

# Formulário de Mecânica Quântica

## Espaços vetoriais com coeficientes complexos

$$\begin{aligned} |\Psi\rangle + (|\Phi\rangle + |\Upsilon\rangle) &= (|\Psi\rangle + |\Phi\rangle) + |\Upsilon\rangle & |\Psi\rangle + |\Phi\rangle &= |\Phi\rangle + |\Psi\rangle \\ |\Psi\rangle + |0\rangle &= |\Psi\rangle & |\Psi\rangle + |-\Psi\rangle &= |0\rangle & z(|\Psi\rangle + |\Phi\rangle) &= z|\Psi\rangle + z|\Phi\rangle \\ z(w|\Psi\rangle) &= zw|\Psi\rangle & (z+w)|\Psi\rangle &= z|\Psi\rangle + w|\Psi\rangle & \langle\Psi|\Phi\rangle &= \langle\Phi|\Psi\rangle^* \\ \langle\Psi|(|\Phi\rangle + |\Upsilon\rangle) &= \langle\Psi|\Phi\rangle + \langle\Psi|\Upsilon\rangle & z\langle\Psi|\Phi\rangle &= \langle\Psi|(z|\Phi\rangle) & \|\Psi\|^2 &= \langle\Psi|\Psi\rangle \end{aligned}$$

## Bases e componentes

$$\begin{aligned} \langle e_j | e_k \rangle &= \delta_{jk} & |\Psi\rangle &= \sum_{j=1}^n \Psi_j |e_j\rangle & \Psi_j &= \langle e_j | \Psi \rangle & \langle \Psi | &= \sum_{j=1}^n \Psi_j^* \langle e_j | \\ \langle \Psi | \Phi \rangle &= \sum_{j=1}^n \Psi_j^* \Phi_j & \|\Psi\|^2 &= \sum_{j=1}^n |\Psi_j|^2 & & (\text{igual a 1 nas próximas linhas}) \\ \hat{\Omega}|\Psi\rangle &= \sum_{j=1}^n \sum_{k=1}^n \Omega_{jk} \Psi_j |e_k\rangle & \Omega_{jk} &= \langle e_j | \hat{\Omega} | e_k \rangle & \langle \hat{\Omega} \rangle &= \langle \Psi | \hat{\Omega} | \Psi \rangle = \sum_{j=1}^n \sum_{k=1}^n \Omega_{jk} \Psi_j^* \Psi_k \end{aligned}$$

## Representação matricial

$$|\Psi\rangle = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{bmatrix} \quad \langle \Psi | = [\Psi_1^* \ \Psi_2^* \ \dots \ \Psi_n^*] \quad \hat{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \dots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} & \dots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn} \end{bmatrix}$$

## Valores e vetores próprios

$$\hat{\Omega} |\Omega_j\rangle = \Omega_j |\Omega_j\rangle \quad \begin{vmatrix} \Omega_{11} - \lambda & \Omega_{12} & \dots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} - \lambda & \dots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn} - \lambda \end{vmatrix} = 0$$

## Síntese de mecânica analítica

$$\{F, G\} = \frac{\partial F}{\partial s} \frac{\partial G}{\partial p} - \frac{\partial F}{\partial p} \frac{\partial G}{\partial s} \quad \frac{dF}{dt} = \{F, H\}$$

## Observáveis (operadores hermiticos)

$$\begin{aligned} \Omega_{jk} &= \Omega_{kj}^* & \text{Valores próprios reais: } \Omega_j &= \Omega_j^* & \text{Base ortonormal: } \langle \Omega_j | \Omega_k \rangle &= \delta_{jk} \\ \text{Possíveis valores medidos: } & \Omega_k \\ \text{Probabilidade de medir } \Omega_k: & P_k = |\langle \Omega_k | \Psi \rangle|^2 \\ \text{Estado após medir } \Omega_k: & |\Omega_k\rangle \end{aligned}$$

## Comutadores

$$\begin{aligned} [\hat{\Omega}, \hat{\Lambda}] &= \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega} & [\hat{\Omega}, \hat{\Lambda}] &= -[\hat{\Lambda}, \hat{\Omega}] & [\hat{\Omega}\hat{\Lambda}, \hat{\Gamma}] &= \hat{\Omega}[\hat{\Lambda}, \hat{\Gamma}] + [\hat{\Omega}, \hat{\Gamma}]\hat{\Lambda} \\ [\hat{F}, \hat{G}] &= i\hbar\{F, G\} & [\hat{x}, \hat{p}] &= i\hbar & \hbar &= 1.055 \times 10^{-34} \frac{\text{kg} \cdot \text{m}^2}{\text{s}} \end{aligned}$$

