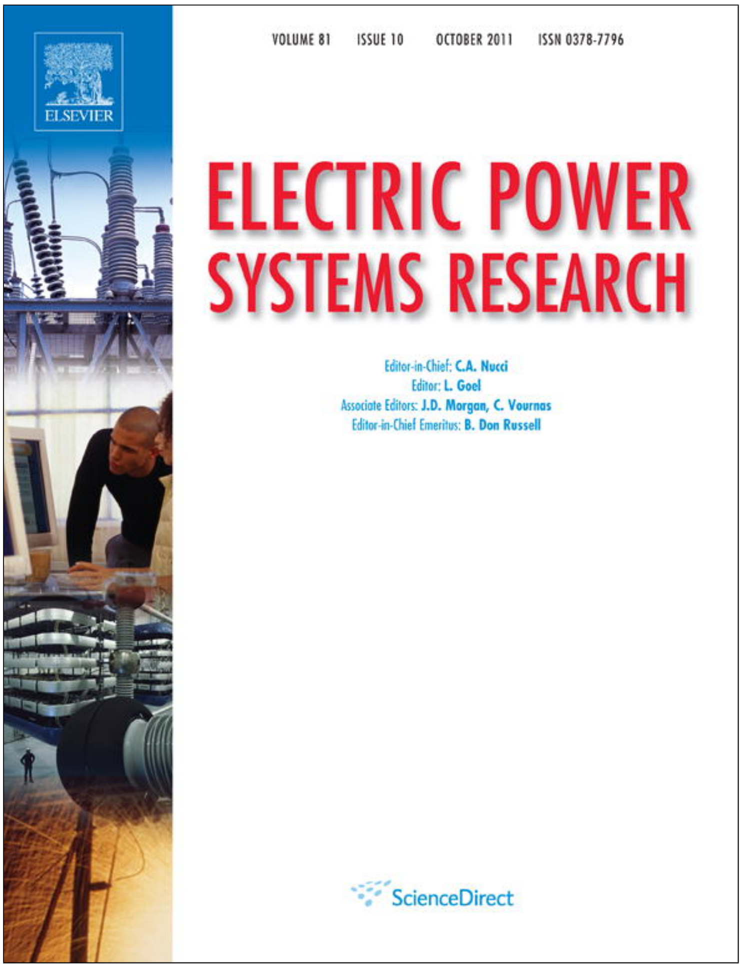
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The effect of wind generation and weekday on Spanish electricity spot price forecasting

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ABSTRACT

This paper empirically compares the predictive accuracy of a set of methods for day-ahead spot price forecasting in the Spanish electricity market. The methods come from time series analysis and artificial intelligence disciplines, and include univariate, multivariate, linear and nonlinear. Within the univariate methods, the double seasonal ARIMA and the recently proposed exponential smoothing for double seasonality are compared and used as benchmarks. They allow us to quantify the improvement on price forecasting when including explanatory variables or using more complex models. Dynamic regression models including the electricity load forecast are then considered. Their good performance in price forecasting has been pointed out by many authors. However, we find evidences of their predictive accuracy can be significantly outperformed by accounting the wind generation forecast provided by the System Operator. Moreover, these forecasts can be even more accurate if changes of price's behavior according with the day of the week are taken into account by means of periodic models. The last of the tested methods are feed-forward neural networks used as multivariate nonlinear regression methods with universal function approximation capabilities. The influence of the wind generation forecast on price prediction is also proved with this approach. Detailed out-of-sample results of the tested methods are given.

1. Introduction

In Spain, the electricity business is organized as a sequence of markets. Generating companies and wholesale purchasers are firstly summoned at the day-ahead spot market to send their bids consisting of pairs of energy-price for the 24 h of the next day. The Market Operator (MO) is in charge of clearing this market and providing the provisional energy schedule of each bidding unit for the 24 h of the next day. Hourly marginal prices are obtained at the intersection of supply and demand curves. The major part of the total energy is traded in the daily spot market, but subsequent short-term market mechanisms (intraday markets, ancillary reserves, and real time markets) are available in order to guarantee the final balance between power generation and consumer demand [1].

Due to this competitive framework, market agents are forced to follow the long-term, medium-term and short-term spot market price movements in order to efficiently operate in the market. In particular, short-term price predictions are a key factor, both for generators when considering adjustments to their pool bidding strategies, and for buyers in their attempts to maximize their utilities or protect themselves against high prices in the pool [2].

Many different short-term forecasting techniques based on time series analysis and artificial intelligence methods have been proposed in the literature to deal with electricity price prediction (see Section 2). In this paper, some of these techniques have been evaluated in order to compare their predictive accuracy. The methods are shown to the reader in a progressive way as the complexity of the model and the number of variables increase. Firstly, two univariate methods are evaluated: the well-known double seasonal ARIMA model [3,4], and the recently proposed exponential smoothing method for double seasonality [5], which has been successfully applied for short-term load forecasting [6]. These two methods provide suitable validation results to be compared with the results of the rest of the methods and so, to enable us to check whether, in terms of predictive accuracy, the inclusion of explanatory variables or the use of more complex models is really justified. Furthermore, comparing both these methods allows us to check how well the exponential smoothing method with double seasonality performs as it has never been used before in short-term electricity price forecasting.

As far as multivariate methods are concerned, two of the evaluated approaches are very frequently used in the literature of short-term electricity price prediction: linear dynamic regression models (in general, linear transfer functions models) and feed-

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forward neural networks. On the one hand, transfer functions models have been pointed out as very accurate methods to predict electricity spot prices in different electricity markets [7–10]. On the other hand, feed-forward neural networks, specially multilayer perceptrons (MLP), have been extensively used to short-term price prediction [11–13]. The improvement in accuracy when considering the wind generation forecast provided by the System Operator is proven in both cases.

The last evaluated method is a periodic dynamic regression model [14] whose parameters switch according to the day of the week. This method arises after observing intra-week differences in the fitting error of its non periodic counterpart. To the best of our knowledge, only a model taking into account these effects has already been reported in [15]. However, daily (instead of hourly) European market prices time series were considered and out-of-sample results were not investigated.

The paper shows the detailed out-of-sample results of all the tested methods. The validation is made in the same way as the market clearing price occurs. Thus, at a set time on day D, the market price values are predicted for each hour of day $D + 1$. A summary and are finally presented. The periodic dynamic regression models have an equivalent formulation in state space form which facilitates the fitting and forecasting process. Appendix A offers a wide explanation of it.

2. State of the art

With the worldwide liberalization of electricity markets, electricity price prediction has become a crucial task in the operational activity of market agents and a new challenge for many researchers. A great number of different modeling approaches have been applied since liberalization, both with respect to the field of knowledge of the applied techniques and in relation to the planning horizon. This section does not attempt to make a thorough presentation of the known techniques used in price prediction, but rather provides a general classification to place the methods herein explained and to show relevant related works. Readers interested in a detailed taxonomy can turn to previous works in [9,16,17].

The different approaches to price prediction in electricity markets can be roughly classified, according to their final use, in (1) models for long and medium term decisions, (2) models for derivative pricing and (3) models for short term decision making. Production cost models, equilibrium analysis based on game theory and model-based approaches are preferred when medium and long term decisions, like planning or investment, are required. Relevant references to these models can be found in [18–20]. Financial modeling is frequently used for valuing derivatives. These models focus on trying to capture the main features of electricity prices, as mean reversion or spikes. Jump-diffusion models are representative within this category [21].

The wide number of techniques for short-term (or day-ahead) price forecasting can be in turn classified in two different groups: (1) models coming from time series analysis, which constitute the parametric approach, and (2) artificial intelligence or non-parametric approaches. ARIMA, Dynamic Regression and Transfer Function are time series models that have been widely used in electricity price forecasting, and have proved to be very accurate for the short-term. In these models, predictions are obtained as linear combinations of past and present values of actual and predicted prices, and, if available, of other exogenous variables. They are able to deal with an important characteristic of market prices: the daily and hourly cycles. Moreover, these models have been subject of a lot of research and a concrete methodology is available as a guide to the fitting, checking and model selection processes [22,23]. Relevant references, among the others, are [2,3], where these models are applied to markets in mainland Spain and California.

