Chapter 2
Motion in a Straight Line

Introduction

The study of motion of objects along a straight line is also known as rectilinear motion .

Average Velocity and Average Speed

Average Velocity

Average velocity is defined as the ratio of total displacement to the total time interval.

$$\text{Average velocity}=\frac{\text{Total displacement}}{\text{Total time interval}}$$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

where x_1 and x_2 are the positions of the object at time t_1 and t_2

Average speed

Average speed is defined as the ratio of total path length (distance travelled) to the total time interval.

$$\text{Average speed}=\frac{\text{Total path length}}{\text{Total time interval}}$$

Uniform motion

If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in uniform motion along a straight line .
In uniform motion velocity of the object remains constant.

2.2 Instantaneous Velocity and Speed

Instantaneous velocity

The average velocity tells us how fast an object has been moving over a given time interval but does not tell us how fast it moves at different instants of time during that interval. For this, we define instantaneous velocity or simply velocity v at an instant t.

The velocity at an instant is called instantaneous velocity and is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$v = \frac{dx}{dt}$$

$\frac{dx}{dt}$ is the differential coefficient of x with respect to t .It is the rate of change of position with respect to time.

Determining velocity from position-time graph. Velocity at t = 4 s is the slope of the tangent to the graph at that instant.

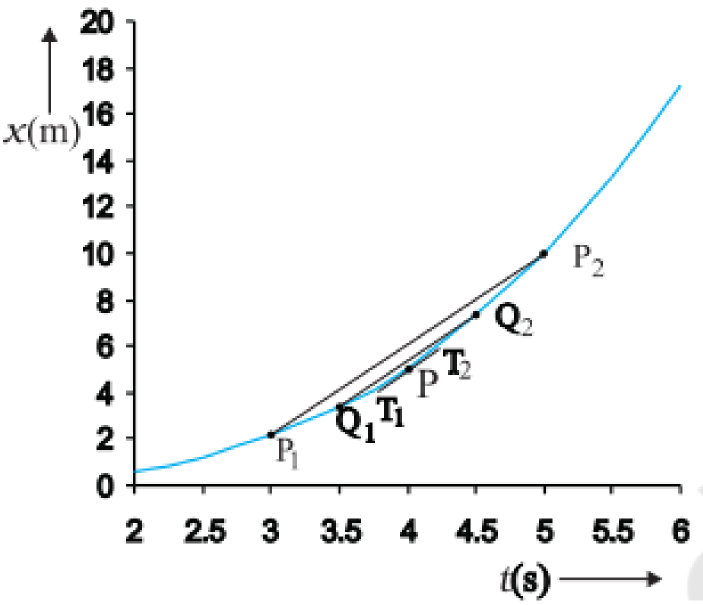


Table 2.1 Limiting value of $\frac{\Delta x}{\Delta t}$ at $t = 4$ s

Δt (s)	t_1 (s)	t_2 (s)	$x(t_1)$ (m)	$x(t_2)$ (m)	Δx (m)	$\Delta x / \Delta t$ (m s^{-1})
2.0	3.0	5.0	2.16	10.0	7.84	3.92
1.0	3.5	4.5	3.43	7.29	3.86	3.86
0.5	3.75	4.25	4.21875	6.14125	1.9225	3.845
0.1	3.95	4.05	4.93039	5.31441	0.38402	3.8402
0.01	3.995	4.005	5.100824	5.139224	0.0384	3.8400

We see from Table 2.1 that as we decrease the value of Δt from 2.0 s to 0.010 s, the value of the average velocity approaches the limiting value 3.84 m s^{-1} which is the value of velocity at t = 4.0 s, i.e. the value of $\frac{dx}{dt}$ at t = 4.0 s.

Instantaneous speed

Instantaneous speed or simply speed is the magnitude of velocity.
For example, a velocity of 24 m s^{-1} and a velocity of – 24 m s^{-1} — both have an associated speed of 24.0 m s^{-1} .

Example

The position of an object moving along x-axis is given by $x = a + bt^2$ where $a = 8.5$ m, $b = 2.5$ m s^{-2} and t is measured in seconds.

(a)What is its velocity at t = 0 s and t = 2.0 s.

(b) What is the average velocity between t = 2.0 s and t = 4.0 s ?

