

$$4 a) \lim_{n \rightarrow \infty} \frac{\log 2 + \log 3 + \dots + \log n}{n \log n} \quad \frac{\infty}{\infty}$$

$$= \lim_{n \rightarrow \infty} \frac{\log n}{n \log n - (n-1) \log(n-1)} = \lim_{n \rightarrow \infty} \frac{\log n}{\log \frac{n^n}{(n-1)^{n-1}}} = (*)$$

Etudions a parte:

$$\lim_{n \rightarrow \infty} \frac{n^n}{(n-1)^{n-1}} = \lim_{n \rightarrow \infty} n \left( \frac{n}{n-1} \right)^{n-1} =$$

$$= \lim_{n \rightarrow \infty} n \left( 1 + \frac{n - (n-1)}{n-1} \right)^{n-1} = \lim_{n \rightarrow \infty} n e$$

$$(*) = \lim_{n \rightarrow \infty} \frac{\log n}{\log n e} = 1$$