MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}$, $z \in C$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points (0, 0), C and (α, 0) is 11 square units, then α^2 equals:

(1) $\frac{81}{25}$ (2) 50

(3) 100 (4) $\frac{121}{25}$

Answer (3)

Sol. Let $z = x + iy \Rightarrow \bar{z} = x - iy$

$3|\bar{z}-i| = 1|2\bar{z}+i|$

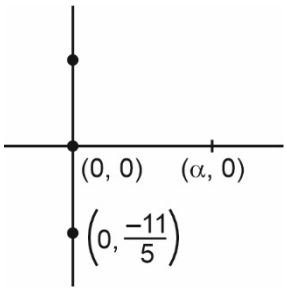
$= 3|(x-(y+1)i| = |2x+i(1-2y)|$

$= 3\sqrt{x^2+(y+1)^2} = \sqrt{(2x)^2+(1-2y)^2}$

$= 9(x^2+y^2+2y+1) = 4x^2+4y^2-4y+1$

$\Rightarrow 5x^2+5y^2+22y+8=0$

$\Rightarrow$ Centre $\equiv \left(0,-\frac{11}{5}\right)$



Area of Δ

$= \frac{1}{2}|\alpha|\left|\frac{-11}{5}\right| = 11$

$\Rightarrow |\alpha| = 10$

$\Rightarrow \alpha^2 = 100$

2. Let $f(x) = \log_e x$ and $g(x)=\frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$.

Then the domain of fog is

(1) $[1, \infty)$ (2) $(0, \infty)$

(3) $\mathbb{R}$ (4) $[0, \infty)$

Answer (3)

Sol. $f(g(x)) = \ln\left(\frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}\right)$

Since $2x^2-2x+1>0 \quad \forall x \in \mathbb{R} \because (-2)^2-4(2)<0$

Consider

$x^4-2x^3+3x^2-2x+2$

$= (x^4-2x^3+x^2)+(x^2-2x+1)+(1+x^2)$

$= x^2(x-1)^2+(x-1)^2+(x^2+1)>0 \forall x \in \mathbb{R}$

$\Rightarrow g(x)>0 \ \forall x \in \mathbb{R}$

$\Rightarrow \ln f((x)),\ f(x)>0 \ \forall x \in \mathbb{R}$

$\Rightarrow x \in \mathbb{R}$ is domain

3. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is

(1) 35000 (2) 37000

(3) 34000 (4) 36000

Answer (4)

Sol. E : all vowel never come together

$\bar{E}$: all vowel come together

$n(E) = n(U) - n(\bar{E})$ $\boxed{AUE}$ DGHTR

= 8! – (6!)(3!)

= 6!(7 × 8 – 6) = 720 (56 – 6)

= 720 × 50 = 36000

4. One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is

(1) $\frac{2}{3}$ (2) $\frac{4}{9}$

(3) $\frac{3}{5}$ (4) $\frac{1}{2}$

Answer (4)

Sol. Die 1 : {1, 1, 2, 2, 3, 4}

Die 2 : {1, 2, 2, 3, 3, 4}

(die 1, die 2) = {(1, 3), (3, 1), (2, 2), (2, 3), (3, 2), (1, 4), (4, 1)}

$$=\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}$$

$$\times\frac{2}{6}+\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}=\frac{1}{2}$$

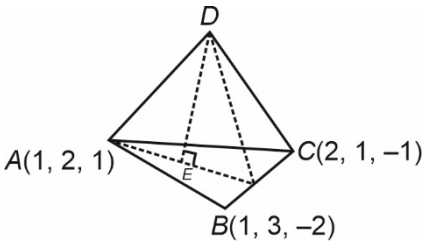
5. Let the position vectors of the vertices A, B and C of a tetrahedron $ABCD$ be $\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

(1) $\frac{1}{12}(7\hat{i}+4\hat{j}+3\hat{k})$ (2) $\frac{1}{6}(7\hat{i}+12\hat{j}+\hat{k})$

(3) $\frac{1}{6}(12\hat{i}+12\hat{j}+\hat{k})$ (4) $\frac{1}{2}(\hat{i}+4\hat{j}+7\hat{k})$

Answer (2)

Sol.



Volume = $\frac{1}{3}\times\text{Base}\times h=\frac{\sqrt{805}}{6\sqrt{2}}$

Base area = $\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$

$$=\frac{1}{2}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&-3\\1&-1&-2\end{vmatrix}=\frac{1}{2}\left|5\hat{i}+3\hat{j}+\hat{k}\right|=\frac{1}{2}\sqrt{35}$$

$$\frac{1}{3}\times\frac{1}{2}\sqrt{35}\times h=\frac{\sqrt{805}}{6\sqrt{2}}$$

$$\Rightarrow\quad h=\sqrt{\frac{23}{2}}$$

Now $AE^2 = AD^2 - DE^2$

$$=\frac{110}{9}-\frac{23}{2}=\frac{13}{18}$$

$$AE=\sqrt{\frac{13}{18}}$$

$$\overline{AE}=|AE|\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right)=\frac{\hat{i}-5\hat{k}}{6}$$

Position vector of E = $\frac{\hat{i}-5\hat{k}}{6}+\hat{i}+2\hat{j}+\hat{k}$

$$=\frac{1}{6}(7\hat{i}+12\hat{j}+\hat{k})$$

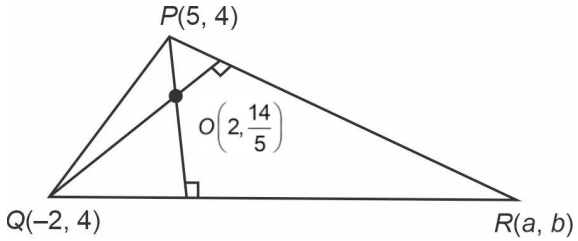
6. Let the area of a ΔPQR with vertices $P(5, 4)$, $Q(-2, 4)$ and $R(a, b)$ be 35 square units. If its orthocentre and centroid are $O\left(2, \frac{14}{5}\right)$ and $C(c, d)$ respectively, then $c + 2d$ is equal to

(1) $\frac{7}{3}$ (2) 2

(3) 3 (4) $\frac{8}{3}$

Answer (3)

Sol.



$QR: y-4=-\left(\frac{5-2}{4-\frac{14}{5}}\right)(x+2)$

$QR: 5x+2y+2=0 \quad \ldots(i)$

$PR: y-4=\frac{4}{\frac{14}{5}-4}(x-4)$

$PR: 10x-3y-38=0 \quad \ldots(ii)$

Solving (i) & (ii)

$R\ (2, -6)$

Centroid $C=\left(\frac{5-2+2}{3}, \frac{4+4-6}{3}\right) \equiv \left(\frac{5}{3}, \frac{2}{3}\right)$

$\Rightarrow \quad c=\frac{5}{3},\ d=\frac{2}{3}$

$c+2d=\frac{5}{3}+\frac{4}{3}=3$

7. Let $R = \{(1,2), (2,3), (3,3)\}$ be a relation defined on the set $\{1, 2, 3, 4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

(1) 8 (2) 7

(3) 9 (4) 10

Answer (2)

Sol. $R = \{(1,2), (2,3), (3,3)\}$ be relation defined on set $\{1,2,3,4\}$ then the elements added to make this equivalence relation are (1,1), (2,2), (2,1), (3,2), (3,1), (1,3), (4, 4).

Total number of elements added = 7.

8. Let $I(x)=\int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$. If $I(37)-I(24)=\frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}}-\frac{1}{c^{\frac{1}{13}}}\right)$, $b, c \in N$, then $3(b + c)$ is equal to

(1) 22 (2) 40

(3) 26 (4) 39

Answer (4)

Sol. $I(x)=\int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$

$= =\int \frac{dx}{(x-11)^2\left(\frac{x+15}{x-11}\right)^{\frac{15}{13}}}$

Let $\frac{x+15}{x-11}=t \Rightarrow \frac{-26}{(x-11)^2}dx=dt$

$\therefore \quad I(x)=\int \frac{-\frac{1}{26}dt}{t^{\frac{15}{13}}}$

$=\frac{1}{4}t^{-\frac{2}{13}}+c=\frac{1}{4}\left(\frac{x-11}{x+15}\right)^{\frac{2}{13}}+c$

$\therefore \quad I(37)-I(24)=\frac{1}{4}\left(\left(\frac{1}{2}\right)^{\frac{2}{13}}-\left(\frac{1}{3}\right)^{\frac{2}{13}}\right)$

$\therefore \quad b = 4, c = 9 \Rightarrow 3(b + c) = 39$

9. If the system of equations

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

has infinitely many solutions, then $\lambda^2 + \lambda$ is equal to

(1) 12 (2) 10

(3) 20 (4) 6

Answer (1)

Sol. The given system of equation are

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

has infinite many solutions, then

$D = D_1 = D_2 = D_3 = 0$

$$\therefore \begin{vmatrix} \lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$

$$\text{or, } \begin{vmatrix} -1 & -3 & 4 \\ -1 & -3 & 2\lambda-2 \\ \lambda+1 & \lambda+2 & -\lambda-2 \end{vmatrix} = 0$$

$$\text{or, } \begin{vmatrix} -1 & -3 & 4 \\ 0 & 0 & 2\lambda-6 \\ \lambda+1 & \lambda+2 & -\lambda-2 \end{vmatrix} = 0$$

$\therefore \quad (6 - 2\lambda)(2\lambda + 1) = 0$

$\therefore \quad \lambda = 3 \text{ or } -\frac{1}{2}$

But $\lambda = 3$ satisfies all other conditions.

$\therefore \quad \lambda^2 + \lambda = 12$

10. If A, B and $(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))$ are non-singular matrices of same order, then the inverse of $A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1} B$, is equal to

(1) $\text{adj}(B^{-1}) + \text{adj}(A^{-1})$

(2) $\frac{1}{|AB|}(\text{adj}(B) + \text{adj}(A))$

(3) $AB^{-1} + A^{-1}B$

(4) $\frac{AB^{-1}}{|A|} + \frac{BA^{-1}}{|B|}$

Answer (2)

Sol. $[A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1}B]^{-1}$

$= B^{-1}(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))A^{-1}$

$$= B^{-1}\left[\frac{A}{|A|} + \frac{B}{|B|}\right]A^{-1}$$

$$= \left[\frac{B^{-1}A}{|A|} + \frac{I}{|B|}\right]A^{-1}$$

$$= \frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|}$$

$$= \frac{\text{adj}(B)}{|AB|} + \frac{\text{adj}(A)}{|AB|}$$

$$= \frac{1}{|AB|}(\text{adj}(B) + \text{adj}(A))$$

11. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

(1) -1080 (2) -1200

(3) -120 (4) -1020

Answer (1)

Sol. Sum of first 4 terms

$= \frac{1}{5} \times$ (sum of next 4 terms)

$$\frac{4}{2}(2a + 3d) = \frac{1}{5}(4a + 22d)$$

$$\Rightarrow \quad 16a = -8d \quad \Rightarrow \quad a = -\frac{d}{2}$$

$a = 3, d = -6$

$$S_{20} = \frac{20}{2}[2(3) + 19(-6)]$$

$= -10(18.6)$

$= -1080$

12. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the are AC, divides the arc AC such that $\frac{\text{length of arc } AB}{\text{length of arc } BC} = \frac{1}{5}$, and $\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$, then $\alpha + \sqrt{2}\left(\sqrt{3} - 1\right)\beta$ is equal to

(1) $5\sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $2\sqrt{3}$ (4) $2 - \sqrt{3}$

Answer (4)

Sol. Let the radius be r

B divides the arc in 1 : 5, so AB would subtend an angle of 15° at origin O and similarly, BC would subtend an angle of 75° at origin.

$\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=\vec{b}, \overrightarrow{OC}=\vec{c}$

$\vec{b}=r(\cos 75°)\hat{i}+r(\sin 75°)\hat{j}$

$$\vec{b}=\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\vec{c}+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\vec{a}$$

$$2\sqrt{2}\vec{b}=\left(\sqrt{3}-1\right)\vec{c}+\left(\sqrt{3}+1\right)\vec{a}$$

$$\overrightarrow{OC}=-\underbrace{\frac{\left(\sqrt{3}+1\right)}{\left|\sqrt{3}-1\right|}}_{\alpha}\overrightarrow{OA}+\underbrace{\frac{2\sqrt{2}}{\left|\sqrt{3}-1\right|}}_{\beta}\overrightarrow{OB}$$

$$\alpha+\sqrt{2}\left(\sqrt{3}-1\right)\beta=2-\sqrt{3}$$

13. If the line $3x-2y+12=0$ intersects the parabola $4y=3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1) $\tan^{-1}\left(\frac{9}{7}\right)$ (2) $\tan^{-1}\left(\frac{11}{9}\right)$

(3) $\frac{\pi}{2}-\tan^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}\left(\frac{4}{5}\right)$

Answer (1)

Sol. Line $L: 3x-2y+12=0$

Parabola $P: 4y=3x^2$

By putting $y=\frac{3x^2}{4}$ in equation of line

We get, $3x-2\left(\frac{3x^2}{4}\right)+12=0$

$\Rightarrow 6x-3x^2+24=0$

$\Rightarrow x^2-2x+8=0$

$\Rightarrow x=4,-2$

for $x=4$, we get $y=12$

for $x=-2$, we get $y=3$

So, points A and B are (4, 12) and (–2, 3)

Now, Vertex of parabola is (0, 0)

$$\Rightarrow \tan\theta=\left|\frac{3-\left(\frac{-3}{2}\right)}{1+3\left(\frac{-3}{2}\right)}\right|$$

$$\tan\theta=\frac{9}{7}$$

$$\Rightarrow \theta=\tan^{-1}\left(\frac{9}{7}\right)$$

$\Rightarrow$ Option (1) is correct

14. Let P be the foot of the perpendicular from the point $Q(10,-3,-1)$ on the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$. Then the area of the right-angled triangle PQR, where R is the point (3, –2, 1), is

(1) $3\sqrt{30}$ (2) $9\sqrt{15}$

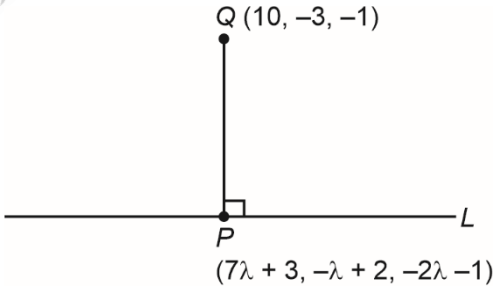
(3) $8\sqrt{15}$ (4) $\sqrt{30}$

Answer (1)

Sol. Q (10, –3, –1)

$$L: \frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$$

General point on $L(7\lambda+3,-\lambda+2,-2\lambda-1)$



DR's of PQ are $7\lambda-7, -\lambda+5, -2\lambda$

$PQ \perp L$

$\Rightarrow (7\lambda-7)7+(-1)(-\lambda+5)+(-2\lambda)(-2)=0$

$\Rightarrow 49\lambda-49+\lambda-5+4\lambda=0$

$\Rightarrow 54\lambda - 54 = 0 \Rightarrow \lambda = 1$

$P(10, 1, -3)$ and $R(3, -2, 1)$

$\overrightarrow{PQ} = -4\hat{j} + 2\hat{k}$, $\overrightarrow{QR} = -7\hat{i} + \hat{j} + 2\hat{k}$,

$\overrightarrow{PR} = -7\hat{i} - 3\hat{j} + 4\hat{k}$

$\overrightarrow{PQ} \cdot \overrightarrow{QR} = 0 \Rightarrow$ right angle at Q

$\Rightarrow \text{ar}(\Delta PQR) = \frac{1}{2} \times PQ \times QR$

$= \frac{1}{2} \times \sqrt{20} \times \sqrt{54}$

$= \frac{1}{2} \times 2 \times 3\left(\sqrt{5} \times \sqrt{6}\right)$

$= 3\sqrt{30}$

$\Rightarrow$ Option (1) is correct

15. The value of $(\sin 70°)(\cot 10° \cot 70° - 1)$ is

(1) 0 (2) 2/3

(3) 3/2 (4) 1

Answer (4)

Sol. $(\sin 70°)(\cot 10° \cot 70° - 1)$

$= \sin 70° \cot 10° \cot 70° - \sin 70°$

$= \cot 10° \cos 70° - \sin 70°$

$= \frac{\cos 10° \cos 70° - \sin 70° \sin 10°}{\sin 10°}$

$= \frac{\cos(10° + 70°)}{\sin 10°}$

$= \frac{\cos 80°}{\sin 10°} = 1$

$\Rightarrow$ Option (4) is correct

16. Let a curve $y = f(x)$ pass through the points $(0, 5)$ and $(\log_e 2, k)$. If the curve satisfies the differential equation $2(3 + y)e^{2x}\,dx - (7 + e^{2x})dy = 0$, then k is equal to

(1) 8 (2) 4

(3) 32 (4) 16

Answer (1)

Sol. $\frac{dy}{dx} = \frac{2(3+y) \cdot e^{2x}}{7 + e^{2x}}$

$\frac{dy}{dx} - \frac{2ye^{2x}}{7 + e^{2x}} = \frac{6 \cdot e^{2x}}{7 + e^{2x}}$

I.F. $= e^{-\int \frac{2e^{2x}}{7+e^{2x}} dx}$

$\Rightarrow e^{-\ln(7+e^{2x})}$

$= \frac{1}{7 + e^{2x}}$

$y \cdot \frac{1}{7 + e^{2x}} = \int \frac{6e^{2x}}{\left(7 + e^{2x}\right)^2} dx$

$\frac{y}{7 + e^{2x}} = \frac{-3}{7 + e^{2x}} + C$

$\therefore y(0) = 5$

$\Rightarrow \frac{5}{8} = \frac{-3}{8} + C$

$\Rightarrow C = 1$

$\therefore y = -3 + 7 + e^{2x}$

$y = e^{2x} + 4$

$\therefore k = 8$

17. If $\frac{\pi}{2} \le x \le \frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$ is equal to

(1) $x - \tan^{-1}\frac{4}{3}$ (2) $x + \tan^{-1}\frac{4}{5}$

(3) $x + \tan^{-1}\frac{5}{12}$ (4) $x - \tan^{-1}\frac{5}{12}$

Answer (4)

Sol. $\frac{12}{13}\cos x + \frac{5}{13}\sin x$

Let $\tan\alpha = \frac{5}{12}, \alpha \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \sin\alpha = \frac{5}{13}, \cos\alpha = \frac{12}{13}$

$$\Rightarrow \frac{12}{13}\cos x + \frac{5}{13}\sin x = \cos\alpha\cos x + \sin\alpha\sin x$$

$$= \cos(x-\alpha)$$

$$\Rightarrow \cos^{-1}[\cos(x-\alpha)] = x-\alpha$$

$$= x - \tan^{-1}\left(\frac{5}{12}\right)$$

18. Marks obtains by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is

(1) 40 (2) 52

(3) 48 (4) 44

Answer (4)

Sol. $M = L + \frac{\frac{n}{2} - Cf}{f} \times h$

$$14 = 12 + \frac{\frac{n}{2} - 18}{12} \times 6$$

$$2 \times 2 = \frac{n}{2} - 18$$

$$\frac{n}{2} = 4 + 18$$

$n = 44$

19. If the function

$$f(x) = \begin{cases} \frac{2}{x}\{\sin(k_1+1)x + \sin(k_2-1)x\}, & x < 0 \\ 4, & x = 0 \\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right), & x > 0 \end{cases}$$

is continuous at $x = 0$ then $k_1^2 + k_2^2$ is equal to

(1) 20 (2) 5

(3) 8 (4) 10

Answer (4)

Sol. $\lim\limits_{x\to0^-} f(x) = \lim\limits_{x\to0^-} 2\left(\frac{\sin(k_1+1)x}{x} + \frac{\sin(k_2-1)x}{x}\right)$

$= 2((k_1+1) + (k_2-1))$

$= 2(k_1 + k_2)$ ...(i)

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} 2\left(\frac{\ln\left(1+\frac{k_1}{2}x\right)}{x} - \frac{\ln\left(1+\frac{k_2}{2}x\right)}{x}\right)$$

$= k_1 - k_2$ ...(ii)

For continuity at $x = 0$

$2(k_1 + k_2) = 4$ and $k_1 - k_2 = 4$

$\Rightarrow k_1 + k_2 = 2$ and $k_1 - k_2 = 4$

$\Rightarrow k_1 = 3$ & $k_2 = -1$

$k_1^2 + k_2^2 = 10$

20. The value of

$$\int_{e^2}^{e^4} \frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}} + e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right)dx \text{ is}$$

(1) 1 (2) e^2

(3) 2 (4) $\log_e 2$

Answer (1)

Sol. Put $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

x	t
e^2	2
e^4	4

$$I = \int_2^4 \frac{e^{(t^2+1)^{-1}}}{e^{(t^2+1)^{-1}} + e^{\left((6-t)^2+1\right)^{-1}}}dt \quad ...(i)$$

$$I = \int_2^4 \frac{e^{\left((6-t)^2+1\right)^{-1}}}{e^{\left((6-t)^2+1\right)^{-1}} + e^{(t^2+1)^{-1}}}dt \quad ...(ii)$$

$\left\{\text{Using}\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right\}$

Adding (i) and (ii) gives

$2I = \int dt \Rightarrow I = 1$

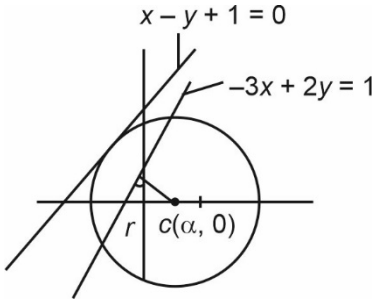
SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let the circle C touch the line $x - y + 1 = 0$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x + 2y = 1$. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to _____.

Answer (19)

Sol.



$$r = \left|\frac{a+1}{\sqrt{2}}\right| \Rightarrow (a+1)^2 = 2r^2$$

Also $\left(\frac{3a-1}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2 = r^2$

$$\Rightarrow \left(\frac{3a-1}{\sqrt{13}}\right)^2 + \frac{4}{13} = \frac{(a+1)^2}{2}$$

$5a^2 - 14a - 3 = 0$

$\therefore \quad a = -\frac{1}{5}, 3$

$\because \quad a \neq -\frac{1}{5} \Rightarrow a = 3$

$\Rightarrow \quad r = 2\sqrt{2}$

$\because$ One focus of $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ is $(3, 0)$

$\Rightarrow \quad \alpha e = 3$ and $2\alpha = 4\sqrt{2}$

$\Rightarrow \quad \alpha = 2\sqrt{2} \Rightarrow \alpha^2 = 8$

$$\alpha^2\left[1 + \frac{\beta^2}{\alpha^2}\right] = 9$$

$\alpha^2 + \beta^2 = 9$

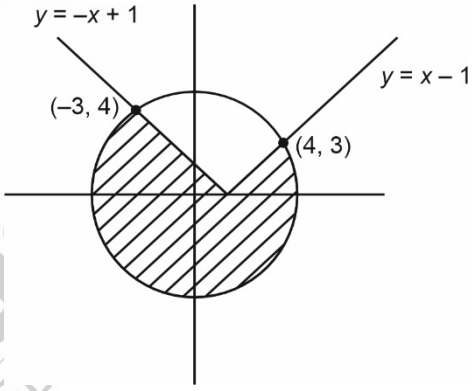
$\Rightarrow \quad \beta^2 = 1$

$\therefore \quad 2\alpha^2 + 3\beta^2 = 19$

22. If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and $y = |x - 1|$ is $\frac{1}{4}(b\pi + c)$, $b, c \in \mathbb{N}$, then $b + c$ is equal to _____.

Answer (77)

Sol.



Area of shaded region

$$= 25\pi - \left[\int_{-3}^{4}\left(\sqrt{25 - x^2} - |x - 1|\right)dx\right]$$

$$= 25\pi - \left[\frac{1}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4} + \left(8 + \frac{9}{2}\right)$$

$$= \frac{1}{4}[75\pi + 2]$$

$\Rightarrow \quad b = 75, c = 2$

$\therefore \quad b + c = 77$

23. The sum of all rational terms in the expansion of $\left(1+2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^6$ is equal to _____

Answer (612)

Sol. The general term of multinomial expansion is

$$\frac{6!}{\alpha!\,\beta!\,\gamma!}(1)^{\alpha}\left(2^{\frac{1}{3}}\right)^{\beta}\left(3^{\frac{1}{2}}\right)^{\gamma}$$

For terms to be rational $3|\beta$ and $2|\gamma$

β	γ	α	Term
0	0	6	1
0	2	4	$15.3 = 45$
0	4	2	$15.3^2 = 135$
0	6	0	$1.3^3 = 27$
3	0	3	$20.2 = 40$
3	2	1	$60.2.3 = 360$
6	0	0	$1.4 = 4$

$\Rightarrow$ Sum of rational terms

$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$

24. If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to _____.

Answer (117)

Sol. Clearly one root is 1, another root is also 1.

Product of roots = 1

$$\frac{c(a-b)}{a(b-c)} = 1$$

$c(a-b) = a(b-c)$

$ac - bc = ab - ac$

$2ac = b(a+c)$

$$2ac = \frac{36}{5}(15)$$

$ac = 18 \times 3 = 54$

$\therefore\ a^2 + c^2 = (a+c)^2 - 2ac$

$= (15)^2 - 2(54)$

$= 225 - 108$

$= 117$

25. If the set of all values of a, for which the equation $5x^3 - 15x - a = 0$ has three distinct real roots, is the interval (α, β), then $\beta - 2\alpha$ is equal to _____.

Answer (30)

Sol. $5x^3 - 15x - a = 0$ has 3 distinct real solution

$\therefore$ Let $f(x) = 5x^3 - 15x - a$

$f'(x) = 15x^2 - 15 = 0$

$15x^2 = 15$

$x^2 = 1$

$\boxed{x = \pm 1}$

$\therefore\ f(1)\, f(-1) < 0$

$(a - 10)(a + 10) < 0$

$a \in (-10, 10)$

$\therefore\ \beta - 2\alpha = 10 + 2(10) = 30$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. A solid sphere of mass 'm' and radius 'r' is allowed to roll without slipping from the highest point of an inclined plane of length 'L' and makes an angle 30° with the horizontal. The speed of the particle at the bottom of the plane is v_1. If the angle of inclination is increased to 45° while keeping L constant. Then the new speed of the sphere at the bottom of the plane is v_2. The ratio $v_1^2 : v_2^2$ is

(1) 1 : 3

(2) $1:\sqrt{2}$

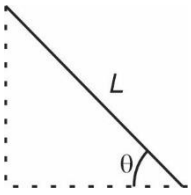
(3) 1 : 2

(4) $1:\sqrt{3}$

Answer (2)

Sol. Loss in P.E. = gain in K.E.

$$mgL\sin\theta = \frac{1}{2}\left(\frac{7}{5}\right)mv^2$$



$$\Rightarrow \quad v^2 \propto \sin\theta$$

$$\frac{v_1^2}{v_2^2} = \frac{\sin 30^\circ}{\sin 45^\circ} = \frac{1}{\sqrt{2}}$$

27. A gun fires a lead bullet of temperature 300 K into a wooden block. The bullet having melting temperature of 600 K penetrates into the block and melts down. If the total heat required for the process is 625 J, then the mass of the bullet is _____ grams.

(Latent heat of fusion of lead = 2.5×10^4 JKg^{-1} and specific heat capacity of lead 125 $JKg^{-1}\,K^{-1}$)

(1) 20 (2) 5

(3) 10 (4) 15

Answer (3)

Sol. $Q = ms\Delta T + mL$

$625 = m \times 125 \times 300 + m \times 2.5 \times 10^4$

$625 = m\{3.75 + 2.5\} \times 10^4$

$$\Rightarrow \frac{625}{6.25}\times 10^{-4}\ \text{kg} = 10\,\text{g} = m$$

28. A radioactive nucleus n_2 has 3 times the decay constant as compared to the decay constant of another radioactive nucleus n_1. If initial number of both nuclei are the same, what is the ratio of number of nuclei of n_2 to the number of nuclei of n_1, after one half-life of n_1?

(1) 1/8 (2) 1/4

(3) 4 (4) 8

Answer (2)

Sol. $\lambda_2 = 3\lambda \qquad \lambda_1 = \lambda$

$$N = N_0 e^{-\lambda t}$$

$$N_1 = N_0 e^{-\lambda t}$$

$$N_2 = N_0 e^{-3\lambda t}$$

$$\frac{N_2}{N_1} = e^{-2\lambda t}$$

for $t = \frac{\ln 2}{\lambda}$

$\Rightarrow \frac{N_2}{N_1} = e^{-2\lambda\frac{\ln 2}{\lambda}}$

$\frac{N_2}{N_1} = \frac{1}{4}$

29. The electric flux is $\phi = \alpha\sigma + \beta\lambda$

Where λ and σ are linear and surface charge density, respectively. $\left(\frac{\alpha}{\beta}\right)$ represents

(1) Area (2) Displacement

(3) Electric field (4) Charge

Answer (2)

Sol. $\alpha \equiv \frac{\phi}{\sigma}$

$\beta = \frac{\phi}{\lambda}$

$\frac{\alpha}{\beta} \equiv \frac{\lambda}{\sigma} \equiv$ displacement

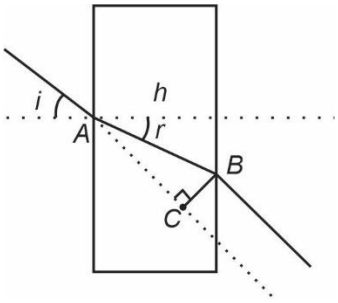
30. What is the lateral shift of a ray refracted through a parallel-sided glass slab of thickness '*h*' in terms of the angle of incidence '*i* ' and angle refraction '*r*', if the glass slab is placed in air medium?

(1) h (2) $\frac{h\sin(i-r)}{\cos r}$

(3) $\frac{h\tan(i-r)}{\tan r}$ (4) $\frac{h\cos(i-r)}{\sin r}$

Answer (2)

Sol.



$AB = h \sec r$

$BC = h \sec r \sin(i - r)$

$BC = \frac{h\sin(i-r)}{\cos r}$

31. Match the **List-I** with **List-II**

List-I		List-II	
A.	Pressure varies inversely with volume of an ideal gas.	I.	Adiabatic process
B.	Heat absorbed goes partly to increase internal energy and partly to do work.	II.	Isochoric process
C.	Heat is neither absorbed nor released by a system.	III.	Isothermal process
D.	No work is done on or by a gas.	IV.	Isobaric process

Choose the correct answer from the options given below:

(1) A-III, B-I, C-IV, D-II (2) A-I, B-IV, C-II, D-III

(3) A-III, B-IV, C-I, D-II (4) A-I, B-III, C-II, D-IV

Answer (3)

Sol. $PV = \mu RT$

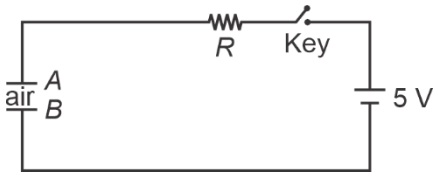
(A) $P \propto \frac{1}{V} \Rightarrow$ isothermal

(B) $\Delta Q = \Delta V + W \Rightarrow$ isobaric

(C) $\Delta Q = 0 \Rightarrow$ adiabatic

(D) $W = 0 \Rightarrow$ isochoric

32. Identify the valid statements relevant to the given circuit at the instant when the key is closed.



A. There will be no current through resistor R.

B. There will be maximum current in the connecting wires.

C. Potential difference between the capacitor plates A and B is minimum.

D. Charge on the capacitor plates is minimum.

Choose the correct answer from the options given below:

(1) C, D only (2) A, B, D only

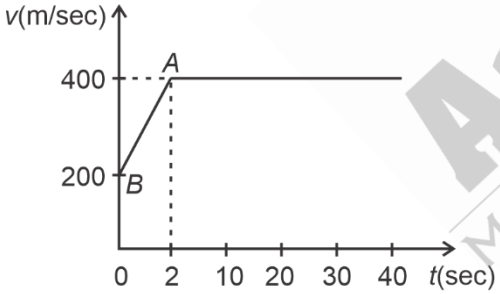
(3) A, C only (4) B, C, D only

Answer (4)

Sol. Capacitor behaves like closed circuit at $t = 0$ and charge is zero.

A ✗, B ✓, C ✓, D ✓

33. The motion of an airplane is represented by velocity-time graph as shown below. The distance covered by airplane in the first 30.5 second is _____ km.



(1) 12 (2) 3

(3) 6 (4) 9

Answer (1)

Sol. Distance = area under the graph

$d = 300 \times 2 + 400 \times 28.5$

$= 600 + 114000$

$= 12000$ m

34. Given below are two statements:

Statement I: The hot water flows faster than cold water.

Statement II: Soap water has higher surface tension as compared to fresh water.

In the light of above statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

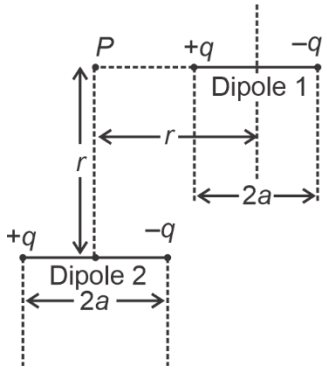
(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Answer (4)

Sol. Hot water flows faster because of less viscosity (✓) and soap water has less surface tension because bubbles are easily formed (✗)

35. A point particle of charge Q is located at P along the axis of an electric dipole 1 at a distance r as shown in the figure. The point P is also on the equatorial plane of a second electric dipole 2 at a distance r. The dipoles are made of opposite charge q separated by a distance $2a$. For the charge particle at P not to experience any net force, which of the following correctly describes the situation?

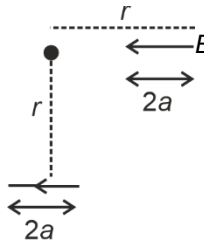


(1) $\frac{a}{r} \sim 3$ (2) $\frac{a}{r} \sim 0.5$

(3) $\frac{a}{r} \sim 10$ (4) $\frac{a}{r} \sim 20$

Answer (1)

Sol.



$E_1 = E_2$, taking $kq = 1$

$$\Rightarrow \frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} = \frac{2a}{\left(a^2+r^2\right)^{3/2}}$$

$$\frac{4ar}{\left(r^2-a^2\right)^2} = \frac{2a}{\left(a^2+r^2\right)^{3/2}}$$

$$\left(r^2-a^2\right)^2 = 2r\left(a^2+r^2\right)^{3/2}$$

$$\left(1-\frac{a^2}{r^2}\right)^2 = 2\left(1+\frac{a^2}{r^2}\right)^{3/2}$$

$$\left(1-x^2\right)^2 = 2\left(1+x^2\right)^{3/2} \quad \left(x = \frac{a}{i}\right)$$

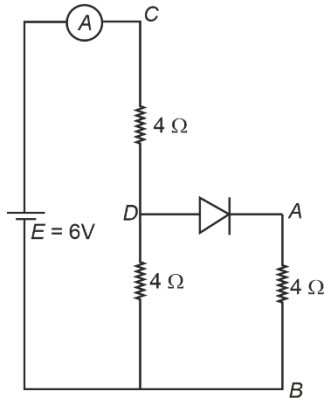
$$\frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^{3/2}} = 2$$

Now for x = 3

We get $\frac{64}{10\sqrt{10}} \approx 2 \Rightarrow \frac{a}{r} \approx 3$

[But for $a > r$ point charge will between the dipole where $\vec{E} \neq 0$]

36. Refer to the circuit diagram given in the figure. which of the following observations are correct?



A. Total resistance of circuit is 6 Ω

B. Current in Ammeter is 1 A

C. Potential across *AB* is 4 volts.

D. Potential across *CD* is 4 volts

E. Total resistance of the circuit is 8 Ω.

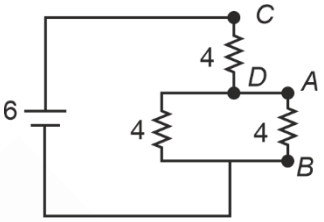
Choose the correct answer from the options given below:

(1) A, B and D Only (2) B, C and E Only

(3) A, B and C Only (4) A, C and D Only

Answer (1)

Sol.



R_{eq} (4 ↑↑ 4) + 4 = 6 Ω

$$i = \frac{6}{6}\text{A}$$

$$V_{CD} = 1 \times 4 = 4\text{V} \Rightarrow V_{AB} = 2\text{V}$$

(A, B, D)

37. Given a thin convex lens (refractive index μ_2), kept in a liquid (refractive index μ_1, $\mu_1 < \mu_2$) having radii of curvatures $|R_1|$ and $|R_2|$. Its second surface is silver polished. Where should an object be placed on the optic axis so that a real and inverted image is formed at the same place?

(1) $\frac{(\mu_2+\mu_1)|R_1|}{(\mu_2-\mu_1)}$

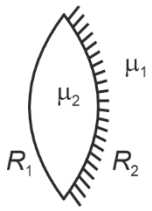
(2) $\frac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_1|}$

(3) $\frac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_2|}$

(4) $\frac{\mu_1|R_1|.|R_2|}{\mu_2(2|R_1|+|R_2|)-\mu_1\sqrt{|R_1|.|R_2|}}$

Answer (3)

Sol.



$$\frac{1}{f_l}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$f_m=-\frac{R_2}{2}$$

$$p_{eq}=2p_l+pm$$

$$\frac{1}{|f|}=\frac{2(\mu_2-\mu_1)}{\mu_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{2}{R_2}$$

$$=\frac{2(\mu_2-\mu_1)(R_1+R_2)}{\mu_1(R_1R_2)}+\frac{2}{R_2}$$

$$\frac{1}{f}=\frac{2\mu_2R_1+2\mu_2R_2-2\mu_1R_1-2\mu_1R_2+2}{\mu_1R_1R_2}$$

$$\Rightarrow\quad f=\frac{\mu_1R_1R_2}{2\mu_2R_1+2\mu_2R_2-2\mu_1R_2}$$

Required distance = 2f.