Multiple-Input Multiple-Output (MIMO) models are an alternative way to forecast the 24 hourly prices of the next day. VARMAX (Vector Auto Regressive Moving Average with Exogenous variables) models are a linear approach suitable for this purpose. These models treat prices as daily observations consisting in 24-component vectors. Each hour in the daily price vector can be differently explained through daily vectors of delayed prices and explanatory variables. Moreover, the 24-dimensional daily price vector is predicted one day-ahead in a single step. Note the difference when considering the hourly approach, in which the 24 h of the next day are predicted in a sequential manner from hour 1 to 24, and therefore the prediction uncertainty of each hour affects the following ones. Despite these advantages, the great number of handled parameters by these models has a negative effect on their generalization capabilities. Some authors have dealt with this drawback by ignoring the relationships among different hours of the day and model the 24 hourly time series separately, [10,24]. In [25], a VARX (Vector autoregressive with exogenous variables) model specified through a sparse autoregressive coefficient matrix and skew t-distributed disturbance is proposed.

Exponential smoothing methods are a family of univariate methods which have been widely used in short-term forecasting [26]. However, they have not been extensively applied in electricity price forecasting, probably due to the fact that they were not originally designed to deal with double seasonality. A version of the Holt-Winters method able to accommodate this double seasonality was introduced in [5], and successfully applied to short-term load forecasting in [6].

In the past, ARIMA and dynamic regression models have been extended in order to incorporate nonlinearities. This is the case in periodic time series models [14] and threshold autoregressive (TAR) models [27]. In these models, a different set of parameters is applied at each time according to the value of an observable variable. In the case of periodic models, a different set of parameters is adjusted for each season. The only references we have found to the use of these models in price prediction are [15], where periodic models are applied to electricity price forecasting in European markets, and [10], where a comparison between different models including threshold models is presented. Another relevant contribution in this field has been made by smooth transition autoregressive (STAR) models [28], where smooth transitions of parameters are applied instead of discrete switches. However, no references to electricity prices have been found in the literature. All these nonlinear models can be classified into a more general group called regime-switching models, in which Markov models are representative. In these models, the different regimes are not directly determined by an observable variable, but by a set of hidden exclusive states whose probability density functions at time t are determined by a set of exogenous variables and the state probabilities at $t - 1$. Each hidden state is associated to an expert model whose output is weighted by the state probability in order to obtain the overall forecast output. Examples of its application in electricity prices are [29–31]. IOHMM (Input Output Hidden Markov Models) with neural networks as expert models were also used for modeling and forecasting Spanish electricity prices in [1].

One of the distinctive universal features of the electricity price stochastic process is its special underlying distribution. There is a lot of literature pointing out the fact that electricity price distributions are typically heavy-tailed, skewed and exhibit heteroskedasticity (see for example [32,16]). GARCH models (Generalized Autoregressive conditional heteroskedasticity) are an attempt to capture this dynamic volatility as a function of variances of previous time periods. They are especially interesting when volatility forecast is required. However, models with GARCH-type

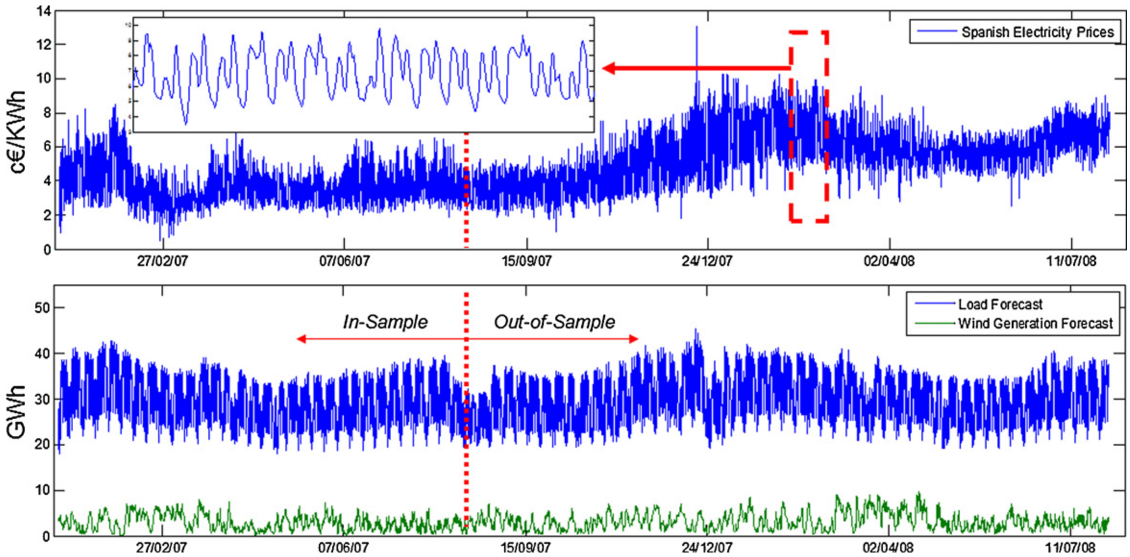

Fig. 1. Spanish day-ahead electricity market prices, and load and wind generation forecasts made by the system operator.

disturbances do not necessarily outperform their homokedastic counterparts in price point forecasting [10,33]. Recent advances trying to find a satisfactory distribution include the work of [34], where a set of non-Gaussian distributions are tested on the price time series of different electricity markets. In [35], a set of semi-parametric models whose density functions are estimated through kernel estimators are proposed.

Artificial neural networks (ANN) are the artificial intelligence-based models that have received most attention in short term electricity load and price forecasting. These non-parametric models are universal function approximators [36] and have been widely applied to model nonlinear dynamic processes. Nevertheless, there is no clear evidence that ANNs outperform parametric methods in price prediction, probably due to their complexity and risk of overfitting. Feed-forward network architectures have been applied to price prediction in [11–13]. In [37], a feed-forward neural network is used to correct the price curve obtained from a similar day approach. But more complex architectures have also been tested to cope with this task. For instance, an Elman recurrent neural network is used in [38]. Fuzzy logic techniques and neural networks are combined in Fuzzy Neural Networks to predict the prices in Spain [39]. Other artificial intelligence techniques have also been applied to electricity prices, as weighted nearest neighbors in [40].

3. The time series data set

The complete dataset used in the comparison consisted of spot prices from the Spanish day-ahead electricity market for the period from Monday, 1st January 2007 to Saturday, 31th July 2008. This time period did not suffer from regulatory changes or external events which could make necessary to refit the involved models. The in-sample period that was selected to fit the models ends on 10th August 2007 and the remainder period, which covers almost a whole year, was used for validation. The in-sample data is therefore comprised of 5328 hourly prices, while the validation period consists of 8544 samples. This large validation data set allows us to test the robustness of the involved models. Hourly electricity load and wind energy production forecasts were used as explanatory variables by the multivariate models. These two time series are predictions made by the System Operator (SO) before competitors send their bids to the market (available at http://www.esios.ree.es/web-publica/). By using these forecasts instead of actual values it is considered the information which market participants actually react to. The three variables are depicted in Fig. 1. Some of the features of electricity prices, as the daily and weekly seasonality, the time varying volatility and the presence of extreme values or spikes, can be noticed. These special features, which make prices erratic and ill-behaved, have already been studied by many authors [15,32].

In contrast to electricity loads, which are commonly used as explanatory variables for short-term electricity price forecasting [2], no references using wind power production forecasts in the Spanish electricity market have been found in the literature before writing this paper. Since the publication in March 2004 of the Royal Decree 436/2004 which established a special regime for wind energy producers, more than 90% of the wind energy in Spain is being sold as any other source of energy in the electricity market (the remainder is remunerated at a fixed price or tariff). This migration from fixed tariff to liberalized market has been stimulated by economic incentives for producers, but has also been accompanied by a significant penalization of energy deviations (differences between produced and programmed energy). In this context, wind energy producers have to predict their hourly resources one day in advance in order to sell their predicted energies in the daily market and the System Operator has to predict the aggregated wind generation in the whole system in order to estimate the effective electricity demand that has to be covered by others resources. In 2008, 11% of the demand in Spain was satisfied by wind generators. The maximum hourly demand in 2008 was attained on December 15th (42,961 MW) and 18% of this demand was fulfilled with wind energy. Therefore, this variable has an outstanding influence in the Spanish day-ahead electricity market clearing process. It determines the percentage of the load that will be covered by wind farms and offered at null price. The reason for this lies in the absence of fuel costs for this technology and a special market regulation that encourages generators to offer their production as price takers, with the result that spot prices are determined by the effective demand that is not covered by wind farms.

