CALCULUS AND LINEAR ALGEBRA

MODULE - 05

LINEAR ALGEBRA

Introduction :-

Matrix :- The Set mn element arranged in a aray of 'm' rows and 'n' columbs is called the matrix of order $m \times n$ and the matrix can be denoted by Capital letters of Alphabets A, B, C ....

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_n \\ a_{21} & a_{22} & a_{23} \cdots & a_n \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

where a_{ij} can be called as element of a matrix of i^{th} row, j^{th} Columbs.

Rank of a matrix:- The rank of a matrix is a positive integer which can be evaluated of a matrix by taking number of non zero rows in a matrix after the elementary transformation.

The rank of a matrix 'A' can be denoted as , $\rho(A) = r$

Note:- If all the elements are zeros it is called zero row.

A. **Find the rank of the matrix.**

①. $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ $\quad R_2 : R_2 - 2R_1$, $R_3 : R_3 + R_1$, $R_4 : R_4 - 2R_1$

$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ $\quad R_3 : R_3 + R_2$

$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad R_3 \longleftrightarrow R_4$

$$\Rightarrow A \sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_3 : \frac{1}{3} R_3$$

$\sim E$ (Echelom form)

$\rho(A)$ = Number of non - zero rows = 4

(2) $$A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$$ By applying elementary row operation.

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -11 & 5 & -3 \\ 0 & -11 & 5 & -3 \end{bmatrix} \qquad \begin{aligned} R_3 &: R_3 - 2R_1 \\ R_4 &: R_4 - 4R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} R_3 &: R_3 + R_2 \\ R_4 &: R_4 + R_2 \end{aligned}$$

$\therefore \rho(A) = 2$

(3) $$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$$ Find the rank of matrix

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & -2 & -10 \\ 0 & 4 & 1 & -5 \end{bmatrix} \qquad \begin{aligned} R_2 &\rightarrow R_2 - R_1 \\ R_3 &\rightarrow R_3 - 3R_1 \\ R_4 &\rightarrow R_4 - 2R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -3 & -11 \\ 0 & 0 & -1 & -7 \end{bmatrix} \quad \begin{array}{l} R_3 \rightarrow R_3 - R_2 \\ R_4 \rightarrow R_4 - 2R_2 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -3 & -11 \\ 0 & 0 & 0 & -10 \end{bmatrix} \quad R_4 \rightarrow 3R_4 - R_3$$

$\rho(A) = 4$

(4) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -1 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -1 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - 6R_1$

$$\sim A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 \to 5R_3 - 4R_2$

$R_4 \to 5R_4 - 9R_2$

$$\sim A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\sim A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 3$

5) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$$

$R_3 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & -2 & -10 \\ 0 & -4 & 1 & -5 \end{bmatrix}$$

$R_3 \to R_3 - R_2$; $R_4 \to R_4 - 2R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$R_4 \to 3R_4 - R_3$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \rho(A) = 3$

→ **System of linear equation by Gauss elimination and Gauss jordan method.**

Consider the system of equation having three unknowns

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$\Rightarrow AX = B$

Here the A is called Co-efficient matrix, X is variable matrix and B is called constant matrix.

The Arrgumented matrix

$$[A : B] = [AB] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & : b_1 \\ a_{21} & a_{22} & a_{23} & : b_2 \\ a_{31} & a_{32} & a_{33} & : b_3 \end{bmatrix}$$

Note :-

① If $\rho(A) = \rho(AB) = r = n$, number of unknowns, then the system of equation have unique solution.

② $\rho(A) = \rho(AB) = r < n$, then the system of equation may have infinet no/. of solution.

③ If $\rho(A) < \rho(AB)$, then there will be no Solution.