38. A sub-atomic particle of mass 10^{-30} kg is moving with a velocity 2.21×10^{6} m/s. Under the matter wave consideration, the particle will behave closely like _____.

($h = 6.63\times10^{-34}$ J.s)

(1) Gamma rays (2) X-rays

(3) Visible radiation (4) Infra-red radiation

Answer (2)

Sol. $$\lambda=\frac{6.63\times10^{-34}}{10^{-30}\times2.21\times10^{6}}$$

$\lambda = 3\times10^{-10}$

$\lambda = 3$ Å

$\Rightarrow$ X Ray

39. Regarding self-inductance:

A. The self-inductance of the coil depends on its geometry.

B. Self-inductance does not depend on the permeability of the medium.

C. Self-induced e.m.f. opposes any change in the current in a circuit.

D. Self-inductance is electromagnetic analogue of mass in mechanics.

E. Work needs to be done against self-induced e.m.f. in establishing the current.

Choose the correct answer from the options given below:

(1) A, C, D, E only (2) A, B, C, E only

(3) A, B, C, D only (4) B, C, D, E only

Answer (1)

Sol. (A) $L = \mu_r\mu_0n^2$ Al ✓

(B) $L = \mu_r\ \mu_vn^2$ Al ✗

(C) ✓

(D) ✓

(E) ✓

40. Consider a moving coil galvanometer (MCG):

A. The torsional constant in moving coil galvanometer has dimentions $[ML^2T^{-2}]$

B. Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

C. If we increase number of turns (N) to its double (2N), then the voltage sensitivity doubles.

D. MCG can be converted into an ammeter by introducing a shunt resistance of large value in parallel with galvanometer.

E. Current sensitivity of MCG depends inversely on number of turns of coil.

Choose the correct answer from the options given below:

(1) A, D Only (2) A, B Only

(3) B, D, E Only (4) A, B, E Only

Answer (2)

Sol. (A) $\tau = C\theta$

$C \equiv ML^2T^{-2}$ ✓

(B) $C\theta = iANB$

$\frac{\theta}{i} = \frac{ANB}{C}$ here $N \uparrow, \frac{\theta}{i} \uparrow$

$\frac{\theta}{v} = \frac{ANB}{CR}$ here $\frac{N}{R}$ cannot increase

✓

(C) X; Theoretical

(D) Shunt of law of resistance X

(E) X; explained in [B]

41. A light hollow cube of side length 10 cm and mass 10 g, is floating in water. It is pushed down and released to execute simple harmonic oscillations. The time period of oscillations is $y\pi \times 10^{-2}$ s, where the value of y is

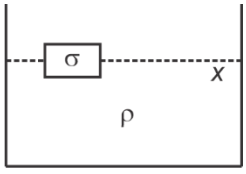
(Acceleration due to gravity, g = 10 m/s^2, density of water = 10^3 kg/m^3)

(1) 2 (2) 6

(3) 4 (4) 1

Answer (1)

Sol. Additional buoyant force



$g\rho\ a^2x = \sigma a^3 A$

$A = \frac{\rho}{\sigma}\frac{g}{a}x$

$T = 2\pi\sqrt{\frac{\sigma a}{\rho g}}$

Now, $\sigma = \frac{10\times10^{-3}}{10^{-3}} = 10$

$\Rightarrow T = 2\pi\sqrt{\frac{10\times0.1}{10^3\times10}}$

$= 2\pi\times10^{-2}$

42. The position of a particle moving on x-axis is given by $x(t) = A\sin t + B\cos^2 t + Ct^2 + D$, where t is time. The dimension of $\frac{ABC}{D}$ is

(1) L^2T^{-2} (2) L^3T^{-2}

(3) L (4) L^2

Answer (1)

Sol. $A \equiv L$

$B \equiv L$

$C \equiv LT^{-2}$

$D \equiv L$

$\Rightarrow \frac{ABC}{D} = \frac{L\times L\times LT^2}{L} = L^2T^{-2}$

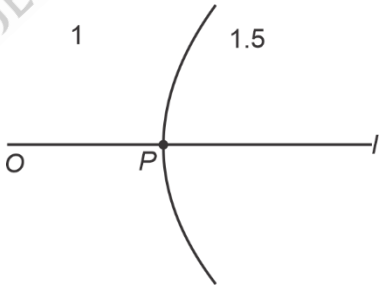
43. A spherical surface of radius of curvature R, separates air from glass (refractive index = 1.5). The centre of curvature is in the glass medium. A point object 'O' placed in air on the optic axis of the surface, so that its real image is formed at 'I' inside glass. The line OI intersects the spherical surface at P and $PO = PI$. The distance PO equals to

(1) $3R$ (2) $2R$

(3) $1.5R$ (4) $5R$

Answer (4)

Sol.



$PO = PI$

$\frac{1.5}{x} - \frac{1}{-x} = \frac{0.5}{R}$

$\frac{2.5}{x} = \frac{0.5}{R}$

$x = 5R$

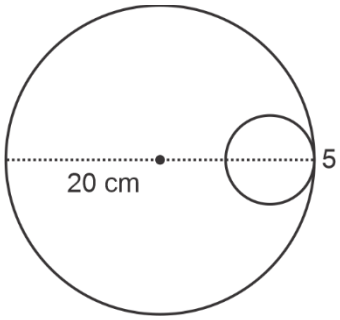
44. Consider a circular disc of radius 20 cm with centre located at the origin. A circular hole of radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of centre of mass of residual or remaining disc from the origin will be

(1) 1.5 cm (2) 0.5 cm

(3) 1.0 cm (4) 2.0 cm

Answer (3)

Sol.



$M \propto A$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{A(0) + \left(-\frac{A}{16}\right)(15)}{A - \frac{A}{16}}$$

$$= \frac{-15}{16-1} = -1\,\text{cm}$$

45. The electric field of an electromagnetic wave in free space is

$\vec{E} = 57\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]$

$\left(4\hat{i} - 3\hat{j}\right)$N/C. The associated magnetic field in Tesla is

(1) $\vec{B} = -\frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(\hat{k}\right)$

(2) $\vec{B} = \frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(5\hat{k}\right)$

(3) $\vec{B} = -\frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(5\hat{k}\right)$

(4) $\vec{B} = \frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(\hat{k}\right)$

Answer (2)

Sol. $E = CB$

and $\vec{E}\times\vec{B} \uparrow\uparrow \vec{C}$

$$B = \frac{57\times5}{3\times10^8}$$

$3\hat{i} + 4\hat{j}$ is direction of propagation

As, $\vec{E}\times\vec{B} \uparrow\uparrow 3\hat{i} + 4\hat{j}$

$$\Rightarrow \left(4\hat{i} - 3\hat{j}\right)\times\left(-\hat{k}\right) = 4\hat{j} + 3\hat{i}$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A positive ion A and a negative ion B has charges 6.67×10^{-19} C and 9.6×10^{-10} C, and masses 19.2×10^{-27} kg and 9×10^{-27} kg respectively. At an instant, the ions are separated by a certain distance r. At that instant the ratio of the magnitudes of electrostatic force to gravitational force is $P \times 10^{-23}$, where the value of P is _____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ Nm^{2}C^{-1} and universal gravitational constant as 6.67×10^{-11} Nm^{2}kg^{-2})

Answer (None)

Sol. $F_e = \frac{kq_1q_2}{r^2}$

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$\frac{F_e}{F_g} = \frac{kq_1q_2}{Gm_1m_2}$$

$$= \frac{9\times10^9 \times 6.67\times10^{-19} \times 9.6\times10^{-10}}{6.67\times10^{-11}\times19.2\times10^{-27}\times9\times10^{-27}}$$

$$= \frac{10^{-20}}{2\times10^{-65}}$$

***Answer does not match with description.**

47. An ideal gas initially at 0°C temperature, is compressed suddenly to one fourth of its volume. If the ratio of specific heat at constant pressure to that at constant volume is 3/2, the change in temperature due to the thermodynamic process is _____ K.

Answer (273)

Sol. $TV^{\gamma-1} = \text{constant}$

$$273 \times V^{0.5} = T\left(\frac{V}{4}\right)^{0.5}$$

$T = 546$

$\Delta T = 273$ K

48. A force $f = x^2y\hat{i} + y^2\hat{j}$ acts on a particle in a plane $x + y = 10$. The work done by this force during a displacement from (0, 0) to (4m, 2m) _____ Joule (round off to the nearest integer)

Answer (152)

Sol. $y = 10 - x$

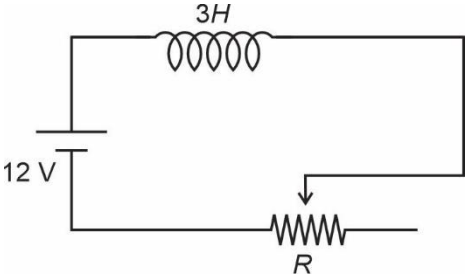
$$w = \int_0^4 x^2(10-x)dx + \int_0^2 y^2 dy$$

$$= \frac{10x^3}{3} - \frac{x^4}{4}\Big|_0^4 + \frac{y^3}{3}\Big|_0^2$$

$$= \frac{640}{3} - \frac{256}{4} + \frac{8}{3}$$

$= 216 \times 64$

$= 152$ J

49. 

In the given circuit the sliding contact is pulled outwards such that electric current in the circuit changes at the rate of 8 A/s. At an instant when R is 12 Ω, the value of the current in the circuit will be _____ A.

Answer (1)

Sol. $\varepsilon - \frac{Ldi}{dt} = iR = 0$

$12 - 3 \times 8 - i\,12 = 0$

$12 = i\,12$

$i = 1$

50. Two particles are located at equal distance from origin. The position vectors of those are represented by $\vec{A} = 2\hat{i} + 3n\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4p\hat{k}$, respectively. If both the vectors are at right angle to each other, the value of n^{-1} is _____.

Answer (3)

Sol. $\vec{A}\cdot\vec{B} = 0$ and $|\vec{A}| = |\vec{B}|$

$\Rightarrow$ $4 - 6n + 8p = 0$

$3n - 4p = 2$ …(i)

Also $4 + 9n^2 + 4 = 4 + 4 + 16p^2$

$3n = \pm 4p$ …(ii)

$\pm 4p - 4p = 2$

Taking –ve sign

$-8p = 2$

$p = -\frac{1}{4}$

$3n + 1 = 2$

$n = \frac{1}{3}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. 2.8×10^{-3} mol of CO_2 is left after removing 10^{21} molecules from its 'x' mg sample. The mass of CO_2 taken initially is

Given : $N_A = 6.02 \times 10^{23}$ mol^{-1}

(1) 98.3 mg (2) 48.2 mg

(3) 150.4 mg (4) 196.2 mg

Answer (4)

Sol. Moles of removed $CO_2 = \frac{10^{21}}{6.02 \times 10^{23}}$ mol

$= 1.66 \times 10^{-3}$ mol

mole of CO_2 left = 2.8×10^{-3} moles

total moles of CO_2 taken initially

$= (2.8 + 1.66) \times 10^{-3}$ mol

mass of CO_2 taken initially

$= 4.46 \times 10^{-3} \times 44$

$= 196.24 \times 10^{-3}$ g

= 196.24 mg

52. $FeO_4^{2-} \xrightarrow{+2.0\,V} Fe^{3+} \xrightarrow{0.8\,V} Fe^{2+} \xrightarrow{-0.5\,V} Fe^0$

In the above diagram, the standard electrode potentials are given in volts (over the arrow).

The value of $E^{\ominus}_{FeO_4^{2-}/Fe^{2+}}$ is

(1) 1.7 V (2) 2.1 V

(3) 1.4 V (4) 1.2 V

Answer (1)

Sol. $FeO_4^{2-} + 3e^- \rightarrow Fe^{3+}$; $E^\circ = 2.0$ eV

$Fe^{3+} + e^- \rightarrow Fe^{2+}$; $E^\circ = 0.8$ eV

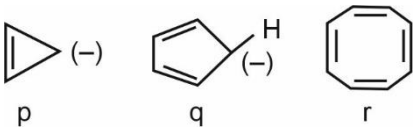
$FeO_4^{2-} + 4e^- \rightarrow Fe^{2+}$; $E^\circ = x$ (say)

$4 \times x = 3 \times 2 + 1 \times 0.8$

$x = \frac{6.8}{4}$ V

$= 1.7$ V

53. The correct stability order of the following species/molecules is



p q r

(1) q > p > r (2) q > r > p

(3) r > q > p (4) p > q > r

Answer (2)

Sol. q is aromatic

r is non-aromatic

p is antiaromatic

q > r > p (order of stability)

Aromatic > non-aromatic > antiaromatic

54. Ice at –5ºC is heated to become vapour with temperature of 110ºC at atmospheric pressure. The entropy change associated with this process can be obtained from

(1) $\int_{268K}^{273K} C_{p,m}dT + \frac{\Delta H_{m,fusion}}{T_f} + \frac{\Delta H_{m,vaporisation}}{T_b} + \int_{273K}^{373K} C_{p,m}dT + \int_{373K}^{383K} C_{p,m}dT$

(2) $\int_{268K}^{383K} C_p\,dT + \frac{\Delta H_{melting}}{273} + \frac{\Delta H_{boiling}}{373}$

(3) $\int_{268K}^{273K} \frac{C_{p,m}}{T}dT + \frac{\Delta H_{m,fusion}}{T_f} + \frac{\Delta H_{m,vaporisation}}{T_b} + \int_{273K}^{373K} \frac{C_{p,m}dT}{T} + \int_{373K}^{383K} \frac{C_{p,m}dT}{T}$

(4) $\int_{268K}^{383K} C_p dT + \frac{q_{rev}}{T}$

Answer (3)

Sol. $\underset{-5^\circ C}{H_2O(s)} \rightarrow \underset{0^\circ C}{H_2O(s)}\ ;\ \Delta S_1 = \int_{268K}^{273K} \frac{C_{p,\,m}dT}{T}$

$\underset{0^\circ C}{H_2O(s)} \rightleftharpoons \underset{0^\circ C}{H_2O(l)}\ ;\ \Delta S_2 = \frac{\Delta H_{m,\,fus}}{273}$

$\underset{0^\circ C}{H_2O(l)} \rightarrow \underset{100^\circ C}{H_2O(l)}\ ;\ \Delta S_3 = \int_{273}^{373} \frac{C_{p,\,m}dt}{T}$

$\underset{100^\circ C}{H_2O(l)} \rightleftharpoons \underset{100^\circ C}{H_2O(g)}\ ;\ \Delta S_4 = \frac{\Delta H_{m,\,vap}}{373}$

$\underset{100^\circ C}{H_2O(g)} \rightarrow \underset{110^\circ C}{H_2O(g)}\ ;\ \Delta S_5 = \int_{373}^{383} \frac{C_{p,\,m}dT}{T}$

$\Delta S_{total} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5$

55. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H atom is suitable for this?

Given: Rydberg constant $R_H = 10^5\ cm^{-1}$, $h = 6.6 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s)

(1) Lyman series, $\infty \rightarrow 1$

(2) Paschen series, $5 \rightarrow 3$

(3) Paschen series, $\infty \rightarrow 3$

(4) Balmer series, $\infty \rightarrow 2$

Answer (3)

Sol. $\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{1}{\lambda} = 10^5 \left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$

$\frac{1}{\lambda} = 10^5 \times \frac{1}{9}$

$\lambda = 9 \times 10^{-5}$

$\lambda = 900 \times 10^{-7} cm$

$= 900$ Å

56. The major product of the following reaction is

$$CH_3CH_2CH = O \xrightarrow[\text{reflux}]{\text{excess HCHO alkali}} ?$$

(1) $CH_3 - CH_2 - CH_2 - OH$

(2) $CH_3 - \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH}}} - CH = O$

(3) $CH_3 - \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{C}}}} - CH_2 - OH$

(4) $CH_3 - \underset{\displaystyle CH_2}{\overset{\displaystyle O}{\overset{\displaystyle ||}{\underset{\displaystyle |}{C}}}} - CH = O$

Answer (3)

Sol. Propanal undergoes aldol condensation with excess of HCHO in presence of OH^- ions to 2,2-dihydroxy methyl propanal which further reacts with HCHO and undergoes Cannizzaro reaction to give 2,2-hydroxymethylpropan-1-ol.

$$CH_3 - CH_2 - \underset{\displaystyle |}{\overset{\displaystyle O}{\overset{\displaystyle ||}{C}}} - H + 2HCHO \xrightarrow{OH^-} \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle CH_2OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH_3 - C - CHO}}}}$$

$$\underset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH_3 - C - CHO}}} + HCHO \xrightarrow[\text{reflux}]{OH^-} \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH_3 - C - CH_2OH}}}} + HCOO^-$$

57. Match the **LIST-I** with **LIST-II**.

	LIST-I (Classification of molecules based on octet rule)		**LIST-II (Example)**
A.	Molecules obeying octet rule	I.	NO, NO_2
B.	Molecules with incomplete octet	II.	BCl_3, $AlCl_3$
C.	Molecules with incomplete octet with odd electron	III.	H_2SO_4, PCl_5
D.	Molecules with expanded octet	IV.	CCl_4, CO_2

Choose the ***correct*** answer from the options given below.

(1) A-IV, B-I, C-III, D-II (2) A-II, B-IV, C-III, D-I

(3) A-IV, B-II, C-I, D-III (4) A-III, B-II, C-I, D-IV

Answer (3)

Sol. $NO = 7e^-$, $NO_2 = 7e^-$ } Incomplete octet with odd electron

$BCl_3 = 6e^-$, $AlCl_3 = 6e^-$ } Incomplete octet

$H_2SO_4 = 12e^-$, $PCl_5 = 10e^- \Rightarrow$ molecules with expanded octet

$CCl_4 = 8e^-$, $CO_2 = 8e^- \Rightarrow$ molecules obeying octet rule

58. Math the **LIST-I** with **LIST-II**.

	LIST-I (Name reaction)		**LIST-II (Product obtainable)**
A.	Swarts reaction	I.	Ethyl benzene
B.	Sandmeyer's reaction	II.	Ethyl iodide
C.	Wurtz-Fittig reaction	III.	Cyanobenzene
D.	Finkelstein reaction	IV.	Ethyl fluoride

Choose the ***correct*** answer from the options given below.

(1) A-II, B-I, C-III, D-IV (2) A-IV, B-I, C-III, D-II

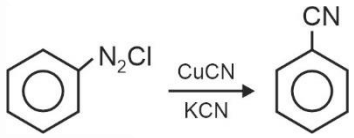
(3) A-IV, B-III, C-I, D-II (4) A-II, B-III, C-I, D-IV

Answer (3)

Sol. Swarts rection : Halogen exchange rxn

$$CH_3 - Br + AgF \xrightarrow{DMF} CH_3F$$

Sandmeyer's reaction :



Wurtz-Fittig reaction :

$$Ph - Br + C_2H_5Br \xrightarrow[\text{Dry ether}]{Na} Ph - C_2H_5$$

Finkelstein reaction : Halogen exchange rxn

$$R - Cl \xrightarrow[\text{Acetone}]{NaI} R - I$$

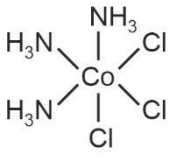
59. The complex that shows Facial - Meridional isomerism is

(1) $[Co(en)_2Cl_2]^+$ (2) $[Co(en)_3]^{3+}$

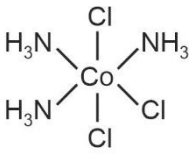
(3) $[Co(NH_3)_4Cl_2]^+$ (4) $[Co(NH_3)_3Cl_3]$

Answer (4)

Sol. Ma_3b_3 type of complex can show Facial-Meridional type isomerism.(where a and b are monodentate ligands)



Facial



Meridional

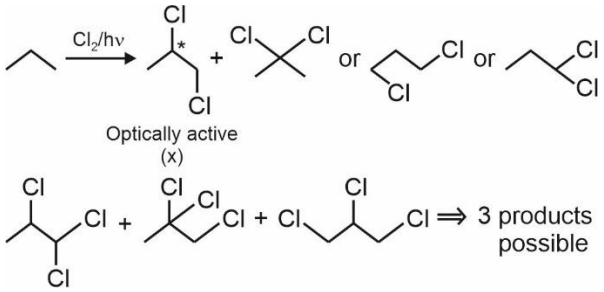
60. Propane molecule on chlorination under photochemical condition gives two di-chloro products, "x" and "y". Amongst "x" and "y", "x" is an optically active molecule. How many tri-chloro products (consider only structural isomers) will be obtained from "x' when it is further treated with chlorine under the photochemical condition?

(1) 4 (2) 3

(3) 2 (4) 5

Answer (2)

Sol.



61. The correct set of ions (aqueous solution) with same colour from the following is:

(1) Ti^{4+}, V^{4+}, Mn^{2+} (2) Zn^{2+}, V^{3+}, Fe^{3+}

(3) V^{2+}, Cr^{3+}, Mn^{3+} (4) Sc^{3+}, Ti^{3+}, Cr^{2+}

Answer (3)

Sol. V^{2+} = Violet

Cr^{3+} = Violet

Mn^{3+} = Violet

62. $CrCl_3.xNH_3$ can exist as a complex. 0.1 molal aqueous solution of this complex shows a depression in freezing point of 0.558°C. Assuming 100% ionisation of this complex and coordination number of Cr is 6, the complex will be

(Given K_f = 1.86 K kg mol^{-1})

(1) $[Cr(NH_3)_4Cl_2]$ Cl (2) $[Cr(NH_3)_3Cl_3]$

(3) $[Cr(NH_3)_6]$ Cl_3 (4) $[Cr(NH_3)_5Cl]$ Cl_2

Answer (4)

Sol. $\Delta T_f = iK_fm$

$0.558 = i \times 1.86 \times 0.1$

$$i = \frac{0.558}{0.186} = 3$$

Number of ions when 100% ionisation takes place = 3

$$[Cr(NH_3)_5Cl]Cl_2 \xrightarrow{\text{aqueous solution}} [Cr(NH_3)_5Cl]^{2+} + 2Cl^-$$

Number of ions = 3

63. What amount of bromine will be required to convert 2 g of phenol into 2,4,6-tribromophenol?

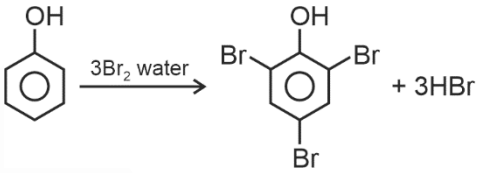
(Given molar mass in g mol^{-1} of C, H, O, Br are 12, 1, 16, 80 respectively)

(1) 4.0 g (2) 6.0 g

(3) 10.22 g (4) 20.44 g

Answer (3)

Sol.



Moles of phenol = $\frac{2}{94}$ mol

Moles of $Br_2 = \frac{3 \times 2}{94}$ moles

Mass of $Br_2 = \frac{3 \times 2}{94} \times 160$ g

= 10.22 g

64. Given below are two statements:

Statement I: Fructose does not contain an aldehydic group but still reduces Tollen's reagent.

Statement II: In the presence of base, fructose undergoes rearrangement to give glucose.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Answer (4)

Sol. Fructose has α-hydroxy ketone group which tautomerise to aldehyde group in presence of base, therefore it reduces Tollen's reagent.

In presence of base, fructose undergo rearrangement to give glucose and mannose.

65. The d-electronic configuration of an octahedral Co(II) complex having magnetic moment of 3.95 BM is:

(1) $t_{2g}^5 e_g^2$ (2) $t_{2g}^3 e_g^0$

(3) $t_{2g}^6 e_g^1$ (4) $e^4 t_2^3$

Answer (1)

Sol. Co^{2+} complex having μ = 3.95 BM

Hence number of unpaired electron = 3

$Co^{2+} \Rightarrow 3d^7 = t_{2g}^5 e_g^2$

66. Which of the following happens when NH_4OH is added gradually to the solution containing 1 M A^{2+} and 1 M B^{3+} ions?

Given : $K_{sp}[A(OH)_2] = 9 \times 10^{-10}$ and $K_{sp}[B(OH)_3] = 27 \times 10^{-18}$ at 298 K

(1) $B(OH)_3$ will precipitate before $A(OH)_2$

(2) $A(OH)_2$ will precipitate before $B(OH)_3$

(3) $A(OH)_2$ and $B(OH)_3$ will precipitate together

(4) Both $A(OH)_2$ and $B(OH)_3$ do not show precipitation with NH_4OH

Answer (1)

Sol. Condition for precipitation IP > K_{sp}

For, $[A(OH)_2]$

$[A^{2+}][OH^-]^2 > 9 \times 10^{-10}$

$[A^{2+}] = 1$ M

$[OH^-] > 3 \times 10^{-5}$

For, $[B(OH)_3]$

$[B^{3+}][OH^-]^3 > 27 \times 10^{-18}$

$[B^{3+}] = 1$ M

$[OH^-] > 3 \times 10^{-6}$

So, $B(OH)_3$ will precipitate first

67. The **incorrect** statement among the following is

(1) PF_3 exists but NF_5 does not

(2) PH_3 shows lower proton affinity than NH_3

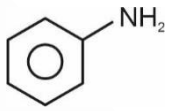
(3) SO_2 can act as an oxidizing agent, but not as a reducing agent

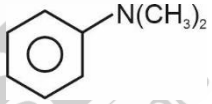
(4) NO_2 can dimerise easily

Answer (3)

Sol. $\overset{+4}{S}O_2$ can acts as both oxidising agent as well reducing agents because due to intermediate oxidation state, it can oxidise and reduce as well.

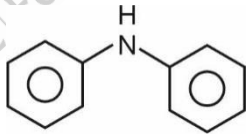
68. Which among the following react with Hinsberg's reagent?

A. 

B. 

C. $CH_3 — NH_2$

D. $N(CH_3)_3$

E. 

Choose the correct answer from the options given below

(1) A, B and E Only

(2) C and D Only

(3) A,C and E Only

(4) B and D Only

Answer (3)

Sol. Primary and secondary amines can reacts with Hinsberg reagent.

69. Given below are two statements

Statement I : In Lassaigne's test, the covalent organic molecules are transformed into ionic compounds.

Statement II: The sodium fusion extract of an organic compound having N and S gives prussian blue colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Answer (4)

Sol. Lassaigne's test is a general test for detection of halogen, nitrogen and sulphur in an organic compound. These elements covalently bonded to the organic compounds, In order to detect them, these have to converted into ionic forms.

$3Na_4[Fe(CN)_6] + 2Fe_2(SO_4)_3 \longrightarrow Fe_4[Fe(CN)_6]_3 + 6\ Na_2SO_4$ (prussian Blue)

70. The element that does not belong to the same period of the remaining elements (modern periodic table) is

(1) Iridium (2) Osmium

(3) Palladium (4) Platinum

Answer (3)

Sol. Ir, Os and Pt belong to 6^{th} period of periodic table while Pd belongs to 5^{th} period.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. For the thermal decomposition of $N_2O_5(g)$ at constant volume, the following table can be formed, for the reaction mentioned below.

$2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

Sr. No.	Time/s	Total pressure/(atm
1	0	0.6
2	100	'x'

$x = \_\_\_ \times 10^{-3}$ atm [nearest integer]

Given : Rate constant for the reaction is $4.606 \times 10^{-2}\ s^{-1}$.

Answer (897)

Sol. $2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

$$k = \frac{2.303}{t}\log\frac{0.9-0.6}{(0.9-x)}$$

$$2 \times 10^{-2} \times 100 = \log\frac{0.3}{(0.9-x)}$$

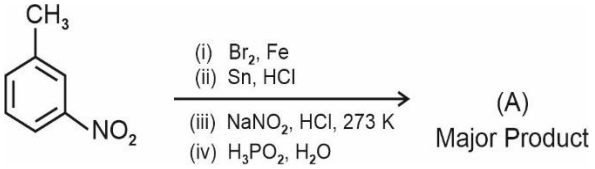
$$100 = \frac{0.3}{(0.9-x)}$$

$$= \frac{0.9-x}{0.3} = 0.01$$

$$0.9 - X = 0.003$$

$$= 897 \times 10^{-3}$$

72. Consider the following sequence of reactions to produce major product (A)

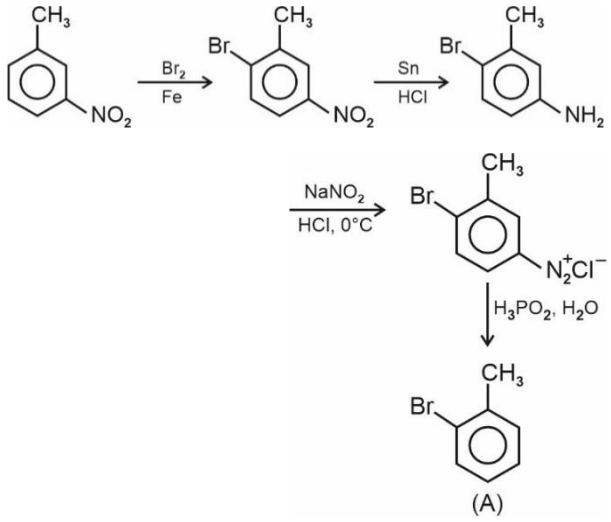


Molar mass of product (A) is ____g mol^{-1}.

(Given molar mass in g mol^{-1} of C : 12, H : 1, O : 16, Br : 80, N : 14, P : 31)

Answer (171)

Sol.



Molar mass of A = 171 g/mol

73. During "S" estimation, 160 mg of an organic compound gives 466 mg of barium sulphate. The percentage of Sulphur in the given compound is ____%.

(Given molar mass in g mol^{-1} of Ba : 137, S : 32, O : 16)

Answer (40)

Sol. m mole of $BaSO_4$ = mmoles of $S = \frac{466}{233}$

Mass of $S = \frac{466}{233} \times 32$ mg

= 64 mg

$\% \ S = \frac{64}{160} \times 100 = 40\%$

74. The standard enthalpy and standard entropy of decomposition of N_2O_4 to NO_2 are 55.0 kJ mol^{-1} and 175.0 J/K/mol respectively. The standard free energy change for this reaction at 25°C in J mol^{-1} is _____ (Nearest integer)

Answer (2850)

Sol. $\Delta H^\circ_{rxn} = 55.0 \text{ kJ/mol}$ T = 298 K

$\Delta S^\circ_{rxn} = 175 \text{ J/mol}$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

= 55000 – 298 × 175

= 2850 J/mol

75. If 1 mM solution of ethylamine produces pH = 9, then the ionization constant (K_b) of ethylamine is 10^{-x}. The value of x is _____ (nearest integer).

[The degree of ionization of ethylamine can be neglected with respect to unity.]

Answer (7)

Sol. pOH + pH = 14

pOH = 14–9

pOH = 5

$[OH^-] = 10^{-5}$ m

$[OH^-] = \sqrt{K_b \cdot C}$

C = concentration of weak base = 1 mM = 10^{-3} M

$10^{-5} = \sqrt{K_b \times 10^{-3}}$

$10^{-10} = K_b \times 10^{-3}$

$K_b = 10^{-7}$

x = 7

❑ ❑ ❑

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by $x\ R\ y$ if and only if $\sec^2 x - \tan^2 y = 1$. Then R is:

(1) both reflexive and transitive but not symmetric

(2) an equivalence relation

(3) reflexive but neither symmetric not transitive

(4) both reflexive and symmetric but not transitive

Answer (2)

Sol. $x\ R\ y : \sec^2 x - \tan^2 y = 1$

Check reflexive:

$x\ R\ x = \sec^2 x - \tan^2 x = 1$

$\forall x \in \left[0, \frac{\pi}{2}\right)$

Check symmetric

$x\ R\ y \Rightarrow y\ R\ x$

$\sec^2 x - \tan^2 y = 1$

$= \sec^2 y - \tan^2 x = (1 + \tan^2 y) - (\sec^2 x - 1)$

$= 2 - (\sec^2 x - \tan^2 y)$

$= 2 - 1 = 1$

$\Rightarrow y\ R\ x$

Check transitive

$x\ R\ y$ and $y\ R\ z$

$\Rightarrow \sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 z = 1$

Add $\Rightarrow \sec^2 x - \tan^2 z + (\sec^2 y - \tan^2 y) = 2$

$\Rightarrow \sec^2 x - \tan^2 z + 1 = 2 \Rightarrow x\ R\ z$

$\Rightarrow$ R is an equivalence relation.

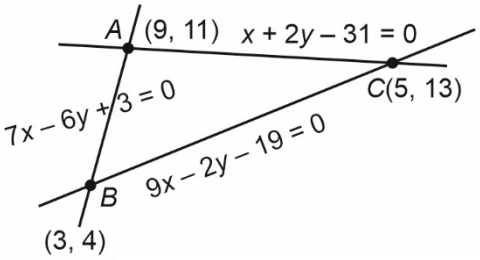
2. Let ABC be a triangle formed by the lines $7x - 6y + 3 = 0$, $x + 2y - 31 = 0$ and $9x - 2y - 19 = 0$. Let the point (h, k) be the image of the centroid of ΔABC in the line $3x + 6y - 53 = 0$. Then $h^2 + k^2 + hk$ is equal to:

(1) 40 (2) 36

(3) 47 (4) 37

Answer (4)

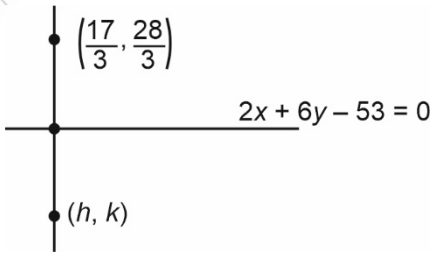
Sol.



Points of intersections are (9, 11), (3, 4), (5, 13)

Centroid of $\Delta ABC = \left(\frac{17}{3}, \frac{28}{3}\right)$

Since during image of ΔABC about line will reflect the whole triangle including centroid, reflected centroid will be image of $\left(\frac{17}{3}, \frac{28}{3}\right)$ about $2x + 6y - 53 = 0$



$$\frac{x - \frac{17}{3}}{2} = \frac{y - \frac{28}{3}}{6} = \frac{-2\left(2\left(\frac{17}{3}\right) + 6\left(\frac{28}{3}\right) - 53\right)}{2^2 + 6^2}$$

$\Rightarrow h = 3, k = 4$

$\Rightarrow h^2 + k^2 + hk = (h + k)^2 - hk$

$= 49 - 12 = 37$

3. Let $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$, $\vec{b}=3\hat{i}-5\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}=\vec{c}\times\vec{b}$ and $(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c})=168$. Then the maximum value of $|\vec{c}|^2$ is

(1) 308 (2) 462

(3) 154 (4) 77

Answer (1)

Sol. $\vec{a}\times\vec{c}=\vec{c}\times\vec{b}=-(\vec{b}\times\vec{c})$

$\Rightarrow (\vec{a}\times\vec{c}+\vec{b}\times\vec{c})=(\vec{a}+\vec{b})\times\vec{c}=0$

$\Rightarrow$ $\vec{c}$ is parallel to $\vec{a}+\vec{b}$.

