

Chapter 6 – Gravitation

1. Introduction

Every object attracts every other object — that's gravitation.
Though weakest, it rules stars, planets, and galaxies.

Newton discovered this universal attraction, explaining both the fall of an apple and the orbit of the Moon with one equation.

2. Newton's Law of Gravitation

$$F = G \frac{m_1 m_2}{r^2} \quad F = G r^2 m_1 m_2$$

Where:

- $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$
- $F \propto m_1 m_2$ (directly), $F \propto 1/r^2$ (inversely).

It's universal — same for all masses and distances.

3. Gravitational Field and Potential

Field:

Force per unit mass at a point:

$$\vec{g} = \frac{\vec{F}}{m} = -G \frac{M}{r^2} \quad \vec{g} = m \vec{F} = -G r^2 M$$

Potential:

Work done per unit mass in bringing body from ∞ to that point:

$$V = -G \frac{M}{r} \quad V = -G r M$$

Unit: J/kg.

Gravitational field = $-\text{gradient of potential}$.

4. Acceleration Due to Gravity

At Earth's surface:

$$g = G \frac{M}{R^2} \quad g = R^2 G M$$

$$M = 5.97 \times 10^{24} \text{ kg}, R = 6.37 \times 10^6 \text{ m} \rightarrow g \approx 9.8 \text{ m/s}^2.$$

Variation of g:

- Height: $g' = g(1 - \frac{2h}{R})$ $g' = g(1 - \frac{2h}{R})$
 - Depth: $g' = g(1 - \frac{d}{R})$ $g' = g(1 - \frac{d}{R})$
 - Latitude: decreases from poles to equator (due to rotation).
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5. Gravitational Potential Energy

$$U = -GMm/r \quad U = -G \frac{Mm}{r} \quad U = -GrMm$$

Zero at infinity; negative for bound systems.

Work done by gravity = decrease in potential energy.

6. Escape Velocity

Minimum velocity to leave Earth's gravity:

$$v_e = \sqrt{2gR} = 11.2 \text{ km/s (Earth)} \quad v_e = \sqrt{2gR} = 11.2 \text{ km/s (Earth)} \quad v_e = 11.2 \text{ km/s (Earth)}$$

For Moon ≈ 2.4 km/s, for Jupiter ≈ 60 km/s.

7. Satellite Motion

For circular orbit:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}} \quad \frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

- Orbital velocity: 7.9 km/s for low Earth orbit.
 - Time period: $T = 2\pi \sqrt{\frac{r^3}{GM}}$ $T = 2\pi \sqrt{\frac{r^3}{GM}}$ (Kepler's third law).
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8. Kepler's Laws

1. Elliptical Law: Planets move in ellipses with Sun at one focus.
2. Area Law: Line joining planet to Sun sweeps equal areas in equal times.
3. Time-Period Law: $T^2 \propto r^3$ $T^2 \propto r^3$

These laws are consequences of Newton's gravitation.

✿ 9. Geostationary Satellite

- **Period = 24 h, orbit radius $\approx 42,000$ km.**
 - **Appears stationary above equator.**
 - **Used for communication, weather forecasting.**
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📖 10. Summary

1. **Gravitational force \propto product of masses / distance².**
2. **g decreases with height & depth.**
3. **Escape velocity on Earth = 11.2 km/s.**
4. **Satellites revolve due to gravitational pull.**
5. **Kepler's laws describe planetary motion.**