4. Forecasting methods

4.1. Notation

The notation used in this section is provided below for easy reference:

q^{-1}: backshift operator ($q^{-1}X_t = X_{t-1}$).
p_t: hourly electricity spot price at time t.
d_t: hourly electricity load forecast at time t provided by the SO.
w_t: hourly wind energy production forecast at time t provided by the SO.
h_t: hour index at time t. $h_t = 1, \ldots, 24$.
S_t: exponential smoothing level component at time t.
D_t: exponential smoothing daily seasonal component at time t.
W_t: exponential smoothing weekly seasonal component at time t.
ε_t: uncorrelated random noise at time t.
$\hat{p}_t(k)$: k-step-ahead spot price forecast made from forecast origin t.
$\alpha, \delta, \omega, \lambda$: exponential smoothing parameters.
a_j: scalar neural network weight of the output neuron coming from the neuron j of the hidden layer (j = 0 for the bias).
b_j: vector neural network weight ($b_{j1}, \ldots, b_{j9}$) of the hidden neuron j.
b_{j0}: scalar neural network weight of the hidden neuron j corresponding to the bias.

4.2. Double seasonal ARIMA model

A well-specified ARIMA model for the in-sample period was identified following the Box-Jenkins methodology [22], and estimated by quasi maximum likelihood.[1] The model is given by (1), where parameters' standard errors appear in brackets (see [45] for further details). Note that the model deals with the double seasonality by including both seasonal autoregressive and moving average terms corresponding to the daily and weekly periods (24 and 168, respectively). Before fitting, log transformation and regular and weekly first differences ($(1-q^{-1})(1-q^{-168})\ln(p_t)$) were applied to induce stationary variance and mean, respectively.

$$(1 - \underset{(.021)}{.825}q^{-1})(1 - \underset{(.027)}{.926}q^{-24})(1-q^{-1})(1-q^{-168})\ln(p_t) = (1 - \underset{(.040)}{.986}q^{-1})(1 - \underset{(.014)}{.769}q^{-24})(1 - \underset{(.011)}{.699}q^{-168})\varepsilon_t \quad (1)$$

The MAPE (mean absolute percentage error) corresponding to the 24-step-ahead forecasts for the whole out-of-sample period is 8.04%, which is better than a naive model whose prediction is the previous week's price, and whose MAPE is 10.88%.

4.3. Exponential smoothing with double seasonality

A description of the identification of exponential smoothing models for double seasonality can be found in [5]. Both, multiplicative and additive exponential smoothing models with no trend were fitted. The level S_0 was initialized as the average of the first 336 observations. The initial values for the within-day seasonal index D_t were set as the ratios between the actual observation and the initial level value, in case of the multiplicative model, and as their differences in case of the additive one. The initial values for the within-week seasonal index W_t were set as the ratios between the actual observation and the initial level value divided by the corresponding value of the within-day seasonal index D_t, in the case of the multiplicative model, and as the differences in the case of the additive one. As the residuals of the tentative models revealed first-order autocorrelation, an autoregressive term AR(1) for the 1-step-ahead errors was added in the same manner as in [5]. The parameters were fitted following a one-stage estimation scheme. Much better fit was obtained with this regular autoregressive term λ, and very similar results were found with the additive and multiplicative versions (see Table 1). As the log transformation applied to the previous ARIMA model involves a multiplicative impact in the price estimation, the multiplicative exponential smoothing with AR term was selected for the study.

Table 1
Double seasonal exponential smoothing models fitted.

	α	δ	ω	λ	Residual sum of squares
Additive (no AR)	0.884	1.000	0.667	–	787.29
Multiplicative (no AR)	0.895	1.000	0.664	–	817.50
Additive (with AR)	0.015	0.217	0.136	0.815	524.88
Multiplicative (with AR)	0.002	0.223	0.132	0.842	527.32

The expression of the model is given by (2),[2] where parameters α, δ, ω and λ must be replaced by the corresponding values in Table 1. In contrast to ARIMA models, Holt-Winters is a decomposition method, and therefore it is possible to separately observe the different components of the model.

$$\begin{aligned} &\text{Level} \quad S_t = \alpha\left(\frac{p_t}{D_{t-24}W_{t-168}}\right) + (1-\alpha)S_{t-1} \\ &\text{Seasonality 1} \quad D_t = \delta\left(\frac{p_t}{S_t W_{t-168}}\right) + (1-\delta)D_{t-24} \\ &\text{Seasonality 2} \quad W_t = \omega\left(\frac{p_t}{S_t D_{t-24}}\right) + (1-\omega)W_{t-168} \\ &\hat{p}_t(k) = S_t D_{t-24+k} W_{t-168+k} + \lambda^k(p_t - (S_{t-1}D_{t-24}W_{t-168})) \end{aligned} \quad (2)$$

Fig. 2 shows the weekly and daily seasonal components for two weeks within the fitting period. The daily component seems to catch properly the average daily profile of the prices, with midday and afternoon peaks, while the weekly component catches the main differences between the days of the week. The 24-step-ahead MAPE for the whole out-of-sample period was a bit better than the ARIMA model: 7.94%, using one less parameter. Further results comparing both univariate methods are provided in Section 5.

4.4. Dynamic regression models

The LTF methodology proposed by Pankratz in [23] was used to identify an appropriate dynamic regression model. In order to evaluate the impact of each explanatory variable, the first fitted models included only the load forecast as explanatory variable. The expression of the model is given by (3).[3] As the log transformation was also applied in the ARIMA model, both residual variances can be compared, and it was observed that the introduction of the electricity load in the model resulted in a decrease of 6.35% in the residual variance. However, the MAPE corresponding to the 24-step-ahead forecast of the out-of-sample period hardly outperformed the ARIMA model and it turned out nearly equal than the exponential smoothing's one: 7.95%.

$$\ln(p_t) = \underset{(.004)}{.519}\ln(d_t) + \frac{(1 - \underset{(.034)}{.972}q^{-1})(1 - \underset{(.015)}{.782}q^{-24})(1 - \underset{(.012)}{.733}q^{-168})}{(1-q^{-1})(1-q^{-168})(1 - \underset{(.020)}{.778}q^{-1})(1 - \underset{(.026)}{.913}q^{-24})}\varepsilon_t \quad (3)$$

The cross correlation function (CCF) of the differenced and log-transformed wind power production forecasts, and the residual of

[1] The quasi maximum likelihood method allows us to make inference about the parameters even if the assumption of normality is violated. All the parameters' standard errors in the paper are calculated by using this approach [45].

[2] Note that the additive version of the exponential smoothing differs from Eq. (2).
[3] Variables p_t and d_t are normalized between the same values.

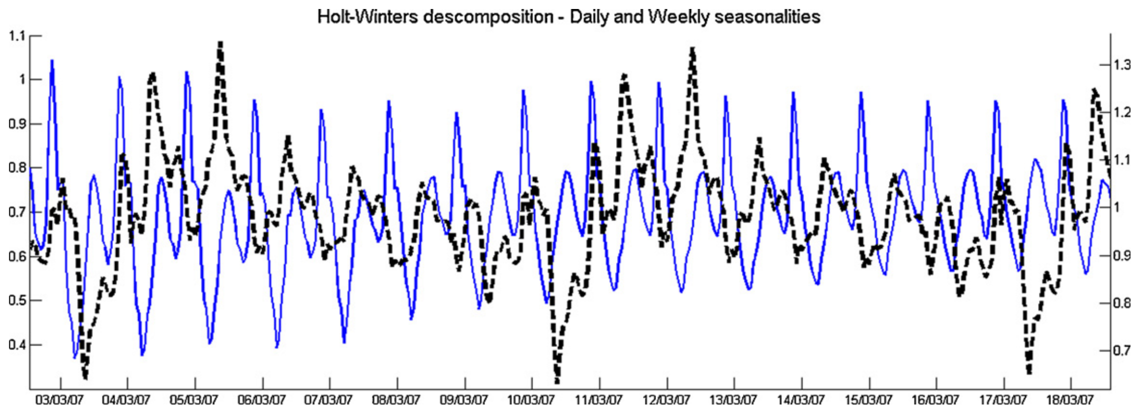

Fig. 2. Daily (solid line) and weekly (dash line) components of the Holt-Winters decomposition.