$\vec{c}=\lambda(5\hat{i}+(-6)\hat{j}+4\hat{k})\Rightarrow|\vec{c}|^2=\lambda^2(\pi)$

$(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+|\vec{c}|^2+\vec{c}\cdot(\vec{a}+\vec{b})$

$=(6+5+3)+|\vec{c}|^2+\lambda(25+36+16)=168$

$\Rightarrow |\vec{c}|^2+\lambda(77)=154$

$\Rightarrow 77\lambda^2+77\lambda-152=0$

$\Rightarrow \lambda^2+\lambda-2=0$

$\Rightarrow (\lambda+2)(\lambda-1)=0$

$|\vec{c}|^2_{max}=(-2)^2(77)=308$

4. Let $y=y(x)$ be the solution of the differential equation $\cos x\left(\log_e(\cos x)\right)^2 dy$ $+(\sin x-3y\sin x\log_e(\cos x))dx=0$, $x\in\left(0,\frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right)=\frac{-1}{\log_e 2}$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $-\frac{1}{\log_e(4)}$ (2) $\frac{1}{\log_e(3)-\log_e(4)}$

(3) $\frac{1}{\log_e(4)-\log_e(3)}$ (4) $\frac{2}{\log_e(3)-\log_e(4)}$

Answer (2)

Sol. $\frac{dy}{dx}-\frac{3\sin x}{\cos x\ln\cos x}y=-\frac{\sin x}{\cos x(\ln\cos x)^2}$

I.F. = $e^{-\int\frac{3\tan x}{\ln\cos x}dx}$

Let ln cos$x$ = t

$-\tan x\,dx = dt$

$e^{3\int\frac{dt}{t}}=e^{3\ln t}=t^3=(\ln\cos x)^3$

$\therefore$ Solution will be

$y(\ln\cos x)^3=-\int(\tan x)(\ln\cos x)dx$

$y(\ln\cos x)^3=\frac{(\ln\cos x)^2}{2}+c$

$\because$ $y\left(\frac{\pi}{4}\right)=-\frac{1}{\ln 2}$

$\Rightarrow$ $c=0$

$\therefore$ $y=\frac{1}{2(\ln\cos x)}$

$y\left(\frac{\pi}{6}\right)=\frac{1}{2}\times\frac{1}{\ln\left(\cos\frac{\pi}{6}\right)}$

$=\frac{1}{2}\left[\frac{1}{\ln\left(\frac{\sqrt{3}}{2}\right)}\right]$

$=\frac{1}{2}\times\frac{1}{\ln\sqrt{3}-\ln 2}$

$=\frac{1}{\ln 3-\ln 4}$

5. The value of $\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}\right)$ is

(1) $\frac{7}{3}$ (2) $\frac{4}{3}$

(3) $\frac{5}{3}$ (4) 2

Answer (3)

Sol. $\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\dfrac{k^3+6k^2+11k+5}{(k+3)!}\right)$

$\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\dfrac{k^3+6k^2+11k+6-1}{(k+3)!}\right)$

$\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\dfrac{(k+1)(k+2)(k+3)-1}{(k+3)!}\right)$

$\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\dfrac{1}{k!}-\dfrac{1}{(k+3)!}\right)$

$= \left(\dfrac{1}{1!}+\dfrac{1}{2!}+\dfrac{1}{3!}+\cdots\infty\right)-\left(\dfrac{1}{4!}+\dfrac{1}{5!}+\dfrac{1}{6!}+\cdots\infty\right)$

$= (e-1)-\left(e-1-\dfrac{1}{1!}-\dfrac{1}{2!}-\dfrac{1}{3!}\right)$

$= 1+\dfrac{1}{2}+\dfrac{1}{6}=\dfrac{10}{6}=\dfrac{5}{3}$

6. Let P be the set of seven digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then the number of elements in the set P is:

(1) 173 (2) 161

(3) 164 (4) 158

Answer (2)

Sol. Case I: 3 2 2 1 1 1 1

$n_1=\dfrac{7!}{4!2!}=105$

Case II: 2 2 2 2 1 1 1

$n_2=\dfrac{7!}{4!3!}=35$

Case III: 3 3 1 1 1 1 1

$n_3=\dfrac{7!}{2!4!}=21$

Total numbers = 105 + 35 + 21 = 161

7. Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix},\ x\in R$$

Then $M^4 - m^4$ is equal to

(1) 1295 (2) 1040

(3) 1280 (4) 1215

Answer (3)

Sol. $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}=f(x)$$

$C_2 \to C_1 + C_2$

$$\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ \sin^2 x & 1 & 1+4\sin 4x\end{vmatrix}=f(x)$$

$f(x) = 1 + 4\sin4x+1 = 2 + 4\sin4x$

$M = 6,\ m = -2$

$M^4 - m^4 = 1280$

8. The integral $80\int\limits_0^{\pi/4}\left(\dfrac{\sin\theta+\cos\theta}{9+16\sin 2\theta}\right)d\theta$ is equal to

(1) $4\log_e 3$ (2) $6\log_e 4$

(3) $2\log_e 3$ (4) $3\log_e 4$

Answer (1)

Sol. $I=\int\limits_0^{\frac{\pi}{4}}\left(\dfrac{\sin\theta+\cos\theta}{9-16\sin 2\theta}\right)d\theta$

Take $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)d\theta = dt$

$(\sin\theta - \cos\theta)^2 = t^2$

$\Rightarrow \sin2\theta = 1 - t^2$

$\theta = 0 \to t = -1$

$\theta=\dfrac{\pi}{4}\to t=0$

$I=\int\limits_{-1}^{0}\dfrac{dt}{9+16(1-t^2)}$

$=\dfrac{1}{16}\int\limits_{-1}^{0}\dfrac{dt}{\frac{25}{16}-t^2}$

$=\dfrac{1}{4}\left[\dfrac{1}{10}\log\left|\dfrac{5+4t}{5-4t}\right|\right]_{-1}^{0}$

$=\dfrac{1}{40}[0+\log_e 9]$

$I=\dfrac{\log_e 9}{40}$

$80I = 2\log_e 9$

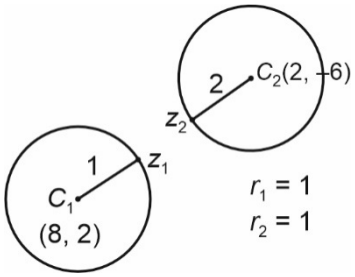
$80I = 4\log_e 3$

9. Let $|z_1 - 8 - 2i| \le 1$ and $|z_2 - 2 + 6i| \le 2$, $z_1, z_2 \in C$. Then the minimum value of $|z_1 - z_2|$ is

(1) 13 (2) 10
(3) 7 (4) 3

Answer (3)

Sol.



$|z_1 - z_2| = C_1C_2 - (r_1 + r_2)$

$= \sqrt{6^2 + 8^2} - (1 + 2)$

$= 10 - 3 = 7$

10. Let $A = [a_{ij}] = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$.

If A_{ij} is the cofactor of $a_{ij}, C_{ij} = \sum_{k=1}^{2} a_{ik} A_{jk}, 1 \le i, j \le 2$ and $C = [C_{ij}]$, then $8|C|$ is equal to

(1) 222 (2) 242
(3) 288 (4) 262

Answer (2)

Sol. $[a_{ij}] = \begin{bmatrix} 7\log_5 2 & \frac{1}{2}\log_2 5 \\ 3\log_5 2 & \log_2 5 \end{bmatrix}$

$[A_{ij}] = \begin{bmatrix} \log_2 5 & -3\log_5 2 \\ -\frac{1}{2}\log_2 5 & 7\log_5 2 \end{bmatrix}$

$C_{ij} = \sum_{k=1}^{2} a_{ik} A_{jk}, \ 1 \le i, j \le 2$

$C = \begin{bmatrix} \frac{11}{2} & 0 \\ 0 & \frac{11}{2} \end{bmatrix}$

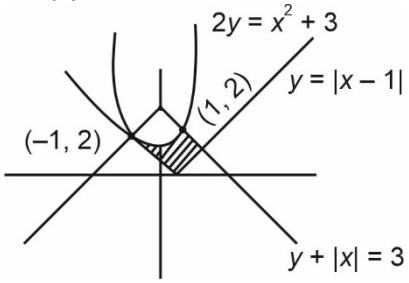
$8|C| = \frac{121}{4} \cdot 8 = 242$

11. Let the area of the region $\left\{(x, y) : 2y \le x^2 + 3, y + |x| \le 3, y \ge |x - 1|\right\}$ be A. Then $6A$ is equal to

(1) 18 (2) 12
(3) 16 (4) 14

Answer (4)

Sol.



$\int_{-1}^{1} \left(\frac{x^2 + 3}{2} + (x - 1)\right) dx + \int_{1}^{2} ((3 - x) - (x - 1)) dx$

$\Rightarrow \frac{x^3}{6} + \frac{3x}{2} + \frac{x^2}{2} - x \Big|_{-1}^{1} + 3x - \frac{x^2}{2} - \frac{x^2}{2} + x \Big|_{1}^{2}$

$A = \frac{7}{3} \Rightarrow 6A = 14$

12. Let $L_1 : \frac{x - 1}{1} = \frac{y - 2}{-1} = \frac{z - 1}{2}$ and $L_2 : \frac{x + 1}{-1} = \frac{y - 2}{2} = \frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ equals

(1) 18 (2) 25
(3) 16 (4) 20

Answer (2)

Sol. $\frac{x - \alpha}{a} = \frac{y - \beta}{b} = \frac{z - \gamma}{c} : L_3$

$\left(a\hat{i} + b\hat{j} + c\hat{k}\right) \| \left(\hat{i} - \hat{j} + 2\hat{k}\right) \times \left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

$(a, b, c) \equiv (5, 3, -1)$

$\Rightarrow \frac{x - \alpha}{5} = \frac{y - \beta}{3} = \frac{z - \gamma}{-1} = \lambda$

Point of intersection

$\alpha = p + 1 - 5\lambda, \beta = -p + 2 - 3\lambda$

$\gamma = 2p + 1 + \lambda$

$\Rightarrow |5\alpha - 11\beta - 8\gamma| = 25$

13. Let $x_1, x_2, \ldots, x_{10}$ be ten observations such that $\sum_{i=1}^{10}(x_i-2)=30$, $\sum_{i=1}^{10}(x_i-\beta)^2=98, \beta>2$, and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1-1)+4\beta, 2(x_2-1)+4\beta, \ldots, 2(x_{10}-1)+4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to

(1) 110 (2) 90

(3) 100 (4) 120

Answer (3)

Sol. $\sum_{i=1}^{10}(x_i-2)=30$

$\sum_{i=1}^{10}x_i=50$

$\Rightarrow$ Mean = 5

Variance = $\frac{4}{5}=\frac{\sum x_i^2}{10}-\left(\bar{x}\right)^2$

$\frac{4}{5}=\frac{\sum x_i^2}{10}-25$

$\Rightarrow \sum x_i^2=258$

Now, $\sum_{i=1}^{10}(x_i-\beta)^2=98$

$\sum_{i=1}^{10}x_i^2-2\beta\sum_{i=1}^{10}x_i+10\beta^2=98$

$\Rightarrow$ $258 - 2\beta(50) + 10\beta^2 = 98$

$\Rightarrow$ $10\beta^2 - 100\beta + 160 = 0$

$\Rightarrow$ $\beta^2 - 10\beta + 16 = 0$

$\Rightarrow$ $\beta = 8$ as $\beta > 2$

Now, as per the question

$2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, \ldots 2(x_{10} - 1) + 4\beta$

Can be simplified as

$2x_1 + 30, 2x_2 + 30, \ldots, 2x_{10} + 30$

$\mu = 2(5) + 30 = 40$

$\sigma^2 = 2^2\left(\frac{4}{5}\right)=\frac{16}{5}$

$\frac{\beta\mu}{\sigma^2}=\frac{8\times40}{\frac{16}{5}}=100$

14. The least value of n for which the number of integral terms in the Binomial expansion of $\left(\sqrt[3]{7}+\sqrt[12]{11}\right)^n$ is 183, is

(1) 2184 (2) 2148

(3) 2196 (4) 2172

Answer (1)

Sol. $\left(\sqrt[3]{7}+\sqrt[12]{11}\right)^n$

$T_{K+1}={}^nC_K 7^{\frac{n-k}{3}}11^{\frac{k}{12}}$

12 divides k and 3 divides $n - k$

For integer terms

$\Rightarrow$ Multiples of 12 for k would work

$\Rightarrow$ $k = 0, 12, 24$

$\Rightarrow$ $k_{max} = 12 \times 182 = 2184$

$\Rightarrow$ Minimum value of n will be 2184

$\Rightarrow$ Option (1) is correct

15. The number of solutions of the equation $\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0$ is

(1) 4 (2) 3

(3) 1 (4) 2

Answer (1)

Sol. $\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0$

Let $\frac{1}{\sqrt{x}}=t$

$(9t^2 - 9t + 2)(2t^2 - 7t + 3) = 0$

$9t^2 - 9t + 2 = 0$

$\Rightarrow t = \frac{9 \pm \sqrt{81-72}}{18}$

$\Rightarrow t = \frac{9 \pm 3}{18}$

$\Rightarrow t = \frac{2}{3}, \frac{1}{3}$

$\Rightarrow 2t^2 - 7t + 3 = 0$

$\Rightarrow 2t^2 - 6t - t + 3 = 0$

$\Rightarrow 2t(t-3) - 1(t-3) = 0$

$t = \frac{1}{2}, 3$

$\Rightarrow \sqrt{x} = \frac{3}{2}, 3, 2, \frac{1}{3}$

$\Rightarrow x = \frac{9}{4}, 9, 4, \frac{1}{9}$

Hence, number of solutions = 4

16. Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is:

(1) 84 (2) 122

(3) 90 (4) 108

Answer (3)

Sol. Let A.P. be a, $a + d$, $a + 2d$

$3a + 3d = 54$

$a + d = 18$ ... (1)

$1600 < \frac{20}{2}[2a + 19d] < 1800$

$160 < 2a + 19d < 180$

$160 < 18 \times 2 + 17d < 180$

$\frac{124}{7} < d < \frac{144}{17}$

$\because d \in$ integer $\Rightarrow d = 8$

$a + d = 18$

$\Rightarrow a = 10$

$a_{11} = a + 10d$

$= 10 + 10 \times 8 = 90$

17. Let the ellipse $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, $A < B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral $ABCD$ equals

(1) $6\sqrt{6}$ (2) $\frac{24\sqrt{6}}{5}$

(3) $\frac{12\sqrt{6}}{5}$ (4) $\frac{18\sqrt{6}}{5}$

Answer (2)

Sol. $2ae = 4$

$\Rightarrow a = 2\sqrt{3}$

$\Rightarrow 1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8$

$\frac{2b^2}{a} \times \frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow \frac{2 \times 8}{2\sqrt{3}} \times \frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow \frac{A^2}{B} = 2 \Rightarrow A^2 = 2B$

$1 - \frac{A^2}{B} = \frac{1}{3}$

$\Rightarrow B = 3 \Rightarrow A^2 = 6$

$E_1 : \frac{x^2}{12} + \frac{y^2}{8} = 1$ ...(i)

$E_1 : \frac{x^2}{6} + \frac{y^2}{9} = 1$ ...(ii)

On solving (i) & (ii)

$(x, y) = \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right)$

Four points are vertices of rectangle area = $\frac{24\sqrt{6}}{5}$

18. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$. Let $L_1 : \vec{r} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\vec{a}, \lambda \in \mathbf{R}$ and $L_2 : \vec{r} = \left(\hat{j} + \hat{k}\right) + \mu\vec{b}, \mu \in \mathbf{R}$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2 and is parallel to $\vec{a} + \vec{b}$, then L_3 passes through the point

(1) (8, 26, 12) (2) (2, 8, 5)

(3) (5, 17, 4) (4) (–1, –1, 1)

Answer (1)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$

$L_1 : \vec{r} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\vec{a}, \lambda \in R$

and $L_2 = \left(\hat{j} + \hat{k}\right) + \mu\vec{b}, \mu \in R$

$\vec{a} + \vec{b} = 3\hat{i} + 9\hat{j} + 4\hat{k}$

$L_1 : \vec{r} = (\lambda - 1)\hat{i} + 2(\lambda + 1)\hat{j} + (\lambda + 1)\hat{k}$

$L_2 : \vec{r} = 2\mu\hat{i} + (7\mu + 1)\hat{j} + (1 + 3\mu)\hat{k}$

For point of intersection of L_1 & L_2

$\lambda - 1 = 2\mu$ and $2(\lambda + 1) = 7\mu + 1$

$\Rightarrow \lambda = 3$ and $\mu = 1$

$L_3 : \vec{r} = 2\hat{i} + 8\hat{i} + 4\hat{k} + \alpha(3\hat{i} + 9\hat{j} + 4\hat{k})$

For $\alpha = 2$

$\vec{r} = 8\hat{i} + 26\hat{j} + 12\hat{k}$

19. Two parabolas have the same focus (4, 3) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to :

(1) 392 (2) 192

(3) 384 (4) 96

Answer (2)

Sol. The parabolas are

$(x - 4)^2 + (y - 3)^2 = x^2$ …(i)

and $(x - 4)^2 + (y - 3)^2 = y^2$ …(ii)

If point of intersection are $A(x_1, y_1)$ and $B(x_2, y_2)$

By solving (i) and (ii), we get

$x_1 + x_2 = 14$ and $x_1 x_2 = 25$

$(AB)^2 = 2((x_1 + x_2)^2 - 4 x_1 x_2) = 192$

20. Let the line $x + y = 1$ meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the mid-point of the chord AB intersects the circle at C and D, then the area of the quadrilateral $ADBC$ is equal to:

(1) $2\sqrt{14}$

(2) $3\sqrt{7}$

(3) $5\sqrt{7}$

(4) $\sqrt{14}$

Answer (1)

Sol. Solving $x = y$ & $x^2 + y^2 = 4$ gives

$C\left(\sqrt{2}, \sqrt{2}\right)$ and $D\left(-\sqrt{2}, -\sqrt{2}\right)$

Solving $x + y = 1$ & $x^2 + y^2 = 4$ gives

$$A\left(\frac{1+\sqrt{7}}{2}, \frac{1-\sqrt{7}}{2}\right) \text{ \& } B\left(\frac{1-\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right)$$

$$\text{Required area} = 2 \times \frac{1}{2}\begin{vmatrix} \sqrt{2} & \sqrt{2} & 1 \\ \frac{1-\sqrt{7}}{2} & \frac{1+\sqrt{7}}{2} & 1 \\ -\sqrt{2} & -\sqrt{2} & 1 \end{vmatrix}$$

$= 2\sqrt{14}$ sq. units

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $[t]$ be the greatest integer less than or equal to t. Then the least value of $p \in N$ for which

$$\lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots\left[\frac{9^2}{x^2}\right]\right)\right)\geq 1$$

is equal to _____.

Answer (24)

Sol. $\lim\limits_{x\to 0^+}\left(x\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots+\left[\frac{9^2}{x^2}\right]\right)\geq 1$

$\Rightarrow (1 + 2 + 3 +\ldots + p) - (1^2 + 2^2 + \ldots + 9^2) \geq 1$

$\Rightarrow \frac{p(p+1)}{2}-\frac{9(10)(19)}{6}\geq 1$

$\Rightarrow p(p + 1) \geq 572$

$\Rightarrow$ Least natural values of p is 24

22. Let $f:(0,\infty)\to \mathbf{R}$ be a twice differentiable function.

If for some $a \neq 0, \int_0^1 f(\lambda x)d\lambda = af(x), f(1) = 1$ and

$f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to____.

Answer (112)

Sol. Given, $\int_0^1 f(\lambda x)d\lambda = af(x)$ ...(1)

Let $\lambda x = u$

$d\lambda = \frac{1}{x}du$

$\therefore$ From (1) $\frac{1}{x}\int_0^x f(u)du = af(x)$

$\Rightarrow \int_0^x f(u)du = axf(x)$

Differentiate both sides

$f(x) = a(xf'(x) + f(x))$

$\Rightarrow f(x) = axf'(x) + af(x)$

$\Rightarrow (1-a)f(x) = axf'(x)$

$\Rightarrow \frac{f'(x)}{f(x)}=\frac{(1-a)}{a}\cdot\frac{1}{x}$

Integrate both side w.r.t. (x)

$\Rightarrow \int\frac{f'(x)}{f(x)}dx=\frac{(1-a)}{a}\int\frac{1}{x}dx$

$\Rightarrow \ln f(x)=\left(\frac{1-a}{a}\right)\ln x + c$

Now at $x = 1$ $f(1) = 1$

$\Rightarrow c = 0$

Also given $f(16)=\frac{1}{8}$

$\therefore \frac{1}{8}=(16)^{\frac{1-a}{a}}$

$\Rightarrow 2^{-3}=2^{\frac{4-4a}{a}}$

$\Rightarrow -3=\frac{4-4a}{a}$

$\Rightarrow -3a = 4 - 4a$

$\Rightarrow a = 4$

$\therefore f(x) = x^{-3/4}$

$f(x)=\frac{-3}{4}x^{-\frac{7}{4}}$

Put $x=\frac{1}{16}$

$f'\left(\frac{1}{16}\right)=\frac{-3}{4}\left(\frac{1}{16}\right)^{-7/4}=\frac{-3}{4}\cdot 2^{-4\times\left(\frac{-7}{4}\right)}=-96$

$\therefore 16-f'\left(\frac{1}{16}\right)\Rightarrow 16-(-96)=112$

23. Let $S = \{x : \cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x+1)\}$.

Then $\sum_{x \in S} (2x-1)^2$ is equal to_____.

Answer (5)

Sol. $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x+1)$.

$$\frac{\pi}{2} - \sin^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x+1)$$

$$-\frac{\pi}{2} - 2\sin^{-1} x = \sin^{-1}(2x+1)$$

$$\sin\left(-\frac{\pi}{2} - 2\sin^{-1} x\right) = \sin\left(\sin^{-1}(2x+1)\right)$$

$-\cos(2\sin^{-1}x) = (2x + 1)$

$-(1 - 2x^2) = 2x + 1$

$-1 + 2x^2 = 2x + 1$

$2x^2 - 2x - 2 = 0$

$$x = \frac{2 \pm \sqrt{4+16}}{4}$$

$$x = \frac{2 \pm 2\sqrt{5}}{4} = \frac{1 \pm \sqrt{5}}{2} \left\{x = \frac{1+\sqrt{5}}{2} \text{ rejected}\right\}$$

So, $\sum_{x \in s} (2x-1)^2 = 5$

24. The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice is

Answer (1405)

Sol. $${}^5C_3 \times \frac{6!}{2!2!2!} + {}^5C_2\left(\frac{6!}{2!4!} \times 2 + \frac{6!}{3!3!}\right) + {}^5C_1 \cdot 1$$

$= 10 \times 90 + 10(15 \times 2 + 20) + 5$

$= 900 + 500 + 5$

$= 1405$

25. Let $S = \left\{m \in \mathbf{Z} : A^{m^2} + A^m = 3I - A^{-6}\right\}$, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$. Then $n(S)$ is equal to_____.

Answer (2)

Sol. $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

Now finding characteristic equation

$$\begin{vmatrix} 2-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (2 - \lambda)(-\lambda) - (-1)(1) = -2\lambda + \lambda^2 + 1 = 0$

$\Rightarrow \lambda^2 - 2\lambda + 1 = 0$

$\Rightarrow (\lambda - 1)^2 = 0$

$\Rightarrow \lambda = 1$

Since A satisfies $(A - I)^2 = 0$

$\therefore$ $A = I + N$ where

$N = A - I$

$$N = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$N^2 = 0$

$A^m = (I + N)^m = I + mN$

$A^m \cdot A^m = (I + mN)(I + mN) = I + 2mN + m^2N^2$

Since $N^2 = 0$

$\Rightarrow A^{m^2} = I + 2mN$

Now putting in given condition

$I + m^2N + I + mN = 3I - A^{-6}$ ...(i)

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$A^{-6} = (A^{-1})^6 = I + (-6)N$

$\therefore$ Putting in (i)

$(m^2 + m)N = I - (I - 6N)$

$(m^2 + m)N = 6N$

Since $N \neq 0$

$\Rightarrow m^2 + m = 6$

$\Rightarrow m^2 + m - 6 = 0$

$\Rightarrow (m - 2)(m + 3) = 0$

$\Rightarrow m = 2, -3$

$\therefore$ Number of elements in S is 2

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. The workdone in an adiabatic change in an ideal gas depends upon only :
    (1) Change in its volume
    (2) Change in its temperature
    (3) Change in its pressure
    (4) Change in its specific heat

Answer (2)

Sol. Work done in adiabatic process = $\frac{nR\Delta T}{1-\gamma}$

So, depends upon change in temperature.

27. Two projectiles are fired with same initial speed from same point on ground at angles of $(45° - \alpha)$ and $(45° + \alpha)$, respectively, with the horizontal direction. The ratio of their maximum heights attained is :
    (1) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha}$ (2) $\frac{1+\sin\alpha}{1-\sin\alpha}$
    (3) $\frac{1-\tan\alpha}{1+\tan\alpha}$ (4) $\frac{1-\sin 2\alpha}{1+\sin 2\alpha}$

Answer (4)

Sol. $H_{max} = \frac{u^2\sin^2\theta}{2g}$

$$\frac{H_1}{H_2} = \frac{\sin^2(45-\alpha)}{\sin^2(45+\alpha)}$$

$$= \frac{(\cos\alpha-\sin\alpha)^2}{(\cos\alpha+\sin\alpha)^2}$$

$$\frac{H_1}{H_2} = \frac{1-\sin 2\alpha}{1+\sin 2\alpha}$$

28. Consider a long straight wire of a circular cross-section (radius *a*) carrying a steady current *I*. The current is uniformly distributed across this cross-section. The distances from the centre of the wire's cross-section at which the magnetic field [inside the wire, outside the wire] is half of the maximum possible magnetic field, anywhere due to the wire, will be
    (1) [a/2, 2a] (2) [a/2, 3a]
    (3) [a/4, 2a] (4) [a/4, 3a/2]

Answer (1)

Sol. Magnetic field inside cylinder at distance *r* from axis

$$= \frac{\mu_0 I}{\pi R^2}\cdot\frac{r}{2}$$

$$B_{max} = \frac{\mu_0 I}{2\pi R}$$

$$\frac{\mu_0 I}{2\pi R^2} r_1 = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right)$$

$$r_1 = \frac{R}{2} = \frac{a}{2}$$

Magnetic field outside cylinder at distance *r* from axis

$$= \frac{\mu_0 I}{2\pi r}$$

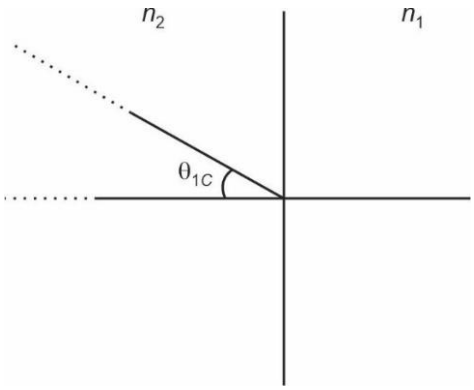
$$\frac{\mu_0 I}{2\pi r_2} = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right)$$

$r_2 = 2R = 2a$

29. At the interface between two materials having refractive indices n_1 and n_2, the critical angle for reflection of an em wave is θ_{1C}. The n_2 material is replaced by another material having refractive index n_3 such that the critical angle at the interface between n_1 and n_3 materials is θ_{2C}. If $n_3 > n_2 > n_1$; $\frac{n_2}{n_3} = \frac{2}{5}$ and $\sin\theta_{2C} - \sin\theta_{1C} = \frac{1}{2}$, then θ_{1C} is
    (1) $\sin^{-1}\left(\frac{1}{3n_1}\right)$ (2) $\sin^{-1}\left(\frac{1}{6n_1}\right)$
    (3) $\sin^{-1}\left(\frac{5}{6n_1}\right)$ (4) $\sin^{-1}\left(\frac{2}{3n_1}\right)$

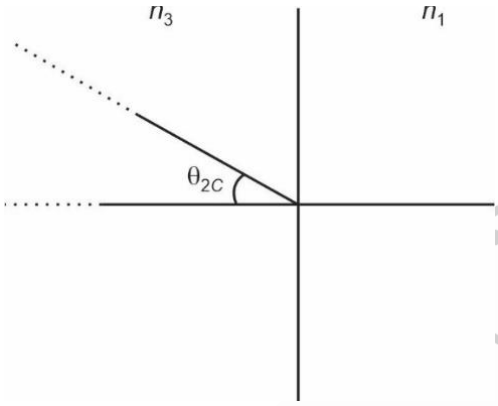
Answer (*)

Sol.



$$n_2 \sin(\theta_{1C}) = n_1$$

$$\sin(\theta_{1C}) = \frac{n_1}{n_2}$$



$$n_3 \sin(\theta_{2C}) = n_1$$

$$\Rightarrow \quad \sin(\theta_{2C}) = \frac{n_1}{n_3}$$

Also, $n_3 = \frac{5n_2}{2} \Rightarrow \sin(\theta_{2C}) = \frac{2n_1}{5n_2}$

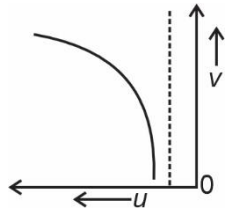
$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{5}{2}.\sin(\theta_{2C}) \Rightarrow \sin(\theta_{2C}) - \sin(\theta_{1c}) = \frac{1}{2}$$

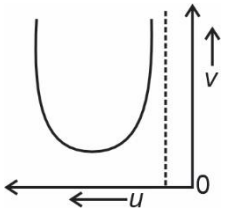
Given, $\frac{2n_1}{5n_2} - \frac{n_1}{n_2} = \frac{1}{2} \Rightarrow \frac{n_1}{n_2}\left(\frac{-3}{5}\right) = \frac{1}{2}$

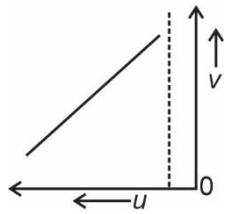
Coming out to be (–ve)

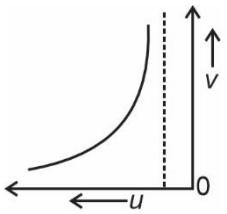
*** None of the answer is matching**

30. Let u and v be the distances of the object and the image from a lens of focal length f. The correct graphical representation of u and v for a convex lens when $|u| > f$, is

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

Since $|\mu| > f$

So, RHS is positive.

31. The pair of physical quantities not having same dimensions is :

(1) Angular momentum and Planck's constant

(2) Torque and energy

(3) Surface tension and impulse

(4) Pressure and Young's modulus

Answer (3)

Sol. [Angular momentum] = ML^2T^{-1}

[Planck's Constant] = ML^2T^{-1}

[Torque] = ML^2T^{-2}

[Energy] = ML^2T^{-2}

[Surface tension] = MT^{-2}

[Impulse]= MLT^{-1}

[Pressure] = $ML^{-1}T^{-2}$

[Young's modulus] = $ML^{-1}T^{-2}$

32. The expression given below shows the variation of velocity (v) with lime (t), $v = At^2 + \frac{Bt}{C+t}$. The dimension of ABC is :

(1) $[M^0L^1T^{-2}]$ (2) $[M^0L^1T^{-3}]$

(3) $[M^0L^2T^{-3}]$ (4) $[M^0L^2T^{-2}]$

Answer (3)

Sol. $v = At^2 + \frac{Bt}{C+t}$

$[v] = [A]\, t^2 = \left[\frac{Bt}{C+t}\right]$

$[A] = LT^{-3}$

$[C] = T$

$[B] = LT^{-1}$

$[ABC] = L^2T^{-3}$

33. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Choke coil is simply a coil having a large inductance but a small resistance. Choke coils are used with fluorescent mercury-tube fittings. If household electric power is directly connected to a mercury tube, the tube will be damaged.

Reason (R) : By using the choke coil, the voltage across the tube is reduced by a factor $\left(R/\sqrt{R^2+\omega^2L^2}\right)$, where ω is frequency of the supply across resistor R and inductor L. If the choke coil were not used, the voltage across the resistor would be the same as the applied voltage.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

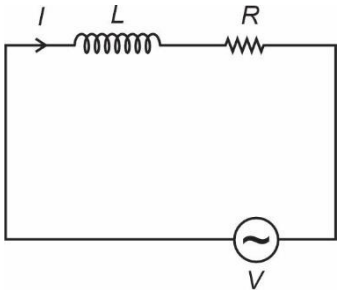
(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of (A)

(3) **(A)** is false but **(R)** is true

(4) **(A)** is true but **(R)** is false

Answer (1)

Sol.

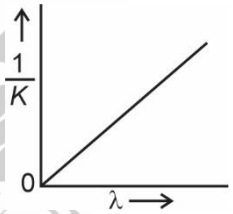


$$I = \frac{V}{\sqrt{R^2+\omega^2L^2}}$$

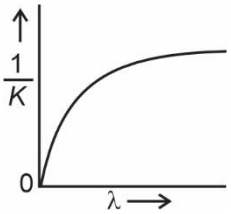
$$V_R = \frac{R}{\sqrt{R^2+\omega^2L^2}}V$$

34. If λ and K are de Broglie wavelength and kinetic energy, respectively, of a particle with constant mass. The correct graphical representation for the particle will be

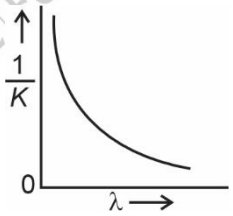
(1)



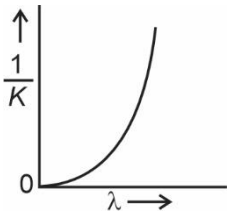
(2)



(3)



(4)



Answer (4)

Sol. $\lambda = \frac{h}{\sqrt{2mK}}$

$$\lambda^2 = \frac{h^2}{2mK}$$

$$\frac{1}{K} = \left(\frac{2m}{h^2}\right)\lambda^2$$

35. Consider I_1 and I_2 are the currents flowing simultaneously in two nearby coils 1 & 2, respectively. If L_1 = sell inductance of coil 1, M_{12} = mutual inductance of coil 1 with respect to coil 2, then the value of induced emf in coil 1 will be

(1) $\varepsilon_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$

(2) $\varepsilon_1 = -L_1\frac{dI_2}{dt} - M_{12}\frac{dI_1}{dt}$

(3) $\varepsilon_1 = -L_1\frac{dI_1}{dt} + M_{12}\frac{dI_2}{dt}$

(4) $\varepsilon_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_1}{dt}$

Answer (1)

Sol. Magnitude of induced emf due to self inductance

$= \frac{LdI_1}{dt}$

Magnitude of induced emf due to mutual inductance

$= \frac{MdI_2}{dt}$

36. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Electromagnetic waves carry energy but not momentum.

Reason (R) : Mass of a photon is zero.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Answer (2)

Sol. EM wave carry both energy and momentum. Rest mass of photon is zero.

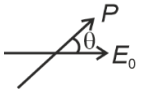
37 An electric dipole of mass m, charge q, and length l is placed in a uniform electric field $\vec{E} = E_0\hat{i}$. When the dipole is rotated slightly from its equilibrium position and released, the time period of its oscillations will be :

(1) $2\pi\sqrt{\frac{ml}{2qE_0}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{ml}{2qE_0}}$

(3) $2\pi\sqrt{\frac{ml}{qE_0}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{2ml}{qE_0}}$

Answer (1)

Sol. $\longrightarrow E_0$



$\tau = PE_0\sin\theta$

If θ is small

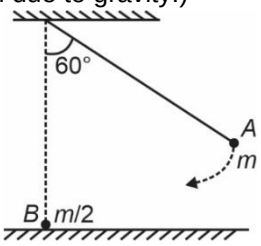
$\tau = -(PE_0)\theta$

$I = m\left(\frac{l}{2}\right)^2\cdot 2 = \frac{ml^2}{2}$

$T = 2\pi\sqrt{\frac{ml^2}{2\cdot PE_0}} = 2\pi\sqrt{\frac{ml^2}{2\cdot qlE_0}}$

$T = 2\pi\sqrt{\frac{ml}{2qE_0}}$

38. As shown below, bob A of a pendulum having massless string of length 'R' is released from 60° to the vertical. It hits another bob B of half the mass that is at rest on a frictionless table in the center. Assuming elastic collision, the magnitude of the velocity of bob A after the collision will be (take g as acceleration due to gravity.)



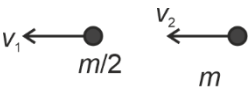
(1) $\frac{1}{3}\sqrt{Rg}$ (2) $\sqrt{Rg}$

(3) $\frac{4}{3}\sqrt{Rg}$ (4) $\frac{2}{3}\sqrt{Rg}$

Answer (1)

Sol. Velocity of A before collision $= \sqrt{2gh}$

$$= \sqrt{2g \times \frac{R}{2}} = \sqrt{Rg}$$

After collision 

COM

$$mu = \frac{m}{2}v_1 + mv_2$$

$2u = v_1 + 2v_2$ …(i)

$e = 1,\ u = v_1 - v_2$ …(ii)

$u = 3v_2$

$$v_2 = \frac{u}{3} = \frac{1}{3}\sqrt{Rg}$$

39. Match **List-I** with **List-II**.

	List-I		List-II
(A)	Electric field inside (distance $r > 0$ from center) of a uniformly charged spherical shell with surface charge density σ, and radius R.	(I)	σ / ε_0
(B)	Electric field at distance $r > 0$ from a uniformly charged infinite plane sheet with surface charge density σ.	(II)	$\sigma / 2\varepsilon_0$
(C)	Electric field outside (distance $r > 0$ from center) of a uniformly charged spherical shell with surface charge density σ, and radius R.	(III)	0
(D)	Electric field between 2 oppositely charged infinite plane parallel sheets with uniform surface charge density σ.	(IV)	$\frac{\sigma}{\varepsilon_0 r^2}$

Choose the **correct** answer from the options given below:

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Answer (*)

Sol. Inside uniformly charged spherical shell, $E = 0$

$\therefore$ A $\rightarrow$ III

For uniformly charged infinite plate

$$E = \frac{\sigma}{2\varepsilon_0}$$

B $\rightarrow$ II

Outside of spherical shell

$$E = \frac{Q}{4\pi\varepsilon_0 r_2} = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

None of the option is matching for C.

Between two plates $E = \frac{\sigma}{\varepsilon_0}$

D $\rightarrow$ I

None of the option is correct

40. The fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the depth of 2.5 km below the sea level is _____ %. Given, the Bulk modulus of water = 2×10^9 Nm^{-2}, density of water = 10^3 kg m^{-3}, acceleration due to gravity = $g = 10$ ms^{-2}.

(1) 1.25

(2) 1.5

(3) 1.0

(4) 1.75

Answer (1)

Sol. $B = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)}$

$$-\left(\frac{\Delta V}{V}\right) = \frac{\Delta P}{B} = \frac{\rho gh}{B}$$

$$= \frac{10^3 \times 10 \times 2.5 \times 10^3}{2 \times 10^9} = 1.25\%$$

41. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Emission of electrons in photoelectric effect can be suppressed by applying a sufficiently negative electron potential to the photoemissive substance.

