Our goal is to calculate the other oscillatory-mode Floquet eigenvector, $v_1(t)$.

It can be easily verified that the adjoint of (2), defined by

$$C^T(t) \frac{d}{dt} y(t) - G^T(t) y(t) = 0, \quad (5)$$

has the state-transition matrix

$$\Psi(t, s) = V(t)D(s-t)U^T(s)C^T(s), \quad (6)$$

which satisfies (5).

Before proceeding to connections with steady state matrices, we establish the following two results (proofs are omitted for brevity):

Lemma 2.1

$$\begin{aligned} u_1(t) &= U(t)e_1 \text{ satisfies (2)} \\ v_1(t) &= V(t)e_1 \text{ satisfies (5)} \end{aligned} \quad (7)$$

where e_1 is the first unit vector, corresponding to $\mu_1 = 0$.

Lemma 2.2

$$\begin{aligned} M &= [\dot{V}^T(t)C(t) - V^T(t)G(t)] U(t) I_m \\ &= I_m [\dot{V}^T(t)C(t) - V^T(t)G(t)] U(t) \\ \text{where } M &= \text{diag}(\mu_1, \dots, \mu_m, 0, \dots, 0). \end{aligned} \quad (8)$$

2.1 Frequency-domain computations

Frequency domain computations are natural for many applications, e.g., mildly nonlinear RF system components. We cast and apply (8) using frequency domain quantities to establish a connection with harmonic balance.

We first develop some useful algebra involving Toeplitz matrices of Fourier components.

Definition 2.1 Given any T -periodic vector or matrix $A(t)$, we denote its Fourier components by A_i , i.e.,

$$A(t) = \sum_i A_i e^{j i \omega_0 t}, \quad \omega_0 = \frac{2\pi}{T} \quad (9)$$

Definition 2.2 Given any vector or matrix $A(t)$, define the block-vector of its Fourier components to be

$$\mathbb{V}_{A(t)}^{FD} = \begin{pmatrix} \vdots \\ A_2 \\ A_1 \\ A_0 \\ A_{-1} \\ A_{-2} \\ \vdots \end{pmatrix}. \quad (10)$$

Definition 2.3 Given any matrix or vector $A(t)$, define the block-Toeplitz matrix of its Fourier components to be

$$\mathbb{T}_{A(t)} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & A_0 & A_1 & A_2 & A_3 & A_4 & \cdots \\ \cdots & A_{-1} & A_0 & A_1 & A_2 & A_3 & \cdots \\ \cdots & A_{-2} & A_{-1} & A_0 & A_1 & A_2 & \cdots \\ \cdots & A_{-3} & A_{-2} & A_{-1} & A_0 & A_1 & \cdots \\ \cdots & A_{-4} & A_{-3} & A_{-2} & A_{-1} & A_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix}. \quad (11)$$

Lemma 2.3 If $X(t)$ and $Y(t)$ are T -periodic vectors or matrices, and $Z(t) = X(t)Y(t)$, then

$$\mathbb{V}_{Z(t)}^{FD} = \mathbb{T}_{X(t)} \mathbb{V}_{Y(t)}^{FD} \quad (12)$$

$$\mathbb{T}_{Z(t)} = \mathbb{T}_{X(t)} \mathbb{T}_{Y(t)} \quad (13)$$

Lemma 2.4 If $X(t)$ is a T -periodic vector or matrix, then

$$\mathbb{V}_{\dot{X}(t)}^{FD} = \mathbb{\Omega}^{FD} \mathbb{V}_{X(t)}^{FD} \quad (14)$$

$$\mathbb{T}_{\dot{X}(t)} = \mathbb{\Omega}^{FD} \mathbb{T}_{X(t)} - \mathbb{T}_{X(t)} \mathbb{\Omega}^{FD} \quad (15)$$

where

$$\mathbb{\Omega}^{FD} = j\omega_0 \begin{pmatrix} \ddots & & & & & \\ & 2I_n & & & & \\ & & I_n & & & \\ & & & 0 & & \\ & & & & -I_n & \\ & & & & & -2I_n \\ & & & & & & \ddots \end{pmatrix} \quad (16)$$

We are now in a position to use the above definitions and lemmas. Applying (12) and (14) to (2), we obtain the linearized harmonic balance equations:

$$\underbrace{[\mathbb{\Omega}^{FD} \mathbb{T}_{C(t)} + \mathbb{T}_{G(t)}] \mathbb{V}_{y(t)}^{FD}}_{\mathbb{H}_J^{FB}} = 0. \quad (17)$$

