Formulário de Mecânica Quântica

Espaços vetoriais com coeficientes complexos

$$|\Psi\rangle + (|\Phi\rangle + |\Upsilon\rangle) = (|\Psi\rangle + |\Phi\rangle) + |\Upsilon\rangle \qquad |\Psi\rangle + |\Phi\rangle = |\Phi\rangle + |\Psi\rangle$$

$$|\Psi\rangle + |0\rangle = |\Psi\rangle \qquad |\Psi\rangle + |-\Psi\rangle = |0\rangle \qquad z(|\Psi\rangle + |\Phi\rangle) = z|\Psi\rangle + z|\Phi\rangle$$

$$z(w|\Psi\rangle) = zw|\Psi\rangle \qquad (z+w)|\Psi\rangle = z|\Psi\rangle + w|\Phi\rangle \qquad \langle\Psi|\Phi\rangle = \langle\Phi|\Psi\rangle^*$$

$$\langle\Psi|(|\Phi\rangle + |\Upsilon\rangle) = \langle\Psi|\Phi\rangle + \langle\Psi|\Upsilon\rangle \qquad z\langle\Psi|\Phi\rangle = \langle\Psi|(z|\Phi\rangle) \qquad \||\Psi\|^2 = \langle\Psi|\Psi\rangle$$

Bases e componentes

$$\langle e_j|e_k\rangle = \delta_{jk} \qquad |\Psi\rangle = \sum_{j=1}^{n}\Psi_j|e_j\rangle \qquad \Psi_j = \langle e_j|\Psi\rangle \qquad \langle\Psi| = \sum_{j=1}^{n}\Psi_j^*\langle e_j|$$

$$\langle\Psi|\Phi\rangle = \sum_{j=1}^{n}\Psi_j^*\Phi_j \qquad \||\Psi\|^2 = \sum_{j=1}^{n}|\Psi_j|^2 \qquad \text{(igual a 1 nas próximas linhas)}$$

$$\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n}\Omega_{jk}\Psi_k|e_j\rangle \qquad \Omega_{jk} = \langle e_j|\hat{\Omega}|e_k\rangle \qquad \langle\hat{\Omega}\rangle = \langle\Psi|\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n}\Omega_{jk}\Psi_j^*\Psi_k$$

Representação matricial

$$|\Psi\rangle = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{bmatrix} \qquad \langle\Psi| = \begin{bmatrix} \Psi_1^* & \Psi_2^* & \ldots & \Psi_n^* \end{bmatrix} \qquad \hat{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \ldots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} & \ldots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \ldots & \Omega_{nn} \end{bmatrix}$$

Valores e vetores próprios

$$\hat{\Omega}|\Omega_j\rangle = \Omega_j|\Omega_j\rangle \qquad \begin{vmatrix} \Omega_{11}-\lambda & \Omega_{12} & \ldots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22}-\lambda & \ldots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \ldots & \Omega_{nn}-\lambda \end{vmatrix} = 0$$

Síntese de mecânica analítica

$$\{F,G\} = \frac{\partial F}{\partial s}\frac{\partial G}{\partial p} - \frac{\partial F}{\partial p}\frac{\partial G}{\partial s} \qquad\qquad \frac{\mathrm{d}F}{\mathrm{d}t} = \{F,H\}$$

Observáveis (operadores hermíticos)

$\Omega_{jk} = \Omega_{kj}^*$ Valores próprios reais: $\Omega_j = \Omega_j^*$ Base ortonormal: $\langle\Omega_j|\Omega_k\rangle = \delta_{jk}$

Possíveis valores medidos: Ω_k

Probabilidade de medir Ω_k: $\mathrm{P}_k = |\langle\Omega_k|\Psi\rangle|^2$

Estado após medir Ω_k: $|\Omega_k\rangle$

Comutadores

$$[\hat{\Omega},\hat{\Lambda}] = \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega} \qquad [\hat{\Omega},\hat{\Lambda}] = -[\hat{\Lambda},\hat{\Omega}] \qquad [\hat{\Omega}\hat{\Lambda},\hat{\Gamma}] = \hat{\Omega}[\hat{\Lambda},\hat{\Gamma}] + [\hat{\Omega},\hat{\Gamma}]\hat{\Lambda}$$

$$[\hat{F},\hat{G}] = i\hbar\{F,G\} \qquad [\hat{x},\hat{p}] = i\hbar \qquad \hbar = 1.055 \times 10^{-34}\,\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}}$$

Oscilador harmónico

$$\hat{H} = \frac{1}{2\,m}\hat{p}^2 + \frac{m\,\omega^2}{2}\hat{x}^2 \qquad \hat{a} = \sqrt{\frac{m\,\omega}{2\,\hbar}}\left(\hat{x} + \frac{i}{m\,\omega}\hat{p}\right) \quad \hat{a}^\dagger = \sqrt{\frac{m\,\omega}{2\,\hbar}}\left(\hat{x} - \frac{i}{m\,\omega}\hat{p}\right)$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad (n = 0, 1, \ldots) \quad \hat{a}\,|E_n\rangle = \sqrt{n}\,|E_{n-1}\rangle \qquad \hat{a}^\dagger|E_n\rangle = \sqrt{n+1}\,|E_{n+1}\rangle$$

