CHAPTER 2

RELATIONS AND FUNCTIONS

Ordered Pair

A pair of numbers or elements grouped together in a definite order is called ordered pair. If a and b are any two numbers, then (a, b) is called ordered pair a,b. Here 'a' is called first element or x element or x co-ordinate or abscissa and 'b' is called second element or y element or y co-ordinate or ordinate.

E.g.: $(2,3), (-1,-2), (x,\ y)$, etc. are ordered pairs.

Note1: $\{\boldsymbol{a}, \boldsymbol{b}\} = \{\boldsymbol{b}, \boldsymbol{a}\}$ $\boldsymbol{but}$ $(\boldsymbol{a}, \boldsymbol{b}) \neq (\boldsymbol{b}, \boldsymbol{a})$ $\boldsymbol{unless}$ $\boldsymbol{a} = \boldsymbol{b}$

E.g.: If $(x, y) = (3,2),\ then\ x = 3\ and\ y = 2$

Cartesian product of sets

Given two non-empty sets A and B. The Cartesian product A × B is the set of all ordered pairs of elements from A and B.

i.e., $A \times B = \{(a\ , b): a \in A, b \in B\ \}$.

If either A or B is a null set, then $A \times B$ will also be a null set, i.e., A × B = ϕ

Note:

- Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.
  i.e., if $(a, b) = (c, d) \Rightarrow a = c\ and\ b = d$
- If there are m elements in A and n elements in B, then there will be mn elements in $A \times B$. i.e., if $n(A) = m$ and $n(B) = n$, then $n(A \times B) = mn$.
- If A and B are non-empty sets and either A or B is an infinite set, then $A \times B$ is also infinite.
- If $A = B$, then $A \times B$ becomes $A \times A$ and is denoted by A^2.
- $A \times A = \{(a, b): a, b \in A\}$. Here (a, b) is called an ordered doublet.
- $A \times A \times A = \{(a, b, c):\ a, b, c\ \in A\}$. Here (a, b, c) is called an ordered triplet.
- The Cartesian product $R \times R = \{(x, y): x, y \in R\}$ represents the coordinates of all points in two dimensional plane and the Cartesian product $R \times R \times R = \{(x, y, z): x, y, z \in R\}$ represents the coordinates of all points in a right handed system or in a 3D space.

RELATIONS

Relation means an association of two objects according to some property possessed by them.

E.g.:

Trivandrum is the capital of Kerala,

Sita is the wife of Rama,

12 is greater than 10,

$\{a\}$ is the subset of $\{a, b\}$, etc..

- If A and B are any two non-empty sets, then the relation from A to B is a subset of $A \times B$.

- If (x, y) is a member of a relation R, then we write xR_y.
- **Domain of R**: The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R
- **Range of R**: The set of all second elements of the ordered pairs in R from A to B is called range of R.

Representation of a relation:

A relation can be expressed in:

a) Roster Method

b) Set-builder Method

c) Arrow diagram and graphical method.

E.g.: Let $A = \{1,2,3,4\}$; $B = \{2,3,4\}$

R is a relation from A to B such that $y = x + 2, x \in A \text{ and } y \in B$.

$R = \{(1,3), (2,4)\}$ - Roster Method

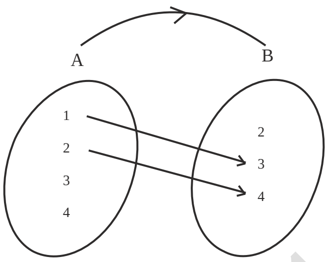
Domain = $\{1,2\}$

Range = $\{3,4\}$

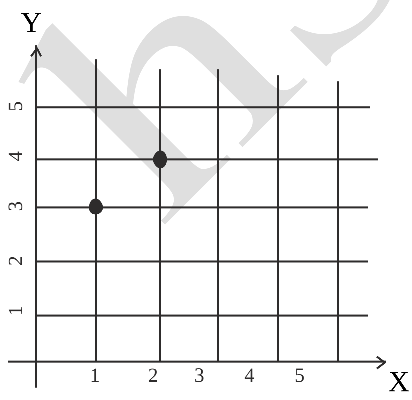
Codomain = $\{2,3,4\}$

$R = \{(x, y): y = x + 2, x \in A and y \in B\}$ - Set-builder Method

Arrow diagram:



Graphical Method



Note: If a set A has m elements and B has n elements, then

- No. of relations from A to B = 2^{mn}
- No. of relations from B to A = $2^{nm} = 2^{mn}$.

