Tema 4: Campo Magnético

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Contenidos

ATENCIÓN !!!

1. Estas notas de ninguna manera sustituyen a los libros recomendados en la bibliografía de la asignatura.
2. Por ello, utilizar SIEMPRE un libro como complemento de estas notas.
3. Estas notas pueden contener errores involuntarios de los que el autor no se responsabiliza.

Introducción

- Las primeras observaciones de las propiedades magnéticas de materiales como la magnetita (Fe_3O_4) se remontan a los antiguos griegos.
- No fue sin embargo hasta el mediados del siglo XIX cuando Oersted, Henry, Faraday y Maxwell entre otros enunciaron de forma rigurosa los orígenes del magnetismo y su síntesis con la electricidad.
- En esta primera parte del tema estudiaremos el efecto de los campos magnéticos sobre cargas eléctricas y corrientes.
- En la segunda parte del tema estudiaremos las fuentes o causas de campo magnético

Fuerza ejercida por un campo magnético: Fuerza de Lorentz #1

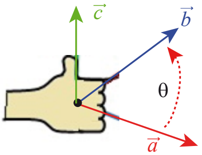
- En el estudio de la electrostática hemos utilizado el concepto de campo eléctrico $\vec{\mathbf{E}}$.

- Sabemos que un campo eléctrico $\vec{\mathbf{E}}$ existe en todos los puntos del espacio cuando al colocar una carga puntual q en un cierto punto P, sobre la carga q se ejerce una fuerza $\vec{\mathbf{F}}_E$ dada por
$$\vec{\mathbf{F}}_E = q\,\vec{\mathbf{E}}$$

- Del mismo modo, en una región del espacio existe un campo magnético $\vec{\mathbf{B}}$ si sobre una partícula de carga q situada en un cierto punto P, el campo $\vec{\mathbf{B}}$ ejerce una fuerza
$$\vec{\mathbf{F}} = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}, \quad \vec{\mathbf{v}} \equiv \text{velocidad de carga } q$$

- Esta fuerza recibe el nombre de Fuerza de Lorentz.

El producto vectorial



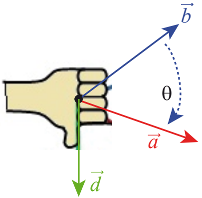
$\vec{c} = \vec{a} \times \vec{b}$

$c = a\, b\, sen\, \theta$

**Tenemos que "tumbar" el primer vector ($\vec{a}$) sobre el segundo vector ($\vec{b}$)*

** Se induce un giro de sentido antihorario (positivo, arco rojo)*

** Dirección y sentido de $\vec{c}$: Regla de la Mano Derecha: el dedo gordo apuntaría hacia arriba*



$\vec{d} = \vec{b} \times \vec{a}$

$d = a\, b\, sen\, \theta$

**Tenemos que "tumbar" el primer vector ($\vec{b}$) sobre el segundo vector ($\vec{a}$)*

** Se induce un giro de sentido horario (negativo, arco azul)*

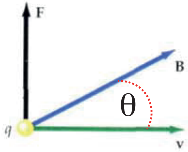
** Dirección y sentido de $\vec{d}$: Regla de la Mano Derecha, el dedo gordo apunta hacia abajo*

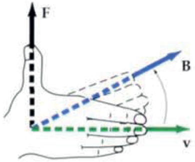
Fuerza ejercida por un campo magnético: Fuerza de Lorentz #2

- La fuerza magnética es perpendicular a $\vec{\mathbf{v}}$ y a $\vec{\mathbf{B}}$ simultáneamente.
- La fuerza magnética es cero cuando la carga está en reposo ($\vec{\mathbf{v}} = 0$) y cuando $\vec{\mathbf{v}}$ y $\vec{\mathbf{B}}$ son paralelos.
- El módulo de $\vec{\mathbf{F}}$ viene dado por

$$F = q\, v\, B \sin\theta$$

- Dirección y sentido de $\vec{\mathbf{F}}$ viene dado regla de la mano derecha, sacacorchos,...





Fuerza ejercida por un campo magnético: Fuerza de Lorentz #3

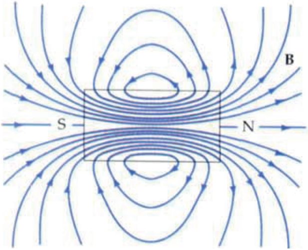
- Las unidades de $\vec{\mathbf{B}}$ en el S.I. son $N/(A\ m)$.
- Esta unidad se llama Tesla (T).
- Otra unidad muy usual de $\vec{\mathbf{B}}$ es el Gauss (G) de forma que: $1\ T = 10^4\ G$
- La fuerza magnética es siempre perpendicular a la velocidad de la carga y *el trabajo que realiza un campo magnético estático* $\vec{\mathbf{B}}$ *es cero.*

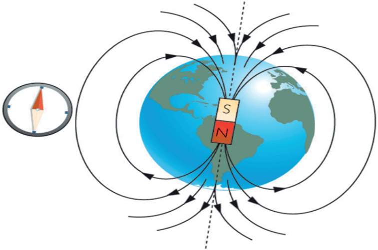
$$\delta W = \vec{\mathbf{F}} \cdot d\vec{l} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}\, dt = 0$$

- **La energía cinética de la carga permanece constante, es decir, el campo $\vec{\mathbf{B}}$ solo es capaz de hacer cambiar la dirección de la velocidad de la carga pero no su módulo.**

Fuerza ejercida por un campo magnético: Fuerza de Lorentz #4

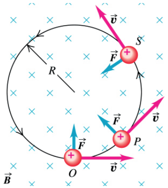
- Al igual que el campo eléctrico, el campo magnético se puede representar por líneas de campo.
- El campo $\vec{\mathbf{B}}$ también es tangente a las líneas y su intensidad es proporcional al número de ellas.
- Al contrario que las líneas de $\vec{\mathbf{E}}$, los polos magnéticos no existen y las líneas de $\vec{\mathbf{B}}$ con siempre cerradas.





Movimiento de una Carga en un Campo Magnético Uniforme #1

- Una partícula de carga q y masa m penetra en una región de campo constante $\vec{\mathbf{B}}$ según el eje z perpendicular al papel y hacia adentro.



- Supongamos que la carga tiene una velocidad inicial en el plano $x - y$, siendo por tanto perpendicular a $\vec{\mathbf{B}}$.

$$\vec{\mathbf{F}} = q\,\vec{v} \times \vec{\mathbf{B}}, \qquad F = q\,v\,B$$

Movimiento de una Carga en un Campo Magnético Uniforme #2

- Esta fuerza es central y produce un movimiento circular uniforme de radio constante dado por:

$$F = q\,v\,B = m\,a_n = m\,\frac{v^2}{R}, \qquad R = \frac{m\,v}{q\,B}$$

- El periodo de giro τ es independiente de la velocidad v:

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{v/R} = \frac{2\pi m}{qB}.$$

Movimiento de una Carga en un Campo Magnético Uniforme #3

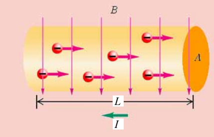
EJEMPLO 1: En el laboratorio hay un aparato que permite determinar la relación entre la carga q y la masa m de un electrón. En pocas palabras, un haz de electrones que se mueven a una cierta velocidad v en línea recta es forzado a moverse en una trayectoria circular de radio $R \approx 5$ cm. Los electrones son acelerados mediante una diferencia de potencial de unos $\Delta V = 250$ voltios. Determinar el valor del campo magnético que se usa en esta experiencia.

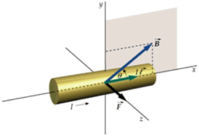
Datos: Carga del electrón: $q \approx -1.6 \times 10^{-19}$ C; masa del electrón: $m \approx 9.1 \times 10^{-31}$ kg.

Solución: ≈ 0.0011 T

Fuerza Magnética sobre un Conductor con Corriente #1

- Sea un conductor recto de longitud L y sección A por el que circula una corriente I y que está sometido a un campo magnético $\vec{\mathbf{B}}$ que suponemos **constante**.





- La fuerza $d\vec{\mathbf{F}}$ que $\vec{\mathbf{B}}$ ejerce sobre un tramo de longitud dx del conductor en el que hay contenido una carga dq:

$$d\vec{\mathbf{F}} = dq\, \vec{\mathbf{v}}_d \times \vec{\mathbf{B}},$$

donde $\vec{\mathbf{v}}_d$ es la velocidad de deriva con que se mueven los portadores de carga (es decir, los electrones) del conductor.

Fuerza Magnética sobre un Conductor con Corriente #2

- La carga dq que hay en dx podemos expresarla en función de la corriente: $dq = I\,dt$.
- La velocidad de deriva podemos expresarla como:$\vec{\mathbf{v}}_d = \frac{d\vec{\mathbf{x}}}{dt}$
- Sustituyendo en la expresión de $d\vec{\mathbf{F}}$ tenemos:

$$d\vec{\mathbf{F}} = I\,\vec{\mathbf{dx}} \times \vec{\mathbf{B}}.$$

- La fuerza total $\vec{\mathbf{F}}$ sobre el conductor será la integral sobre x entre 0 y la longitud L del conductor quedando:

$$\vec{\mathbf{F}} = \int_{x=0}^{x=L} I\,\vec{\mathbf{dx}} \times \vec{\mathbf{B}} = I\,\vec{L} \times \vec{\mathbf{B}}, \qquad F = I\,L\,B\sin\theta.$$

- Siendo $\vec{L}$ un vector de módulo la longitud L del conductor y cuyo sentido es el de la corriente que lo recorre.

Fuerza Magnética sobre un Conductor con Corriente #3

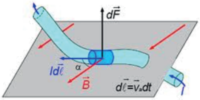
- Para el caso de un conductor que no es recto dentro de un campo magnético no uniforme, dividimos el alambre en trozos rectos de longitud dl donde asumimos que $\vec{\mathbf{B}}$ no varía en forma apreciable.

- La fuerza $d\vec{\mathbf{F}}$ sobre cada uno de estos trozos viene dada por

$$d\vec{\mathbf{F}} = I\, d\vec{l} \times \vec{\mathbf{B}}, \qquad dF = I\, dl\, B \sin\theta$$

- La fuerza total $\vec{\mathbf{F}}$ sobre el conductor vendrá dada por integral sobre el conductor:

$$\vec{\mathbf{F}} = \int_l I\, d\vec{l} \times \vec{\mathbf{B}}$$



Fuerza Magnética sobre un Conductor con Corriente: Ejemplos #4

- **EJEMPLO 2**: Un alambre recto horizontal transporta una corriente de 10 A de Oeste a Este en el campo magnético terrestre. El alambre está colocado en un lugar donde $\vec{\mathbf{B}}$ es paralelo a la superficie de la Tierra y tiene un valor de 0.04 mT. a) Calcular la fuerza magnética sobre un 1 m de ese alambre; b) si la masa del trozo de alambre es de 50 g, determinar la corriente que debe circular por el alambre para que éste quede levitando en el aire.

  Solución: a) $\mathbf{F} = 4 \times 10^{-4}$ N $\mathbf{k}$; b) $I = 12250$ A.

Fuerza Magnética sobre un Conductor con Corriente: Ejemplos #5

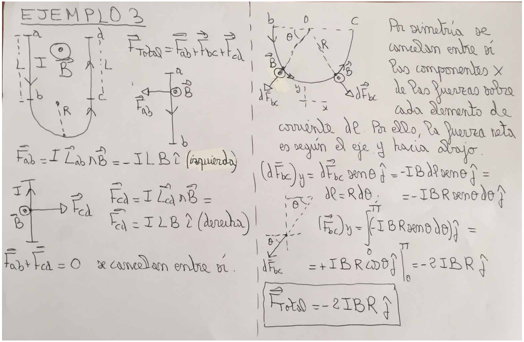
- **EJEMPLO 3**: Un conductor en forma de U transporta una corriente I y está contenido en un plano perpendicular a un campo magnético uniforme. La parte curva es un semicírculo de radio R. Determinar la fuerza neta sobre dicho conductor.

  Pista: La fuerza total es la suma de la fuerza sobre cada uno de los dos tramos rectos de la U y sobre de tramo curvo de la U. Suponer la fuerza sobre cada tramo aplicada sobre el centro de masas del tramo correspondiente.

  Solución: Ver página siguiente

Fuerza Magnética sobre un Conductor con Corriente: Ejemplos #5

- **EJEMPLO 3**: Solución



EJEMPLO 3

$\vec{F}_{Total} = \vec{F}_{ab} + \vec{F}_{bc} + \vec{F}_{cd}$

$\vec{F}_{ab} = I\,\vec{L}_{ab} \wedge \vec{B} = -ILB\,\hat{\imath}$ (izquierda)

$\vec{F}_{cd} = I\,\vec{L}_{cd} \wedge \vec{B} =$

$\vec{F}_{cd} = ILB\,\hat{\imath}$ (derecha)

$\vec{F}_{ab} + \vec{F}_{cd} = 0$ se cancelan entre sí.

Por simetría se cancelan entre sí las componentes x de las fuerzas sobre cada elemento de corriente dl. Por ello, la fuerza neta es según el eje y hacia abajo.

$(d\vec{F}_{bc})_y = d\vec{F}_{bc}\,\text{sen}\,\theta\,\hat{\jmath} = -IB\,dl\,\text{sen}\,\theta\,\hat{\jmath} =$

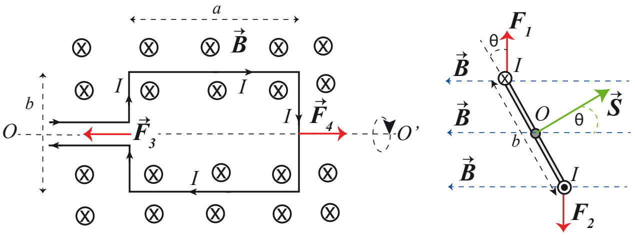
$dl = R\,d\theta$; $= -IBR\,\text{sen}\,\theta\,d\theta\,\hat{\jmath}$

$(\vec{F}_{bc})_y = \int_0^{\pi} (-IBR\,\text{sen}\,\theta\,d\theta)\,\hat{\jmath} = +IBR\cos\theta\,\hat{\jmath}\Big|_0^{\pi} = -2IBR\,\hat{\jmath}$

$\boxed{\vec{F}_{Total} = -2IBR\,\hat{\jmath}}$

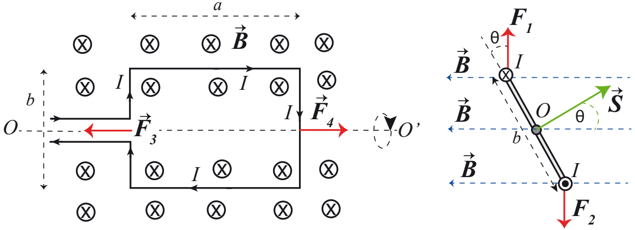
Momento de Fuerza sobre una Espira con Corriente #1

- Sea una espira rígida de área $S = a \cdot b$ por la que circula una corriente I y que está inmersa en un campo magnético constante $\vec{\mathbf{B}}$ según la dirección z.
- La espira está obligada a girar alrededor del eje OO'



- Las fuerzas $\vec{\mathbf{F}}_3$ y $\vec{\mathbf{F}}_4$ sobre las corrientes en los lados de longitud b se cancelan entre sí y están aplicadas sobre el eje de giro, luego no producen ningún efecto.

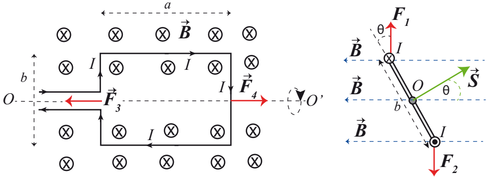
Momentos de Fuerza sobre una Espira con Corriente #2



- Sin embargo, las fuerzas $\vec{\mathbf{F}}_1$ y $\vec{\mathbf{F}}_2$ sobre las corrientes en los lados de longitud a aunque se cancelan entre sí, producen giro en la espira alrededor del eje OO'.

$$\vec{\mathbf{F_1}} = -\vec{\mathbf{F_2}} = I\,\vec{a} \times \vec{\mathbf{B}}, \quad F_1 = F_2 = I\,a\,B$$

Momentos de Fuerza sobre una Espira con Corriente #3



- El efecto de girar alrededor de un eje fijo se caracteriza por una magnitud física llamada momento $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
- Los momentos $\vec{\tau}_1$ y $\vec{\tau}_2$ que crean $\vec{\mathbf{F}}_1$ y $\vec{\mathbf{F}}_2$ son iguales y se suman:

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_1 = 2\,\frac{\vec{\mathbf{b}}}{2} \times \vec{\mathbf{F}}_{1,2},$$

$$\tau = I\;a\;b\;B\sin\theta = I\;S\;B\sin\theta$$

Momentos de Fuerza sobre una Espira con Corriente #4

- Siendo $\vec{\mathbf{S}}$ el vector superficie de la espira, expresamos $\vec{\tau}$:

$$\vec{\tau} = I\,\vec{\mathbf{S}} \times \vec{\mathbf{B}}$$

- Si la espira tiene N vueltas, el momento total es N veces el de una sola espira:

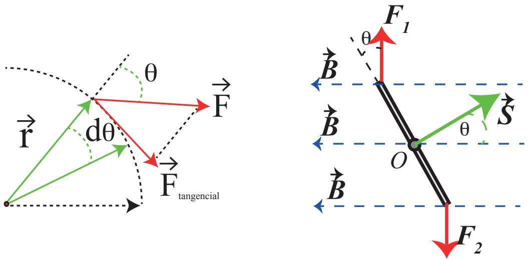
$$\vec{\tau} = N\,I\,\vec{\mathbf{S}} \times \vec{\mathbf{B}}, \qquad \tau = N\,I\,S\,B\sin\theta$$

- Se define el **momento magnético** o **dipolo magnético** $\vec{\mu}$ de una espira con corriente como:

$$\vec{\mu} = N\,I\,S\,\widehat{\mathbf{n}}, \quad \vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

- Unidades de $\vec{\mu}$: Amperio·metro^2 ($A \cdot m^2$).
- Este efecto es la base del funcionamiento de los motores eléctricos.

Energía Potencial de un dipolo magnético en un campo magnético #1



- El trabajo de rotación δW viene dado por:

$$\delta W = \vec{\mathbf{F}} \cdot \vec{\mathbf{dl}} = F\ r\ \sin\theta\ d\theta = -\tau d\theta.$$

- En en caso de un dipolo magnético $\vec{\mu}$ en presencia de un campo magnético $\vec{B}$:

$$\delta W = -\tau d\theta = -\mu B \sin\theta d\theta.$$

Energía Potencial de un dipolo magnético en un campo magnético #2

- Este trabajo es igual a menos la variación de energía potencial dU del dipolo:

$$dU = \mu B \sin\theta d\theta.$$

- La variación de energía potencial ΔU cuando el dipolo gira entre los ángulos inicial θ_o y final θ es:

$$\Delta U = \int_{\theta_o}^{\theta} \mu B \sin\theta d\theta = -\mu B \cos\theta + \mu B \cos\theta_o$$

- Por tanto, la energía potencial del dipolo es:

$$U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{\mathbf{B}}$$

- La energía potencial del dipolo es mínima cuando $\theta = 0$. Es decir, cuando la espira está situada perpendicular al campo magnético $\vec{B}$.

