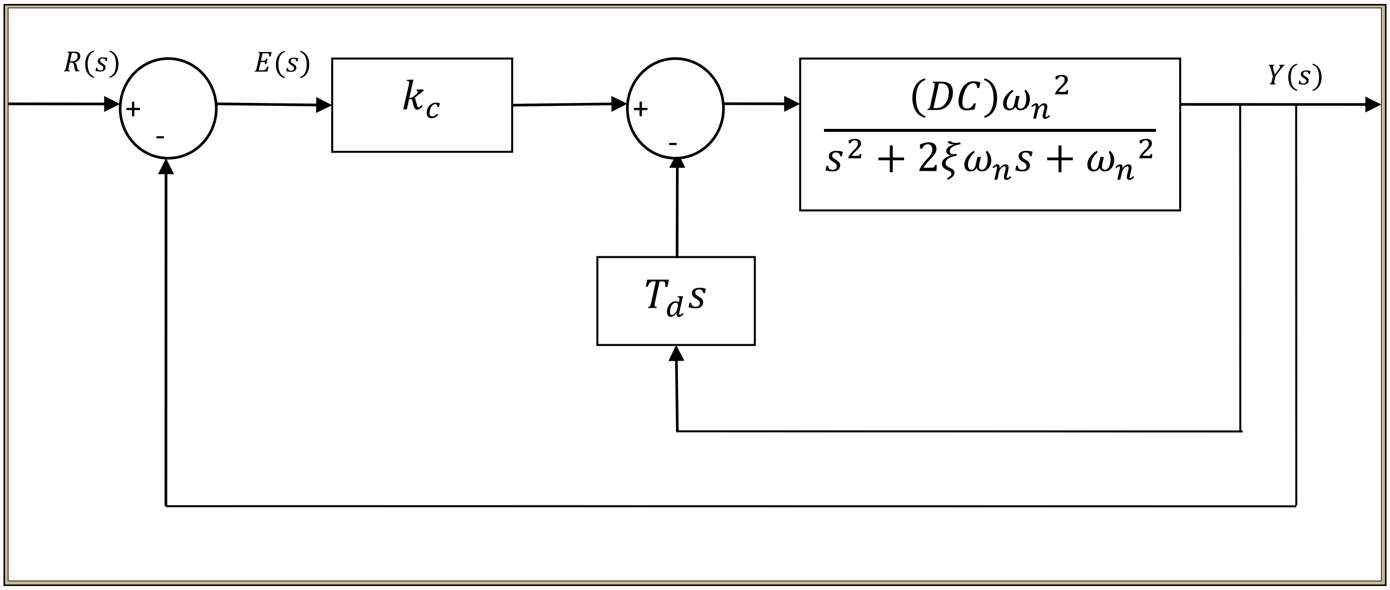
Theory

We now design the PD controller to place the closed loop poles at desired location. The PD controller that we use is the one shown in below figure.



In above figure, the derivative mode acts on the output, and not on the error. This prevents the "*derivative kick*" which happens when the set point $R(s)$ is changed by a step signal. (The derivative of a step signal is the impulse signal, so if $R(s)$ changes by a step, $E(s) = R(s) - Y(s)$ will change instantly by an impulse.) This gives the "derivative kick".

The PD controller in this configuration has the form:

Controller output $= K_c e(t) - T_d . \frac{dy(t)}{dt}$, (1)

The closed loop transfer function can be derived from the above figure as

$$CLTF = \frac{Y(s)}{R(s)} = \frac{K_c (DC)\omega_n^2}{s^2 + \{2\xi\omega_n + T_d (DC)\omega_n^2\}s + \omega_n^2(1 + K_c(DC))} \ , \tag{2}$$

Let us design the PD controller to place closed loop poles at desired location. Suppose we want to place the closed loop poles at $(-a' \pm jb')$. Then, the desired closed loop denominator is

$$(s + a' + jb')(s + a' - jb') = s^2 + 2a's + a'^2 + b'^2$$

Compare the $CLTF$ denominator in (2) to get

$$s^2 + 2a's + a'^2 + b'^2 = s^2 + \{2\xi\omega_n + T_d(DC)\omega_n^2\}s + \omega_n^2(1 + K_c(DC))$$

$$2a' = 2\xi\omega_n + T_d(DC)\omega_n^2 \,,$$

$$\mathrm{a}'^2 + \mathrm{b}'^2 = \omega_n^2\left(1 + \mathrm{K_c(DC)}\right) ,$$

which gives

$$\boxed{\begin{aligned} T_d &= \frac{2(a' - \xi\omega_n)}{\omega_n^2(DC)} \,, \\ K_c &= \frac{a'^2 + b'^2 - \omega_n^2}{\omega_n^2(DC)} \,. \end{aligned}} \qquad (3)$$