FUNCTIONS

A function from a non-empty set A to a non-empty set B is the one to one correspondence between the elements. In other words, a relation from a non-empty set A to a non-empty set B is said to be a function, if and only if

i) if every x element has unique y element

ii) the x element cannot be repeated (or) if every x in A has unique image in B.

E.g.: Let $A = \{0, 1, 2, 3, 4\}$; $B = \{1, 2, 3, 5, 7, 9\}$
Let $R = \{(x, y): y = 2x + 1, x \in A, y \in B\}$

Note: If a set A has 'm' elements and set B has 'n' elements, then,
i. No. of functions from A to B $= n(B)^{n(A)}$
ii. No. of functions from B to A $= n(A)^{n(B)}$

Domain, Range and codomain of a function:

If $f : A \to B$ is a function from A to B, then
i) Domain of *f* = set A

ii) Range of *f* = set of all images of elements of A is known as range.

iii) Codomain of f = set B

Thus $\boxed{range \subseteq codomain}$

Equal functions: If two functions *f* and *g* are said to be equal, then,

i. domain of *f* = domain of *g*

ii. codomain of *f* = codomain of *g*

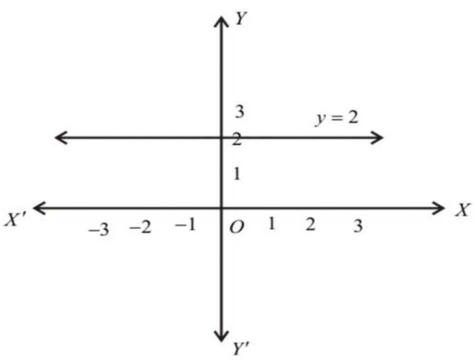
Note: The terms map or mapping are also used to denote function.

If *f* is a function from A to B, we denote f: A → B or $A \xrightarrow{f} B$. If *f* is a function from A to B and (a, b) ∈ f, then f(a) = b, where 'b' is called the image of 'a' under *f* and 'a' is called the pre-image of 'b' under *f*.

Types of functions:

Real function: A function $f : R \to R$ is said to be a real function, if its domain is a real constant.

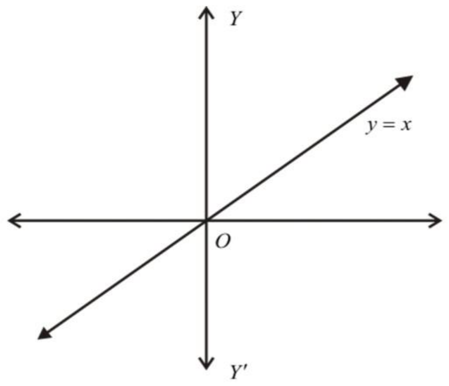
Constant function: A function $f : R \to R$ is said to be a constant function if $f(x) = c$, where 'c' is a constant.
Domain: R
Range : c (a constant)



Identity function: A function $f : R \to R$ is said to be an identity function if $f(x) = x$.

Domain: R

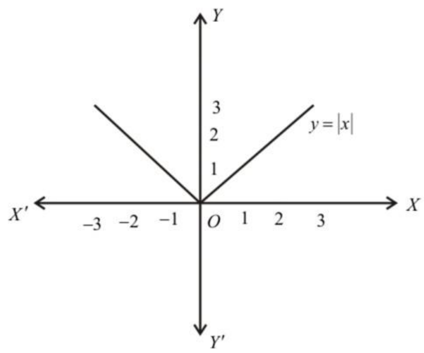
Range : R



Modulus function: A function $f : R \to R$ is said to be a modulus function if $f(x) = |x|$, where $|x| = \begin{cases} x, if x \geq 0 \\ -x, if x < 0 \end{cases}$

