

Funciones-II-1-bach.pdf



paula5



Matemáticas Aplicadas a las Ciencias Sociales I



1º Bachillerato



Estudios España



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grado que quieras sin
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Apertura plazo de
matriculación 3 de julio.



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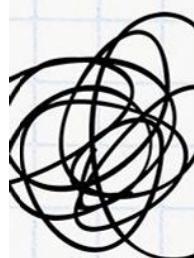
TEMA 5 Funciones II

10/Enero/2022

pierdo
espacio



(2)



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1) Funciones a trozos

$$f(x) = \begin{cases} f_1(x) & x \leq a \\ f_2(x) & a \leq x \leq b \\ f_3(x) & x > b \end{cases}$$

$$\text{ej: } \begin{cases} 2x+1 & x \leq 1 \\ x^2 & x > 1 \end{cases}$$

$$y_1 = 2x+1$$

$$y = x^2$$

x	y
1	3
0	1
-1	-1
-2	-3

x	y
1	1
2	4
3	9
4	16
5	25

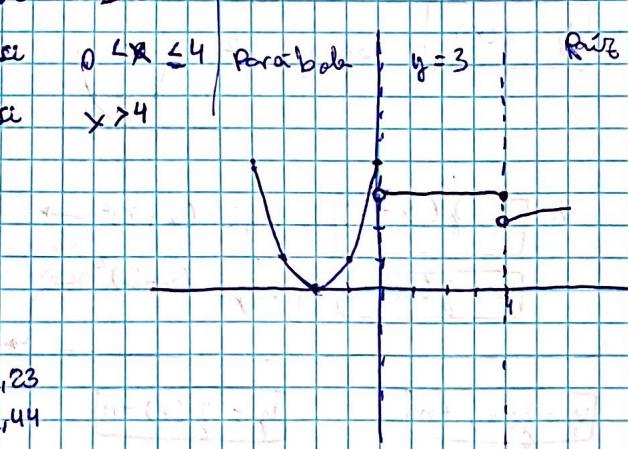
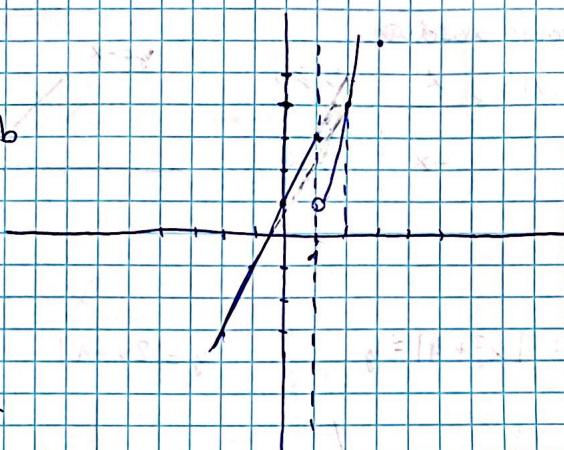
Recta Parábola

$$\text{ej 2: } \begin{cases} x^2 + 4x + 4 & \text{si } x \leq 0 \\ 3 & \text{si } 0 < x \leq 4 \\ \sqrt{x} & \text{si } x > 4 \end{cases}$$

$$x^2 + 4x + 4 = y \quad y = \sqrt{x}$$

x	y
0	4
-1	1
-2	0
-3	1
-4	4

x	y
4	2
5	$\sqrt{5} = 2,23$
6	$\sqrt{6} = 2,44$



$$V(x_V, y_V) \quad \left\{ \begin{array}{l} x_V = \frac{-b}{2a} = \frac{-4}{2 \cdot 1} = -2 \\ y_V = (-2)^2 + 4(-2) + 4 = 0 \end{array} \right. \quad (-2, 0)$$