Next, we state an important intermediate result:

Lemma 2.5

$$\left(\mathbb{T}_{V^T} \mathbb{H}_J^{FB} \mathbb{T}_U + [\mathbb{T}_M - \mathbb{\Omega}^{FD}] \right) \mathbb{T}_{I_m} = 0, \text{ and} \quad (18)$$

$$\mathbb{T}_{I_m} \left(\mathbb{T}_{V^T} \mathbb{H}_J^{FB} \mathbb{T}_U + [\mathbb{T}_M - \mathbb{\Omega}^{FD}] \right) = 0. \quad (19)$$

We concentrate on a single row of (19) by premultiplying by $\mathbb{V}_{e_1}^{FD T}$, where e_1 is a unit vector (of size $m+k$) chosen to correspond to the oscillatory mode ($\mu_1 = 0$) of the system.

Theorem 2.1

$$\mathbb{V}_{e_1}^{FD T} \mathbb{T}_{V^T} \mathbb{H}_J^{FB} = 0 \quad (20)$$

Remark 2.1 From (20), we observe that $\mathbb{T}_{VT}^T \mathbb{V}_{e_1}^{FD}$ (i.e., the vector of Fourier components of $v_1(t)$) is in the null space of J_{HB}^T , and that \mathbb{J}^{HB} is singular.

Next, consider the augmented harmonic balance matrix:

Definition 2.4 (Augmented HB matrix for oscillators)

$$\tilde{\mathbb{J}}^{HB} = \begin{pmatrix} \mathbb{J}^{HB} & p \\ q^T & r \end{pmatrix}, \quad \text{with } \tilde{\mathbb{J}}^{HB^{-1}} = \begin{pmatrix} A & b \\ l^T & d \end{pmatrix}, \quad (21)$$

where p, q, b and l are column vectors, and r and d scalars.

$\tilde{\mathbb{J}}^{HB}$ is the harmonic balance matrix augmented with a row and column, which are chosen to make it nonsingular. The following theorem establishes a simple means of computing the last row of its inverse.

Theorem 2.2 If $p = \mathbb{T}_C \mathbb{T}_U \mathbb{V}_{e_1}^{FD}$ and $\tilde{\mathbb{J}}^{HB}$ is nonsingular, then

$$l^T = \mathbb{V}_{e_1}^{FD T} \mathbb{T}_{VT}. \quad (22)$$

Remark 2.2 $p = \mathbb{T}_C \mathbb{T}_U \mathbb{V}_{e_1}^{FD}$ is the vector of the Fourier coefficients of $C(t)\dot{x}_s(t)$, i.e., $p = \mathbb{V}_{C(t)\dot{x}_s(t)}^{FD}$.

Remark 2.3 $l = \mathbb{T}_{VT}^T \mathbb{V}_{e_1}^{FD}$ is the conjugated vector of the Fourier components of $v_1(t)$, i.e., $\bar{l} = \mathbb{V}_{v_1(t)}^{FD}$.

Corollary 2.1 From (21), l is the solution of the system

$$\tilde{\mathbb{J}}^{HB T} [l^T, d]^T = [\cdots, 0, \cdots, 0, \cdots, 1]^T, \quad (23)$$

hence $\bar{l} = \mathbb{V}_{v_1(t)}^{FD}$ (i.e., the Fourier coefficients of $v_1(t)$) is the solution of

$$\tilde{\mathbb{J}}^{HB} [\bar{l}^T, \bar{d}]^T = [\cdots, 0, \cdots, 0, \cdots, 1]^T. \quad (24)$$

Remark 2.4 The augmented harmonic balance matrix $\tilde{\mathbb{J}}^{HB}$, with $p = \mathbb{T}_C \mathbb{T}_U \mathbb{V}_{e_1}^{FD}$, arises naturally as the Jacobian matrix of the oscillator's steady-state equations augmented by a phase condition, with the frequency of oscillation as an additional unknown (e.g., [15]). Hence, from (24), the Fourier coefficients of $v_1(t)$ can be obtained from a single solution of the hermitian of the augmented harmonic balance Jacobian of the oscillator, with right-hand-side equal to a unit vector with value 1 in the phase condition equation. By exploiting circulant approximations to \mathbb{J}^{HB} and applying iterative linear methods to solve (23) (e.g., [15]), this computation becomes approximately linear in the system size.