CHAPTER 26

THE AURORA BOREALIS APPEARS WHEN THE SOLAR WIND, CHARGED PARTICLES PRODUCED BY NUCLEAR FUSION REACTIONS IN THE SUN, BECOMES TRAPPED BY EARTH'S MAGNETIC FIELD. *(Atlas Photo Bank/Photo Researchers, Inc.)*

The Magnetic Field

? How does Earth's magnetic field act on subatomic particles? (See Example 26-1.)

More than 2000 years ago, the Greeks were aware that a certain type of stone (now called magnetite) attracts pieces of iron, and written references exist which describe the use of magnets for navigation dating from the twelfth century.

In 1269, Pierre de Maricourt discovered that a needle laid at various positions on a spherical natural magnet orients itself along lines that pass through points at opposite ends of the sphere. He called these points the *poles of the magnet*. Subsequently, many experimenters noted that every magnet of any shape has two poles, called the north and the south pole, where the force exerted by the magnet is strongest. It was also noted that the *like poles* of two magnets repel each other and the *unlike poles* of two magnets attract each other.

In 1600, William Gilbert discovered that Earth is a natural magnet and has magnetic poles near the north and south geographic poles. Because the north pole of a compass needle points toward the south pole of a given magnet, what we call the north pole of Earth is actually a south magnetic pole, as illustrated in Figure 26-1. Thus, the north and south poles of a magnet are sometimes referred to as the north-seeking and south-seeking poles, respectively.

Although electric charges and magnetic poles are similar in many respects, there is an important difference: Magnetic poles always occur in pairs. When a magnet is broken in half, equal and opposite poles appear at either side of the break point. The result is two magnets, each with a north and a south pole.

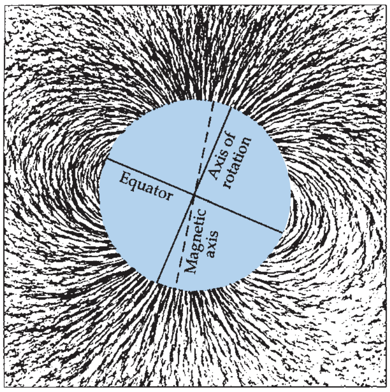


FIGURE 26-1 Magnetic field lines of Earth depicted by iron filings around a uniformly magnetized sphere. The field lines exit from the north magnetic pole, which is near the south geographic pole, and enter the south magnetic pole, which is near the north geographic pole.

There has long been speculation about the existence of an isolated magnetic pole, and in recent years considerable experimental effort has been made to find such an object. Thus far, there is no conclusive evidence that an isolated magnetic pole exists.

In this chapter, we consider the effects of a given magnetic field on moving charges and on wires carrying currents. The sources of magnetic fields are discussed in the next chapter.

26-1 THE FORCE EXERTED BY A MAGNETIC FIELD

The existence of a magnetic field $\vec{B}$ at some point in space can be demonstrated using a compass needle. If there is a magnetic field, the needle will align itself in the direction of the field.*

It has been experimentally observed that, when a particle that has charge q and velocity $\vec{v}$ is in a region with a magnetic field $\vec{B}$, a force acts on the particle that is proportional to q, to v, to B, and to the sine of the angle between the directions of $\vec{v}$ and $\vec{B}$. Surprisingly, the force is perpendicular to both the velocity and the megnetic field. These experimental results can be summarized as follows: When a particle that has a charge q and a velocity $\vec{v}$ is in a region with a magnetic field $\vec{B}$, the magnetic force $\vec{F}$ on the particle is

$$\vec{F} = q\vec{v} \times \vec{B} \qquad 26\text{-}1$$

MAGNETIC FORCE ON A MOVING CHARGED PARTICLE

Because $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$, $\vec{F}$ is perpendicular to the plane defined by these two vectors. The direction of $\vec{v} \times \vec{B}$ is given by the right-hand rule as $\vec{v}$ is rotated into $\vec{B}$, as illustrated in Figure 26-2. If q is positive, then $\vec{F}$ is in the same direction as $\vec{v} \times \vec{B}$.

Examples of the direction of the forces exerted on moving charged particles when the magnetic field vector $\vec{B}$ is in the vertical direction are shown in Figure 26-3.

The direction of any particular magnetic field $\vec{B}$ can be found experimentally by measuring $\vec{F}$ and $\vec{v}$ for several velocities in different directions and then applying Equation 26-1.

Equation 26-1 defines the **magnetic field** $\vec{B}$ in terms of the force exerted on a moving charged particle. The SI unit of magnetic field is the **tesla** (T). A particle that has a charge of one coulomb and is moving with a velocity of one meter per second perpendicular to a magnetic field of one tesla experiences a force of one newton:

$$1\ \text{T} = 1\frac{\text{N}}{\text{C}\cdot\text{m/s}} = 1\ \text{N/(A}\cdot\text{m)} \qquad 26\text{-}2$$

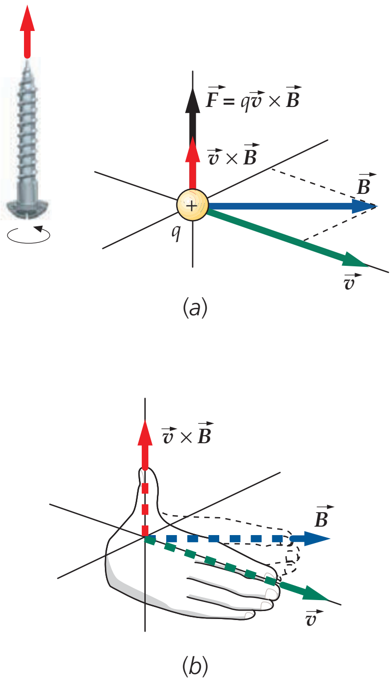
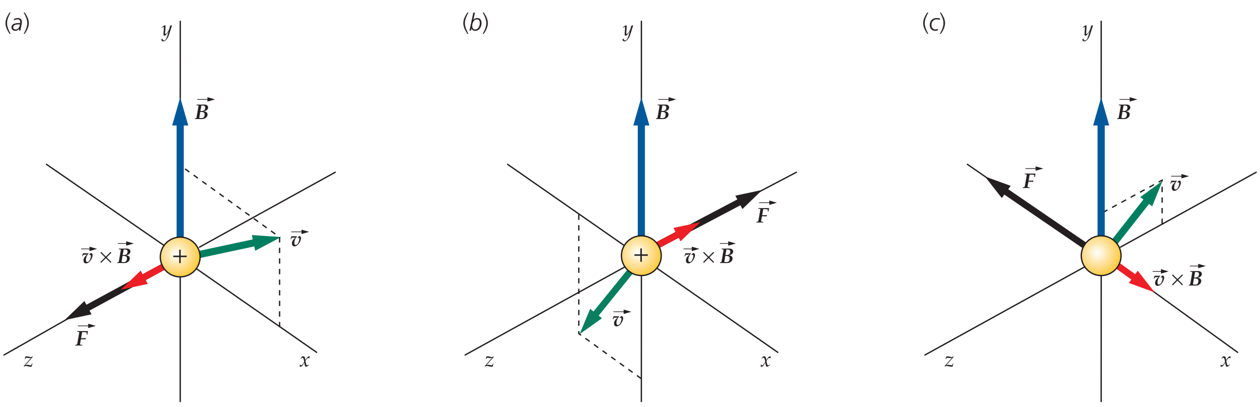


FIGURE 26-2 Right-hand rule for determining the direction of a force exerted on a charged particle moving in a magnetic field. If $q$is positive, then $\vec{F}$, is in the same direction as $\vec{v} \times \vec{B}$. (*a*) The vector product $\vec{v} \times \vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{B}$ and is in the direction of the advance of a right-hand-threaded screw if turned in the same direction as to rotate $\vec{v}$ into $\vec{B}$. (*b*) If the fingers of the right hand are in the direction of $\vec{v}$ so that they can be curled toward $\vec{B}$, the thumb points in the direction of $\vec{v} \times \vec{B}$.

FIGURE 26-3 Part (*a*) and (*b*) show the direction of the magnetic force on a positively charged particle moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. In Concept Check 26-2 (see page 889), you are asked to find the sign of the charge on the particle shown in part (*c*) of this figure.



* Compass needles are suspended so they remain horizontal. This results in the compass needle aligning itself in the horizontal component of the magnetic field. A compass needle with an unrestricted suspension would align itself in the magnetic field.

CONCEPT CHECK 26-1

The direction of any magnetic field $\vec{B}$ is specified as the direction that the north pole of a compass needle points toward when the needle is aligned in the field. Suppose that the direction of the magnetic field $\vec{B}$ were instead specified as the direction pointed toward by the south pole of a compass needle aligned in the field. Would the right-hand rule shown in Figure 26-2 then give the direction of the magnetic force on the moving positive charge, or would a left-hand rule be required? Explain your answer.

CONCEPT CHECK 26-2

The particle in Figure 26-3(*c*) (*a*) is positively charged, (*b*) is negatively charged, (*c*) could be either positively or negatively charged. Explain your answer.

Like the farad, the tesla is a large unit. The magnetic field strength of Earth has a magnitude somewhat less than 10^{-4} T on Earth's surface. The magnetic field strengths near powerful permanent magnets are about 0.1 T to 0.5 T, and powerful laboratory and industrial electromagnets produce fields of 1 T to 2 T. Fields greater than 10 T are extremely difficult to produce because the resulting magnetic forces will either tear the magnets apart or crush the magnets. A commonly used unit, derived from the cgs system, is the **gauss** (G), which is related to the tesla as follows:

$$1\text{ G} = 10^{-4}\text{ T} \qquad 26\text{-}3$$

DEFINITION—GAUSS

Because magnetic fields are often given in gauss, which is not an SI unit, you need to remember to convert from gauss to teslas when making calculations.

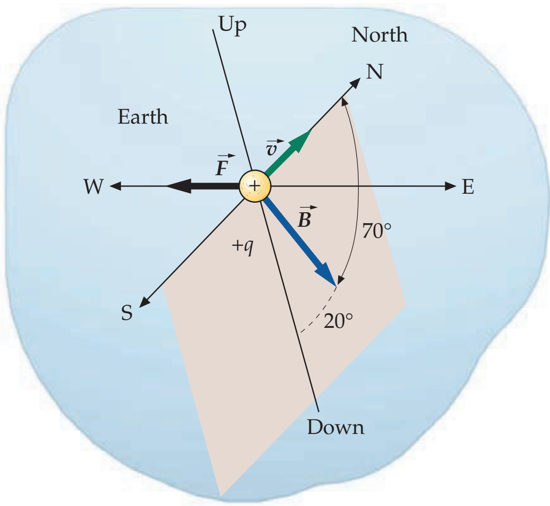


FIGURE 26-4

Example 26-1 Force on a Proton Going North

The magnetic field strength of Earth is measured at a point on the surface to have a magnitude of about 0.6 G and is tilted downward in the northern hemisphere, making an angle of about 70° with the horizontal, as shown in Figure 26-4. (Earth's magnetic field varies from place to place. These data are approximately correct for the central United States.) A proton ($q = +e$) is moving horizontally in the northward direction with speed $v = 1.0 \times 10^7$ m/s. Calculate the magnetic force on the proton (*a*) using $F = qvB\sin\theta$ and (*b*) by first expressing $\vec{v}$ and $\vec{B}$ in terms of the unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, and then computing $\vec{F} = q\vec{v} \times \vec{B}$.

PICTURE Let the x and y directions be to the east and to the north, respectively, and let the z direction be vertically upward (Figure 26-5). The velocity vector is then in the $+y$ direction.

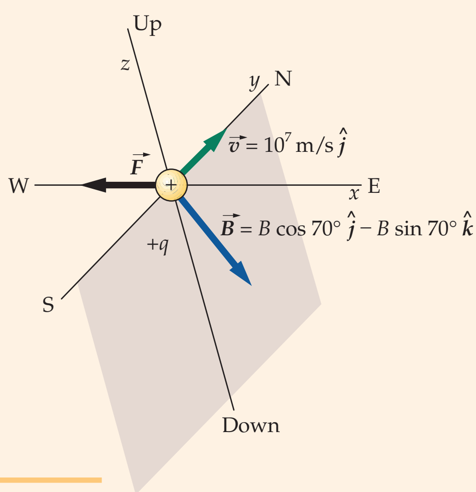


FIGURE 26-5

SOLVE

(*a*) Calculate $F = qvB\sin\theta$ using $\theta = 70°$. From Figure 26-4, we see that the direction of the force is westward.

$$F = qvB\sin 70°$$
$$= (1.6 \times 10^{-19}\text{ C})(10 \times 10^6\text{ m/s})(0.6 \times 10^{-4}\text{ T})(0.94)$$
$$= \boxed{9.0 \times 10^{-17}\text{ N}}$$

(*b*) 1. The magnetic force is the vector product of $q\vec{v}$ and $\vec{B}$:

$$\vec{F} = q\vec{v} \times \vec{B}$$

2. Express $\vec{v}$ and $\vec{B}$ in terms of their components:

$$\vec{v} = v_y\hat{j}$$
$$\vec{B} = B_y\hat{j} + B_z\hat{k}$$

3. Write $\vec{F} = q\vec{v} \times \vec{B}$ in terms of these components:

$$\vec{F} = q\vec{v} \times \vec{B} = q(v_y\hat{j}) \times (B_y\hat{j} + B_z\hat{k})$$
$$= qv_yB_y(\hat{j} \times \hat{j}) + qv_yB_z(\hat{j} \times \hat{k}) = qv_yB_z\hat{i}$$

4. Evaluate $\vec{F}$:

$$\vec{F} = qv(-B\sin\theta)\hat{i}$$
$$= -(1.6 \times 10^{-19}\text{C})(10^7\text{ m/s})(0.6 \times 10^{-4}\text{ T})\sin 70^\circ\,\hat{i}$$
$$= \boxed{-9.0 \times 10^{-17}\text{ N}\hat{i}}$$

CHECK The Part (*a*) result is equal to the magnitude of the Part (*b*) result.

TAKING IT FURTHER Note that the direction of $\hat{i}$ is to the east, so the force is directed to the west as shown in Figure 26-5.

PRACTICE PROBLEM 26-1 Find the force on a proton moving with velocity $\vec{v} = 4 \times 10^6\text{ m/s}\,\hat{i}$ in a magnetic field $\vec{B} = 2.0\text{ T}\hat{k}$.

When a current-carrying wire is in a region that has a magnetic field, there is a force on the wire that is equal to the sum of the magnetic forces on the individual charge carriers in the wire. Figure 26-6 shows a short segment of wire that has cross-sectional area A, length L, and current I. If the wire is in a magnetic field $\vec{B}$, the magnetic force on each charge is $q\vec{v}_d \times \vec{B}$, where $\vec{v}_d$ is the drift velocity of the charge carriers (the drift velocity is the same as the average velocity). The number of charges in the wire segment is the number n per unit volume multiplied by the volume AL. Thus, the total force on the wire segment is

$$\vec{F} = (q\vec{v}_d \times \vec{B})nAL$$

From Equation 25-3, the current in the wire is

$$I = nqv_dA$$

Hence, the force can be written

$$\vec{F} = I\vec{L} \times \vec{B} \qquad 26\text{-}4$$

MAGNETIC FORCE ON A STRAIGHT SEGMENT OF CURRENT-CARRYING WIRE

where $\vec{L}$ is a vector whose magnitude is the length of the segment and whose direction is the same as that of the current.* For the current in the $+x$ direction (Figure 26-7) and the magnetic field vector at the segment in the xy plane, the force on the wire is in the $+z$ direction.

When using Equation 26-4, it is assumed that the wire segment is straight and that the magnetic field does not vary over its length. The equation can be generalized for an arbitrarily shaped wire in any magnetic field. If we choose a very short wire segment that has length $d\vec{\ell}$ and write the force on this segment as $d\vec{F}$, we have

$$d\vec{F} = I\,d\vec{\ell} \times \vec{B} \qquad 26\text{-}5$$

MAGNETIC FORCE ON A CURRENT ELEMENT

where $\vec{B}$ is the magnetic field vector at the location of the segment. The quantity $I\,d\vec{\ell}$ is called a **current element.** We find the total magnetic force on a current-carrying wire by summing (integrating) the magnetic forces due to all the current elements in the wire. (Note that Equation 26-5 is the same as Equation 26-1 with the current element $I\,d\vec{\ell}$ replacing $q\vec{v}$.)

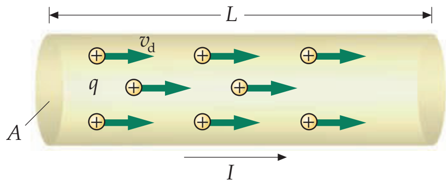


FIGURE 26-6 Wire segment that has a length L and carries a current I. If the wire is in a magnetic field $\vec{B}$, there will be a force on each charge carrier resulting in a force on the wire.

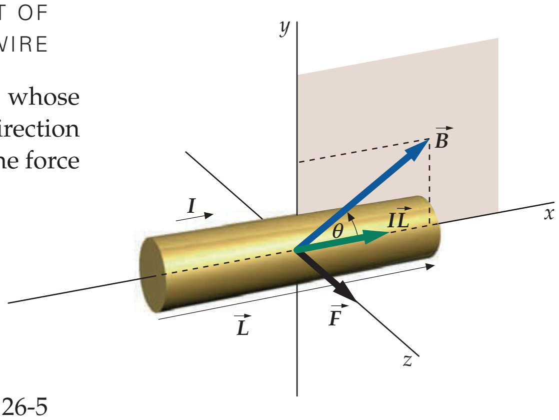


FIGURE 26-7 Magnetic force on a current-carrying segment of wire in a magnetic field. The current is in the x direction, and the magnetic field is in the xy plane and makes an angle θ with the $+x$ direction. The force $\vec{F}$ is in the $+z$ direction, perpendicular to both $\vec{B}$ and $\vec{L}$, and has magnitude $ILB\sin\theta$.

* By the direction of the current we mean the direction of the current-density vector $\vec{J}$.

Just as the electric field $\vec{E}$ can be represented by electric field lines, the magnetic field $\vec{B}$ can be represented by **magnetic field lines.** In both cases, the direction of the field is indicated by the direction of the field lines and the magnitude of the field is indicated by the density (number per unit area) of the lines on surface perpendicular to the lines. There are, however, two important differences between electric field lines and magnetic field lines:

1. Electric field lines are in the direction of the electric force on a positive charge, but the magnetic field lines are perpendicular to the magnetic force on a moving charge.
2. Electric field lines begin on positive charges and end on negative charges; magnetic field lines neither begin nor end.

Figure 26-8 shows the magnetic field lines both inside and outside a bar magnet.

! Do not think the field lines for the magnetic field of a magnet begin on magnetic south poles and end on magnetic north poles. In reality, they neither begin nor end. Instead they enter the magnet at one end and exit the magnet at the other end.