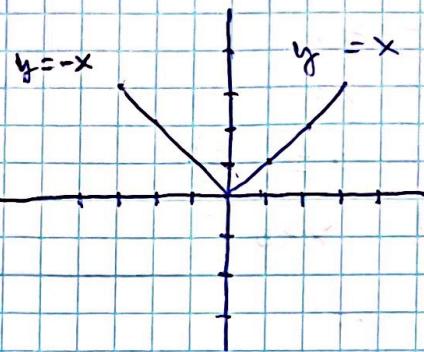
$$\text{PC} \quad \left\{ \begin{array}{l} \text{eje } x : y = 0 \rightarrow x^2 + 4x + 4 = 0 \rightarrow x = -2 \quad (-2, 0) \\ \text{eje } y : x = 0 \rightarrow y = 0^2 + 4 \cdot 0 + 4 = 4 \quad (0, 4) \end{array} \right.$$

wuolah

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1.1) Valor absoluto

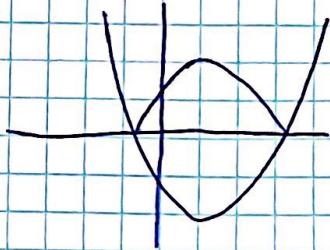
$$\text{Ej: } y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



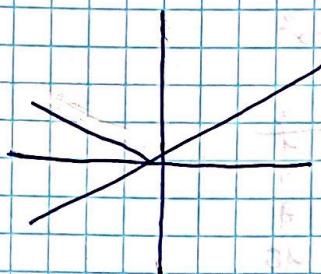
$y = x$
x
0
1
2

$y = -x$
x
0
-1
-2

$$\text{Ej: } |x^2 + 4| = y$$



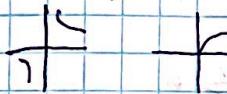
$$y = |2x + 1|$$



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2) Transformaciones de funciones

2.1) Translaciones



$$y = f(x)$$

$$\text{ej: } y = x^2$$



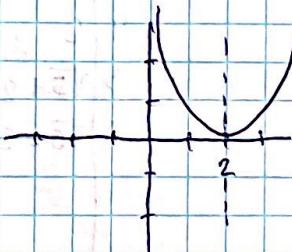
$$* \boxed{y = f(x+n)}$$

$$\text{ej: } y = (x+2)^2$$



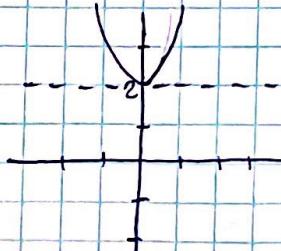
$$\boxed{y = g(x-n)}$$

$$\text{ej: } y = (x-2)^2$$



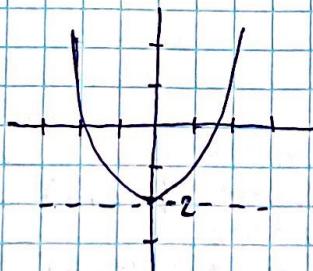
$$\boxed{y = f(x) + n}$$

$$\text{ej: } y = x^2 + 2$$



$$\boxed{y = f(x) - n}$$

$$\text{ej: } y = x^2 - 2$$



D
D

D

D

2.2) Estiramientos y contracciones

$$y = f(x)$$

ej: $y = x^2$

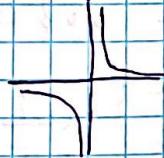


$$y = k \cdot f(x)$$

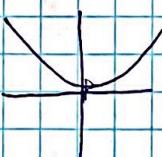
$\hookrightarrow |k| > 1$



$$y = 4x^2$$



$\hookrightarrow 0 < |k| < 1$



$$y = \frac{1}{4}x^2$$



2.3) Simetrías

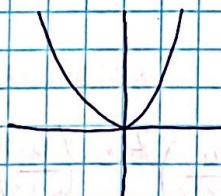
Par

$$y = f(x) \rightarrow y = f(-x) \rightarrow [f(x) = f(-x)]$$

Impar

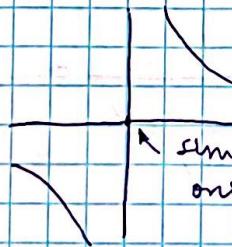
$$y = f(x) \rightarrow y = f(-x) = -f(x) \rightarrow [f(x) = -f(-x)]$$

