Chapter 5 – Motion of System of Particles and Rigid Body

🚀 1. Introduction

So far, we've studied how single objects move. But in the real world, everything is made up of **many particles** — a football, a car, even your own body.
How do we describe the motion of such systems?
That's where this chapter comes in: **Motion of System of Particles and Rigid Bodies.**

We'll explore:

- Center of mass
- Linear momentum of a system
- Rotational motion and torque
- Moment of inertia (the rotational twin of mass)

⚖️ 2. System of Particles and Center of Mass

When many particles form a system, each particle moves differently. Instead of tracking each one, we define a single point that represents the whole body's motion — the **center of mass (C.M.)**.

Formula:

For n particles of masses $m_1, m_2, \ldots m_n$ with positions $x_1, x_2, \ldots x_n$:

xcm=m1x1+m2x2+⋯+mnxnm1+m2+⋯+mnx_{cm} = \frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1 + m_2 + \dots + m_n}xcm=m1+m2+⋯+mnm1x1+m2x2+⋯+mnxn

For continuous bodies:

r⃗cm=∫r dm∫dm\vec{r}_{cm} = \frac{\int r\,dm}{\int dm}rcm=∫dm∫rdm

Examples:

- Uniform rod → C.M. at midpoint.
- Solid sphere → at center.
- Hemisphere → at 3R/8 from flat face.

The center of mass moves as if the entire mass were concentrated there and all external forces act on it.

💥 3. Linear Momentum of a System

For N particles:

P⃗=∑i=1nmivi⃗\vec{P} = \sum_{i=1}^{n} m_i \vec{v}_iP=i=1∑nmivi

Differentiating:

dP⃗dt=∑F⃗ext\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}dtdP=∑Fext

Internal forces cancel (Newton’s Third Law).
So, only external forces change total momentum.
If external force = 0 → total momentum constant.

🔁 4. Motion of Center of Mass

Ma⃗cm=F⃗extM \vec{a}_{cm} = \vec{F}_{ext}Macm=Fext

This means: the center of mass moves like a single particle under external forces.
Internal collisions or explosions don’t affect C.M. motion.

Example:
When a bomb explodes mid-air, fragments scatter, but the C.M. follows the same parabolic path as before explosion.

🧠 5. Torque and Angular Momentum

Torque (τ)

The rotational equivalent of force:

τ⃗=r⃗×F⃗\vec{τ} = \vec{r} × \vec{F}τ=r×F

Unit: N·m.

Direction: perpendicular to the plane (right-hand rule).
It causes rotation.

Angular Momentum (L)

L⃗=r⃗×p⃗\vec{L} = \vec{r} × \vec{p}L=r×p

Unit: kg·m²/s.

For rotation:

L=IωL = I ωL=Iω

⚙️ 6. Relation between Torque and Angular Momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Analogous to F = dp/dt in linear motion.

♻️ 7. Conservation of Angular Momentum

If external torque = 0:

$$\vec{L} = \text{constant}$$

Example:

- A skater spins faster when pulling in arms → I decreases, ω increases.
- Planets move faster when closer to the Sun.

🧩 8. Rigid Body and Rotational Motion

A **rigid body** doesn’t deform; distance between any two points remains fixed.

Rotational motion is described by:

- **Angular displacement (θ)**
- **Angular velocity (ω = dθ/dt)**
- **Angular acceleration (α = dω/dt)**

Linear ↔ Angular analogies:

Linear	Rotational
Displacement (s)	θ
Velocity (v)	ω
Acceleration (a)	α
Force (F)	Torque (τ)
Mass (m)	Moment of Inertia (I)

🛰️ 9. Moment of Inertia (I)

It measures how difficult it is to change rotation:

$$I = \sum m_i r_i^2$$

Unit: $kg \cdot m^2$.

Examples:

Object	**Axis**	**I**
Rod about center	$I = (1/12)ML^2$	
Rod about end	$I = (1/3)ML^2$	
Solid cylinder	$(1/2)MR^2$	
Hollow cylinder	MR^2	
Solid sphere	$(2/5)MR^2$	
Thin ring	MR^2	

Theorems:

- **Parallel Axis:** I=Icm+Md2I = I_{cm} + M d^2I=Icm+Md2
- **Perpendicular Axis:** Iz=Ix+IyI_z = I_x + I_yIz=Ix+Iy

🌀 10. Rotational Kinetic Energy

KErot=12Iω2KE_{rot} = \frac{1}{2} I ω^2KErot=21Iω2

For rolling without slipping:

v=ωRv = ω Rv=ωR

Total energy = translational + rotational.

🌍 11. Rolling Motion

Rolling combines translation and rotation.
Example: A wheel rolling on road → C.M. moves forward; point of contact momentarily at rest.

📃 12. Summary

1. C.M. represents overall motion.
2. Torque = rotational force.
3. τ=dL/dtτ = dL/dtτ=dL/dt, angular momentum conserved if $\tau = 0$.
4. Moment of inertia → rotational analog of mass.

5. Rolling motion → both rotation + translation.