Domain: R

Range : $R^+ \cup \{0\}$



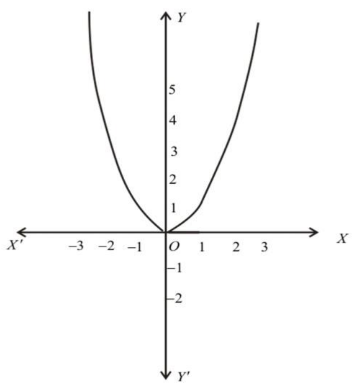
Polynomial function: A function $f : R \to R$ is said to be a polynomial function if $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_{n-2}x^2 + a_{n-1}x + a_n$, where $a_0, a_1, a_2, a_3, \ldots$ are constants, is known as a polynomial function of degree 'n'.

E.g.: $f(x) = x^3 - 2x + 5$; $g(x) = 2x^2 + 3x - 1$, etc.

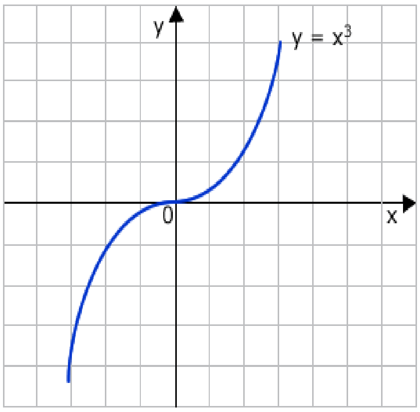
Domain : R

Range : R

i. $f(x) = x^2$

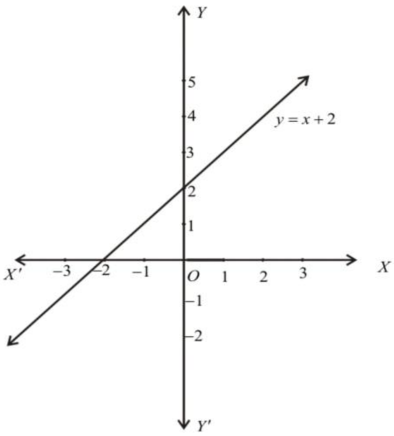


ii. $f(x) = x^3$



ii. $f(x) = x + 2 \Rightarrow y = x + 2$

x	0	-2
y	2	0



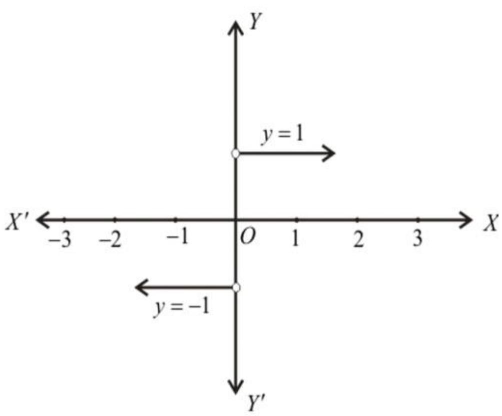
Signum Function: A function $f: R \to R$ is said to be a signum function,

if $f(x) = \begin{cases} -1, \ if \ x < 0 \\ 0, \ if \ x = 0 \\ 1, \ if \ x > 0 \end{cases}$

or $f(x) = \dfrac{|x|}{x}, x \neq 0 \ and \ 0 \ \ for \ \ x = 0$ is known as signum function.