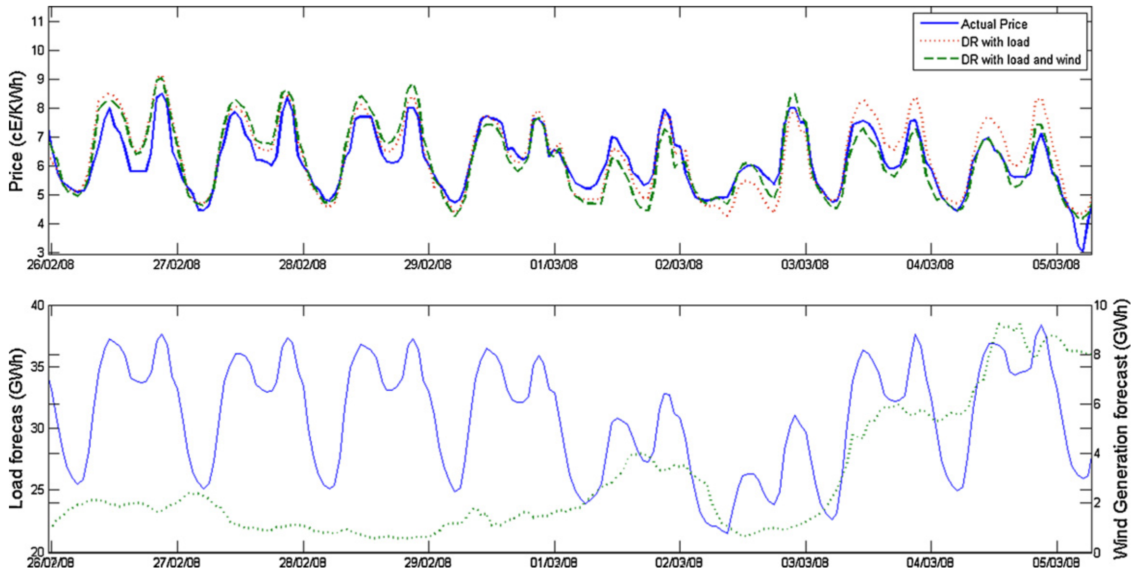

Fig. 3. Actual price and estimation of the dynamic regression models in the upper picture. Load (solid line) and wind generation (dash line) forecasts.

the model in (3) was calculated. The significant values observed in the CCF for the first delays led us to refit the tentative model including the wind power production as explanatory variable. The resulting model is shown in (4).[4] It should be mentioned that the adequacy of the transfer function structure corresponding to the explanatory variables of the two models was validated by calculating the cross correlation function of the residuals and the residuals from well-defined ARIMA models for the input explanatory series (residual cross correlation), and identifying significant delays which could show an improper identification of the corresponding transfer function [23].

$$\ln(p_t) = \underset{(.006)}{.475}\ln(d_t) - \underset{(.015)}{.122}\ln(w_t) + \frac{(1-\underset{(.035)}{.980}q^{-1})(1-\underset{(.015)}{.771}q^{-24})(1-\underset{(.012)}{.726}q^{-168})}{(1-q^{-1})(1-q^{-168})(1-\underset{(.018)}{.747}q^{-1})(1-\underset{(.025)}{.907}q^{-24})}\varepsilon_t \quad (4)$$

The residual variance of this model is 9.23% lower than the residual variance of the ARIMA model. But the relevance of the wind energy production in the model is evidenced when the 24-step-ahead out-of-sample MAPE is calculated: 7.12%. Fig. 3 shows a situation with stable load and changing wind conditions. In this case, the increase of the wind generation will imply a reduction in the spot price, because there is less effective load. Note that, for example at 3rd or 4th of May 2008, the error of the dynamic regression model which does not incorporate this information is higher than that of the alternative model, overestimating the spot price.

4.5. *Periodic dynamic regression model*

It seems more realistic to assume that the underlying price model does not behave in the same way for every day of the week. For example, it would be expected that the autoregressive coefficient affecting to the same hour on the day before would be more relevant to predict a Wednesday than a Sunday [14]. This hypothesis is confirmed when the residual variances of the dynamic regression model for each day of the week are calculated. Table 2 lists the mean of squared errors of the fitted dynamic regression model for each day of the week. Note that the days of the week with the largest variances are Monday, Saturday and Sunday, and they are significantly[5] different to the other days of the week. These different behaviors can be explained in part by the effect of non

[4] Variables *pt*, *dt* and *wt* are normalized between the same values.

[5] The application of the statistical *F*-test identifies significant differences among variances of Saturdays, Sundays, Mondays and the remaining days of the week.

Table 2
Mean of squared errors of the dynamic regression model by day of the week.

	Mean of square errors (DR model) $\times 10^{-2}$
Sunday	0.338
Monday	0.306
Tuesday	0.253
Wednesday	0.219
Thursday	0.240
Friday	0.231
Saturday	0.299

working-days. The price on these days is affected by the prices of the previous day and of the previous week in a different way than the rest of days (case of Saturdays and Sundays), and they also affect the estimation of the following day (case of Mondays).

$$\ln(p_t)=\begin{cases}
\underset{(.020)}{.482}\ln(d_t)-\underset{(.012)}{.092}\ln(w_t)+\dfrac{(1-\underset{(.007)}{.982}q^{-1})(1-\underset{(.022)}{.808}q^{-24})(1-\underset{(.016)}{.879}q^{-168})}{(1-\underset{(.002)}{1.746}q^{-1}+\underset{(.002)}{.747}q^{-2})(1-\underset{(.023)}{1.103}q^{-24}+\underset{(.018)}{.137}q^{-48})(1-\underset{(.011)}{.967}q^{-168}+\underset{(.017)}{.054}q^{-336})}\varepsilon_t, & \text{for Monday}\\
\underset{(.019)}{.504}\ln(d_t)-\underset{(.010)}{.122}\ln(w_t)+\dfrac{(1-\underset{(.004)}{.977}q^{-1})(1-\underset{(.013)}{.822}q^{-24})(1-\underset{(.014)}{.892}q^{-168})}{(1-\underset{(.001)}{1.750}q^{-1}+\underset{(.001)}{.751}q^{-2})(1-\underset{(.017)}{1.053}q^{-24}+\underset{(.014)}{.093}q^{-48})(1-\underset{(.014)}{.896}q^{-168}-\underset{(.006)}{.007}q^{-336})}\varepsilon_t, & \text{for Tuesday to Friday}\\
\underset{(.027)}{.587}\ln(d_t)-\underset{(.014)}{.157}\ln(w_t)+\dfrac{(1-\underset{(.009)}{.980}q^{-1})(1-\underset{(.028)}{.997}q^{-24})(1-\underset{(.027)}{.709}q^{-168})}{(1-\underset{(.004)}{1.743}q^{-1}+\underset{(.004)}{.745}q^{-2})(1-\underset{(.035)}{1.176}q^{-24}+\underset{(.032)}{.065}q^{-48})(1-\underset{(.030)}{.839}q^{-168}-\underset{(.024)}{.071}q^{-336})}\varepsilon_t, & \text{for Saturday}\\
\underset{(.030)}{.582}\ln(d_t)-\underset{(.015)}{.135}\ln(w_t)+\dfrac{(1-\underset{(.008)}{.983}q^{-1})(1-\underset{(.020)}{.910}q^{-24})(1-\underset{(.030)}{.644}q^{-168})}{(1-\underset{(.002)}{1.749}q^{-1}+\underset{(.002)}{.750}q^{-2})(1-\underset{(.028)}{.922}q^{-24}+\underset{(.019)}{.027}q^{-48})(1-\underset{(.027)}{.819}q^{-168}-\underset{(.023)}{.085}q^{-336})}\varepsilon_t, & \text{for Sunday}
\end{cases}\tag{5}$$