① Solve the System of equation by using Gauss elemination method $x+y+z=9$, $2x+y+z=0$, $2x+5y+7z=52$

Given,

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 52 \end{bmatrix}$$

$\Rightarrow AX - B$

$$\therefore (AB) = \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 2 & 1 & -1 & : 0 \\ 2 & 5 & 7 & : 52 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 0 & -1 & -3 & : -18 \\ 0 & 3 & 5 & : 34 \end{bmatrix} \quad \begin{matrix} R_2 : R_2 - 2R_1 \\ R_3 : R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 0 & -1 & -3 & : -18 \\ 0 & 0 & -4 & : -20 \end{bmatrix} \quad R_3 : R_3 + 3R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 0 & 1 & 3 & : 18 \\ 0 & 0 & 1 & : 5 \end{bmatrix}$$

$\therefore \rho(A) = \rho(AB) = 3$ Number of unknown

$\therefore AX = B$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 18 \\ 5 \end{bmatrix}$$

$\Rightarrow x + y + z = 9 \longrightarrow ①$

$y + 3z = 18 \longrightarrow ②$

$z = 5$

$\therefore$ (2) $\Rightarrow y + 3z = 18$

$y + 3(5) = 3$

$y = 3$

(1) $\Rightarrow x + 3 + 5 = 9$

$\Rightarrow x = 9 - 8$

$\Rightarrow x = 1$

$\therefore x = 1, \; y = 3, \; z = 5$

(2) For what values of λ and μ, the System of equation $x + y + z = 6$, $x + 2y - 3z = 10$, $x + 2y + \lambda z = \mu$

a) unique Solution b) Infinete Solution c) No Solution

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$\Rightarrow AX = B$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & -3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & -4 & : & 4 \\ 0 & 1 & (\lambda - 0) & : & (\mu - 6) \end{bmatrix} \quad \begin{matrix} R_2 : R_2 - R_1 \\ R_3 : R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & -4 & : & 4 \\ 0 & 0 & (\lambda + 3) & : & (\mu - 10) \end{bmatrix} \quad R_3 : R_3 - R_2$$

$\rightarrow$ For $\lambda \neq -3$ and $\mu \neq 10$, It may have unique Solution

$\rightarrow$ For $\lambda = -3$ and $\mu = 10$, then the System have infinte Solution

$\rightarrow$ For $\lambda = -3$ and $\mu \neq 10$ the system of equation may have no Solution.

③ For what values of λ and μ the System of equation $x+2y+3z=6$, $x+3y+5z=9$, $2x+5y+\lambda z=\mu$ ① unique Solution ② Infinete Solution ③ No Solution.

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ \mu \end{bmatrix}$$

$\Rightarrow AX = B$

$$[AB] = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & \lambda \end{bmatrix} \begin{bmatrix} 6 \\ 9 \\ \mu \end{bmatrix} = \left[\begin{array}{ccc:c} 1 & 2 & 3 & 6 \\ 1 & 3 & 5 & 9 \\ 2 & 5 & \lambda & \mu \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & (\lambda-6) & (\mu-12) \end{array}\right] \quad \begin{array}{l} R_2 : R_2 - R_1 \\ R_3 : R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & (\lambda-8) & (\mu-15) \end{array}\right] \quad R_3 : R_3 - R_2$$

→ For $\lambda \neq 8$ and $\mu \neq 15$. It may have unique Solution

→ For $\lambda = 8$ and $\mu = 15$, It may have infinete Solution

→ For $\lambda = 8$ and $\mu \neq 15$ the system of equation may have no Solution.

③ Solve the System of equation Gauss elemination method $x+2y+z=3$, $3x+2y+z=3$, $x-2y-5z=1$

Given,

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

$\Rightarrow AX = B$

$$[AB] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 3 & 2 & 1 & 3 \\ 1 & -2 & -5 & 1 \end{array}\right] \qquad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -4 & -2 & -6 \\ 0 & -4 & -6 & -2 \end{array}\right] \qquad R_2 \to (-1)R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 4 & 2 & 6 \\ 0 & -4 & -6 & -2 \end{array}\right] \qquad R_3 \to R_3 + R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & -4 & 4 \end{array}\right]$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 4 \end{bmatrix}$$

$-4z = 4$

$\boxed{z = -1}$

$x + 2(1) - 1 = 3$

$x + 2 - 1 = 3$

$x + 1 = 3$

$x = 3 - 1$

$\boxed{x = 2}$

$x + 2y - z = 3$

$2 + 2y - (-1) = 3$

$2 + 2y + 1 = 3$

$2y = 3 + 3$

$\boxed{y = 2}$

(4) Solve the following system of equations by Gauss Jordan method $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 52 \end{bmatrix}$$

