#ODSC

BOSTON
APR 30 - MAY 3

When the Bootstrap Breaks

Ryan Harter

Senior Staff Data Scientist,
Mozilla

Format

- 40m: Harter - Intro to Bootstrap and Advantages
- 10m: Break
- 40m: Saptarshi - The Bootstrap at Mozilla, Real Firefox Data
- 15m: Questions

What are we talking about?

We’re going to strip some of the varnish off the bootstrap.

We recently did a deep dive while building out our experiments pipeline.

We want to share what we learned with you.

The Bootstrap: Definitions

What is it for?

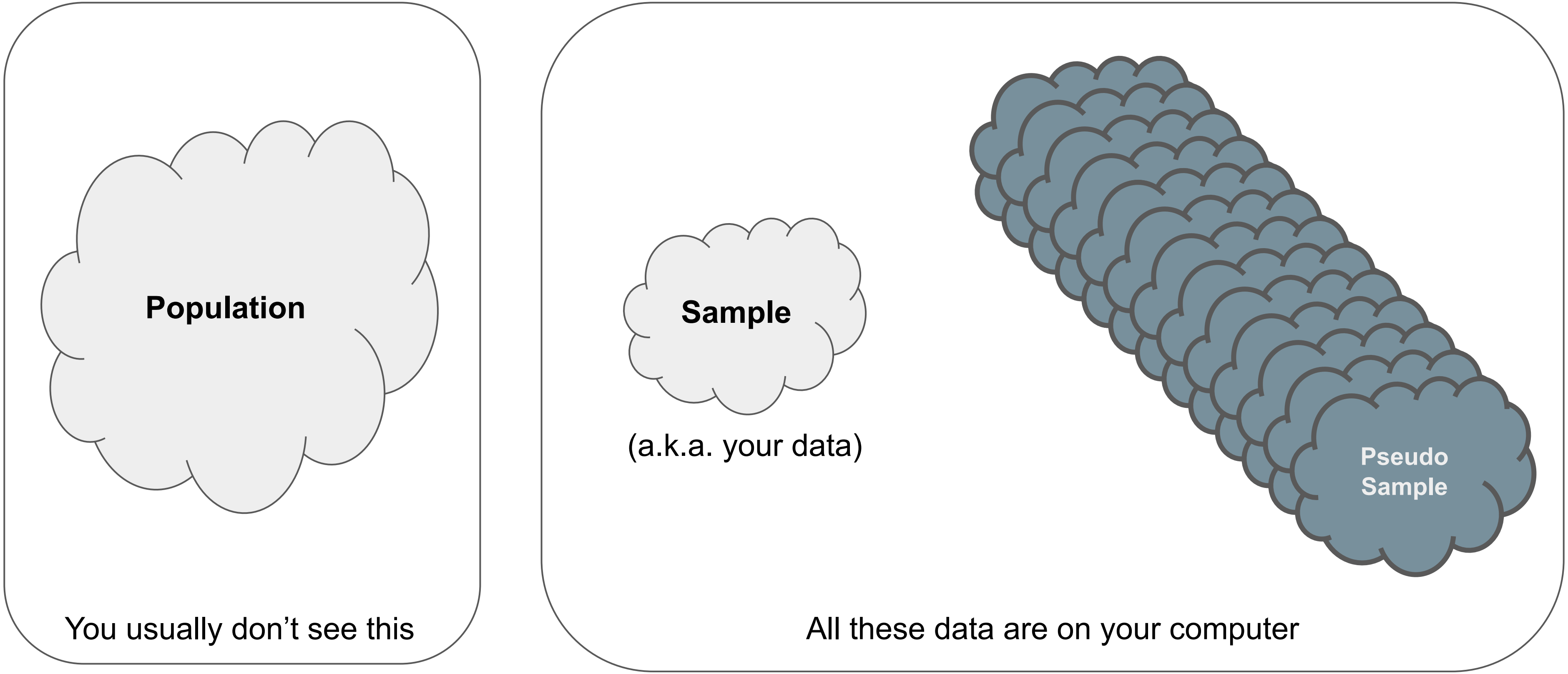
Understanding the distribution of estimators.