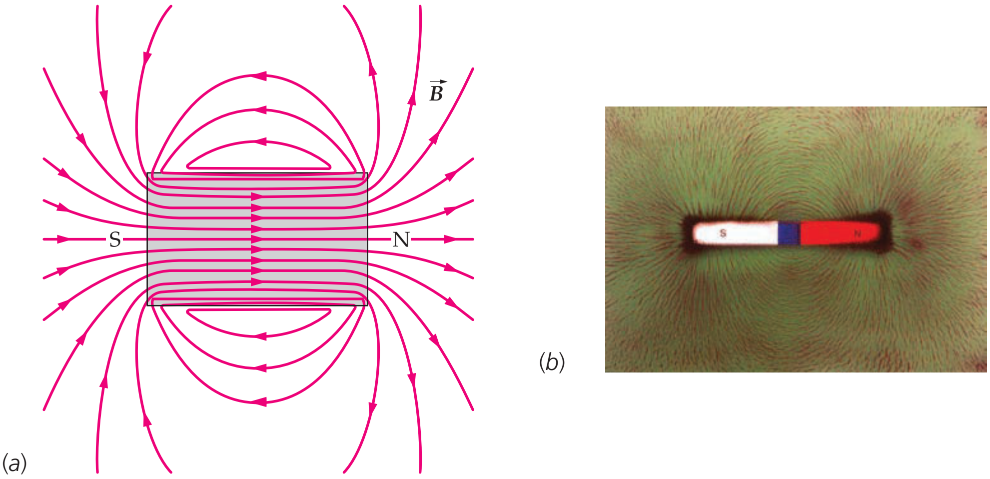
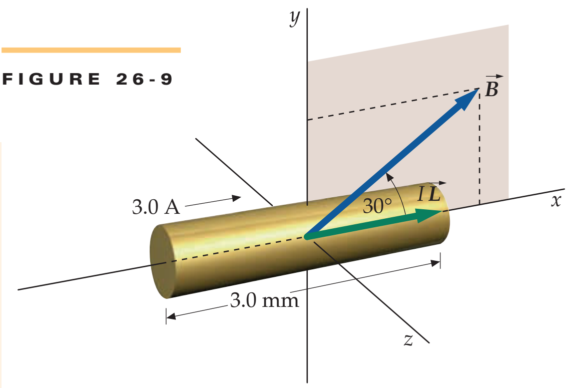


FIGURE 26-8 (*a*) Magnetic field lines inside and outside a bar magnet. The lines emerge from the north pole and enter the south pole, but they have no beginning or end. Instead, they form closed loops. (*b*) Magnetic field lines outside a bar magnet as indicated by iron filings. *(© 1995 Tom Pantages.)*

Example 26-2 Force on a Straight Wire

A 3.0-mm-long segment of wire carries a current of 3.0 A in the $+x$ direction. It lies in a magnetic field of magnitude 0.020 T that is in the xy plane and makes an angle of 30° with the $+x$ direction, as shown in Figure 26-9. What is the magnetic force exerted on the wire segment?

FIGURE 26-9



PICTURE The magnetic force is in the direction of $\vec{L} \times \vec{B}$, which we see from Figure 26-9 is in the $+z$ direction.

SOLVE

1. The magnetic force is given by Equation 26-4:

$$\vec{F} = I\vec{L} \times \vec{B} = ILB \sin 30° \,\hat{k}$$
$$= (3.0\text{ A})(0.0030\text{ m})(0.020\text{ T})(\sin 30°)\hat{k}$$
$$= \boxed{9.0 \times 10^{-5}\text{ N}\hat{k}}$$

CHECK The force is perpendicular to the wire, as expected.

Example 26-3 Force on a Bent Wire

A wire bent into a semicircular loop of radius R lies in the xy plane. It carries a current I from point a to point b, as shown in Figure 26-10. Throughout the region there is a uniform magnetic field $\vec{B} = B\hat{k}$ that is perpendicular to the plane of the loop. Find the magnetic force acting on the semicircular loop section of the wire.

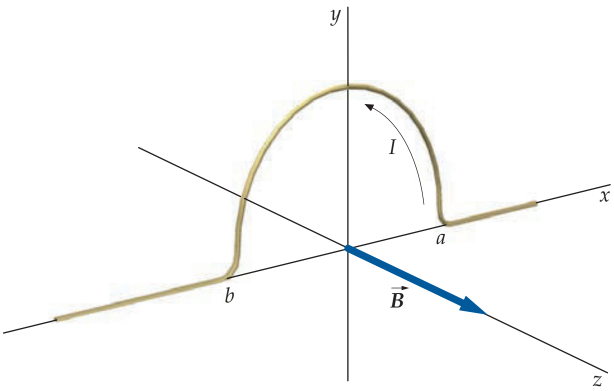


FIGURE 26-10

PICTURE The magnetic force $d\vec{F}$ is exerted on a segment of the semicircular wire that lies in the xy plane, as shown in Figure 26-11. We find the total magnetic force by expressing the x and y components of $d\vec{F}$ in terms of θ and integrating them separately from $\theta = 0$ to $\theta = \pi$.

SOLVE

1. Write the force $d\vec{F}$ on a current element $I\,d\vec{\ell}$.
$$d\vec{F} = I\,d\vec{\ell} \times \vec{B}$$

2. Express $d\vec{\ell}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$:
$$d\vec{\ell} = -d\ell\sin\theta\,\hat{i} + d\ell\cos\theta\,\hat{j}$$

3. Compute $I\,d\vec{\ell}$ using $d\ell = R\,d\theta$ and $\vec{B} = B\hat{k}$:
$$\begin{aligned} d\vec{F} &= I\,d\vec{\ell} \times \vec{B} \\ &= I(-R\sin\theta\,d\theta\,\hat{i} + R\cos\theta\,d\theta\,\hat{j}) \times B\hat{k} \\ &= IRB\sin\theta\,d\theta\,\hat{j} + IRB\cos\theta\,d\theta\,\hat{i} \end{aligned}$$

4. Integrate each component of $d\vec{F}$ from $\theta = 0$ to $\theta = \pi$.
$$\begin{aligned} \vec{F} &= \int d\vec{F} = IRB\hat{i}\int_0^\pi \cos\theta\,d\theta + IRB\hat{j}\int_0^\pi \sin\theta\,d\theta \\ &= IRB\hat{i}(0) + IRB\hat{j}(2) = \boxed{2IRB\hat{j}} \end{aligned}$$

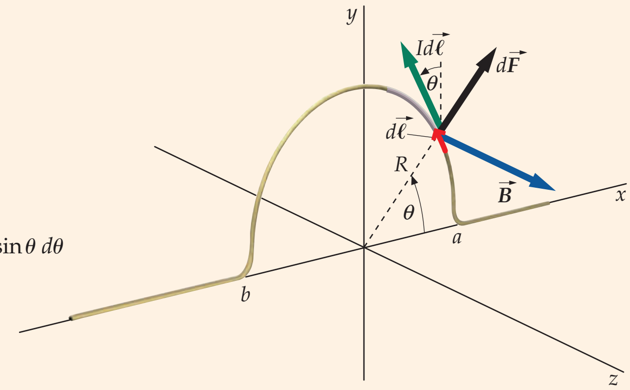


FIGURE 26-11

CHECK The result that the x component of $\vec{F}$ is zero can be seen from symmetry. For the right half of the loop, $d\vec{F}$ tilts to the right; for the left half of the loop, $d\vec{F}$ tilts to the left.

TAKING IT FURTHER The net force on the semicircular wire is the same as if the semicircle were replaced by a straight-line segment of length $2R$ connecting points a and b. (This is a general result that is derived in Problem 26.)

26-2 MOTION OF A POINT CHARGE IN A MAGNETIC FIELD

The magnetic force on a charged particle moving through a region with a magnetic field is always perpendicular to the velocity of the particle. The magnetic force thus changes the direction of the velocity but not the magnitude of the velocity (the speed). Therefore, *magnetic forces do no work on particles and do not change their kinetic energy*.

In the special case where the velocity of a charged particle is perpendicular to a uniform magnetic field, as shown in Figure 26-12, the particle moves in a circular orbit. The magnetic force provides the force in the centripetal direction that is necessary for circular motion. We can use Newton's second law to relate the radius of the circle to the magnetic field and the speed of the particle. If the velocity is $\vec{v}$, the magnetic force on a particle that has charge q is given by $\vec{F} = q\vec{v} \times \vec{B}$. The magnitude of the net force is equal to qvB, because $\vec{v}$ and $\vec{B}$ are perpendicular.

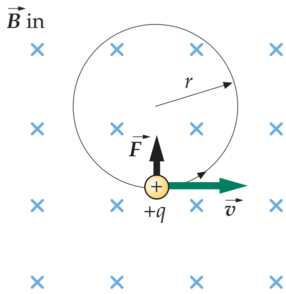


FIGURE 26-12 Charged particle moving in a plane perpendicular to a uniform magnetic field. The magnetic field is into the page as indicated by the crosses. (Each cross represents the tail feathers of an arrow. A field out of the plane of the page would be indicated by dots, each dot representing the point of an arrow.) The magnetic force is perpendicular to the velocity of the particle, causing it to move in a circular orbit.

Newton's second law gives

$$F = ma$$

$$qvB = m\frac{v^2}{r}$$

or

$$r = \frac{mv}{qB} \qquad \text{26-6}$$

where m is the mass of the particle.

The period of the circular motion is the time it takes the particle to travel once around the circumference of the circle. The period is related to the speed by

$$T = \frac{2\pi r}{v}$$

Substituting $mv/(qB)$ for r (Equation 26-6), we obtain the period of the particle's circular orbit, which is called the **cyclotron period:**

$$T = \frac{2\pi(mv/qB)}{v} = \frac{2\pi m}{qB} \qquad \text{26-7}$$

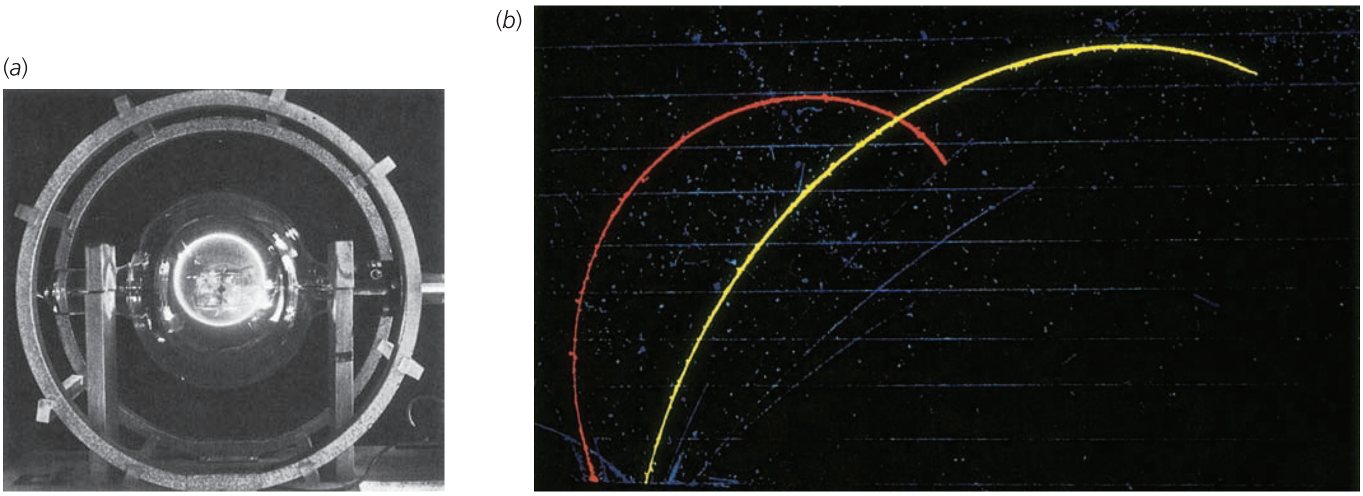
CYCLOTRON PERIOD

The frequency of the circular motion, called the **cyclotron frequency**, is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{qB}{2\pi m} \quad \text{so} \quad \omega = 2\pi f = \frac{q}{m}B \qquad \text{26-8}$$

CYCLOTRON FREQUENCY

Note that the period and the frequency given by Equations 26-7 and 26-8 depend on the charge-to-mass ratio q/m, but the period and the frequency are independent of the velocity v or the radius r. Two important applications of the circular motion of charged particles in a uniform magnetic field, the mass spectrometer and the cyclotron, are discussed later in this section.



(*a*) Circular path of electrons moving in the magnetic field produced by the current in two large coils. The electrons ionize the dilute gas in the tube, causing it to give off a glow that indicates the path of the beam. (*b*) False-color photograph showing tracks of a 1.6-MeV proton (red) and a 7-MeV α particle (yellow) in a cloud chamber. The radius of curvature is proportional to the momentum and inversely proportional to the charge of the particle. For these energies, the momentum of the α particle, which has twice the charge of the proton, is about four times that of the proton and so its radius of curvature is greater. *((a) Larry Langrill. (b) © Lawrence Berkeley Laboratory/Science Photo Library.)*

Example 26-4 Cyclotron Period

A proton has a mass equal to 1.67×10^{-27} kg, has a charge equal to 1.60×10^{-19} C, and moves in a circle of radius $r = 21.0$ cm perpendicular to a magnetic field equal to 4000 G. Find (*a*) the speed of the proton and (*b*) the period of the motion.

PICTURE Apply Newton's second law to find the speed, and use distance equals speed multiplied by time to find the period.

SOLVE

(*a*) 1. Apply Newton's second law ($F = ma$):

$$F = ma \quad \Rightarrow \quad qvB = m\frac{v^2}{r}$$

2. Solve for the speed:

$$v = \frac{rqB}{m} = \frac{(0.210\ \text{m})(1.60 \times 10^{-19}\ \text{C})(0.400\ \text{T})}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= \boxed{8.05 \times 10^6\ \text{m/s} = 0.0268c}$$

(*b*) Use distance equals speed multiplied by time and solve for the period.

$$2\pi r = vT$$

so

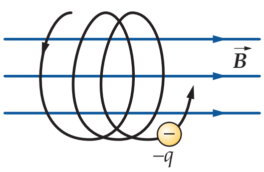
$$T = \frac{2\pi r}{v} = \frac{2\pi(0.210\ \text{m})}{(8.05 \times 10^6\ \text{m/s})} = 1.64 \times 10^{-7}\ \text{s} = \boxed{164\ \text{ns}}$$

TAKING IT FURTHER The radius of the circular orbit is proportional to the speed, but the period of the orbit is independent of both the speed and radius.

Suppose that a charged particle is in a region that has a uniform magnetic field and is moving with a velocity that is not perpendicular to $\vec{B}$. There is no magnetic force component, and thus no acceleration component, parallel to $\vec{B}$, so the component of the velocity that is parallel to $\vec{B}$ remains constant. The magnetic force on the particle is perpendicular to $\vec{B}$, so the change in motion of the particle due to this force is the same as that just discussed. The path of the particle is thus a helix, as shown in Figure 26-13.

The motion of charged particles in nonuniform magnetic fields can be quite complex. Figure 26-14 shows a *magnetic bottle,* an interesting magnetic field configuration in which the field is weak at the center and strong at both ends. A detailed analysis of the motion of a charged particle in such a field shows that the particle spirals around the field lines and becomes trapped, oscillating back and forth between points P_1 and P_2 in the figure. Such magnetic field configurations are used to confine dense beams of charged particles, called plasmas, in nuclear fusion research. A similar phenomenon is the oscillation of ions back and forth between Earth's magnetic poles in the Van Allen belts (Figure 26-15).

(*a*)



(*b*)

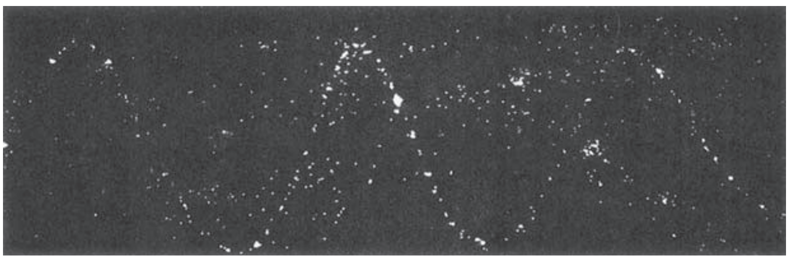


FIGURE 26-13 (*a*) When a particle has a velocity component parallel to a magnetic field as well as a velocity component perpendicular to the magnetic field the particle moves in a helical path around the field lines. (*b*) Cloud-chamber photograph of the helical path of an electron moving in a magnetic field. The path of the electron is made visible by the condensation of water droplets in the cloud chamber. *(Carl E. Nielson.)*

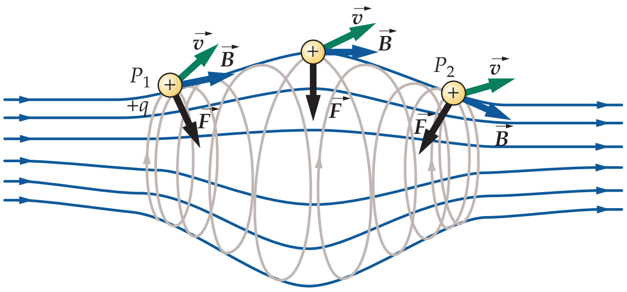


FIGURE 26-14 Magnetic bottle. When a charged particle moves in such a field, which is strong at both ends and weak in the middle, the particle becomes trapped and moves back and forth, spiraling around the field lines.

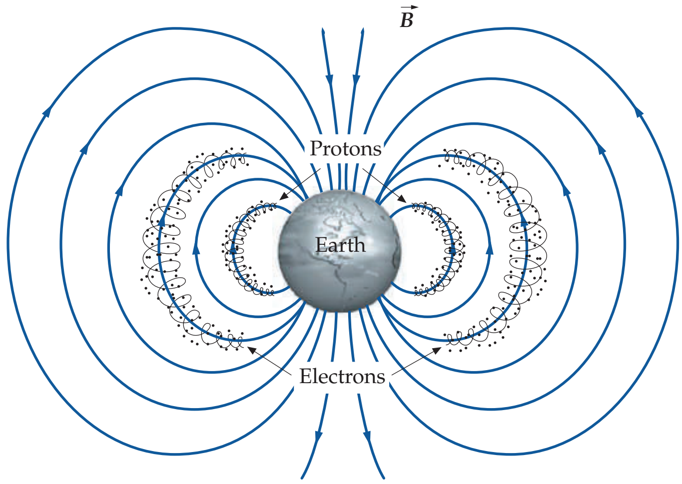


FIGURE 26-15 Van Allen belts. Protons (inner belts) and electrons (outer belts) are trapped in Earth's magnetic field and spiral around the field lines between the north and south poles.