Ej: $y = x^2 \xrightarrow{x \mapsto -x} y = (-x)^2 = x^2$ simetría par



→ eje y = eje de simetría

Ej: $y = \frac{1}{x} \xrightarrow{x \mapsto -x} y = -\frac{1}{x} = \frac{1}{-x}$



simetría respecto del origen $(0,0)$

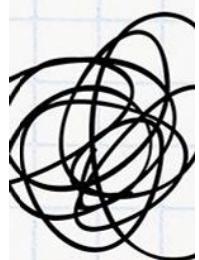
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3) Composición de funciones

Dados dos funciones f, g , la función compuesta de $f \circ g$ es

$g \circ f$ y se lee f compuesta con g

$$x \xrightarrow{g \circ f} g \circ f(x) = g[f(x)]$$

$$\left. \begin{array}{l} f(x) = x+2 \\ g(x) = \sqrt{x} \end{array} \right\} x \xrightarrow{f} f(x) = x+2 \xrightarrow{g} g(x+2) = \sqrt{x+2}$$

$$g \circ f(x) = g(x+2) = \sqrt{x+2}$$

$$\left. \begin{array}{l} f(x) = x^2 \\ g(x) = \sqrt{x} \end{array} \right\} g \circ f(x) = \sqrt{x^2}$$

$$g \circ f(x) = g(f(x)) = \sqrt{x^2} = |x|$$

Normalmente: $g \circ f \neq f \circ g$

$$\text{ej: } f(x) = x^2 - x$$

$$g(x) = \frac{4}{x+1}$$

$$\text{calcular: } f \circ g(x) = f[g(x)] = f\left(\frac{4}{x+1}\right) = \left(\frac{4}{x+1}\right)^2 - \left(\frac{4}{x+1}\right) = \dots$$

$$g \circ f(x) = g[f(x)] = g(x^2 - x) = \frac{4}{(x^2 - x) + 1} = \frac{4}{x^2 - x + 1} = \dots$$

$$f \circ g(x) = f[g(x)] = f\left(\frac{4}{x+1}\right) = \left(\frac{4}{x+1}\right)^2 - \left(\frac{4}{x+1}\right) = \dots$$

$$g \circ g(x) = g[g(x)] = g\left(\frac{4}{x+1}\right) = \frac{4}{\left(\frac{4}{x+1}\right) + 1} = \dots$$

$$g \circ f(x) = \sqrt{x+2}$$

$$g \circ f(2) = \sqrt{2+2} = \sqrt{4} = 2$$

$$g \circ f(-1) = \sqrt{-1+2} = \sqrt{1} = 1$$

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$$f \circ g(x) = \sqrt{x} + 2$$

$$f \circ g(2) = \sqrt{2} + 2 =$$

$$f \circ g(-1) = \sqrt{-1} + 2$$

$$f \circ g(x) = \left(\frac{4}{x+1}\right)^2 - \frac{4}{x+1}$$

$$f \circ g(2) = \left(\frac{4}{2+1}\right)^2 - \frac{4}{2+1} = \left(\frac{4}{3}\right)^2 - \frac{4}{3} = \frac{16}{9} - \frac{4}{3} = \frac{16}{9} - \frac{12}{9} = \frac{4}{9}$$

$$f \circ g(-1) = \left(\frac{4}{-1+1}\right)^2 - \frac{4}{-1+1} = \frac{8}{0} - \frac{4}{0} = \text{D}$$

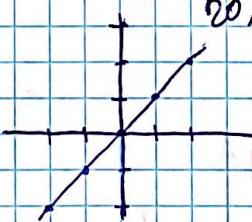
$$g \circ f(x) = \frac{4}{x^2 - x + 1}$$

$$g \circ f(2) = \frac{4}{2^2 - 2 + 1} = \frac{4}{3}$$

$$g \circ f(-1) = \frac{4}{(-1)^2 + 1 + 1} = \frac{4}{3}$$

4) Función inversa

Función identidad $f(x) = x$
 $y = x$



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Se llama función inversa de f a la función f^{-1} que cumple
 $f(a) = b \rightarrow f^{-1}(b) = a$