In practice:

- How sure are we in this number? (confidence intervals)
- Are these two quantities different? (hypothesis testing)

Most common alternative is asymptotic normality

Bootstrap Outline



Conventions

- **Sample** - a.k.a. “your data”
- **Pseudo-sample** - An artificial dataset created by sampling with replacement from our sample

- **Statistic** - A metric computed from the sample data
- **Resampled Statistic** - The same metric, but applied to a pseudo-sample

- **Confidence Intervals**: use the format: `point(lower_bound, upper_bound)`
  - `6.0 (1.1, 0.9)` roughly means `6.0 ±4.9`

The Bootstrap: Advantages

Applies to Many Statistics

Asymptotic normality mostly applies to means.

- Means are easy
- What about Medians?
- What about learned parameters?

Particularly nice when you don’t know what metrics you’ll be using

Bootstrap Example

```
def bootstrap(data, f):
  bootstraps = [
    f(data.resample(frac=1, resample=True))
    for x in 100
  ]

  return(
    f(data)
    bootstraps.quantile(0.05),
    bootstraps.quantile(0.95)
  )
```

Bootstrap Example

```
# Schema:
#   client_id:           String UUID, dataset key
#   experiment_branch:   String: "control" or "experiment"
#   page_views:          Integer count of page loads
data = pd.read_csv('data.csv')

# Compute the mean
bootstrap(data, lambda d: mean(data['page_views']))

# Compute the median
bootstrap(data, lambda d: data['page_views'].quantile(0.5))
```

Bootstrap Example

```
# Compute experiment summaries (!)
def change_in_page_views(data):
  branch_summaries = (
    data
    .groupby('experiment_branch')
    .mean()
  )

  return(branch_summaries['experiment'] / branch_summaries['control'])

bootstrap(data, change_in_page_views)
# (0.04, 0.01, 0.07) -> 4.0% (1.0%, 7.0%)
```

Bootstrap Example

Pseudo-code here: http://go.harterrt.com/bootstrap_pseudo

The Bootstrap: ~~Advantages~~

No Normality Assumptions

- Totally true!
- But not super motivating

The CLT - Surprisingly Robust

- Remember! CLT says the *sample mean* is normally distributed
- Does not require the source distribution to be normal
- E.g. 100 coin flips
    - Each flip is not normal
    - The mean of many flips approaches normal

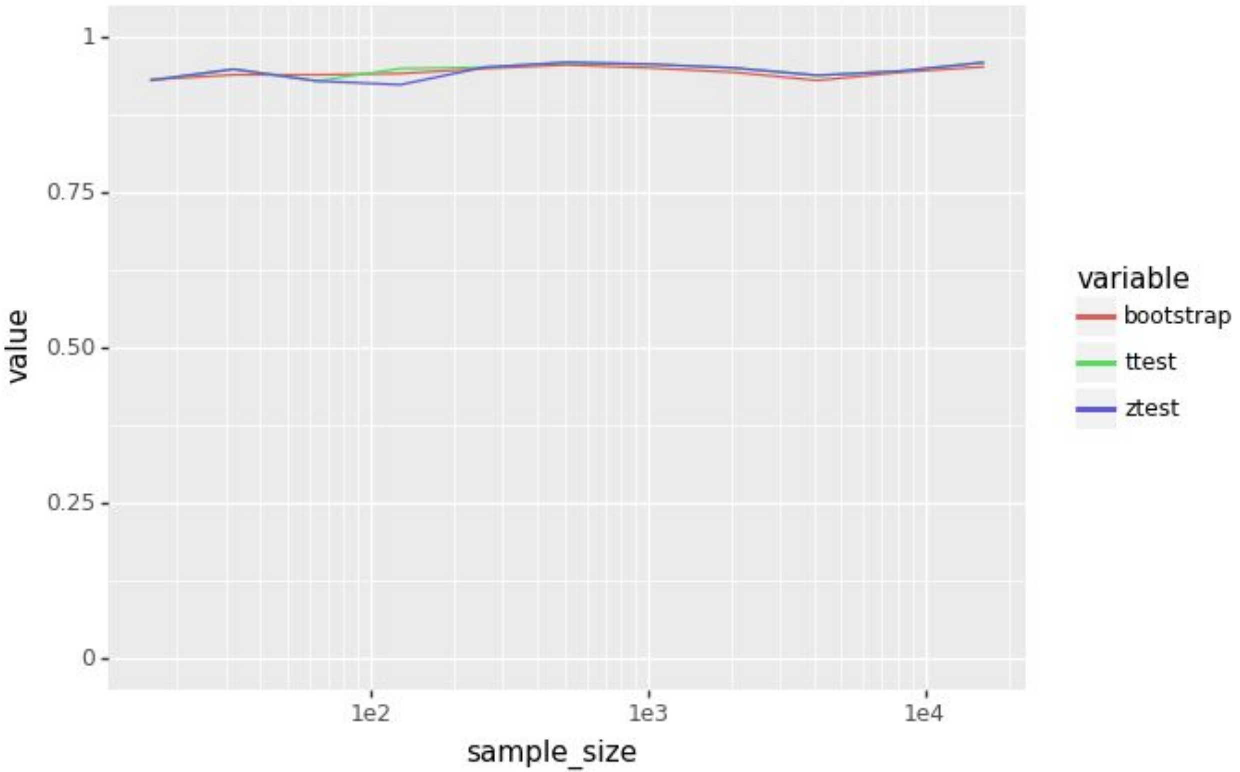
- CLT does require:
    - Finite Variance
    - Big N

