

Esercizio 4

C.Resto $\lim_{n \rightarrow \infty} (\sqrt{n+3} - \sqrt{n}) \sin \frac{1}{n^2} =$

$$= \lim_{n \rightarrow \infty} \frac{(\sqrt{n+3} - \sqrt{n})(\sqrt{n+3} + \sqrt{n})}{(\sqrt{n+3} + \sqrt{n})} \frac{1}{n^2} =$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n^2(\sqrt{n+3} + \sqrt{n})} = 0$$

La serie puede converger.

C. Comparación

$$\lim_{n \rightarrow \infty} \frac{(\sqrt{n+3} + \sqrt{n}) \sin \frac{1}{n^2}}{\frac{1}{n^{5/2}}} =$$

$$= \lim_{n \rightarrow \infty} \frac{3 n^{5/2}}{n^2(\sqrt{n+3} + \sqrt{n})} = \frac{3}{2} \in (0, +\infty)$$

Como $\frac{1}{n^{5/2}} < +\infty$ ($5/2 > 1$) \Rightarrow

$$\sum (\sqrt{n+3} - \sqrt{n}) \sin \frac{1}{n^2} < +\infty //$$