(a) $x = a + bt^2$

$$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 2bt$$

At $t = 0$, $v = 0$

At $t = 2$, $v = 2 \times 2.5 \times 2 = 10 \text{ m s}^{-1}$

(b) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x_4 - x_2}{4-2}$$

$$= \frac{a + 16b - a - 4b}{2}$$

$$= \frac{12b}{2} = \frac{12 \times 2.5}{2} = 15 \text{ m s}^{-1}$$

2.3 Acceleration

Suppose the velocity itself is changing with time. In order to describe its effect on the motion of the particle, we require another physical quantity called acceleration. The rate of change of velocity of an object is called acceleration.

Average Acceleration

The average acceleration a over a time interval is defined as the change of velocity divided by the time interval .

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- Unit of acceleration is ms^{-2} , $[a] = LT^{-2}$
- Acceleration is a vector quantity.
- If velocity is increasing with time, acceleration is +ve.
- If velocity is decreasing with time, acceleration is -ve.
- -ve acceleration is called retardation or deceleration.

Uniform acceleration

If the velocity of an object changes by equal amounts in equal intervals of time, it has uniform acceleration.

Instantaneous acceleration

The acceleration of a particle at any instant of its motion is called instantaneous acceleration. **It is defined as the limit of the average acceleration as the time interval Δt becomes infinitesimally small.**

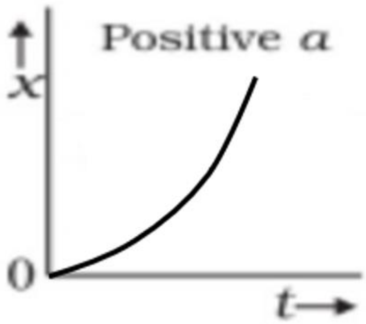
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

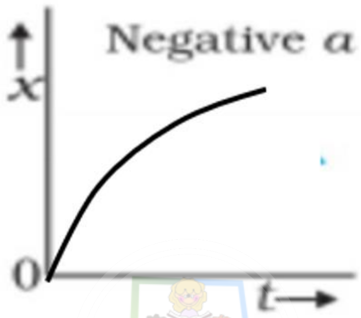
$$a = \frac{dv}{dt}$$

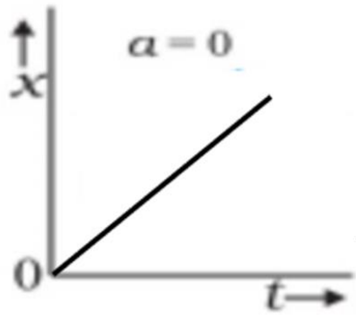
or $$a = \frac{d^2x}{dt^2}$$

Position-time graph for motion with

(a)positive acceleration (b) negative acceleration c)zero acceleration

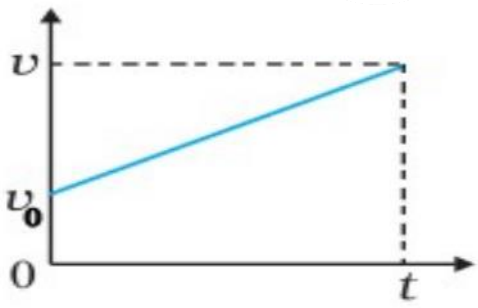




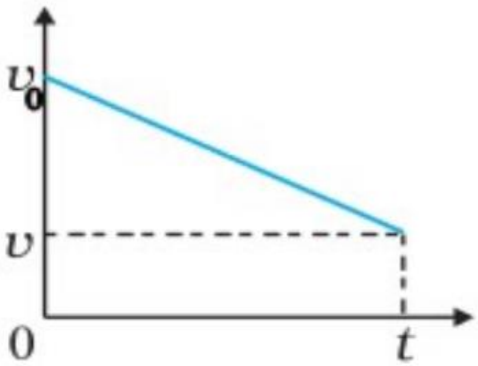


Velocity–time graph for motions with constant acceleration

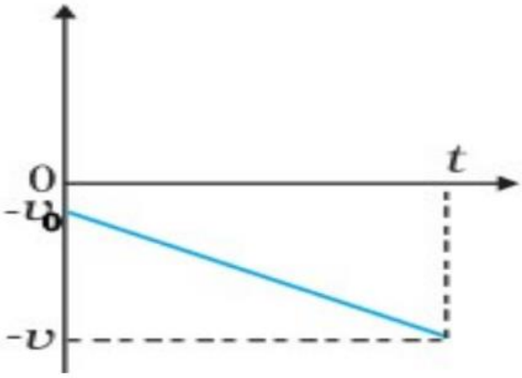
(a)Motion in positive direction with positive acceleration