Bootstrap vs CLT - Simulations

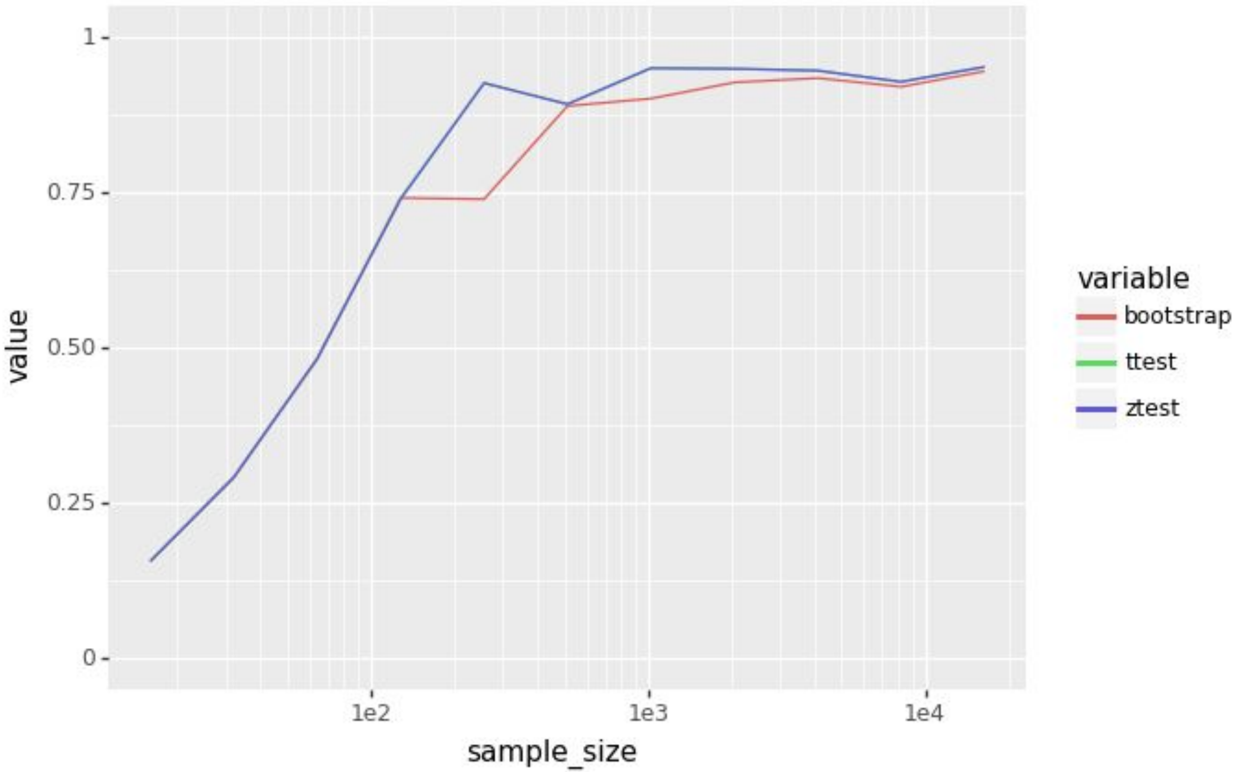
- Simulate Type I error for increasing sample sizes
    - Sample sizes: Powers of 2 between 16 through 16384
    - For each sample size, take 1000 samples (meta-iterations)
        - Bootstrap with 200 resamples
        - Calculate T-dist and Z-dist confidence intervals
        - Test whether the CI contains the sample mean
- We want to see Type I error ~95%