*THE VELOCITY SELECTOR

The magnetic force on a charged particle moving in a uniform magnetic field can be balanced by an electric force if the magnitudes and directions of the magnetic field and the electric field are properly chosen. Because the electric force is in the direction of the electric field (for particles with positive charge) and the magnetic force is perpendicular to the magnetic field, the electric and magnetic fields in the region through which the particle is moving must be perpendicular to each other if the forces are to balance. Such a region is said to have **crossed fields.**

Figure 26-16 shows a region of space between the plates of a capacitor where there is an electric field and a perpendicular magnetic field (produced by a magnet that has one pole on each side of this sheet of paper). Consider a particle that has charge q entering this space from the left. The net force on the particle is

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

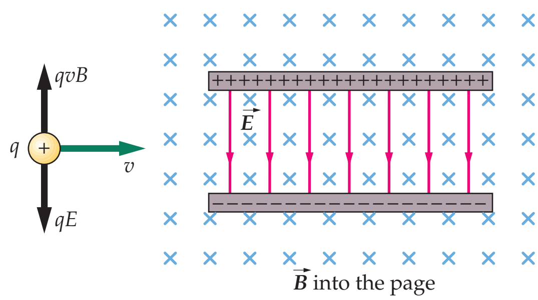


FIGURE 26-16 Crossed electric and magnetic fields. When a particle that has a positive charge moves to the right, the particle experiences a downward electric force and an upward magnetic force. These forces balance if the speed of the particle is related to the field strengths by $vB = E$.

If q is positive, the electric force of magnitude qE is down the page and the magnetic force of magnitude qvB is up the page. If the charge is negative, the direction of each of these forces is reversed. The two forces balance if $qE = qvB$ that is, if

$$v = \frac{E}{B} \qquad \text{26-9}$$

For given magnitudes of the electric and magnetic fields, the forces balance only for particles that have the exact speed given by Equation 26-9. Any particle that has this speed, regardless of its mass or charge, will traverse the space undeflected. A particle that has a greater speed will be deflected toward the direction of the magnetic force, and a particle that has a lesser speed will be deflected in the direction of the electric force. This arrangement of fields is often used as a **velocity selector,** which is a device that allows only particles with the speed specified by Equation 26-9 to pass.

PRACTICE PROBLEM 26-2

A proton is moving in the $+x$ direction in a region of crossed fields where $\vec{E} = 2.00 \times 10^5$ N/C $\hat{k}$ and $\vec{B} = 0.300$ T $\hat{j}$. (*a*) What is the speed of the proton if it is not deflected? (*b*) If the proton moves with twice this speed, in which direction will it be deflected?

*THOMSON'S MEASUREMENT OF q/m FOR ELECTRONS

An example of the use of crossed electric and magnetic fields is the famous experiment performed by J. J. Thomson in 1897 where he showed that the rays of a cathode-ray tube can be deflected by electric and magnetic fields, indicating that they must consist of charged particles. By measuring the deflections of these particles, Thomson showed that all the particles have the same charge-to-mass ratio q/m. He also showed that particles that have this charge-to-mass ratio can be obtained using any material for a source, which means that these particles, now called electrons, are a fundamental constituent of all matter.

Figure 26-17 shows a schematic diagram of the cathode-ray tube Thomson used. Electrons are emitted from the cathode C, which is at a negative potential relative to the potential at slits A and B. An electric field in the direction from A toward C accelerates the electrons, and some of the electrons pass through slits A and B into a field-free region. The electrons then enter the electric field between the capacitor plates D and F that is perpendicular to the velocity of the electrons. This field accelerates the electrons vertically for the short time that they are between the plates. The electrons are deflected and strike the phosphorescent screen S at the far right side of the tube at some deflection Δy from the point at which they strike when there is no electric field between the plates. The screen glows where the electrons strike the screen, indicating the location of the beam. The speed of the electrons v_0 is determined by introducing a magnetic field $\vec{B}$ between the plates in a direction that is perpendicular to both the electric field and the initial velocity of the electrons. The magnitude of $\vec{B}$ is adjusted until the beam is not deflected. The speed is then found from Equation 26-9.

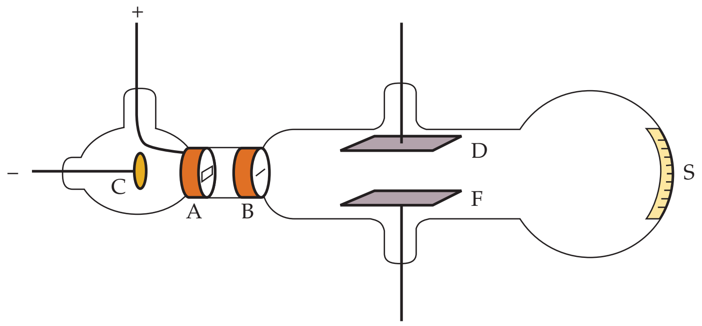


FIGURE 26-17 Thomson's tube for measuring q/m for the particles of cathode rays (electrons). Electrons from the cathode C pass through the slits at A and B and strike a phosphorescent screen S. The beam can be deflected by an electric field between plates D and F or by a magnetic field (not shown).

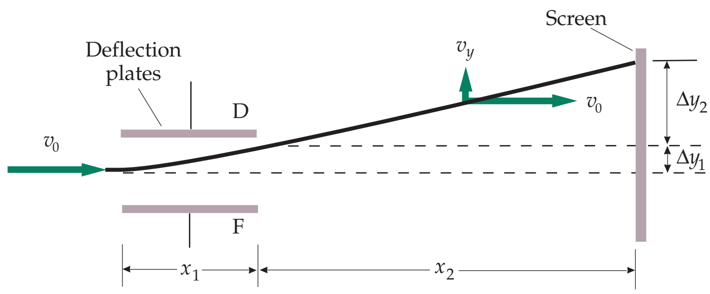


FIGURE 26-18 The total deflection of the beam in the J. J. Thomson experiments consists of the deflection Δy_1 while the electrons are between the plates plus the deflection Δy_2 that occurs in the field-free region between the plates and the screen.

With the magnetic field turned off, the beam is deflected by an amount Δy, which consists of two parts: the deflection Δy_1, which occurs while the electrons are between the plates, and the deflection Δy_2, which occurs after the electrons leave the region between the plates (Figure 26-18).

Let x_1 be the horizontal distance across the deflection plates D and F. If the electron is moving horizontally with speed v_0 when it enters the region between the plates, the time spent between the plates is $t_1 = x_1/v_0$, and the vertical velocity when it leaves the plates is

$$v_y = a_y t_1 = \frac{qE_y}{m} t_1 = \frac{qE_y}{m}\frac{x_1}{v_0}$$

where E_y is the upward component of the electric field between the plates. The deflection in this region is

$$\Delta y_1 = \frac{1}{2} a_y t_1^2 = \frac{1}{2}\frac{qE_y}{m}\left(\frac{x_1}{v_0}\right)^2$$

The electron then travels an additional horizontal distance x_2 in the field-free region from the deflection plates to the screen. Because the velocity of the electron is constant in this region, the time to reach the screen is $t_2 = x_2/v_0$, and the additional vertical deflection is

$$\Delta y_2 = v_y t_2 = \frac{qE_y}{m}\frac{x_1}{v_0}\frac{x_2}{v_0}$$

The total deflection at the screen is therefore

$$\Delta y = \Delta y_1 + \Delta y_2 = \frac{1}{2}\frac{qE_y}{mv_0^2}x_1^2 + \frac{qE_y}{mv_0^2}x_1x_2 \qquad 26\text{-}10$$

The measured deflection Δy can be used to determine the charge-to-mass ratio, q/m, from Equation 26-10.

Example 26-5 Electron Beam Deflection

Electrons pass undeflected through the plates of Thomson's apparatus when the electric field is 3000 V/m and there is a crossed magnetic field of 0.140 mT. If the plates are 4.00 cm long and the ends of the plates are 30.0 cm from the screen, find the deflection on the screen when the magnetic field is turned off.

PICTURE The mass and charge of the electron are known: $m = 9.11 \times 10^{-31}$ kg and $q = -e = -1.60 \times 10^{-19}$ C. The speed of the electron can be found from the ratio of the magnetic and electric fields.

SOLVE

1. The total deflection of the electron is given by Equation 26-10:

$$\Delta y = \Delta y_1 + \Delta y_2 = \frac{1}{2}\frac{qE_y}{mv_0^2}x_1^2 + \frac{qE_y}{mv_0^2}x_1x_2$$

2. The speed v_0 equals E/B:

$$v_0 = \frac{E}{B} = \frac{3000\ \text{V/m}}{1.40 \times 10^{-4}\ \text{T}} = 2.14 \times 10^7\ \text{m/s}$$

3. Substitute the value for v_0 determined in step 2, the given value of E, and the known values for m and q into Equation 26-10 to find Δy:

$$\Delta y_1 = \frac{1}{2}\frac{(-1.60 \times 10^{-19}\ \text{C})(-3000\ \text{V/m})}{(9.11 \times 10^{-31}\ \text{kg})(2.14 \times 10^7\ \text{m/s})^2}(0.0400\ \text{m})^2$$
$$= 9.20 \times 10^{-4}\ \text{m}$$

$$\Delta y_2 = \frac{(-1.60 \times 10^{-19}\ \text{C})(-3000\ \text{V/m})}{(9.11 \times 10^{-31}\ \text{kg})(2.14 \times 10^7\ \text{m/s})^2}(0.0400\ \text{m})(0.300\ \text{m})$$
$$= 1.38 \times 10^{-2}\ \text{m}$$

$$\Delta y = \Delta y_1 + \Delta y_2$$
$$= 9.20 \times 10^{-4}\ \text{m} + 1.38 \times 10^{-2}\ \text{m}$$
$$= 0.92\ \text{mm} + 13.8\ \text{mm} = \boxed{14.7\ \text{mm}}$$

CHECK As expected, Δy_2 is an order of magnitude greater than Δy_1. This was expected because the distance from the plates to the screen is an order of magnitude greater than the length of the plates.

*THE MASS SPECTROMETER

The **mass spectrometer,** first designed by Francis William Aston in 1919, was developed as a means of measuring the masses of isotopes. Such measurements are important in determining both the presence of isotopes and their abundance in nature. On Earth, for example, naturally occurring magnesium has been found to consist of 78.7 percent ^{24}Mg, 10.1 percent ^{25}Mg, and 11.2 percent ^{26}Mg. These isotopes have masses in the approximate ratio 24:25:26.

Figure 26-19 shows a simple schematic drawing of a mass spectrometer. Positive ions are formed by bombarding atoms with X rays or a beam of electrons. (Electrons are knocked out of the atoms by the X rays or bombarding electrons to form positive ions.) The ions are accelerated by an electric field and enter a uniform magnetic field. If the positive ions start from rest and move through a potential difference ΔV, the ions' kinetic energy when they enter the magnetic field equals their loss in potential energy, $q|\Delta V|$:

$$\tfrac{1}{2}mv^2 = q|\Delta V| \qquad 26\text{-}11$$

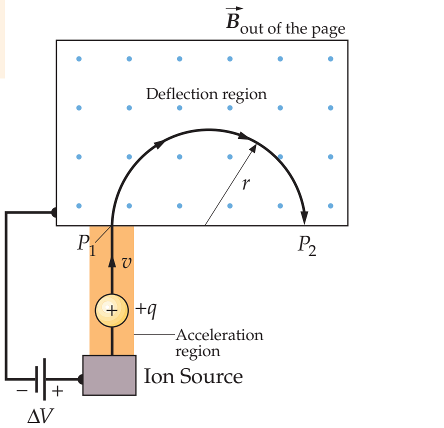


FIGURE 26-19 Schematic drawing of a mass spectrometer. Positive ions from an ion source are accelerated through a potential difference ΔV and enter a uniform magnetic field at P_1. The magnetic field is out of the page as indicated by the dots. The ions are bent into a circular arc and emerge at P_2. The radius r of the circle varies with the mass of the ion.

The ions move in a semicircle of radius r given by Equation 26-6, $r = mv/qB$, and strike a photographic plate at point P_2, a distance $2r$ from the point P_1 where the ions entered the magnetic field.

The speed v can be eliminated from Equations 26-6 and 26-11 to find m/q in terms of the known quantities V, B, and r. We first solve Equation 26-6 for v and square each term, which gives

$$v^2 = \frac{r^2q^2B^2}{m^2}$$

Substituting this expression for v^2 into Equation 26-11, we obtain

$$\frac{1}{2}m\left(\frac{r^2q^2B^2}{m^2}\right) = q|\Delta V|$$

Simplifying this equation and solving for m/q, we obtain

$$\frac{m}{q} = \frac{B^2r^2}{2|\Delta V|} \qquad \text{26-12}$$

In Aston's original mass spectrometer, mass differences could be measured to a precision of about 1 part in 10,000. The precision has been improved by introducing a velocity selector between the ion source and the magnet, which increases the degree of accuracy with which the speeds of the incoming ions can be determined.

Example 26-6 Separating Isotopes of Nickel

A ^{58}Ni ion that has a charge equal to $+e$ and a mass equal to 9.62×10^{-26} kg is accelerated through a potential drop of 3.00 kV and deflected in a magnetic field of 0.120 T. (*a*) Find the radius of curvature of the orbit of the ion. (*b*) Find the difference in the radii of curvature of ^{58}Ni ions and ^{60}Ni ions. (Assume that the mass ratio is 58:60.)

PICTURE The radius of curvature r can be found using Equation 26-12. Using the mass dependence of r, we can find the radius of curvature for the orbit of the ^{60}Ni ions from the radius of curvature for the orbit of the ^{58}Ni ions, and then take the difference between the two radii.

SOLVE

(*a*) Solve Equation 26-12 for r:

$$r = \sqrt{\frac{2m|\Delta V|}{qB^2}} = \left[\frac{2(9.62 \times 10^{-26}\ \text{kg})(3000\ \text{V})}{(1.60 \times 10^{-19}\ \text{C})(0.120\ \text{T})^2}\right]^{1/2}$$
$$= \boxed{0.501\ \text{m}}$$

(*b*) 1. Let r_1 and r_2 be the radius of the orbit of the ^{58}Ni ion and the ^{60}Ni ion, respectively. Use the result in Part (*a*) to find the ratio of r_2 to r_1:

$$\frac{r_2}{r_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{60}{58}} = 1.017$$

2. Use the result of the previous step to calculate r_2 for ^{60}Ni:

$$r_2 = 1.017r_1 = (1.017)(0.501\ \text{m}) = 0.510\ \text{m}$$

3. The difference in orbital radii is $r_2 - r_1$:

$$r_2 - r_1 = 0.510\ \text{m} - 0.501\ \text{m} = \boxed{9\ \text{mm}}$$

CHECK The difference in the orbital radii is less than 2 percent of the radius of curvature of either orbit. This result is expected for two ions whose masses differ by less than 4 percent.

THE CYCLOTRON

The cyclotron was invented by E. O. Lawrence and M. S. Livingston in 1934 to accelerate particles, such as protons or deuterons, to large kinetic energies.* The high-energy particles are used to bombard atomic nuclei, causing nuclear reactions that

* A deuteron is the nucleus of heavy hydrogen, ^{2}H, which consists of a proton and neutron tightly bound together.

are then studied to obtain information about nuclei. High-energy protons and deuterons are also used to produce radioactive materials and for medical purposes.

Figure 26-20 is a schematic drawing of a cyclotron. The particles move in two semicircular metal containers called *dees* (because they are the shape of the letter "D"). The dees are housed in a vacuum chamber that is in a region with a uniform magnetic field provided by an electromagnet. The region in which the particles move must be evacuated so that the particles will not be scattered in collisions with air molecules. A potential difference ΔV, which alternates in time with a period T, is maintained between the dees. The period is chosen to be the cyclotron period $T = 2\pi m/(qB)$ (Equation 26-7). The potential difference creates an electric field across the gap between the dees. At the same time, there is no electric field within each dee because the metal dees act as shields.

Positively charged particles are initially injected into dee$_1$ with a small velocity from an ion source S near the center of the dees. They move in a semicircle in dee$_1$ and arrive at the gap between dee$_1$ and dee$_2$ after a time $\frac{1}{2}T$. The potential is adjusted so that dee$_1$ is at a higher potential than dee$_2$ when the particles arrive at the gap between them. Each particle is therefore accelerated across the gap by the electric field and gains kinetic energy equal to $q\Delta V$.

Because the particle now has more kinetic energy, the particle moves in a semicircle of larger radius in dee$_2$. It arrives at the gap again after a time $\frac{1}{2}T$, because the period is independent of the particle's speed. By this time, the potential difference between the dees has been reversed so that dee$_2$ is now at the higher potential. Once more the particle is accelerated across the gap and gains additional kinetic energy equal to $q\Delta V$. Each time the particle arrives at the gap, it is accelerated and gains kinetic energy equal to $q\Delta V$. Thus, the particle moves in larger and larger semicircular orbits until it eventually leaves the magnetic field. In the typical cyclotron, each particle may make 50 to 100 revolutions and exit with energies of up to several hundred megaelectron volts.

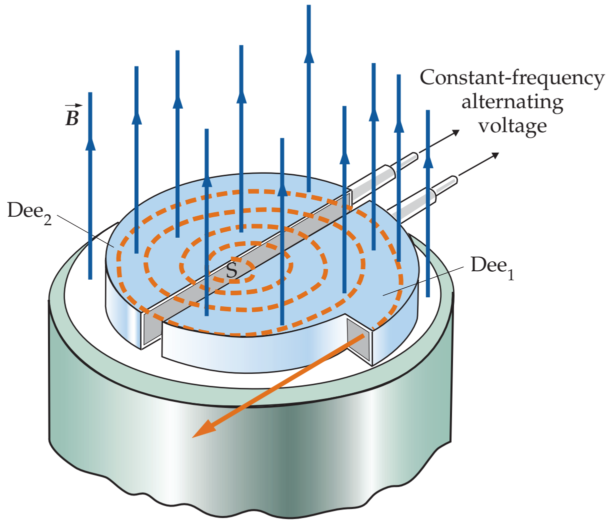


FIGURE 26-20 Schematic drawing of a cyclotron. The upper-pole face of the magnet has been omitted. Charged particles, such as protons, are accelerated from a source S at the center by the potential difference across the gap between the dees. When the charged particles arrive at the gap again the potential difference has changed sign so they are again accelerated across the gap and move in a larger circle. The potential difference across the gap alternates with the cyclotron frequency of the particle, which is independent of the radius of the circle.

The kinetic energy of a particle leaving a cyclotron can be calculated by setting r in Equation 26-6 equal to the maximum radius of the dees and solving the equation for v:

$$r = \frac{mv}{qB} \quad \Rightarrow \quad v = \frac{qBr}{m}$$

Then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{q^2B^2}{m}\right)r^2 \qquad \text{26-13}$$

Example 26-7 Energy of Accelerated Proton

A cyclotron for accelerating protons has a magnetic field of 0.150 T and a maximum radius of 0.500 m. (*a*) What is the cyclotron frequency? (*b*) What is the kinetic energy of the protons when they emerge?

PICTURE Apply Newton's second law ($F = ma$) with $F = |q\vec{v} \times \vec{B}|$. Use $v = r\omega$ and solve for the frequency and the speed.

