Mössbauer spectroscopy of spacetimes

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Abstract

The Mössbauer effect has proven an indispensable tool in the study of systems where $\hbar$ is important. In this paper a program for its application to theoretical problems involving gravitation is presented. After refering the appropiate literature, a framework is proposed to implement the scheme. A relation between the Mössbauer effect and the lapse function is introduced and used later to depict the spectral lines and bands of some typical spacetimes. Special attention is devoted to NHEK geometry for its appeal as *alter ego* of the nuclear resonance in samples.

Keywords: spacetime - mössbauer - decay

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1 Introduction

This is the first of a set of papers exploiting the similarities between the Mössbauer effect -MBE- and black holes as an entry point for tackling more general problems. In the quest for shedding light on branches such as the information loss paradox, whatever analog system over which experiments could be conducted may be elucidating, particularly if these do not exclusively belong to the *gedaken* family.

MBE is a familiar technique whose study has been part of any Physics student curriculum and a very rich literature is available for a detailed study, we suggest [1, 2]. It can be safely argued that crystals undergoing MBE transitions share some of the properties of black holes. In some sense they also "have no hair"; they are asked for very little to give such a simple behaviour, when solid state physics and nuclear physics acting together can prove to be very complicated; and in both cases the process is a considerable perturbation if taken by one of the constitutients of the system, but the system reacts collectively and no component of it can be singled out. Take the Hawking effect as the channel for relaxation in the high temperature regime -continuous side of the spectrum, when thermodynamics applies- of what might be a much more intrincate spectrum which presumably extends to manifesting discrete levels and perhaps bands. Also in the MBE, as in QFT on general curved spacetimes, time translation symmetry is not expected: the dominant energy condition does not need to hold, there is no well defined total energy at a "sharp" moment of time [3], and even so, in many insteresting scenarios, a scattering matrix can be written [4]. To avoid mission creep, in this first paper the focus will be on providing a succint, straightforward introduction to the key elements of the program which will be addressed on the papers to come.

2 Setting up the framework

We need a representation for the spacetime suitable for our purposes. It often happens that situations that can be described purely in terms of spins give rise to pictures reasonably explained using semiclassical arguments ([1], p.206). As this is our case, the murky waters where GR and QM meet, we will follow that advice and pursue an implementation of the states in terms of angular momentum carrying quanta.

2.1 The Fock Space: Schwinger bosons in curved spacetime

Schwinger half integer spin bosons can provide an adequate *ansatz* to construct the Fock space as they can naturally accomodate angular momentum related issues such as its conservation. No more justification for this choice is needed since the closest thing we can have as vacuum state would be a quasi-free Hadamard state and the requirement for our machinery would essentially be that the singularity structure of our two point correlation function matches that of a spacetime with Killing time vector [5, 6]. Oscillators for the plethora of other groups beyond rotations should be investigated.

According to past literature [4], this bosonic case of spin 1/2 could be defined in the steps below:

1. Construct a compound system consisting of a two-state quantum subsystem, and a background field. The background field is to be taken as the arena where the scattering of "lumped" spacetimes take place.

2. Bring this to general spacetimes if needed.

After this we add:

3. Use the "Energy shift = function of lapse function" equation to resolve the spectrum of the spacetimes involved.

The first step is accounted for in the seminal work by Arecchi et al [7]. The second step is done pedagogically in chapters 3 and 4 of Wald's [4]. At this stage reelaborating both would be nonsensical and the author could not do it better, in fact, probably would just make the reader run away. Copying it and hiding it under different notations, as it is usually seen, is out of the question. In this paper we take that construction as a given. So basically all one has to do is the spectroscopy of the spacetimes. For brevity, and having found no foreseeable issue in the development of the first two steps, we jump directly to the third one, just bringing into our discussion the indispensable keypoints of the previous steps as needed.