https://github.com/harterrt/when_the_bootstrap_breaks

Bootstrap vs CLT - Coin Flip



Bootstrap vs CLT - Binomial P=0.01



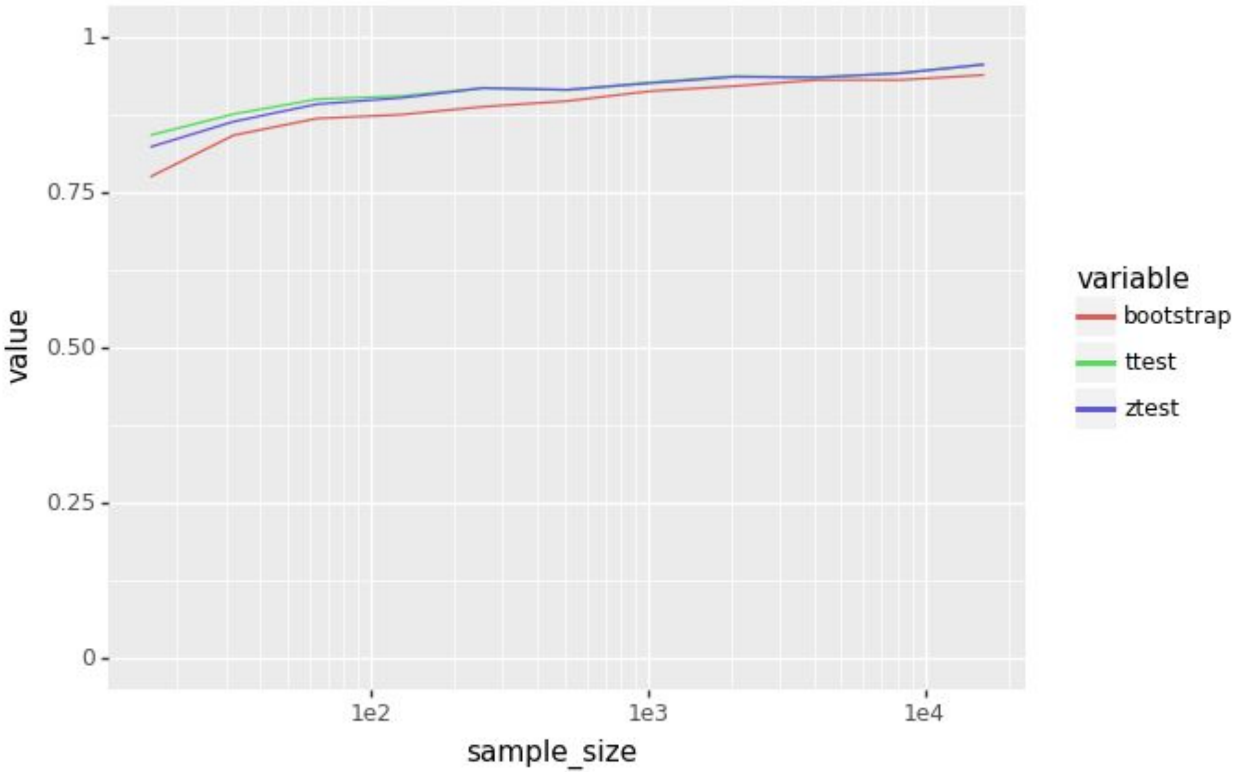
The CLT - Surprisingly Robust

- CLT does require:
  - Finite Variance
  - Big N

Bootstrap vs CLT - Small N

- Bootstrap is often recommended for small-n
  - Tighter intervals!
  - Looks better!
- All false confidence