$$\ln(p_t)=\begin{cases}
\underset{(.004)}{.580}\ln(d_t)-\underset{(.011)}{.117}\ln(w_t)+\dfrac{(1-\underset{(.032)}{.973}q^{-1})(1-\underset{(.008)}{.810}q^{-24})(1-\underset{(.006)}{.702}q^{-168})}{(1-q^{-1})(1-\underset{(.013)}{.701}q^{-1})(1-\underset{(.016)}{1.023}q^{-24}+\underset{(.017)}{.076}q^{-48})(1-q^{-168})(1+\underset{(.006)}{.103}q^{-168})}\varepsilon_t, & \text{for Monday}\\
\underset{(.007)}{.428}\ln(d_t)-\underset{(.021)}{.115}\ln(w_t)+\dfrac{(1-\underset{(.047)}{.979}q^{-1})(1-\underset{(.013)}{.776}q^{-24})(1-\underset{(.009)}{.736}q^{-168})}{(1-q^{-1})(1-\underset{(.020)}{.732}q^{-1})(1-\underset{(.024)}{1.013}q^{-24}+\underset{(.025)}{.100}q^{-48})(1-q^{-168})(1-\underset{(.006)}{.004}q^{-168})}\varepsilon_t, & \text{for Tuesday to Friday}\\
\underset{(.003)}{.470}\ln(d_t)-\underset{(.008)}{.110}\ln(w_t)+\dfrac{(1-\underset{(.029)}{.978}q^{-1})(1-\underset{(.009)}{.867}q^{-24})(1-\underset{(.004)}{.696}q^{-168})}{(1-q^{-1})(1-\underset{(.015)}{.746}q^{-1})(1-\underset{(.012)}{.969}q^{-24}+\underset{(.011)}{.023}q^{-48})(1-q^{-168})(1+\underset{(.004)}{.020}q^{-168})}\varepsilon_t, & \text{for Saturday}\\
\underset{(.003)}{.535}\ln(d_t)-\underset{(.010)}{.165}\ln(w_t)+\dfrac{(1-\underset{(.028)}{.998}q^{-1})(1-\underset{(.008)}{.984}q^{-24})(1-\underset{(.006)}{.713}q^{-168})}{(1-q^{-1})(1-\underset{(.013)}{.760}q^{-1})(1-\underset{(.012)}{1.005}q^{-24}+\underset{(.012)}{.025}q^{-48})(1-q^{-168})(1+\underset{(.004)}{.062}q^{-168})}\varepsilon_t, & \text{for Sunday}
\end{cases}\tag{6}$$

Taking this into account, a periodic dynamic regression model whose parameters switch according to the day of the week was the next tentative model.[6] The periodic model consisted of four inner models: three different models for Saturdays, Sundays and Mondays and another one for the days from Tuesday to Friday. Different structures were tested when fitting the model. During this process, it was observed that the sum of square residuals was significantly reduced when the unit roots were substituted by free autoregressive parameters. This alternative unconstrained model provided the smaller value for the SBC (Schwarz Bayesian Criterion), and the same criterion served to validate the parsimony of this switching model against the most simple dynamic regression model, as it was also smaller (see first and third rows in Table 3). The equation of the model is shown in (5).[7] When calculating the denominator roots of the different disturbance inner models, it can be observed that those corresponding to the Saturdays' model lie inside the unit circle, which means that the whole model does not fulfill the stationary condition. As it is indicated in [41], this non-fulfillment of the stationary condition would be a problem if the model was used for simulation or long-term forecasting, but it is not a problem when short term predictions are involved, and therefore it was decided to test[8] the significance of the free new parameters against the equivalent model constrained with unit roots described in (6). The null hypothesis is that all the inner models inside the switching model present two fixed unit roots: one regular and another corresponding to the within-week seasonal part. This test revealed that the null hypothesis can be rejected with a confidence level of 99%. Therefore, taking into account the SBC criterion, the unconstrained model was chosen. It is also important to point out that the MAPE of the 24 step-ahead forecasts in the out-of-sample period is another result which bears out the adequacy of this choice: the constrained stationary model resulted in a MAPE of 7.18%, whereas the MAPE of the unconstrained model was 6.98%.

Focusing on the parameters' values, note that those affecting load and wind energy production forecasts are significantly different from inner model to inner model. In particular, the parameters corresponding to the load and wind generation are larger for Sundays than for Tuesdays–Fridays, that is, these variables have more effect on the price estimation on Sundays than on Tuesdays or Wednesdays. This fact could be explained by the differences that can be observed in the supply curve slopes in the market clearing price on these days. In Fig. 4, two different market clearing points are illustrated: Sunday, 18th March 2007 and Wednesday, 21st March 2007, both at 12 a.m. Both supply curves could be segmented in a piece-wise linear manner in order to identify the different clearing conditions corresponding to different supply slopes. These differences could be explained by taking into account the nature of the marginal generating units and the bidding strategies of the agents. As the load on Sundays is lower than on Wednesdays, the clearing point is displaced to the left in the figure, and enters a segment of higher slope. A higher slope is equivalent to a more

[6] Details of the fitting and prediction process can be found in Appendix A.
[7] Variables p_t, d_t and w_t are normalized between the same values.

[8] The F-value corresponding to this test is $((\mathrm{RSS0}-\mathrm{RSS1})/(p-q))/(\mathrm{RSS1}/(n-p-1))$, where RSS0 and RSS1 are the residual square sum of the restricted and unrestricted models, respectively, p is the number of parameters corresponding to the unrestricted model, q is the number of parameters corresponding to the restricted model, and n is the number of effective samples which the models were fitted with.

Table 3
Summary of the fitted periodic models.

Models	Periods	Sum of squared residuals	Number of params	SBC	Out-of-sample MAPE
Non periodic	–	13.2796	7	−5.90	7.12
Constrained Model	Mon, Tue–Fri, Sat, Sun (4)	12.9828	36	−5.87	7.18
Unconstrained Model 1	Mon, Tue–Fri, Sat, Sun (4)	11.1318	44	−6.01	6.98
Unconstrained Model 2	Hours of the day (24)	11.8770	264	−5.57	7.04
Unconstrained Model 3	Hours of the day for each Mon, Tue–Fri, Sat, Sun (96)	11.8349	1056	−4.21	7.04
Unconstrained Model 4	Peak, valley and plateau hours (3)	11.5007	33	−6.00	7.15
Unconstrained Model 5	Peak, valley and plateau hours for each Mon, Tue–Fri, Sat, Sun (12)	10.5313	132	−5.92	7.47

4940 samples.

significant increase in price for the same increase in the effective demand.

The forecasting ability of the previous periodic model led us to test if intra-day dynamics can also be captured and provide even better forecasting results. Therefore, a set of periodic models were fitted and tested. The parameters of the periodic models reported in Table 3 switch according to the classification of the days of the week (Mon, Tue–Fri, Sat and Sun) and/or the definition of different types of demand hours (peak, valley and plateau hours). Five periodic models have been tested according to this definition. Model 1, the periodic dynamic regression reference model, has 4 periods according to the classification of the days of the week. Model 2 considers 24 periods, one for each hour of the day. Model 3 has 96 periods, as a result of the combination of the periods of the two previous models. Model 4 admits 3 periods, one for each type of demand hour. Model 5 uses 12 periods, as a combination of model 1 with model 4. None of these models outperforms the reference model 1 in terms of SBC and out-of-sample MAPE.

4.6. Neural network approach

Neural networks have been tested in this study in order to handle nonlinearities among prices, demand and wind power generation. These nonlinearities may appear in the form of spikes, saturations (see Fig. 5) and nonlinear causal relationships. A set of variables similar to those used in the previous models was considered: delayed prices and actual and delayed demand and wind generation forecasts. The hour of the day and the day of the week were also considered as input variables candidates. In order to be able to distinguish between high and low absolute values, input variables were previously neither regular nor seasonal differentiated. However, notice that price takes values in the validation set never reached in the training period. This fact could lead a neural network to inefficiently predict the output, i.e., the price. Therefore, all the input and output time series were detrended with a moving average filter.

A fully connected multilayer perceptron (MLP) with one hidden layer was the feed-forward neural network topology selected. The activation function of the neurons in the hidden layer was the hyperbolic tangent, and the linear function for the output layer. Several configurations differing in the number of hidden neurons and autoregressive and exogenous orders were tested to get the best network in terms of generalization capability. The selection of the most appropriate input variables is made via a statistical sensitivity analysis, allowing us to identify which variables are the most significant and which of them do not contribute to improve the fitting and should be eliminated.