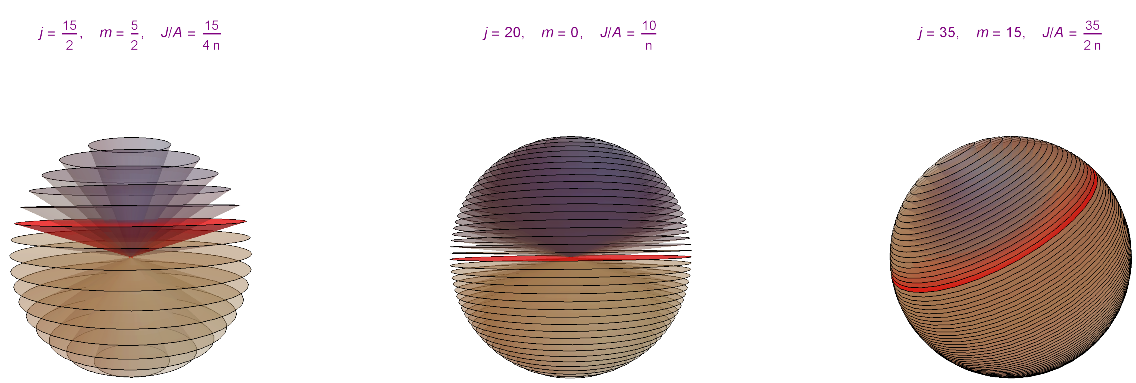

Fig. 1. Sampled some different configurations of angular momentum according to different numbers of the Schwinger representation. Displayed is also the ratio of angular momentum to area according to an adapted version of Bekenstein's relation where we set J as the one coming from the Schwinger model, in terms of the oscillators, whereas A comes from Bekenstein´s. The Mathematica code is elaborated as a modification of Blinder's [8].

2.2 The "energy shift = function of lapse function"

First, let α be the lapse function, as defined in eq. 2.3b [9] or eq. 4.2.1 in [4]. We choose to stick to the first notation because even if it would be natural to reserve that symbol to label the coherent state, whenever this may lead to confusion, it will be explicitly pointed out what the symbol means.

An energy shift can be seen as a time shift through the uncertainty relatation, thence, relate the energy shift to a function of the lapse function. Let us take it to be linear or quadratic, and may Nature *concedeth*.
The unconcious first guess would be:

$\delta_e\left(1-\frac{\delta_e}{2M}\right)=f(\alpha)$ (1)

for that to be dimensionally correct, the function should provide some energy to the dimensionless lapse function. As the natural scale around is the gravitational radius of the lump, $2M$, when seen from the point of view of gravitation, but it is $\hbar$ when seen from the MBE, this calls for a two-lenghtscale expansion (see Box 3.3 and 7.2 [10]) on $\hbar/2M$. Instead, now we just introduce a function f that will take care of providing the right dimensions and coping with undesirable behaviours, as explained below.
In the linear case, our expressions come to be:

$\delta_e\left(1-\frac{\delta_e}{2M}\right)=f\alpha$, $\delta_a\left(1+\frac{\delta_a}{2M}\right)=f\alpha$ (2)

(we introduced an index e for emission and a for absorption respectively). In the quadratic case it makes sense to use the same function times g_{00}:

$\delta_e\left(1-\frac{\delta_e}{2M}\right)=fg_{00}$, $\delta_e\left(1+\frac{\delta_e}{2M}\right)=fg_{00}$ (3)

The solutions for the linear case are:

$\delta_e=M\pm\sqrt{M^2-2Mf\alpha}$, $\delta_a=-M\pm\sqrt{M^2+2Mf\alpha}$ (4)

and the ones for the quadratic case are obtained by plain replacement of α for g_{00}.
From the simplicity of these equations we inmediatly infer some structure.
In the general case there is a splitting of the shifts, which sometimes is absent for certain values of the lapse function and the gravitational radius, which can be reminiscent of Zeeman effect or the Jaynes-Cummings ladder. The solutions contain sharp lines and can prove themselves very insensitive to the actual geometry of the spacetimes arranging them in equivalence classes according to the whether lapse function takes the same form in the metric. This method will not notice the difference as it only sees the gross structure of the spectra. For realizing the fine and hyperfine structure this scheme would need to be extended to off-diagonal terms of the metric of the form: rt, ϕt, θt, $\phi\theta t$...interplaying also with the scales on $\hbar$ and *2M*.
Obviously we would arrive at different expressions and conclusions following a different but somewhat parallel scheme where the identification didn´t link the lapse function to the energy shift but to the excitation energy, to some other similar parameter, or to some other way of representing the energy of the spacetime. In the same way, *2M* could be substituted by Christodoulou's irreducible mass, or simply M, or something completely different like a function of the Ricci scalar and $T_{\mu\nu}$.
With these equations at hand we can apply them now as a guidance for discovering possible constraints in the spirit of the discussion following theorem 10.2.2 in Wald [11], where we refer the reader to. But before we do so with some canonical lapse functions, for the cases of Minkowski, Rindler, Rindler at surface gravity of 1/4M, Schwarzschild, Kerr and NHEK, we would like to pause to comment on some related elements: the partition function, the recoiless fraction and the populating mechanism.

