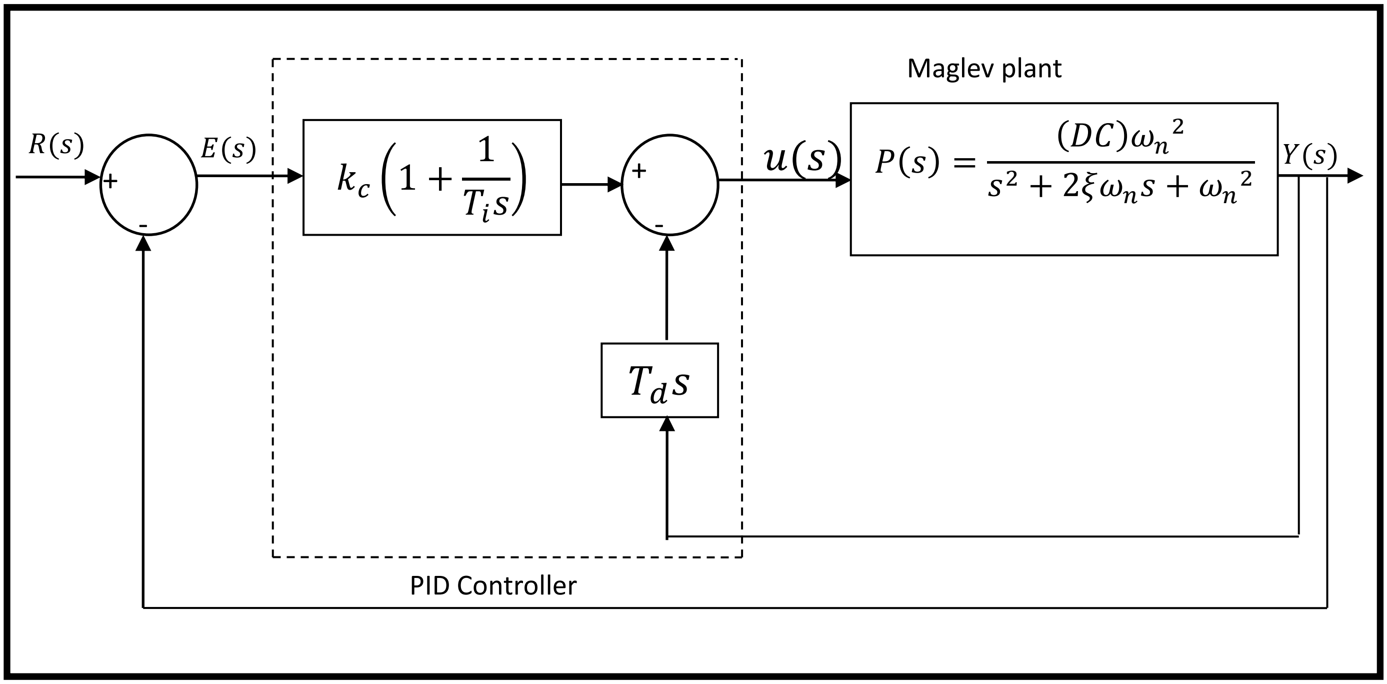
Theory

Consider the PID controller configuration with the Maglev plant shown in Figure:



The PID controller in above figure has the derivative mode acting on the output $Y(s)$ instead of the error $E(s)$. This is commonly done in industry, to prevent the "derivative kick" from happening for step changes in set point signal $R(s)$.

$$u(s) = k_c\left(1+\frac{1}{T_i s}\right)E(s) - T_d s Y(s)\,, \tag{1}$$

We can derive the closed loop transfer function CLTF between $Y(s)$ and $R(s)$as

$$CLTF = \frac{Y(s)}{R(s)} = \frac{(DC)\omega_n^2 K_c(s+\frac{1}{T_i})}{s^3+\left(2\xi\omega_n+T_d(DC)\omega_n^2\right)s^2+\omega_n^2\left(1+K_c(DC)\right)s+K_c\omega_n^2(DC)/T_i}\,, \tag{2}$$

Remark 1: *A word of caution in using T_d: In most PID implementation, we use the form*

$k_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d\ s\ \ Y(s)$,

so our T_d value may have to be suitably adjusted.