Reason (R) : A negative electric potential, which stops the emission of electrons from the surface of a photoemissive substance, varies linearly with frequency of incident radiation.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **(A)** is true but **(R)** is false

(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

Answer (4)

Sol. Negative potential will slow the electrons and if it is sufficient, it will make the photocurrent zero.

$eVs = hf - \phi_0$

42. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Time period of a simple pendulum is longer at the top of a mountain than that at the base of the mountain.

Reason (R) : Time period of a simple pendulum decreases with increasing value of acceleration due to gravity and vice-versa.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **(A)** is false but **(R)** is true

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Answer (4)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

At top of mountain $g \downarrow$, $\therefore T \uparrow$

43. A coil of area A and N turns is rotating with angular velocity ω in a uniform magnetic field $\vec{B}$ about an axis perpendicular to $\vec{B}$. Magnetic flux φ and induced emf ε across it, at an instant when $\vec{B}$ is parallel to the plane of coil, are :

(1) $\varphi = 0, \varepsilon = 0$

(2) $\varphi = AB, \varepsilon = 0$

(3) $\varphi = AB, \varepsilon = NAB\omega$

(4) $\varphi = 0, \varepsilon = NAB\omega$

Answer (3)

Sol. $\phi = NBA\cos\theta$

$$\varepsilon = -\frac{d\phi}{dt} = -NBA\frac{d\cos\theta}{dt}$$

$\theta = \omega t$

$\varepsilon = -NBA\omega\sin\omega t$

if B is parallel to plane of coil

$\theta = 90°$

$\phi = 0, E = BA\omega N$

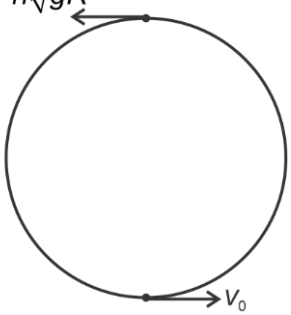
44. A body of mass 'm' connected to a massless and unstretchable string goes in verticle circle of radius 'R' under gravity g. The other end of the string is fixed at the center of circle. If velocity at top of circular path is $n\sqrt{gR}$, where, $n \geq 1$, then ratio of kinetic energy of the body at bottom to that at top of the circle is

(1) $\frac{n^2+4}{n^2}$ (2) $\frac{n}{n+4}$

(3) $\frac{n+4}{n}$ (4) $\frac{n^2}{n^2+4}$

Answer (1)

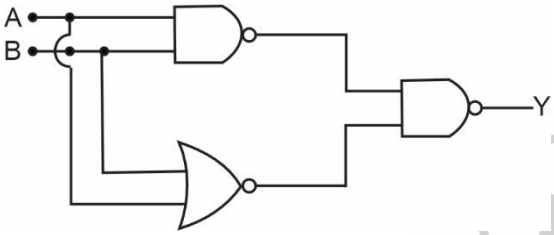
Sol. $v = n\sqrt{gR}$



$$v_0 = \sqrt{v^2 + 2g(2R)}$$

$$v_0 = \sqrt{n^2 gR + 4gR}$$

$$\therefore \quad \frac{k_{bottom}}{k_{top}} = \frac{v_0^2}{v^2} = \frac{n^2+4}{n^2}$$

45. 

For the circuit shown above, equivalent GATE is :

(1) AND gate

(2) OR gate

(3) NOT gate

(4) NAND gate

Answer (2)

Sol. $Y = \overline{AB}\left(\overline{A+B}\right)$

$= AB + A + B$

$= A(B+1) + B$

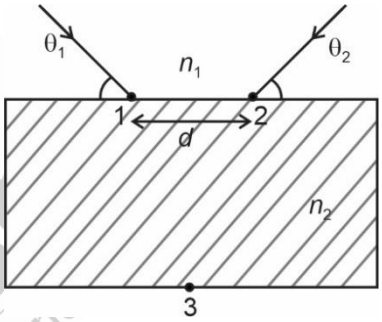
$= A + B$

or GATE

SECTION - B

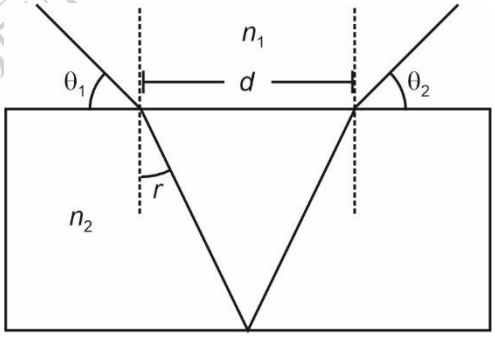
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. Two light beams fall on a transparent material block at point 1 and 2 with angle θ_1 and θ_2, respectively, as shown in figure. After refraction, the beams intersect at point 3 which is exactly on the interface at other end of the block. Given : the distance between 1 and 2, $d = 4\sqrt{3}$ cm and $\theta_1 = \theta_2 = \cos^{-1}\left(\frac{n_2}{2n_1}\right)$, where refractive index of the block n_2 > refractive index of the outside medium n_1, then the thickness of the block is _____ cm.



Answer (6)

Sol.



$$n_1 \sin(90 - \theta_1) = n_2 \sin r$$

$$n_1 \times \frac{n_2}{2n_1} = n_2 \sin r$$

$$\sin r = \frac{1}{2}$$

$r = 30°$

$$\tan r = \left(\frac{d/2}{t}\right)$$

$$t = \frac{d}{2\tan r} = \frac{d\sqrt{3}}{2} = \frac{\left(4\sqrt{3}\right)\sqrt{3}}{2}$$

= 6 cm

47. A container of fixed volume contains a gas at 27° C. To double the pressure of the gas, the temperature of gas should be raised to ____ °C.

Answer (327)

Sol. V = constant

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$P_2 = 2P_1$

$T_2 = 2T_1$

= 2 × 300 = 600 K

$\therefore$ = 327°C

48. The maximum speed of a boat in still water is 27 km/h. Now this boat is moving downstream in a river flowing at 9 km/h. A man in the boat throws a ball vertically upwards with speed of 10 m/ s. Range of the ball as observed by an observer at rest on the river bank, is _____ cm. (Take g = 10 m/s^2)

Answer (2000)

Sol. $v_y = 10$ m/s

$v_x = v_{river} + v_{boat} = 27 + 9$

= 36 km/h = 10 m/s

$$R = \left(\frac{2v_y}{g}\right)v_x = \frac{2\times10\times10}{10} = 20 \text{ m}$$

= 2000 cm

49. In a hydraulic lift, the surface area of the input piston is 6 cm^2 and that of the output piston is 1500 cm^2. If 100 N force is applied to the input piston to raise the output piston by 20 cm, then the work done is _____ kJ.

Answer (5)

Sol. According to Pascal's law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \left(\frac{A_2}{A_1}\right)F_1 = \frac{1500}{6}\times100\text{N}$$

$$W = F_2 d_2 = \frac{1500}{6}\times100\times\frac{20}{100}$$

= 5 kJ

50. The coordinates of a particle with respect to origin in a given reference frame is (1, 1, 1) meters. If a force of $\vec{F} = \hat{i} - \hat{j} + \hat{k}$ acts on the particle, then the magnitude of torque (with respect to origin) in z-direction is _____.

Answer (2)

Sol. $\tau = \vec{r} \times \vec{F}$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

Along $\hat{k}$

$\tau_2 = -2\hat{k}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. For a $Mg|Mg^{2+}(aq) || Ag^{+}(aq) | Ag$ the correct Nernst equation is :

(1) $E_{cell} = E^{\circ}_{cell} + \frac{RT}{2F}\ln\frac{[Ag^{+}]^2}{[Mg^{2+}]}$

(2) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F}\ln\frac{[Ag^{+}]^2}{[Mg^{2+}]}$

(3) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F}\ln\frac{[Ag^{+}]}{[Mg^{2+}]}$

(4) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F}\ln\frac{[Mg^{2+}]}{[Ag^{+}]}$

Answer (1)

Sol.

Cathode	$(Ag^{+}(aq) + e^{-}$	$\rightarrow Ag) \times 2$
Anode	Mg	$\rightarrow Mg^{2+}(aq) + 2e^{-}$
Cell reaction	$2Ag^{+} + Mg$	$\rightarrow 2Ag + Mg^{2+}$

$$Q = \frac{[Mg^{2+}]}{[Ag^{+}]^2}$$

By Nernst equation

$$E_{cell} = E^{o}_{cell} - \frac{RT}{nF}\ln Q$$

$$E_{cell} = E^{o}_{cell} - \frac{RT}{nF}\ln\frac{[Mg^{2+}]}{[Ag^{+}]^2}$$

$$= E^{o}_{cell} + \frac{RT}{2F}\ln\frac{[Ag^{+}]^2}{[Mg^{2+}]}$$

52. 1.24 g of AX_2 (molar mass 124 g mol^{-1}) is dissolved in 1 kg of water to form a solution with boiling point of 100.0156°C, while 25.4 g of AY_2 (molar mass 250 g mol^{-1}) in 2 kg of water constitutes a solution with a boiling point of 100.0260°C.

$K_b(H_2O) = 0.52$ K kg mol^{-1}

Which of the following is **correct** ?

(1) AX_2 and AY_2 (both) are fully ionised

(2) AX_2 is fully ionised while AY_2 is completely unionised

(3) AX_2 and AY_2 (both) are completely unionised

(4) AX_2 is completely unionised while AY_2 is fully ionised

Answer (2)

Sol. For AX_2

$\Delta T_b = iK_bm$

$$0.0156 = i \times 0.52 \times \frac{1.24}{124 \times 1}$$

$3 = i$

$3 = 1 + 2\alpha$

$1 = \alpha$

For AY_2

$\Delta T_b = iK_bm$

$$0.0260 = i \times 0.52 \times \frac{25.4}{250 \times 2}$$

$i \approx 1$

$\therefore$ AX_2 is completely ionised & AY_2 is completely unionised

53. The standard reduction potential values of some of the p-block ions are given below. Predict the one with the strongest oxidising capacity.

(1) $E^{\ominus}_{Al^{3+}/Al} = -1.66$ V

(2) $E^{\ominus}_{Sn^{4+}/Sn^{2+}} = +1.15$ V

(3) $E^{\ominus}_{Tl^{3+}/Tl} = +1.26$ V

(4) $E^{\ominus}_{Pb^{4+}/Pb^{2+}} = +1.67$ V

Answer (4)

Sol. The element having strongest oxidising capacity will have highest value of standard reduction potential

54. The reaction $A_2 + B_2 \rightarrow 2AB$ follows the mechanism

$$A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} A + A \text{ (fast)}$$

$$A + B_2 \xrightarrow{k_2} AB + B \text{ (slow)}$$

$A + B \rightarrow AB$ (fast)

The overall order of the reaction is :

(1) 1.5 (2) 2.5

(3) 3 (4) 2

Answer (1)

Sol. Since, second step is slow step

$r = k_2[A][B_2]$ …(i)

Also, $\frac{k_1}{k_{-1}} = \frac{[A]^2}{[A_2]}$

$$[A] = \left(\frac{[A_2]k_1}{k_{-1}}\right)^{\frac{1}{2}} \quad \text{…(ii)}$$

$$r = k_2\left(\frac{k_1}{k_{-1}}\right)^{\frac{1}{2}}[A_2]^{\frac{1}{2}}[B_2]$$

$$\text{order} = \frac{1}{2} + 1 = \frac{3}{2}$$

55. The molar conductivity of a weak electrolyte when plotted against the square root of its concentration, which of the following is expected to be observed?

(1) A small increase in molar conductivity is observed at infinite dilution

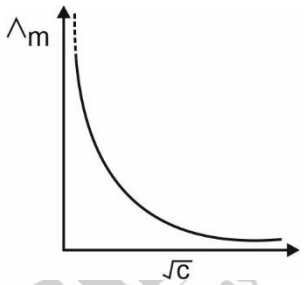
(2) Molar conductivity increases sharply with increase in concentration

(3) A small decrease in molar conductivity is observed at infinite dilution

(4) Molar conductivity decreases sharply with increase in concentration.

Answer (4)

Sol. For weak electrolyte, variation of $\wedge_m$ with $\sqrt{c}$ is



56. At temperature T, compound $AB_2(g)$ dissociates as $AB_2(g) \rightleftharpoons AB(g) + \frac{1}{2}B_2(g)$ having degree of dissociation x (small compared to unity). The correct expression for x in terms of K_p and p is

(1) $\sqrt[3]{\frac{2K_p^2}{p}}$ (2) $\sqrt[3]{\frac{2K_p}{p}}$

(3) $\sqrt[4]{\frac{2K_p}{p}}$ (4) $\sqrt{K_p}$

Answer (1)

Sol.

	$AB_2(g)$	$\rightleftharpoons$	$AB(g)$	+	$\frac{1}{2}B_2(g)$
$t = 0$	p_0				
$t = t_{eq}$	$p_0(1-x)$		p_0x		$\frac{p_0x}{2}$

$$p = p_0x - p_0x + p_0x + \frac{p_0x}{2}$$

$$p = p_0\left(1+\frac{x}{2}\right)$$

$$p_0 = \frac{p}{\left(1+\frac{x}{2}\right)}$$

$$K_p = \frac{(p_{AB})(p_{B_2})^{1/2}}{(p_{AB_2})}$$

$$K_p = \frac{(p_0 x)\left(\frac{p_0 x}{2}\right)^{1/2}}{p_0(1-x)}$$

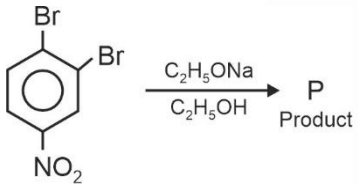
$$K_p = \frac{\frac{px}{\left(1+\frac{x}{2}\right)}\left(\frac{p}{1+\frac{x}{2}}\times\frac{x}{2}\right)^{1/2}}{\frac{p(1-x)}{\left(1+\frac{x}{2}\right)}}$$

Since x << 1

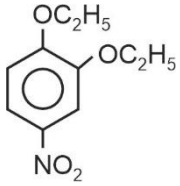
$$K_p = \frac{p^{1/2}x^{3/2}}{2^{1/2}}$$

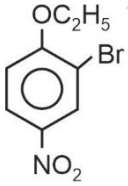
$$x = \sqrt[3]{\frac{2K_p^2}{p}}$$

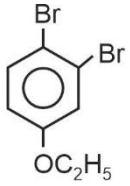
57. In the following substitution reaction :

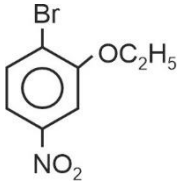


Product 'P' formed is

(1) 

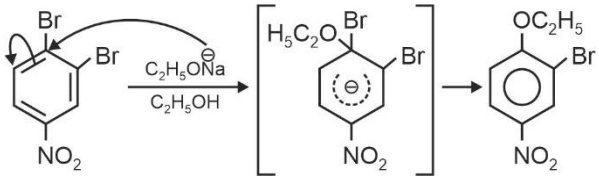
(2) 

(3) 

(4) 

Answer (2)

Sol. Br at the para of NO_2 will undergo aromatic nucleophilic substitution by nucleophile C_2H_5ONa.



58. Total number of nucleophiles from the following is

NH_3, PhSH, $(H_3C)_2S$, $H_2C{=}CH_2$, $\bar{O}H$, H_3O^+, $(CH_3)_2CO$, 

(1) 4 (2) 7

(3) 5 (4) 6

Answer (3)

Sol. Any specie having electrons available for donation can act as nucleophile.

Total 5 are present.

59. Choose the **correct** statements :

(A) Weight of a substance is the amount of matter present in it.

(B) Mass is the force exerted by gravity on an object.

(C) Volume is the amount of space occupied by a substance.

(D) Temperatures below 0°C are possible in Celsius scale, but in Kelvin scale negative temperature is not possible.

(E) Precision refers to the closeness of various measurements for the same quantity.

Choose the **correct** answer from the options given below.

(1) (A), (B) and (C) only (2) (A), (D) and (E) only

(3) (C), (D) and (E) only (4) (B), (C) and (D) only

Answer (3)

Sol. Mass of substance is amount of matter present in it. Weight is force exerted by gravity on object.

60. Match **List-I** with **List-II**.

List-I (Carbohydrate)	**List-II (Linkage Source)**
(A) Amylose	(I) β-C_1-C_4, plant
(B) Cellulose	(II) α-C_1-C_4, animal
(C) Glycogen	(III) α-C_1-C_4, α-C_1-C_6, plant
(D) Amylopectin	(IV) α-C_1-C_4, plant

Choose the **correct** answer from the options given below.

(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Answer (4)

Sol. Amylose $\Rightarrow$ It is a plant based starch it has α-C_1-C_4 glycosidic linkage.

Cellulose $\Rightarrow$ It has β-C_1-C_4 glycosidic linkage

Glycogen $\Rightarrow$ It has α-C_1-C_4 and glycosidic linkage (animal starch)

Amylopectin $\Rightarrow$ It is a plant based with α-C_1-C_4 and C_1-C_6 glycosidic linkage

61. If a_0 is denoted as the Bohr radius of hydrogen atom, then what is the de-Broglie wavelength (λ) of the electron present in the second orbit of hydrogen atom? [n : any integer]

(1) $\frac{2a_0}{n\pi}$ (2) $\frac{4n}{\pi a_0}$

(3) $\frac{4\pi a_0}{n}$ (4) $\frac{8\pi a_0}{n}$

Answer (4)

Sol. $r_n = \frac{a_0 n^2}{z}$

Also, $2\pi r_n = n\lambda$

Where λ is de-Broglie wavelength

$$\frac{2\pi a_0 n^2}{z} = n\lambda$$

For second orbit of H-atom

$$\lambda = \frac{8\pi a_0}{n}$$

62. Match **List - I** with **List - II.**

List - I (Complex)	**List - II (Hybridisation & Magnetic characters)**
(A) $[MnBr_4]^{2-}$	(I) d^2sp^3 & diamagnetic
(B) $[FeF_6]^{3-}$	(II) sp^3d^2 & paramagnetic
(C) $[Co(C_2O_4)_3]^{3-}$	(III) sp^3 & diamagnetic
(D) $[Ni(CO)_4]$	(IV) sp^3 & paramagnetic

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(2) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (1)

Sol. $[MnBr_4]^{2-}$

Mn^{2+}, Br^- is WFL

d^5, so it is sp^3 and paramagnetic.

$[FeF_6]^{3-}$

Fe^{3+}, F^- is WFL

d^5, so it is sp^3d^2 and paramagnetic.

$[Co(C_2O_4)_3]^{3-}$

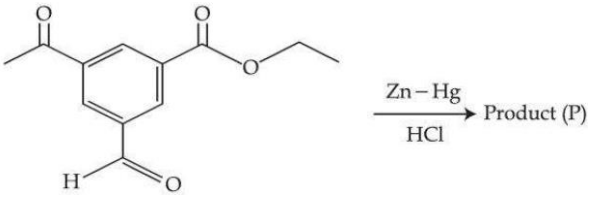
Co^{3+}, $C_2O_4^{2-}$ is a SFL

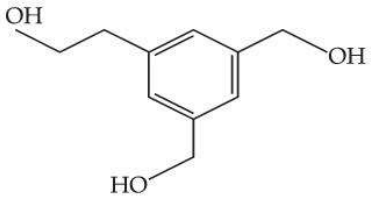
d^6, so it is d^2sp^3 and diamagnetic.

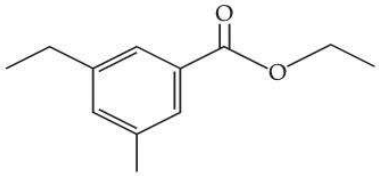
$[Ni(CO)_4]$, CO is a SFL

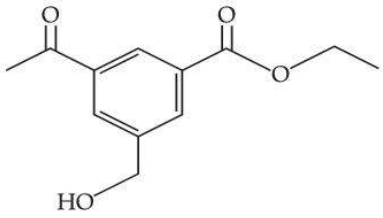
d^{10}, it is sp^3 and diamagnetic.

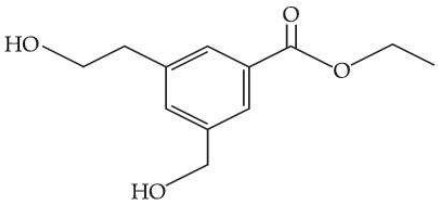
63. The product (P) formed in the following reaction is:



(1) 

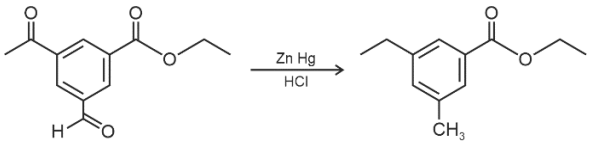
(2) 

(3) 

(4) 

Answer (2)

Sol. It is Clemmensen reduction, it will not reduce ester,



Ester cannot be reduced by clemmensen reduction

64. The correct increasing order of stability of the complexes based on Δ_0 value is:

I. $[Mn(CN)_6]^{3-}$ II. $[Co(CN)_6]^{4-}$

III. $[Fe(CN)_6]^{4-}$ IV. $[Fe(CN)_6]^{3-}$

(1) III < II < IV < I (2) II < III < I < IV

(3) IV < III < II < I (4) I < II < IV < III

Answer (4)

Sol. Neglecting pairing energy

I. $[Mn(CN)_6]^{3-} \Rightarrow Mn^{3+}$, t_{2g}^4, CFSE = $-0.4 \times 4\,\Delta_o = -1.6\Delta_o$

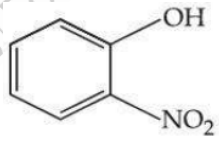
II. $[Co(CN)_6]^{4-} \Rightarrow Co^{2+}$, $t_{2g}^6 e_g^1$, CFSE = $-0.4 \times 6 + 0.6 \times 1 = -1.8\Delta_o$

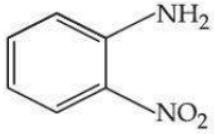
III. $[Fe(CN)_6]^{4-} \Rightarrow Fe^{2+}$, $t_{2g}^6 e_g^0$, CFSE = $-0.4 \times 6 = -2.4\Delta_o$

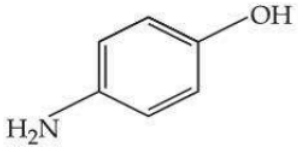
IV. $[Fe(CN)_6]^{3-} \Rightarrow Fe^{3+}$, $t_{2g}^5 e_g^0$, CFSE = $-0.4 \times 5 = -2\Delta_o$

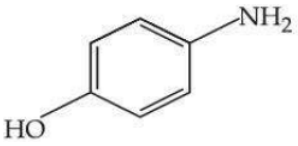
Order of stability III > IV > II > I

65. The steam volatile compounds among the following are :

(A) 

(B) 

(C) 

(D) 

Choose the **correct** answer from the options given below :

(1) (B) and (D) Only

(2) (A) and (C) Only

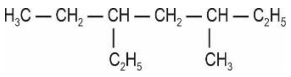
(3) (A), (B) and (C) Only

(4) (A) and (B) Only

Answer (4)

Sol. Ortho nitro phenol and ortho nitro aniline will be steam volatile as they will show intra molecular H-bonding.

66. Match List I with List-II

	List-I (Structure)	**List-II (IUPAC Name)**
(A)		(I) 4-Methylpent-l-ene
(B)	$(CH_3)_2C(C_3H_7)_2$	(II) 3-Ethyl-5-methylheptane
(C)		(III) 4, 4-Dimethylheptane
(D)		(IV) 2-Methyl-1, 3-pentadiene

Choose the correct answer from the options given below

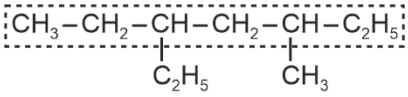
(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(2) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

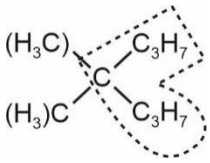
(3) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (2)

Sol. (A) 

3-Ethyl-5-methylheptane

(B) 

4,4-Dimethylheptane

(C) 

2-Methyl-1,3-pentadiene

(D) 

4-Methylpent-1-ene

67. 500 J of energy is transferred as heat to 0.5 mol of Argon gas at 298 K and 1.00 atm. The final temperature and the change in internal energy respectively are: Given: R = 8.3 JK^{-1} mol^{-1}

(1) 378 K and 500 J (2) 368 K and 500 J

(3) 378 K and 300 J (4) 348 K and 300 J

Answer (4)

Sol. At cons. P

$Q = nC_p \Delta T$

$500 = 0.5 \times \frac{5}{2} \times 8.3 (T_f - 298)$

$346.2\ K = T_f$

$\Delta U = nC_v \Delta T$

$= \frac{1}{2} \times \frac{3}{2} \times 8.3 \times (346.2 - 298)$

$= 300\ J$

68. The correct option with order of melting points of the pairs (Mn, Fe), (Tc, Ru) and (Re, Os) is:

(1) Fe < Mn, Ru < Tc and Re < Os

(2) Mn < Fe, Tc < Ru and Os < Re

(3) Fe < Mn, Ru < Tc and Os < Re

(4) Mn < Fe, Tc < Ru and Re < Os

Answer (2)

Sol. Melting point order

Fe > Mn

Ru > Tc

Re > Os

69. An element 'E' has the ionisation enthalpy value of 374 kJ mol^{-1}.'E' reacts with elements A, B, C and D with electron gain enthalpy values of –328, –349, –325 and –295 kJ mol^{-1}, respectively. The correct order of the products EA, EB, EC and ED in terms of ionic character is:

(1) ED > EC > EA > EB (2) EA > EB > EC > ED

(3) EB > EA > EC > ED (4) ED > EC > EB > EA

Answer (3)

Sol. The element having high value of Electron gain enthalpy (magnitude) will form a compound having higher ionic character so order of ionic character

EB > EA > EC > ED

70. Given below are two statements :

Statement (I) : The radii of isoelectronic species increases in the order.

$Mg^{2+} < Na^{+} < F^{-} < O^{2-}$

Statement (II) : The magnitude of electron gain enthalpy of halogen decreases in the order.

Cl > F > Br > I

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are correct

(3) **Statement I** is incorrect but **Statement II** is **correct**

(4) Both **Statement I** and **Statement II** are incorrect

Answer (2)

Sol. $r \propto q^{-}$ (for isoelectronic species)

$$\propto \frac{1}{q^{+}}$$

$\therefore$ Statement I is correct

Magnitude of electron gain enthalpy

Cl > F > Br > I

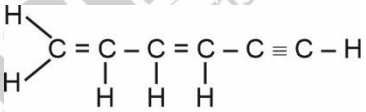
SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. The sum of sigma (σ) and pi(π) bonds in Hex-1, 3-dien-5-yne is _____.

Answer (15)

Sol.



Hex-1, 3-dien-5-yne

No. of π bond = 4

No. of σ bond = 11

72. If A_2B is 30% ionised in an aqueous solution, then the value of van't Hoff factor (i) is _____ × 10^{-1}.

Answer (16)

Sol.

AB_2	$\rightleftharpoons$	A^{2+}	+	$2B^{-}$
1		0		0
$1-\alpha$		α		2α

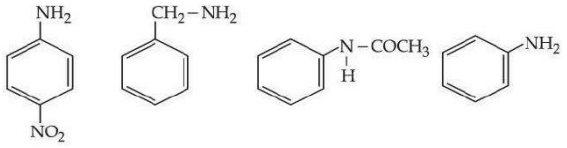
$i = 1 + 2\alpha$

$= 1 + 2 \times (0.3)$

$= 1.6$

$= 16 \times 10^{-1}$

73. Given below are some nitrogen containing compounds

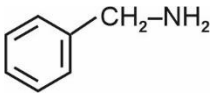


Each of them is treated with HCl separately, 1.0 g of the most basic compound will consume _____ mg of HCl.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16, Cl : 35.5)

Answer (341)

Sol. The most basic compound will be aliphatic amine due to localised electrons



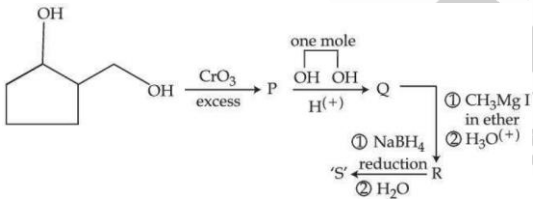
1 mole of this will consume 1 mole HCl

So mass of HCl consumed for 1 g of this compound

$$= \frac{1}{107} \times 36.5$$

= 0.341 gm

= 341 mg

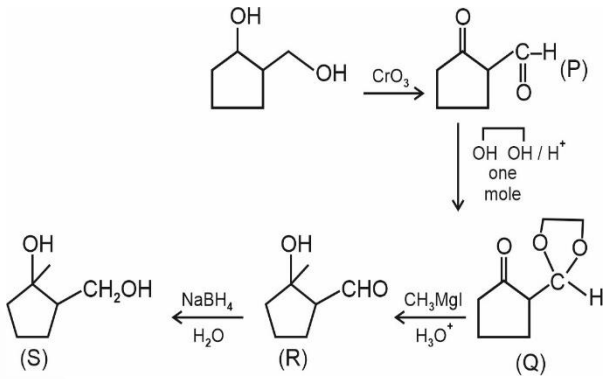
74. 

0.1 mole of compound 'S' will weight _____ g.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16)

Answer (13)

Sol.



mass of 0.1 mole (S) = 0.1(84 + 32 +14)

= 13 g

75. The molar mass of the water insoluble product formed from the fusion of chromite ore ($FeCr_2O_4$), with Na_2CO_3 in presence of O_2 is _____ g mol^{-1}.

Answer (160)

Sol. $FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow Na_2CrO_4 + Fe_2O_3 + CO_2$

Insoluble product will be Fe_2O_3

molar mass = 56 × 2 + 16 × 3

= 112 + 48

= 160

❐ ❐ ❐

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right)\cdot {}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta,\ \alpha,\ \beta \in \mathbb{N}$, then $(\alpha+\beta)^2$ is equal to

(1) 18 (2) 27

(3) 81 (4) 9

Answer (3)

Sol. $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right)\cdot {}^9C_r = \sum_{r=1}^{9}\frac{r}{2^r}\cdot\frac{9}{r}\cdot {}^8C_{r-1} + \sum_{r=1}^{9}3\cdot {}^9C_r\left(\frac{1}{2}\right)^r$

$= \frac{9}{2}\sum_{r=1}^{9} {}^8C_{r-1}\left(\frac{1}{2}\right)^{r-1} + 3\sum_{r=1}^{9} {}^9C_r\left(\frac{1}{2}\right)^r$

$= \frac{9}{2}\sum_{r=0}^{8} {}^8C_{r-1}\left(\frac{1}{2}\right)^{r} + 3\sum_{r=1}^{9} {}^9C_r\left(\frac{1}{2}\right)^r$

$= \frac{9}{2}\left(1+\frac{1}{2}\right)^8 + 3\left[\left(1+\frac{1}{2}\right)^9 - {}^9C_0\left(\frac{1}{2}\right)^0\right]$

$= \frac{9}{2}\cdot\frac{3^8}{2^8} + 3\left[\frac{3^9}{2^9} - 1\right]$

$= \frac{3^{10}}{2^9} + \frac{3^{10}}{2^9} - 3 = 4\cdot\frac{3^{10}}{2^{10}} - 3$

$= 4\left(\frac{3}{2}\right)^{10} - 3$

$= 6\left(\frac{3}{2}\right)^9 - 3$

$\alpha = 6, \beta = 3 \Rightarrow (\alpha+\beta)^2 = 81$

2. Let $z \in \mathbb{C}$ be such that $\frac{z^2+3i}{z-2+i} = 2+3i$. Then the sum of all possible values of z^2 is

(1) $-19-2i$ (2) $-19+2i$

(3) $19-2i$ (4) $19+2i$

Answer (1)

Sol. $\frac{z^2+3i}{z-2+i} = 2+3i$

$z^2 + 3i = (z-2+i)(2+3i)$

$z^2 + 3i = 2z - 4 + 2i + 3iz - 6i - 3$

$z^2 + 3i = (2z-7) + i(3z-4)$

$z^2 - (2+3i)z + (7+7i) = 0$

This is a quadratic in z.

$z_1 + z_2 = 2 + 3i$

$z_1 + z_2 = 7 + 7i$

$z_1^2 + z_2^2 = (z_1+z_2)^2 - 2z_1z_2$

$= (2+3i)^2 - 2(7+7i)$

$= 4 - 9 + 12i - 14 - 14i$

$= -19 - 2i$

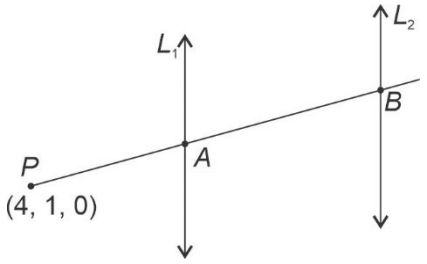
3. Let a line passing through the point (4, 1, 0) intersect the line $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point $A(\alpha, \beta, \gamma)$ and the line $L_2 : x-6 = y = -z+4$ at the point $B(a, b, c)$. Then $\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$ is equal to

(1) 16 (2) 6

(3) 12 (4) 8

Answer (4)

Sol.



