Theory

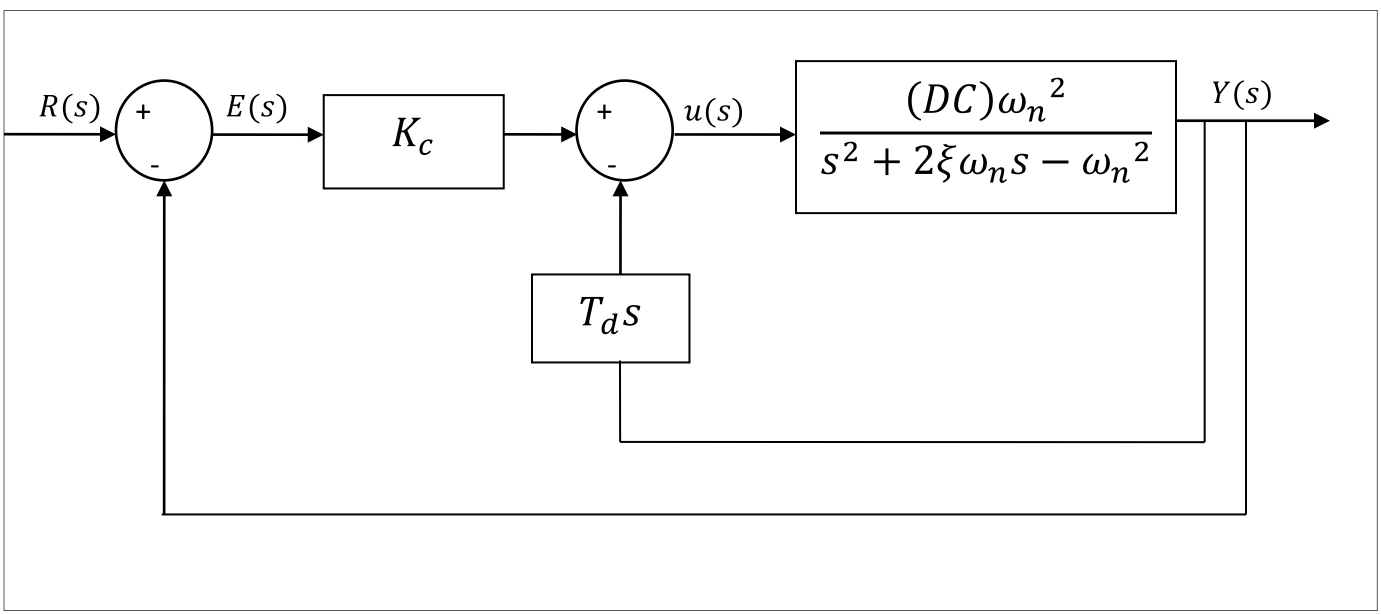
In the unstable configuration of the Maglev system, the upper electromagnetic coil is used to balance the magnetic disc at a desired height.

In this case, even a small disturbance at the equilibrium position makes the magnetic disc move completely away from the equilibrium position. Hence, a stabilizing controller is needed.

The transfer function of the Maglev plant in the unstable configuration can be derived as

$$P(s) = \frac{(DC)\omega_n^2}{s^2 + 2\xi\omega_n s - \omega_n{}^2} \ ,$$

Let us design a PD controller (in the "derivative kick" free form) to control the <u>unstable</u> Maglev plant. The closed loop block diagram is shown below:



The closed loop transfer function (CLTF) can be derived from the above block diagram as

$$CLTF = \frac{Y(s)}{R(s)} = \frac{K_c(DC)\omega_n^2}{s^2+\{2\xi\omega_n+T_d\omega_n^2(DC)\}s+\omega_n^2(K_c(DC)-1)} \ , \tag{1}$$

For comparison purposes, we recall the CLTF for the stable Maglev plant as

$$CLTF(s)_{forstablecase} \ = \frac{K_c(DC)\omega_n^2}{s^2 + \{2\xi\omega_n + T_d\omega_n^2(DC)\}s + \omega_n^2(K_c(DC) - 1)} \ ,$$

Put the CLTF in (1) in standard form:

$$CLTF(s) = \left\{\frac{K_c(DC)}{K_c(DC)-1}\right\} = \frac{\omega_n^2(K_c(DC)-1)}{s^2+\{2\xi\omega_n+T_d(DC)\omega_n^2\}s+\omega_n^2(K_c(DC)-1)} , \tag{2}$$

which shows that for the CLTF

$$(DC)_{CLTF} = \frac{K_c(DC)}{K_c(DC)-1} \ , \tag{3}$$

$$\omega_{n_{CLTF}}^2 = \omega_n^2(K_c(DC) - 1) \ , \tag{4}$$

$$2\xi_{CLTF}\omega_{n_{CLTF}} = 2\xi\omega_n + T_d(DC)\omega_n^2 , \tag{5}$$

Suppose we want the closed loop poles to be a pair of complex poles $(-a' \pm jb')$.That is, suppose we want the denominator of the desired CLTF to be the polynomial

$$(s + a' + jb')(s + a' - jb') = s^2 + 2a's + \left({a'}^2 + {b'}^2\right) , \tag{6}$$

Comparing (6) with the denominator of CLTF in (2) gives

$$s^2 + \{2\xi\omega_n + T_d(DC)\omega_n^2\}s + \omega_n^2(K_c(DC) - 1) = s^2 + 2a's + \left(a'^2 + b'^2\right) \ , \quad (7)$$

Or

$$2a' = 2\xi\omega_n + T_d(DC)\omega_n^2 \ , \quad (8)$$

$$\mathrm{a}'^2 + \mathrm{b}'^2 = \omega_n^2(\mathrm{K_c}(\mathrm{DC}) - 1) \ , \quad (9)$$

Solving for $\mathrm{K_c}$and T_d gives

$$K_c = \frac{a'^2 + b'^2 + \omega_n^2}{\omega_n^2(DC)} \ , \quad (10)$$

$$T_d = \frac{2a' - 2\xi\omega_n}{\omega_n^2(DC)} \ . \quad (11)$$