$\Rightarrow AX = B$

$$[AB] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array}\right] \quad \begin{array}{l} R_2 : R_2 - 2R_1 \\ R_3 : R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right] \quad \begin{array}{l} R_1 : R_1 + R_2 \\ R_3 : R_3 + 3R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad R_3 : \left(\tfrac{1}{4}\right) R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad \begin{array}{l} R_1 : R_1 + 2R_3 \\ R_2 : R_2 + 3R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad R_2 : (-1)R_2$$

$\therefore \rho(A) = \rho(AB) = 3 = n$

$\therefore AX = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$x = 1, \quad y = 3, \quad z = 5$

⑤ Solve the System of equation by Gauss Jordan method. $x + y + z = 8$, $-x - y + 2z = -4$, $3x + 5y - 7x = 14$

Given,

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 3 & 5 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 8 \\ -4 \\ 14 \end{bmatrix}$$

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$AX = B$

$$AB = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ -1 & 1 & 2 & -4 \\ 3 & 5 & -7 & 14 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ -1 & 1 & 2 & -4 \\ 3 & 5 & -7 & 14 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & -10 & -10 \end{array}\right] \quad R_2 \longleftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 2 & -10 & -10 \\ 0 & 0 & 3 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 1 & -5 & -5 \\ 0 & 0 & 3 & 4 \end{array}\right] \quad R_2 = \frac{1}{2} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 6 & 13 \\ 0 & 1 & -5 & -5 \\ 0 & 0 & 3 & 4 \end{array}\right] \quad R_1 : R_1 - R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 6 & 13 \\ 0 & 1 & -5 & -5 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \quad R_3 : \frac{1}{3} R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \quad \begin{array}{l} R_1 : R_1 - 6R_3 \\ R_2 : R_2 + 5R_3 \end{array}$$

$AX = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5/3 \\ 4/3 \end{bmatrix}$$

$x = 5$, $y = 5/3$, $z = 4/3$

$AX = B$

$$AB = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ -1 & 1 & 2 & -4 \\ 3 & 5 & -7 & 14 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ -1 & 1 & 2 & -4 \\ 3 & 5 & -7 & 14 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & -10 & -10 \end{array}\right] \quad R_2 \longleftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 2 & -10 & -10 \\ 0 & 0 & 3 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 1 & -5 & -5 \\ 0 & 0 & 3 & 4 \end{array}\right] \quad R_2 = \frac{1}{2} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 6 & 13 \\ 0 & 1 & -5 & -5 \\ 0 & 0 & 3 & 4 \end{array}\right] \quad R_1 : R_1 - R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 6 & 13 \\ 0 & 1 & -5 & -5 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \quad R_3 : \frac{1}{3} R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \quad \begin{array}{l} R_1 : R_1 - 6R_3 \\ R_2 : R_2 + 5R_3 \end{array}$$

$AX = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5/3 \\ 4/3 \end{bmatrix}$$

$x = 5, \; y = 5/3, \; z = 4/3$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

$AX = B$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \\ 3 & -5 & 5 & 2 \end{array}\right]$$

$R_2 : R_2 - 2R_1$

$R_3 : R_3 - 3R_1$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & -11 & 2 & -7 \end{array}\right]$$

$R_3 : R_3 - 11R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

$\rho(A) < \rho(AB)$

The system of equation have no solution.

→ Gauss Seidal iterative method :-

① consider system of equation

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \longrightarrow ①$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \longrightarrow ②$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \longrightarrow ③$

② Check the system of equation are diagonally dominate or not by the following way.

$|a_{11}| > |a_{12}| + |a_{13}|$

$|a_{22}| > |a_{21}| + |a_{23}|$

$|a_{33}| > |a_{31}| + |a_{32}|$

③ write $x_1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3) \Rightarrow$ ①

$x_2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3) \Rightarrow$ ②

$x_3 = \frac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}x_2) \Rightarrow$ ③

④ take the intial values of x_1, x_2, x_3 and Continue the interation untill to get accuracy value.

⑤ If the given equation are not diagonal domin-at the rearrange the given equation according to diagonal dominate porperly.