$A = (\alpha, \beta, 4) = (2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

and $B = (a, b, c) = (\mu + 6, \mu, -\mu + 4)$

$\because \dfrac{\mu+2}{2\lambda-3} = \dfrac{\mu-1}{3\lambda+1} = \dfrac{-\mu+4}{4\lambda+3}$

$\therefore \quad \lambda\mu + 8\lambda + 4\mu = 1 \quad \ldots(1)$

and $7\lambda\mu - 16\lambda + 4\mu = 7 \quad \ldots(2)$

from equation (1) and (2)

$\lambda = -1$ and $\mu = 3$

Or $\lambda = -\dfrac{1}{3}$ and $\mu = 1$

By taking, $\lambda = -1$ and $\mu = 3$, we get

$\therefore \quad A(\alpha, \beta, 4) = (-1, -1, -1)$

and $B(a, b, c) = (9, 3, 1)$

$$\therefore \begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 8$$

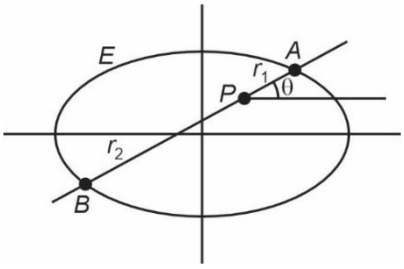
4. A line passing through the point $P(\sqrt{5}, \sqrt{5})$ intersects the ellipse $\dfrac{x^2}{36} + \dfrac{y^2}{25} = 1$ at A and B such that $(PA)\cdot(PB)$ is maximum. Then $5(PA^2 + PB^2)$ is equal to

(1) 377 (2) 218

(3) 338 (4) 290

Answer (3)

Sol.



$$\frac{x-\sqrt{5}}{\cos\theta} = \frac{y-\sqrt{5}}{\sin\theta} = r$$

$x = r\cos\theta + \sqrt{5}$

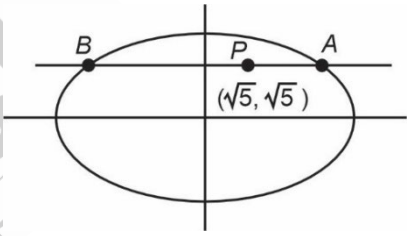
$y = r\sin\theta + \sqrt{5}$

$25x^2 + 36y^2 = 900$, (x, y) lie on E

$$r^2\left[25\cos^2\theta + 36\sin^2\theta\right] + r\left[50\sqrt{5}\cos\theta + 72\sqrt{5}\sin\theta\right] + 25[5] + 36[5] = 900$$

$$r_1 r_2 = \frac{900-305}{25+11\sin^2\theta}, \left(r_1 r_2\right)_{max} = \frac{595}{25} = \frac{119}{5}$$

$$\left|r_1 \cdot r_2\right|_{max} = \left(\frac{595}{25}\right) = \left(\frac{119}{5}\right) \text{ at } \theta = 0$$



$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

At $y = \sqrt{5}$

$$\frac{x^2}{36} + \frac{1}{5} = 1$$

$$\Rightarrow \quad \frac{x^2}{36} = \frac{4}{5} \quad \Rightarrow x = \pm\frac{12}{\sqrt{5}}$$

$$PA^2 + PB^2 = (PA + PB)^2 - 2PA\cdot PB = \left(\frac{24}{\sqrt{5}}\right)^2 - 2\cdot\frac{119}{5}$$

$$5\left(PA^2 + PB^2\right) = 5\left(\frac{24^2}{5} - \frac{2.119}{5}\right) = 24^2 - 238$$

$= 338.$

5. Let $A = (-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $0 \le x^2 + 2y \le 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. Then $l + m$ is equal to

(1) 18 (2) 19

(3) 17 (4) 20

Answer (1)

Sol. $0 \le x^2 + 2y \le 4$

For $y = -3$, $x = \{3, -3\}$

For $y = -2$, $x = \{-2, 2\}$

For $y = -1$, $x = \{-2, 2\}$

For $y = 0$, $x = \{-2, -1, 0, 1, 2\}$

For $y = 1$, $x = \{-1, 0, 1\}$

For $y = 2$, $x = \{0\}$

$R = \{(3, -3), (-3, -3), (-2, -2), (2, -2), (-2, -1), (2, -1), (-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0), (-1, 1), (0, 1), (1, 1), (0, 2)\}$

$l = 15$

We need to add $(-1, -1)$, $(2, 2)$, $(3, 3)$ to make it reflexive

$m = 3 \Rightarrow l + m = 18$

6. Let A be a matrix of order 3×3 and $|A| = 5$. If $|2adj(3A\ adj(2A))| = 2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}$, $\alpha, \beta, \gamma \in \mathbb{N}$, then $\alpha + \beta + \gamma$ is equal to

(1) 25 (2) 26

(3) 27 (4) 28

Answer (3)

Sol. $|2adj(3A\ adj(2A))| = 2^3|3A(adj(2A))|^2$

$= 2^3(3^3)^2|A|^2|adj(2A)^2|$

$= 2^3 \cdot 3^6 \cdot 5^2(|(2A)|^2)^2$

$= 2^3 \cdot 3^6 \cdot 5^2|2A|^4$

$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (2^3)^4 \cdot |A|^4$

$= 2^{15} \cdot 3^6 \cdot 5^6$

7. Let $f(x) = \begin{cases} (1+ax)^{1/x} & , x < 0 \\ 1+b & , x = 0 \\ \dfrac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2} & , x > 0 \end{cases}$

be continuous at $x = 0$. Then $e^a bc$ is equal to:

(1) 48 (2) 72

(3) 36 (4) 64

Answer (1)

Sol. $f(x) = \begin{cases} (1+ax)^{1/x} & , x < 0 \\ 1+b & , x = 0 \\ \dfrac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2} & , x > 0 \end{cases}$

Hence, $f(0) = 1 + 6$

$$\text{RHL} = \lim_{x \to 0^+} \frac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2} \quad \left[\text{For } \frac{0}{0} \text{ form, } c = 8\right]$$

$$= \lim_{x \to 0} \frac{2\left(1+\frac{x}{4}\right)^{1/2}-2}{\left(1+\frac{x}{8}\right)^{1/3}-2}$$

$$= \lim_{x \to 0} \frac{1+\frac{x}{8}-1}{1+\frac{x}{8}\cdot\frac{1}{3}-1} = \frac{\frac{1}{8}}{\frac{1}{8}\cdot\frac{1}{3}} = 3$$

$\therefore \quad 1 + b = 3$

$\Rightarrow \quad b = 2$

$$\text{LHL} = \lim_{x \to 0} (1+ax)^{1/x} = \lim_{x \to 0} e^{\frac{1+ax-1}{x}} = e^a = 3$$

$\therefore \quad e^a \cdot bc = 3.2.8 = 48$

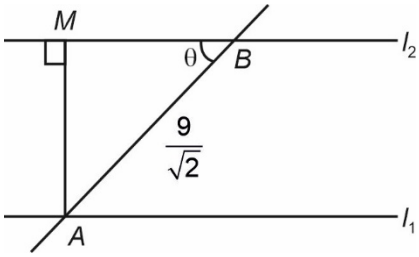
8. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines $L_1 : 2x + y + 6 = 0$ and $L_2 : 4x + 2y - p = 0$, $p > 0$, at the points A and B, respectively. If $AB = \dfrac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\dfrac{AM}{BM}$ is equal to

(1) 3 (2) 5

(3) 2 (4) 4

Answer (1)

Sol.



$$\tan\theta = \left|\frac{-2-1}{1+(-2)(1)}\right| = \frac{3}{1}$$

$$\sin\theta = \frac{3}{\sqrt{10}}$$

$$AM = \frac{3}{\sqrt{10}} \times \frac{9}{\sqrt{2}} = \frac{27}{2\sqrt{5}}$$

Dist. between l_1 & l_2, $AM = \left|\frac{\frac{P}{2}+6}{\sqrt{5}}\right| = \frac{27}{2\sqrt{5}}$

$$\frac{P}{2}+6 = \pm\frac{27}{2}$$

$$\frac{P}{2} = \frac{27}{2} - 6 \Rightarrow P = 15, \text{ As } P > 0$$

$$BM = \sqrt{\frac{81}{2} - \left(\frac{27}{2\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{810-729}{20}} = \sqrt{\frac{81}{20}} = \frac{9}{2\sqrt{5}}$$

Now, $\frac{AM}{BM} = \frac{\frac{27}{2\sqrt{5}}}{\frac{9}{2\sqrt{5}}} = 3$

9. The sum of all rational terms in the expansion of $\left(2+\sqrt{3}\right)^8$ is

(1) 3763 (2) 18817
(3) 33845 (4) 16923

Answer (2)

Sol. $S = {}^8C_0(2)^8 + {}^8C_1 2^7\left(\sqrt{3}\right) + \ldots + {}^8C_8\left(\sqrt{3}\right)^8$

Sum of rational terms

$$= {}^8C_0(2)^8 + {}^8C_2 2^6\left(\sqrt{3}\right)^2 + {}^8C_4(2)^4\left(\sqrt{3}\right)^4 + {}^8C_6(2)^2\left(\sqrt{3}\right)^2 + {}^8C_8\left(\sqrt{3}\right)^8$$

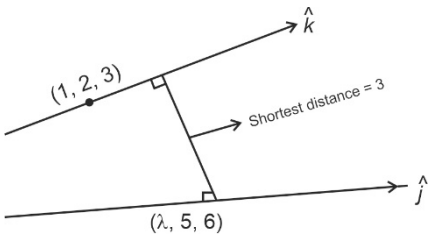
= 18,817

10. Line L_1 passes through the point (1, 2, 3) and is parallel to z-axis. Line L_2 passes through the point $(\lambda, 5, 6)$ and is parallel to y-axis. Let for $\lambda = \lambda_1, \lambda_2$, $\lambda_2 < \lambda_1$, the shortest distance between the two lines be 3. Then the square of the distance of the point $(\lambda_1, \lambda_2, 7)$ from the line L_1 is

(1) 32 (2) 25
(3) 37 (4) 40

Answer (2)

Sol.

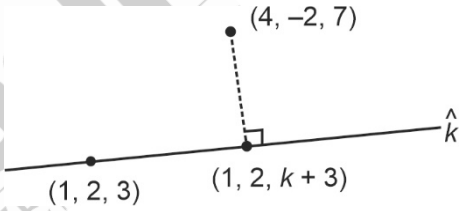


$$3 = \frac{\left|[(\lambda-1)\hat{i}+3\hat{j}+3\hat{k}].(\hat{k}\times\hat{j})\right|}{\left|\hat{k}\times\hat{j}\right|} = |\lambda-1| = 3$$

$\Rightarrow$ $\lambda = 4$ or -2, $\lambda_2 < \lambda_1 \Rightarrow \lambda_2 = -2$

$\lambda_1 = 4$

$$L_1 : \frac{x-1}{0} = \frac{y-2}{0} = \frac{z-3}{1} = k$$



$(3, -4, 4-k) \perp \hat{k}$

$4 - k = 0 \Rightarrow k = 4$

$\Rightarrow$ Distance between (4, –2, 7) & (1, 2, 7)

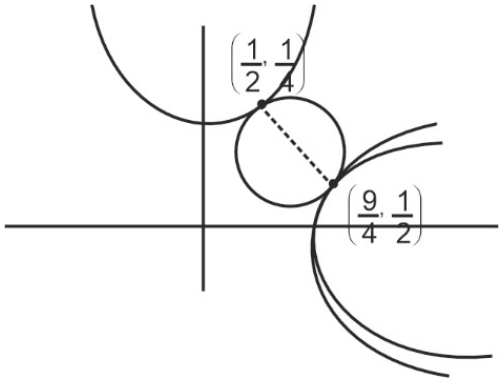
$\Rightarrow$ $\sqrt{3^2+4^2} = 5$

11. The radius of the smallest circle which touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is

(1) $\frac{7\sqrt{2}}{4}$ (2) $\frac{7\sqrt{2}}{16}$
(3) $\frac{7\sqrt{2}}{2}$ (4) $\frac{7\sqrt{2}}{8}$

Answer (4)

Sol.



The circle will have centre on $x = y$ line since parabolas are symmetric about $y = x$ line

The slope of tangent at closest point $y^2 = x - 2$

$$\Rightarrow \quad 2y\left(\frac{dy}{dx}\right)=1 \Rightarrow y=\frac{1}{2} \Rightarrow \text{point will be } \left(\frac{9}{4},\frac{1}{2}\right)$$

Similarly, on $x^2 = y - 2 \Rightarrow 2x = \frac{dy}{dx} = 1 \Rightarrow \left(\frac{1}{2},\frac{9}{4}\right)$

$$2r=\sqrt{\left(\frac{9}{4}-\frac{1}{2}\right)^2+\left(\frac{1}{2}-\frac{9}{4}\right)^2}=\sqrt{2}.\left|\frac{9}{4}-\frac{1}{2}\right|$$

$$=\frac{14}{8}\sqrt{2}=\frac{7\sqrt{2}}{4}$$

$$\Rightarrow \quad r=\frac{7\sqrt{2}}{8}$$

12. The sum $1 + 3 + 11 + 25 + 45 + 71 + \ldots$ upto 20 terms, is equal to

(1) 8124 (2) 7240

(3) 7130 (4) 6982

Answer (2)

Sol. $T_r = 3r^2 - 7r + 5$ using second order difference

$$\sum_{r=1}^{20}(3r^2-7r+5)=3\,\Sigma r^2-7\Sigma r+5\Sigma(1)$$

$$=\frac{3(n)(n+1)(2n+1)}{6}-\frac{7\,n(n+1)}{2}-5n,\ n=20$$

$= 7240$

13. The number of solutions of the equation $2x + 3\tan x = \pi$, $x \in [-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$ is:

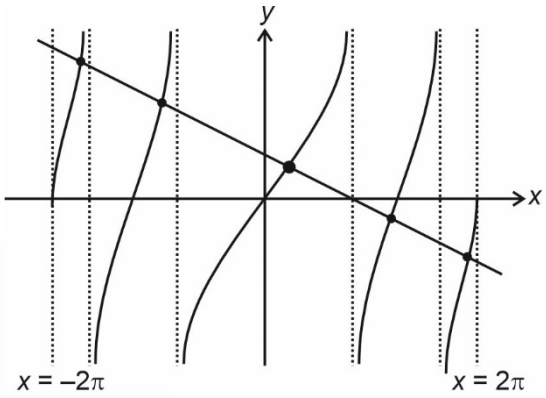
(1) 5 (2) 3

(3) 4 (4) 6

Answer (1)

Sol. $2x + 3\tan x = \pi$

$$\Rightarrow \quad \tan x=\frac{\pi-2x}{3},\ x\in[-2\pi,2\pi]-\left\{\pm\frac{\pi}{2},\pm\frac{3\pi}{2}\right\}$$



5 solutions

14. Let the domain of the function $f(x) = \log_2 \log_4 \log_6 (3 + 4x - x^2)$ be (a, b). If $\int_0^{b-a}[x^2]dx = p - \sqrt{q} - \sqrt{r}$, $p, q, r \in \mathrm{N}$, $\gcd(p, q, r) = 1$, where $[\cdot]$ is the greatest integer function, then $p + q + r$ is equal to

(1) 8 (2) 10

(3) 11 (4) 9

Answer (2)

Sol. $f(x) = \log_2(\log_4 (\log_6(3 + 4x - x^2)))$

$\log_4(\log_6(3 + 4x - x^2)) > 0$

$\Rightarrow \log_6(3 + 4x - x^2) > 1$

$\Rightarrow 3 + 4x - x^2 > 6$

$\Rightarrow x^2 - 4x + 3 < 0$

$\Rightarrow (x - 1)(x - 3) < 0$

$\Rightarrow x \in (1, 3)$

$a = 1, b = 3$

$$\int_1^2[x^2]dx=\int_1^{\sqrt{2}}1dx+\int_{\sqrt{2}}^{\sqrt{3}}2dx+\int_{\sqrt{3}}^{2}3dx=5-\sqrt{2}-\sqrt{3}$$

$p + q + r = 10$

15. If the domain of the function

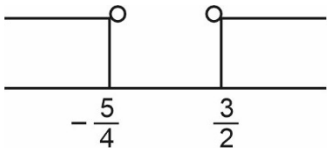
$f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is $[\alpha, \beta)$, then $\alpha^2 + 4\beta$ is equal to

(1) 3 (2) 7

(3) 4 (4) 5

Answer (3)

Sol. $\frac{2x-3}{4x+5}>0$



$\therefore \quad x\in\left(-\infty,-\frac{5}{4}\right)\cup\left(\frac{3}{2},\infty\right)$ ...(i)

$-1\le\frac{3x+4}{2-x}\le 1$

$\frac{3x+4}{2-x}\le 1$

$\Rightarrow \quad \frac{3x+4}{2-x}-1\le 0$

$\Rightarrow \quad \frac{3x+4-2+x}{x-2}\ge 0$

$\Rightarrow \quad \frac{4x+2}{x-2}\ge 0$

$\Rightarrow \quad x\in\left(-\infty,-\frac{1}{2}\right]\cup(2,\infty)$ ...(ii)

$\frac{3x+4}{2-x}\ge -1$

$\Rightarrow \quad \frac{3x+4}{2-x}+1\ge 0$

$\Rightarrow \quad \frac{3x+4+2-x}{2-x}\ge 0$

$\Rightarrow \quad \frac{2x+6}{x-2}\le 0$

$\therefore \quad x\in[-3, 2)$ ...(iii)

Taking intersection of (i), (ii) and (iii)

$x\in\left[-3,-\frac{5}{4}\right)$

$\alpha=-3, \beta=-\frac{5}{4}$

$\alpha^2 + 4\beta = 4$

16. Let $f(x)=\int x^3\sqrt{3-x^2}dx$. If $5f\left(\sqrt{2}\right)=-4$, then $f(1)$ is equal to

(1) $-\frac{6\sqrt{2}}{5}$ (2) $-\frac{4\sqrt{2}}{5}$

(3) $-\frac{2\sqrt{2}}{5}$ (4) $-\frac{8\sqrt{2}}{5}$

Answer (1)

Sol. $\int x^3\sqrt{3-x^2}dx$

$3 - x^2 = t^2$

$-2xdx = 2tdt$

$=-\int t^2(3-t^2)dt=\int t^4-3t^2dt$

$=\frac{t^5}{5}-t^3+c$

$f(x)=\frac{(3-x^2)^{\frac{5}{2}}}{5}-(3-x^2)^{\frac{3}{2}}+c$

$\because \quad f(\sqrt{2})=-\frac{4}{5}$

$\Rightarrow \quad -\frac{4}{5}=\frac{1}{5}-1+c \Rightarrow c=0$

$\therefore \quad f(x)=\frac{(3-x^2)^{\frac{5}{2}}}{5}-(3-x^2)^{\frac{3}{2}}$

Now $f(1)=\frac{2^{\frac{5}{2}}}{5}-2^{\frac{3}{2}}=2^{\frac{3}{2}}\left[\frac{2}{5}-1\right]$

$=2\sqrt{2}\left(-\frac{3}{5}\right)=-\frac{6}{5}\sqrt{2}$

17. Let $a_1, a_2, a_3, \ldots$ be a G.P of increasing positive numbers. If $a_3a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

(1) 130 (2) 131
(3) 129 (4) 128

Answer (3)

Sol. $a_2 + a_4 = \frac{111}{4}$

$a_3a_5 = 729$

$[ar^2][ar^4] = 729 = (27)^2$

$\Rightarrow ar^3 = 27$ ...(i)

$\Rightarrow a_4 = 27$

Now $a_2 = \frac{111}{4} - 27$

$a_2 = \frac{3}{4} = ar$ ...(ii)

By (i) & (ii)

$r^2 = \frac{4\times 27}{3} = 4\times 9 = 36$

$\Rightarrow r = 6$ ($\because$ increasing GP)

$a = \frac{1}{8}$

Now $24[a_1 + a_2 + a_3] = 24\,[a + ar + ar^2]$

$= 24\times\frac{1}{8}[1+r+r^2] = 3[1+6+6^2] = 3 \times 43 = 129$

18. Let α and β be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q^n = \gamma^n + \delta^n$, then $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$ is equal to

(1) 4 (2) 3
(3) 7 (4) 5

Answer (4)

Sol. $x^2 + 3x - 1 = 0 \rightarrow \gamma, \delta \Rightarrow x^2 - 1 = -3x$

$\Rightarrow P^n = \gamma^n + \delta^n$

$P_{25} - P_{23} = (\gamma^{25} - \gamma^{23}) + (\delta^{25} - \delta^{23})$

$= \gamma^{23}(\gamma^2 - 1) + \delta^{23}(\delta^2 - 1)$

$= \gamma^{23}(-3\gamma) + \delta^{23}(-3\delta) = -3[\gamma^{24} + \delta^{24}]$

$\Rightarrow \frac{P_{25} - P_{23}}{P_{24}} = (-3)$

Similarly,

$x^2 + \sqrt{3}x - 16 = 0 \rightarrow \alpha, \beta$ $Q_n = \alpha^n + \beta^n$

$\Rightarrow Q_{25} + \sqrt{3}Q_{24} = (\alpha^{25} + \sqrt{3}\alpha^{24}) + (\beta^{25} + \sqrt{3}\beta^{24})$

$= \alpha^{23}(\alpha^2 + \sqrt{3}\alpha) + \beta^{23}(\beta^2 + \sqrt{3}\beta)$

$= \alpha^{23}(16) + 16\beta^{23}$

$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2\cdot Q_{23}} = \frac{16(\alpha^{23} + \beta^{23})}{2(\alpha^{23} + \beta^{23})} = 8$

$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2Q_{23}} + \frac{(P_{25} - P_{23})}{P_{24}} = 8 + (-3) = 5$

19. Let g be a differentiable function such that $\int_0^x g(t)dt = x - \int_0^x tg(t)dt$, $x \geq 0$ and let $y = y(x)$ satisfy the differential equation $\frac{dy}{dx} - y\tan x = 2(x + 1)\sec x$ $g(x)$, $x \in \left[0, \frac{\pi}{2}\right)$. If $y(0) = 0$, then $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{4\pi}{3}$
(3) $\frac{2\pi}{3\sqrt{3}}$ (4) $\frac{4\pi}{3\sqrt{3}}$

Answer (2)

Sol. $\int_0^x g(t)dt = x - \int_0^x tg(t)dt$

Differentiate both sides w.r.t x.

$g(x) = 1 - xg(x)$

$g(x)(1 + x) = 1$

$g(x) = \frac{1}{1+x}$

Now $\frac{dy}{dx} - y\tan x = 2(x+1)\sec x g(x)$

I.F = $e^{-\int \tan x dx}$

I.F = $e^{-(-\ln\cos x)}$

I.F = $\cos x$

$$y.\cos x = \int 2(x+1)\sec x \cdot \frac{1}{(1+x)}\cos x\,dx$$

$y\cos x = 2x + c$ …(i)

$y(0) = 0$

$\Rightarrow \boxed{c = 0}$

$\therefore \quad y\cos x = 2x$

Put $x = \frac{\pi}{3}$

$\Rightarrow \quad y\left(\frac{1}{2}\right) = \frac{2\pi}{3} \Rightarrow \boxed{y = \frac{4\pi}{3}}$

20. If $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$, $x \in \mathbb{R}$,

then $\frac{d^2y}{dx^2} + y$ is equal to

(1) 28 (2) 1

(3) –1 (4) 27

Answer (3)

Sol. $f(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$

$f(x) = \sin x(1) - \cos x(0) + (\sin x + \cos x + 1)(-1)$

$f(x) = \sin x - \sin x - \cos x - 1$

$f(x) = -\cos x - 1$

$f'(x) = \sin x$

$f''(x) = \cos x$

$\Rightarrow f''(x) + f(x) = -\cos x - 1 + \cos x = -1$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$. if $\vec{c}$ is perpendicular to $\vec{a}$, then $\left|3\lambda\hat{d} + \mu\vec{c}\right|^2$ is equal to _____

Answer (5)

Sol. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = 3\hat{c} + 2\hat{j} - \hat{k}, \ \vec{c} = \lambda\hat{j} + \mu\hat{k}$

$\vec{a} \times \vec{d} = \vec{b} \times \vec{d}$

$\Rightarrow (\vec{a} - \vec{b}) \times \vec{d} = 0$

$\Rightarrow \vec{d} \,||\, (\vec{a} - \vec{b}) = (-2\hat{i} + \hat{j} + 2\hat{k})$

$\Rightarrow \hat{d} = \frac{-2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

Given $\hat{c} \cdot \hat{d} = 1$

$\frac{-\lambda}{3} + \frac{2\mu}{3} = 1$

$\Rightarrow -\lambda + 2\mu = 3$ …(i)

Also $\vec{a}$ is perpendicular to $\vec{c}$

$\Rightarrow \vec{c} \cdot \vec{a} = 0$

$\Rightarrow \lambda + \mu = 0$

$\Rightarrow \boxed{\lambda = -\mu}$

Put in (i)

$\mu + 2\mu = 3 \ \Rightarrow 3\mu = 3$

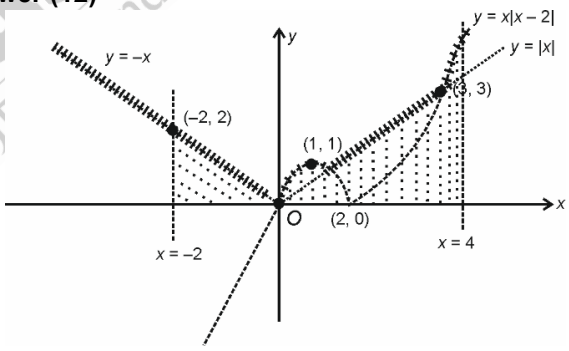
$\Rightarrow \mu = 1 \ \Rightarrow \lambda = -1$

$\therefore \left|3\lambda\hat{d} + \mu\hat{c}\right|^2 \Rightarrow 9\lambda^2\left|\hat{d}\right|^2 + \mu^2\left|\vec{c}\right|^2 + 2 \cdot 3 \cdot \lambda \cdot \mu\vec{c} \cdot \vec{d}$

22. The area of the region bounded by the curve $y = \max\{|x|, |x-2|\}$, the x-axis and the lines $x = -2$ and $x = 4$ is equal to _____

Answer (12)

Sol.



Area =

$$\frac{1}{2} \times 2 \times 2 + \int_0^1 (2x - x^2)dx + \int_1^3 x\,dx + \int_3^4 (x^2 - 2x)dx$$

$$= 2 + \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^3 + \left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_3^4$$

= 12 square units

23. Let the product of the focal distances of the point $P(4,2\sqrt{3})$ on the hyperbola $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be 32. Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then p^2+q^2 is equal to _____.

Answer (120)

Sol. $P(4,2\sqrt{3})$ lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\Rightarrow\frac{16}{a^2}-\frac{12}{b^2}=1\ldots(i)$

$SP=e\left(4-\frac{a}{e}\right),S'P=e\left(4+\frac{a}{e}\right)$

$\Rightarrow SP\cdot S'P=16e^2-a^2=32$

$\because e^2=1+\frac{b^2}{a^2}$

$\Rightarrow 16\left(1+\frac{b^2}{a^2}\right)-a^2=32\quad\ldots(ii)$

From (i) and (ii), $a^2=8, b^2=12$

$\Rightarrow\because p^2+q^2=(2b)^2+\left(\frac{2b^2}{9}\right)^2=4b^2+\frac{4b^4}{a^2}$

$=48\left(1+\frac{12}{8}\right)=120$

24. All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by W_n. Let the probability $P(W_n)$ of choosing the word W_n satisfy $P(W_n) = 2P(W_n-1)$, $n > 1$.

If $P(CDBEA)=\frac{2^\alpha}{2^\beta-1},\alpha,\beta\in\mathbb{N}$, then $\alpha+\beta$ is equal to : _____

Answer (183)

Sol. We are given that the probability of choosing W_n is:

$P(W_n)=2P(W_{n-1})$ for $n>1$

$\Rightarrow P(W_1)=p,P(W_2)=2p,P(W_3)=4p,\ldots,P(W_n)=2^{n-1}p$

To find the value of p, we use:

$\sum_{n-1}^{120}P(W_n)=1$ (total probability)

So, $p(1+2+2^2+\ldots+2^{119})=1$

$p(2^{120}-1)=1\Rightarrow p=\frac{1}{2^{120}-1}$

Thus, $P(W_n)=\frac{2^{n-1}}{2^{120}-1}\quad\ldots(i)$

Since the first letter of $CDBEA$ is C

Words starting with A : 4! = 24

Words starting with B : 4! = 24

$CA\_\_\_$: 3! = 6

$CB\_\_\_$: 3! = 6

$CDA\_\_$: 2! = 2

$CDBA\_$: 1! = 1

Total before $CDBEA$ = 63

Position of $CDBEA$ = 64^{th}

Putting in (i)

$P(CDBEA)=P(W_{64})=\frac{2^{63}}{2^{120}-1}$

Compare with the given form:

$P(CDBEA)=\frac{2^\alpha}{2^\beta-1}$

So, $\alpha = 63$, $\beta = 120$

$\alpha + \beta = 63 + 120 = 183$

25. If the number of seven-digit numbers, such that the sum of their digits is even, is $m\cdot n\cdot 10^n$, $m,n\in\{1,2,3,\ldots,9\}$, then $m + n$ is equal to _____.

Answer (14)

Sol. When numbers are uniformly distributed, half of them have even digit sums and half have odd digits sums.

Number of 7-digit numbers with even digit sum = $\frac{1}{2}\cdot 9\cdot 10^6=4.5\cdot 10^6$

Note that $9\cdot 5\cdot 10^5=4.5\cdot 10^6$

$m + n = 9 + 5 = 14$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. Match the **List-I** with **List-II**

	List-I		List-II
A.	${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^{1}_{0}n$	I.	Chemical reaction
B.	$2H_2 + O_2 \rightarrow 2H_2O$	II.	Fusion with +ve Q value
C.	${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$	III.	Fission
D.	${}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{2}_{1}H + {}^{2}_{1}H$	IV.	Fusion with –ve Q value

Choose the **correct** answer from the options given below:

(1) A-III, B-I, C-IV, D-II

(2) A-II, B-I, C-IV, D-III

(3) A-III, B-I, C-II, D-IV

(4) A-II, B-I, C-III, D-IV

Answer (3)

Sol. (A) is fission reaction

(B) is a chemical reaction

(C) deuteron + deuteron $\rightarrow$ helium 3 is exothermic process

(D) H + 3He $\rightarrow$ 2deuteron is endothermic process

27. A particle is released from height S above the surface of the earth. At certain height its kinetic energy is three times its potential energy. The height from the surface of the earth and the speed of the particle at that instant are respectively.

(1) $\frac{S}{2}, \sqrt{\frac{3gS}{2}}$ (2) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

(3) $\frac{S}{4}, \frac{3gS}{2}$ (4) $\frac{S}{2}, \frac{3gS}{2}$

Answer (2)

Sol. PE = mgh

KE = $mg\,(S - h)$

given $mg(S - h) = 3mgh$

$S = 4h$

$h = \frac{S}{4}$

$U = \sqrt{\frac{2g(3S)}{4}} = \sqrt{\frac{3gS}{2}}$

28. Consider following statements for refraction of light through prism, when angle of deviation is minimum.

A. The refracted ray inside prism becomes parallel to the base.

B. Larger angle prisms provide smaller angle of minimum deviation.

C. Angle of incidence and angle of emergence becomes equal.

D. There are always two sets of angle of incidence for which deviation will be same except at minimum deviation setting.

E. Angle of refraction becomes double of prism angle.

Choose the **correct** answer from the options given below:

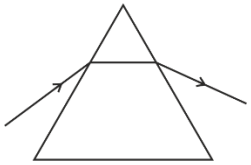
(1) A, B and E only (2) B, D and E only

(3) B, C and D only (4) A, C and D only

Answer (4)

Sol. Because of symmetry where $i = r$, and $r = \frac{A}{2}$

$\delta = (\mu - 1)A$



(A) is correct

(B) is incorrect

(C) is correct

(D) is correct as $\delta = i + e - A$

(E) is incorrect

29. The radiation pressure exerted by a 450 W light source on a perfectly reflecting surface placed at 2 m away from it, is

(1) 1.5×10^{-8} Pascals (2) 0

(3) 3×10^{-8} Pascals (4) 6×10^{-8} Pascals

Answer (4)

Sol. $P = \frac{2I}{C}$

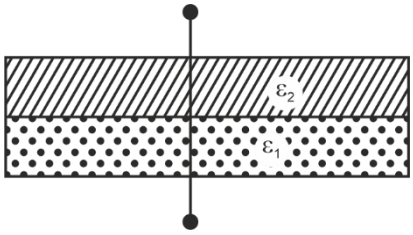
$I = \frac{P}{A} = \frac{2 \times 450}{4\pi \times 2^2}$

Pressure = $\frac{2 \times 450}{16\pi \times 3 \times 10^8}$

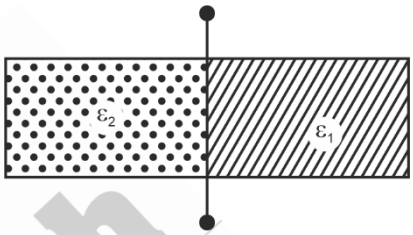
$\approx 6 \times 10^{-8}$

30. A parallel plate capacitor is filled equally (half) with two dielectrics of dielectric constants ε_1 and ε_2, as shown in figures. The distance between the plates is d and area of each plate is A. If capacitance in first configuration and second configuration are C_1 and C_2 respectively, then $\frac{C_1}{C_2}$ is

First Configuration



Second Configuration



(1) $\frac{4\varepsilon_1\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)^2}$ (2) $\frac{\varepsilon_0(\varepsilon_1 + \varepsilon_2)}{2}$

(3) $\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$ (4) $\frac{\varepsilon_1\varepsilon_2^2}{(\varepsilon_1 + \varepsilon_2)^2}$

Answer (1)

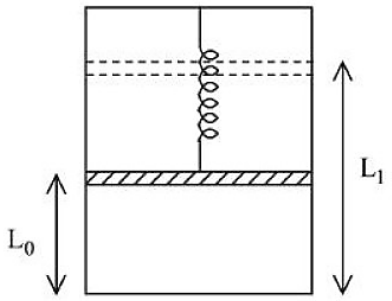
Sol. $C = \frac{k\varepsilon_0 A}{d}$

$$C_1 = \frac{\varepsilon_0 A}{\frac{d}{2\varepsilon_1} + \frac{d}{2\varepsilon_2}} = \frac{\varepsilon_0 A}{d}\left\{\frac{1}{\frac{\varepsilon_2 + \varepsilon_1}{2\varepsilon_1\varepsilon_2}}\right\}$$

$$C_2 = \frac{\varepsilon_0}{d}\left(\varepsilon_1\frac{A}{2} + \varepsilon_2\frac{A}{2}\right) = \frac{\varepsilon_0 A}{d}\left\{\frac{\varepsilon_1}{2} + \frac{\varepsilon_2}{2}\right\}$$

$$\frac{C_1}{C_2} = \frac{2\varepsilon_1\varepsilon_2}{(\varepsilon_2 + \varepsilon_1)\frac{(\varepsilon_1 + \varepsilon_2)}{2}} = \frac{4\varepsilon_1\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)^2}$$

31. A piston of mass M is hung from a massless spring whose restoring force law goes as $F = -kx^3$, where k is the spring constant of appropriate dimension. The piston separates the vertical chamber into two parts, where the bottom part is filled with 'n' moles of an ideal gas. An external work is done on the gas isothermally (at a constant temperature T) with the help of a heating filament (with negligible volume) mounted in lower part of the chamber, so that the piston goes up from a height L_0 to L_1, the total energy delivered by the filament is: (Assume spring to be in its natural length before heating)



(1) $nrT\ln\left(\frac{L_1}{L_0}\right)+Mg(L_1-L_0)+\frac{k}{4}(L_1^4-L_0^4)$

(2) $3nrT\ln\left(\frac{L_1}{L_0}\right)+2Mg(L_1-L_0)+\frac{k}{3}(L_1^3-L_0^3)$

(3) $nrT\ln\left(\frac{L_1^2}{L_0^2}\right)+\frac{Mg}{2}(L_1-L_0)+\frac{k}{4}(L_1^4-L_0^4)$

(4) $nrT\ln\left(\frac{L_1}{L_0}\right)+Mg(L_1-L_0)+\frac{3k}{4}(L_1^4-L_0^4)$

Answer (None option matches)

Sol. From energy conservation

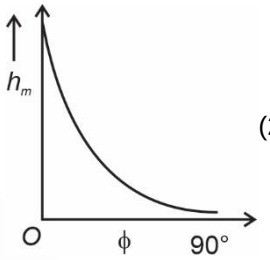
heat given = ($\Delta U + \Delta W$) for gas + change in G.P.E. of piston + energy stored in spring

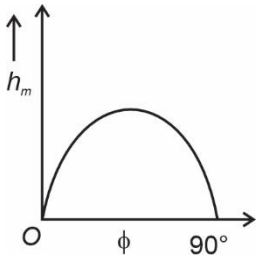
$$= 0+\mu RT\ln\frac{V_2}{V_1}+Mg(h_2-h_1)+\frac{1}{4}k(x_2^4-x_1^4)$$

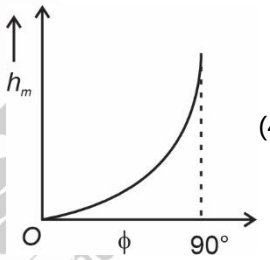
$$= nRT\ln\frac{L_1}{L_0}+Mg(L_1-L_0)+\frac{1}{4}k(L_1-L_0)^4$$

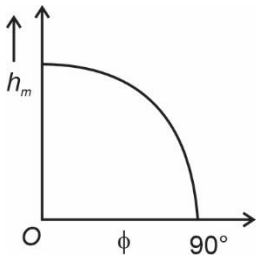
None option matches.

32. The angle of projection of a particle is measured from the vertical axis as ϕ and the maximum height reached by the particle is h_m. Here h_m as function of ϕ can be presented as

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. $h=\frac{u^2\sin^2\theta}{2g}$ $\qquad \theta = 90° - \phi$

$$\Rightarrow\ h=\frac{u^2\cos^2\theta}{2g}$$

From $\phi = 0$ to $\phi = 90$

h decreases with $\cos^2$ function.

33. A person measures mass of 3 different particles as 435.42 g, 226.3 g and 0.125 g. According to the rules for arithmetic operations with significant figures, the addition of the masses of 3 particles will be

(1) 661.8 g (2) 661.84 g

(3) 662 g (4) 661.845 g

Answer (1)

Sol.
$$\begin{array}{r} 435.42 \\ 226.3 \\ 0.125 \\ \hline 661.845 \end{array}$$

For addition, minimum decimal number is considered for reporting the measured value

$\Rightarrow$ 661.8

34. During the melting of a slab of ice at 273 K at atmospheric pressure:

(1) Internal energy of ice-water system remains unchanged

(2) Internal energy of the ice-water system decreases

(3) Positive work is done by the ice-water system on the atmosphere

(4) Positive work is done on the ice-water system by the atmosphere

Answer (4)

Sol. Melting of ice requires heat therefore internal energy increases.

But because of decrease in volume the work done on atmosphere is negative or atmosphere does positive work on ice-water system.

35. The work function of a metal is 3 eV. The color of the visible light that is required to cause emission of photoelectrons is

(1) Red (2) Yellow

(3) Green (4) Blue

Answer (4)

Sol. $\lambda = \frac{12400}{3}\text{Å} = 4133\ \text{Å}$

and $\lambda_{\text{visible}} \in (3800 - 7600)\ \text{Å}$
VIBGYOR

Therefore, most appropriate response is blue.

36. Match the **List-I** with **List-II**

	List-I		**List-II**
A.	Gravitational constant	I.	$[LT^{-2}]$
B.	Gravitational potential energy	II.	$[L^2T^{-2}]$
C.	Gravitational potential	III.	$[ML^2T^{-2}]$
D.	Acceleration due to gravity	IV.	$[M^{-1}L^3T^{-2}]$

Choose the **correct** answer from the options given below:

(1) A-II, B-IV, C-III, D-I

(2) A-IV, B-III, C-II, D-I

(3) A-I, B-III, C-IV, D-II

(4) A-III, B-II, C-I, D-IV

Answer (2)

Sol. (A) $F = \frac{Gm^2}{r^2} \equiv G = \frac{Fr^2}{m^2} \equiv \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$

$A \rightarrow$ IV

(B) $G.P\ E \equiv$ Energy $\equiv ML^2T^{-2}$

$B \rightarrow$ III

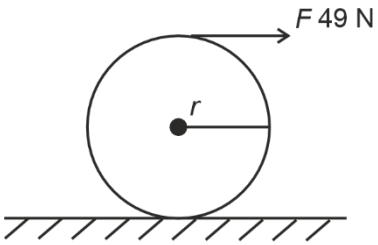
(C) $G.P = \frac{mgh}{m} = gh \equiv V^2 \equiv L^2T^{-2}$

$C \rightarrow$ II

(D) $g \equiv LT^{-2}$

D $\rightarrow$ I

37. A force of 49 N acts tangentially at the highest point of a sphere (solid) of mass 20 kg, kept on a rough horizontal plane. If the sphere rolls without slipping, then the acceleration of the center of the sphere is



(1) 2.5 m/s^2

(2) 0.25 m/s^2

(3) 3.5 m/s^2

(4) 0.35 m/s^2

Answer (3)

Sol. $\tau = I\alpha \Rightarrow 49 \times 2r = \frac{7}{5}mr^2\alpha$

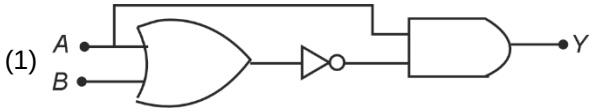
$49 \times 2r = \frac{7}{5}mr\,a$

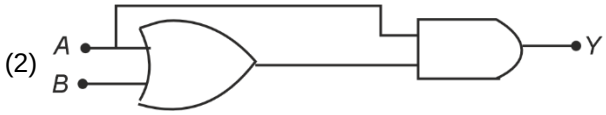
$\frac{49 \times 2 \times 5}{7 \times 20} = a$

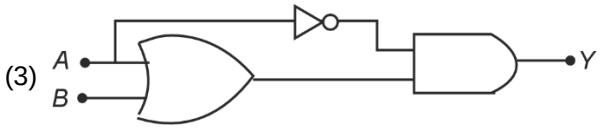
$a = 3.5$ m/s^2

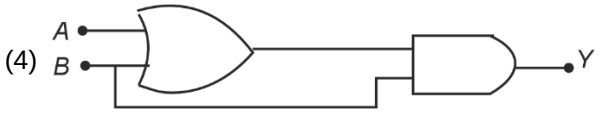
38. Choose the correct logic circuit for the given truth table having inputs A and B.