SOLVE

(*a*) 1. Apply $F = ma$, where F is the magnetic force and a is the centripetal acceleration. Substitute ωr for v and solve for ω:

$$F = ma$$

$$qvB = m\frac{v^2}{r}$$

$$q\omega rB = m\frac{\omega^2 r^2}{r}$$

$$\omega = \frac{qB}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(0.150\ \text{T})}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= 1.44 \times 10^{7}\ \text{rad/s}$$

2. Use $2\pi f = \omega$ to calculate the frequency in cycles per second (hertz):

$$f = \frac{\omega}{2\pi} = \frac{1.44 \times 10^{7}\ \text{rad/s}}{2\pi\ \text{rad}}$$

$$= 2.29 \times 10^{6}\ \text{Hz} = \boxed{2.29\ \text{MHz}}$$

(*b*) 1. Calculate the kinetic energy:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 r^2$$

$$= \frac{1}{2}(1.67 \times 10^{-27}\ \text{kg})(1.44 \times 10^{7}\ \text{rad/s})^2(0.500\ \text{m})^2$$

$$= 4.33 \times 10^{-14}\ \text{J}$$

2. The energies of protons and other elementary particles are usually expressed in electron volts. Use $1\ \text{eV} = 1.60 \times 10^{-19}$ J to convert to eV:

$$K = 4.33 \times 10^{-14}\ \text{J} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}} = \boxed{271\ \text{keV}}$$

CHECK The exit speed of the proton is $v = r\omega = (0.500\ \text{m})(1.44 \times 10^{7}\ \text{rad/s}) = 7.20 \times 10^{6}$ m/s. The speed of light is 3.00×10^{8} m/s. Our calculated value of 1.44×10^{7} rad/s for the angular frequency is plausible because it is a high speed that is less than ten percent of the speed of light.

26-3 TORQUES ON CURRENT LOOPS AND MAGNETS

A current-carrying loop experiences no net force in a uniform magnetic field, but it does experience a net torque. The orientation of the loop can be described conveniently by a unit vector $\hat{n}$ that is normal to the plane of the loop, as illustrated in Figure 26-21. If the fingers of the right hand curl around the loop in the direction of the current, the thumb points in the direction of $\hat{n}$.

Figure 26-22 shows the forces exerted by a uniform magnetic field on a rectangular current-carrying loop whose vector $\hat{n}$ makes an angle θ with the direction of the magnetic field $\vec{B}$. The net force on the loop is zero. The forces $\vec{F}_1$ and $\vec{F}_2$ have the magnitude

$$F_1 = F_2 = IaB$$

The forces form a couple, so the torque they exert is the same about any point. Point P in Figure 26-22 is a convenient point about which to compute the torque. The magnitude of the torque is

$$\tau = F_2 b \sin\theta = IaBb \sin\theta = IAB \sin\theta$$

where $A = ab$ is the area of the loop. For a loop that has N turns, the torque has the magnitude

$$\tau = NIAB \sin\theta$$

This torque tends to twist the loop so that $\hat{n}$ is in the same direction as $\vec{B}$.

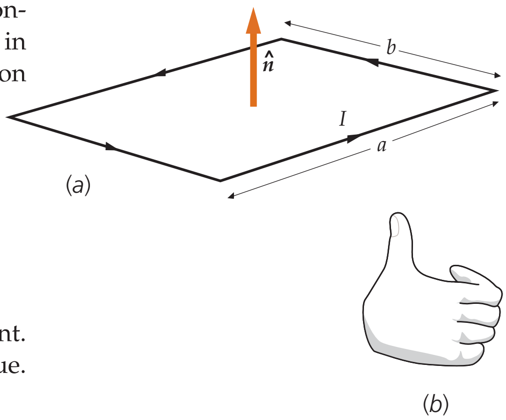


FIGURE 26-21 (*a*) The orientation of a current loop is described by the unit vector $\hat{n}$ perpendicular to the plane of the loop. (*b*) Right-hand rule for determining the direction of $\hat{n}$. If the fingers of the right hand curl around the loop in the direction of the current, the thumb points in the direction of $\hat{n}$.

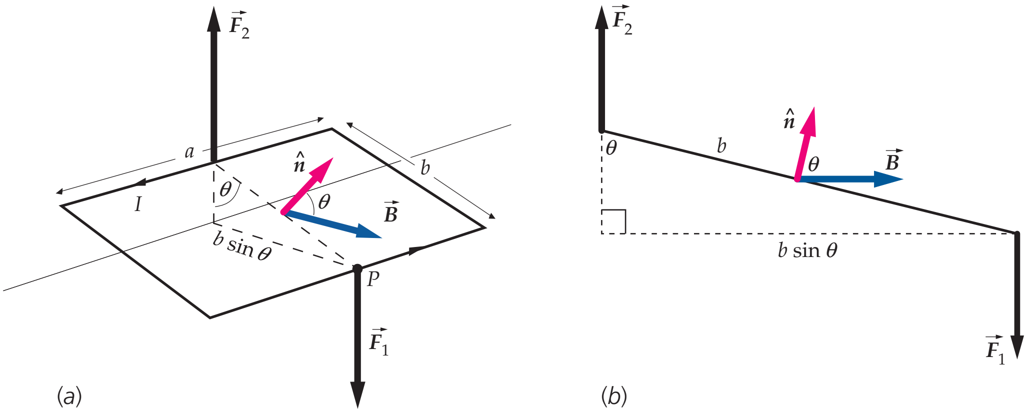


FIGURE 26-22 (*a*) Rectangular current loop whose unit normal $\hat{n}$ makes an angle θ with a uniform magnetic field $\vec{B}$. (*b*) An edge-on view of the current loop. The torque on the loop has magnitude $IAB\sin\theta$ and is in the direction such that $\hat{n}$ tends to rotate so as to align itself with $\vec{B}$.

The torque can be written conveniently in terms of the **magnetic dipole moment** $\vec{\mu}$ (also referred to simply as the **magnetic moment**) of the current loop, which is defined as

$$\vec{\mu} = NIA\hat{n} \qquad 26\text{-}14$$

MAGNETIC DIPOLE MOMENT OF A CURRENT LOOP

The SI unit of magnetic moment is the ampere-square meter ($A\cdot m^2$). In terms of the magnetic dipole moment, the torque on the current loop is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad 26\text{-}15$$

TORQUE ON A CURRENT LOOP

Equation 26-15, which we have derived for a rectangular loop, holds in general for a loop of any shape that lies in a single plane. The torque on any such loop is the vector product of the magnetic moment $\vec{\mu}$ of the loop and the magnetic field $\vec{B}$, where the magnetic moment (Figure 26-23) is defined as a vector that has a magnitude equal to NIA and has the same direction as $\hat{n}$. Comparing Equation 26-15 with Equation 21-11 ($\vec{\tau} = \vec{p} \times \vec{E}$) for the torque on an electric dipole, we see that the expression for the torque on a magnetic dipole in a magnetic field has the same form as that for the torque on an electric dipole in an electric field.

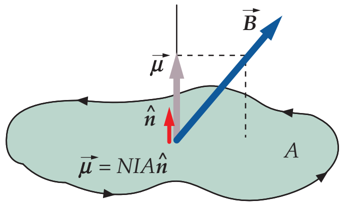


FIGURE 26-23 A flat current loop of arbitrary shape is described by its magnetic moment $\vec{\mu} = NIA\hat{n}$. In a magnetic field $\vec{B}$, the loop experiences a torque $\vec{\mu} \times \vec{B}$.

Example 26-8 Torque on a Current Loop

A circular loop has a radius equal to 2.00 cm, has 10 turns of wire, and carries a current equal to 3.00 A. The axis of the loop makes an angle of 30.0° with a magnetic field of 8000 G. Find the magnitude of the torque on the loop.

PICTURE The torque on a current loop is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$ (Equation 26-15) where $\vec{\mu} = NIA\hat{n}$ (Equation 26-14).

SOLVE

The magnitude of the torque is given by Equation 26-15:

$$\tau = |\vec{\mu} \times \vec{B}| = \mu B\sin\theta = NIAB\sin\theta$$
$$= (10.0)(3.00\text{ A})\pi(0.0200\text{ m})^2(0.800\text{ T})\sin 30.0°$$
$$= \boxed{1.51 \times 10^{-2}\text{ N}\cdot\text{m}}$$

CHECK From $\vec{F} = I\vec{L} \times \vec{B}$ (Equation 26-4) we can see that the SI unit for magnetic field (the tesla) must have units of $N/(A\cdot m)$. With this in mind, one can see by inspection that the units for the right-hand side of the equation in the solution work out to $N\cdot m$, which are SI units for torque.

Example 26-9 Tilting a Loop

Try It Yourself

A circular wire loop that has a radius R, a mass m, and a current I lies on a horizontal surface (Figure 26-24). There is a horizontal magnetic field $\vec{B}$. How large can the current I be before one edge of the loop will lift off the surface?

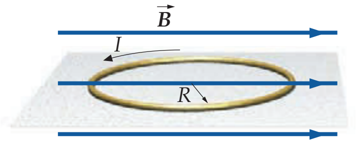


FIGURE 26-24

PICTURE The loop (Figure 26-25) will start to rotate when the magnitude of the net torque on the loop is greater than zero. To eliminate the torque due to the normal force, we calculate torques about the point of contact between the surface and the loop. The magnetic torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$. The magnetic torque is the same about any point because the magnetic torque consists of couples. The lever arm for the gravitational torque is the radius of the loop.

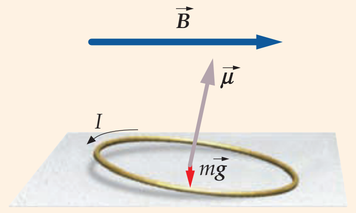


FIGURE 26-25

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
1. Find the magnitude of the magnetic torque acting on the loop.	$\tau_m = \mu B \sin(90°) = I\pi R^2 B$
2. Find the magnitude of the gravitational torque exerted on the loop.	$\tau_g = mgR$
3. Equate the magnitudes of the torques and solve for the current I.	$I = \boxed{\dfrac{mg}{\pi RB}}$

CHECK The current is directly proportional to the mass for constant B, which makes sense. The larger the mass, the more current is needed to start to rotate the ring.

POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A MAGNETIC FIELD

When a torque is exerted on a rotating object, work is done. When a magnetic dipole is rotated through an angle $d\theta$, the work done is

$$dW = -\tau\, d\theta = -\mu B \sin\theta\, d\theta$$

where θ is the angle between $\vec{\mu}$ and $\vec{B}$. The minus sign arises because the magnetic torque tends to decrease θ. Setting this work equal to the decrease in potential energy U, we have

$$dU = -dW = +\mu B \sin\theta\, d\theta$$

Integrating, we obtain

$$U = -\mu B \cos\theta + U_0$$

We choose the potential energy to be zero when $\theta = 90°$. Then $U_0 = 0$ and the potential energy of the dipole is given by

$$U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B} \qquad 26\text{-}16$$

POTENTIAL ENERGY OF A MAGNETIC DIPOLE

Equation 26-16 gives the potential energy of a magnetic dipole at an angle θ to the direction of a magnetic field.

Example 26-10 Torque on a Coil

A square 12-turn coil has an edge-length equal to 40.0 cm and carries a current of 3.00 A. It lies in the $z = 0$ plane, as shown in a uniform magnetic field $\vec{B} = 0.300\ \mathrm{T}\ \hat{i} + 0.400\ \mathrm{T}\ \hat{k}$. The current is counterclockwise when viewed from a point on the positive z axis. Find (*a*) the magnetic moment of the coil and (*b*) the torque exerted on the coil. (*c*) Find the potential energy of the coil.

PICTURE From Figure 26-26, we see that the magnetic moment of the loop is in the $+z$ direction.

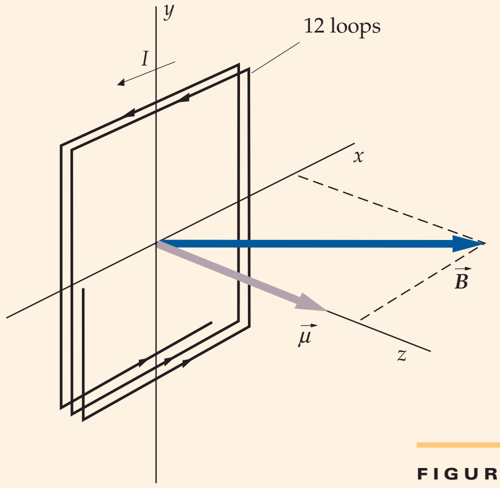


FIGURE 26-26

SOLVE

(a) Calculate the magnetic moment of the loop:

$$\vec{\mu} = NIA\hat{k} = (12)(3.00\ \mathrm{A})(0.400\ \mathrm{m})^2\hat{k}$$
$$= \boxed{5.76\ \mathrm{A\cdot m^2}\ \hat{k}}$$

(b) The torque on the current loop is given by Equation 26-15:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$= (5.76\ \mathrm{A\cdot m^2}\ \hat{k}) \times (0.300\ \mathrm{T}\ \hat{i} + 0.400\ \mathrm{T}\ \hat{k})$$
$$= \boxed{1.73\ \mathrm{N\cdot m}\ \hat{j}}$$

(c) The potential energy is the negative dot product of $\vec{\mu}$ and $\vec{B}$:

$$U = -\vec{\mu}\cdot\vec{B}$$
$$= -(5.76\ \mathrm{A\cdot m^2}\ \hat{k})\cdot(0.300\ \mathrm{T}\ \hat{i} + 0.400\ \mathrm{T}\ \hat{k})$$
$$= \boxed{-2.30\ \mathrm{J}}$$

CHECK The torque in the Part (*b*) result is perpendicular to both the magnetic moment vector and the magnetic field vector, as is expected for a vector product.

PRACTICE PROBLEM 26-3 The potential energy of a current-carrying coil in a uniform magnetic field $\vec{B}$ is equal to zero when the angle between the magnetic dipole moment of the coil $\vec{\mu}$ and the magnetic field is 90°. Calculate the potential energy of the system if the coil is oriented so $\vec{B}$ and $\vec{\mu}$ are (*a*) in the same direction and (*b*) in opposite directions.

When a permanent magnet, such as a compass needle or a bar magnet, is placed in a region where there is a magnetic field $\vec{B}$, the field exerts a torque on the magnet that tends to rotate the magnet so that it lines up with the field. (This effect also occurs with previously unmagnetized iron filings, which become magnetized in the presence of a field $\vec{B}$.) The bar magnet is characterized by a magnetic moment $\vec{\mu}$, a vector that points in the same direction as an arrow drawn from the south pole of the magnet to the north pole of the magnet. A short bar magnet thus behaves like a current loop.

Example 26-11 $\vec{\mu}$ of a Spinning Disk

A thin nonconducting disk that has a mass m, a radius a, and a uniform surface charge per unit area σ spins with angular velocity $\vec{\omega}$ about an axis through the center of the disk and perpendicular to the plane of the disk. Find the magnetic moment of the spinning disk.

PICTURE We find the magnetic moment of a circular element that has a radius R and a width dR and integrate (Figure 26-27). The charge on the element is $dq = \sigma\, dA = \sigma 2\pi R\, dR$. If the charge is positive, the magnetic moment is in the direction of $\vec{\omega}$, so we need only calculate its magnitude.

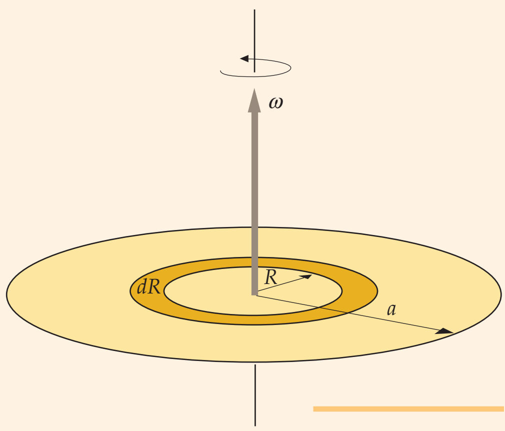


FIGURE 26-27

SOLVE

1. The magnitude of the magnetic moment of the strip shown is the current multiplied by the area of the loop:

$$d\mu = A\,dI = \pi R^2\,dI$$

2. The current in the strip is the total charge dq on the strip divided by the period T. During one period the charge dq passes by a point not rotating with the strip. The period is equal to the reciprocal of the frequency f of rotation $1/T = f = \omega/(2\pi)$:

$$dI = \frac{dq}{T} = \frac{\omega}{2\pi}dq = \frac{\omega}{2\pi}\sigma\,dA$$
$$= \frac{\omega}{2\pi}\sigma 2\pi R\,dR = \sigma\omega R\,dR$$

3. Substitute to obtain the magnitude of the magnetic moment of the strip $d\mu$ in terms of r and dr:

$$d\mu = \pi R^2\,dI = \pi R^2\sigma\omega R\,dR = \pi\sigma\omega R^3\,dR$$

4. Integrate from $r = 0$ to $r = a$:

$$\mu = \int_0^a \pi\sigma\omega R^3\,dR = \frac{1}{4}\pi\sigma\omega a^4$$

5. Use the fact that $\vec{\mu}$ is parallel to $\vec{\omega}$ (if σ is positive) to express the magnetic moment as a vector:

$$\vec{\mu} = \boxed{\tfrac{1}{4}\pi\sigma a^4\vec{\omega}}$$

CHECK Consider a thin spinning ring, also of radius a, carrying the same charge, $Q = \sigma\pi a^2$, as the disk. The magnitude of the magnetic moment of the ring is given by $\mu = IA = \dfrac{Q}{T}\pi a^2 = \dfrac{\sigma\pi a^2}{2\pi/\omega}\pi a^2 = \tfrac{1}{2}\pi\sigma a^4\omega$, which is twice the step-5 result. The step-5 result is smaller than the magnitude of the magnetic moment of the ring, which is what one would expect.

TAKING IT FURTHER In terms of the total charge $Q = \sigma\pi a^2$, the magnetic moment is $\vec{\mu} = \tfrac{1}{4}Qa^2\vec{\omega}$. The angular momentum of the disk is $\vec{L} = (\tfrac{1}{2}ma^2)\vec{\omega}$, so the magnetic moment can be written $\vec{\mu} = \dfrac{Q}{2m}\vec{L}$, which is a more general result. (See Problem 57.)

26-4 THE HALL EFFECT

As we have seen, charges moving in a region where there is a magnetic field each experience a force perpendicular to their motion. When these charges are traveling in a conducting wire, they will be pushed to one side of the wire. This results in a separation of charge in the wire—a phenomenon called the **Hall effect.** This phenomenon allows us to determine the sign of the charge on the charge carriers and the number of charge carriers per unit volume n in a conductor. The Hall effect also provides a convenient method for measuring magnetic fields.

Figure 26-28 shows two conducting strips; each conducting strip carries a current I to the right because the left sides of the strips are connected to the positive terminal of a battery and the right sides are connected to the negative terminal. A magnetic field $\vec{B}$ is directed into the paper. Let us suppose the current in the strip is due to positively charged particles moving to the right, as shown in Figure 26-28*a*. On average, the magnetic force on these particles is $q\vec{v}_\mathrm{d} \times \vec{B}$ (where $\vec{v}_\mathrm{d}$ is the drift velocity). This force is directed up the page. The positively charged particles therefore move up the page to the top edge of the strip, leaving the bottom edge of the strip with an excess negative charge. This separation of charge produces an electric field $\vec{E}$ in the strip that exerts a force on the particles that opposes the magnetic force on them. When the electric and magnetic forces balance, the charge carriers no longer drift up the page. Because the electric field points in the direction of decreasing potential, the upper edge of the strip is at a higher potential than is the lower edge of the strip. This potential difference can be measured using a sensitive voltmeter.