Matrizes de Pauli

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\langle z_+|z_-\rangle = 0 \qquad \langle z_+|z_+\rangle = \langle z_-|z_-\rangle = 1 \qquad \hat{\sigma}_z|z_\pm\rangle = \pm|z_\pm\rangle$$

$$\hat{\sigma}_x|z_k\rangle = |z_{-k}\rangle \qquad \hat{\sigma}_y|z_k\rangle = k\,i|z_{-k}\rangle \qquad (k = +, -)$$

$$\hat{\sigma}_j\hat{\sigma}_k = -\hat{\sigma}_k\hat{\sigma}_j \qquad \hat{\sigma}_x\hat{\sigma}_y = i\hat{\sigma}_z \qquad \hat{\sigma}_y\hat{\sigma}_z = i\hat{\sigma}_x \qquad \hat{\sigma}_z\hat{\sigma}_x = i\hat{\sigma}_y$$

Combinação de dois spins

Base ortonormal:

$$\{|z_+, z_+\rangle, |z_+, z_-\rangle, |z_-\rangle, z_+\rangle, |z_-, z_-\rangle\} \qquad \langle z_j, z_k|z_l, z_m\rangle = \delta_{jl}\delta_{km} \qquad (j, k, l, m = +, -)$$

$$\hat{\sigma}_{1z}\hat{\sigma}_{2z}|z_j, z_k\rangle = jk|z_j, z_k\rangle \qquad\qquad \hat{\sigma}_{1x}\hat{\sigma}_{2x}|z_j, z_k\rangle = |z_{-j}, z_{-k}\rangle \quad \text{etc} \ldots$$

Forma geral dos estados:
$$|\Upsilon\rangle = \Upsilon_{++}|z_+, z_+\rangle + \Upsilon_{+-}|z_+, z_-\rangle + \Upsilon_{-+}|z_-, z_+\rangle + \Upsilon_{--}|z_-, z_-\rangle$$

Dois estados independentes:
$$|\Psi\rangle = \Psi_+|z_+\rangle + \Psi_-|z_+\rangle \qquad |\Phi\rangle = \Phi_+|z_+\rangle + \Phi_-|z_+\rangle$$

$$|\Psi, \Phi\rangle = \Psi_+\Phi_+|z_+, z_+\rangle + \Psi_+\Phi_-|z_+, z_-\rangle + \Psi_-\Phi_+|z_-, z_+\rangle + \Psi_-\Phi_-|z_-, z_-\rangle$$

Estados entrelaçados:
$$\frac{\Upsilon_{++}}{\Upsilon_{+-}} \neq \frac{\Upsilon_{-+}}{\Upsilon_{--}} \quad \text{ou:} \quad \frac{\Upsilon_{++}}{\Upsilon_{-+}} \neq \frac{\Upsilon_{+-}}{\Upsilon_{--}}$$

Matrizes:
$$|\Upsilon\rangle = \begin{bmatrix} \Upsilon_{++} \\ \Upsilon_{+-} \\ \Upsilon_{-+} \\ \Upsilon_{--} \end{bmatrix} \qquad \langle\Upsilon| = \begin{bmatrix} \Upsilon_{++}^* & \Upsilon_{+-}^* & \Upsilon_{-+}^* & \Upsilon_{+-}^* \end{bmatrix}$$

$$\hat{\Omega} = \begin{bmatrix} \Omega_{++} & \Omega_{+-} \\ \Omega_{-+} & \Omega_{--} \end{bmatrix} \quad \hat{\Lambda} = \begin{bmatrix} \Lambda_{++} & \Lambda_{+-} \\ \Lambda_{-+} & \Lambda_{--} \end{bmatrix} \quad \hat{\Omega}_1\hat{\Lambda}_2 = \begin{bmatrix} \Omega_{++}\Lambda_{++} & \Omega_{++}\Lambda_{+-} & \Omega_{+-}\Lambda_{++} & \Omega_{+-}\Lambda_{+-} \\ \Omega_{++}\Lambda_{-+} & \Omega_{++}\Lambda_{--} & \Omega_{+-}\Lambda_{-+} & \Omega_{+-}\Lambda_{--} \\ \Omega_{-+}\Lambda_{++} & \Omega_{-+}\Lambda_{+-} & \Omega_{--}\Lambda_{++} & \Omega_{--}\Lambda_{+-} \\ \Omega_{-+}\Lambda_{-+} & \Omega_{-+}\Lambda_{--} & \Omega_{--}\Lambda_{-+} & \Omega_{--}\Lambda_{--} \end{bmatrix}$$