## Oscilador harmónico

$$\hat{H} = \frac{1}{2m} \hat{p}^2 + \frac{m\omega^2}{2} \hat{x}^2 \quad \hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \left( \hat{x} + \frac{i}{m\omega} \hat{p} \right) \quad \hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left( \hat{x} - \frac{i}{m\omega} \hat{p} \right)$$

$$E_n = \hbar\omega \left( n + \frac{1}{2} \right) \quad (n = 0, 1, \dots) \quad \hat{a} |E_n\rangle = \sqrt{n} |E_{n-1}\rangle \quad \hat{a}^\dagger |E_n\rangle = \sqrt{n+1} |E_{n+1}\rangle$$

## Matrizes de Pauli

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\langle z_+ | z_- \rangle = 0 \quad \langle z_+ | z_+ \rangle = \langle z_- | z_- \rangle = 1 \quad \hat{\sigma}_z |z_\pm\rangle = \pm |z_\pm\rangle$$

$$\hat{\sigma}_x |z_k\rangle = |z_{-k}\rangle \quad \hat{\sigma}_y |z_k\rangle = k i |z_{-k}\rangle \quad (k = +, -)$$

$$\hat{\sigma}_j \hat{\sigma}_k = -\hat{\sigma}_k \hat{\sigma}_j \quad \hat{\sigma}_x \hat{\sigma}_y = i \hat{\sigma}_z \quad \hat{\sigma}_y \hat{\sigma}_z = i \hat{\sigma}_x \quad \hat{\sigma}_z \hat{\sigma}_x = i \hat{\sigma}_y$$

## Combinação de dois spins

Base ortonormal:

$$\{|z_+, z_+\rangle, |z_+, z_-\rangle, |z_-, z_+\rangle, |z_-, z_-\rangle\} \quad \langle z_j, z_k | z_l, z_m \rangle = \delta_{jl} \delta_{km} \quad (j, k, l, m = +, -)$$

$$\hat{\sigma}_{1z} \hat{\sigma}_{2z} |z_j, z_k\rangle = j k |z_j, z_k\rangle \quad \hat{\sigma}_{1x} \hat{\sigma}_{2x} |z_j, z_k\rangle = |z_{-j}, z_{-k}\rangle \quad \text{etc} \dots$$

Forma geral dos estados:  $|Y\rangle = Y_{++} |z_+, z_+\rangle + Y_{+-} |z_+, z_-\rangle + Y_{-+} |z_-, z_+\rangle + Y_{--} |z_-, z_-\rangle$

Dois estados independentes:  $|\Psi\rangle = \Psi_+ |z_+\rangle + \Psi_- |z_-\rangle \quad |\Phi\rangle = \Phi_+ |z_+\rangle + \Phi_- |z_-\rangle$

$$|\Psi, \Phi\rangle = \Psi_+ \Phi_+ |z_+, z_+\rangle + \Psi_+ \Phi_- |z_+, z_-\rangle + \Psi_- \Phi_+ |z_-, z_+\rangle + \Psi_- \Phi_- |z_-, z_-\rangle$$

Estados entrelaçados:  $\frac{Y_{++}}{Y_{+-}} \neq \frac{Y_{-+}}{Y_{--}} \quad \text{ou:} \quad \frac{Y_{++}}{Y_{-+}} \neq \frac{Y_{+-}}{Y_{--}}$

Matrizes:  $|Y\rangle = \begin{bmatrix} Y_{++} \\ Y_{+-} \\ Y_{-+} \\ Y_{--} \end{bmatrix} \quad \langle Y| = [Y_{++}^* \ Y_{+-}^* \ Y_{-+}^* \ Y_{--}^*]$

$$\hat{\Omega} = \begin{bmatrix} \Omega_{++} & \Omega_{+-} \\ \Omega_{-+} & \Omega_{--} \end{bmatrix} \quad \hat{\Lambda} = \begin{bmatrix} \Lambda_{++} & \Lambda_{+-} \\ \Lambda_{-+} & \Lambda_{--} \end{bmatrix} \quad \hat{\Omega}_1 \hat{\Lambda}_2 = \begin{bmatrix} \Omega_{++} \Lambda_{++} & \Omega_{++} \Lambda_{+-} & \Omega_{+-} \Lambda_{++} & \Omega_{+-} \Lambda_{+-} \\ \Omega_{++} \Lambda_{-+} & \Omega_{++} \Lambda_{--} & \Omega_{+-} \Lambda_{-+} & \Omega_{+-} \Lambda_{--} \\ \Omega_{-+} \Lambda_{++} & \Omega_{-+} \Lambda_{+-} & \Omega_{--} \Lambda_{++} & \Omega_{--} \Lambda_{+-} \\ \Omega_{-+} \Lambda_{-+} & \Omega_{-+} \Lambda_{--} & \Omega_{--} \Lambda_{-+} & \Omega_{--} \Lambda_{--} \end{bmatrix}$$