Además se cumplen las siguientes propiedades

$$f^{-1} \circ f(x) = x$$

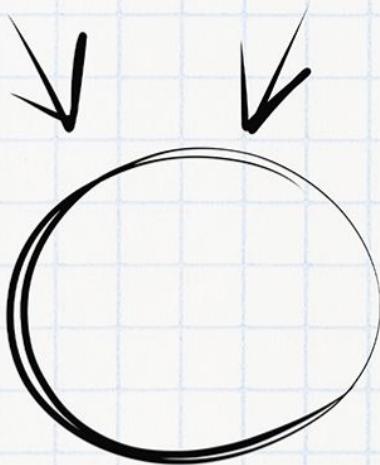
$$f \circ f^{-1}(x) = x$$

Imagínate aprobando el examen

Necesitas tiempo y concentración

Planes	PLAN TURBO	PLAN PRO	PLAN PRO+
diamond Descargas sin publi al mes	10 🟡	40 🟡	80 🟡
clock Elimina el video entre descargas	✓	✓	✓
folder Descarga carpetas	✗	✓	✓
download Descarga archivos grandes	✗	✓	✓
circle Visualiza apuntes online sin publi	✗	✓	✓
glasses Elimina toda la publi web	✗	✗	✓
€ Precios	Anual <input type="checkbox"/>	0,99 € / mes	3,99 € / mes
			7,99 € / mes

Ahora que puedes conseguirlo,
¿Qué nota vas a sacar?



WUOLAH

$$\text{Ej 1: } f(x) = \sqrt{x}$$

$$g(x) = x^2$$

Comprobar si $g = f^{-1}$

$$f \circ g(x) = f(g(x)) = f(x^2) = \sqrt{x^2} = x$$

$$g \circ f(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x$$

g es la inversa de f

$$\text{Ej 2: } f(x) = x^3 - 6$$

Comprobar Si $g = f^{-1}$

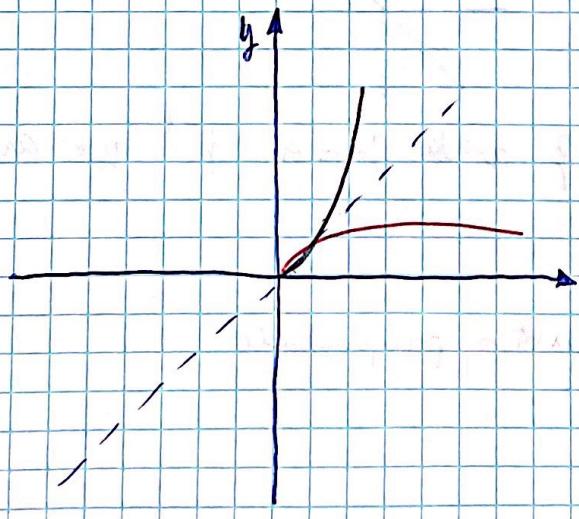
$$g(x) = \sqrt[3]{x+6}$$

$$f \circ g(x) = f(g(x)) = f(\sqrt[3]{x+6}) = (\sqrt[3]{x+6})^3 - 6 = x+6-6 = x$$

$$g \circ f(x) = g(f(x)) = g(x^3 - 6) = \sqrt[3]{(x^3 - 6)} =$$

4.1) Gráficamente

24/Enero/2022



$$f(x) = x^2, \text{ si } x \geq 0$$

$$f^{-1}(x) = \sqrt{x}$$

Al representarlas gráficamente, son simétricas respecto al eje $y = x$

Importante

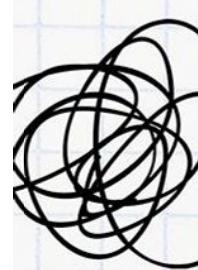
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1.2) Calcular función inversa

$$\text{Ej: } f(x) = 2x - 3$$

Calcular f^{-1} :

$$y = 2x - 3 \rightarrow x = 2y - 3 \rightarrow x + 3 = 2y \rightarrow y = \frac{x+3}{2}$$