On the other hand, suppose the current is due to negatively charged particles moving to the left, as shown in Figure 26-28*b*. (The negatively charged particles in the strip must move to the left because the current, as before, is to the right.) The magnetic force $q\vec{v}_d \times \vec{B}$ is again up the page, because the signs of both q and the direction of $\vec{v}_d$ have been reversed. Again the carriers are forced to the upper edge of the strip, but the upper edge of the strip now carries a negative charge (because the charge carriers are negative) and the lower edge of the strip now carries a positive charge.

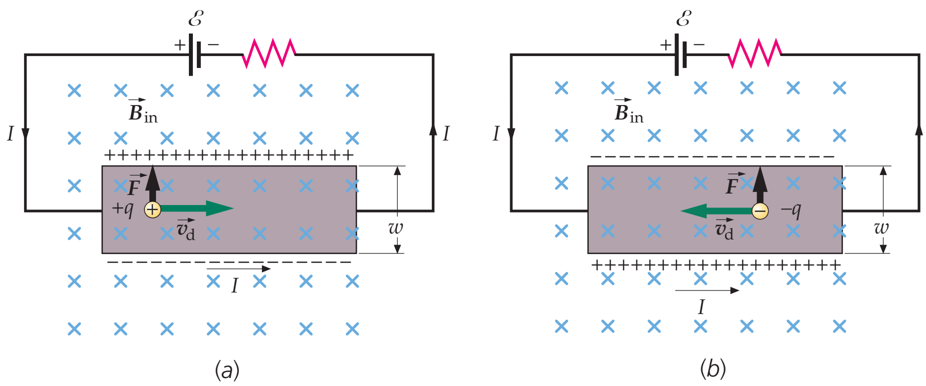


FIGURE 26-28 The Hall effect. The magnetic field is directed into the plane of the page as indicated by the crosses. The magnetic force on a charged particle is upward for a current to the right whether the current is due to (*a*) positive particles moving to the right or (*b*) negative particles moving to the left.

A measurement of the sign of the potential difference between the upper and lower parts of the strip tells us the sign of the charge carriers. In semiconductors, the charge carriers may be negative electrons or positive "holes." A measurement of the sign of the potential difference tells us which are dominant for a particular semiconductor. For a metal strip, we find that the upper edge of the strip in Figure 26-28*b* is at a lower potential than is the lower edge of the strip—which means that the upper part must carry a negative charge. Thus, Figure 26-28*b* is the correct illustration of the current in a metal strip. It was a measurement like this which led to the discovery that the charge carriers in metals are negatively charged.

The potential difference between the top of the strip and the bottom of the strip is called the **Hall voltage**. We can calculate the magnitude of the Hall voltage in terms of the drift velocity. The magnitude of the magnetic force on the charge carriers in the strip is qv_dB. This magnetic force is balanced by the electrostatic force of magnitude qE_H, where E_H is the electric field due to the charge separation. Thus, we have $E_H = v_dB$. If the width of the strip is w, the potential difference is E_Hw. The Hall voltage is therefore

$$V_H = E_Hw = v_dBw \qquad 26\text{-}17$$

PRACTICE PROBLEM 26-4

A conducting strip of width $w = 2.0$ cm is placed in a magnetic field of 0.80 T. The Hall voltage is measured to be 0.64 μV. Calculate the drift velocity of the electrons.

Because the drift velocity for ordinary currents is very small, we can see from Equation 26-17 that the Hall voltage is very small for ordinary-sized strips and magnetic fields. From measurements of the Hall voltage for a strip of a given size, we can determine the number of charge carriers per unit volume in the strip.

The magnitude of the current is given by Equation 26-3:

$$|I| = |q|nv_{\mathrm{d}}A$$

where A is the cross-sectional area of the strip. For a strip of width w and thickness t, the cross-sectional area is $A = wt$. Because the charge carriers are electrons, the quantity $|q|$ is the charge on one electron e. The number density of charge carriers n is thus given by

$$n = \frac{|I|}{A|q|v_{\mathrm{d}}} = \frac{|I|}{wtev_{\mathrm{d}}} \qquad 26\text{-}18$$

Substituting V_{H}/B for $v_{\mathrm{d}}w$ (Equation 26-17), we have

$$n = \frac{|I|B}{teV_{\mathrm{H}}} \qquad 26\text{-}19$$

Example 26-12 Charge Carrier Number Density in Silver

A silver slab has a thickness equal to 1.00 mm, a width equal to 1.50 cm, and a current equal to 2.50 A in a region where there is a magnetic field of magnitude 1.25 T perpendicular to the slab. The Hall voltage is measured to be 0.334 μV. (*a*) Calculate the number density of the charge carriers. (*b*) Calculate the number density of atoms in silver, which has a mass density of $\rho = 10.5\ \mathrm{g/cm^3}$ and a molar mass of $M = 107.9$ g/mol, and compare the number density of atoms in silver with the Part (*a*) result.

PICTURE We can use Equation 26-19 to find the number density of charge carriers. The number density of atoms can be obtained from knowledge of the density and the molar mass.

SOLVE

(*a*) Substitute numerical values into Equation 26-19 to find n:

$$n = \frac{|I|B}{teV_{\mathrm{H}}} = \frac{(2.50\ \mathrm{A})(1.25\ \mathrm{T})}{(1.00 \times 10^{-3}\ \mathrm{m})(1.60 \times 10^{-19}\ \mathrm{C})(3.34 \times 10^{-7}\ \mathrm{V})}$$

$$= \boxed{5.85 \times 10^{28}\ \mathrm{electrons/m^3}}$$

(*b*) 1. The number of atoms per unit volume is $\rho N_{\mathrm{A}}/M$:

$$n_{\mathrm{a}} = \rho\frac{N_{\mathrm{A}}}{M} = (10.5\ \mathrm{g/cm^3})\frac{6.02 \times 10^{23}\ \mathrm{atoms/mol}}{107.9\ \mathrm{g/mol}}$$

$$= \boxed{5.86 \times 10^{22}\ \mathrm{atoms/cm^3} = 5.86 \times 10^{28}\ \mathrm{atoms/m^3}}$$

2. Compare the Part (*b*) step-1 result with the Part (*a*) result:

These results indicate that the number of charge carriers in silver is very nearly one per atom.

CHECK We should expect the number density of charge carriers and the number density of atoms in a metal to be the same order of magnitude. Our results validate that expectation.

The Hall voltage provides a convenient method for measuring magnetic fields. If we rearrange Equation 26-19, we can write for the Hall voltage

$$V_{\mathrm{H}} = \frac{|I|}{nte}B \qquad 26\text{-}20$$

A given strip can be calibrated by measuring the Hall voltage for a given current in a known magnetic field. The strip can then be used to measure an unknown magnetic field B by measuring the Hall voltage for a given current.

*THE QUANTUM HALL EFFECTS

According to Equation 26-20, the Hall voltage should increase linearly with magnetic field strength B for a given current in a given slab. In 1980, while studying the Hall effect in semiconductors at very low temperatures and very large magnetic fields, Klaus von Klitzing discovered that a plot of V_H versus B resulted in a series of plateaus, as shown in Figure 26-29, rather than a straight line. That is, the Hall voltage is quantized. For the discovery of the integer quantum Hall effect, von Klitzing won the Nobel Prize in Physics in 1985.

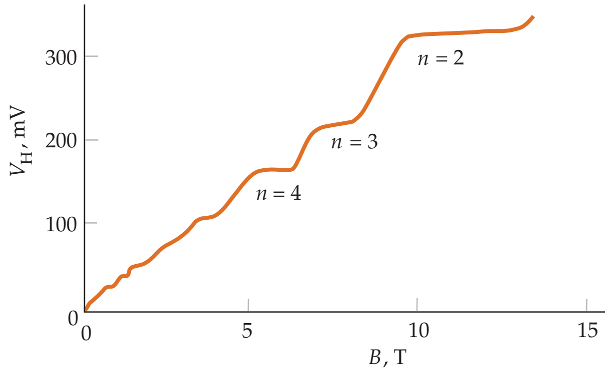


FIGURE 26-29 A plot of the Hall voltage versus applied magnetic field shows plateaus, indicating that the Hall voltage is quantized. The data were taken at a temperature of 1.39 K with the current I held fixed at 25.52 μA.

In the theory of the integer quantum Hall effect, the Hall resistance, defined as $R_H = V_H/I$, can take on only the values

$$R_H = \frac{V_H}{I} = \frac{R_K}{n} \qquad n = 1, 2, 3, \ldots \qquad \text{26-21}$$

where n is an integer, and R_K, called the **von Klitzing constant,** is related to the fundamental electric charge e and Planck's constant h by

$$R_K = \frac{h}{e^2} \qquad \text{26-22}$$

Because the von Klitzing constant can be measured to an accuracy of a few parts per billion, the quantum Hall effect is now used to define a standard of resistance. As of January 1990, the **ohm** is defined in terms of the conventional value* of the von Klitzing constant R_{K-90}, which has the value

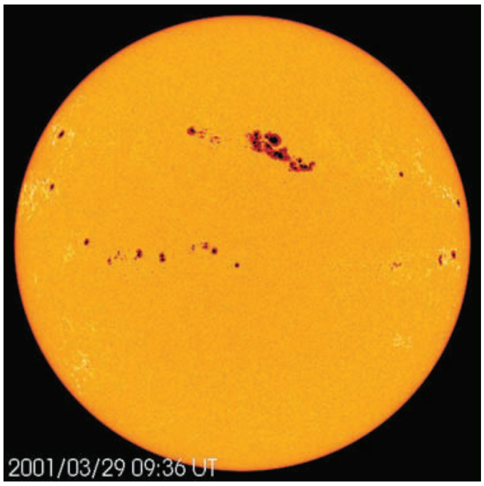
$$R_{K-90} = 25\,812.8076\ \Omega \text{ (exact)} \qquad \text{26-23}$$

In 1982, it was observed that under certain special conditions the Hall resistance is given by Equation 26-22 but with the integer n replaced by a series of rational fractions. This is called the *fractional quantum Hall effect*. For the discovery and explanation of the fractional quantum Hall effect, American professors Laughlin, Störmer, and Tsui won the Nobel Prize in Physics in 1998.

* The value of R_{K-90} differs only slightly from that of R_K. The currently used value of the von Klitzing constant is $R_K = (25\,812.807\,572 \pm 0.000\,095)\ \Omega$.

Physics Spotlight

Earth and the Sun—Magnetic Changes



Sunspots are regions where the magnetic field strength is very high. They are darker than the surrounding surface because the temperature in the sunspot is cooler than the temperature of the surrounding area. *(SOHO/NASA.)*

The magnetic fields of the Sun and Earth have been measured almost constantly in recent years by satellite and ground-based magnetic observatories.[*] Geologists and physicists have collaborated to study the paleomagnetic fields of both Earth[†] and the Sun.[‡] The paleomagnetic studies and the ongoing observations show that the magnetic fields of the Earth and Sun are continuously changing.

Earth's magnetic field has been used as a navigational aid for over 900 years.[#] Navigators were soon aware that magnetic north does not coincide with celestial north, and that the magnetic declination (the difference in direction between magnetic north and celestial north) varied from place to place. Measurements of magnetic declination taken in the same places dating from the sixteenth century[°] showed that the apparent location of magnetic north varied with time at the same place.[§] These measurements are the first evidence that Earth's magnetic field is dynamic.

In the 1960s, drill cores showed many layers of magnetic reversals in volcanic rocks.[¶] It became clear that Earth's magnetic field reverses around every 200 000 years, but there have been durations of over six million years during which there were no geomagnetic reversals. Immediately surrounding the reversal, the record shows that the field strength decreases, reverses, and then increases over a period of a few thousand years.[**] The last geomagnetic reversal was 700 000 years ago. Lately, Earth's magnetic field strength has been decreasing. From 1840 to the present, Earth's magnetic field has decreased by 15 nT/y,[††] which is a decrease of 3% per century, and reconstruction of data from ships' logs shows a decrease of about 2 nT/y from 1590 to 1840.

In the early twentieth century, G. E. Hale noted that sunspots, which had been observed for hundreds of years, had magnetic fields. He demonstrated that during a 22-year sunspot cycle, the Sun's magnetic field gradually decreased, reversed, increased, and returned back to the original configuration.[‡‡] Sunspots themselves have been measured with a magnetic field strength in excess of 200 mT.[##] Recent observation has shown that sunspots are magnetically powered vortices in the Sun. Although the surface of the Sun has an apparent average field of 0.10 mT in regions without sunspots, small areas of such regions have magnetic strengths varying from below 20 mT up to 100 mT.[°°]

The solar wind, which consists of charged sub-atomic particles ejected from the Sun at around 400 km/s,[§§] carries a magnetic field. Satellite data show that the interplanetary magnetic field is complex and dynamic.[¶¶,**] Near Earth, the strength of the interplanetary magnetic field varies between 1 and 37 nT. Sometimes, the Sun ejects a large burst of charged particles. When a large burst arrives at Earth's magnetic field, it causes a magnetic storm that can block radio communications and cause widespread power blackouts. The *Voyager 1* spacecraft was more than 94 AU from the Sun when it measured the strength of the interplanetary magnetic field as 0.03 nT.[††,‡‡] The solar wind still carries a measurable magnetic field well beyond the orbit of Pluto.

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* "Geomagnetic Frequently Asked Questions." United States National Geophysical Data Center, National Oceanic and Atmospheric Administration. http://www.ngdc.noaa.gov/seg/geomag/faqgeom.shtml As of Sept., 2006.

† Yamazaki, T., and Oda, H., "Orbital Influence on Earth's Magnetic Field: 100,000-Year Periodicity in Inclination." *Science*, Mar. 29, 2002, Vol. 294 pp. 2435–2437.

‡ Solanki, S. K., et al., "11,000 Year Sunspot Number Reconstruction." *IBGP Pages/World Data Center for Paleoclimatology Data Contribution Series #2005-015*. 2005. ftp://ftp.ncdc.noaa.gov/pub/data/paleo/climate-forcing/solar_variability/solanki2004-ssn.txt As of Sept., 2006.

\# Hellemans, A., and Bunch, B., *The Timetables of Science*. New York: Simon and Schuster, 1988. p. 75.

° Kono, M., "Ships' Logs and Archeomagnetism." *Science*, May 12, 2006, Vol. 312, pp. 865–66.

§ Hermanus Magnetic Observatory, "Detailed History." http://www.hmo.ac.za/detailed-history.html As of Sept., 2006.

¶ Dunn, J. R., et al., "Paleomagnetic Study of a Reversal of the Earth's Magnetic Field." *Science*, May 21, 1971, Vol. 172, pp. 840–844.

** Merrill, R. T., and McFadden, P. L., "Geomagnetic Polarity Transitions." *Reviews of Geophysics*, May 1999, Vol. 37, No. 2, pp. 201–226.

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‡‡ Abbot, C. G., "Sun-Spots and Weather." *Science*, Dec. 8, 1933, Vol. 78, pp. 518–519.

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§§ Hathaway, D., "The Solar Wind." *Solar Physics*, Marshall Space Flight Center, NASA http://solarscience.msfc. nasa.gov/SolarWind.shtml. Jun. 1, 2006, As of Oct., 2006.

¶¶ Smith, E. J., et al., "The Sun and Heliosphere at Solar Maximum." *Science*, Nov. 14, 2003, Vol. 302, pp. 1165–1168.

*** Arnold, N., and Lyons, A., "Granta MIST: Meeting Report." *Astronomy and Geophysics*, Aug. 2006, Vol. 46, pp. 4.18–4.21.

††† Gurnett, D. A., and Kurth, W. S., "Electron Plasma Oscillations Upstream of the Solar Wind Termination Shock." *Science*, Sept. 23, 2005, Vol. 309, pp. 2025–2027.

‡‡‡ Burlaga, L. F., et al., "Crossing the Termination Shock into the Helosheath: Magnetic Fields." *Science*, Sept. 23, 2005, Vol. 309, pp. 2027–2029.

Summary

1. The magnetic field describes the condition in space in which moving charges experience a force perpendicular to their velocity.
2. The magnetic force is part of the electromagnetic interaction, one of the three known fundamental interactions in nature.
3. The magnitude and direction of a magnetic field $\vec{B}$ are defined by the formula $\vec{F} = q\vec{v} \times \vec{B}$, where $\vec{F}$ is the force exerted on a particle with charge q moving with velocity $\vec{v}$.

TOPIC	RELEVANT EQUATIONS AND REMARKS	
1. Magnetic Force		
On a moving charge	$\vec{F} = q\vec{v} \times \vec{B}$	26-1
On a current element	$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$	26-5
Unit of the magnetic field	The SI unit of magnetic fields is the tesla (T). A commonly used unit is the gauss (G), which is related to the tesla by $1\text{ G} = 10^{-4}\text{ T}$	26-3
2. Motion of Point Charges	A particle of mass m and charge q moving with speed v in a plane perpendicular to a uniform magnetic field moves in a circular orbit. The period and frequency of the circular motion are independent of the radius of the orbit and of the speed of the particle.	
Newton's second law	$qvB = m\dfrac{v^2}{r}$	26-6
Cyclotron period	$T = \dfrac{2\pi m}{qB}$	26-7
Cyclotron frequency	$f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$	26-8
*Velocity selector	A velocity selector consists of crossed electric and magnetic fields so that the electric and magnetic forces balance for a particle moving with speed v. $v = \dfrac{E}{B}$	26-9
*Thomson's measurement of q/m	The deflection of a charged particle in an electric field depends on the speed of the particle and is proportional to the charge-to-mass ratio q/m of the particle. J. J. Thomson used crossed electric and magnetic fields to measure the speed of cathode rays and then measured q/m for these particles by deflecting them in an electric field. He showed that all cathode rays consist of particles which all have the same charge-to-mass ratio. These particles are now called electrons.	
*Mass spectrometer	The mass-to-charge ratio of an ion of known speed can be determined by measuring the radius of the circular path taken by the ion in a known magnetic field.	
3. Current Loops		
Magnetic dipole moment	$\vec{\mu} = NIA\hat{n}$	26-14
Torque	$\vec{\tau} = \vec{\mu} \times \vec{B}$	26-15
Potential energy of a magnetic dipole	$U = -\vec{\mu} \cdot \vec{B}$	26-16
Net force	The net force on a current loop in a *uniform* magnetic field is zero.	