2.3 Partition function, recoiless fraction and populating mechanism

Even though the elements we introduce now are unnecesary for the application of our result*, they are an essential part of the formal description of the MBE. It must be said, that they may not be needed, and they may just be wrong, only the facts matter [12]-chapter 15 and preface-.
The reader can refresh of the partition function and the recoiless fraction regarding the MBE refering directly to [1, 2]. How should we interpret those in the case of spacetimes?. The author is not sure at the time of this writing.

If the representation on Schwinger bosons prevails -that is much to expect-, being bosons, the gravitational collapse should be radically different. The role of the recoiless fraction is clearer. In this scenario it has no meaning unless confronted with solutions like the multi-extremal-Nördstrom where the "sample" is the lattice of black holes that "just sit there".
As to accommodate the significant gaps that appear, which are on the scale of M, a mechanism to "populate" those bands in terms of Schwinger bosons (as proposed here as a possible option, or, anyhow, in terms of "swarms" of $\tilde{}\hbar$ contributing "particles", in the sense this term is to be taken [4]) is needed.
On account of the partition function, Strominger provided in his notes [13] a partition function and a complementary analysis for the case of shock-wave, Vaidya metric, which is one of the simplest and most graphical example to view excitation and decay for the discrete levels.
To provide one just from our prescription is a work in progress.
[*]Strominger once referred in his recorded lectures on Black Holes to Christodoulou's M_{irr}as "just pulled this out of a hat", we understand the equations here rely on the same foundations.

3 Generation and decay of the excited state of the spacetime

Before we embark on calculations, it is worth sharing the physical intuition for this paradigm so that we may anticipate what the result should look like. The following could be a "standard" example. Dangerous as it is thinking in Schwarzschild coordinates in highly dynamical situations[9], consider nevertheless a "lump spacetime", meaning an isolated submanifold -a Schwarzschild-like region f.ex.- "tightly bound" to a flat background. This could be seen as one of the states of the crystal in the lab. We stress again that the analogy is not between the crystal and a spacetime, but between a state of the crystal and a spacetime or between a transition in crystal states and a transition from a set of spacetimes. Then that Schwarzschild-like region can be excited through accretion (absorption with recoil in the case of the crystal) but at a rate that it makes it impossible for it to transit to another discrete excited state so it just can stimulate an ensemble of Schwinger bosons that account for this excitation. This presumably gives a Lorentzian shape to the distributions related to the lump, bands. If the accretion somehow were asymmetrical enough, this Schwinger bosons could "frame drag" the lump making it rotate. The dexcitation of this process comes through Hawking radiation, and we are witnessing a rotational band, an ergosphere, although if the rotation becomes extremal, it really comes to excite an overlapping discrete level, that of the NHEK[14, 15] geometry which can be then considered the upper bound of the rotational band which overlaps with an independent AdS_2xS^2 spacetime. Therefore NHEK could be regarded as an isomeric state of a Kerr geometry or a spacetime on the verge of being emitted along the decay of an extreme Kerr geometry to a Penrose-mined Kerr.
Now, when analyzing spacetimes with an event horizon, a detailed knowledge of all the spacetime is needed to properly talk about what is going on, but let's go ahead and think of a process in which a shock-wave has just the right energy to excite the first excited state of the spacetime. Decay does not come then as a "thunderbolt" at the endpoint of evaporation, using the terminology in [7], because the system does not decay this way. This is analogous to the case of electrons not falling to the nucleus due to emitted radiation. There is no radiation emitted in the Bohr stable atom, nor in the channels under discussion here or in any case it is not the main channel for falling to lower states. The system decays, if it does, through a shock-wave, be it a GRB, be it Dark Matter, Dark Energy or pure curvature (the area theorem is not considered to hold in this view, only in the regime where thermodynamics applies).

4 Spectroscopy of some metrics

We will be very "tolerant", impossing only the restrictions that can be mathematically suggested.

Minkowski:

$\delta_e = M \pm \sqrt{M(M+2f)}$ (5) This is acceptable if $|f| < \infty$; if we let f be complex valued, then solutions would oscillate.

$\delta_a = -M \pm \sqrt{M(M-2f)}$ (6) Oscillations show up for $2f > M$. It is nice to see that in every case, there is a $-M$ term that favors or enhances absorption.

For values such as $\delta_a > 0$, absorption comes at a price. Those are for f strictly negative. At $f = 0$, there is absorption: $\delta_a = -M \pm M$ and for $f \epsilon \left[0^+, \frac{M}{2}\right]$solutions are bounded and real. Being complex from there up.