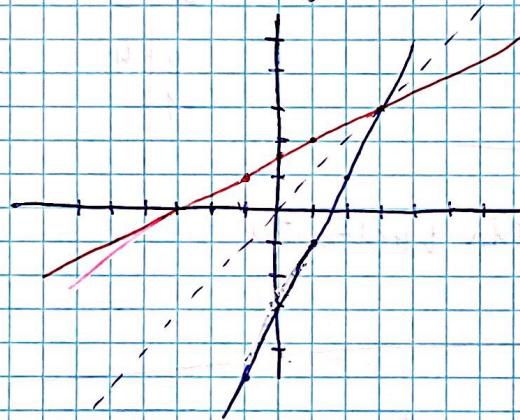
Cambiar

$$x \leftrightarrow y$$

despejar

$$y$$

$$f^{-1}(x) = \frac{x+3}{2}$$



x	y = 2x - 3
-1	-5
0	-3
1	-1

x	y = (x+3)/2
-1	1
0	-3/2
1	2

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$$\text{Ej 1: } y = 2x - 3$$

$$\text{Ej 2: } y = x^2 + 3x$$

Calcular función inversa: $x \leftrightarrow y$

$$x = y^2 + 3y$$

$$x = y^2 + 3y$$

$$y^2 + 3y - x = 0$$

a

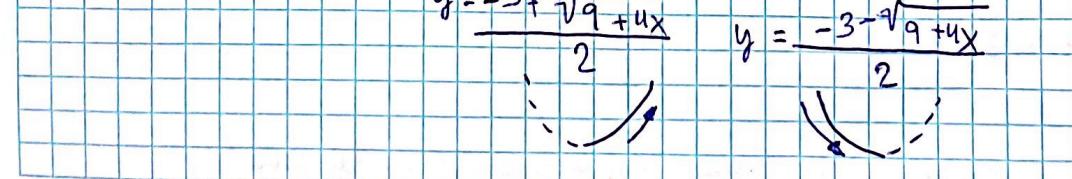
b

c

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-x)}}{2 \cdot 1} = \frac{-3 \pm \sqrt{9 + 4x}}{2}$$

$$y = \frac{-3 + \sqrt{9 + 4x}}{2}$$

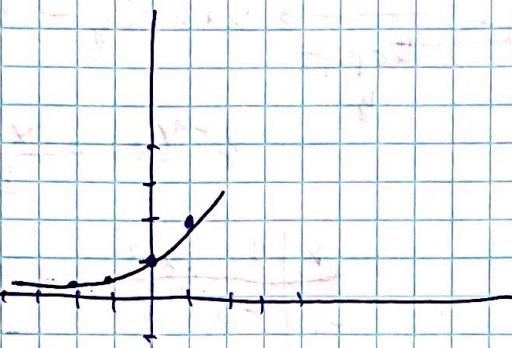
$$y = \frac{-3 - \sqrt{9 + 4x}}{2}$$



5) Funciones exponenciales

$$y = a^x, \quad a = \text{número real positivo} (a \neq 1)$$

Passan siempre por $(0, 1)$, $(1/a)$



$$\text{Eq: } y = 2^x$$

x	y
0	1
1	2
2	4
-1	

$a > 1$ crece

$0 < a < 1$ decrece

Hay una asíntota horizontal

$$\text{Eq: } y = \left(\frac{1}{2}\right)^x$$

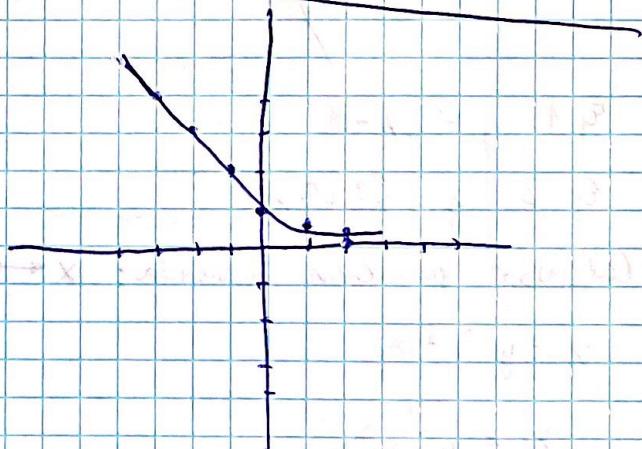
$$-2 \quad \left(\frac{1}{2}\right)^{-2} = -2$$