TOPIC	RELEVANT EQUATIONS AND REMARKS	
4. The Hall Effect	When a conducting strip carrying a current is placed in a magnetic field, the magnetic force on the charge carriers causes a separation of charge called the Hall effect. This results in a voltage V_H, called the Hall voltage. The sign of the charge carriers can be determined from a measurement of the sign of the Hall voltage, and the number of carriers per unit volume can be determined from the magnitude of V_H.	
Hall voltage	$V_H = E_H w = v_d B w = \frac{\lvert I \rvert}{nte} B$	26-17, 26-20
*Quantum Hall effects	Measurements at very low temperatures in very large magnetic fields indicate that the Hall resistance $R_H = V_H/I$ is quantized and can take on only the values given by $R_H = \frac{V_H}{I} = \frac{R_K}{n} \quad n = 1, 2, 3, \ldots$	26-21
*Conventional von Klitzing constant (definition of ohm)	$R_{K-90} = 25\,812.8076\ \Omega$ (exact)	26-23

Answers to Concept Checks

26-1 A left-hand rule is one way to answer the question. The definition for the direction of $\vec{B}$ is a convention. If the definition for the direction of $\vec{B}$ were changed as described in the question statement, a correct force law could be written $\vec{F} = q\vec{v} \otimes \vec{B}$, where the symbol $\otimes$ denotes the same operation as the symbol $\times$, except the product denoted by $\otimes$ requires using the left-hand rule instead of the right-hand rule. Alternatively, the force law could be revised to $\vec{F} = \vec{B} \times q\vec{v}$ and then you could stay with the right-hand rule.

26-2 (*b*) Negatively charged. The force $\vec{F}$ and the vector $\vec{v} \times \vec{B}$ are in opposite directions only if the particle is negatively charged. This is consistent with the relation $\vec{F} = q\vec{v} \times \vec{B}$.

Answers to Practice Problems

26-1 $-1.3 \times 10^{-12}\ \mathrm{N}\ \hat{j}$

26-2 (*a*) 667 km/s, (*b*) in the $-z$ direction

26-3 (*a*) -2.88 J. Note that this potential energy is lower than the potential energy calculated in the example. (The potential energy is lowest when $\vec{\mu}$ and $\vec{B}$ are in the same direction.) (*b*) $+2.88$ J

26-4 4.0×10^{-5} m/s

Problems

In a few problems, you are given more data than you actually need; in a few other problems, you are required to supply data from your general knowledge, outside sources, or informed estimate.

Interpret as significant all digits in numerical values that have trailing zeros and no decimal points.

- • Single-concept, single-step, relatively easy
- •• Intermediate-level, may require synthesis of concepts
- ••• Challenging
- SSM Solution is in the *Student Solutions Manual*
- Consecutive problems that are shaded are paired problems.

CONCEPTUAL PROBLEMS

1 • When the axis of a cathode-ray tube is horizontal in a region in which there is a magnetic field that is directed vertically upward, the electrons emitted from the cathode follow one of the dashed paths to the face of the tube in Figure 26-30. The correct path is (*a*) 1, (*b*) 2, (*c*) 3, (*d*) 4, (*e*) 5. SSM

2 •• We define the direction of the electric field to be the same as the direction of the force on a positive test charge. Why then do we *not* define the direction of the magnetic field to be the same as the direction of the magnetic force on a moving positive test charge?

3 • A *flicker bulb* is a lightbulb that has a long, thin flexible filament. It is meant to be plugged into an ac outlet that delivers current at a frequency of 60 Hz. There is a small permanent magnet

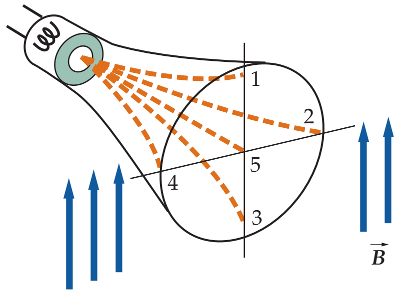

FIGURE 26-30 Problem 1

inside the bulb. When the bulb is plugged in the filament oscillates back and forth. At what frequency does it oscillate? Explain your answer. SSM

4 • In a cyclotron, the potential difference between the dees oscillates with a period given by $T = 2\pi m/(qB)$. Show that the expression to the right of the equal sign has units of seconds if q, B, and m have units of coulombs, teslas, and kilograms, respectively.

5 • A ^{7}Li nucleus has a charge equal to $+3e$ and a mass that is equal to the mass of seven protons. A ^{7}Li nucleus and a proton are both moving perpendicular to a uniform magnetic field $\vec{B}$. The magnitude of the momentum of the proton is equal to the magnitude of the momentum of the nucleus. The path of the proton has a radius of curvature equal to R_p and the path of the ^{7}Li nucleus has a radius of curvature equal to R_{Li}. The ratio R_p/R_{Li} is closest to (*a*) 3/1, (*b*) 1/3, (*c*) 1/7, (*d*) 7/1, (*e*) 3/7, (*f*) 7/3.

6 • An electron moving in the $+x$ direction enters a region that has a uniform magnetic field in the $+y$ direction. When the electron enters this region, it will (*a*) be deflected toward the $+y$ direction, (*b*) be deflected toward the $-y$ direction, (*c*) be deflected toward the $+z$ direction, (*d*) be deflected toward the $-z$ direction, (*e*) continue undeflected in the $+x$ direction.

7 • In a velocity selector, the speed of the undeflected charged particle is given by the ratio of the magnitude of the electric field to the magnitude of the magnetic field. Show that E/B in fact does have the units of m/s if E and B are in units of volts per meter and teslas, respectively. SSM

8 • How are the properties of magnetic field lines similar to the properties of electric field lines? How are they different?

9 • True or false:

(*a*) The magnetic moment of a bar magnet points from its north pole to its south pole.
(*b*) Inside the material of a bar magnet, the magnetic field due to the bar magnet points from the magnet's south pole toward its north pole.
(*c*) If a current loop simultaneously has its current doubled and its area cut in half, then the magnitude of its magnetic moment remains the same.
(*d*) The maximum torque on a current loop placed in a magnetic field occurs when the plane of the loop is perpendicular to the direction of the magnetic field.

10 •• Show that the von Klitzing constant, h/e^2, gives the SI unit for resistance (the ohm) if h and e are in units of joule-seconds and coulombs, respectively.

11 ••• The theory of relativity states that no law of physics can be described using the absolute velocity of an object, which is in fact impossible to define due to a lack of an absolute reference frame. Instead, the behavior of interacting objects can only be described by the relative velocities between the objects. New physical insights result from this idea. For example, in Figure 26-31 a magnet moving at high speed relative to some observer passes by an electron that is at rest relative to the same observer. Explain why you are sure that a force must be acting on the electron. In what direction will the force point at the instant the north pole of the magnet passes directly underneath the electron? Explain your answer.

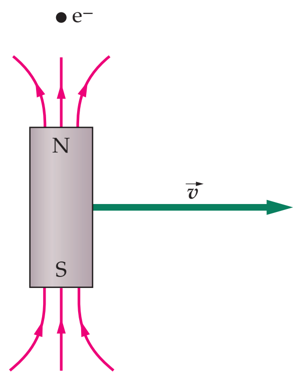

FIGURE 26-31 Problem 11

ESTIMATION AND APPROXIMATION

12 • Estimate the maximum magnetic force per meter that Earth's magnetic field could exert on a current-carrying wire in a 20-A circuit in your house.

13 •• **CONTEXT-RICH** Your friend wants to be a magician and intends to use Earth's magnetic field to suspend a current-carrying wire above the stage. He asks you to estimate the minimum current needed to suspend the wire just above Earth's surface at the equator (where Earth's magnetic field is horizontal). Assume the wire has a linear mass density of 10 g/m. Would you advise him to proceed with his plans for this act?

THE FORCE EXERTED BY A MAGNETIC FIELD

14 • Find the magnetic force on a proton moving in the $+x$ direction at a speed of 0.446 Mm/s in a uniform magnetic field of 1.75 T in the $+z$ direction.

15 • A point particle has a charge equal to -3.64 nC and a velocity equal to 2.75×10^3 m/s $\hat{i}$. Find the force on the charge if the magnetic field is (*a*) 0.38 T $\hat{j}$, (*b*) 0.75 T $\hat{i}$ + 0.75 T $\hat{j}$, (*c*) 0.65 T $\hat{i}$, and (*d*) 0.75 T $\hat{i}$ + 0.75 T $\hat{k}$.

16 • A uniform magnetic field equal to 1.48 T $\hat{k}$ is in the $+z$ direction. Find the force exerted by the field on a proton if the velocity of the proton is (*a*) 2.7 km/s $\hat{i}$, (*b*) 3.7 km/s $\hat{j}$, (*c*) 6.8 km/s $\hat{k}$, and (*d*) 4.0 km/s $\hat{i}$ + 3.0 km/s $\hat{j}$.

17 • A straight wire segment that is 2.0 m long makes an angle of 30° with a uniform 0.37-T magnetic field. Find the magnitude of the force on the wire if the wire has a current of 2.6 A.

18 • A straight segment of a current-carrying wire has a current element $I\vec{L}$, where $I = 2.7$ A and $\vec{L} = 3.0$ cm $\hat{i}$ + 4.0 cm $\hat{j}$. The segment is in a region with a uniform magnetic field given by 1.3 T $\hat{i}$. Find the force on the segment of wire.

19 • What is the force on an electron that has a velocity equal to 2.0×10^6 m/s $\hat{i}$ − 3.0×10^6 m/s $\hat{j}$ when it is in a region with a magnetic field given by 0.80 T $\hat{i}$ + 0.60 T $\hat{j}$ − 0.40 T $\hat{k}$?

20 •• The section of wire shown in Figure 26-32 carries a current equal to 1.8 A from a to b. The segment is in a region that has a magnetic field whose value is $1.2\ \mathrm{T}\ \hat{k}$. Find the total force on the wire and show that the total force is the same as if the wire were in the form of a straight wire directly from a to b and carrying the same current.

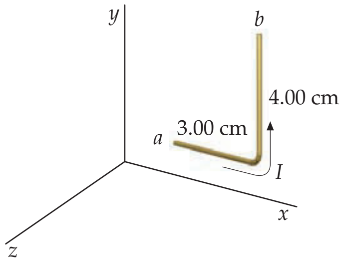

FIGURE 26-32 Problem 20

21 •• A straight, stiff, horizontal 25-cm-long wire that has a mass equal to 50 g is connected to a source of emf by light, flexible leads. A magnetic field of 1.33 T is horizontal and perpendicular to the wire. Find the current necessary to "float" the wire, that is, when the wire is released from rest it remains at rest.

22 •• **ENGINEERING APPLICATION** In your physics laboratory class, you have constructed a simple *gaussmeter* for measuring the horizontal component of magnetic fields. The setup consists of a stiff 50-cm wire that hangs vertically from a conducting pivot so that its free end makes contact with a pool of mercury in a dish below (Figure 26-33). The mercury provides an electrical contact without constraining the movement of the wire. The wire has a mass of 5.0 g and conducts a current downward. (*a*) What is the equilibrium angular displacement of the wire from vertical if the horizontal component of the magnetic field is 0.040 T and the current is 0.20 A? (*b*) What is the sensitivity of this gaussmeter? That is, what is the ratio of the output to the input (in radians per tesla)?

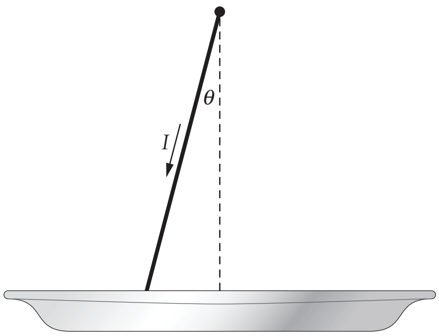

FIGURE 26-33 Problem 22

23 •• A 10-cm-long straight wire is parallel with the x axis and carries a current of 2.0 A in the $+x$ direction. The force on this wire due to the presence of a magnetic field $\vec{B}$ is $3.0\ \mathrm{N}\ \hat{j} + 2.0\ \mathrm{N}\ \hat{k}$. If this wire is rotated so that it is parallel with the y axis with the current in the $+y$ direction, the force on the wire becomes $-3.0\ \mathrm{N}\ \hat{j} - 2.0\ \mathrm{N}\ \hat{k}$. Determine the magnetic field $\vec{B}$. SSM

24 •• A 10-cm-long straight wire is parallel with the z axis and carries a current of 4.0 A in the $+z$ direction. The force on this wire due to a uniform magnetic field $\vec{B}$ is $-0.20\ \mathrm{N}\ \hat{i} + 0.20\ \mathrm{N}\ \hat{j}$. If this wire is rotated so that it is parallel with the x axis with the current in the $+x$ direction, the force on the wire becomes $0.20\ \hat{k}\ \mathrm{N}$. Find $\vec{B}$.

25 •• A current-carrying wire is bent into a closed semicircular loop of radius R that lies in the xy plane (Figure 26-34). The wire is in a uniform magnetic field that is in the $+z$ direction, as shown. Verify that the force acting on the loop is zero. SSM

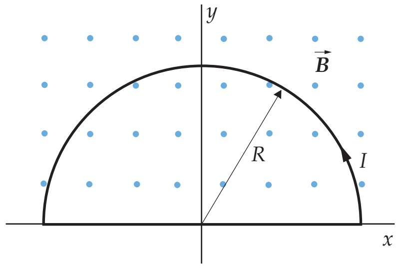

FIGURE 26-34 Problem 25

26 ••• A wire bent in some arbitrary shape carries a current I. The wire is in a region with a uniform magnetic field $\vec{B}$. Show that the total force on the part of the wire from some arbitrary point on the wire (designated as a) to some other arbitrary point on the wire (designated as b) is $\vec{F} = I\vec{L} \times \vec{B}$, where $\vec{L}$ is the vector from point a to point b. In other words, show that the force on an arbitrary section of the bent wire is the same as the force on a straight section wire carrying the same current and connecting the two endpoints of the arbitrary section.

MOTION OF A POINT CHARGE IN A MAGNETIC FIELD

27 • A proton moves in a 65-cm-radius circular orbit that is perpendicular to a uniform magnetic field of magnitude 0.75 T. (*a*) What is the orbital period for the motion? (*b*) What is the speed of the proton? (*c*) What is the kinetic energy of the proton? SSM

28 • A 4.5-keV electron (an electron that has a kinetic energy equal to 4.5 keV) moves in a circular orbit that is perpendicular to a magnetic field of 0.325 T. (*a*) Find the radius of the orbit. (*b*) Find the frequency and period of the orbital motion.

29 •• A proton, deuteron, and an alpha particle in a region with a uniform magnetic field each follow circular paths that have the same radius. The deuteron has a charge that is equal to the charge a proton has, and an alpha particle has a charge that is equal to twice the charge a proton has. Assume that $m_\alpha = 2m_d = 4m_p$. Compare (*a*) their speeds, (*b*) their kinetic energies, and (*c*) the magnitudes of their angular momenta about the centers of the orbits.

30 •• A particle has a charge q, a mass m, a linear momentum of magnitude p, and a kinetic energy K. The particle moves in a circular orbit of radius R perpendicular to a uniform magnetic field $\vec{B}$. Show that (*a*) $p = BqR$ and (*b*) $K = \frac{1}{2}B^2q^2R^2/m$.

31 •• A beam of particles with velocity $\vec{v}$ enters a region that has a uniform magnetic field $\vec{B}$ in the $+x$ direction. Show that when the x component of the displacement of one of the particles is $2\pi(m/qB)v\cos\theta$, where θ is the angle between $\vec{v}$ and $\vec{B}$, the velocity of the particle is in the same direction as it was when the particle entered the field. SSM

32 •• A proton that has a speed equal to 1.00×10^6 m/s enters a region with a uniform magnetic field that has a magnitude of 0.800 T and points into the page, as shown in Figure 26-35. The proton enters the region at an angle $\theta = 60°$. Find the exit angle ϕ and the distance d.

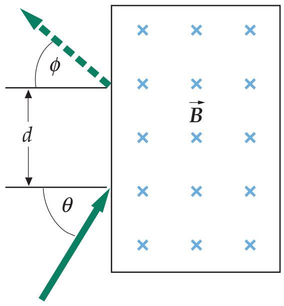

FIGURE 26-35
Problems 32 and 33

33 •• Suppose that in Figure 26-35, the magnetic field has a magnitude of 60 mT, the distance d is 40 cm, and θ is 24°. Find the speed v at which a particle enters the region and the exit angle ϕ if the particle is (*a*) a proton and (*b*) a deuteron. Assume that $m_d = 2m_p$. SSM

34 •• The galactic magnetic field in some region of interstellar space has a magnitude of 1.00×10^{-9} T. A particle of interstellar dust has a mass of 10.0 μg and a total charge of 0.300 nC. How many years does it take for the particle to complete a revolution of the circular orbit caused by its interaction with the magnetic field?

APPLICATIONS OF THE MAGNETIC FORCE ACTING ON CHARGED PARTICLES

35 • A velocity selector has a magnetic field that has a magnitude equal to 0.28 T and is perpendicular to an electric field that has a magnitude equal to 0.46 MV/m. (*a*) What must the speed of a particle be for that particle to pass through the velocity selector undeflected? What kinetic energy must (*b*) protons and (*c*) electrons have in order to pass through the velocity selector undeflected? SSM

36 •• A beam of protons is moving in the $+x$ direction with a speed of 12.4 km/s through a region in which the electric field is perpendicular to the magnetic field. The beam is not deflected in this region. (*a*) If the magnetic field has a magnitude of 0.85 T and points in the $+y$ direction, find the magnitude and direction of the electric field. (*b*) Would electrons that have the same velocity as the protons be deflected by these fields? If so, in what direction would they be deflected? If not, why not?

37 •• The plates of a Thomson q/m apparatus are 6.00 cm long and are separated by 1.20 cm. The end of the plates is 30.0 cm from the tube screen. The kinetic energy of the electrons is 2.80 keV. If a potential difference of 25.0 V is applied across the deflection plates, by how much will the point where the beam of electrons strikes the screen be displaced?

38 •• Chlorine has two stable isotopes, ^{35}Cl and ^{37}Cl. Chlorine gas which consists of singly ionized ions is to be separated into its isotopic components using a mass spectrometer. The magnetic field strength in the spectrometer is 1.2 T. What is the minimum value of the potential difference through which these ions must be accelerated so that the separation between them, after they complete their semicircular path, is 1.4 cm?

39 •• In a mass spectrometer, a singly ionized ^{24}Mg ion has a mass equal to 3.983×10^{-26} kg and is accelerated through a 2.50-kV potential difference. It then enters a region where it is deflected by a magnetic field of 557 G. (*a*) Find the radius of curvature of the ion's orbit. (*b*) What is the difference in the orbital radii of the ^{26}Mg and ^{24}Mg ions? Assume that their mass ratio is 26:24. SSM

40 •• A beam of singly ionized ^{6}Li and ^{7}Li ions passes through a velocity selector and enters a region of uniform magnetic field with a velocity that is perpendicular to the direction of the field. If the diameter of the orbit of the ^{6}Li ions is 15 cm, what is the diameter of the orbit for ^{7}Li ions? Assume their mass ratio is 7:6.

41 •• Using Example 26-6, determine the time required for a ^{58}Ni ion and a ^{60}Ni ion to complete the semicircular path.

42 •• Before entering a mass spectrometer, ions pass through a velocity selector consisting of parallel plates that are separated by 2.0 mm and have a potential difference of 160 V. The magnetic field strength is 0.42 T in the region between the plates. The magnetic field strength in the mass spectrometer is 1.2 T. Find (*a*) the speed of the ions entering the mass spectrometer and (*b*) the difference in the diameters of the orbits of singly ionized ^{238}U and ^{235}U. The mass of a ^{235}U ion is 3.903×10^{-25} kg.