Inputs		Output
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. Option (1) = $\left(\overline{A+B}\right)\cdot A$

$= \left(\overline{A}+\overline{B}\right)\cdot A \equiv 0$

Option (2) = $(A + B) \cdot A$

$A \cdot A + A \cdot B = A$

Option (3) $(A+B)\cdot\overline{A}$

$A \cdot \overline{A} + \overline{A} \cdot B = \overline{A}B$

Option (4)

$(A + B) \cdot (B)$

$A \cdot B + B \cdot B = B$

$\Rightarrow$ option 2 matches the truth table

39. The electrostatic potential on the surface of uniformly charged spherical shell of radius R = 10 cm is 120 V. The potential at the centre of shell, at a distance r = 5 cm from centre, and at a distance r = 15 cm from the centre of the shell respectively, are:

(1) 120V, 120V, 80V (2) 40V, 40V, 80V

(3) 0V, 120V, 40V (4) 0V, 0V, 80V

Answer (1)

Sol. For $r \le R \quad V = \frac{kQ}{R}$

For $r > R \quad V = \frac{kQ}{r}$

(i) For r = 0 and r = 5 cm

(< R = 10 cm)

V = 120 V

(ii) For r = 15 cm $V = \frac{kQ}{R}\frac{R}{r}$

$= 120 \times \frac{10}{15}$

= 80 V

40. Consider a completely full cylindrical water tank of height 1.6 m and of cross-sectional area 0.5 m^2. It has a small hole in its side at a height 90 cm from the bottom. Assume, the cross-sectional area of the hole to be negligibly small as compared to that of the water tank. If a load 50 kg is applied at the top surface of the water in the tank then the velocity of the water coming out at the instant when the hole is opened is:

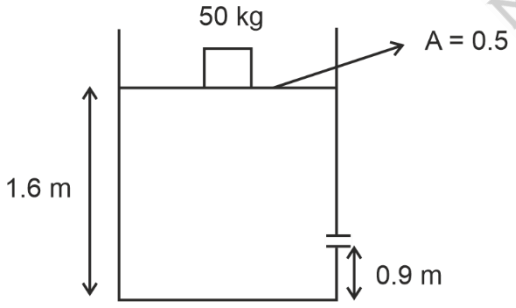
(g = 10 m/s^2)

(1) 3 m/s (2) 2 m/s

(3) 5 m/s (4) 4 m/s

Answer (4)

Sol.



$$\Delta P = \frac{1}{2}\rho v^2$$

$$\frac{mg}{A} + \rho g \Delta h = \frac{1}{2}\rho v^2$$

$$\frac{500}{0.5} + 10^3 \times 10 \times 0.7 = \frac{1}{2} \times 10^3 v^2$$

$$1 + 7 = \frac{1}{2}v^2$$

v = 4 m/s

41. A gas is kept in a container having walls which are thermally non-conducting. Initially the gas has a volume of 800 cm^3 and temperature 27°C. The change in temperature when the gas is adiabatically compressed to 200 cm^3 is :

(Take γ = 1.5; γ is the ratio of specific heats at constant pressure and at constant volume)

(1) 300 K

(2) 522 K

(3) 327 K

(4) 600 K

Answer (1)

Sol. Being adiabatic process PV^{γ} = constant, where γ = 1.5 and also $TV^{\gamma-1}$ = constant

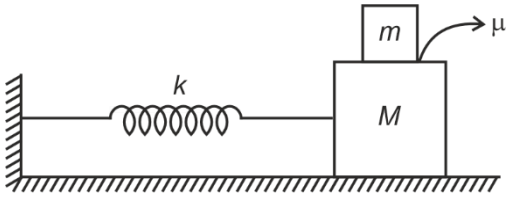
$300\{800\}^{0.5} = T\{200\}^{0.5}$

$T = 300 \times 2 = 600$ K

$\Delta T = 600 - 300 = 300$ K

42. Two blocks of masses m and M, ($M > m$) are placed on a frictionless table as shown in figure. A massless spring with spring constant k is attached with the lower block. If the system is slightly displaced and released, then

(μ = coefficient of friction between the two blocks)



A. The time period of small oscillation of the two blocks is $T = 2\pi\sqrt{\frac{(m+M)}{k}}$

B. The acceleration of the blocks is $a = -\frac{kx}{M+m}$ (x = displacement of the blocks from the mean position)

C. The magnitude of the frictional force on the upper block is $\frac{m\mu|x|}{M+m}$

D. The maximum amplitude of the upper block, if it does not slip, is $\frac{\mu(M+m)g}{k}$

E. Maximum frictional force can be $\mu(M+m)g$

Choose the **correct** answer from the options given below :

(1) B, C, D only (2) A, B, C only

(3) A, B, D only (4) C, D, E only

Answer (3)

Sol. A. Assuming no slipping, $T = 2\pi\sqrt{\frac{m_{total}}{k}}$

A is correct.

B. Assuming no slipping, $a = \frac{|F|}{m}$

B is correct.

C. $f = (m)(a) = \frac{m \times kx}{m+M}$

C is correct.

D. For no slipping $\frac{kx_0}{m+M} \le \mu g$

D is correct.

E. $f_{max} = \mu mg$

E is incorrect.

43. The radii of curvature for a thin convex lens are 10 cm and 15 cm respectively. The focal length of the lens is 12 cm. The refractive index of the lens material is

(1) 1.5 (2) 1.2

(3) 1.4 (4) 1.8

Answer (1)

Sol. For biconvex

$R_1 = 10$

$R_2 = 15$

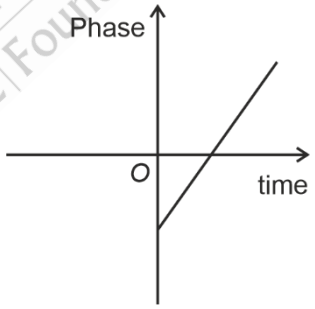
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{12} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = (\mu - 1)\left(\frac{1}{6}\right)$$

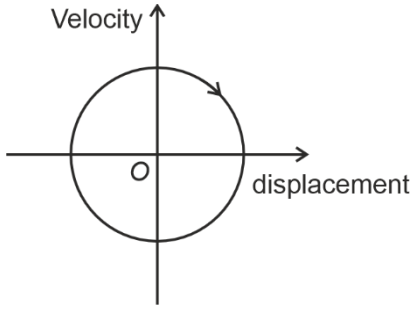
$$\mu - 1 = \frac{1}{2} \Rightarrow \mu = 1.5$$

44. Which of the following curves possibly represent one-dimensional motion of a particle?

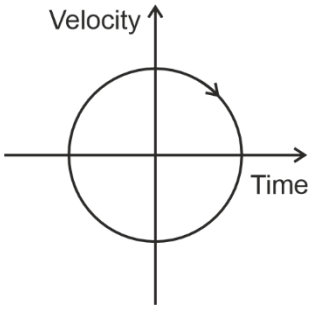
A.



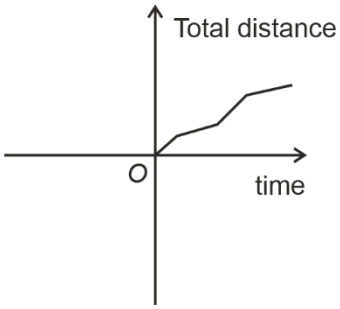
B.



C.



D.



Choose the **correct** answer from the options given below :

(1) A and B only (2) A, C and D only

(3) A, B and C only (4) A, B and D only

Answer (4)

Sol. A. Phase increase with time in SHM

$\Rightarrow$ Correct

B. Velocity and displacement are related in elliptical/circular relation

i.e. $\frac{v^2}{v_0^2}+\frac{x^2}{x_0^2}=1$

$\Rightarrow$ Correct

C. At same time particle can't have two velocities

$\Rightarrow$ Incorrect

D. Distance always increases

$\Rightarrow$ Correct

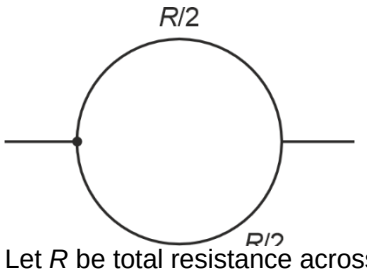
45. A wire of length 25 m and cross-sectional area 5 mm^2 having resistivity $2 \times 10^{-6}\ \Omega$ m is bent into a complete circle. The resistance between diametrically opposite points will be

(1) 100 Ω (2) 50 Ω

(3) 12.5 Ω (4) 25 Ω

Answer (None matches the options)

Sol.



Let R be total resistance across ends of wire, then

$$R_{eq}=\frac{R}{4}=\frac{\rho\ell}{4A}=\frac{2\times10^{-6}\times25}{4\times5\times10^{-6}}$$

$= 2.5\ \Omega$

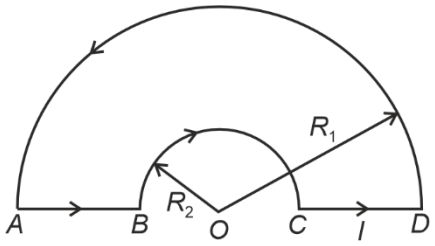
None matches the options.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A loop *ABCDA*, carrying current I = 12 A, is placed in a plane, consists of two semi-circular segments of radius $R_1 = 6\pi$ m and $R_2 = 4\pi$ m. The magnitude of the resultant magnetic field at center O is $k \times 10^{-7}$ T. The value of k is _____.

(Given $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1})



Answer (1)

Sol. $B = \frac{\mu_0 I}{2r}\frac{\theta}{2\pi}$ for an arc

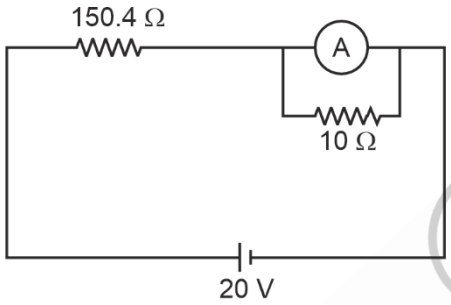
For semicircle $B = \frac{\mu_0 I}{4r}$

$$B_{net} = \frac{\mu_0 I}{4\{4\pi\}} - \frac{\mu_0 I}{4\{6\pi\}}$$

$$= \frac{4_0 I}{4\pi}\left\{\frac{1}{4} - \frac{1}{6}\right\}$$

$$= 10^{-7} \times 12 \times \frac{2}{24} = 10^{-7}\,\text{T} \Rightarrow k = 1$$

47. In the figure shown below, a resistance of 150.4 Ω is connected in series to an ammeter A of resistance 240 Ω. A shunt resistance of 10 Ω is connected in parallel with the ammeter. The reading of the ammeter is _____ mA.



Answer (5)

Sol. $i_0 = \frac{20}{150.4 + \frac{240 \times 10}{250}}$

$$= \frac{20}{150.4 + 9.6} = \frac{20}{160} = \frac{1}{8}\ \text{A}$$

$$i_A = \frac{1}{8} \times \frac{10}{250} \times 1000\ \text{mA}$$

= 5 mA

48. Three identical spheres of mass m, are placed at the vertices of an equilateral triangle of length a. When released, they interact only through gravitational force and collide after a time T = 4 seconds. If the sides of the triangle are increased to length 2a and also the masses of the spheres are made 2m, then they will collide after _____ seconds.

Answer (8)

Sol. As limiting case of elliptical path is straight line therefore proportionality $T^2 \propto \frac{a^3}{M}$ will holds from $\left(T^2 = \frac{4\pi^2}{GM} r^3\right)$

$$\frac{4^2}{T^2} = \frac{a^3}{m}\frac{2m}{(2a)^3} = \frac{2}{8} = \frac{1}{4}$$

$$\frac{4}{T} = \frac{1}{2}$$

T = 8 seconds

49. Two coherent monochromatic light beams of intensities 4I and 9I are superimposed. The difference between the maximum and minimum intensities in the resulting interference pattern is xI. The value of x is _____.

Answer (24)

Sol. $I_{max} = \left(\sqrt{4I} + \sqrt{9I}\right)^2 = 25I$

$$I_{min} = \left(\sqrt{4I} - \sqrt{9I}\right)^2 = I$$

$\Delta I = 24I$

50. A 4.0 cm long straight wire carrying a current of 8 A is placed perpendicular to a uniform magnetic field of strength 0.15 T. The magnetic force on the wire is _____ mN.

Answer (48)

Sol. $F = i\,l\,B$

= 8 × 4 × 10^{-2} × 0.15 newton

= 8 × 4 × 10^{-2} × 0.15 × 10^3 mN

= 32 × 1.5 mN

= 48 mN

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. 2 moles each of ethylene glycol and glucose are dissolved in 500 g of water. The boiling point of the resulting solution is:

(Given : Ebullioscopic constant of water = 0.52 K kg mol^{-1})

(1) 377.3 K (2) 375.3 K

(3) 379.2 K (4) 277.3 K

Answer (1)

Sol. $\Delta T_b = K_b(m)$

$= (0.52)\left(\frac{4}{0.5}\right)$

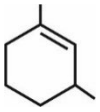
$= (0.52)(8)$

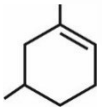
$= 4.16$ K

$T_b = 373.15 + 4.16$

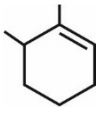
$= 377.3$ K

52. Which compound would give 3-methyl-6-oxoheptanal upon ozonolysis?

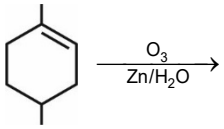
(1) 

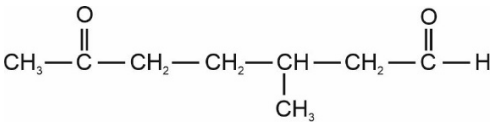
(2) 

(3) 

(4) 

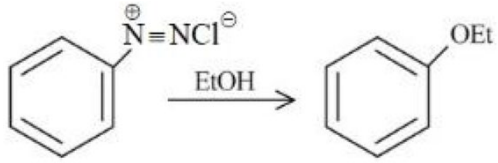
Answer (3)

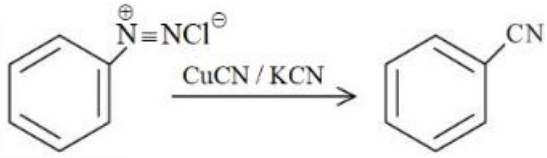
Sol. 

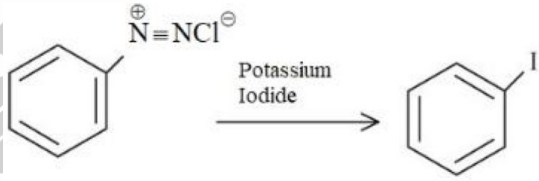


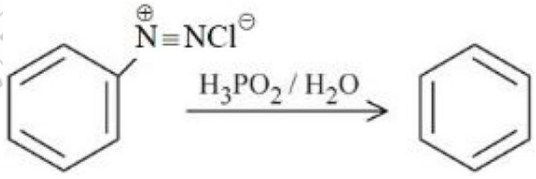
3-methyl-6-oxoheptanal

53. In the following reactions, which one is NOT correct?

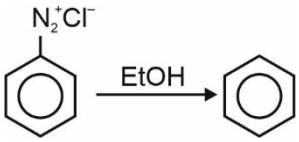
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

Option (1) is incorrect

54. Among 10^{-9} g (each) of the following elements, which one will have the highest number of atoms?

Element: Pb, Po, Pr and Pt

(1) Po (2) Pb

(3) Pt (4) Pr

Answer (4)

Sol. Highest atoms in case where atomic mass is lowest

Po = 209

Pb = 207

Pt = 195

Pr = 140

55. Correct order of limiting molar conductivity for cations in water at 298 K is :

(1) $H^+ > Na^+ > K^+ > Ca^{2+} > Mg^{2+}$

(2) $Mg^{2+} > H^+ > Ca^{2+} > K^+ > Na^+$

(3) $H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$

(4) $H^+ > Na^+ > Ca^{2+} > Mg^{2+} > K^+$

Answer (3)

Sol. λ_m^0 order

$H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$

56. Which of the following postulate of Bohr's model of hydrogen atom is not in agreement with quantum mechanical model of an atom?

(1) The electron in a H atom's stationary state moves in a circle around the nucleus

(2) An atom in a stationary state does not emit electromagnetic radiation as long as it stays in the same state.

(3) When an electron makes a transition from a higher energy stationary state to a lower energy stationary state, then it emits a photon of light.

(4) An atom can take only certain distinct energies E_1, E_2, E_3, etc. These allowed states of constant energy are called the stationary states of atom.

Answer (1)

Sol. The electron in a H-atom's stationary state moves in a circle around the nucleus. This is not in agreement with Quantum Mechanical model of an atom.

57. Number of molecules from below which cannot give iodoform reaction is :

Ethanol, Isopropyl alcohol, Bromoacetone, 2-Butanol, 2-Butanone, Butanal, 2-Pentanone, 3-Pentanone, Pentanal and 3-Pentanol.

(1) 2 (2) 5

(3) 3 (4) 4

Answer (4)

Sol. The compounds which doesn't give iodoform test are –

Butanal, 3-pentanone, pentanal, 3-pentanol

58. Given below are two statements :

Statement (I) : A catalyst cannot alter the equilibrium constant (K_C) of the reaction, temperature remaining constant.

Statement (II) : A homogenous catalyst can change the equilibrium composition of a system, temperature remaining constant.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(1) Both **Statement-I** and **Statement-II** are false

(2) Both **Statement-I** and **Statement-II** are true

(3) **Statement-I** is true but **Statement-II** is false

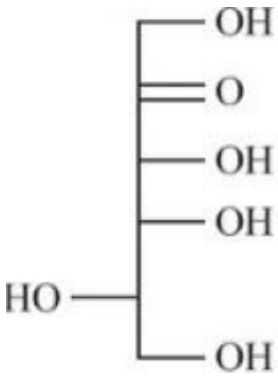
(4) **Statement-I** is false but **Statement-II** is true

Answer (3)

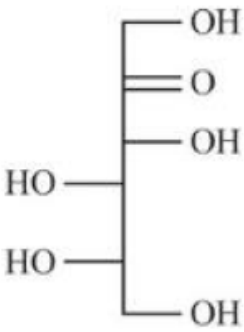
Sol. Catalyst doesn't change K_{eq} catalyst doesn't change composition statement-I is true But statement-II is false.

59. Which of the following is the correct structure of L-Fructose?

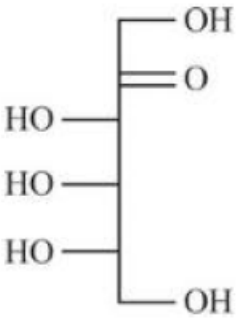
(1)



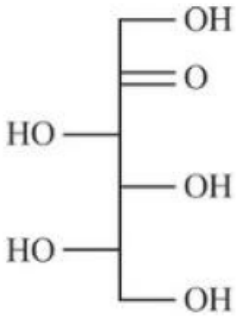
(2)



(3)

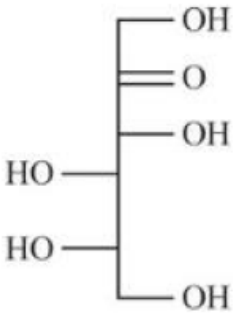


(4)



Answer (2)

Sol.



60. Match the **LIST-I** with **LIST-II**

	LIST-I (Molecules/ion)		**LIST-II (Hybridisation of central atmn)**
A.	PF_5	I.	dsp^2
B.	SF_6	II.	sp^3d
C.	$Ni(CO)_4$	III.	sp^3d^2
D.	$[PtCl_4]^{2-}$	IV.	sp^3

Choose the ***correct*** answer from the options given below:

(1) A-III, B-I, C-IV, D-II (2) A-II, B-III, C-IV, D-I

(3) A-I, B-II, C-III, D-IV (4) A-IV, B-I, C-II, D-III

Answer (2)

Sol. (A) PF_5 – sp^3d (II) (B) SF_6 – sp^3d^2 (III)

(C) $Ni(CO)_4$ – sp^3 (IV) (D) $[PtCl_4]^{2-}$ – dsp^2 (I)

61. In the following system, $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ at equilibrium, upon addition of xenon gas at constant T & p, the concentration of

(1) Cl_2 will decrease

(2) PCl_3 will increase

(3) PCl_5 will increase

(4) PCl_5, PCl_3 & Cl_2 remain constant

Answer (2)

Sol. According to Le-Chatelier's principle

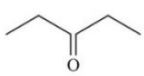
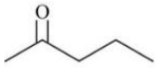
Reaction will shift in forward direction.

Cl_2 will increase

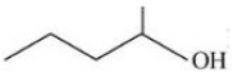
PCl_3 will increase

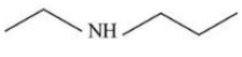
PCl_5 will decrease

62. Identify the correct statements from the following.

A.  and  are metamers

B.  and  are functional isomers

C.  and  are position isomers

D.  and  are homologous

Choose the ***correct*** answer from the options given below:

(1) A, B & C Only (2) B & C Only

(3) A & B Only (4) C & D Only

Answer (3)

Sol. A and B are correct. C ⇒ Homologous

63. In a reaction A + B → C, initial concentrations of A and B are related as $[A]_0 = 8[B]_0$. The half lives of A and B are 10 min and 40 min, respectively, If they start to disappear at the same time, both following first order kinetics, after how much time will the concentration of both the reactants be same?

(1) 40 min (2) 20 min

(3) 80 min (4) 60 min

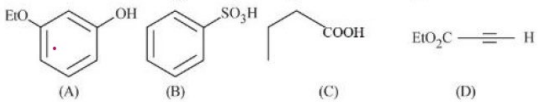
Answer (1)

Sol. $\underset{8B_0}{A} \xrightarrow{10} 4B_0 \xrightarrow{10} 2B_0 \xrightarrow{10} B_0 \xrightarrow{10} \frac{B_0}{2}$

$\underset{B_0}{B} \xrightarrow{40} \frac{B_0}{2}$

t = 40

64. The least acidic compound, among the following is



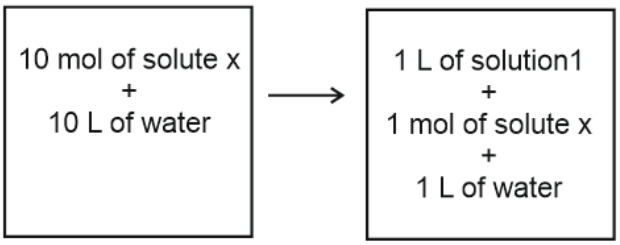
(1) A (2) D

(3) C (4) B

Answer (2)

Sol. Terminal alkyne is least Acidic among given compound

65. Which of the following properties will change when system containing solution 1 will become solution 2



(Solution 1) (Solution 2)

(1) Density (2) Concentration

(3) Gibbs free energy (4) Mol heat capacity

Answer (3)

Sol. Gibbs free energy will change.

66. The metal ions that have the calculated spin only magnetic moment value of 4.9 B.M. are

A. Cr^{2+} B. Fe^{2+}

C. Fe^{3+} D. Co^{2+}

E. Mn^{3+}

Choose the correct answer from the options given below:

(1) B and E only (2) A, B and E only

(3) A, D and E only (4) A, C and E only

Answer (2)

Sol. μ = 4.9 BM

n = 4 for Cr^{2+}; Fe^{2+}; Mn^{3+} (where n = unpaired electron)

67. Given below are two statements:

Statement I: The N – N single bond is weaker and longer than that of P – P single bond.

Statement II: Compounds of group 15 elements in + 3 oxidation states readily undergo disproportionation reactions.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Answer (4)

Sol. Bond energy : (N – N) < (P – P)

Bond length : (N – N) < (P – P)

Statement-I is false.

Not all elements of Group-15 undergo disproportionation reaction.

Statement-II is false.

68. Which of the following statements are correct?

A. The process of adding an electron to a neutral gaseous atom is always exothermic.

B. The process of removing an electron from an isolated gaseous atom is always endothermic.

C. The 1st ionization energy of boron is less than that of beryllium.

D. The electronegativity of C is 2.5 in CH_4 and CCl_4

E. Li is the most electropositive among elements of group I.

Choose the correct answer from the options given below:

(1) B and D Only (2) B, C and E Only

(3) A, C and D Only (4) B and C Only

Answer (4)

Sol. B and C are correct

IE : B < Be

69. The correct order of the complexes $[Co(NH_3)_5(H_2O)]^{3+}$ (A), $[Co(NH_3)_6]^{3+}$ (B), $[Co(CN)_6]^{3-}$ (C) and $[CoCl(NH_3)_5]^{2+}$ (D) in terms of wavelength of light absorbed is

(1) C > B > A > D (2) D > C > B > A

(3) C > B > D > A (4) D > A > B > C

Answer (4)

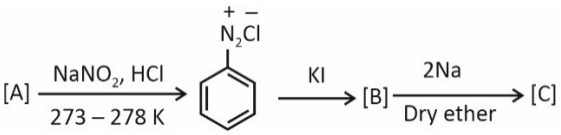
Sol. D > A > B > C

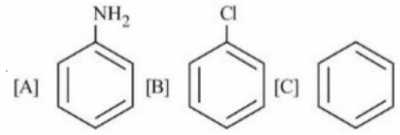
$$\Delta_o = \frac{hc}{\lambda}$$

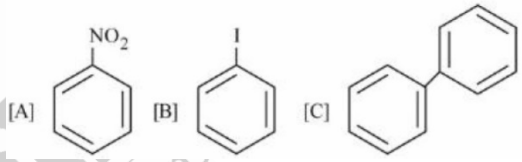
More the value of Δ_o more energy will be obseved

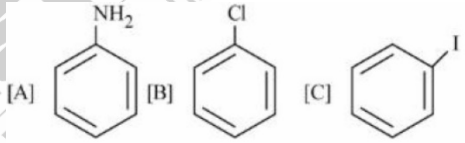
Δ_o increases and λ decreases.

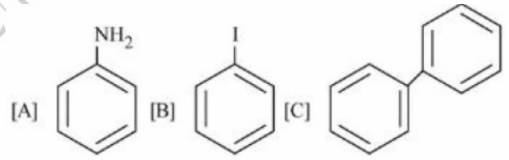
70. Identify [A], [B] and [C], respectively in the following reaction sequence:



(1) 

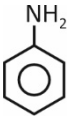
(2) 

(3) 

(4) 

Answer (4)

Sol.

A ; 

B : 

C : Ph – Ph

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. The number of optical isomers exhibited by the iron complex (A) obtained from the following reaction is _____.

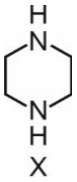
$FeCl_3 + KOH + H_2C_2O_4 \rightarrow A$

Answer (2)

Sol. Complex obtained is $[Fe(C_2O_4)_3]^{3-}$

Total isomers = 2 (d/l form)

72. During estimation of nitrogen by Dumas' method of compound X (0.42 g)



_____ mL of N_2 gas will be liberated at STP. (nearest integer)

(Given molar mass in g mol^{-1} : C : 12, H : 1, N : 14)

Answer (109)

Sol. Formula of compound = $C_4H_{10}N_2$

= 86

Moles of compound = 0.00488

Moles of N_2 = 0.00488

Volume of N_2 = 0.1093 L

= 109.3 mL

Nearest integer = 109

73. Given :

$\Delta H^{\Theta}_{sub}[C(graphite)] = 710\ kJ\ mol^{-1}$

$\Delta_{C-H}H^{\Theta} = 414\ kJ\ mol^{-1}$

$\Delta_{H-H}H^{\Theta} = 436\ kJ\ mol^{-1}$

$\Delta_{C=C}H^{\Theta} = 611\ kJ\ mol^{-1}$

The ΔH^{Θ}_f for $CH_2 = CH_2$ is _____ kJ mol^{-1} (nearest integer value)

Answer (25)

Sol. $\Delta H^{\Theta}_r = 2(710) + 2 \times 436 - 611 - 4(414)$

$= 1420 + 872 - 611 - 1656$

$= 25\ kJ\ mole^{-1}$

74. 0.5 g of an organic compound on combustion gave 1.46 g of CO_2 and 0.9 g of H_2O. The percentage of carbon in the compound is _____. (Nearest integer)

[Given : Molar mass (in g mol^{-1}) C : 12, H : 1, O : 16]

Answer (80)

Sol. Mass of C = $\frac{1.46}{44} \times 12 = 0.398$ g

% by mass of C = $\frac{0.398}{0.5} \times 100$

= 79.63%

Nearest integer = 80

75. Consider the following reactions

$A + \underset{\text{Little amount}}{NaCl} + H_2SO_4 \rightarrow CrO_2Cl_2 + \text{Side Products}$

$CrO_2Cl_{2(Vapour)} + NaOH \rightarrow B + NaCl + H_2O$

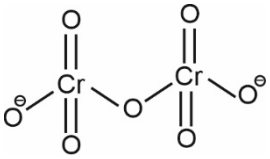
$B + H^+ \rightarrow C + H_2O$

The number of terminal 'O' present in the compound 'C' is _____.

Answer (6)

Sol. B : CrO_4^{2-}

C : $Cr_2O_7^{2-}$



Terminal oxygen atoms = 6

❐ ❐ ❐

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to

(1) 68 (2) 74
(3) 70 (4) 78

Answer (3)

Sol. $S : x^2 + y^2 - 2x - 4y - 11 = 0$

Centre $C(1, 2)$

radius $= \sqrt{1+4+11}$

$= 4$

Ellipse $3x^2 + py^2 = 4$ passes through $(1, 2)$

$3(1) + p(4) = 4$

$4p = 1$

$p = \frac{1}{4}$

$E : 3x^2 + \frac{y^2}{4} = 4$

or $\frac{x^2}{\frac{4}{3}} + \frac{y^2}{16} = 1$

$e = \sqrt{1 - \frac{\frac{4}{3}}{16}}$

$e = \sqrt{1 - \frac{1}{12}}$

$e = \sqrt{\frac{11}{12}}$

Focus $= \left(0, \pm 2\sqrt{\frac{11}{3}}\right)$

$f_1 = \sqrt{1 + \left(2 - 2\sqrt{\frac{11}{3}}\right)^2}$ and $f_2 = \sqrt{1 + \left(2 + 2\sqrt{\frac{11}{3}}\right)^2}$

$f_1 f_2 = \frac{37}{3}$

$\Rightarrow 6f_1f_2 - r = 74 - 7$

$= 70$

2. The integral $\int_{-1}^{\frac{3}{2}} \left(\left|\pi^2 x \sin(\pi x)\right|\right) dx$ is equal to:

(1) $3 + 2\pi$ (2) $2 + 3\pi$
(3) $4 + \pi$ (4) $1 + 3\pi$

Answer (4)

Sol. $I = \int_{-1}^{3/2} |\pi^2 x \sin(\pi x)| \, dx$

$= \int_{-1}^{1} |\pi^2 x \sin(\pi x)| \, dx + \int_{1}^{3/2} |\pi^2 x \sin(\pi x)| \, dx$

$= 2\int_{0}^{1} |\pi^2 x \sin(\pi x)| \, dx - \pi^2 \int_{1}^{3/2} |x \sin(\pi x)| \, dx$

$= 2\pi^2 \int_{0}^{1} |x \sin(\pi x)| \, dx - \pi^2 \int_{1}^{3/2} |x \sin(\pi x)| \, dx$

$\because \int x \sin(\pi x) dx = x\left(\frac{-\cos \pi x}{\pi}\right) - \int \frac{-\cos \pi x}{\pi} dx$

$= -\frac{x}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x + C$

$\therefore I = 2\pi^2\left(\frac{1}{\pi}\right) - \pi^2\left(-\frac{1}{\pi^2} - \frac{1}{\pi}\right)$

$= 2\pi + 1 + \pi$

$= 3\pi + 1$

3. The value of

$$\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)}\right)-\cot^{-1}\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)}+1}{\tan\left(\frac{1}{2}\right)}\right)$$

is equal to

(1) $\pi-\frac{3}{2}$ (2) $\pi-\frac{5}{4}$

(3) $\pi+\frac{3}{2}$ (4) $\pi+\frac{5}{2}$

Answer (2)

Sol. $\cot^{-1}\left(\frac{\sqrt{1+\tan^2 2}-1}{\tan 2}\right)-\cot^{-1}\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)}+1}{\tan\frac{1}{2}}\right)$

$$=\cot^{-1}\left(\frac{|\sec 2|-1}{\tan 2}\right)-\cot^{-1}\left(\frac{\left|\sec\left(\frac{1}{2}\right)\right|+1}{\tan\frac{1}{2}}\right)$$

$$=\cot^{-1}\left(\frac{-\sec 2-1}{\tan 2}\right)-\cot^{-1}\left(\frac{\sec\frac{1}{2}+1}{\tan\frac{1}{2}}\right)$$

$$=\pi-\cot^{-1}\left(\frac{1+\cos 2}{\sin 2}\right)-\cot^{-1}\left(\frac{1+\cos\frac{1}{2}}{\sin\frac{1}{2}}\right)$$

$$=\pi-\cot^{-1}\left(\frac{2\cos^2 1}{2\sin 1\cdot\cos 1}\right)-\cot^{-1}\left(\frac{2\cos^2\frac{1}{4}}{2\sin\frac{1}{4}\cdot\cos\frac{1}{4}}\right)$$

$$=\pi-\cot^{-1}(\cot 1)-\cot^{-1}\left(\cot\frac{1}{4}\right)$$

$$=\pi-1-\frac{1}{4}$$

$$=\pi-\frac{5}{4}$$

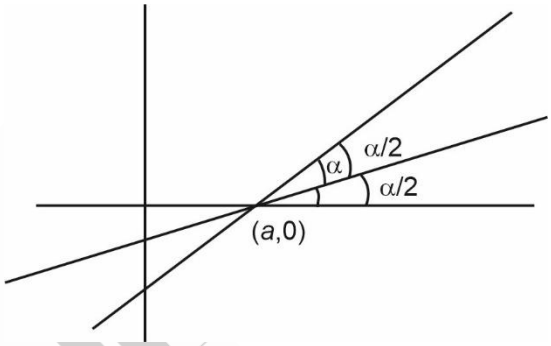
4. A line passing through the point $P(a, 0)$ makes an acute angle α with the positive x-axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2-\sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2\tan^2\alpha-2\sqrt{3}$ is

(1) 8 (2) 6

(3) 5 (4) 4

Answer (4)

Sol.