We note that the accuracy of the calculation (24) is dominated primarily by the smallest of the non-oscillatory eigenvalues $\mu_2, \cdots, \mu_m^{-1}$. For high-Q oscillators, some of these eigenvalues can be very close to zero themselves. Since finding the

¹ The $\mu_1 = 0$ eigenvalue of the non-augmented HB Jacobian is shifted to a non-zero value by the augmentation, resulting in a non-singular augmented HB Jacobian.

steady-state solution of the oscillator is itself dependent on accurate solutions with the augmented HB matrix, it is to be expected that $v_1(t)$ will also be found to a similarly acceptable accuracy. This indicates that the main issue in calculating $v_1(t)$ by (24) is the accurate formation of, and solution with, the augmented HB matrix – a task that has already been accomplished during steady-state solution.

Direct approaches to calculating $v_1(t)$, based on finding the 1-eigenpair of the system's state-transition or monodromy matrix, do not exploit the accuracy of the steady-state calculation to the same extent as (24). In the absence of a periodicity condition, transient integration errors can accumulate in computing the monodromy matrix, causing the oscillatory eigenvalue to become numerically indistinguishable from other eigenvalues close to 1. Hence several eigenvectors corresponding to multiple eigenvalues close to 1 often need to be found, followed by subsequent selection of v_1 using the criterion of orthogonality with $C(t)\dot{x}_s(t)$. It is interesting to note that this orthogonality criterion is effectively embedded into (24), due to augmentation with p ; as a result, calculation of multiple eigenvectors and subsequent selection is eliminated.

2.2 Time-domain computations

Time-domain computations are useful for systems with strong nonlinearities, such as ring oscillators. We first establish some notation.

Definition 2.5 Denote by $\{t_0, \cdots, t_{N-1}\}$ a set of N ordered sample points of the interval $[0, T)$.

Definition 2.6 Given any T -periodic vector or matrix $A(t)$, define

$$\mathbb{V}_{A(t)}^{TD} = \begin{pmatrix} A(t_0) \\ A(t_1) \\ \vdots \\ A(t_{N-1}) \end{pmatrix} \quad (25)$$

Definition 2.7 given any T -periodic matrix or vector $A(t)$, define the block-diagonal matrix of its samples to be

$$\mathbb{D}_{A(t)} = \begin{pmatrix} A(t_0) & & & \\ & A(t_1) & & \\ & & \ddots & \\ & & & A(t_{N-1}) \end{pmatrix} \quad (26)$$

Lemma 2.6 If $X(t)$ and $Y(t)$ are T -periodic vectors or matrices, and $Z(t) = X(t)Y(t)$, then

$$\mathbb{V}_{Z(t)}^{TD} = \mathbb{D}_{X(t)} \mathbb{V}_{Y(t)}^{TD} \quad (27)$$

$$\mathbb{D}_{Z(t)} = \mathbb{D}_{X(t)} \mathbb{D}_{Y(t)} \quad (28)$$

Lemma 2.7 If $X(t)$ is a T -periodic vector or matrix, then

$$\mathbb{V}_{\dot{X}(t)}^{TD} = \mathbb{\Omega}^{TD} \mathbb{V}_{X(t)}^{TD} \quad (29)$$

where $\mathbb{\Omega}^{TD}$ is a time-domain matrix that approximates differentiation, corresponding to a linear multistep formula. For example, $\mathbb{\Omega}^{TD}$ for the Backward Euler method is:

$$\mathbb{\Omega}_{BE}^{TD} = \begin{pmatrix} (T-t_{N-1})I_n & & & \\ & (t_1-t_0)I_n & & \\ & & \ddots & \\ & & & (t_{N-1}-t_{N-2})I_n \end{pmatrix}^{-1} \begin{pmatrix} I_n & & -I_n \\ -I_n & I_n & \\ & \ddots & \ddots \\ & & -I_n & I_n \end{pmatrix}$$

We now establish the time-domain analogue of (20):

