

Global

$$S \subseteq \mathbb{R}^3$$

$$T_p S \subseteq \mathbb{R}^3$$

$$N: S \rightarrow \mathbb{R}^3$$

$$dN_p: T_p S \rightarrow T_p S$$

$$I_p: T_p S \times T_p S \rightarrow \mathbb{R}$$

$$I_p = \langle \cdot, \cdot \rangle$$

$$\mathbb{I}_p: T_p S \times T_p S \rightarrow \mathbb{R}$$

$$\mathbb{I}_p = - \langle dN_p(\cdot), \cdot \rangle$$

$$K(p) = \det(dN_p)$$

$$H(p) = -\frac{1}{2} \operatorname{tr}(dN_p)$$

Local

$$\varphi: \mathbb{R}^2 \rightarrow S$$

$$T_{\varphi(u,v)} S \simeq \operatorname{span} \underbrace{\{\varphi_u, \varphi_v\}}_{\text{basis}}$$

$$N \circ \varphi: \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$(N \circ \varphi)(u,v) := \frac{(\varphi_u \wedge \varphi_v)(u,v)}{\|\varphi_u \wedge \varphi_v\|}$$

$$A(u,v): \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$M(u,v) = \begin{pmatrix} E(u,v) & F(u,v) \\ F(u,v) & G(u,v) \end{pmatrix}$$

$$\Rightarrow E(u,v) = \langle \varphi_u(u,v), \varphi_u(u,v) \rangle = \|\varphi_u(u,v)\|^2$$

$$F(u,v) = \langle \varphi_u(u,v), \varphi_v(u,v) \rangle$$

$$G(u,v) = \langle \varphi_v(u,v), \varphi_v(u,v) \rangle = \|\varphi_v(u,v)\|^2$$

$$\Sigma(u,v) = \begin{pmatrix} e(u,v) & f(u,v) \\ f(u,v) & g(u,v) \end{pmatrix}$$

$$\Rightarrow e(u,v) = \langle (N \circ \varphi)(u,v), \varphi_{uu}(u,v) \rangle$$

$$f(u,v) = \langle (N \circ \varphi)(u,v), \varphi_{uv}(u,v) \rangle$$

$$g(u,v) = \langle (N \circ \varphi)(u,v), \varphi_{vv}(u,v) \rangle$$

$$\Rightarrow A = \frac{-1}{EG - F^2} \begin{pmatrix} G & -F \\ -F & E \end{pmatrix} \begin{pmatrix} e & f \\ f & g \end{pmatrix}$$

$$(K \circ \varphi)(u,v) = \det A(u,v)$$

$$= \frac{eg - f^2}{EG - F^2}$$

$$(H \circ \varphi)(u,v) = -\frac{1}{2} \operatorname{tr}(A)$$

$$= \frac{1}{2} \frac{eG + gE - 2fF}{EG - F^2}$$