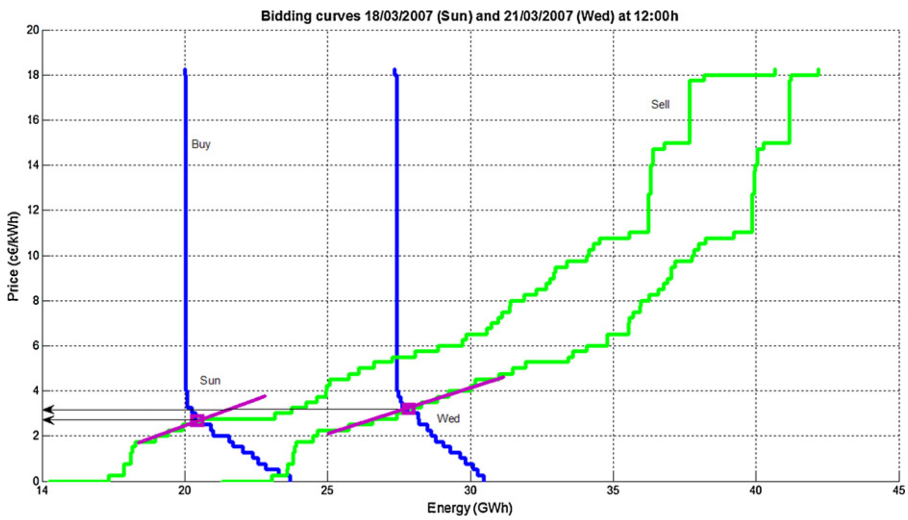

Fig. 4. Bidding curves for 18 and 21 March 2003 (Sunday and Wednesday, respectively). Note the intersection point of supply and demand curves.

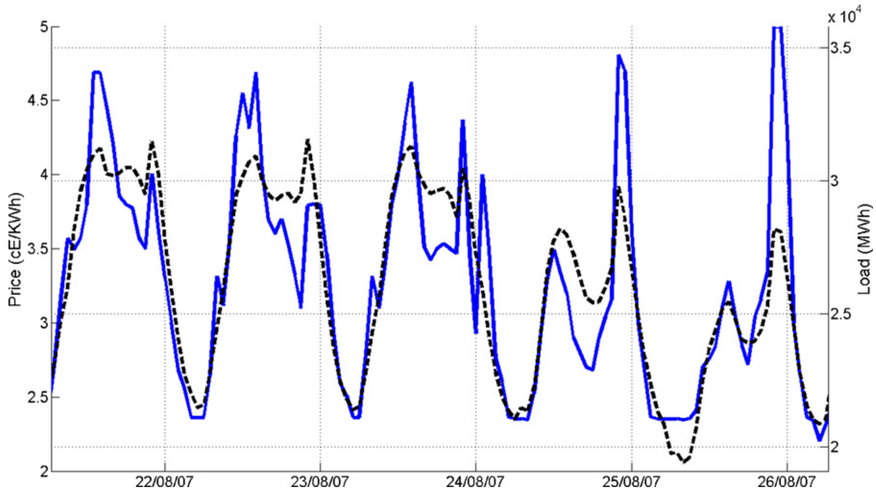

Fig. 5. Price (solid line) and demand (dash line, in different scales) in some days of August. Note the spikes in the price when demand is high, and saturations when demand is low.

In each fitting process, the in-sample data was randomly divided into two different sets: 80% for training and 20% for testing (used for cross-validation with early stopping). The fitting criterion was the quadratic error minimization with a regularization term that penalizes complex networks. The process was repeated several times with random weights initialization in order to avoid spurious local minima.

The final model is given by (7). The analysis of the input/output sensitivities ($\partial y/\partial x$) is shown in Fig. 6. The upper picture illustrates the 95%-percentiles of the sensitivity absolute values normalized between 0 and 1. This analysis reveals that the neural network is using wind generation as a relevant variable to predict prices. Although to a lesser extent, the hour of the day is also relevant. The day of the week did not reveal itself as relevant and, therefore, it was not included in the final model. This puzzling result is possibly due to the fact that the information provided by the day of the week is contained into the electricity load forecast, and therefore, the day of the week is deemed a relevant variable in model specifications in which the effect of the load has a linear impact on price, but it does not if this information can be captured from load by means of a nonlinear model specification.

$$p_t = a_0 + \sum_{j=1}^{12} a_j \tanh(b_j x_t + b_{j0}) + \varepsilon_t$$
$$x_t = (d_t, d_{t-1}, w_t, w_{t-1}, p_{t-1}, p_{t-2}, p_{t-24}, p_{t-168}, h_t)^T \quad (7)$$

The lower picture in Fig. 6 depicts the histograms of the sensitivities [42]. A linear input/output relationship would lead to a spiky histogram centered on the equivalent linear regression coefficient (this is the case of price(t-168) sensitivities). A non significant input variable would lead to a spiky histogram centered on zero (in Fig. 6, the input variable price(t-2) seems to be the least significant one). Finally, nonlinear input/output relationships give rise to non-spiky histograms, like the ones that have been obtained for the load sensitivities.

The MAPE for the whole out of sample period was 7.77%. This global result is not as competitive as those obtained with periodic or non-periodic dynamic regression models.

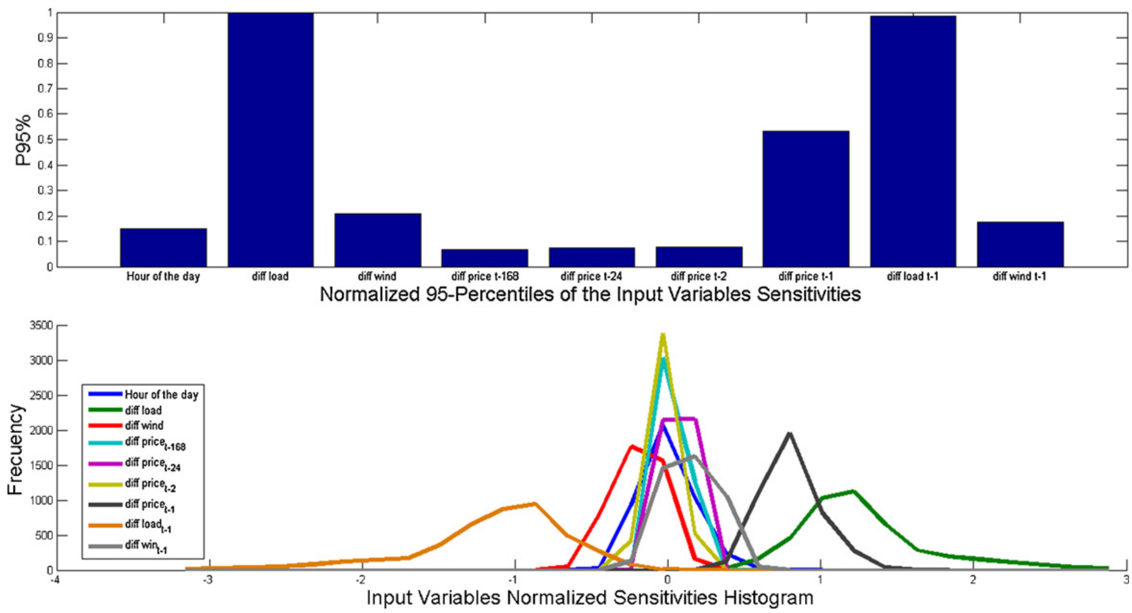

Fig. 6. Analysis of the input/output sensitivities for the neural network.

Table 4
Out-of-sample mape and relmae per month.

Month	MAPE (%)							RelMAE						
	ARIMA	HW	DR L	DR L + W	PER	MLP	NAIVE	ARIMA	HW	DR L	DR L + W	PER	MLP	NAIVE
2007/08	10.30	10.57	9.03	8.70	8.30	9.61	12.01	0.88	0.89	0.77	0.75	0.72	0.80	1.00
2007/09	7.44	7.35	7.67	6.61	6.35	7.04	9.67	0.76	0.75	0.04	0.69	0.66	0.74	1.00
2007/10	8.82	8.54	8.34	8.09	8.16	8.33	11.95	0.76	0.74	0.72	0.71	0.72	0.72	1.00
2007/11	9.49	9.11	9.84	8.43	8.19	9.07	12.19	0.75	0.74	0.78	0.67	0.65	0.71	1.00
2007/12	12.44	12.25	10.63	9.10	9.18	11.54	16.65	0.73	0.71	0.63	0.54	0.54	0.67	1.00
2008/01	9.42	9.41	9.32	7.87	7.35	9.13	13.17	0.71	0.71	0.70	0.58	0.55	0.67	1.00
2008/02	7.86	7.49	7.95	6.75	6.30	8.38	11.73	0.68	0.64	0.69	0.58	0.54	0.71	1.00
2008/03	8.42	8.32	8.68	7.69	7.63	7.36	11.12	0.77	0.75	0.80	0.70	0.70	0.68	1.00
2008/04	6.88	6.83	7.43	6.33	6.02	6.29	9.62	0.72	0.71	0.78	0.66	0.63	0.65	1.00
2008/05	5.82	5.90	6.54	6.11	6.21	6.35	7.86	0.74	0.75	0.84	0.78	0.80	0.81	1.00
2008/06	5.33	5.29	5.31	5.27	5.32	5.87	7.51	0.70	0.70	0.70	0.70	0.71	0.76	1.00
2008/07	4.91	4.96	4.90	4.88	5.09	4.83	7.40	0.66	0.67	0.67	0.66	0.69	0.66	1.00
Total	8.04	7.94	7.95	7.12	6.98	7.77	10.88	0.74	0.73	0.74	0.64	0.63	0.70	1.00

