CHAPTER 1
SETS

Points to be remember:

A well-defined collection of objects/facts.

The numbers constituting a set is called elements/members of the set.

The symbol '$\in$' represents element of/member of.

Sets are generally represented by Capital letters and elements are denoted by small letters.

E.g.:

A = set of natural numbers less than 10.

$B = \{a, b, c, d\}$

There are two methods to represent a set. They are:

1. Roster Method (Tabular method or Listing method)
2. Set-builder method (Rule method/Property method).

In roster method, elements are written one by one, separated by commas and enclosed between braces or curly brackets { }.

E.g.: A = {1,2,3,4,5,6,7,8,9}.

But in set builder form, the elements of a set are described by their characterising property.

E.g.: $A = \{x: x, is\ a\ natural\ number < 10\}$ or

$A = \{x/x, is\ a\ natural\ number < 10\}$ *or*

$A = \{x: n, n \in N, n < 10\}$.

Note:

a) Repetition of elements are not considered while writing elements in a set.

   E.g.: A = Set of letters in the word "MALAYALAM"
   Roster method: A = {M, A, L, Y}

b) Order of elements are not considered while writing elements in a set.

   E.g.: $\{a, b\} = \{b.a\}$

   $\{a, b, c\} = \{b, \mathrm{c}, a\} = \{\mathrm{c, a, b}\} = \{\mathrm{a, c, b}\} = \{\mathrm{b, a, c}\} = \{\mathrm{c, b, a}\}$

Letter denoted by set:

N - Set of natural numbers
Z - Set of integers
Z^+ - Set of all positive integers
Z^- - Set of –ve integers
Q - Set of all rational numbers
Q^+ - Set of all positive rational numbers
R - Set of all real numbers

R^+ - Set of all positive real numbers
C - Set of all complex numbers
$\overline{Q}$ *or* T - Set of irrational numbers

Notations commonly used in sets

:	- (or) / Such that
$\subseteq$	- Proper subset
$=$	- Equal sets
$\in$	- Element of
$\approx$	- Equivalent sets
$\notin$	- Not an element of
$\supset$	- Superset
U	- Universal set
ϕ / { }	- Null set
$\subset$	- Subset
A' or A^c	- Complement of a set A
n(A)	- No. of element of set A
$\cap$	- Intersection
$\cup$	- Union
– (or) \	- Difference of sets
Δ	- Symmetric difference of sets

Types of sets

1.	Null set	{ }
2.	Singleton set	{ 5 }
3.	Finite set	{1, 2, 3, … , 100}
4.	Infinite set	{1, 2, 3, … }
5.	Equivalent sets	$n(A) = n(B)$, if A and B be any two sets.
6.	Equal sets	Elements of both the sets are same ($A = B$)
7.	Disjoint sets	Two or more sets having different elements

Subset and Superset: Consider the sets $A = \{1,2,3\}$ and $B = \{2,3\}$. Here every element of B is an element of A. Therefore, B is known as subset of A, denoted by $B \subset A$ and A is known as super set of B, denoted by $A \supset B$.

Note:

1. Number of subsets if a set has n elements = 2^n
2. No. of proper subsets = $2^n - 1$
3. Every set is a subset of itself.
4. Empty set (ϕ) is a subset of every set.
5. No. of elements in the power set of A = $n[P(A)] = 2^n$, where $n = n(A)$
6. If A and B are disjoint sets,

   a) $A \cap B = \phi$ b) $A - B \neq B - A$

7. $The\ number\ of\ elements\ of\ a\ power\ set = No. of\ subsets.$

No. of elements of a set	No. of subsets
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
$\vdots$	$\vdots$
n	2^n

E.g.:

1. $A = \phi$

   Subsets: ϕ

2. $A = \{\phi\}$

   Subsets: $\{\phi\}, \phi$

3. $A = \{1, 2\}$

   Subsets: $\{1,2\}, \{1\}, \{2\}, \phi$

4. $A = \{1, 2, \{3\}\}$

   Subsets: $\{1, 2, \{3\}\}\ \{1, 2\}, \{1, \{3\}\}, \{2, \{3\}\}\ \{1\}, \{2\}, \{\{3\}\}, \phi$

5. $A = \{1, 2, 3, 4\}$

   Subsets: $\{1,2,3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2\},$

   $\{1,3\}, \{1,4\}\ \{2,3\}, \{2,4\}, \{3,4\}, \{1\}, \{2\}, \{3\}, \{4\}, \phi$

Proper subset: A set excluding the given set is known as proper subset.