Let us compare the denominator of CLTF in (2) in terms of the standard second order system along with a pole p_{CLTF}. That is, let us express

$$s^3 + \left(2\xi\omega_n + T_d\omega_n^2(DC)\right)s^2 + \omega_n^2\left(1 + K_c(DC)\right)s + \frac{K_c\omega_n^2(DC)}{T_i}$$

$$= (s + p_{CLTF})\left(s^2 + 2\xi_{CLTF}\omega_{n_{CLTF}}s + \omega_{n\,CLTF}^2\right)\ , \qquad (3)$$

Where the term $(s + p_{CLTF})$ is chosen (or designed) to cancel the numerator term $\left(s + {}^{1}/_{T_i}\right)$ In equation (2) (leading to a standard second order form that is easy to use for design).

The RHS of (3) can be expressed as

$$s^3 + \left(2\xi_{CLTF}\omega_{n_{CLTF}} + p_{CLTF}\right)s^2 + \left(\omega_{n\,CLTF}^2 + 2\xi_{CLTF}\omega_{n_{CLTF}}p_{CLTF}\right)s + \omega_{n\,CLTF}^2 p_{CLTF}\ \ ,(4)$$

From (3) and (4)

$$s^3 + \left(2\xi\omega_n + T_d\omega_n^2(DC)\right)s^2 + \omega_n^2\left(1 + K_c(DC)\right)s + K_c\omega_n^2\ {}^{(DC)}/_{T_i}$$

$$= s^3 + \left(2\xi_{CLTF}\omega_{n_{CLTF}} + p_{CLTF}\right)s^2 + \left(\omega_{n\,CLTF}^2 + 2\xi_{CLTF}\omega_{n_{CLTF}}p_{CLTF}\right)s + \omega_{n\,CLTF}^2 p_{CLTF}\ , \qquad (5)$$

Comparing the coefficients of corresponding powers of s on both sides gives

- Constant: $\omega_{n_{CLTF}}^2 p_{CLTF} = K_c\omega_n^2(DC)/T_i$, (6)

- s term: $\omega_n^2\left(1 + K_c(DC)\right) = \omega_{n\,CLTF}^2 + 2\xi_{CLTF}\omega_{n_{CLTF}}p_{CLTF}$, (7)

- s^2term: $$2\xi\omega_n + T_d(DC)\omega_n^2 = 2\xi_{CLTF}\omega_{n_{CLTF}} + p_{CLTF} \ , \tag{8}$$

Note that p_{CLTF} is chosen such that the pole cancels the zero at s=-1/T_i.

Thus, $p_{CLTF} = \frac{1}{T_i} \rightarrow p_{CLTF} * T_i = 1$.

From equations (6), (7), (8), we obtain

$$K_c = \frac{\omega_{n_{CLTF}}^2}{\omega_n^2(DC)} \ , \tag{9}$$

$$T_i = \frac{2\xi_{CLTF}\omega_{CLTF}}{\omega_n^2} \ , \tag{10}$$

$$T_d = \frac{2\xi_{CLTF}\omega_{n_{CLTF}} - 2\xi\omega_n + 1/T_i}{\omega_n^2(DC)} \ , \tag{11}$$

From the given PO and using the well-known relation

$$\xi_{CLTF} = \sqrt{\frac{(lnPO)^2}{(lnPO)^2 + \pi^2}} \ , \tag{12}$$

we can obtain the corresponding ξ_{CLTF}. From the given t_s and using the approximate relation

$$t_s \approx \frac{4}{\xi_{CLTF}\omega_{n_{CLTF}}} \ , \tag{13}$$

we get

$$\omega_{n_{CLTF}} = \frac{4}{(t_s)(\xi_{CLTF})} \ , \tag{14}$$

where ξ_{CLTF} is found from (12).

Having obtained ξ_{CLTF} from (12) and $\omega_{n_{CLTF}}$ from (14), we can find the required K_c,T_i,T_d, using equations (9), (10), (11). The steady state error (or offset) to step input is zero, due to the presence of integral action.

Remark 2: *It is important to note that the relation (13) is quite approximate. See any standard book on control systems, or the references link on the main page of this site.*