$$-1 \quad \left(\frac{1}{2}\right)^{-1} = 2$$

$$0 \quad \left(\frac{1}{2}\right)^0 = 1$$

$$1 \quad \left(\frac{1}{2}\right)^1 = 0,5$$

$$2 \quad \left(\frac{1}{2}\right)^2 = 0,25$$



$$\text{Eq: } y = e^x$$

$$x \mid y$$

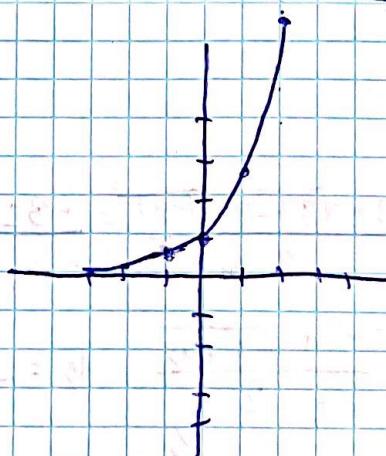
$$-2 \quad 0,135$$

$$-1 \quad 0,368$$

$$0 \quad 1$$

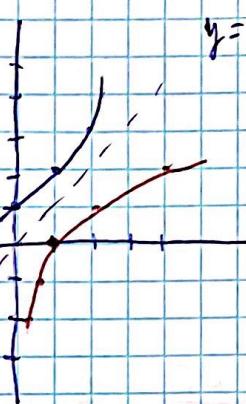
$$1 \quad e = 2,718$$

$$2 \quad 7,39$$



¿Dónde aparecen funciones exponenciales en la vida?

6) Función logarítmica



$y = 2^x$ calendar inversa

$$x = 2^y, 2^y = x$$

Def $\log_a x = y \Leftrightarrow a^y = x$

$$\log_2 x = y$$

$$y = \log_2 x$$

$$\begin{array}{|c|c|} \hline x & y \\ \hline \end{array}$$

$$-1 \quad \log_2(-1) = ?$$

$$0 \quad \log_2 0 = ?$$

$$1 \quad \log_2 1 = 0$$

$$2 \quad \log_2 2 = 1$$

$$4 \quad 2$$

Propiedades

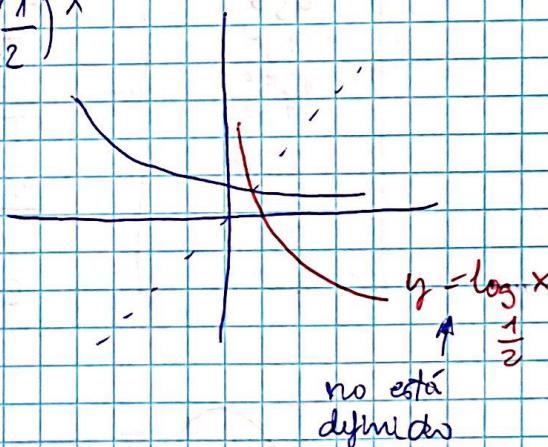
- Pasa por los puntos $(1, 0)$ y $(a, 1)$

- $a > 1 \rightarrow f$ crece



- $a < 1 \rightarrow f$ no está definida

$$y = \left(\frac{1}{2}\right)^x$$



Importante

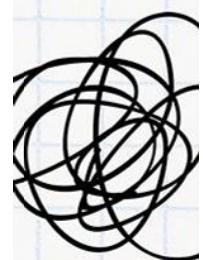
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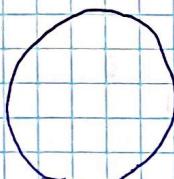
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7) Funciones trigonométricas

7.1) Pasar de grados a radianes



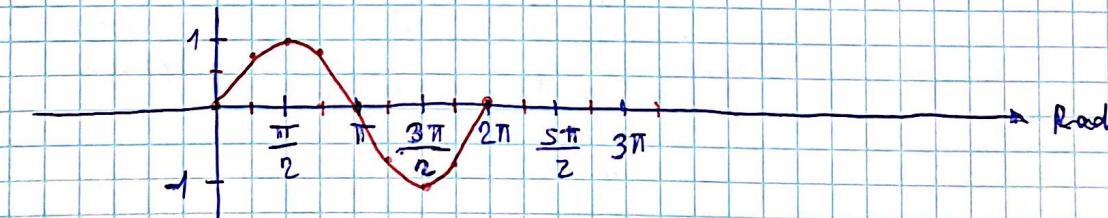
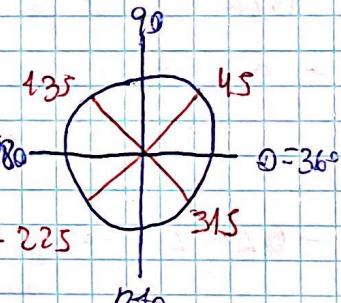
$$360^\circ = 2\pi \text{ rad}$$