$\tan\frac{\alpha}{2}=2-\sqrt{3}$

$\Rightarrow \tan\alpha=\frac{1}{\sqrt{3}}$

Equation of new line: $(y-0)=(2-\sqrt{3})(x-a)$

$y=(2-\sqrt{3})x-(2-\sqrt{3})a$

Distance from origin $=\frac{1}{\sqrt{2}}$

$$\left|\frac{-(2-\sqrt{3})a}{4+3-4\sqrt{3}+1}\right|=\frac{1}{\sqrt{2}}$$

$$|a|=\frac{\sqrt{8-4\sqrt{3}}}{\sqrt{2}(2-\sqrt{3})}$$

$$|a|=\frac{2\sqrt{2-\sqrt{3}}}{\sqrt{2}(2-\sqrt{3})}$$

$$|a| = \frac{\sqrt{2}}{\sqrt{2-\sqrt{3}}}$$

$$a^2 = \frac{2}{2-\sqrt{3}} = 2(2+\sqrt{3})$$

$$3a^2\tan^2\alpha - 2\sqrt{3} = 3(4+2\sqrt{3})\times\frac{1}{3} - 2\sqrt{3}$$

$= 4$

5. Let $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$.

If $\det(\text{adj}(\text{adj}(3A))) = 2^m \cdot 3^n$, $m, n \in \mathbb{N}$, then $m + n$ is equal to

(1) 20 (2) 26

(3) 22 (4) 24

Answer (4)

Sol. $\begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$

$$= \begin{vmatrix} 2 & 2 & 2+p+q \\ 4 & 6 & 8+3p+2q \\ 6 & 12 & 20+6p+3q \end{vmatrix} + \underbrace{\begin{vmatrix} 2 & p & 2+p+q \\ 4 & 2p & 8+3p+2q \\ 6 & 3p & 20+6p+3q \end{vmatrix}}_{=0}$$

$$= 2\times 2\begin{vmatrix} 1 & 1 & 2+p+q \\ 2 & 3 & 8+3p+2q \\ 3 & 6 & 20+6p+3q \end{vmatrix}$$

$C_3 \to C_3 \to pC_2$

$$= 4\begin{vmatrix} 1 & 1 & 2+q \\ 2 & 3 & 8+2q \\ 3 & 6 & 20+3q \end{vmatrix} = 4\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 8 \\ 3 & 6 & 20 \end{vmatrix} + 0$$

$$= 4\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

$= 8\,(1(6) - 1(8) + 1(3))$

$= 8$

$$|\text{adj}(\text{adj}(3A))| = (|3A|)^{2^2} = |3A|^4$$

$$= (3^3|A|)^4 = 3^{12}\cdot|A|^4$$

$$= 3^{12}\cdot(2^3)^4$$

$$= 3^{12}\cdot 2^{12}$$

6. Given below are two statements:

Statement I: $\lim\limits_{x\to 0}\left(\dfrac{\tan^{-1}x + \log_e\sqrt{\dfrac{1+x}{1-x}} - 2x}{x^5}\right) = \dfrac{2}{5}$

Statement II: $\lim\limits_{x\to 1}\left(x^{\frac{2}{1-x}}\right) = \dfrac{1}{e^2}$

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Answer (3)

Sol. $\lim\limits_{x\to 0}\left(\dfrac{\tan^{-1}x + \ln\sqrt{\dfrac{1+x}{1-x}} - 2x}{x^5}\right)$

$$\Rightarrow \lim_{x\to 0}\frac{\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\right) + \frac{1}{2}\left(+x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \ldots\right)}{-\frac{1}{2}\left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} + \ldots\right) - 2x}$$

$$\Rightarrow \lim_{x\to 0}\frac{\left(x + \frac{1}{2}(x+x) - 2x\right) + x^3\left(\frac{-1}{3} + \frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times\frac{1}{3}\right)}{+x^5\left(\frac{1}{5} + \frac{1}{2}\times\frac{1}{5} + \frac{1}{2}\times\frac{1}{5}\right) + \ldots}$$

$$\Rightarrow \lim_{x\to 0}\frac{\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{10}\right)x^5}{x^5} = \frac{2}{5}$$

$$\Rightarrow \lim_{x\to 1} x^{\left(\frac{2}{1-x}\right)} = \lim_{x\to 1}\left([1+(x-1)]^{\frac{1}{x-1}}\right)^{\frac{(x-1)2}{(1-x)}}$$

$$= e^{-2} = \frac{1}{e^2}$$

7. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{c}$ be a unit vector in the plane of the vector $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is:

(1) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$

(2) $\frac{1}{\sqrt{5}}\left(\hat{j} - 2\hat{k}\right)$

(3) $\frac{1}{\sqrt{2}}\left(-\hat{i} + \hat{k}\right)$

(4) $\frac{1}{\sqrt{3}}\left(-\hat{i} + \hat{j} - \hat{k}\right)$

Answer (3)

Sol. $\vec{c} = x\vec{a} + y\vec{b}$

$\vec{c} = x(\hat{i} + 2\hat{j} + \hat{k}) + y(2\hat{i} + \hat{j} - \hat{k})$

$\vec{a} \cdot \vec{c} = (\hat{i} + 2\hat{j} + \hat{k}) \cdot (x(\hat{i} + 2\hat{j} + \hat{k}) + y(2\hat{i} + \hat{j} - \hat{k}))$

$(\hat{i} + 2\hat{j} + \hat{k}) \cdot (x\hat{i} + 2x\hat{j} + x\hat{k}) + 2y\hat{i} + y\hat{j} - y\hat{k} = 0$

$\Rightarrow (x + 2y) + 2(x + 9) + (x - y) = 0$

$\Rightarrow y = -2x$

$\therefore \vec{c} = x(-3\hat{i} + 3\hat{k})$

$|\vec{c}| = |x|\sqrt{9+9} = 3|x|\sqrt{2}$

$\therefore |\vec{c}| = 1$

$3|x|\sqrt{2} = 1$

$|x| = \frac{1}{3\sqrt{2}}$

Let $x = \frac{1}{3\sqrt{2}}$

$\vec{c} = \frac{1}{3\sqrt{2}}(-3\hat{i} + 3\hat{k})$

or $\vec{c} = \frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

8. Let the value of λ for which the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-\lambda}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2. Then the radius of the circle passing through the points (0, 0), (λ_1, λ_2) and (λ_2, λ_1) is

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{5\sqrt{2}}{3}$

(3) 3 (4) 4

Answer (2)

Sol. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ ...(1)

$\frac{x-\lambda}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ ...(2)

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$

$= \hat{i}(15-16) - \hat{j}(10-12) + \hat{k}(8-9)$

$= -\hat{i} + 2\hat{j} - \hat{k}$

L_1 passing through (1, 2, 3) and L_2 through (λ, 4, 5)

$d = \frac{1}{\sqrt{6}}$

$\Rightarrow \frac{|(\lambda-1)(-1) - 2(-2) + 2(-1)|}{\sqrt{1^2 + 4^2 + 1}} = \frac{1}{\sqrt{6}}$

$|-\lambda + 1 + 4 - 2| = 1$

$|-\lambda + 3| = 1$

$\lambda - 3 = \pm 1$

$\lambda = 4, 2$

Circle passing through (0, 0), (1, 4) (4, 1)

$\therefore$ Area $= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 4 & 1 \\ 4 & 1 & 1 \end{vmatrix}$

$= \left|\frac{1}{2}(1-16)\right| = \frac{15}{2}$

$\therefore r = \frac{abc}{4\Delta}$

$= \frac{5\sqrt{2}}{3}$

9. Let $A = \{0, 1, 2, 3, 4, 5\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $\max\{x, y\} \in \{3, 4\}$. Then among the statements

(S_1) : The number of elements in R is 18, and

(S_2): The relation R is symmetric but neither reflexive nor transitive.

(1) only (S_2) is true (2) both are false

(3) only (S_1) is true (4) both are true

Answer (1)

Sol. Let's write the pairs (x, y) in R

(0, 3), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (3, 0), (3, 1), (3, 2), (3, 4), (3, 5), (0, 4), (1, 4), (2, 4), (4, 4), (5, 4), (4, 0), (4, 1), (4, 2), (4, 5)

There are total 20 pairs

If $(x, y) \in R$, then $\max(x, y) \in \{3, 4\}$

This means, $\max(y, x) \in \{3, 4\}$. So, $(y, x) \in R$.

Thus R is symmetric

Since $\max(5, 5) = 5 \notin \{3, 4\}$, $(5, 5) \notin R$

Thus R is not reflexive

$(3, 4) \in R$ & $(4, 2) \in R$, but $(3, 2) \notin R$, $2 \notin \{3, 4\}$

Thus, R is not transitive

Therefore, only S_2 is true

10. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\infty=\frac{\pi^4}{90}$,

$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\infty=\alpha$

$\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\ldots\infty=\beta$,

Then $\frac{\alpha}{\beta}$ is equal to

(1) 14 (2) 15

(3) 18 (4) 23

Answer (2)

Sol. $\alpha=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots$

$\beta=\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\ldots$

$=\frac{1}{2^4}\left[\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\ldots\right]$

$\Rightarrow 16\beta=\left[\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\right]+\left[\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\ldots\right]$

$=\alpha+\beta \Rightarrow 15\beta=\alpha \Rightarrow \frac{\alpha}{\beta}=15$

11. Let the function $f(x)=\frac{x}{3}+\frac{3}{x}+3$, $x \neq 0$ be strictly increasing in $(-\infty, \alpha_1) \cup (\alpha_2, \infty)$ and strictly decreasing in $(\alpha_3, \alpha_4) \cup (\alpha_4, \alpha_5)$. Then $\sum_{i=1}^{5}\alpha_i^2$ is equal to

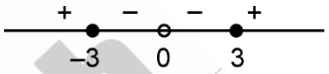
(1) 48 (2) 40

(3) 36 (4) 28

Answer (3)

Sol. $f(x)=\frac{x}{3}+\frac{3}{x}+3$, $x \neq 0$

$f'(x)=\frac{1}{3}-\frac{3}{x^2}=\left(\frac{x^2-9}{3x^2}\right)=\frac{(x-3)(x+3)}{3x^2}$



$\Rightarrow f'(x) > 0 \ \forall \ x \in (-\infty, -3) \cup (3, \infty)$

$f'(x) < 0 \ \forall \ x \in (-3, 0) \cup (0, 3)$

$\Rightarrow \alpha_1 = -3, \alpha_2 = 3, \alpha_3 = -3, \alpha_4 = 0, \alpha_5 = 3$

$\Rightarrow \sum_{i=1}^{5}\alpha_i^2=(-3)^2+(3^2)+0^2+(-3)^2+(3)^2$

$= 4(9) = 36$

12. If A and B are two events such that $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \cap \overline{B}) = 0.5$, where $\overline{B}$ denotes the complement of B, then $P\left(B\middle|\left(A \cup \overline{B}\right)\right)$ is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{1}{3}$ (4) $\frac{1}{6}$

Answer (2)

Sol. $P(A) = 0.7$

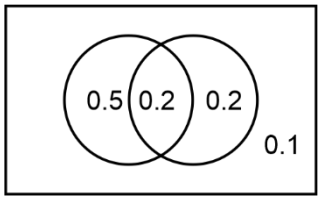
$P(B) = 0.4$

$P(A \cap B^C) = 0.5$

$$P\left(\frac{B}{A\cup B^C}\right)=\frac{P(B\cap(A\cup B^C))}{P(A\cup B^C)}$$

$$=\frac{P(A\cap B)}{P(A\cup B^C)}$$

Using Venn diagram



$$\Rightarrow \frac{P(A\cap B)}{P(A\cup B^C)}=\frac{0.2}{0.5+0.2+0.1}=\frac{2}{8}=\frac{1}{4}$$

13. Let $f(x) = x - 1$ and $g(x) = e^x$ for $x \in \mathbb{R}$. If $\frac{dy}{dx}=\left(e^{-2\sqrt{x}}g(f(f(x)))-\frac{y}{\sqrt{x}}\right)$, $y(0) = 0$, then $y(1)$ is

(1) $\frac{2e-1}{e^3}$ (2) $\frac{1-e^2}{e^4}$

(3) $\frac{1-e^3}{e^4}$ (4) $\frac{e-1}{e^4}$

Answer (4)

Sol. $f(x) = x - 1$

$f(f(x)) = (x - 1) - 1 = x - 2$

$g(f(f(x))) = e^{x-2}$

$$\frac{dy}{dx}=e^{-2\sqrt{x}}(x-2)-\frac{y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{\sqrt{x}}=e^{-2\sqrt{x}}e^{x-2}$$

$$\text{IF}=e^{\int x^{-1/2}dx}=e^{2\sqrt{x}}$$

$$y.e^{2\sqrt{x}}=\int e^{-2\sqrt{x}}e^{x-2}e^{2\sqrt{x}}dx$$

$$=y.e^{2\sqrt{x}}=e^{x-2}+c$$

$y(0) = 0 \Rightarrow c = e^{-2}$

$$\therefore\ y.e^{2\sqrt{x}}=e^{x-2}-e^{-2}$$

$$y.e^2=e^{-1}-e^{-2}=\frac{1}{e}-\frac{1}{e^2}=\frac{e-1}{e^2}$$

$$y=\frac{e-1}{e^4}$$

14. The sum of the squares of the roots of $|x - 2|^2 + |x - 2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x - 3| - 5 = 0$, is

(1) 24 (2) 30

(3) 36 (4) 26

Answer (3)

Sol. $|x - 2|^2 + |x - 2| - 2 = 0$

$\Rightarrow (|x - 2| + 2)(|x - 2| - 1) = 0$

$\Rightarrow |x - 2| = 1$

$\Rightarrow x = 3, 1$

$x^2 - 2|x - 3| - 5 = 0$

$x \geq 3$

$x^2 - 2(x - 3) - 5 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1$ (rejected)

$x < 3$

$x^2 + 2(x - 3) - 5 = 0$

$\Rightarrow x^2 + 2x - 11 = 0$

$x = -1 \pm 2\sqrt{3}$

Sum of square of roots

$$=3^2+1^2+\left(-1+2\sqrt{3}\right)^2+\left(-1-2\sqrt{3}\right)^2$$

$$=10+(1+12)2$$

= 36

15. Let

$$A=\left\{\theta\in[0,\,2\pi]:1+10\,\text{Re}\left(\frac{2\cos\theta+i\sin\theta}{\cos\theta-3i\sin\theta}\right)=0\right\}.$$

Then $\sum_{\theta\in A}\theta^2$ is equal to

(1) $\frac{27}{4}\pi^2$ (2) $8\pi^2$

(3) $6\pi^2$ (4) $\frac{21}{4}\pi^2$

Answer (4)

Sol. $\text{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right)$

$\text{Re}\left(\frac{(2\cos\theta + i\sin\theta)(\cos\theta + 3i\sin\theta)}{\cos^2\theta + 9\sin^2\theta}\right)$

$= \frac{2\cos^2\theta - 3\sin^2\theta}{1 + 8\sin^2\theta}$

Now, $1 + 10\left(\frac{2\cos^2\theta - 3\sin^2\theta}{1 + 8\sin^2\theta}\right) = 0$

$= 1 + 8\sin^2\theta + 20\cos^2\theta - 30\sin^2\theta = 0$

$= 1 - 22\sin^2\theta + 20\cos^2\theta = 0$

$= 1 + 20(\cos 2\theta) - 2\sin^2\theta = 0$

$= 20\cos 2\theta + \cos 2\theta = 0$

$= 21\cos 2\theta = 0$

$2\theta = (2n+1)\frac{\pi}{2},\ n \in I$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\sum_{\theta \in A} \theta^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} + \frac{25\pi^2}{16} + \frac{49\pi^2}{16}$

$= \frac{84\pi^2}{16}$

$= \frac{21\pi^2}{4}$

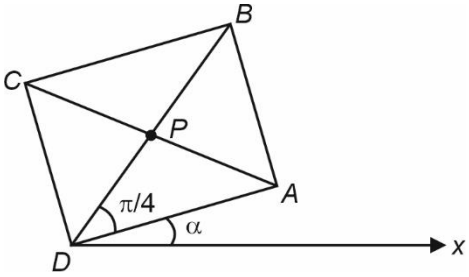
16. Let a be the length of a side of a square $OABC$ with O being the origin. Its side OA makes an acute angle α with the positive x-axis and the equations of its diagonals are $\left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 0$ and $\left(\sqrt{3}-1\right)x - \left(\sqrt{3}+1\right)y + 8\sqrt{3} = 0$. Then a^2 is equal to

(1) 24 (2) 32

(3) 16 (4) 48

Answer (4)

Sol.



$OB : \left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 0$

$AC : \left(\sqrt{3}-1\right)x - \left(\sqrt{3}+1\right)y + 8\sqrt{3} = 0$

$\Rightarrow (x, y) \equiv P\left(3-\sqrt{3}, 3+\sqrt{3}\right)$

Let $AB = a = OA$

$\Rightarrow OA^2 + AB^2 = OB^2$

$2a^2 = 4\left[\left(3-\sqrt{3}\right)^2 + \left(3-\sqrt{3}\right)^2\right]$

$a^2 = 2 \times 24 = 48$

17. Let $f(x)$ be a positive function and $I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))dx$ and $I_2 = \int_{-1}^{2} f(x(1-x))dx$. Then the value of $\frac{I_2}{I_1}$ is equal to _____.

(1) 12

(2) 4

(3) 6

(4) 9

Answer (2)

Sol. $I_1 = \int_{-\frac{1}{2}}^{1} 2xf\left(2x(1-2x)dx\right)$

$I_1 = \int_{-\frac{1}{2}}^{1} 2\left(\frac{1}{2} - x\right)f\left(2\left(\frac{1}{2} - x\right)\left(1 - 2\left(\frac{1}{2} - x\right)\right)\right)dx$

$I_1 = \int_{-\frac{1}{2}}^{1} (1-2x)f\left((1-2x)(2x)\right)dx$

$$I_1 = \int_{-\frac{1}{2}}^{1} f\left((1-2x)(2x)\right)dx - \int_{-\frac{1}{2}}^{1} 2x \underbrace{f\left((1-2x)(2x)\right)dx}_{I_1}$$

$$2I_1 = \int_{-\frac{1}{2}}^{1} f\left((1-2x)(2x)\right)dx$$

Put $2x = t$

$2dx = dt$

$$dx = \frac{dt}{2}$$

$$2I_1 = \frac{1}{2}\int_{-1}^{2} f\left((1-t)(t)\right)dt$$

$$I_1 = \frac{1}{4}\int_{-1}^{2} f\left((1-x)(x)\right)dx$$

$$I_1 = \frac{1}{4} I_2$$

$$4 \Rightarrow \frac{I_2}{I_1}$$

18. The number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1016}$ is

(1) 127

(2) 128

(3) 129

(4) 130

Answer (2)

Sol. $T_{r+1} = 1016_{C_r} \cdot \left(7^{\frac{1}{8}}\right)^4 \cdot 5^{\frac{1}{2}(1016-r)}$

$\Rightarrow \frac{r}{8}$ and $\frac{1016-r}{2} \in$ Integer

$\Rightarrow 8 / r \Rightarrow r = 0, 8, 16, ...1016$

$\Rightarrow r = 0 \times 8, 1 \times 8, ... 127 \times 8$

$\Rightarrow$ Total 128 r such that T_{r+1} is rational

19. There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is

(1) 230 (2) 200

(3) 220 (4) 210

Answer (4)

Sol.

Number of triangle = ${}^5C_1 \times {}^7C_2 + {}^7C_3 + {}^5C_2 + {}^7C_1$

= 210

20. Let α be a solution of $x^2 + x + 1 = 0$, and for some a and b in **R**, $\begin{bmatrix}4 & a & b\end{bmatrix}\begin{bmatrix}1 & 16 & 13\\ -1 & -1 & 2\\ -2 & -14 & -8\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}$.

If $\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$, then $m + n$ is equal to _____.

(1) 7

(2) 11

(3) 3

(4) 8

Answer (2)

Sol. $x^2 + x + 1 = 0$

$w \quad w^2$

$$\begin{bmatrix}4 & a & b\end{bmatrix}\begin{bmatrix}1 & 16 & 13\\ -1 & -1 & 2\\ -2 & -14 & -8\end{bmatrix}_{3\times3} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}$$

$\Rightarrow [4 - a - 2b, 64 - a - 14b, 52 + 2a - 8b] = [0\ 0\ 0]$

$a + 2b = 4$ …(i)

$a + 14b = 64$ …(ii)

Solving (i) and (ii)

We get $a = -6$, $b = 5$

$\therefore \frac{4}{\alpha^4}+\frac{m}{\alpha^a}+\frac{n}{\alpha^b}=3$

$\Rightarrow \frac{4}{w^4}+\frac{m}{w^{-6}}+\frac{n}{w^5}=3$

$\Rightarrow 4w^2 + m + nw = 3$ …(i)

For $\alpha = w^2$,

$\frac{4}{w^8}+\frac{m}{w^{-12}}+\frac{n}{w^{10}}=3$

$\Rightarrow \frac{4}{w^2}+m+\frac{n}{w}=3$

$\Rightarrow 4w + m + nw^2 = 3$ ….(ii)

Adding (i) & (ii)

$\Rightarrow 4(w^2 – w) + n(w – w^2) = 0$

$\Rightarrow (w^2 – w)(4 – n) = 0$

$\Rightarrow n = 4$

$\therefore 4w + m + 4w^2 = 3$

$\Rightarrow –4 + m = 3$

$\Rightarrow m = 7$

$\therefore m + n = 11$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The product of the last two digits of $(1919)^{1919}$ is _____.

Answer (63)

Sol. $(1919)^{1919} = (1920 – 1)^{1919}$

$= {}^{1919}C_0(1920)^{1919} – {}^{1919}C_1(1920)^{1918} + ... + {}^{1919}C_{1918}(1920) – {}^{1919}C_{1919}$

For last two digit $\Rightarrow {}^{1919}C_{1919}(1920) – 1$

= 3684479

$\therefore$ Product of last two digit = 63

22. Let the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha, \beta]$ and the domain of $g(x) = \log_2(2 – 6\log_{27}(2x + 5))$ be (γ, δ).

Then $|7(\alpha + \beta) + 4(\gamma + \delta)|$ is equal to _____.

Answer (96)

Sol. $f(x)=\cos^{-1}\left(\frac{4x+5}{3x-7}\right)$

$-1\le\frac{4x+5}{3x-7}\le 1$

$\frac{7x-2}{3x-7}\ge 0,\ \frac{x+12}{3x-7}\le 0$

$x\in\left(-\infty,\frac{2}{7}\right]\cup\left(\frac{7}{3},\infty\right),\ x\in\left[-12,\frac{7}{3}\right)$

$\Rightarrow x\in\left[-12,\frac{2}{7}\right]\Rightarrow \alpha=-12,\ \beta=\frac{2}{7}$

$g(x) = \log_2(2 – 6\log_{27}(2x+5))$

$\Rightarrow 2-6\log_{27}(2x+5)>0,\ \boxed{2x+5>0}$

$\log_{27}(2x+5)<\frac{1}{3}\Rightarrow(2x+5)^3<27$

$\Rightarrow 2x+5<3\Rightarrow \boxed{x<-1}$

$\Rightarrow x\in\left(-\frac{5}{2},-1\right)\Rightarrow \gamma=-\frac{5}{2},\ \delta=-1$

$\Rightarrow |7(\alpha + \beta) + 4(\gamma + \delta)| = 96$

23. Let r be the radius of the circle, which touches x-axis at point $(a, 0)$, $a < 0$ and the parabola $y^2 = 9x$ at the point $(4, 6)$. Then r is equal to _____.

Answer (30)

Sol. Equation of tangent to $y^2 = gx$ at (4, 6) is $3x - 4y + 12 = 0$

Equation of circle is $(x-4)^2 + (y-6)^2 + \lambda(3x - 4y + 12) = 0$

$\Rightarrow x^2 + y^2 + (3\lambda - 8)x + (-12 - 4\lambda)y + 52 + 12\lambda = 0$

$\because \quad 2\sqrt{g^2 - c} = 0 \Rightarrow g^2 = c$

$\Rightarrow \left(-\frac{3\lambda - 8}{2}\right)^2 = 52 + 12\lambda$

$\Rightarrow 9\lambda^2 + 64 - 48\lambda = 208 + 48\lambda \Rightarrow 9\lambda^2 - 96\lambda - 144 = 0$

$\Rightarrow \lambda = 12, -\frac{2}{3} \Rightarrow f = -30, -\frac{14}{3}$

$\Rightarrow r = \sqrt{g^2 + f^2 - c} = |f| = |-(2\lambda + 6)|$

$\because$ centre lies in 2nd quadrant

$\Rightarrow 3\lambda - 8 > 0 \Rightarrow \lambda > \frac{8}{3}$

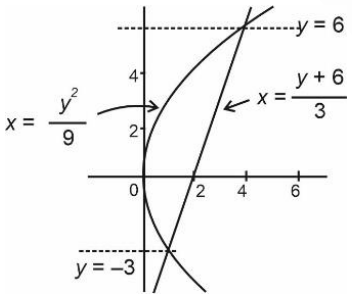
$\Rightarrow \lambda = 12, f = -30, r = 30$

24. Let the area of the bounded region $\{(x, y) : 0 \le 9x \le y^2, y \ge 3x - 6\}$ be A. Then $6A$ is equal to _____.

Answer (Bonus)

Sol. The given problem has area bounded $\to \infty$

However, if we correct in to $y^2 \le 4x$ then here is the solution.



Required area

$$= \int_{-3}^{3}\left(\frac{y+6}{3} - \frac{y^2}{9}\right)dy$$

$$= \frac{27}{2} \text{ sq. units}$$

25. Let the area of the triangle formed by the lines $x + 2 = y - 1 = z$, $\frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1}$ and $\frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1}$ be A. Then A^2 is equal to _____.

Answer (56)

Sol. $L_1 = \frac{x+2}{1} = \frac{y-1}{1} = \frac{z}{1} = \lambda$, any point on it $(\lambda - 2, \lambda + 1, \lambda)$

$L_2 = \frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1} = \mu$, any point on it $(5\mu + 3, -\mu, \mu + 1)$

$L_3 = \frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1} = k$, any point on it $(-3k, 3k + 3, k + 2)$

$P \equiv$ point of intersection of L_1 and $L_2 = (-2, 1, 0)$

$Q \equiv$ point of intersection of L_1 and $L_3 = (0, 3, 2)$

$R \equiv$ point of intersection of L_2 and $L_3 = (3, 0, 1)$

$\overrightarrow{PQ} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$\overrightarrow{PR} = 5\hat{i} - \hat{j} + \hat{k}$

$A = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{56}$

$A^2 = 56$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answers :

26. A rod of linear mass density 'λ' and length 'L' is bent to form a ring of radius 'R'. Moment of inertia of ring about any of its diameter is

(1) $\frac{\lambda L^3}{4\pi^2}$ (2) $\frac{\lambda L^3}{8\pi^2}$

(3) $\frac{\lambda L^3}{12}$ (4) $\frac{\lambda L^3}{16\pi^2}$

Answer (2)

Sol. $2\pi R = L \Rightarrow M = \lambda L$

$$R = \frac{L}{2\pi}$$

Moment of Inertia about diameter

$$= \frac{MR^2}{2} = \frac{M}{2}\left(\frac{L}{2\pi}\right)^2 = \frac{\lambda L^3}{8\pi^2}$$

27. In a Young's double slit experiment, the source is white light. One of the slits is covered by red filter and another by a green filter. In this case

(1) There shall be alternate interference fringes of red and green.

(2) There shall be an interference pattern for red distinct from that for green.

(3) There shall be no interference fringes.

(4) There shall be an interference pattern, where each fringe's pattern center is green and outer edges is red.

Answer (3)

Sol. Frequency of Green and frequency of Red will be different. No interference pattern is observed for two lights of different frequencies as phase difference does not remain constant.

28. Two balls with same mass and initial velocity, are projected at different angles in such a way that maximum height reached by first ball is 8 times higher than that of the second ball. T_1 and T_2 are the total flying times of first and second ball, respectively, then the ratio of T_1 and T_2 is

(1) $2\sqrt{2}:1$ (2) $\sqrt{2}:1$

(3) $4:1$ (4) $2:1$

Answer (1)

Sol. $H_1 = \frac{u^2\sin^2\theta_1}{2g}$ $H_2 = \frac{u^2\sin^2\theta_2}{2g}$

$T_1 = \frac{2u\sin\theta_1}{g}$ $T_2 = \frac{2u\sin\theta_2}{g}$

$\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2}$ $\frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2}$

$8 = \left(\frac{T_1}{T_2}\right)^2 \Rightarrow \frac{T_1}{T_2} = 2\sqrt{2}$

$T_1 : T_2 = 2\sqrt{2}:1$

29. For a nucleus of mass number A and radius R, the mass density of nucleus can be represented as

(1) A^3 (2) $A^{\frac{2}{3}}$

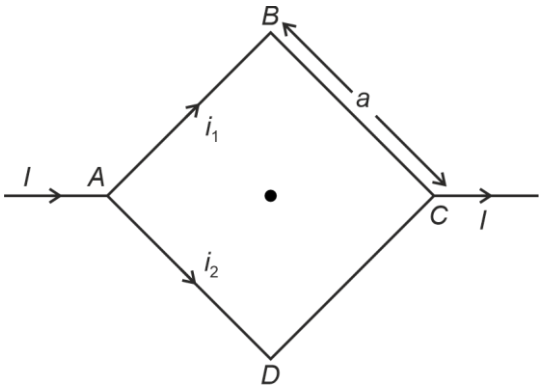
(3) $A^{\frac{1}{3}}$ (4) Independent of A

Answer (4)

Sol. $R = R_0A^{1/3}$

$$\text{Density} = \frac{\text{Mass number}}{\text{Volume}} = \frac{A}{\frac{4}{3}\pi R_0^3 A} = \text{constant}$$

30. Figure shows a current carrying square loop *ABCD* of edge length is '*a*' lying in a plane. If the resistance of the *ABC* part is *r* and that of *ADC* part is 2*r*, then the magnitude of the resultant magnetic field at centre of the square loop is

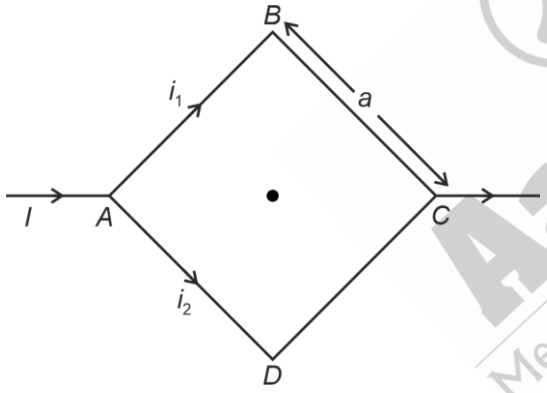


(1) $\frac{\sqrt{2}\mu_0 I}{3\pi a}$ (2) $\frac{\mu_0 I}{2\pi a}$

(3) $\frac{2\mu_0 I}{3\pi a}$ (4) $\frac{3\pi\mu_0 I}{\sqrt{2}a}$

Answer (1)

Sol.



$R_{ABC} = r$ $R_{ADC} = 2r$

$i_1 = \frac{2I}{3}$ $i_2 = \frac{I}{3}$

$$B_{\text{centre}} = \frac{2\left(\mu_0\sqrt{2}\right)}{4\pi\left(\frac{a}{2}\right)}\left[\frac{2I}{3} - \frac{I}{3}\right] = \sqrt{2}\frac{\mu_0 I}{3\pi a}$$

31. A convex lens of focal length 30 cm is placed in contact with a concave lens of focal length 20 cm. An object is placed at 20 cm to the left of this lens system. The distance of the image from the lens in cm is _____.

(1) 30 (2) $\frac{60}{7}$

(3) 15 (4) 45

Answer (3)

Sol. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{-20} = -\frac{1}{60}$

$$v = \frac{Fu}{u+F} = \frac{-60\times -20}{-20+-60} = -15 \text{ cm}$$

32. The amplitude and phase of a wave that is formed by the superposition of two harmonic travelling waves, $y_1(x, t) = 4\sin(kx - \omega t)$ and $y_2(x, t) = 2\sin\left(kx - \omega t + \frac{2\pi}{3}\right)$, are :

(Take the angular frequency of initial waves same as ω)

(1) $\left[2\sqrt{3}, \pi/6\right]$ (2) $\left[6, 2\pi/3\right]$

(3) $\left[6, \pi/3\right]$ (4) $\left[\sqrt{3}, \pi/6\right]$

Answer (1)

Sol. $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\phi = \frac{2\pi}{3}$

$$A = \sqrt{4^2 + 2^2 + 2\times 4\times 2\cos\frac{2\pi}{3}} = 2\sqrt{3}$$

$\tan\alpha = \frac{2\sin\phi}{4+2\cos\phi} = \frac{1}{\sqrt{3}}$

$\alpha = \pi/6$

33. A body of mass 2 kg moving with velocity of $\vec{v}_{in} = 3\hat{i} + 4\hat{j}$ ms^{-1} enters into a constant force field of 6N directed along positive z-axis. If the body remains in the field for a period of $\frac{5}{3}$ seconds, then velocity of the body when it emerges from force field is :

(1) $3\hat{i} + 4\hat{j} + 5\hat{k}$ (2) $3\hat{i} + 4\hat{j} - 5\hat{k}$

(3) $4\hat{i} + 3\hat{j} + 5\hat{k}$ (4) $3\hat{i} + 4\hat{j} + \sqrt{5}\hat{k}$

Answer (1)

Sol. $\vec{F} \cdot t = m(\vec{v}_2 - \vec{v}_1)$

$$\frac{6}{2} \times \frac{5}{3}\hat{k} = \vec{v}_2 - \vec{v}_1$$

$$\vec{v}_2 = \vec{v}_1 + 5\hat{k} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

34. A quantity Q is formulated as $X^{-2}Y^{+\frac{3}{2}}Z^{-\frac{2}{5}}$. X, Y and Z are independent parameters which have fractional errors of 0.1, 0.2 and 0.5, respectively in measurement. The maximum fractional error of Q is

(1) 0.1 (2) 0.6

(3) 0.7 (4) 0.8

Answer (3)

Sol. $$\frac{\Delta Q}{Q} = -2\frac{(\pm\Delta x)}{x} + \frac{3}{2}\frac{(\pm\Delta y)}{y} - \frac{2}{5}\frac{(\pm\Delta z)}{z}$$

For maximum fractional error

$$\frac{\Delta Q}{Q} = \frac{2\Delta x}{x} + \frac{3}{2}\frac{\Delta y}{y} + \frac{2}{5}\frac{\Delta z}{z}$$

$$= 2 \times 0.1 + \frac{3}{2} \times 0.2 + \frac{2}{5} \times 0.5$$

$$= 0.7$$

35. A monoatomic gas having $\gamma = \frac{5}{3}$ is stored in a thermally insulated container and the gas is suddenly compressed to $\left(\frac{1}{8}\right)^{th}$ of its initial volume. The ratio of final pressure and initial pressure is (γ is the ratio of specific heats of the gas at constant pressure and at constant volume)

(1) 40 (2) 28

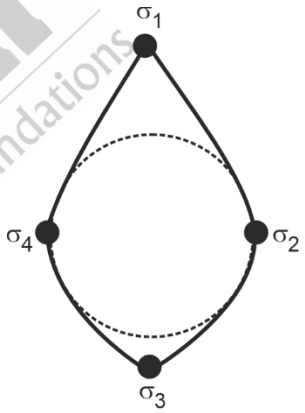
(3) 32 (4) 16

Answer (3)

Sol. $P_1 v_1^{\gamma} = P_2 v_2^{\gamma}$ $v_2 = \frac{v_1}{8}$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (8)^{\frac{5}{3}} = 32$$

36. Electric charge is transferred to an irregular metallic disk as shown in figure. If σ_1, σ_2, σ_3 and σ_4 are charge densities at given points then, choose the correct answer from the options given below:



A. $\sigma_1 > \sigma_3$; $\sigma_2 = \sigma_4$

B. $\sigma_1 > \sigma_2$; $\sigma_3 > \sigma_4$

C. $\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$

D. $\sigma_1 < \sigma_3 < \sigma_2 = \sigma_4$

E. $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$

(1) A and C Only (2) D and E Only

(3) A, B and C Only (4) B and C Only

Answer (3)

Sol. $\sigma \propto \frac{1}{R}$ $R \rightarrow$ radius of curvature

$R_2 = R_4 > R_3 > R_1$

$\Rightarrow \sigma_1 > \sigma_3 > \sigma_4 = \sigma_2$

37. Two strings with circular cross section and made of same material are stretched to have same amount of tension. A transverse wave is then made to pass through both the strings. The velocity of the wave in the first string having the radius of cross section R is v_1, and that in the other string having radius of cross section $R/2$ is v_2. Then $\frac{v_2}{v_1} =$

(1) 8 (2) 2

(3) $\sqrt{2}$ (4) 4

Answer (2)

Sol. $v = \sqrt{\frac{T}{\mu}}$ $\mu = \rho\pi R^2$

$\frac{v_1}{v_2} = \sqrt{\frac{R_2^2}{R_1^2}} = \frac{R_2}{R_1} = \frac{1}{2}$

$\frac{v_2}{v_1} = 2$

38. An infinitely long wire has uniform linear charge density λ = 2 nC/m. The net flux through a Gaussian cube of side length $\sqrt{3}$ cm, if the wire passes through any two corners of the cube, that are maximally displaced from each other, would be x Nm^2C^{-1}, where x is

[Neglect any edge effects and use $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ SI units]

(1) 0.72 π (2) 2.16 π

(3) 1.44 π (4) 6.48 π

Answer (2)

Sol. $\phi = \frac{q_{enclosed}}{\varepsilon_0}$ $l = \sqrt{3}a = 3$

$q = \lambda l$

$\phi = \frac{\lambda l}{\varepsilon_0} = 2\times10^{-9}\times3\times10^{-2}\times4\pi\times9\times10^9$

$= 2.16\ \pi$

39. A block of mass 2 kg is attached to one end of a massless spring whose other end is fixed at a wall. The spring-mass system moves on a frictionless horizontal table. The spring's natural length is 2 m and spring constant is 200 N/m. The block is pushed such that the length of the spring becomes 1 m and then released. At distance x m (x < 2) from the wall, the speed of the block will be

(1) $10\left[1-(2-x)^2\right]^2$ m/s

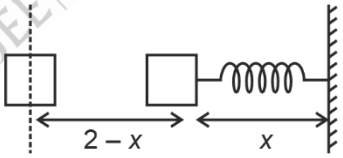
(2) $10\left[1-(2-x)^2\right]^{1/2}$ m/s

(3) $10[1-(2-x)]^{3/2}$ m/s

(4) $10\left[1-(2-x)^2\right]$ m/s

Answer (2)

Sol.



Energy conservation

$\frac{1}{2}k(1)^2 = \frac{1}{2}mv^2 + \frac{1}{2}k(2-x)^2$

Compression in the spring = $(2 - x)$

$\Rightarrow \frac{1}{2}k(1-(2-x)^2) = \frac{1}{2}mv^2$

$\Rightarrow v = [100(1-(2-x)^2)]^{1/2}$

$v = 10[1-(2-x)]^{1/2}$

40. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Work done in moving a test charge between two points inside a uniformly charged spherical shell is zero, no matter which path is chosen.