Theorem 2.3

$$\underbrace{\left[\overset{TD}{\Omega} \overset{TD}{\mathbb{D}}_{C(t)} + \overset{TD}{\mathbb{D}}_{G(t)} \right]}_{\overset{TD}{J}} \overset{TD}{V}_{u_1(t)} = 0 \quad (30)$$

$$\underbrace{\left[\overset{TD}{\mathbb{D}}_{C^T(t)} \overset{TD}{\Omega} - \overset{TD}{\mathbb{D}}_{G^T(t)} \right]}_{\overset{TD}{J}} \overset{TD}{V}_{v_1(t)} = 0 \quad (31)$$

$\overset{TD}{J}$ and $\overset{TD}{J}$ are the forward and reverse time-domain Jacobian matrices, respectively. Next, consider the augmented Jacobian:

Definition 2.8

$$\overset{TD}{J}_r = \begin{pmatrix} \overset{TD}{J} & p \\ q^T & r \end{pmatrix} \quad (32)$$

where p and q are column vectors and r is a scalar. $\overset{TD}{J}_r$ is the reverse time-domain Jacobian matrix augmented with a row and column, chosen to make it nonsingular.

Solving the following augmented Jacobian system results directly in the PPV $v_1(t)$:

Theorem 2.4 If $q = \overset{TD}{V}_{(C(t)u_1(t))} = \overset{TD}{\mathbb{D}}_C \overset{TD}{\mathbb{D}}_U \overset{TD}{V}_{e_1}$ and $\overset{TD}{J}_r$ is nonsingular,

$$\overset{TD}{J}_r \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ N \end{pmatrix} \quad (33)$$

has solution $x = \overset{TD}{V}_{v_1(t)}, y = 0$.

As in the frequency-domain case, the equation system (33) can be solved efficiently with iterative methods, as a final step after solving the time-domain steady-state equations of the system.

3 Evaluation of the new technique

To evaluate the new method, it was compared against the established method that uses monodromy matrix eigendecomposition. The steady-state of a tank-circuit-based oscillator was computed using Harmonic Balance with $m = 31$ harmonics, resulting in $N = 63$ distinct frequency components. The frequency of oscillation f_0 was 159154.853364298Hz. The time-domain voltage waveform at the tank capacitor, and the currents through the inductor and power supply are shown in Figure 1, Figure 2, and Figure 3 respectively.

The PPV $v_1(t)$ was first determined through the time-domain monodromy matrix by computing its 1-eigenpair using iterative linear methods followed by manual selection from among candidate eigenpairs. The eigenvector thus obtained was then used as an initial condition for a transient simulation of the adjoint system, using a time-step corresponding to an oversampling factor of 4 (i.e., $4N$ timepoints) to limit accuracy loss from linear multistep formulae for DAE solution. The result of this transient simulation, after normalization, is the conventionally computed PPV. We refer to as $v_{1m}(t)$.

The new method described above simply computes the system (24) directly from the oscillator's harmonic balance Jacobian, with a single iterative linear solve. No oversampling is