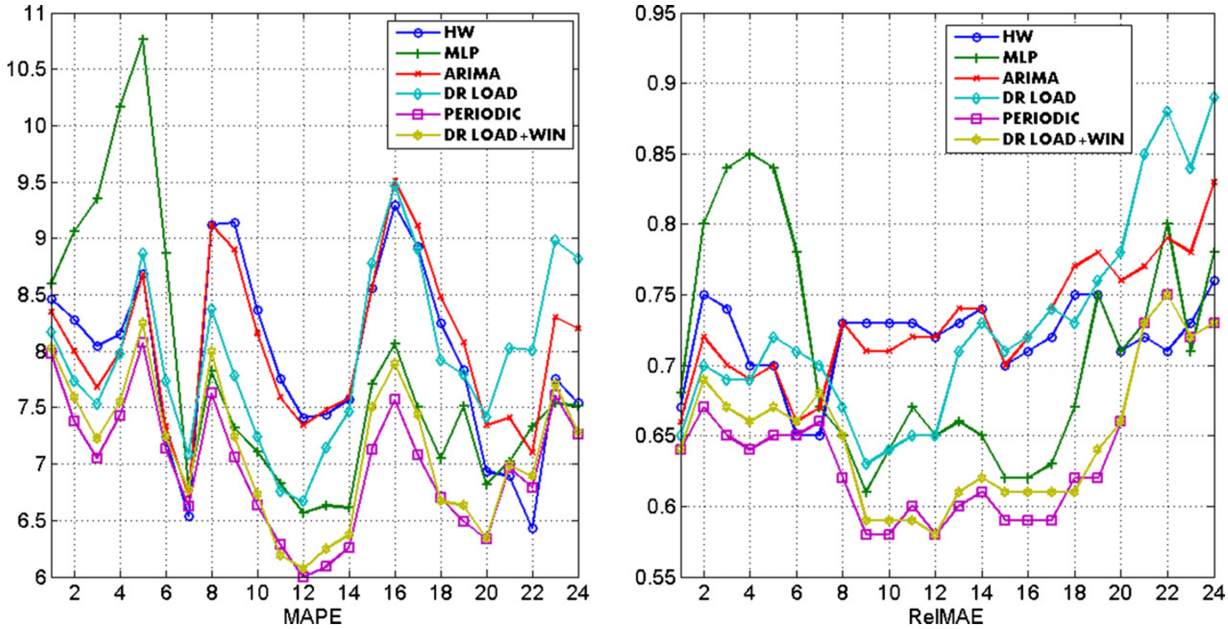

Fig. 7. MAPE and RelMAE of the out-of-sample period per each hour of the day.

5. Detailed out-of-sample forecasting results

The period ranging from 11th August 2007 to 31th July 2008 was used to evaluate the performance of the previously fitted models. The validation was made according to the way in which the market clearing process occurs. Thus, at a set time on day D, the market price values were predicted for each hour of day $D+1$, and therefore each hour of the day was forecast with a different horizon, from 1 to 24. The MAPE (Mean Absolute Percentage Error) was one of the selected error measures as it is frequently used by traders. A relative measure was also considered [43]. This measure responds to (8), where the denominator is the MAE (Mean Absolute Error) of a naive model whose prediction is the previous week's price. Table 4 summarizes the validation results for each month in the out-of-sample period. Table 5 summarizes the results of the Diebold–Mariano statistical tests used to compare the models' predictive accuracy[9] [44]. Fig. 7 shows the validation results for each hour of the day. Fig. 8 illustrates the cumulative MAPE of all models in the out-of-sample period.

$$\text{RelMAE} = \frac{\text{MAE}}{\text{MAE}_{\text{naive}}} \tag{8}$$

A first analysis of the results reveals that the wind energy production is a fundamental driver in the Spanish day-ahead market. The forecasting ability of the dynamic regression model is dramatically outperformed if this variable is included as explanatory variable. Moreover, even the MLP, which was the worst method including this variable, performed better than the dynamic regression model which did not use it.

The second result is that the periodic dynamic regression model outperforms the rest of models for both criteria in the whole out-of-sample period. Although the dynamic regression model with load and wind generation forecasts as explanatory variables obtained very good results, the identification of the different intra-week dynamics in the spot prices by means of a periodic model pro-

[9] To determine if model 1 predicts better than model 2, the accuracy of each forecast is measured by a loss function $L(e_t^i) = \left|e_t^i\right|$ where $e_t^i = y_t - \hat{y}_t^i$ is the forecast error of the model i. The null hypothesis of the Diebold–Mariano test $H_0 : [L(e_t^1)] = E[L(e_t^2)]$ is tested against the alternative $H_1 : [L(e_t^1)] < E[L(e_t^2)]$. Based on the loss differential $d_t = L(e_t^2) - L(e_t^1)$, the test statist is $S = \bar{d}_t/(LRV_{\bar{d}}/T)^{1/2}$, where $LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{23}\gamma_j$, $\gamma_j = cov(d_t, d_{t-j})$. Under the null hypothesis, S is distributed as a standard normal distribution.

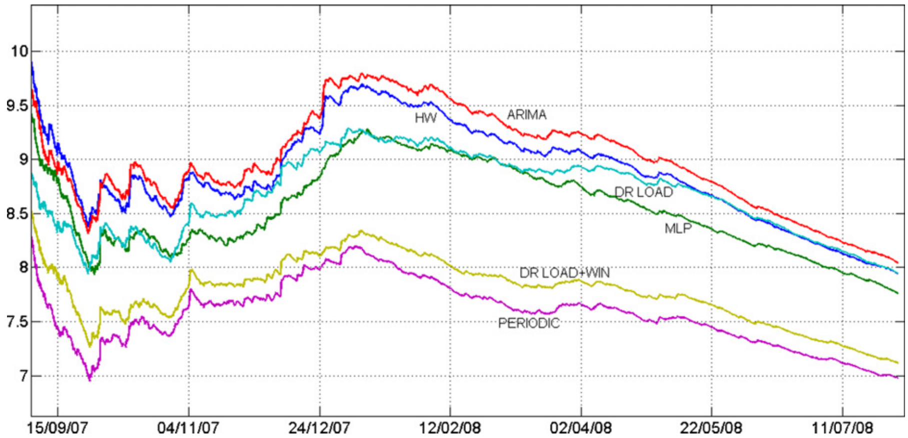

Fig. 8. Cumulative MAPE of all tested methods along the whole out-of-sample period.

Table 5
Summary of the Diebold–Mariano statistical test.

Model 1	Model 2	Statistics	*p*-Value
ARIMA	Naive	9.972	0.000
HW	ARIMA	1.338	0.090
DR (load)	HW	−0.503	–
MLP	DR (load)	1.729	0.042
DR (load + wind)	MLP	4.269	0.000
Periodic	DR (load + wind)	1.300	0.097

vides not only a better in-sample fit of the model, but a significant improvement in the out-of-sample prediction of day-ahead electricity prices (see Table 5). Note that the periodic model shows better cumulative MAPE than its non-periodic counterpart for each hour of the day in the whole out-of-sample period (see Fig. 8).

Regarding the univariate tested methods, exponential smoothing performed slightly better than ARIMA, and both outperformed the naive method for the whole out-of-sample period. The ARIMA model was more accurate for shorter horizons, while the Holt-Winters worked better for longer prediction horizons. In fact, the Holt-Winters method surprisingly outperformed the rest of methods at the hour 22. In general, the methods coming from the Box–Jenkins methodology (ARIMA and periodic and non periodic dynamic regression models) performed better than the others (Holt Winters and Multilayer Perceptron) for shorter horizons.

