Due 23:59 March 10 (Sunday). There are 100 points in this assignment. Submit your answers (**must be typed**) in pdf file to CourSys
`https://coursys.sfu.ca/2024sp-cmpt-307-d1/`.
Submissions received after 23:59 will get penalty of reducing points: 20 and 50 points deductions for submissions received at $[00:00, 00:10]$ and $(00:10, 00:30]$ of March 11, respectively; no points will be given to submissions after 00 : 30 of March 11.

1. 15 points

   There are two machines A and B and a job J. In each time interval t_i of $t_1, .., t_n$, J can be executed on A by $a_i > 0$ steps or on B by $b_i > 0$ steps or moved between machines (from A to B or from B to A, J is executed 0 step). Design a dynamic programming algorithm (optimal solution structure, Bellman equation, pseudo code and running time) which, given $a_1, .., a_n$ and $b_1, .., b_n$, finds a plan that decides run J on A or on B or move between machines for every interval such that J is executed by a maximum number of steps. At t_1, J can be executed on either A or B. (Hint: Let $\text{opt}(i, A)$ (resp. $\text{opt}(i, B)$) be the number of steps executed in an optimal plan in $t_1, .., t_i$ that runs job J at time t_i on machin A (resp. B). Then the maximum number of steps executed is $\max\{\text{opt}(n, A), \text{opt}(n, B)\}$.)

2. 10 points (Ex 14.3-2 of text book)

   Describe the recursion tree for the Merge-Sort procedure for an array of n elements. Explain why memoization fails to speed up a good divide-and-conquer algorithm such as Merge-Sort.

3. 10 points (Ex 14.5-2 of text book)

   Determine the cost and structure of an optimal binary search tree for an input instance shown below:

i	0	1	2	3	4	5	6	7
p_i	0	0.04	0.06	0.08	0.02	0.10	0.12	0.14
q_i	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05

   Your answer for the tree structure may look like: the root of the tree is xxx with left child yyy and right child zzz, the left child of yyy is ....

4. 20 points Interval scheduling with p recourses

   Given a set $S = \{a_1, .., a_n\}$ of proposed jobs and $p > 1$ resources, each job requires a resource and each resource can serve exactly one job at a time. Each job a_i has a start time s_i and a finish time f_i with $0 \leq s_i < f_i < \infty$. If job a_i is assigned to a resource r, then r serves a_i in time interval $[s_i, f_i)$. Jobs a_i and a_j are compatible if $[s_i, f_i) \cap [s_j, f_j) = \emptyset$. Design a greedy algorithm which assigns a maximum number of jobs from S to resources so that the jobs assigned to a same resource are mutually compatible, and analyze your algorithm.

5. 15 points (Ex 15.2-5 of text book)

   Give a greedy algorithm which, given a set $X = \{x_1, .., x_n\}$ of real numbers in the interval $[0, 1000]$, finds minimum number of unit-length closed intervals $[a_1, b_1], .., [a_k, b_k]$ (e.g., $b_i - a_i = 1$) such that every number of X is contained in some interval $[a_i, b_i]$; prove the correctness and analyze the algorithm.

6. 15 points (Ex 15.3-7 of text book)

   Suppose that a data file contains a sequence of 8-bit characters such that all 256 characters are about equally common: the maximum character frequency is less than twice the minimum character frequency. Prove that Huffman coding in this case is no more efficient than using an ordinary 8-bit fixed-length code.

7. 15 points (Ex 16-1.3, 16-2.2, 16-3.2 of text book)

   Suppose we perform a sequence of n operations on a data structure in which the ith operation costs i if i is an exact power of 2, and 1 otherwise. (a) Use aggregate analysis to determine the amortized cost per operation. (b) Use an accounting method of analysis to determine the amortized cost per operation. (c) Use a potential method of analysis to determine the amortized cost per operation.