E.g.: $A = \{1, 2\}$

Subsets: $\{1,2\}, \{1\}, \{2\}, \phi$

Proper subsets: $\{1\}, \{2\}, \phi$

Power Set: Set of subsets is called power set. P(A) denotes power set of the set A.

E.g.: $A = \{\phi\}$

Subsets : $\{\phi\}, \phi$

Power set of A, P(A) = $\{\{\phi\}, \phi\}$

Symmetric Difference of Sets: If A and B are any two sets, then

$A \Delta B = (A - B) \cup (B - A)$

Subsets of set of real numbers

- Set of all natural numbers: $\{1, 2, 3, 4, ...\}$

- The set of all integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Set of all rational numbers: $\left\{x : x = \frac{a}{b},\ a, b \in Z, b \neq 0\right\}$

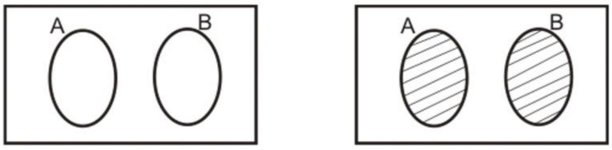
Laws of Algebra for operations on sets:

1. $A \cup A = A$ [Idempotent law]
2. $A \cup \phi = A$ [Identity law]
3. $A \cup U = U$ [Universal law]
4. $A \cup B = B \cup A$ [Commutative law]
5. $A \cup (B \cup C) = (A \cup B) \cup C$ [Associative law]
6. $A \cap A = A$ [Idempotent law]
7. $A \cap \phi = \phi$ [Law of ϕ]
8. $A \cap U = A$ [Universal law]
9. $A \cap B = B \cap A$ [Commutative law]
9. $A \cap (B \cap C) = (A \cap B) \cap C$ [Associative Law]
10. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Distributive of $\cup$ over $\cap$]
    $A \cap (B \cup C) = (A \cup B) \cup (A \cap C)$ [Distributive of $\cap$ over $\cup$]
11. $A \cup A' = U$: $A \cap A' = \phi$ [Complement law]
12. $(A')' = A$ [Involution or Double complement law]
13. $\phi' = U$ [law of ϕ]
14. $U = \phi$ [Universal law]
15. De Morgan's Law
    For any two sets A and B, we have
    $(A \cup B)' = A' \cap B'$
    $(A \cap B)' = A' \cup B'$
16. $A \cap B' = A - B$
17. If $A = B$ then $A \cup B = A \cap B$
18. If $B \subset A$, $(A - B) \cup B = A$
19. If $A \subset B$, $A \cup B = B$ *and* $A \cap B = A$
20. If $B \subset A$, $A \cup B = A$ *and* $A \cap B = B$

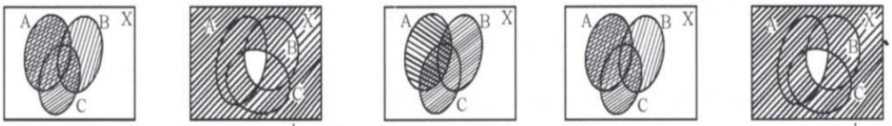
Venn diagram

It is a pictorial representation of sets. It consists of two closed figures – a rectangle for universal set and circles or oval shaped circles for sets. It was introduced by two mathematicians John Venn and Euler. Hence it is known as Venn-Euler diagram or simply Venn diagram.