(b) Motion in positive direction with negative acceleration

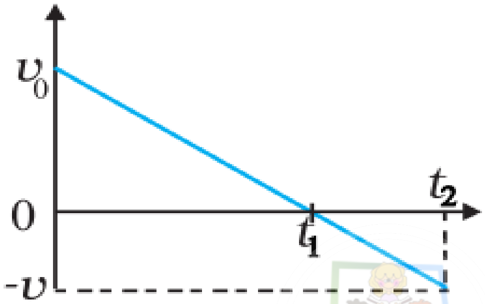


(c)Motion in negative direction with negative acceleration



(d)Motion of an object with negative acceleration that changes direction at time t_1.

(Between times 0 to t_1, its moves in positive x - direction and between t_1 and t_2 it moves in the opposite direction.)

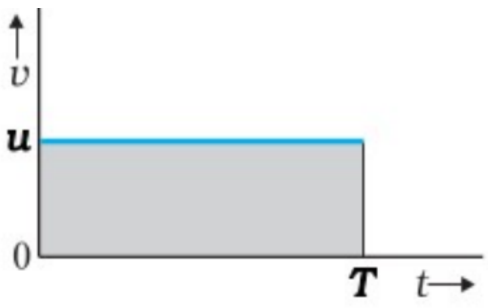


Importance of Velocity - time graph for a moving object

An interesting feature of velocity - time graph for any moving object is that **the area under the velocity - time graph is equal to the displacement of the particle.**

Proof for this statement :-

In uniform motion, velocity is the same at any instant of motion. Therefore, the velocity - time graph is a straight line parallel to the time axis.

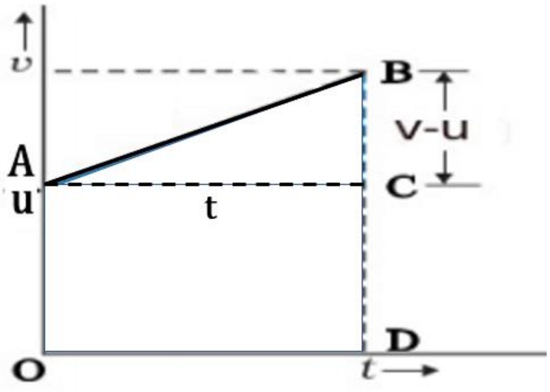


Area =uT = Displacement

i.e.,the area under the velocity - time graph is equal to the displacement of the particle.

2.4 Kinematic Equations for Uniformly Accelerated Motion

Consider a body moving with uniform acceleration . The velocity – time graph is as shown in figure



(1) Velocity – time relation

From the graph , acceleration = slope

$$a = \frac{BC}{AC}$$

$$a = \frac{v-u}{t}$$

$$v-u = at$$

$$\mathbf{v = u + at} \text{ ------------------- } (1)$$

or $(v = v_0 + at)$

(2) Position-time relation

Displacement = Area under the graph

s = Area of ▬ + Area of ▲

$$s = ut + \tfrac{1}{2}(v-u)\,t$$

But from equation (1)

$$v - u = at$$

$$s = ut + \tfrac{1}{2}\, at \times t$$

$$\mathbf{s = ut + \tfrac{1}{2} at^2} \text{ ------------------ } (2)$$

or $(s = v_0 t + \tfrac{1}{2} at^2)$

(3)Position – velocity relation

Displacement = Average velocity x time

$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{v^2-u^2}{2a}\right)$$

$$v^2 - u^2 = 2as$$

$$\mathbf{v^2 = u^2 + 2as} \text{ ---------------- } (3)$$

Or $(v^2 = {v_0}^2 + 2as)$

Stopping distance of vehicles

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

$$v^2 = u^2 + 2as$$
$$0 = u^2 + 2as$$
$$-u^2 = 2as$$
$$s = \frac{-u^2}{2a}$$

Motion of an object under Free Fall

Free fall is a case of motion with uniform acceleration.

