Theory

Let F_u be the attractive force experienced by the magnetic disc when current is passed through the electromagnetic coil. Considering the interaction of electrical current with magnetic fields, we can show that the force equation for magnet-coil interaction is of the form

$$F_u = \frac{i}{\tilde{a}(y+\tilde{b})^N} \ , \tag{1}$$

Where, y is the distance of the magnet from the coil, $\tilde{a}, \tilde{b}, N$ are constants that depend on the magnet and its configuration, and i is the coil current. If the coil current is adjusted by a computer algorithm via a digital-to- analog converter, then we can replace the coil current with the control effort u in counts. The coefficients $\tilde{a}, \tilde{b}$ in (1) need to be appropriately scaled, giving the force equation for magnet-coil interaction in the form

$$F_u = \frac{u}{a(y+b)^N} \ , \tag{2}$$

From fundamental physics, considering the forces acting on the magnet such as F_u and gravity, we can write the governing differential equation for F_u as

$$F_u = m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + mg \ , \tag{3}$$

Where, m is the mass of disc, c is the frictional coefficient, mg is weight of the disc, and y is displacement of the magnetic disc from the coil. Substituting the expression for F_u from equation (2) in equation (3), we get the governing differential equation as

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + mg = \frac{u}{a(y+b)^N} \ , \tag{4}$$

Denote the nonlinear term on the RHS of equation (4) as $f(u, y)$. Equation (4) can be rewritten as

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + mg = f(u, y) \ , \tag{5}$$

Now we shall linearize equation (5) around an equilibrium point. Let the equilibrium pointbe (u_0, y_0),

i. e., corresponding to the control input u_0 we have displacement y_0. At equilibrium point, the governing differential equation is

$$m\frac{d^2y_0}{dt^2} + c\frac{dy_0}{dt} + mg = f(u_0, y_0) \ , \tag{6}$$

Suppose a small perturbation of Δu about the input results in a small variation Δy about the output y. That is, let

$$y = y_0 + \Delta y; u = u_0 + \Delta u \ , \tag{7}$$

Substituting (7) in (5) gives the governing differential equation around the equilibrium as

$$m\frac{d^2}{dt^2}(y_0 + \Delta y) + c\frac{d}{dt}(y_0 + \Delta y) + mg = f(u_0 + \Delta u, y_0 + \Delta y) \ , \tag{8}$$

For a small perturbation, the RHS term can be linearized using the Taylor series approximation around the equilibrium point (u_0, y_0) as

$$f(u_0 + \Delta u, y_0 + \Delta y) \approx f(u_0 + y_0) + \left(\frac{\partial f}{\partial u}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) \Delta u + \left(\frac{\partial f}{\partial y}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) \Delta y \ , \qquad (9)$$

Substitute (9) in (8) gives

$$m \frac{d^2}{dt^2}(y_0 + \Delta y) + c \frac{d}{dt}(y_0 + \Delta y) + mg = f(u_0 + y_0) + \left(\frac{\partial f}{\partial u}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) \Delta u + \left(\frac{\partial f}{\partial y}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) \Delta y \ , (10)$$

Using (6), we can write (10) as

$$m \frac{d^2 \Delta y}{dt^2} + c \frac{d\Delta y}{dt} = \left(\frac{\partial f}{\partial u}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) \Delta u + \left(\frac{\partial f}{\partial y}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) \Delta y \ , \qquad (11)$$

where,

$$\left(\frac{\partial f}{\partial u}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) = \left(\frac{1}{a(y_0+b)^N} \right) and \left(\frac{\partial f}{\partial y}\bigg|_{\substack{u=u_0,\\ y=y_0}} \right) = \left(\frac{-Nu_0}{a(y_0+b)^{N+1}} \right) \ , \qquad (12)$$

Substituting (12) in (11) gives

$$m \frac{d^2 \Delta y}{dt^2} + c \frac{d\Delta y}{dt} = \left(\frac{1}{a(y_0+b)^N} \right) \Delta u + \left(\frac{-Nu_0}{a(y_0+b)^{N+1}} \right) \Delta y \ , \qquad (13)$$

Thus, the differential equation of the system for small perturbations around the equilibrium point (u_0, y_0) is

$$m\frac{d^2\Delta y}{dt^2} + c\frac{d\Delta y}{dt} + r\Delta y = q\Delta u \ , \tag{14}$$

where,

$$r = \left(\frac{Nu_0}{a(y_0+b)^{N+1}}\right), q = \left(\frac{1}{a(y_0+b)^{N}}\right) \ , \tag{15}$$

The above equation serves the perturbation model of the system.

Taking the Laplace transform of (14) gives the transfer function of the maglev plant as

$$P(s) = \frac{q}{ms^2+cs+r} = \frac{\Delta y(s)}{\Delta u(s)} \ , \tag{16}$$

Maglev Plant Parameters

In the particular Maglev plant under consideration, the various parameters have the following values

Parameter	Symbol	Units	Default value	Reference equation
Actuator parameter a	a	-	1.33	
Actuator parameter b	b	-	6.2	
Actuator parameter N	N	-	4	
Mass of magnet m	m	Kgs	Kgs	
Viscous friction coefficient	c	-	0.4	
Initial Height of Magnet	y_o	Cms	2.0	
Initial Control effort	u_0	Counts	7137.8	

In the last row of the table, the initial control effort u_0 is calculated from (2) as

$$u_0 = 1.187a(y_0 + b)^N, F_u = mg \ , \tag{17}$$

The initial control effort u_0 is also called the “gravity offset”.

The default values of the plant parameters are used in the user interfaces for all experiments.

However, you must use parameter values that are different from the default. The exact values will be provided to you.