

$$r) \lim_{n \rightarrow \infty} \frac{2^{2n} (n!)^2 \sqrt{n}}{(2n+1)!} = (*)$$

$$n! \sim n^n \cdot e^{-n} \sqrt{2\pi n}$$

$$(2n+1)! \sim (2n+1)^{2n+1} e^{-(2n+1)} \sqrt{2\pi(2n+1)}$$

$$(*) = \lim_{n \rightarrow \infty} \frac{2^{2n} n^{2n} \cdot e^{-2n} 2\pi n \sqrt{n}}{(2n+1)^{2n+1} e^{-2n-1} \sqrt{2\pi(2n+1)}} =$$

$$= \lim_{n \rightarrow \infty} \frac{(2n)^{2n}}{(2n+1)^{2n}} \cdot \frac{e \cdot 2\pi n \sqrt{n}}{(2n+1) \sqrt{2\pi(2n+1)}} =$$

$$\downarrow$$

$$\frac{2\pi e}{4\sqrt{\pi}} = \frac{\sqrt{\pi} e}{2}$$

$$= \frac{\sqrt{\pi} e}{2} \lim_{n \rightarrow \infty} \left(\frac{2n}{2n+1} \right)^{2n} = \frac{\sqrt{\pi} e}{2} \lim_{n \rightarrow \infty} \left(1 + \frac{2n-2n-1}{2n+1} \right)^{2n} =$$

$$= \frac{\sqrt{\pi} e}{2} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{-(2n+1)} \right)^{2n} = \frac{\sqrt{\pi} e}{2} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{-(2n+1)} \right)^{-(2n+1) \frac{2n}{-(2n+1)}} =$$

$$= \frac{\sqrt{\pi} e}{2} \lim_{n \rightarrow \infty} e^{\frac{-2n}{2n+1}} = \frac{\sqrt{\pi} e}{2} \cdot e^{-1} = \frac{\sqrt{\pi}}{2}$$