43 •• A cyclotron for accelerating protons has a magnetic field strength of 1.4 T and a radius of 0.70 m. (*a*) What is the cyclotron's frequency? (*b*) Find the kinetic energy of the protons when they emerge. (*c*) How will your answers change if deuterons are used instead of protons? SSM

44 •• A certain cyclotron that has a magnetic field whose magnitude is 1.8 T is designed to accelerate protons to a kinetic energy of 25 MeV. (*a*) What is the cyclotron frequency for this cyclotron? (*b*) What must the minimum radius of the magnet be to achieve this energy? (*c*) If the alternating potential difference applied to the dees has a maximum value of 50 kV, how many revolutions must the protons make before emerging with kinetic energies of 25 MeV?

45 •• Show that for a given cyclotron the cyclotron frequency for accelerating deuterons is the same as the frequency for accelerating alpha particles and is half the frequency for accelerating protons in the same magnetic field. The deuteron has a charge that is equal to the charge a proton has, and an alpha particle has a charge that is equal to twice the charge a proton has. Assume that $m_\alpha = 2m_d = 4m_p$.

46 ••• Show that the radius of the orbit of a charged particle in a cyclotron is proportional to the square root of the number of orbits completed.

TORQUES ON CURRENT LOOPS, MAGNETS, AND MAGNETIC MOMENTS

47 • A small circular coil consisting of 20 turns of wire lies in a region with a uniform magnetic field whose magnitude is 0.50 T. The arrangement is such that the normal to the plane of the coil makes an angle of 60° with the direction of the magnetic field. The radius of the coil is 4.0 cm, and the wire carries a current of 3.0 A. (*a*) What is the magnitude of the magnetic moment of the coil? (*b*) What is the magnitude of the torque exerted on the coil? SSM

48 • What is the maximum torque on a 400-turn circular coil of radius 0.75 cm that carries a current of 1.6 mA and is in a region with a uniform magnetic field of 0.25 T?

49 • A current-carrying wire is in the shape of a square of edge length 6.0 cm. The square lies in the $z = 0$ plane. The wire carries a current of 2.5 A. What is the magnitude of the torque on the wire if it is in a region with a uniform magnetic field of magnitude 0.30 T that points in the (*a*) $+z$ direction and (*b*) $+x$ direction? SSM

50 • A current-carrying wire is in the shape of an equilateral triangle of edge length 8.0 cm. The triangle lies in the $z = 0$ plane. The wire carries a current of 2.5 A. What is the magnitude of the torque on the wire if it is in a region with a uniform magnetic field of magnitude 0.30 T that points in the (*a*) $+z$ direction and (*b*) $+x$ direction?

51 •• A rigid wire is in the shape of a square of edge length L. The square has mass m and the wire carries current I. The square lies on a flat horizontal surface in a region where there is a magnetic field of magnitude B that is parallel to two edges of the square. What is the minimum value of B so that one edge of the square will lift off the surface?

52 •• A rectangular current-carrying 50-turn coil, as shown in Figure 26-36, is pivoted about the z axis. (*a*) If the wires in the $z = 0$ plane make an angle $\theta = 37°$ with the y axis, what angle does the magnetic moment of the coil make with the unit vector $\hat{i}$? (*b*) Write an expression for $\hat{n}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$, where $\hat{n}$ is a unit vector in the direction of the magnetic moment. (*c*) What is the magnetic moment of the coil? (*d*) Find the torque on the coil when there is a uniform magnetic field $\vec{B} = 1.5\text{ T}\,\hat{j}$ in the region occupied by the coil. (*e*) Find the potential energy of the coil in this field. (The potential energy is zero when $\theta = 0$.)

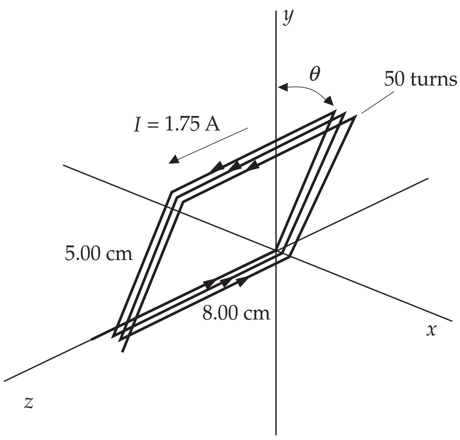

FIGURE 26-36
Problems 52 and 53

53 •• For the coil in Problem 52 the magnetic field is now $\vec{B} = 2.0\text{ T}\,\hat{j}$. Find the torque exerted on the coil when $\hat{n}$ is equal to (*a*) $\hat{i}$, (*b*) $\hat{j}$, (*c*) $-\hat{j}$, and (*d*) $(\hat{i} + \hat{j})/\sqrt{2}$. SSM

54 •• A small bar magnet has a length equal to 6.8 cm and its magnetic moment is aligned with a uniform magnetic field of magnitude 0.040 T. The bar magnet is then rotated through an angle of 60° about an axis perpendicular to its length. The observed torque on the bar magnet has a magnitude of 0.10 N · m. (*a*) Find the magnetic moment of the magnet. (*b*) Find the potential energy of the magnet.

55 •• A wire loop consists of two semicircles connected by straight segments (Figure 26-37). The inner and outer radii are 0.30 m and 0.50 m, respectively. A current of 1.5 A is in this wire and the current in the outer semicircle is in the clockwise direction. What is the magnetic moment of this current loop?

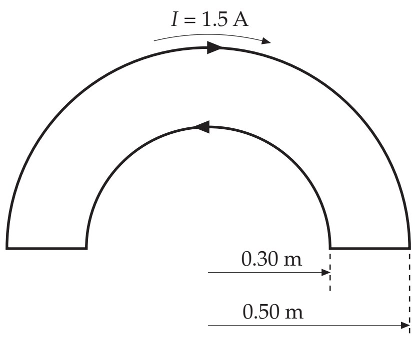

FIGURE 26-37
Problem 55

56 •• A wire of length L is wound into a circular coil that has N turns. Show that when the wire carries a current I, the magnetic moment of the coil has a magnitude given by $IL^2/(4\pi N)$.

57 •• A particle that has a charge q and a mass m moves with angular velocity ω in a circular path of radius r. (*a*) Show that the average current created by this moving particle is $\omega q/(2\pi)$ and that the magnetic moment of its orbit has a magnitude of $\frac{1}{2}q\omega r^2$. (*b*) Show that the angular momentum of this particle has the magnitude of $mr^2\omega$ and that the magnetic moment and angular momentum vectors are related by $\vec{\mu} = \frac{1}{2}(q/m)\,\vec{L}$, where $\vec{L}$ is the angular momentum about the center of the circle. SSM

58 ••• A uniformly charged nonconducting cylindrical shell (Figure 26-38) has length L, inner and outer radii R_i and R_o, respectively, a charge density ρ, and an angular velocity ω about its axis. Derive an expression for the magnetic moment of the cylinder.

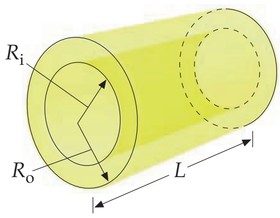

FIGURE 26-38 **Problem 58**

59 ••• A uniform nonconducting thin rod of mass m and length L has a uniform charge per unit length λ and rotates with angular speed ω about an axis through one end and perpendicular to the rod. (*a*) Consider a small segment of the rod of length dx and charge $dq = \lambda\,dr$ at a distance r from the pivot (Figure 26-39). Show that the average current created by this moving segment is $\omega\,dq/(2\pi)$ and show that the magnetic moment of this segment is $\frac{1}{2}\lambda\omega r^2\,dx$. (*b*) Use this to show that the magnitude of the magnetic moment of the rod is $\frac{1}{6}\lambda\omega L^3$. (*c*) Show that the magnetic moment $\vec{\mu}$ and angular momentum $\vec{L}$ are related by $\vec{\mu} = \frac{1}{2}(Q/m)\vec{L}$, where Q is the total charge on the rod. SSM

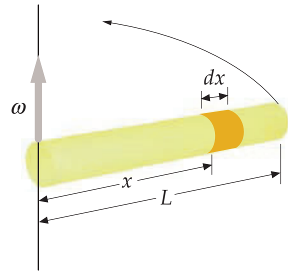

FIGURE 26-39
Problem 59

60 ••• A nonuniform, nonconducting thin disk of mass m, radius R, and total charge Q has a charge per unit area σ that varies as $\sigma_0 r/R$ and a mass per unit area σ_m that is given by $(m/Q)\sigma$. The disk rotates with angular speed ω about its central axis. (*a*) Show that the magnetic moment of the disk has a magnitude $\frac{1}{5}\pi\omega\sigma_0 R^4$, which can be alternatively rewritten as $\frac{3}{10}\omega QR^2$. (*b*) Show that the magnetic moment $\vec{\mu}$ and angular momentum $\vec{L}$ are related by $\vec{\mu} = \frac{1}{2}(Q/m)\vec{L}$.

61 ••• A spherical shell of radius R carries a constant surface charge density σ. The shell rotates about its diameter with angular speed ω. Find the magnitude of the magnetic moment of the rotating spherical shell. SSM

62 ••• A uniform, solid, uniformly charged sphere of radius R has a volume charge density ρ. The sphere rotates about an axis through its center with angular speed ω. Find the magnitude of the magnetic moment of the rotating sphere.

63 ••• A uniform, thin, uniformly charged disk of mass m, radius R, and uniform surface charge density σ rotates with angular speed ω about an axis through its center and perpendicular to the disk (Figure 26-40). The disk is in a region with a uniform magnetic field $\vec{B}$ that makes an angle θ with the rotation axis. Calculate (*a*) the magnitude of the torque exerted on the disk by the magnetic field and (*b*) the precession frequency of the disk in the magnetic field.

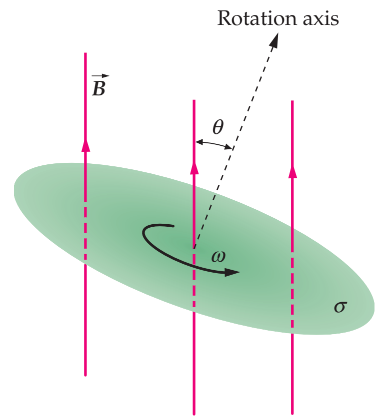

FIGURE 26-40 Problem 63

THE HALL EFFECT

64 • A metal strip that is 2.00 cm wide and 0.100 cm thick carries a current of 20.0 A in a region with a uniform magnetic field of 2.00 T, as shown in Figure 26-41. The Hall voltage is measured to be 4.27 μV. (*a*) Calculate the drift speed of the free electrons in the strip. (*b*) Find the number density of the free electrons in the strip. (*c*) Is point *a* or point *b* at the higher potential? Explain your answer.

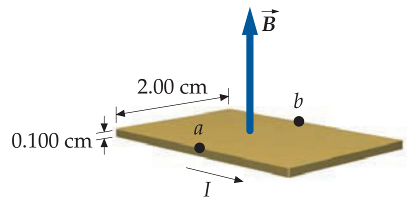

FIGURE 26-41 Problems 64 and 65

65 •• The number density of free electrons in copper is 8.47×10^{22} electrons per cubic centimeter. If the metal strip in Figure 26-41 is copper and the current is 10.0 A, find (*a*) the drift speed v_d and (*b*) the potential difference $V_a - V_b$. Assume that the magnetic field strength is 2.00 T. SSM

66 •• **Engineering Application** A copper strip has 8.47×10^{22} free electrons per cubic centimeter, is 2.00 cm wide, is 0.100 cm thick, and is used to measure the magnitudes of unknown magnetic fields that are perpendicular to it. Find the magnitude B when the current is 20.0 A and the Hall voltage is (*a*) 2.00 μV, (*b*) 5.25 μV, and (*c*) 8.00 μV.

67 •• **Biological Application** Because blood contains ions, moving blood in the presence of a magnetic field develops a Hall voltage across the diameter of an artery. A large artery that has a diameter of 0.85 cm can have blood flowing through it with a maximum speed of 0.60 m/s. If a section of the artery is in a magnetic field of 0.20 T, what is the maximum potential difference across the diameter of the artery?

68 •• The Hall coefficient R_H is a property of conducting material (just as resistivity is). It is defined as $R_H = E_y/(J_xB_z)$, where J_x is the x component of the current density in the material, B_z is the z component of the magnetic field, and E_y is the y component of the resulting Hall electric field. Show that the Hall coefficient is equal to $1/(nq)$, where q is the charge of the charge carriers ($-e$ if they are electrons). (The Hall coefficients of monovalent metals, such as copper, silver, and sodium, are therefore negative.)

69 •• Aluminum has a density of 2.7×10^3 kg/m^3 and a molar mass of 27 g/mol. The Hall coefficient of aluminum is $R = -0.30 \times 10^{-10}$ m^3/C. (See Problem 68 for the definition of R.) What is the number of conduction electrons per aluminum atom? SSM

GENERAL PROBLEMS

70 • A long wire parallel to the x axis carries a current of 6.50 A in the $+x$ direction. The wire occupies a region that has a uniform magnetic field $\vec{B} = 1.35\text{ T}\,\hat{j}$. Find the magnetic force per unit length on the wire.

71 • An alpha particle (charge $+2e$) travels in a circular path of radius 0.50 m in a region with a magnetic field whose magnitude is 0.10 T. Find (*a*) the period, (*b*) the speed, and (*c*) the kinetic energy (in electron volts) of the alpha particle. (The mass of an alpha particle is 6.65×10^{-27} kg.)

72 •• The pole strength q_m of a bar magnet is defined by $\vec{\mu} = q_m\vec{\ell}$, where $\vec{\mu}$ is the magnetic moment of the magnet and $\vec{\ell}$ is the position of the north-pole end of the magnet relative to the south-pole end. Show that the torque exerted on a bar magnet in a uniform magnetic field $\vec{B}$ is the same as if a force $+q_m\vec{B}$ is exerted on the north pole of the magnet and a force $-q_m\vec{B}$ is exerted on the south pole.

73 •• A particle of mass m and charge q enters a region where there is a uniform magnetic field $\vec{B}$ parallel with the x axis. The initial velocity of the particle is $\vec{v} = v_{0x}\hat{i} + v_{0y}\hat{j}$, so the particle moves in a helix. (*a*) Show that the radius of the helix is $r = mv_{0y}/qB$. (*b*) Show that the particle takes a time $\Delta t = 2\pi m/qB$ to complete each turn of the helix. (*c*) What is the x component of the displacement of the particle during the time given in Part (*b*)? SSM

74 •• A metal cross-bar of mass m rides on a parallel pair of long horizontal conducting rails separated by a distance L and connected to a device that supplies constant current I to the circuit, as shown in Figure 26-42. The circuit is in a region with a uniform magnetic field $\vec{B}$ whose direction is vertically downward. There is no friction and the bar starts from rest at $t = 0$. (*a*) In which direction will the bar start to move? (*b*) Show that at time t the bar has a speed of $(BIL/m)t$.

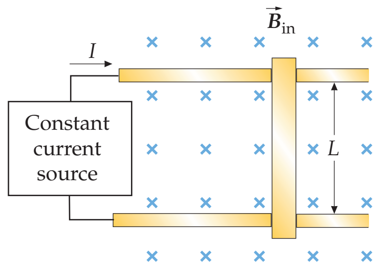

FIGURE 26-42 Problems 74 and 75

75 •• Assume that the rails Problem 74 are frictionless but tilted upward so that they make an angle θ with the horizontal, and with the current source attached to the low end of the rails. The magnetic field is still directed vertically downward. (*a*) What minimum value of B is needed to keep the bar from sliding down the rails? (*b*) What is the acceleration of the bar if B is twice the value found in Part (*a*)? SSM

76 •• A long, narrow bar magnet that has magnetic moment $\vec{\boldsymbol{\mu}}$ parallel to its long axis is suspended at its center as a frictionless compass needle. When placed in region with a horizontal magnetic field $\vec{\boldsymbol{B}}$, the needle lines up with the field. If it is displaced by a small angle θ, show that the needle will oscillate about its equilibrium position with frequency $f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$, where I is the moment of inertia of the needle about the point of suspension.

77 •• A straight 20-m-long conducting wire is parallel to the y axis and is moving in the $+x$ direction with a speed of 20 m/s in a region that has a magnetic field given by 0.50 T $\hat{\boldsymbol{k}}$. (*a*) A magnetic force acting on the conduction electrons leaves one end negatively charged due to an electron surplus and the other end positively charged due to an electron deficit. This charge separation process continues until the electric field due to the accumulated positive and negative charges exerts forces on the remaining conduction electrons that exactly balance the magnetic forces acting on them. Find the magnitude and direction of this electric field in the steady-state situation. (*b*) Which end of the wire is positively charged and which end is negatively charged? (*c*) Suppose the moving wire is 2.0 m long. What is the potential difference between its two ends due to this electric field?

78 ••• A circular loop of wire that has a mass m and a constant current I is in a region with a uniform magnetic field. It is initially in equilibrium and its magnetic moment is aligned with the magnetic field. The loop is given a small angular displacement about an axis through its center and perpendicular to the magnetic field and then released. What is the period of the subsequent motion? (Assume that the only torque exerted on the loop is due to the magnetic field and that there are no other forces acting on the loop.)

79 ••• A small bar magnet has a magnetic moment $\vec{\boldsymbol{\mu}}$ that makes an angle θ with the x axis. The magnet is in a region that has a *nonuniform* magnetic field given by $\vec{\boldsymbol{B}} = B_x(x)\hat{\boldsymbol{i}} + B_y(y)\hat{\boldsymbol{j}}$. Using $F_x = -\partial U/\partial x$, $F_y = -\partial U/\partial y$, and $F_z = -\partial U/\partial z$, show that there is a net magnetic force on the magnet that is given by

$$\vec{\boldsymbol{F}} = \mu_x \frac{\partial B_x}{\partial x}\hat{\boldsymbol{i}} + \mu_y \frac{\partial B_y}{\partial y}\hat{\boldsymbol{j}}$$

80 •• A proton, a deuteron, and an alpha particle all have the same kinetic energy. They are moving in a region with a uniform magnetic field that is perpendicular to each of their velocities. Let R_p, R_d, and R_α be the radii of their circular orbits, respectively. The deuteron has a charge that is equal to the charge a proton has, and an alpha particle has a charge that is equal to twice the charge a proton has. Find the ratios R_d/R_p and R_α/R_p. Assume that $m_\alpha = 2m_d = 4m_p$.

81 ••• **ENGINEERING APPLICATION, CONTEXT-RICH** Your forensic chemistry group, working closely with local law enforcement agencies, has acquired a mass spectrometer similar to that discussed in the text. It employs a uniform magnetic field that has a magnitude of 0.75 T. To calibrate the mass spectrometer, you decide to measure the masses of various carbon isotopes by measuring the position of impact of the various singly ionized carbon ions that have entered the spectrometer with a kinetic energy of 25 keV. A wire chamber with position sensitivity of 0.50 mm is part of the apparatus. What will be the limit on its mass resolution (in kg) for ions in this mass range, that is, those whose mass is on the order of that of a carbon atom?