① Solve the System of equation by using gauss Seidal iterative method $10x+y+z=12$, $x+10y+z=12$ $x+y+10z=12$

Given,

$10x + y + z = 12 \longrightarrow$ ①

$x + 10y + z = 12 \longrightarrow$ ②

$10x + y + z = 12 \longrightarrow$ ③

∴ The given equation is diagonally dominate.

① $\Rightarrow x = \frac{1}{10}[12 - y - z]$

② $\Rightarrow y = \frac{1}{10}[12 - x - z]$

③ $\Rightarrow z = \frac{1}{10}[12 - x - y]$

Let $x=0,\ y=0,\ z=0$

Iterative ① $\Rightarrow$ $x^{(1)} = \frac{1}{10}[12-0-0] = 1.2$

$y^{(1)} = \frac{1}{10}[12-1.2-0] = 1.08$

$z^{(1)} = \frac{1}{10}[12-1.2-1.08] = 0.972$

Iterative ② $\Rightarrow$ $x^{(2)} = \frac{1}{10}[12-1.08-0.972] = 0.9948$

$y^{(2)} = \frac{1}{10}[12-0.9948-0.972] = 1.0033$

$z^{(2)} = \frac{1}{10}[12-0.9948-1.0033] = 1.002$

Iterative ③ $\Rightarrow$ $x^{(3)} = \frac{1}{10}[12-1.0033-1.0002] = 0.999$

$y^{(3)} = \frac{1}{10}[12-1-1.0002] = 1$

$z^{(3)} = \frac{1}{10}[12-1-1] = 1$

$\therefore$ The Solution $x=1,\ y=1,\ z=1$

② Solve the System of equation by Gauss Seidal method $2x+y-2z=17$, $3x+20y-z=18$, $2x-3y+2z=25$ Carry out three iterations

Given,

$2x+y-2z=17 \longrightarrow ①$

$3x+20y-z=-18 \longrightarrow ②$

$2x-3y+2z=25 \longrightarrow ③$

$\therefore$ The equation are diagonally dominate.

$$① \Rightarrow x = \frac{1}{20}[17 - y + 2z]$$

$$② \Rightarrow y = \frac{1}{20}[-18 - 3x + z]$$

$$③ \Rightarrow z = \frac{1}{20}[25 - 2x + 3y]$$

Let $x=0,\ y=0,\ z=0$

Iterative ① :- $x^{(1)} = \frac{1}{20}[17 - 0 + 0] = 0.85$

$$y^{(1)} = \frac{1}{20}[-18 - 2.55 + 0] = -1.0275$$

$$z^{(1)} = \frac{1}{20}[20 - 1.7 - 3(0.825)] = 1.0109$$

Iterative ② :- $x^{(2)} = \frac{1}{20}[17 + 1.025 - 12(1.0109)] = 1.0025$

$$y^{(2)} = \frac{1}{20}[-18 - 3(1.0025) + 1.0109] = -0.999$$

$$z^{(2)} = \frac{1}{20}[25 - 2(1.0025) + 3(-0.9998)] = 0.9998$$

Iterative ③ :- $x^{(3)} = \frac{1}{20}[17 + 0.9998 + 2(0.9998)] = 1$

$$y^{(3)} = \frac{1}{20}[-18 - 3(1) + 0.9998] = -1$$

$$z^{(3)} = \frac{1}{20}[25 - 2(1) + 3(-1)] = 1$$

$\therefore x = 1,\ y = -1,\ z = 1$

③ Solve the System of equation by using Gauss Seidal Iterative method $12x + y + z = 31$, $2x + 8y + z = 24$, $3x + 4y + 10z = 58$