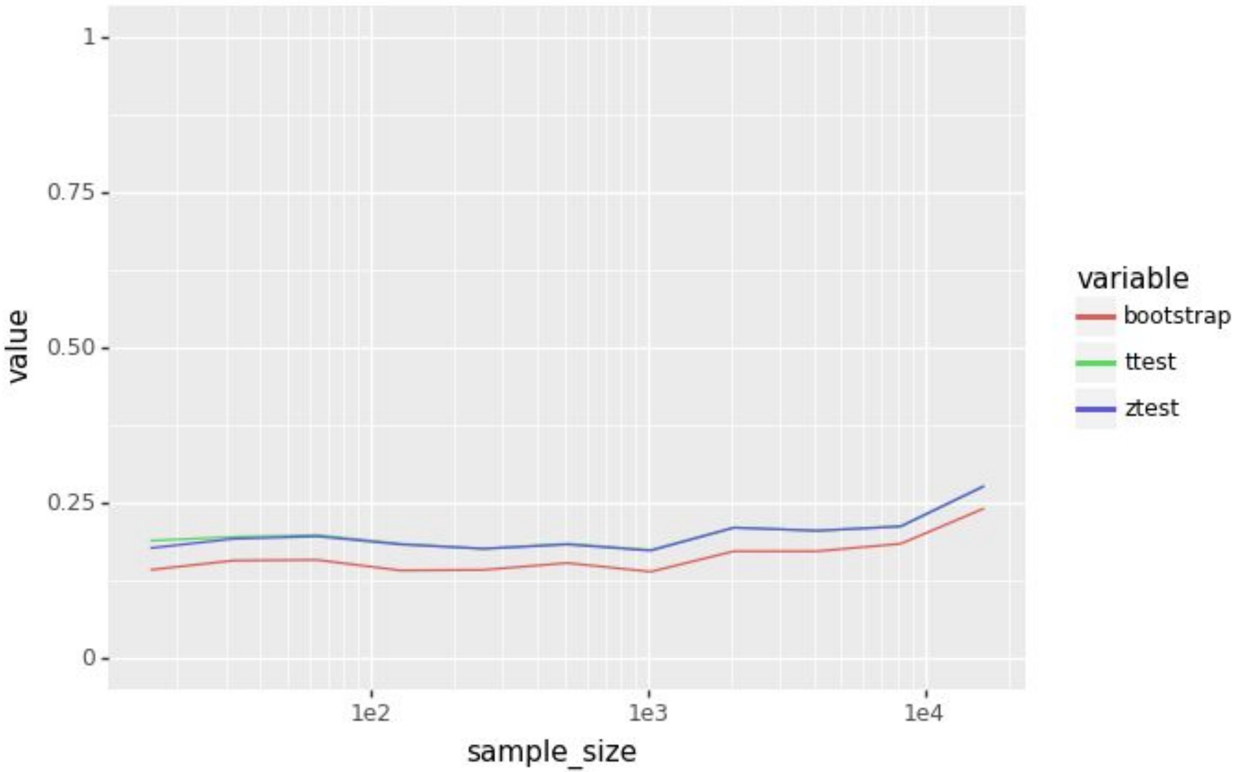
Bootstrap vs CLT - Pereto 3



Bootstrap vs CLT - Infinite Variance

- Most common argument I've seen for the bootstrap
  - "Our data is too skewed to use asymptotic normality"
- Unfortunately, the bootstrap also requires finite variance
  - Otherwise it converges to the wrong value

Bootstrap vs CLT - Pereto 1



Bootstrap vs CLT

Again the bootstrap works for more metrics but doesn’t have many other advantages over the CLT.

Keep in mind that the limitations of the CLT are well known and clearly documented. Can we say the same about the bootstrap?

Also, the CLT is very computationally efficient. Calculate all you need with one pass through the data.

Easy to Understand

Totally true!

- More likely to produce more robust results from non-statisticians
- Taught in a way that produces better understanding
  - A tool, not an incantation
  - Less likely to have to swim through p-values
- No need for arcane formulas

(Do you really understand it though?)

Easy to Understand - Small N

- We see this confusion IRL!
- Unclear assumptions → broken applications
- In practice, bootstrap seems magic

Easy to Understand - The basic bootstrap

- Percentile bootstrap doesn’t necessarily converge to the sample metric for skewed distributions.
  - Confidence intervals may not contain point estimate!
  - e.g. 50% (55%, 58%)
- Instead, use the “basic bootstrap”
  - Bootstrap deviations from the sample metric
  - Intervals stay centered around the sample metric
  - Fixed!

Easy to Understand - The basic bootstrap

In doing this we reverse the skew of the bootstrap samples. WTH?

Sure it makes sense in my gut, but I didn't know that would happen a priori. I'm rolling with the punches not calling my shot. Dangerous place to be.

$$(2\hat{\theta} - \theta^*_{(1-\alpha/2)}, 2\hat{\theta} - \theta^*_{(\alpha/2)})$$

https://en.wikipedia.org/wiki/Bootstrapping_(statistics)