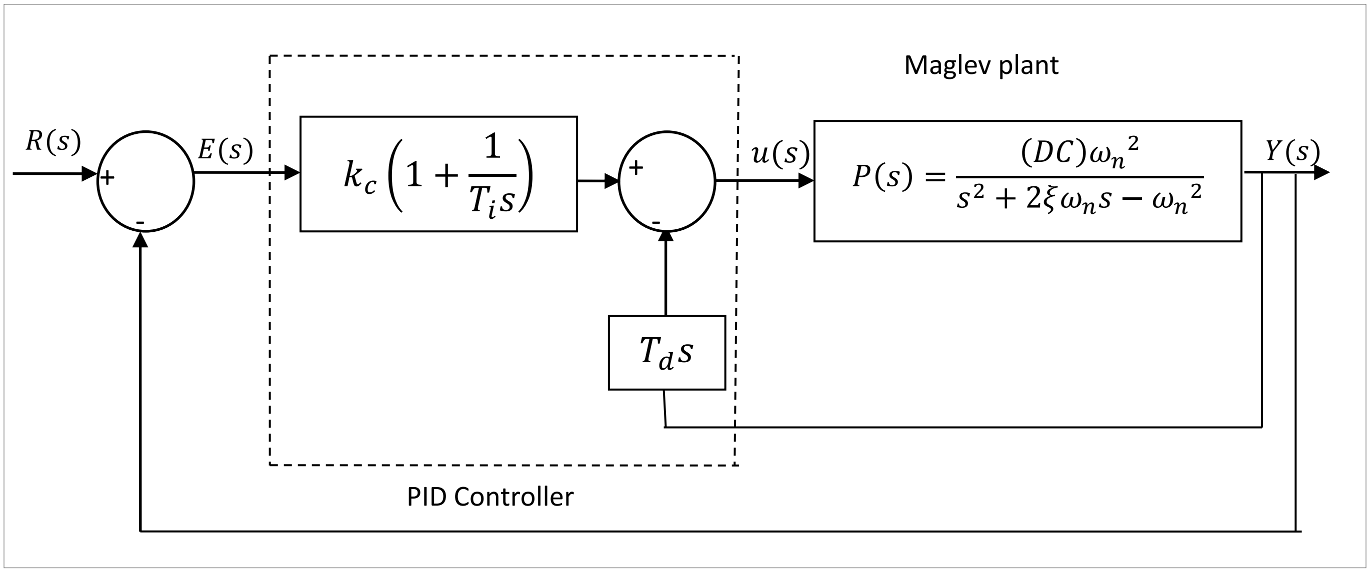
Theory

Consider the PID controller configuration with the unstable Maglev plant shown in Figure:



The PID controller in above figure has the derivative mode acting on the output $Y(s)$ instead of the error $E(s)$. As described earlier, this is commonly done in industry, to prevent the "derivative kick" from happening for step changes in setpoint signal $R(s)$.

We can derive the closed loop transfer function CLTF between $Y(s)$ and $R(s)$as

$$CLTF = \frac{Y(s)}{R(s)} = \frac{(DC)\omega_n^2 K_c\left(s + \frac{1}{T_i}\right)}{s^3 + \left(2\xi\omega_n + T_d(DC)\omega_n^2\right)s^2 + \omega_n^2\left(K_c(DC)-1\right)s + (DC)K_c\,\omega_n^2/T_i} \qquad (1)$$

If we compare the denominator of CLTF in (1) in terms of the standard second order system along with a pole p_{CLTF} , we have

$$\begin{aligned} & s^3 + \left(2\xi\omega_n + T_d\omega_n^2(DC)\right)s^2 + \omega_n^2\left(K_c(DC)-1\right)s + \frac{K_c\omega_n^2(DC)}{T_i} \\ & = \left(s + p_{CLTF}\right)\left(s^2 + 2\xi_{CLTF}\omega_{n_{CLTF}}s + \omega_{n\,CLTF}^2\right) \end{aligned} \qquad (2)$$

where the term $(s+p_{CLTF})$ is chosen (or designed) to cancel the numerator term$\left(s + {}^{1}/_{T_i}\right)$ in equation 2 (so that we get a standard second order form that is easy to use for design).

The RHS of (2) can be expressed as

$$s^3 + \left(2\xi_{CLTF}\omega_{n_{CLTF}}+p_{CLTF}\right)s^2 + \left(\omega_{n\,CLTF}^2 + 2\xi_{CLTF}\omega_{n_{CLTF}}+p_{CLTF}\right)s + \omega_{n\,CLTF}^2 p_{CLTF} \tag{3}$$

From (2) and (3)

$$\begin{aligned} &s^3 + \left(2\xi\omega_n + T_d\omega_n^2(DC)\right)s^2 + \omega_n^2\left(K_c\left(DC\right)-1\right)s + K_c\omega_n^2(DC)/T_i \\ &= s^3 + \left(2\xi_{CLTF}\omega_{n_{CLTF}} + p_{CLTF}\right)s^2 + \left(\omega_{n\,CLTF}^2 + 2\xi_{CLTF}\omega_{n_{CLTF}} + p_{CLTF}\right)s + \omega_{n\,CLTF}^2 p_{CLTF} \end{aligned} \tag{4}$$

Comparing the coefficients of corresponding powers of s on both sides of equation 4 gives

- Constant: $\omega_{n\,CLTF}^2 p_{CLTF} = K_c\omega_n^2(DC)/T_i$ (5)
- s term: $\omega_n^2(K_c(DC) - 1) = \omega_{n\,CLTF}^2 + 2\xi_{CLTF}\omega_{n_{CLTF}}p_{CLTF}$ (6)
- s^2term: $2\xi\omega_n + T_d(DC)\omega_n^2 = 2\xi_{CLTF}\omega_{n_{CLTF}} + p_{CLTF}$ (7)

From equations 5, 6, 7, we obtain

$$K_c = \frac{\omega_{n\,CLTF}^2}{\omega_n^2(DC)} \tag{8}$$

$$T_i = \frac{-2\xi_{CLTF}\omega_{n_{CLTF}}}{\omega_n^2} \tag{9}$$

$$T_d = \frac{2\xi_{CLTF}\omega_{n_{CLTF}} - 2\xi\omega_n + {}^{1}/_{T_i}}{\omega_n^2(DC)} \tag{10}$$

From the given PO and using the well-known relation

$$\xi_{CLTF} = \sqrt{\frac{(\ln PO)^2}{(\ln PO)^2+\pi^2}} \tag{11}$$

we can obtain the corresponding value of ξ_{CLTF}.

From the given t_s and using the approximate relation

$$t_s \approx \frac{4}{\xi_{CLTF}\omega_{n_{CLTF}}} \tag{12}$$

we get

$$\omega_{n_{CLTF}} = \frac{4}{(t_s)(\xi_{CLTF})} \tag{13}$$

where ξ_{CLTF} is found from (11).

Having obtained ξ_{CLTF} from (11) and $\omega_{n_{CLTF}}$ from (13), we can find the required K_c, T_i, T_d using equations 8, 9, 10.

The steady state error (or offset) to step inputs is zero, due to the presence of integral action.

Remark: *It is important to note that the relation (12) is quite approximate. See any standard book on control systems, or the references link on the main page of this site.*