Scanned with CamScanner

$\Rightarrow$ Given equation are diagonally dominate

$$x = \frac{1}{12}[31 - y - z]$$

$$y = \frac{1}{8}[24 - 2x + z]$$

$$z = \frac{1}{10}[58 - 3x - 4y]$$

Iterative ① :- $x^{(1)} = \frac{1}{12}[31 - 0 - 0] = \frac{31}{12} = 2.583$

$$y^{(1)} = \frac{1}{8}[24 - 2(2.583) + 0] = 2.354$$

$$z^{(1)} = \frac{1}{10}[58 - 3(2.583) - 4(2.354)] = 4.0835$$

Iterative ② :- $x^{(2)} = \frac{1}{12}[31 - (2.354) - (4.0835)] = 2.046$

$$y^{(2)} = \frac{1}{8}[24 - 2(2.046) + 4.0835] = 2.99 = 3$$

$$z^{(2)} = \frac{1}{10}[58 - 3(2.046) - 4(3)] = 3.98 = 4$$

Iterative ③ :-

$$x^{(3)} = \frac{1}{12}[31 - (3) - (4)] = 2$$

$$y^{(3)} = \frac{1}{8}[24 - 2(2) + 4] = 3$$

$$z^{(3)} = \frac{1}{10}[58 - 3(2) - 4(3)] = 4$$

$\therefore x = 2, \; y = 3, \; z = 4$

→ Eigen value and Eigen Vectors

Suppose 'A' be a Square matrix of the Order 2×2 or 3×3 for any 'λ' Such that $(A - \lambda I)$ is the Eigen value and x is called the Eigen Vector To elvaluate the eigen values and respective Eigen Vectors to write the Characterstics equation

$|A - \lambda I| = 0$

① Diagnolysi the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

$$\therefore A - \lambda I = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

$$\therefore |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (5-\lambda)(2-\lambda) - 4 = 0$$

$$\Rightarrow 10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 6 = 0$$

$$\Rightarrow \lambda^2 - \lambda - 6\lambda + 6 = 0$$

$$\Rightarrow \lambda(\lambda - 1) - 6(\lambda - 1) = 0$$

$$\Rightarrow (\lambda - 1)(\lambda - 6) = 0$$

$$\Rightarrow \lambda = 1, \lambda = 6$$

If, $\lambda = 1$

$$(A - \lambda_1 I)x_1 = 0$$

$$\Rightarrow \begin{bmatrix} 5-1 & 4 \\ 1 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 4x_1 + 4x_2 = 0$

$\Rightarrow x_1 + x_2 = 0$

$\Rightarrow x_1 = -x_2$

$\Rightarrow \frac{x_1}{1} = \frac{x_2}{1}$

$\therefore \lambda_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

if $\lambda = 6$

$(A - \lambda_2 I) X_2 = 0$

$$\Rightarrow \begin{bmatrix} 5-6 & 4 \\ 1 & 2-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -x_1 + 4x_2 = 0$

$\Rightarrow x_1 = 4x_2$

$\Rightarrow \frac{x_1}{4} = \frac{x_2}{1}$

$\therefore \lambda_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\therefore P = [X_1 X_2] = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$

$P^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

$P^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

$$\therefore P^{-1}AP = \frac{1}{5}\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{5}\begin{bmatrix} 1 & -4 \\ 6 & 6 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{5}\begin{bmatrix} 5 & 0 \\ 0 & 30 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$$

(2) diagnolysi the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$

$$\therefore A - \lambda I = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -19-\lambda & 7 \\ -42 & 16-\lambda \end{bmatrix}$$

$$\therefore |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -(\lambda+19) & 7 \\ -42 & (-\lambda-16) \end{vmatrix} = 0$$

$$\Rightarrow (\lambda+19)(\lambda-16) + 294 = 0$$

$$\Rightarrow \lambda^2 + 19\lambda - 16\lambda - 304 + 294 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda + 5\lambda - 10 = 0$$

$$\Rightarrow \lambda(\lambda-2) + 5(\lambda-2) = 0$$

$$\Rightarrow (\lambda-2)(\lambda+5) = 0$$

$$\Rightarrow \lambda = 2, -5$$

Where $\lambda = 2$

$(A - \lambda_1 I) X_1 = 0$

$$\Rightarrow \begin{bmatrix} -21 & 7 \\ -42 & 14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -21x_1 + 7x_2 = 0$

$\Rightarrow -3x_1 + x_2 = 0$

$\Rightarrow 3x_1 = x_2$

$\Rightarrow \frac{x_1}{1} = \frac{x_2}{3}$

$\therefore \lambda_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

when $\lambda = -5$

$(A - \lambda_2 5) X_2 = 0$

$$\Rightarrow \begin{bmatrix} -14 & 7 \\ -42 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -14x_1 + 7x_2 = 0$

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow \frac{x_1}{1} = \frac{x_2}{2}$

$\therefore X_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\therefore P = [X_1 \ X_2]$