Reason R : Electrostatic potential inside a uniformly charged spherical shell is constant and is same as that on the surface of the shell.

In the light of the above statements, choose the **correct** answer from the options given below

(1) **A** is false but **R** is true

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(4) **A** is true but **R** is false

Answer (2)

Sol. Both A & R are correct and R is correct explanation

$\Delta w = q(\Delta v)$

$\Delta v = 0$

$\Rightarrow \Delta w = 0$

41. Two metal spheres of radius R and $3R$ have same surface charge density σ. If they are brought in contact and then separated, the surface charge density on smaller and bigger sphere becomes σ_1 and σ_2, respectively. The ratio $\frac{\sigma_1}{\sigma_2}$ is

(1) 9 (2) 3

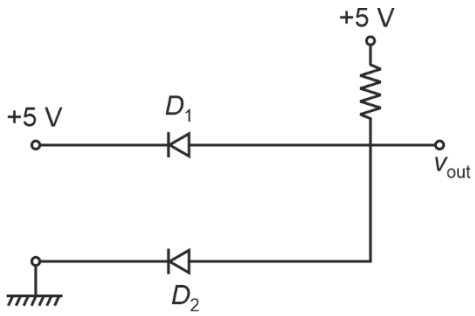
(3) $\frac{1}{9}$ (4) $\frac{1}{3}$

Answer (2)

Sol. $v_1 = v_2 \Rightarrow \frac{Q_1}{R_1} = \frac{Q_2}{R_2} \Rightarrow \frac{\sigma_1 R_1^2}{R_1} = \frac{\sigma_2 R_2^2}{R_2}$

$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = 3$

42. The output voltage in the following circuit is (Consider ideal diode case)

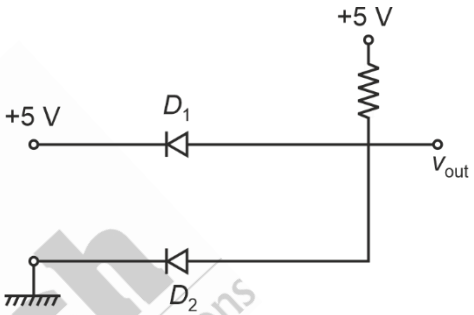


(1) 10 V (2) 0 V

(3) +5 V (4) –5 V

Answer (2)

Sol.



D_1 = Reverse biased

D_2 = Forward biased

$V_{(out)}$ = 0 V

43. Water falls from a height of 200 m into a pool. Calculate the rise in temperature of the water assuming no heat dissipation from the water in the pool.

(Take g = 10 m/s^2, specific heat of water = 4200 J/(kg K))

(1) 0.36 K (2) 0.23 K

(3) 0.48 K (4) 0.14 K

Answer (3)

Sol. $mgh = ms\Delta T$

$\Delta T = \frac{gh}{S} = \frac{10 \times 200}{4200} = 0.48$ K

44. A 3 m long wire of radius 3 mm shows an extension of 0.1 mm when loaded vertically by a mass of 50 kg in an experiment to determine Young's modulus. The value of Young's modulus of the wire as per this experiment is $P \times 10^{11}$ Nm^{-2}, where the value of P is: (Take $g = 3\pi$ m/s^2)

(1) 2.5 (2) 25

(3) 5 (4) 10

Answer (3)

Sol. $Y = \dfrac{F/A}{\Delta L/L} = \dfrac{FL}{A\Delta L} = \dfrac{(500N)(3)}{\pi \times 9 \times 10^{-6} \times 1 \times 10^{-4}}$

$= 5 \times 10^{11}$ N/m^2

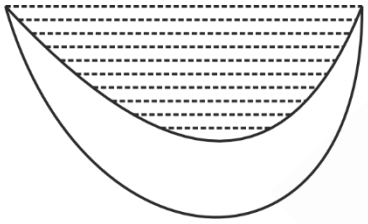
45. A concave-convex lens of refractive index 1.5 and the radii of curvature of its surfaces are 30 cm and 20 cm, respectively. The concave surface is upwards and is filled with a liquid of refractive index 1.3. The focal length of the liquid-glass combination will be

(1) $\dfrac{600}{11}$ cm (2) $\dfrac{500}{11}$ cm

(3) $\dfrac{700}{11}$ cm (4) $\dfrac{800}{11}$ cm

Answer (1)

Sol.



$\dfrac{1}{F} = \dfrac{1}{F_1} + \dfrac{1}{F_2}$ $\qquad \dfrac{1}{F_1} = 1.3 - 1\left(-\left(\dfrac{-1}{30}\right)\right)$

$\dfrac{1}{F} = \dfrac{1}{120} + \dfrac{1}{100} = \dfrac{10+12}{1200}$ $\qquad = \dfrac{0.3}{30} = \dfrac{1}{100}$

$\dfrac{1}{F} = \dfrac{22}{1200}$

$\dfrac{1}{F_2} = (1.5-1)\left(\dfrac{-1}{30} - \dfrac{1}{-20}\right)$

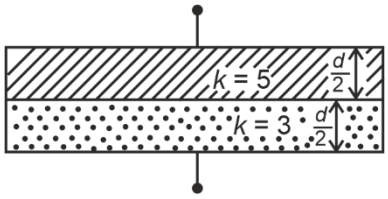
$F = \dfrac{600}{11}$ $\qquad = 0.5\left(\dfrac{3}{60} - \dfrac{2}{60}\right)$

$= \dfrac{1}{120}$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46.



Space between the plates of a parallel plate capacitor of plate area 4 cm^2 and separation of (d) 1.77 mm, is filled with uniform dielectric materials with dielectric constants (3 and 5)as shown in figure. Another capacitor of capacitance 7.5 pF is connected in parallel with it. The effective capacitance of this combination is _____ pF.

(Given $\in_0 = 8.85 \times 10^{-12}$ F/m)

Answer (15)

Sol. $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{d/2}{Ak_1\varepsilon_0} + \dfrac{d/2}{Ak_2\varepsilon_0}$

$= \left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)\dfrac{d}{2A\varepsilon_0}$

$= \left(\dfrac{1}{3} + \dfrac{1}{5}\right)\dfrac{d}{2A\varepsilon_0}$

$\dfrac{1}{C} = \dfrac{4}{15}\dfrac{d}{A\varepsilon_0}$

$C = \dfrac{15}{4} \times \dfrac{A\varepsilon_0}{d} = \dfrac{15}{4} \times \dfrac{4 \times 10^{-4} \times 8.85 \times 10^{-12}}{1.77 \times 10^{-3}}$

= 7.5 pF

$C + C_3 = (7.5 + 7.5)$ pF = 15 pF

47. An electron is released from rest near an infinite non-conducting sheet of uniform charge density '$-\sigma$'. The rate of change of de-Broglie wave length associated with the electron varies inversely as n^{th} power of time. The numerical value of n is _____.

Answer (2)

Sol. $F = qE = q\left(\frac{\sigma}{2\epsilon_0}\right) = \frac{\Delta p}{\Delta t}$

$\lambda = \frac{h}{p} \quad \frac{d\lambda}{dt} = \frac{h}{p^2} \times \frac{\Delta p}{\Delta t} = \frac{h}{p^2} \times \frac{q\sigma}{2\epsilon_0}$

$\frac{d\lambda}{dt} \propto \frac{1}{p^2} \propto \frac{l}{t^2} \propto \frac{1}{t^n}$

$p = \frac{q\sigma}{2\epsilon_0} t$

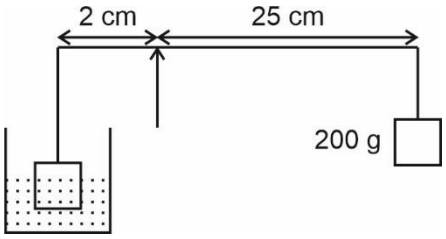
$n = 2$

48. A cube having a side of 10 cm with unknown mass and 200 gm mass were hung at two ends of an uniform rigid rod of 27 cm long. The rod along with masses was placed on a wedge keeping the distance between wedge point and 200 gm weight as 25 cm. Initially the masses were not at balance. A beaker is placed beneath the unknown mass and water is added slowly to it. At given point the masses were in balance and half volume of the unknown mass was inside the water.

(Take the density of unknown mass is more than that of the water, the mass did not absorb water and water density is 1 gm/cm^3.) The unknown mass is _____ kg.

Answer (3)

Sol.



$25 \times 0.2 \times g = 2 \times (m - \rho \times v)\, g$

$m - \rho v = 2.5$ kg

$\rho v = \frac{1 \times 10^{-3} \text{ kg}}{\text{cm}^3} \times \frac{10^3 \text{ cm}^3}{2} = \frac{1}{2}$ kg

$m = 3$ kg

49. A sample of a liquid is kept at 1 atm. It is compressed to 5 atm which leads to change of volume of 0.8 cm^3. If the bulk modulus of the liquid is 2 GPa, the initial volume of the liquid was _____ litre.

(Take 1 atm = 10^5 Pa)

Answer (4)

Sol. $-\frac{\Delta v B}{v} = \Delta P \Rightarrow 4 \times 10^5 \text{ Pa} = \frac{2 \times 10^9 \text{Pa} \times 10^{-6} \times 0.8}{v}$

$v = \frac{2 \times 10^9 \times 10^{-6} \times 0.8}{4 \times 10^5} = 4 \times 10^{-3} \text{ m}^3$

$\Rightarrow$ 4 litre

50. A thin solid disk of 1 kg is rotating along its diameter axis at the speed of 1800 rpm. By applying an external torque of 25π Nm for 40s, the speed increases to 2100 rpm. The diameter of the disk is _____ m.

Answer (40)

Sol. $\tau\, dt = I\Delta w$

$\Rightarrow \quad 25\pi \times 40 = I(300) \times \frac{2\pi}{60}$

$I = \frac{25 \times 60 \times 40}{300 \times 2} = 100 = \frac{MR^2}{4}$

$R^2 = 400$

$R = 20$ m

$D = 40$ m

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. Correct statements for an element with atomic number 9 are

A. There can be 5 electrons for which $m_s = +\frac{1}{2}$ and 4 electrons for which $m_s = -\frac{1}{2}$

B. There is only one electron in p_z orbital

C. The last electron goes to orbital with $n = 2$ and $l = 1$

D. The sum of angular nodes of all the atomic orbitals is 1

Choose the correct answer from the options given below :

(1) A and B only (2) C and D only

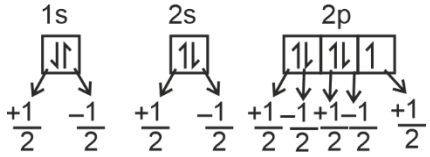
(3) A and C only (4) A, C and D only

Answer (3)

Sol. Z = 9

Electronic configuration $\Rightarrow$

$1s^2\ 2s^2\ 2p^5$



- There can be $5e^-$ for which $m_s = +\frac{1}{2}$ and $4e^-$ for which $m_s = -\frac{1}{2}$
- There can be $2e^-$ in p_z orbital
- Last e^- enters 2p (n = 2, l = 1)
- Angular nodes = l

$\therefore$ Sum of angular nodes = 0 + 0 + 3 = 3

52. Match List-I with List-II :

	List-I		List-II
A.	$[Ni(CO)_4]$	I.	Tetrahedral, 2.8 BM
B.	$[Ni(CN)_4]^{2-}$	II.	Square planar, 0 BM
C.	$[NiCl_4]^{2-}$	III.	Tetrahedral, 0 BM
D.	$[MnBr_4]^{2-}$	IV.	Tetrahedral, 5.9 BM

Choose the correct answer from the options given below

(1) A(III), B(II), C(I), D(IV)

(2) A(I), B(II), C(III), D(IV)

(3) A(IV), B(I), C(III), D(II)

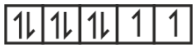
(4) A(III), B(IV), C(II), D(I)

Answer (1)

Sol. A) $Ni(CO)_4 \rightarrow Ni(0) \Rightarrow sp^3$, tetrahedral, 0 BM $(3d^{10})$(pairing)

B) $[Ni(CN)_4]^{2-} \rightarrow Ni^{2+} \Rightarrow dsp^2$, square planar, 0 BM $(3d^8)$(pairing)

C) $[NiCl_4]^{2-} \rightarrow Ni^{2+}$(no pairing) $\Rightarrow sp^3$, tetrahedral, 2.8 BM

$3d^8$ 

D) $[MnBr_4]^{2-} \Rightarrow Mn^{2+} \Rightarrow 3d^5$ (no pairing)

1	1	1	1	1

μ = 5.9 BM

(A-III, B-II, C-I, D-IV)

53. $HA(aq) \rightleftharpoons H^+(aq) + A^-(aq)$

The freezing point depression of a 0.1 m aqueous solution of a monobasic weak acid HA is 0.20°C. The dissociation constant for the acid is

Given: $K_f(H_2O) = 1.8$ K kg mol^{-1}, molality $\equiv$ molarity

(1) 1.1×10^{-2} (2) 1.38×10^{-3}

(3) 1.90×10^{-3} (4) 1.89×10^{-1}

Answer (2)

Sol. $\Delta T_f = iK_fm$

$$i = \frac{\Delta T_f}{K_f.m}$$

$$i = \frac{0.20}{1.8 \times 0.1} = 1.11$$

i = 1.11

$$\alpha = \frac{i-1}{n-1} \text{ (for HA, n = 2)}$$

$$\alpha = \frac{1.11-1}{1} = 0.11$$

$$K_a = \frac{c\alpha^2}{1-\alpha} = \frac{0.1 \times (0.11)^2}{1-0.11} = 1.38 \times 10^{-3}$$

54. Given below are two statements :

Statement-I : A homoleptic octahedral complex, formed using monodentate ligands, will not show stereoisomerism

Statement-II : cis and trans platin are heteroleptic complexes of Pd

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statemnet II are true

(2) Statement I is true but Statement II is false

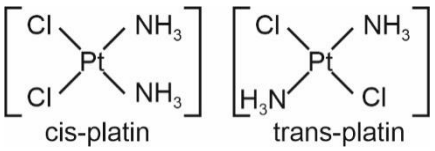
(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Answer (2)

Sol. $[Ma_6]$ type complex will not show stereoisomerism, where a is monodentate ligand

Cis and trans-platin are heterolytic complexes of Pt(Platinum). Formula is $[Pt(NH_3)_2 Cl_2]$



55. Which of the following binary mixture does not show the behaviour of minimum boiling azeotropes?

(1) $C_6H_5OH + C_6H_5NH_2$ (2) $H_2O + CH_3COC_2H_5$

(3) $CH_3OH + CHCl_3$ (4) $CS_2 + CH_3COCH_3$

Answer (1)

Sol. The solution showing positive deviation from Raoult's law will form minimum boiling azeotrope.

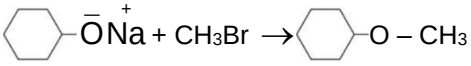
Phenol + Aniline shows negative deviation, so they will not form minimum boiling azeotrope

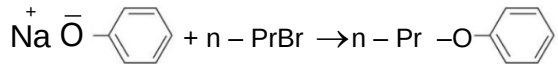
56. Which one of the followng reactions will not lead to the desired ether formation in major proportion ?

(iso-Bu $\Rightarrow$ isobutyl, sec-Bu $\Rightarrow$ sec-butyl, nPr $\Rightarrow$ n-propyl, ^{t}Bu $\Rightarrow$ tert-butyl, Et $\Rightarrow$ ethyl)

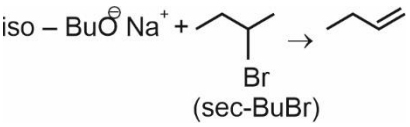
(1) iso-Bu$\bar{O}\overset{+}{Na}$ + sec-BuBr $\rightarrow$ sec-Bu–O–iso-Bu

(2) ^{t}Bu$\bar{O}\overset{+}{Na}$ + EtBr $\rightarrow$ ^{t}Bu – O – Et

(3) 

(4) 

Answer (1)

Sol. iso – $BuO^{\ominus}Na^{+}$ + 

(sec-BuBr)

For 2° RX $\Rightarrow$ elimination would be more favourable than substitution

For Williamson's synthesis of ether

$RO^{\ominus}Na^{+} + R'X \rightarrow ROR'$ (undergo S_N2 reaction)
(1° alkyl halide)

57. Match the List-I with List-II

	List-I		List-II
A.	Carbocation	I.	Species that can supply a pair of electrons
B.	C-Free radical	II.	Species that can receive a pair of electrons
C.	Nucleophile	III.	sp^2 hybridized carbon with empty p-orbital
D.	Electrophile	IV.	sp^2/sp^3 hybridised carbon with one unpaired electron

Choose the correct answer from the options given below:

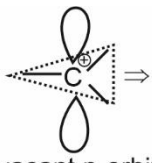
(1) A(III), B(IV), C(II), D(I)

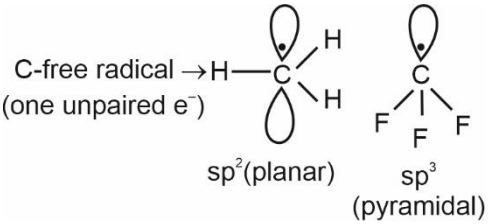
(2) A(II), B(III), C(I), D(IV)

(3) A(IV), B(II), C(III), D(I)

(4) A(III), B(IV), C(I), D(II)

Answer (4)

Sol. Carbocation $\Rightarrow$  $\Rightarrow sp^2$, planar
vacant p-orbital



Nucleophile $\Rightarrow e^-$ rich species like anions etc.

$\Rightarrow$ can supply a pair of electrons

Electrophile $\Rightarrow e^-$ deficient species $\Rightarrow$ can receive a pair of electrons.

58. The correct decreasing order of spin only magnetic moment values (BM) of Cu^+, Cu^{2+}, Cr^{2+} and Cr^{3+} ions is :

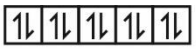
(1) $Cu^+ > Cu^{2+} > Cr^{3+} > Cr^{2+}$

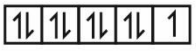
(2) $Cu^{2+} > Cu^+ > Cr^{2+} > Cr^{3+}$

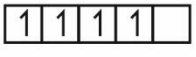
(3) $Cr^{3+} > Cr^{2+} > Cu^+ > Cu^{2+}$

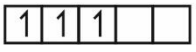
(4) $Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$

Answer (4)

Sol. $Cu^+ \Rightarrow 3d^{10} \Rightarrow$  $\Rightarrow$ 0 BM

$Cu^{2+} \Rightarrow 3d^9 \Rightarrow$  $\Rightarrow$ 1.73 BM

$Cr^{2+} \Rightarrow 3d^4 \Rightarrow$  $\Rightarrow$ 4.89 BM

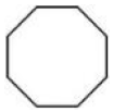
$Cr^{3+} \Rightarrow 3d^3 \Rightarrow$  $\Rightarrow$ 3.87 BM

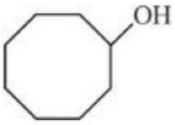
So order :

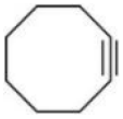
$Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$

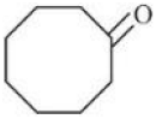
59. 1, 2-dibromocyclooctane $\xrightarrow[\text{(ii) } NaNH_2 \\ \text{(iii) } Hg^{2+}/H^+ \\ \text{(iv) } Zn\text{-}Hg/H^+]{\text{(i) KOH(alc.)}}$ P (major product)

'P' is

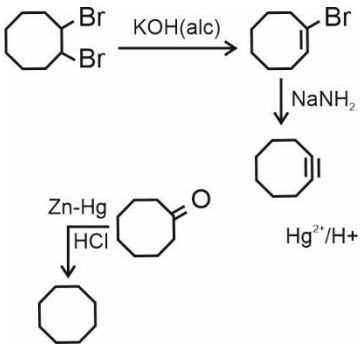
(1) 

(2) 

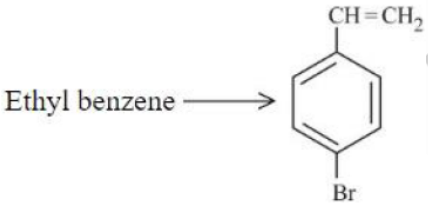
(3) 

(4) 

Answer (1)

Sol. 

60. Choose the correct set of reagents for the following converson.



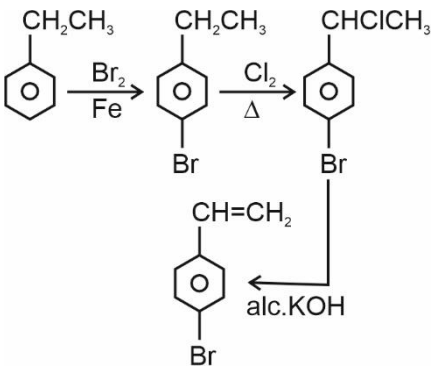
(1) Cl_2/anhy.$AlCl_3$; Br_2/Fe ; alc. KOH

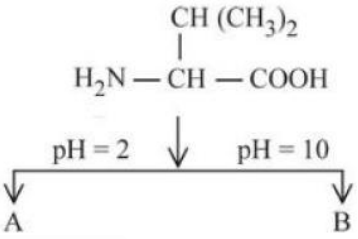
(2) Br_2/Fe ; Cl_2, Δ ; alc. KOH

(3) Br_2/anhy.$AlCl_3$; Cl_2, Δ ; aq. KOH

(4) Cl_2/Fe ; Br_2/anhy.$AlCl_3$; aq. KOH

Answer (2)

Sol. 

61. 

Choose the correct option for structures of A and B, respectively

(1) $H_2N\text{-}CH(CH(CH_3)_2)\text{-}COO^-$ and $H_3N^+\text{-}CH(CH(CH_3)_2)\text{-}COOH$

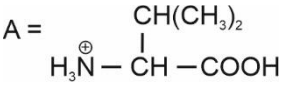
(2) $H_2N\text{-}CH(CH(CH_3)_2)\text{-}COO^{\ominus}$ and $H_3N^+\text{-}CH(CH(CH_3)_2)\text{-}COO^-$

(3) $H_3N^+\text{-}CH(CH(CH_3)_2)\text{-}COO^-$ and $H_3N^+\text{-}CH(CH(CH_3)_2)\text{-}COOH$

(4) $H_3N^+\text{-}CH(CH(CH_3)_2)\text{-}COOH$ and $H_2N\text{-}CH(CH(CH_3)_2)\text{-}COO^-$

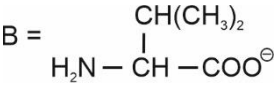
Answer (4)

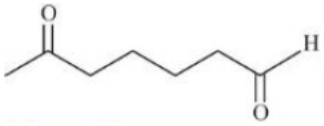
Sol. At pH = 2 $\Rightarrow$ Medium is acidic so $-NH_2$ group will convert to $-NH_3^+$

So A = 

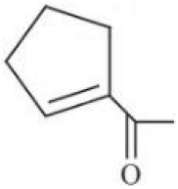
$\therefore$ At pH = 10 $\Rightarrow$ medium is basic

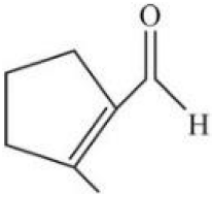
So –COOH group will convert to $-COO^{\ominus}$

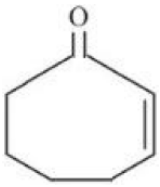
So B = 

62. When  undergoes intramolecular aldol condensation, the major product formed is :

(1) 

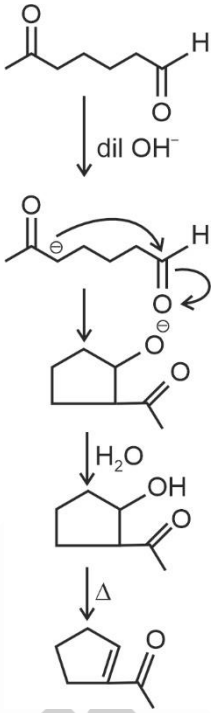
(2) 

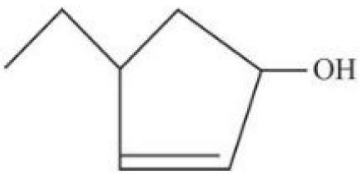
(3) 

(4) 

Answer (2)

Sol.



63. What is the correct IUPAC name of  ?

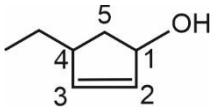
(1) 4-Ethylcyclopent-2-en-1-ol

(2) 4-Ethyl-1-hydroxycyclopent-2-ene

(3) 1-Ethyl-3-hydroxycyclopent-2-ene

(4) 1-Ethylcyclopent-2-en-3-ol

Answer (1)

Sol. 

4-Ethylcyclopent-2-en-1-ol

64. Given below are two statements:

Statement I : H_2Se is more acidic than H_2Te

Statement II : H_2Se has higher bond enthalpy for dissociation than H_2Te

In the light of the above statement, choose the correct answer from the options given

(1) Both Statement I and Statement II are false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Answer (3)

Sol. $\therefore$ H_2Te is more acidic than H_2Se

because of lesser bond dissociation energy of H_2Te

It can release H^+ more easily

pK_{a_1} : $H_2Se(3.89) > H_2Te(2.6)$

65. In a first order decomposition reaction, the time taken for the decomposition of reactant to one fourth and one eighth of its initial concentration are t_1 and t_2(s), respectively. The ratio t_1/t_2 will be:

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$

(3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Answer (4)

Sol. $t_1 = t_{\frac{1}{4}} = \frac{1}{k}\ln\frac{A_0}{\frac{A_0}{4}} = \frac{1}{k}\ln 4$

$t_2 = t_{\frac{1}{8}} = \frac{1}{k}\ln\frac{A_0}{\frac{A_0}{8}} = \frac{1}{k}\ln 8$

$\frac{t_1}{t_2} = \frac{\ln 4}{\ln 8} = \frac{2\ln 2}{3\ln 2} = \frac{2}{3}$

66. On combustion of 0.210 g of an organic compound containing C, H and O gave 0.127 g H_2O and 0.307 g CO_2. The percentage of hydrogen and oxygen in the given organic compound respectively are:

(1) 53.41, 39.6 (2) 6.72, 39.87

(3) 6.72, 53.41 (4) 7.55, 43.85

Answer (3)

Sol. Mass of organic compound = 0.210 g

Mass of water formed = 0.127 g

Mass of CO_2 formed = 0.307 g

Mass of hydrogen = $\frac{0.127\times 2}{18} = 0.014$ g

Percentage of hydrogen = $\frac{0.014\times 100}{0.210} = 6.72\%$

Mass of carbon $= \frac{0.307\times 12}{44} = 0.084$ g

Percentage of carbon = $= \frac{0.084\times 100}{0.210} = 39.87\%$

Percentage of oxygen = 53.41%

67. The number of species from the following that are involved in sp^3d^2 hybridization is $[Co(NH_3)_6]^{3+}$, SF_6,$[CrF_6]^{3-}$, $[CoF_6]^{3-}$, $[Mn(CN)_6]3^-$and $[MnCl_6]^{3-}$

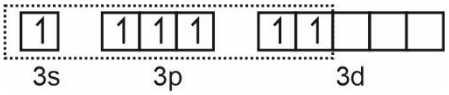
(1) 4 (2) 6

(3) 5 (4) 3

Answer (4)

Sol. (i) $[Co(NH_3)_6]^{3+}$

$Co^{3+} \Rightarrow 3d^6$; $t_{2g}^6e_g^0$ d^2sp^3 hybridisation

(ii) SF_6 

sp^3d^2 hybridisation

(iii) $[CrF_6]^{3-}$

$Cr^{3+} \Rightarrow 3d^3$; $t_{2g}^3e^{g0}$ d^2sp^3 hybridisation

(iv) $[CoF_6]^{3-}$

$Co^{3+} \Rightarrow 3d^6$; $t_{2g}^4 e_g^2$ sp^3d^2 hybridisation

(v) $[Mn(CN)_6]^{3-}$

$Mn^{3+} \Rightarrow 3d^4$; $t_{2g}^4 e_g^0$ d^2sp^3 hybridisation

(vi) $[MnCl_6]^{3-}$

$Mn^{3+} \Rightarrow 3d^4$; $t_{2g}^3 e_g^1$ sp^3d^2 hybridisation

68. The atomic number of the element from the following with lowest 1st ionisation enthalpy is

(1) 35 (2) 19

(3) 32 (4) 87

Answer (4)

Sol.

Atomic No.	Period No.	Group No.
35	4	17
19	4	1
32	4	14
87	6	1

First ionisation energy of an element generally decreases down the group and increases from left to right along a period. Therefore, element having atomic number 87 has the lowest first ionisation energy.

69. Match the List-I with List-II

List-I (Reagent)		List-II (Functional Group detected)	
A.	Sodium bicarbonate solution	I.	Double bond/unsaturation
B.	Neutral ferric chloride	II.	Carboxylic acid
C.	Ceric ammonium nitrate	III.	Phenolic - OH
D.	Alkaline $KMnO_4$	IV.	Alcoholic - OH

Choose the correct answer from the options given below:

(1) A-III, B-II, C-IV, D-I

(2) A-II, B-IV, C-III, D-I

(3) A-II, B-III, C-IV, D-I

(4) A-II, B-III, C-I, D-IV

Answer (3)

Sol.

	Reagent		Functional group
(A)	Sodium bicarbonate	(II)	Carboxylic acid
(B)	Neutral ferric chloride	(III)	Phenolic-OH
(C)	Ceric Ammonium nitrate	(IV)	Alcoholic-OH
(D)	Alkaline $KMnO_4$	(I)	Double bond/Unsaturation

(A) Carboxylic acid gives effervescence with sodium bicarbonate

$$R-COOH + NaHCO_3 \rightarrow RCOONa + CO_2 \uparrow + H_2O$$

(B) Phenolic-OH gives characteristic colour with neutral $FeCl_3$

(C) Alcoholic-OH gives red colour with ceric ammonium nitrate

$$(NH_4)_2\left[Ce(NO_3)_6\right] + 2ROH \rightarrow \underset{(Red)}{\left[Ce(ROH)_2(NO_3)_4\right]} + 2NH_4NO_3$$

(D) Purple colour of alkaline $KMnO_4$ is discharged by multiple bond of alkenes and alkynes

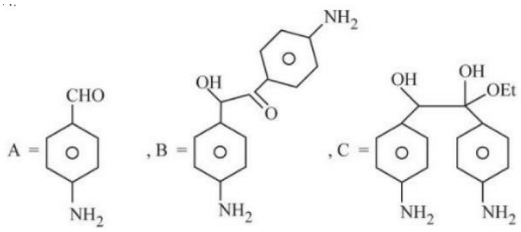
70. $A \xrightarrow[(ii)\ H_3O^+]{(i)\ NaOH} B \xrightarrow[(ii)\ H_2SO_4,\Delta]{(i)\ EtOH} C$

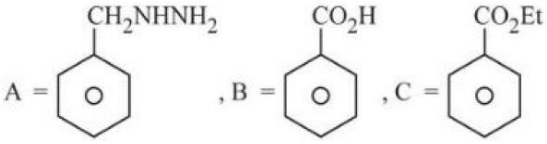
'A' shows positive Lassaigne's test for N and its molar mass is 121.

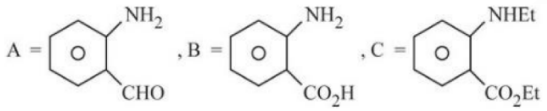
'B' gives effervescence with aq $NaHCO_3$

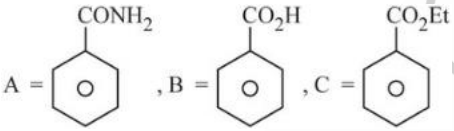
'C' gives fruity smell

Identify A, B and C from the following

(1) 

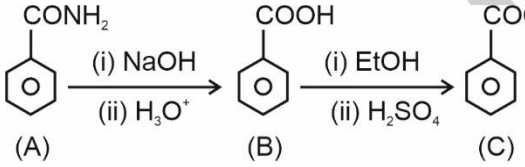
(2) 

(3) 

(4) 

Answer (4)

Sol. Compound (A) is likely to be amide which gives carboxylic acid (B) after hydrolysis. Compound (B) reacts with alcohol to give ester



SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. Consider the following half cell reaction

$$Cr_2O_7^{2-}(aq) + 6e^- + 14H^+(aq) \rightarrow 2Cr^{3+}(aq) + 7H_2O(l)$$

The reaction was conducted with the ratio of $\frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}]} = 10^{-6}$. The pH value at which the EMF of the half cell will become zero is _____. (nearest integer value)

[Given : standard half cell reduction potential $E^o_{Cr_2O_7^{2-},H^+/Cr^{3+}} = 1.33V, \frac{2.303RT}{F} = 0.059V$]

Answer (10)

Sol.

$$Cr_2O_7^{2-}(aq) + 6e^- + 14H^+(aq) \rightarrow 2Cr^{3+}(aq) + 7H_2O(l)$$

Using Nernst equation

$$E_{Cr_2O_7^{2-}/H^+/Cr^{3+}} = E^0_{Cr_2O_7^{2-}/H^+/Cr^{3+}} - \frac{2.303RT}{6F}\log\frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}][H^+]}14$$

$$0 = E^0_{Cr_2O_7^{2-},H^+/Cr^{3+}} - \frac{0.059}{6}\log\left(10^{-16}[H^+]^{-14}\right)$$

$$0 = 1.33 + 0.059 - \frac{0.059\times14}{6}pH$$

$$pH = \frac{1.389\times6}{0.059\times14} = 10.10 \simeq 10$$

72. The equilibrium constant for decomposition of $H_2O(g)$

$$H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)\left(\Delta G^o = 92.34\ kJ\,mol^{-1}\right)$$

is 8.0 x 10^{-3} at 2300 K and total pressure at equilibrium is 1 bar. Under this condition, the degree of dissociation (α) of water is _____ x 10^{-2} (nearest integer value)

[Assume α is negligible with respect to 1]

Answer (5)

Sol. $H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)$

Initial mole	1	–	–
Moles at equil.	$1-\alpha$	α	$\frac{\alpha}{2}$

Equilibrium pressure = 1 bar

$$K_p = \frac{\left(\frac{2\alpha}{2+\alpha}\right)\left(\frac{\alpha}{2+\alpha}\right)^{\frac{1}{2}}}{2\left(\frac{1-\alpha}{2+\alpha}\right)} = 8.0\times10^{-3}$$

$$\frac{\alpha^{\frac{3}{2}}}{\sqrt{2}} = 8.0\times10^{-3}$$

$$\alpha = \left(8\sqrt{2}\times10^{-3}\right)^{\frac{2}{3}}$$

$$= \left(2^{\frac{7}{2}}\times10^{-3}\right)^{\frac{2}{3}} = 2^{\frac{7}{3}}\times10^{-2} \simeq 5\times10^{-2}$$

73. 20 mL of sodium iodide solution gave 4.74 g silver iodide when treated with excess of silver nitrate solution. The molarity of the sodium iodide solution is _____ M. (Nearest Integer Value)
(Given : Na = 23, I = 127, Ag = 108, N = 14, O = 16 g mol^{-1})

Answer (1)

Sol. Let molarity of NaI solution be x M

$NaI + AgNO_3 \rightarrow AgI + NaNO_3$

Moles of AgI formed = $\frac{4.74}{235} = 0.02$

Moles of NaI = $\frac{20\times x}{1000} = 0.02x$

$0.02x = 0.02$

$x = 1$

$\therefore$ Molarity of NaI solution = 1 M

74. The energy of an electron in first Bohr orbit of H-atom is –13.6 eV. The magnitude of energy value of electron in the first excited state of Be^{3+} is _____ eV (nearest integer value)

Answer (54)

Sol. E_1 of H-atom = –13.6 eV

E_2 of $Be^{3+} = \frac{-13.6\times Z^2}{n^2}$

$= \frac{-13.6\times(4)^2}{(2)^2}$

= –54.4 eV

$|E_2|$ of Be^{3+} = 54 eV

75. Resonance in X_2Y can be represented as

$$\ddot{\underset{\cdot\cdot}{X}}^{\ominus} = \overset{\oplus}{X} = \ddot{\underset{\cdot\cdot}{Y}} \leftrightarrow : X \equiv \overset{\oplus}{X} - \ddot{\underset{\cdot\cdot}{Y}}:^{\ominus}$$

The enthalpy of formation of X_2Y $\left(X \equiv X(g) + \frac{1}{2}Y = Y(g) \rightarrow X_2Y(g)\right)$ is 80 kJ mol^{-1}.
The magnitude of resonance energy of X_2Y is _____ kJ mol^{-1} (nearest integer value)
Given : Bond energies of $X \equiv X$, $X = X$, $Y = Y$ and $X = Y$ are 940, 410, 500 and 602 kJ mol^{-1} respectively.
Valence X : 3, Y : 2

Answer (98)

Sol. $X \equiv X(g) + \frac{1}{2}Y = Y(g) \rightarrow \overset{(-)}{X} = \overset{(+)}{X} = Y(g)$

$[\Delta H_f(X_2Y)]_{Actual}$ = 80 kJ mol^{-1}

$[\Delta H_f(X_2Y)]_{Theoretical} = 940 + \frac{1}{2}(500) - (410 + 602)$

= 1190 – 1012

= 178 kJ mol^{-1}

Resonance energy = $|(\Delta H_f)_{Actual} - (\Delta H_f)_{Theoretical}|$

= |80 – 178| = 98 kJ mol^{-1}

❑ ❑ ❑