Rindler:

$\delta_e = M \pm \sqrt{M(M+2g_H^2 z^2 f)}$ (7) g_H is the surface gravity[9], z is the spatial dimension along where the acceleration is observed. As we have too much freedom, set $g_H = \frac{1}{4M}$which takes us to the geometry of a Schwarzschild spacetime as seen just above the horizon ([9], eqs. 2.35-2.37).

$\delta_e = M \pm \sqrt{M(M+\frac{z^2}{8M^2}f)}$ (8) for consistency, as if the surface gravity is that for being just by the horizon, $z \sim 2M$, and the expression is further simplified to:

$\delta_e = M \pm \sqrt{M(M+\frac{f}{2})}$ (9) very similar to the Minkowski excitation energy (eq. 5). From (8) we see that f needs to compensate for the blowing up of z^2at large values of z.

Expressions analogous to the preceding can be obtained for absorption.

Kerr:

$\delta_e = M \pm \sqrt{M^2 + 2Mf\frac{r^2-2Mr+a^2(2-\cos^2\theta)}{r^2+a^2\cos^2\theta}}$ (10), as expected, we have bands for different values of a and of r,θ.

At the equator:

$\delta_{e_{\theta=\frac{\pi}{2}}} = M \pm \sqrt{M^2 + 2Mf\frac{r^2-2Mr+2a^2}{r^2}}$ (11)

At the poles:

$\delta_{e_{\theta=\{0,\pi\}}} = M \pm \sqrt{M^2 + 2Mf\frac{r^2-2Mr+a^2}{r^2+a^2}}$ (12)

Sibling equations for absorption are obtained easily.

NHEK:

This might be the most interesting case because the geometry is closely related to a crystal, and in particular to monolayers, as we can map the cylinder to the plane, of the metrics analyzed here, is the one that would connect better with the MBE physics.

We employ Poincaré coordinates as found in Lupsasca's dissertation [14]:

$g_{00} = J(1+\cos^2\theta)\left[-r^2 + \left(\frac{2r\sin\theta}{1+\cos^2\theta}\right)^2\right]$(13)

$\delta_e = M \pm \sqrt{M^2 - 2MJf(1+\cos^2\theta)\left[-r^2 + \left(\frac{2r\sin\theta}{1+\cos^2\theta}\right)^2\right]}$(14)

Regarding θ this is well behaved everywhere, so we can safely simplify without too much loss of generality.

At the equator:

$\delta_{e_{\theta=\frac{\pi}{2}}} = M \pm \sqrt{M^2 - 6MJfr^2}$(15)

At the poles:

$\delta_{e_{\theta=\{0,\pi\}}} = M \pm \sqrt{M^2 + 4MJfr^2}$(16)

At the equator, varying r we get the following lines and bands:

$E_\infty \equiv \lim_{r\to\infty} \delta_{e\frac{\pi}{2}} = M \pm \sqrt{-\infty}$(17) Emission to infinity. A result that might be interpreted as impossible. This decay mode is prohibited which may indicate some group theoretical question ruling it out. Physically reasonable. If the divergence is not to be taken seriously but a mere artifact of the coordinates and we ought to consider this just as a bare complex number, it might represent an oscillation in between two levels.

$E_H \equiv \lim_{r\to 2M} \delta_{e\frac{\pi}{2}} = M \pm \sqrt{M^2 - 24M^3Jf}$ (18) Emission to the horizon.

$E_{Planck} \equiv \lim_{r\to l_P} \delta_{e\frac{\pi}{2}} = M \pm \sqrt{M^2 - 6Ml_P^2Jf}$ (19) Emission to Planck length. Natural is to write too its value for $M = M_P$.

$E_s \equiv \lim_{r\to 0} \delta_{e\frac{\pi}{2}} = M \pm M$(20) Emission to singularity.

For its relevance we give the absorption lines and bands:

$A_\infty \equiv \lim_{r\to\infty} \delta_{a\frac{\pi}{2}} = \pm\infty$(21) Absorption from infinity.

$A_H \equiv \lim_{r\to 2M} \delta_{a\frac{\pi}{2}} = -M \pm \sqrt{M^2 + 24M^3Jf}$ (22) Absorption from the horizon.

$A_{Planck} \equiv \lim_{r\to l_P} \delta_{a\frac{\pi}{2}} = -M \pm \sqrt{M^2 + 6Ml_P^2Jf}$ (23) Absorption from Planck length. Equally it is natural to make $M = M_P$ in (23).

$A_s \equiv \lim_{r\to 0} \delta_{a\frac{\pi}{2}} = -M \pm M$(24) Absorption from the singularity.

Now all these can be taken as impossing conditions on f freeing us of some of the burden of leaving it unrestricted and undetermined.