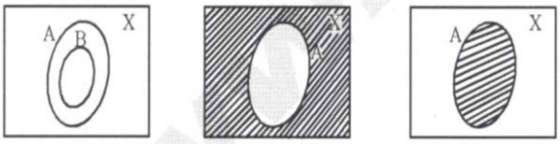
The following are some examples: If A and B are disjoint sets,



$A \cap B$ $A \cup B$

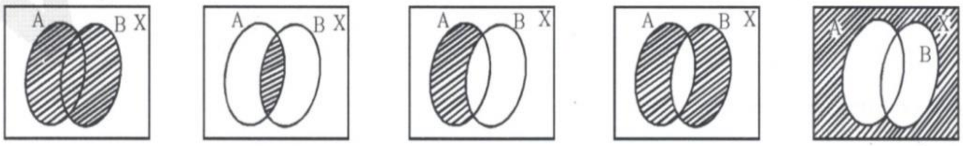


$(A\cup B)\cap(A\cup C)$ $(A\cap B\cap C)'$ $A\cap(B\cup C)$ $(A\cup B)\cap(A\cup C)$ $(A\cap B\cap C)'$

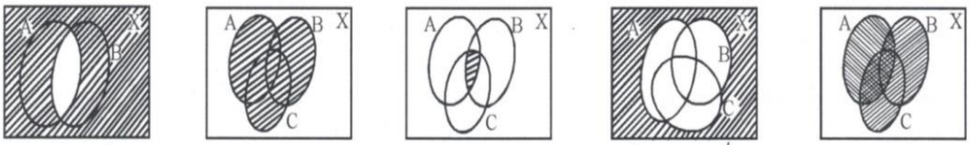


$B\subseteq A$ A' $(A')'$

If A and B and A, B and C are not disjoint sets



$A\cup B$ $A\cap B$ $A-B$ $A\Delta B$ $(A\cup B)'$



$(A\cap B)'$ $A\cup B\cup C$ $A\cap B\cap C$ $(A\cup B\cup C)'$ $A\cap(B\cup C)$

Subset as intervals of R

Let a and b be any two real numbers. If $a < b$, then

i. $\{x : x \in R, a < x < b\}$ is known as open interval a,b. It is denoted as (a,b).

Graph: 

ii. $\{x : x \in R, a \le x \le b\}$ is known as closed interval a,b. It is denoted as [a,b].

Graph: 

iii. $\{x : x \in R, a \le x < b\}$ is known as semi-closed interval a,b. It is denoted as [a,b).

Graph: 

iv. $\{x : x \in R, a < x \le b\}$ is known as semi-open interval a,b. It is denoted as (a,b].

Graph: 

Infinite intervals

Set builder form	Roster form	Graph
$\{x : x \in R, -\infty < x < \infty\}$	$(-\infty, \infty)$	
$\{x : x \in R, -\infty < x < 0\}$	$(-\infty, 0)$	
$\{x : x \in R, -\infty < x \le 0\}$	$(-\infty, 0]$	
$\{x : x \in R, 0 < x < \infty\}$	$(0, \infty)$	
$\{x : x \in R, 0 \le x < \infty\}$	$[0, \infty)$	

Tips:

1. $|x| = \begin{cases} x, \ if \ x > 0 \\ 0, \ if \ x = 0 \\ -x, \ if \ x < 0 \end{cases}$
2. $|x|^2 = |-x|^2 = x^2$
3. $|xy| = |x||y|$
4. $\left|\frac{x}{y}\right|^2 = \frac{|x|}{|y|}, y \neq 0$
5. $|x + y| \le |x| + |y|$
6. $|x - y| \le |x| + |y|$
7. $|x + y| \ge |x| - |y|$
8. $|x - y| \ge |x| - |y|$
9. If $|\boldsymbol{x}| \le \boldsymbol{a} \Rightarrow -\boldsymbol{a} \le \boldsymbol{x} \le \boldsymbol{a}$
10. If $|\boldsymbol{x}| \ge \boldsymbol{a} \Rightarrow -\boldsymbol{a} \ge \boldsymbol{x} \ge \boldsymbol{a}$, for $\boldsymbol{k} > \boldsymbol{0}$