The global results obtained with the dynamic regression model using only the load forecast as explanatory variable seem to point out that the contribution of this variable is not relevant for the sake of price forecasting. However, a deeper analysis shows the failure of univariate models when forecasting special days, as public holidays or strikes. The behavior of prices in these special days is similar to Saturdays and Sundays due to the reduction in system load, but it cannot be captured through the univariate models' seasonal component. The inclusion of the electricity load forecast as an explanatory variable permits to take into account these calendar effects (see Fig. 9).

In spite of the universal approximation capability of the Multilayer Perceptron, it provided the worst global results among the multivariate models. This fact can be explained by its poor performance in the first hours of the day (see Fig. 7).

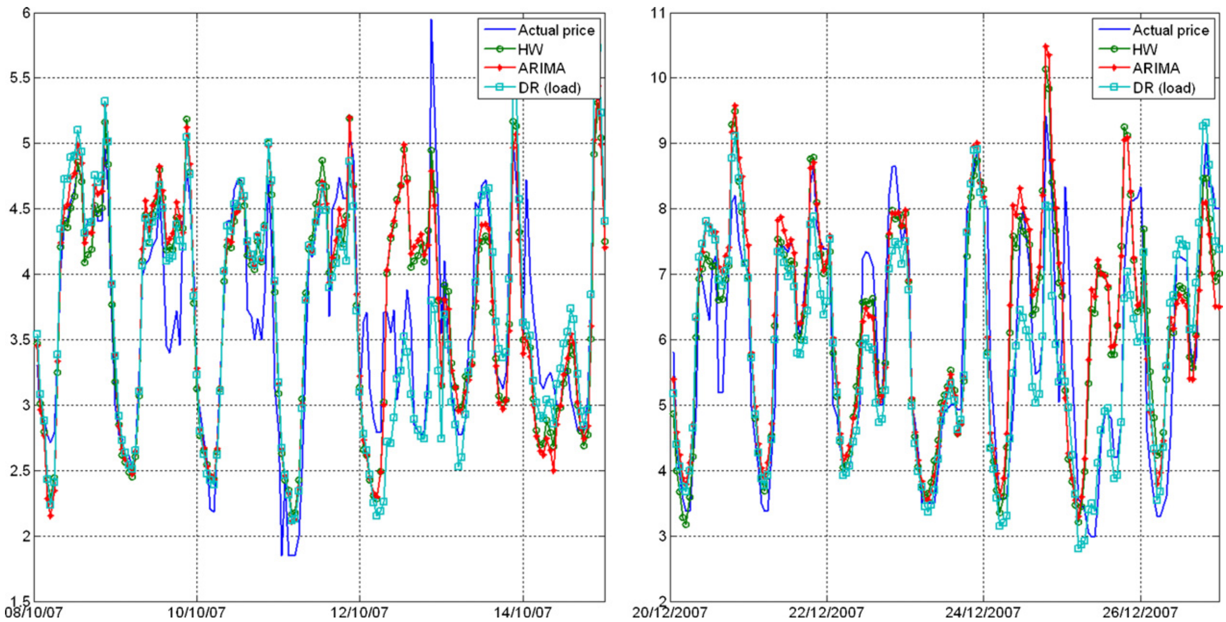

Fig. 9. Actual and predicted prices of the Holt-Winters', ARIMA and DR with load methods during two different weeks with holidays (12th October on the left and 25th December on the right).

6. Summary and conclusions

In this paper, a set of methods for short-term forecasting Spanish day-ahead electricity market prices have been evaluated in order to compare their performance. The methods most commonly studied in the literature of electricity price prediction and most commonly used in the industry have been included among the tested methods.

With respect to univariate methods, the exponential smoothing model for double seasonality showed slightly better global performance than the ARIMA one, although they performed differently as the prediction horizon increases. In fact the exponential smoothing method performed better than the ARIMA for longer forecasting horizons, while the ARIMA outperformed Holt Winters for shorter ones. This fact should be taken into account to improve the predictive accuracy when no explanatory variables are available.

The results obtained in this comparison demonstrate that the inclusion of the electricity load and wind generation forecasts provided by the System Operator significantly improves the predictive capabilities of the forecasting methods in the Spanish electricity market. The electricity load forecast allows us to account for calendar effects which in other way should be captured through dummy variables or any other special treatment. The wind generation is a fundamental driver of Spanish electricity spot prices due to its high level of penetration and its special market regulation. The inclusion of this variable dramatically improved the predictive accuracy of the dynamic regression model.

Finally, a significant increase in accuracy, relative to the dynamic regression model, was obtained when considering different dynamics for each day of the week through a novel periodic model. This regime switching model is able to account for the different effects of load, wind generation and recent past prices on the actual price as a function of the day of the week.

Appendix A. Periodic dynamic regression model

The general formulation of the periodic model used in this research can be written as shown in (A1),

$$\begin{aligned} y_t &= G_s(q^{-1})u_t + H_s(q^{-1})\varepsilon_t \\ & s \in \left\{1, 2, \dots, n_s\right\} \end{aligned} \tag{A1}$$

where u_t and y_t are the input and output variables, respectively, s is the intra-period stage or season at time t, n_s the number of different seasons inside the period, $G_1, \dots, G_{n_s}$ and $H_1, \dots, H_{n_s}$ a set of transfer functions, and ε_t is an independent random noise which, in general, has different variance for each intra-period stage or season. The parameter vector which defines this model is then defined as follows in (A2), θ_s being the parameter vector which affects the season s, and $g_{s1} \dots g_{sn_g}$ and $h_{s1} \dots h_{sn_g}$ the parameters defining the transfer functions G_s and H_s, respectively.

$$\begin{aligned} &\theta = [\theta_1 \dots \theta_{n_s}] \\ &\theta_s = [g_{s1} \dots g_{sn_g} h_{s1} \dots h_{sn_h}]^T \end{aligned} \tag{A2}$$

In a wider definition, s could depend not only on time but on others variables such as u_t or past values of y_t. However, it is important to notice that it does not depend on the parameters vector θ or any previous value of the model output, so the different seasons or intra-period stages are determined before any evaluation or fitting of the model.

The prediction process of the model in (A1) is, therefore, equivalent to applying different transfer functions to the inputs and errors in each instant of time according to the corresponding season. Also, it can be seen as a dynamic regression model which switches its parameters inside a period when the season changes. A computationally efficient way of calculating the model output is by using an equivalent state-space representation. The model in this representation follows as (A3),[10]

$$\begin{aligned} x_{t+1} &= A_s x_t + B_s u_t + K_s \varepsilon_t \\ y_t &= C_s x_t + D_s u_t + \varepsilon_t \end{aligned} \tag{A3}$$

where x_t is the state vector containing all the necessary information to estimate the model output at time t, and each set of matrices $\{A_s, B_s, C_s, D_s, K_s\}$ corresponding to the season s depends on the parameter vector θ_s, and is related with the transfer functions in (9) through the equations in (A4).

$$\begin{aligned} &G_s(q^{-1}) = C_s(q^{-1}I - A_s)^{-1}B_s + D_s \quad H_s(q^{-1}) = C_s(q^{-1}I - A_s)^{-1}K_s + 1 \\ &s = s(t) \in \left\{1, 2, \dots, n_s\right\} \end{aligned} \tag{A4}$$

The steps to recursively calculate the one step-ahead prediction appear in (A5). For the k step-ahead prediction ($k > 1$) it is necessary to jump the third step as the real value of the output is unknown, and take the error to be its expected value $\varepsilon_t = 0$.

$$\begin{aligned} &(1)\,\text{determine the season or regime } s = s(t) \text{ and} \\ &\left\{A_s(\theta_s), B_s(\theta_s), C_s(\theta_s), D_s(\theta_s), K_s(\theta_s)\right\} \\ &(2)\,\text{calculate the model estimation } \hat{y}_t = C_s\hat{x}_t + D_s u_t \\ &(3)\,\text{calculate the error } \varepsilon_t = y_t - \hat{y}_t \\ &(4)\,\text{update the state } \hat{x}_{t+1} = A_s\hat{x}_t + B_s u_t + K_s\varepsilon_t \end{aligned} \tag{A5}$$

In this study, the different seasons or intra-period stages inside the week are determined by the day of week, though intra-period stages inside the day were also considered. The routine used to fit the model is based on nonlinear least squares. The fitting process for the periodic dynamic regression model uses as initial parameter vector the solution reached in the fitting process of the corresponding non-periodic dynamic regression model.

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