E_H is perfectly fine as long as $\frac{1}{24MJ} \geq f$ (25), which for $J = M$ gives $\frac{1}{24M^2} \geq f$

E_P poses no problem for $M \geq 6l_P^2Jf$, solving for f at $J = M$: $\frac{1}{6l_P^2} \geq f$ (26)

A_H is identically well defined except for J negative, where there is a restriction that mimics the one for $E_H$but taking the absolute value of J in (25). And the same can be said of $A_P$with respect to its counterpart E_P.

A_s has a possible jump to $-2M$, and absorption is possible.

5 Experimental prospects

It is worth stating that the MBE was chosen as a paradigmatic example well known by anyone from the undergraduate student years. Newcoming techniques from Quantum Optics, “moving mirrors” or Time Domain Mössbauer Spectroscopy / Nuclear Forward Scattering of Synchrotron Radiation, could be even more suitable for experimental purposes. This last one provides short pulses on the scale of picoseconds generating nuclear excitons. If we were to prepare a Planck mass sample of absorber/emitter as a disk-shaped monolayer, this would be of a few centimeters in diameter and it would take ~nanoseconds for light to get to the edge from its center, and twice that in case the decay “took place” in one edge to cross through a diameter - for iron, the disk would extend roughly, 8cm in diameter, ~13ns-light . Whatever coherent behaviour would be demanded from the sample, for example, angular momentum conservation as described in the MBE, if the sample really acted as a whole, correlations might become spacelike. The Planck mass of the sample could be considered to be the one corresponding, not naively to the sample *per se* but, to the ensemble of the states of the sample that is acting in the process. Hence, it could serve to study quantum states on this mass scale experimentally. If the angular momentum were to be equally distributed along this state, a Berry phase could presumably be observed. The author is aware that all this could well end as problem 13.4 in [16].

Conclusions

In this first work we have argued that black holes and crystals undergoing recoiless gamma absorption or emission share enough physics as to devise a scheme towards relating the knowledge gained in one system to the other. In particular, the spectrum of black holes, and beyond, extending this to general spacetimes is analyzed by referring to a theoretical tool, a map between the energy shifts in a system -as seen in the Mössbauer effect-, and the lapse function of the system. By this procedure we arrive at spectral lines and bands, and we explore their meaning. Hopefully this baby steps will draw the attention of more skilled and talented researchers who will fix all that is wrong and fill all that is missing. Nevertheless, the core principle behind this approach might help a bit those in the search for the many-fingered nature of time and truth if it exists[10](above Exercise 28.24).

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References

[1] "Statistical Mechanics: a Set of Lectures", Richard Feynman, 1972

[2] "Mössbauer Effect in Lattice Dynamics: Experimental Techniques and Applications", Yi-Long Chen, De-Ping Yang, 2007

[3] "Extreme Kerr Throat geometry: A vacuum analog of AdS2xS2", James Bardeen, Gary Horowitz, 1999

[4] "Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics", Robert Wald, 1994

[5] "Singularity Structure of the Two-Point Function in Quantum Field Theory in Curved Spacetime", Fulling, Sweeny, Wald, 1978

[6] "The Formulation of Quantum Field Theory in Curved Spacetime", Robert Wald, arXiv:0907.0416 [gr-qc]

[7] "Atomic Coherent States in Quantum Optics", F.T. Arecchi, Eric Courtens, Robert Gilmore and Harry Thomas, 1972

[8] http://demonstrations.wolfram.com/SchwingersOscillatorModelForAngularMomentum/ S.M. Blinder, 2020

[9] "Black Holes: The Membrane Paradigm", Kip. Thorne, Richard Price and Douglas MacDonald, 1986

[10] "Modern Classical Physics", Kip Thorne, Roger Blandford, 2017

[11] "General Relativity", Robert Wald, 1984

[12] "The Theory of Fundamental Processes", Richard Feynman, 1961

[13] "Les Houches Lectures on Black Holes", Andrew Strominger, 1995

[14] "The Maximally Rotating Black Hole as a Critical Point in Astronomy", Phd Thesis dissertation, Alexandru Lupsasca, 2017

[15] "Rotating black holes: Locally nonrotating frames, energy extraction, and scalar synchrotron radiation", James Bardeen, William Press, Saul Teukolsky, 1972

[16] "Problem Book in Relativity and Gravitation", Alan Lightman, William Press, Richard Price, Saul Teukolsky, 1975

[*] "Lecture #12, 2019, Phys 211, Black Holes [41:28]Youtube video", Andrew Strominger, 2019