Total number of printed pages–8

53 (MA 301) ENMA-III

2018

ENGINEERING MATHEMATICS–III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Form the partial differential equation : ***(any two)*** 3+3=6

   (a) $(x-a)^2+(y-b)^2+z^2=a^2$

   (b) $z=(x+y)\,\phi\left(x^2-y^2\right)$

   (c) $F\left(x^2-yz,\ y^2-xz\right)=0$

   (b) A covariant tensor has components $2x,\ y^2,\ z^2x$ in rectangular coordinates. Determine its covariant components in cylindrical coordinates. 5

(c) Expand : 2+2=4

$$f(z)=\frac{1}{(z-1)(z-2)}$$

in the region (i) $|z|>2$ (ii) $1<|z|<2$

(d) If $L\{F(t)\}=f(s)$, prove that

$$L\{e^{at}F(t)\}=f(s-a),\ s>a.$$

5

2. (a) If $a_{ijk}\,dx^i\,dx^j\,dx^k=0$ for all values of a_{ij}, then show that

$a_{123}+a_{132}+a_{213}+a_{231}+a_{312}+a_{321}=0$. 6

(b) Find Laplace transform of the following functions : ***(any three)*** 3×3=9

(i) $\sin(wt+\theta)$

(ii) $(t^3+2)^3$

(iii) $e^{-at}\sin wt$

(iv) te^{2t}

where w, θ, a are constants.

(c) Find the poles and residues of

$$f(z)=\frac{z^3}{(z-1)^4\,(z-2)\,(z-3)}$$

and hence evaluate $\int_C f(z)\,dz$ where C is $|z|=\frac{3}{2}$. 1+3+1=5

3. (a) Solve the following equations : ***(any two)*** 4+4=8

(i) $x(y^2-z^2)\,p+y(z^2-x^2)\,q-z(x^2-y^2)=0$

(ii) $(x^2-y^2-z^2)\,p+2xyq=2xz$

(iii) $x^2p+y^2q=(x+y)\,z$

(b) Show that the function $u=x^2-y^2+2y$ is harmonic. Find the conjugate harmonic function v and express $f(z)=u(x,y)+iv(x,y)$ in terms of complex variable z where $z=x+iy$. 2×3=6

(c) (i) If S_{ik} is symmetric and A_{ik} is skew-symmetric, then prove that $S_{ik} A_{ik} = 0$. 3

(ii) If A_{ij} is skew-symmetric then show that $\left(B_j^j B_n^m + B_n^i B_j^m\right) A_{im} = 0$. 3

4. (a) Show that

$$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2} \log \frac{s^2 + b^2}{s^2 + a^2}$$

6

(b) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{4 - 3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = \frac{3}{2}$. 4

(ii) Evaluate the integral $\int_{1-i}^{1+i} (2x + iy + 1)\, dz$ along the path (i) $y = x^2$ and (ii) $y = x$. 4

(c) Using Charpit's method solve :

$$2z + p^2 + qy + 2y^2 = 0$$

6

5. (a) (i) Assume $\phi = a_{jk} A^j A^k$. Then show that $\phi = b_{jk} A^j A^k$ where b_{jk} is symmetric.

(ii) If $C(m, n)$ is the cofactor of A_{mn} in the $det(A_{mn}) = d \neq 0$ and $A^{mn} = \frac{C(m, n)}{d}$ then show that

$$A_{mn} A^{in} = \delta_m^i$$

3+3=6

(b) Evaluate : ***(any two)*** 4+4=8

(i) $L^{-1}\left\{\frac{3s + 7}{s^2 - 2s - 3}\right\}$

(ii) $L^{-1}\left\{\log\left(1 + \frac{1}{s^2}\right)\right\}$

(iii) $L^{-1}\left\{\frac{1}{s^2 (s+1)^2}\right\}$

(c) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$. 6

6. (a) (i) When is a complex function $f(z) = u(x, y) + iv(x, y)$ analytic?
Is the function $\frac{x - iy}{x^2 + y^2}$ analytic?
Justify your answer. 1+4=5

(ii) Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. 3

(b) Solve (using Laplace transform) : 6

$$Y'' + 2Y' + 5Y = e^{-t} \sin t,$$
$$Y(0) = 0, Y'(0) = -1$$

(c) Solve : ***(any two)*** 3+3=6

(i) $z = p^2 + q^2$

(ii) $p^2 + q^2 = x + y$

(iii) $p(1 - q^2) = q(1 - z)$

7. (a) Show that the function $f(z) = z^3$ is analytic everywhere in the complex plane. 4

(b) Solve :

$$\frac{\partial^2 z}{\partial x^2} = z,$$

given that when $x = 0$, $z = e^y$ and $\frac{\partial y}{\partial x} = e^{-y}$. 4

(c) (i) Find the line element in cylindrical system. 4

(ii) If the metric is given by $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$ find the conjugate metric tensor g^{ij}. 5

(iii) Find Z-transform of

$$U(k) = \begin{cases} 0, & k < 0 \\ 1, & k \geq 0 \end{cases}$$

3