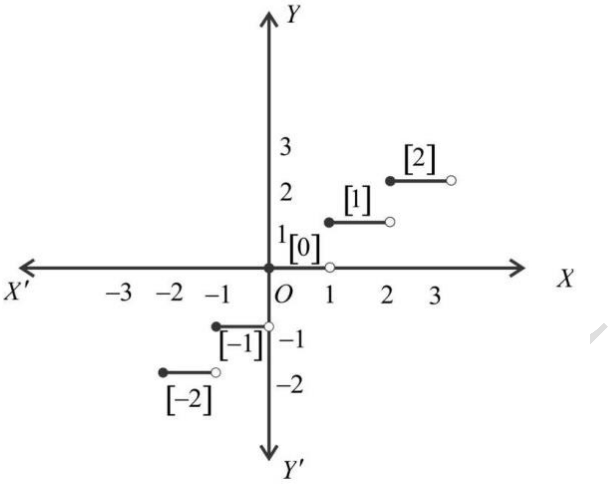
Domain: R

Range: $\{-1, 0, 1\}$, if $x < 0, x = 0 \, and \, x > 0$



Greatest Integer Function: A function $f : R \to R$ is said to be a greatest integer function, if $f(x) = [x], x \in R$.

Domain : R
Range : Integer.



Note: The above graph is also known as step graph.

Note:

[0]	$0 \le x < 1 = 0$
[1]	$1 \le x < 2 = 1$
[-1]	$-1 \le x < 0 = -1$
[1.3]	$1 \le x < 1.3 = 1$
[2.999]	$2 \le x < 2.999 = 2$
[-2.3]	$-3 \le x < -2.3 = -3$

Rational Function: A function $f : R \to R$ is said to be a greatest integer function, if $f(x) = \dfrac{ax+b}{cx+d}, x \ne -\dfrac{d}{c}$.

E.g.: $f(x) = \dfrac{2x+1}{x-2}, x \ne 2$; $g(x) = \dfrac{x-5}{x+1}, x \ne -1$, etc..

Graph: $f(x) = \dfrac{1}{x},\ x \ne 0$

1. Find the domain of the rational function $f(x) = \dfrac{2x-3}{1-x}$:

   $f(x)$ is defined, if $1 - x = 0 \Rightarrow x = 1$.

2. Find the domain of the rational function $f(x) = \dfrac{x^2-3x+5}{x^2-5x+6}$:

   $f(x)$ is defined, if $x^2 - 5x + 6 = 0 \Rightarrow (x-3)(x-2) = 0 \Rightarrow x = 3 \ or \ x = 2$
   $\therefore domain = R - \{2,3\}$

3. Find the domain and range of the function: $f(x) = \sqrt{4 - x^2}$

   Let $f(x) = \sqrt{4 - x^2}$

   i.e, $y = \sqrt{4 - x^2}$ …………….. (1)

   In order to find the domain, let $4 - x^2 \ge 0$

   $4 \ge x^2 \Rightarrow x^2 \le 4 \Rightarrow x \le \pm 2$

   $\Rightarrow x \ge -2 and x \le 2$

   $\therefore$ domain of f is $[-2,2] or -2 \le x \le 2$

   From (1), $y \ge 0$…………. (2)

To find the range:

Let $y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2 \Rightarrow x^2 = 4 - y^2$

$x = \sqrt{4 - y^2}$

In order to define x, let $4 - y^2 \geq 0$

$4 \geq y^2 \Rightarrow y^2 \leq 4 \Rightarrow y \leq \pm 2$

$\Rightarrow y \geq -2 and y \leq 2 \ldots\ldots\ldots\ldots\ldots (3)$

From (2) and (3), we have

Range of f is $[0,2] or 0 \leq x \leq 2$

Algebra of functions:

Let $f(x)$ and $g(x)$ be any two functions of x, then

1. $f + g = f(x) + g(x)$
2. $f - g = f(x) - g(x)$
3. $f.g = f(x) \times g(x)$
4. $\frac{f}{g} = \frac{f(x)}{g(x)}, provided g(x) \neq 0$

E.g.: If $f(x) = x^2$ and $g(x) = 2x + 1$, then

$f + g = f(x) + g(x) = x^2 + 2x + 1 = (x + 1)^2$

$f - g = f(x) - g(x) = x^2 - (2x + 1) = x^2 - 2x - 1$

$f.g = f(x) \times g(x) = x^2(2x + 1) = 2x^3 + x^2$

$\frac{f}{g} = \frac{f(x)}{g(x)} = \frac{x^2}{2x+1}, x \neq -\frac{1}{2}$