$P = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$

$\therefore P^{-1}AP = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} -4 & 2 \\ -15 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

$P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix} = D$

③ diagnolysi the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(A - \lambda I) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix}$

$|A - \lambda I| = (1-\lambda)(1-\lambda) - 4 = 0$

$= 1 - \lambda - \lambda + \lambda^2 - 4 = 0$

$= \lambda^2 - 2\lambda - 3 = 0$

$= \lambda^2 - 3\lambda + \lambda - 3 = 0$

$= \lambda(\lambda - 3) + 1(\lambda - 3) = 0$

$\lambda = -1, \lambda = 3$

when $\lambda = -1$

$[A - \lambda_1 I] X_1 = 0$

$\begin{bmatrix} 1+1 & 2 \\ 2 & 1+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x_1 + 2x_2 = 0$

$$\frac{x_1}{1} = \frac{x_2}{1}$$

$$X_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

When $\lambda = 3$

$$\begin{bmatrix} 1-3 & 2 \\ 2 & 1-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 2x_2 = 0$$

$$\frac{x_1}{1} = \frac{x_2}{1}$$

$$X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$P[X_1 \ X_2] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -3 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -2 & 0 \\ 0 & 6 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} = D$$

④ Diagnolysi the matrix, $A = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -1-\lambda & 3 \\ -3 & 4-\lambda \end{bmatrix}$$

$$(A - \lambda I) = (1+\lambda)(4-\lambda)+9 = 0$$

$$= (4-\lambda+4\lambda-\lambda^2)+9=0$$

$$= (-\lambda^2+3\lambda+4)+9=0$$

$$= -3\lambda-4\lambda^2+6=0$$

$$= \lambda^2-2\lambda-\lambda+2=0$$

$$= \lambda(\lambda-2)-1(\lambda-2)=0$$

$$\lambda=1, \quad \lambda=2$$

If $\lambda = 2$

$$\begin{bmatrix} 1 & -2-3 \\ 2 & 4-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1+3x_2=0$$

$$\frac{x_1}{1} = \frac{x_2}{1}$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

If $\lambda = 1$

$$\begin{bmatrix} -1 & -1 & 3 \\ 2 & 4 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 3x_2 = 0$$

$$2x_1 = 3x_2$$

$$\frac{x_1}{3} = \frac{x_2}{2}$$

$$X_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$P = [X_1 \; X_2] = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2-3} \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 6 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = D$$

Rayleigh's power method :-

Step ① :- let A be a square matrix of order $m \times m$ consider the intial eigen vector of the order $m \times 1$

Step ② :- $A x^{(0)} = \lambda^{(1)} x^{(1)}$ where λ indicates here numeri-cally large in the product of two matrix and X is respectively eigen vector.

Step ③ :- $A x^{(1)} = \lambda^{(2)} x^{(2)}$

$A x^{(2)} = \lambda^{(3)} x^{(3)}$ ....

Continue the process untill to get the value is accuratelly equal and which is said to be largest eigen value , and which is said to be largest eigen

① Find the largest eigen value $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ with $x^{(0)} = [1\ 0\ 0]^T$ by using power value

Given,

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}, \quad x^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$Ax^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^{(1)} x^{(1)}$$

$$Ax^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)} x^{(2)}$$

$$Ax^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8\begin{bmatrix} 1 \\ 0 \\ 0.9286 \end{bmatrix} = \lambda^{(3)} x^{(3)}$$

$$Ax^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.9228 \end{bmatrix} = \begin{bmatrix} 2.9286 \\ 0 \\ 2.8572 \end{bmatrix} = 2.9286\begin{bmatrix} 1 \\ 0 \\ 0.9756 \end{bmatrix}$$

$\lambda^{(4)}\ (x)^{(4)}$

$$Ax^{(4)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.9756 \end{bmatrix} = \begin{bmatrix} 2.9756 \\ 0 \\ 2.9512 \end{bmatrix} = 2.9756\begin{bmatrix} 1 \\ 0 \\ 0.9918 \end{bmatrix}$$

$\lambda^{(5)}\ x^{(5)}$

$$Ax^{(5)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.9918 \end{bmatrix} = \begin{bmatrix} 2.9918 \\ 0 \\ 2.9836 \end{bmatrix} = 2.9918\begin{bmatrix} 1 \\ 0 \\ 0.9973 \end{bmatrix}$$

$\lambda^{(6)}\ x^{(6)}$

$\therefore$ The large Given value $\lambda = 2.9918 \sim 3$ and it's eigen value $\begin{bmatrix} 1 \\ 0 \\ 0.9973 \end{bmatrix} \sim \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

② Find the largest eigen value Corresponding eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking intial eigen vector for $X = [1 \ 1 \ 1]^T$.