$$30^\circ = x \text{ rad}$$

$$x = \frac{30 \cdot 2\pi}{360} = \frac{60\pi}{360} = \frac{6\pi}{36} = \frac{\pi}{6} \text{ radianes}$$

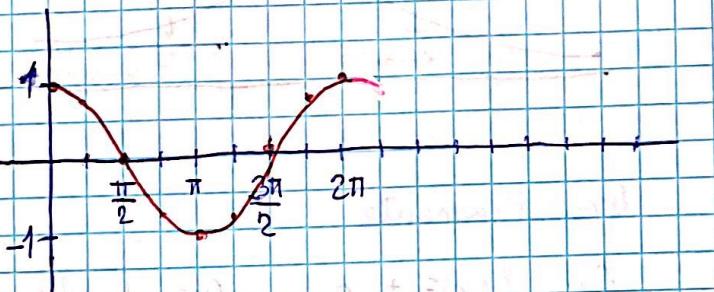
7.2) Seno

Grados	x	y	seno
	Rad		
0°	0	1	1
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} = 0.7$
90°	$\frac{\pi}{2}$	1	1
135°	$\frac{3\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2} = -0.7$
180°	π	-1	-1
225°	$\frac{5\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2} = -0.7$
270°	$\frac{3\pi}{2}$	-1	-1
315°	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} = 0.7$
360°	2π	1	1



7.3) Coseno

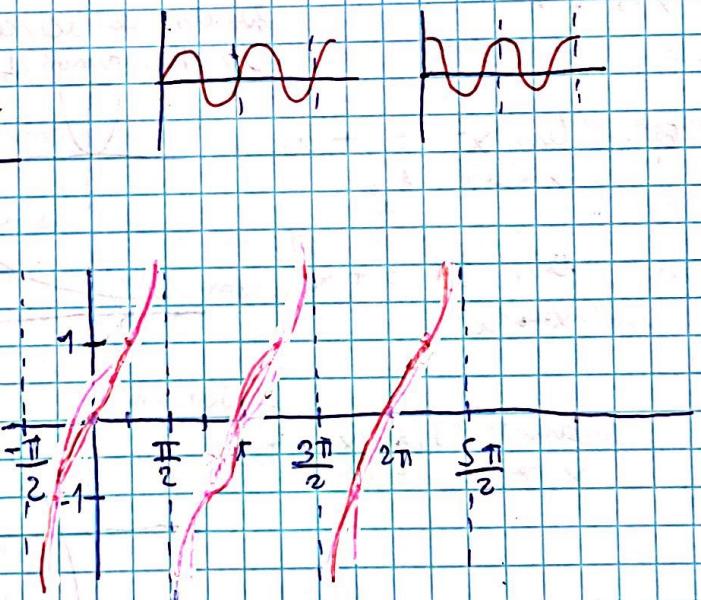
Grado	Rad	Coseno
0°	0	1
45°	$\pi/4$	$0,7$
90°	$\pi/2$	0
135°	$3\pi/4$	-0,7
180°	π	-1
225°	$5\pi/4$	-0,7
270°	$3\pi/2$	0
315°	$7\pi/4$	0,7
360°	2π	1



Las funciones seno y coseno son periódicas (su gráfica se repite cada cierto tiempo) de periodo $T = 2\pi$

7.4) Tangente

Grado	Rad	Sen	Tan
0°	0	0	0
45°	$\pi/4$	1	-1
90°	$\pi/2$	A	A
135°	$3\pi/4$	-1	1
180°	π	0	0
225°	$5\pi/4$	1	-1
270°	$3\pi/2$	#	#
315°	$7\pi/4$	-1	1
360°	2π	0	0



la tangente es periódica de periodo $T = \pi$