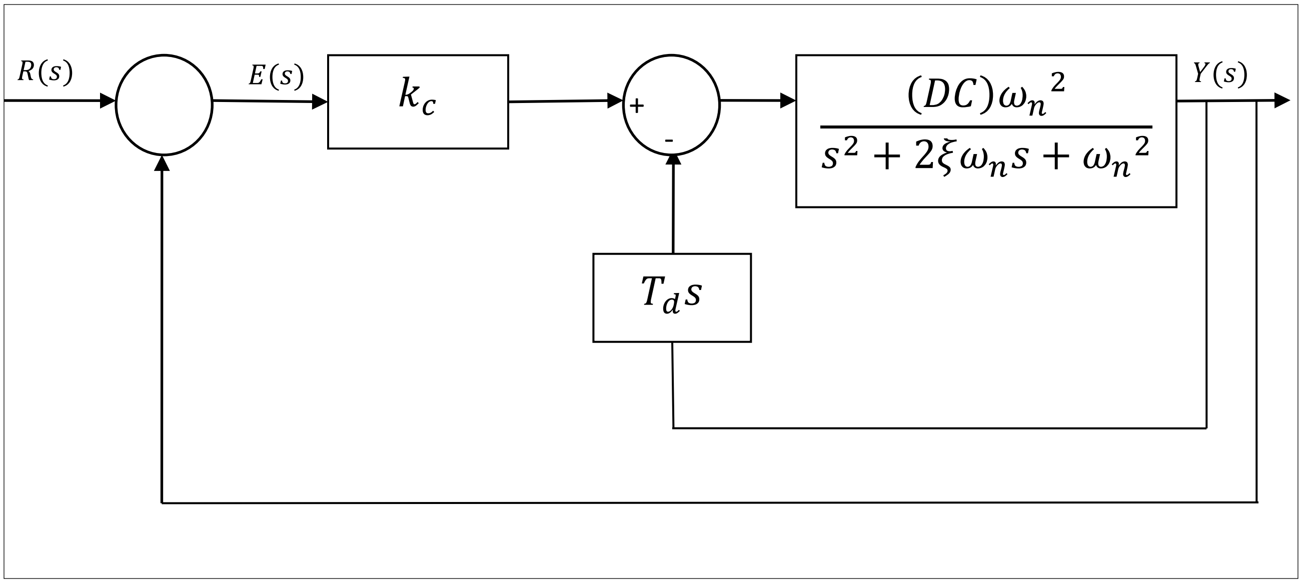
Theory

The PD controller that we use is the one shown below:



As we saw earlier, the derivative mode acts on the output, and not on the error. This prevents the "*derivative kick*" which happens when the setpoint R(s) is changed by a step signal. (The derivative of the *step* signal is the *impulse* signal. So if R(s) changes by a step, then E(s) =R(s)-Y(s) will change instantly by an impulse, giving a "derivative kick").

The PD controller in this configuration has the form:

Controller output = $K_c e(t) - T_d . \frac{dy(t)}{dt}$, (1)

The closed loop transfer function can be derived from the Figure as

$$CLTF = \frac{Y(s)}{R(s)} = \frac{K_c(DC)\omega_n^2}{s^2 + \{2\xi\omega_n + T_d(DC)\omega_n^2\}s + \omega_n^2(1 + K_c(DC))} , \quad (2)$$

We see how to design the PD controller to achieve desired setting time t_s and peak overshoot.

Compare (2) with standard form of transfer function

$$CLTF = \frac{K_c(DC)}{K_c(DC)+1} \cdot \frac{\omega_n^2(1+K_c(DC))}{s^2+\{2\xi\omega_n+T_d(DC)\omega_n^2\}s+\omega_n^2(1+K_c(DC))} \ , \tag{3}$$

$$CLTF = \underbrace{\frac{K_c(DC)}{K_c(DC)+1}}_{DC_{CLTF}} \cdot \frac{\omega_n^2(1+K_c(DC))}{s^2+\underbrace{\{2\xi\omega_n+T_d(DC)\omega_n^2\}}_{2\xi_{CLTF}\ \omega n_{CLTF}}s+\underbrace{\omega_n^2(1+K_c(DC))}_{\omega^2_{n\,CLTF}}} \ , \tag{4}$$

From standard relations for a second order transfer function, we know that

$$\xi_{CLTF} = \sqrt{\frac{(\ln PO)^2}{(\ln PO)^2+\pi^2}} \ , \tag{5}$$

$$t_s = \frac{4}{\xi_{CLTF}\omega_{n_{CLTF}}} \ . \tag{6}$$

Remark 1: It is important to note that the relation (6) is quite approximate. See any standard book on control systems.

Continuing, from (4),

$$2\xi_{CLTF}\omega_{n_{CLTF}} = 2\xi\omega_n + T_d(DC)\omega_n^2 \ , \tag{7}$$

Using (6), we can rewrite the LHS of (7) as

$$2\xi_{CLTF}\omega_{n_{CLTF}} = \frac{8}{t_s} \ , \tag{8}$$

Thus (7) becomes

$$\frac{8}{t_s} = 2\xi\omega_n + T_d(DC)\omega_n^2 \ , \tag{9}$$

Solving for T_d gives

$$\boxed{T_d = \frac{\left(\frac{8}{t_s} - 2\xi\omega_n\right)}{\omega_n^2(DC)} \ ,} \tag{10}$$

The above T_d achieves the desired settling time t_s. We next design K_c to achieve desired peak overshoot (PO). From (4), we have the following relation

$$\omega_{n_{CLTF}}^2 = \omega_n^2(1 + K_c(DC)) \ ,$$

or

$$\omega_{n_{CLTF}} = \sqrt{1 + K_c(DC)} \ , \tag{11}$$

Substituting for $\omega_{n_{CLTF}}$ in (7) gives

$$2\xi_{CLTF}\omega_n\sqrt{1 + K_c(DC)} = 2\xi\omega_n + T_d(DC)\omega_n^2 \ , \tag{12}$$

Solving (12) for K_c gives

$$K_c = \frac{\xi^2 - \xi^2{}_{CLTF} + \frac{T_d^2 (DC)^2 \omega_n^2}{4} + T_d (DC) \omega_n \xi}{(DC) \xi_{CLTF}^2} \ , \tag{13}$$

The above K_c achieves a desired PO (note that ξ_{CLTF} can be found from PO using equation 5).