Given,

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & 1 & 3 \end{bmatrix}, \quad X^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0 \\ 0.6667 \end{bmatrix} \quad \lambda^{(1)} \ X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.3636 \\ 0.5455 \end{bmatrix} = \begin{bmatrix} 7.8182 \\ -3.6363 \\ 4.0003 \end{bmatrix} = 7.8182\begin{bmatrix} 1 \\ 0.4651 \\ 0.5117 \end{bmatrix} \quad \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.4631 \\ 0.5117 \end{bmatrix} = \begin{bmatrix} 7.9536 \\ -3.9070 \\ 4.0002 \end{bmatrix} = 7.9536\begin{bmatrix} 1 \\ -0.4912 \\ 0.5029 \end{bmatrix} \quad \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.4631 \\ 0.5117 \end{bmatrix} = \begin{bmatrix} 7.9536 \\ -3.9070 \\ 4.0002 \end{bmatrix} = 7.9536\begin{bmatrix} 1 \\ -0.4912 \\ 0.5029 \end{bmatrix} \quad \lambda^{(4)} X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.4912 \\ 0.5029 \end{bmatrix} = \begin{bmatrix} 7.9882 \\ -3.9765 \\ 4 \end{bmatrix} = 7.9882\begin{bmatrix} 1 \\ -0.4978 \\ 0.5007 \end{bmatrix} \quad \lambda^{(5)} X^{(5)}$$

$$AX^{(5)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.4978 \\ 0.5007 \end{bmatrix} = \begin{bmatrix} 7.9970 \\ -3.9941 \\ 4 \end{bmatrix} = 7.9970\begin{bmatrix} 1 \\ -0.4994 \\ 0.5002 \end{bmatrix} \sim \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}$$

∴ The large Given value of $\lambda = 7.9970 \sim 8$ and it's eigen value $\begin{bmatrix} 1 \\ -0.4994 \\ 0.5002 \end{bmatrix}$

Q.3) Find the largest Eign value of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by taking $X^{0} = [1,0\ 0]^{T}$ as intial eign value.

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \qquad X^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$AX^{(0)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 25 \\ 1 \\ 2 \end{bmatrix} = 25 \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} \qquad \lambda^{(1)}\, X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 25.2 \\ 1.12 \\ 1.66 \end{bmatrix} = 25.2 \begin{bmatrix} 1 \\ 0.0454 \\ 0.0668 \end{bmatrix} \qquad \lambda^{(2)}\, X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0444 \\ 0.0666 \end{bmatrix} = \begin{bmatrix} 25.1778 \\ 1.1332 \\ 1.7332 \end{bmatrix} = 25.1778 \begin{bmatrix} 1 \\ 0.0450 \\ 0.0668 \end{bmatrix}$$

$$AX^{(3)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0450 \\ 0.0668 \end{bmatrix} = \begin{bmatrix} 25.1526 \\ 1.1350 \\ 1.7246 \end{bmatrix} = \overset{\lambda^{(3)}}{25.1526} \overset{X^{(3)}}{\begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix}}$$

$$AX^{(4)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix} = \begin{bmatrix} 25.1826 \\ 1.1350 \\ 1.7248 \end{bmatrix} = \overset{\lambda^{(4)}}{25.1826} \overset{X^{(4)}}{\begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix}}$$

$$AX^{(5)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix} = \begin{bmatrix} 25.1820 \\ 1.1350 \\ 1.7260 \end{bmatrix} = \overset{\lambda^{(5)}}{25.1820} \overset{X^{(5)}}{\begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix}}$$

$\therefore$ The largest Eign value $\lambda = 25.1820$ and Eign vecotor $X = \begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix}$