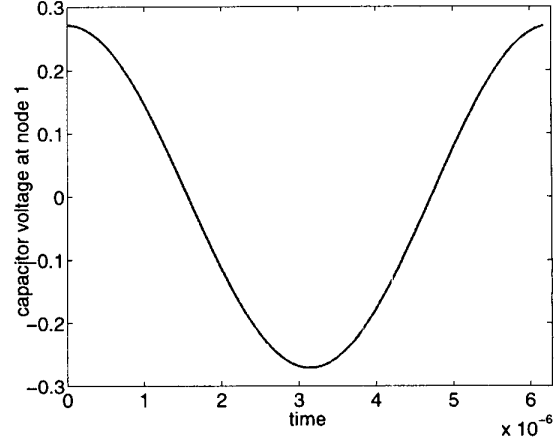


Figure 1: Oscillator steady-state: voltage at capacitor

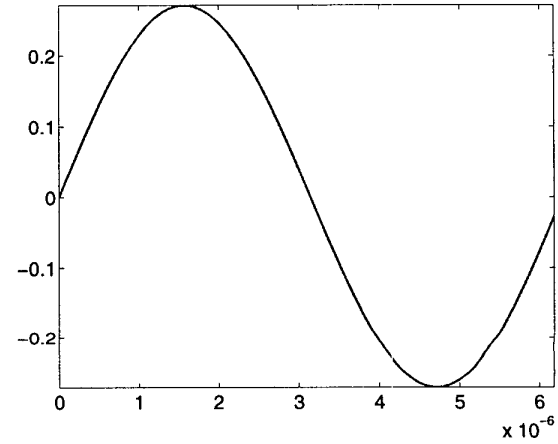


Figure 2: Oscillator steady-state: current through inductor

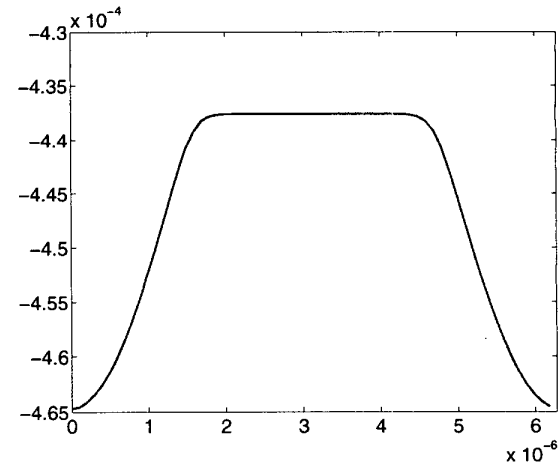


Figure 3: Oscillator steady-state: current through vdd

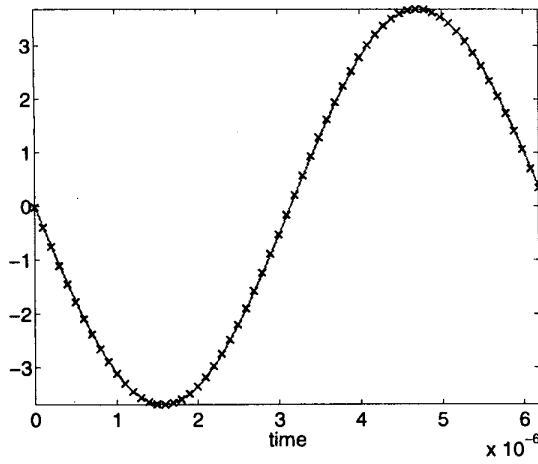


Figure 4: Capacitor node of PPVs v_{1d} and v_{1m}

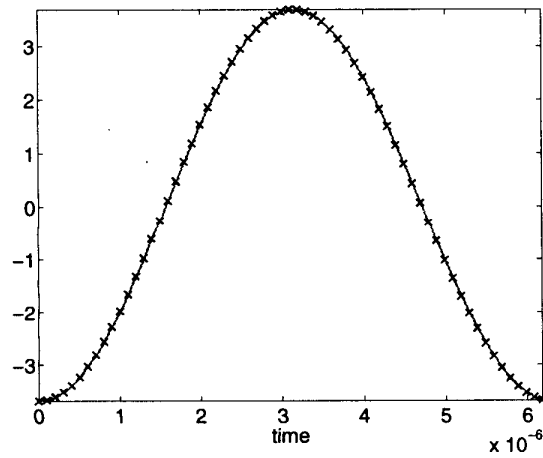


Figure 5: Inductor current component of PPVs v_{1d} and v_{1m}

used by the method. The PPV obtained in this manner is denoted by $v_{1d}(t)$.

Figure 4, Figure 5, and Figure 6 depict the components of $v_{1d}(t)$ (solid red line) and $v_{1m}(t)$ (blue \times marks) corresponding to the capacitor node, the inductor current and the power supply current, respectively. It can be seen that the PPV waveforms produced by the two methods are visually indistinguishable from each other.

A more critical assessment of the two methods can be made using the fact that $u_1^T(t)v_1(t) \equiv 1^2$. We plot the error $\epsilon_d(t) = |u_1^T(t)v_{1d}(t) - 1|$ vs $\epsilon_m(t) = |u_1^T(t)v_{1m}(t) - 1|$ in Figure 7. The solid red line indicates $\epsilon_d(t)$, the error of the new method, while the blue \times marks indicate $\epsilon_m(t)$. The new method is about 2 orders of magnitude more accurate than monodromy matrix eigendecomposition, despite the $4\times$ oversampling used by the latter method.

