PRACTICA 6 LOGICA

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Contents

Ejercicio 3: Felapton

$\exists x G x, \forall x (G x \rightarrow \neg F x), \forall x (G x \rightarrow H x) \therefore \exists x (H x \wedge \neg F x)$

1	$\exists x G x$	
2	$\forall x (G x \rightarrow \neg F x)$	
3	$\forall x (G x \rightarrow H x)$	
4	$\quad G\,a$	
5	$\quad G\,a \rightarrow H\,a$	$\forall E\ 3$
6	$\quad H\,a$	$\rightarrow E\ 5, 4$
7	$\quad G\,a \rightarrow \neg F\,a$	$\forall E\ 2$
8	$\quad \neg F\,a$	$\rightarrow E\ 7, 4$
9	$\quad H\,a \wedge \neg Fa$	$\wedge I\ 6, 8$
10	$\quad \exists x (H\,x \wedge \neg Fx)$	$\exists I\ 9$
11	$\exists x (H\,x \wedge \neg Fx)$	$\exists E\ 1, 4\text{--}10$

⊩ NEW LINE	⊩ NEW SUBPROOF

☺ Congratulations! This proof is correct.

Ejercicio 1.6

6. $\vdash \forall x R x x \rightarrow \exists x \exists y R x y$

Construct a proof for the argument: $\therefore \forall x R x x \rightarrow \exists x \exists y R x y$

1	$\quad \forall x R x x$	
2	$\quad R\,aa$	$\forall E\ 1$
3	$\quad \exists y\, R\,ay$	$\exists I\ 2$
4	$\quad \exists x \exists y\, R\,xy$	$\exists I\ 3$
5	$\forall x\, R\,xx \rightarrow \exists x \exists y\, R\,xy$	$\rightarrow I\ 1\text{--}4$

⊩ NEW LINE	⊩ NEW SUBPROOF

☺ Congratulations! This proof is correct.

Ejercicio 1.8

8. $Na \rightarrow \forall x(Mx \leftrightarrow Ma), Ma, \neg Mb \vdash \neg Na$

Construct a proof for the argument: $Na \rightarrow \forall x(Mx \leftrightarrow Ma), Ma, \neg Mb$ $\therefore \neg Na$

1	$Na \rightarrow \forall x(Mx \leftrightarrow Ma)$	
2	Ma	
3	$\neg Mb$	
4	$\quad Na$	
5	$\quad \forall x(Mx \leftrightarrow Ma)$	$\rightarrow$E 1, 4
6	$\quad Mb \leftrightarrow Ma$	$\forall$E 5
7	$\quad Mb$	$\leftrightarrow$E 6, 2
8	$\quad \perp$	$\neg$E 7, 3
9	$\neg Na$	$\neg$I 4–8

[NEW LINE]   [NEW SUBPROOF]

☺ Congratulations! This proof is correct.

Ejercicio 3 Camestrop:

Camestrop: $\exists x Hx, \forall x(Fx \rightarrow Gx), \forall x(Hx \rightarrow \neg Gx) \therefore \exists x(Hx \wedge \neg Fx)$

Construct a proof for the argument: $\exists x Hx, \forall x(Fx \rightarrow Gx), \forall x(Hx \rightarrow \neg Gx) \therefore \exists x(Hx \wedge \neg Fx)$

1	$\exists x Hx$	
2	$\forall x(Fx \rightarrow Gx)$	
3	$\forall x(Hx \rightarrow \neg Gx)$	
4	Ha	
5	$Ha \rightarrow \neg Ga$	$\forall$E 3
6	$\neg Ga$	$\rightarrow$E 5, 4
7	$Fa \rightarrow Ga$	$\forall$E 2
8	$\neg Fa$	MT 7, 6
9	$Ha \wedge \neg Fa$	$\wedge$I 4, 8
10	$\exists x(Hx \wedge \neg Fx)$	$\exists$I 9
11	$\exists x(Hx \wedge \neg Fx)$	$\exists$E 1, 4–10

[NEW LINE] [NEW SUBPROOF]

☺ Congratulations! This proof is correct.

Ejercicio 3 Ferio:

Ferio: $\forall x(Gx \rightarrow \neg Fx), \exists x(Hx \wedge Gx) \vdash \exists x(Hx \wedge \neg Fx)$

Construct a proof for the argument: $\forall x(Gx \rightarrow \neg Fx), \exists x(Hx \wedge Gx) \therefore \exists x(Hx \wedge \neg Fx)$

1	$\forall x(Gx \rightarrow \neg Fx)$	
2	$\exists x(Hx \wedge Gx)$	
3	$Ha \wedge Ga$	
4	Ha	$\wedge E\ 3$
5	Ga	$\wedge E\ 3$
6	$Ga \rightarrow \neg Fa$	$\forall E\ 1$
7	$\neg Fa$	$\rightarrow E\ 6, 5$
8	$Ha \wedge \neg Fa$	$\wedge I\ 4, 7$
9	$\exists x\ (Hx \wedge \neg Fx)$	$\exists I\ 8$
10	$\exists x\ (Hx \wedge \neg Fx)$	$\exists E\ 2, 3\text{--}9$

⊩ NEW LINE	⊩ NEW SUBPROOF

☺ Congratulations! This proof is correct.

Ejercicio 1.9

9. $\forall x \forall y(Gxy \rightarrow Gyx) \vdash \forall x \forall y(Gxy \leftrightarrow Gyx)$

Construct a proof for the argument: $\forall x \forall y(Gxy \rightarrow Gyx)$ ∴
$\forall x \forall y(Gxy \leftrightarrow Gyx)$

1	$\forall x \forall y(Gxy \rightarrow Gyx)$	
2	Gab	
3	$\forall y(Gay \rightarrow Gya)$	$\forall$E 1
4	$Gab \rightarrow Gba$	$\forall$E 3
5	Gba	$\rightarrow$E 4, 2
6	Gba	
7	$\forall y(Gby \rightarrow Gyb)$	$\forall$E 1
8	$Gba \rightarrow Gab$	$\forall$E 7
9	Gab	$\rightarrow$E 8, 6
10	$Gab \leftrightarrow Gba$	$\leftrightarrow$I 2–5, 6–9
11	$\forall y(Gay \leftrightarrow Gya)$	$\forall$I 10
12	$\forall x \forall y(Gxy \leftrightarrow Gyx)$	$\forall$I 11

⊩+ NEW LINE	⊩+ NEW SUBPROOF

☺ Congratulations! This proof is correct.