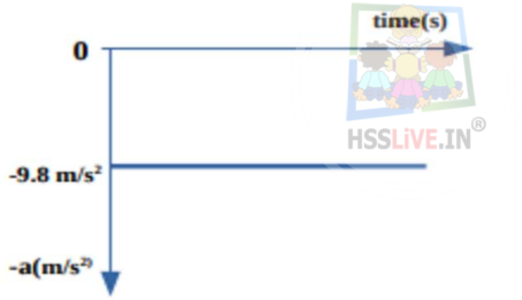
Since the acceleration due to gravity is always downward, a=–g =–9.8ms^{-2}

The object is released from rest at y = 0. Therefore, u = 0

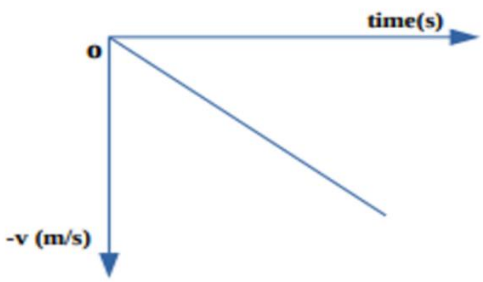
Then the equations of motion become

$$v = 0 - g\,t \quad = -9.8\,t$$
$$y = 0 - \tfrac{1}{2}\,g\,t^2 \quad = -4.9\,t^2$$
$$v^2 = 0 - 2\,g\,y \quad = -19.6\,y$$

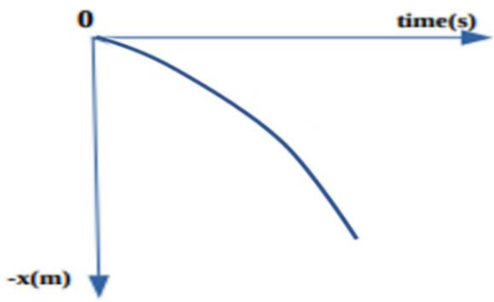
(a)Variation of acceleration with time



(b)Variation of velocity with time



(c)Variation of distance with time

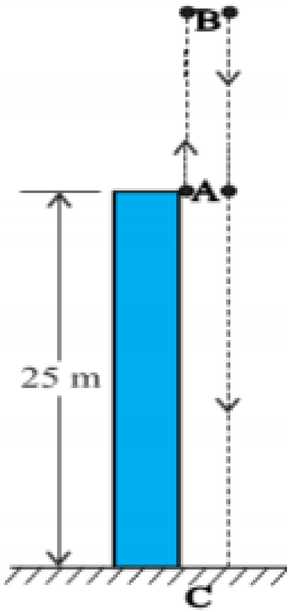


Example

A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25.0 m from the ground.

(a) How high will the ball rise ? and

(b) how long will it be before the ball hits the ground? Take g = 10 m s^{-2}



a) $u = 20$ m/s

$v = 0$

$a = -10 m/s^2$

$v^2 - u^2 = 2as$

$0 - 20^2 = 2 \times -10 \times s$

$-400 = -20\ s$

$s = -400/-20 = 20$ m

Total height = 20+ 25 = 45 m

(b) Total time = time fro upward motion + time for downward motion

For upward motion ,

$v = 0$

$u = 20$ m/s

$a = -10 m/s^2$

$v = u + at$

$0 = 20 + -10t$

$10\ t = 20$ $t = 20/10 = 2$ s

For downward motion,

$u = 0$

$s = -45m$

$a = -10 m/s^2$

$s = ut + ½ at^2$

$-45 = 0 - ½ \times 10 \times t^2$

$-45 = -5\ t^2$ $t^2 = 9$, $t = 3$ s

Total time = 2 + 3 = 5s

Example

Galileo's law of odd numbers :"The distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [namely, 1: 3: 5: 7......]." Prove it.

Answer: Let us divide the time interval of motion of an object under free fall into many equal intervals τ and find out the distances traversed during successive intervals of time. Since initial velocity is zero, we have

$$y = -\tfrac{1}{2} g t^2$$

t	y	y in terms of y_0 [$=(-\tfrac{1}{2}) g \tau^2$]	Distance traversed in successive intervals	Ratio of distances traversed
0	0	0		
τ	$-(1/2) g \tau^2$	y_0	y_0	1
2τ	$-4(1/2) g \tau^2$	$4 y_0$	$3 y_0$	3
3τ	$-9(1/2) g \tau^2$	$9 y_0$	$5 y_0$	5
4τ	$-16(1/2) g \tau^2$	$16 y_0$	$7 y_0$	7
5τ	$-25(1/2) g \tau^2$	$25 y_0$	$9 y_0$	9
6τ	$-36(1/2) g \tau^2$	$36 y_0$	$11 y_0$	11

We find that the distances are in the simple ratio 1: 3: 5: 7: 9: 11… as shown in the last column. This law was established by Galileo Galilei.