The Tow-Thomas oscillator in Figure 8 [] was also ana-

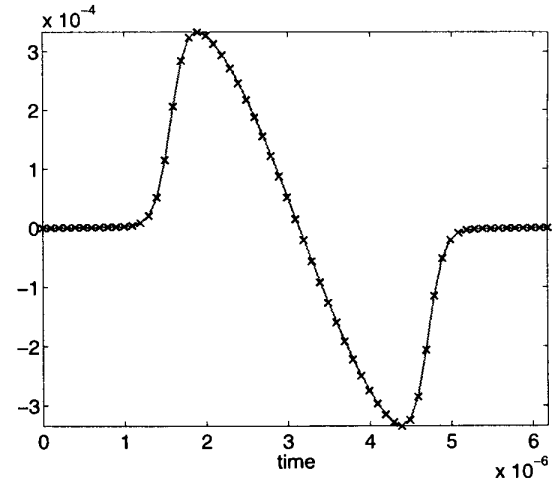


Figure 6: Power supply current component of PPVs v_{1d} and v_{1m}

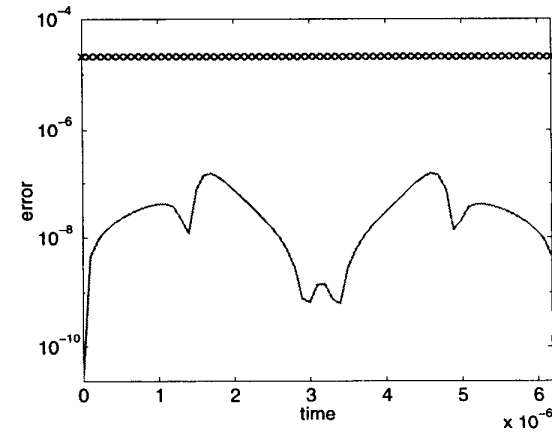


Figure 7: Errors in the PPV obtained using the monodromy and new methods

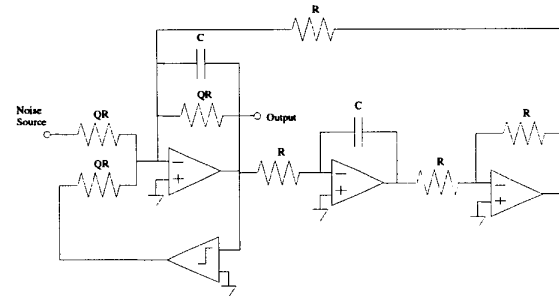


Figure 8: Oscillator with a band-pass filter and a comparator []

²See, e.g., []. Note that $u_1(t)$ defined in (7) is identical to $p(t) = \frac{p(t)}{V_{C(t)}\delta_1(t)}$.

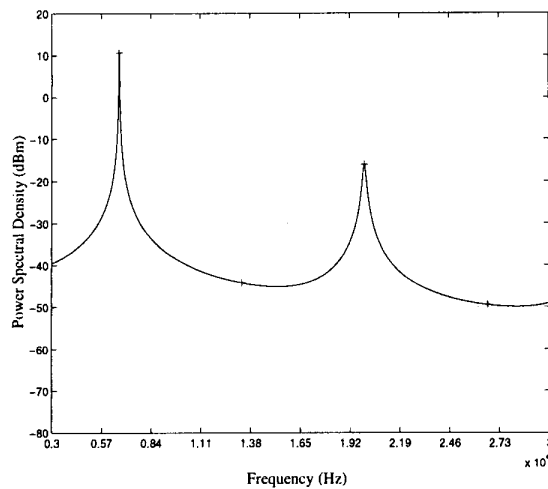


Figure 9: Computed PSD (4 harmonics)

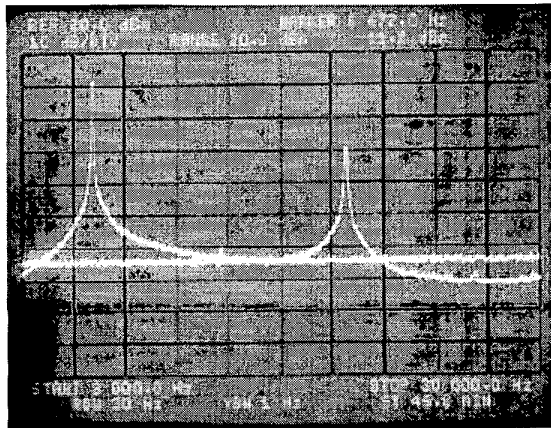


Figure 10: Measured PSD []

lyzed using the new method, and its phase noise spectrum plotted in Figure 9. The measured spectrum [] is shown in Figure 10.

Acknowledgments

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