Formulário de Mecânica Quântica

Espaços vetoriais com coeficientes complexos

$$|\Psi\rangle + (|\Phi\rangle + |\Upsilon\rangle) = (|\Psi\rangle + |\Phi\rangle) + |\Upsilon\rangle \qquad |\Psi\rangle + |\Phi\rangle = |\Phi\rangle + |\Psi\rangle$$

$$|\Psi\rangle + |0\rangle = |\Psi\rangle \qquad |\Psi\rangle + |-\Psi\rangle = |0\rangle \qquad z(|\Psi\rangle + |\Phi\rangle) = z|\Psi\rangle + z|\Phi\rangle$$

$$z(w|\Psi\rangle) = zw|\Psi\rangle \qquad (z+w)|\Psi\rangle = z|\Psi\rangle + w|\Phi\rangle \qquad \langle\Psi|\Phi\rangle = \langle\Phi|\Psi\rangle^*$$

$$\langle\Psi|(|\Phi\rangle + |\Upsilon\rangle) = \langle\Psi|\Phi\rangle + \langle\Psi|\Upsilon\rangle \qquad z\langle\Psi|\Phi\rangle = \langle\Psi|(z|\Phi\rangle) \qquad \|\Psi\|^2 = \langle\Psi|\Psi\rangle$$

Bases e componentes

$$\langle \mathrm{e}_j|\mathrm{e}_k\rangle = \delta_{jk} \qquad |\Psi\rangle = \sum_{j=1}^{n} \mathit{\Psi}_j|\mathrm{e}_j\rangle \qquad \mathit{\Psi}_j = \langle \mathrm{e}_j|\Psi\rangle \qquad \langle\Psi| = \sum_{j=1}^{n} \mathit{\Psi}_j^*\langle \mathrm{e}_j|$$

$$\langle\Psi|\Phi\rangle = \sum_{j=1}^{n} \mathit{\Psi}_j^*\mathit{\Phi}_j \qquad \|\Psi\|^2 = \sum_{j=1}^{n} |\mathit{\Psi}_j|^2 \qquad \text{(igual a 1 nas próximas linhas)}$$

$$\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n} \mathit{\Omega}_{jk}\mathit{\Psi}_k|\mathrm{e}_j\rangle \qquad \mathit{\Omega}_{jk} = \langle \mathrm{e}_j|\hat{\Omega}|\mathrm{e}_k\rangle \qquad \langle\hat{\Omega}\rangle = \langle\Psi|\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n} \mathit{\Omega}_{jk}\mathit{\Psi}_j^*\mathit{\Psi}_k$$

Representação matricial

$$|\Psi\rangle = \begin{bmatrix} \mathit{\Psi}_1 \\ \mathit{\Psi}_2 \\ \vdots \\ \mathit{\Psi}_n \end{bmatrix} \qquad \langle\Psi| = \begin{bmatrix} \mathit{\Psi}_1^* & \mathit{\Psi}_2^* & \ldots & \mathit{\Psi}_n^* \end{bmatrix} \qquad \hat{\Omega} = \begin{bmatrix} \mathit{\Omega}_{11} & \mathit{\Omega}_{12} & \ldots & \mathit{\Omega}_{1n} \\ \mathit{\Omega}_{21} & \mathit{\Omega}_{22} & \ldots & \mathit{\Omega}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathit{\Omega}_{n1} & \mathit{\Omega}_{n2} & \ldots & \mathit{\Omega}_{nn} \end{bmatrix}$$

Valores e vetores próprios

$$\hat{\Omega}\,|\Omega_j\rangle = \mathit{\Omega}_j\,|\Omega_j\rangle \qquad \begin{vmatrix} \mathit{\Omega}_{11}-\lambda & \mathit{\Omega}_{12} & \ldots & \mathit{\Omega}_{1n} \\ \mathit{\Omega}_{21} & \mathit{\Omega}_{22}-\lambda & \ldots & \mathit{\Omega}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathit{\Omega}_{n1} & \mathit{\Omega}_{n2} & \ldots & \mathit{\Omega}_{nn}-\lambda \end{vmatrix} = 0$$

Síntese de mecânica analítica

$$\{F,G\} = \frac{\partial F}{\partial s}\frac{\partial G}{\partial p} - \frac{\partial F}{\partial p}\frac{\partial G}{\partial s} \qquad \frac{\mathrm{d}F}{\mathrm{d}t} = \{F,H\} \qquad |\Psi(0)\rangle = \sum_{j=1}^{n} \mathit{\Psi}_j|\mathrm{E}_j\rangle$$

Observáveis (operadores hermíticos)

$\mathit{\Omega}_{jk} = \mathit{\Omega}_{kj}^*$ Valores próprios reais: $\mathit{\Omega}_j = \mathit{\Omega}_j^*$ Base ortonormal: $\langle\mathit{\Omega}_j|\mathit{\Omega}_k\rangle = \delta_{jk}$

Possíveis valores medidos: $\mathit{\Omega}_k$

Probabilidade de medir $\mathit{\Omega}_k$: $\mathrm{P}_k = |\langle\Omega_k|\Psi\rangle|^2$

Estado após medir $\mathit{\Omega}_k$: $|\Omega_k\rangle$

Comutadores

$$[\hat{\Omega},\hat{\Lambda}] = \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega} \qquad [\hat{\Omega},\hat{\Lambda}] = -[\hat{\Lambda},\hat{\Omega}] \qquad [\hat{\Omega}\,\hat{\Lambda},\hat{\Gamma}] = \hat{\Omega}[\hat{\Lambda},\hat{\Gamma}] + [\hat{\Omega},\hat{\Gamma}]\hat{\Lambda}$$

$$[\hat{\mathrm{F}},\hat{\mathrm{G}}] = \mathrm{i}\hbar\{F,G\} \qquad [\hat{\mathrm{x}},\hat{\mathrm{p}}] = \mathrm{i}\hbar \qquad \hbar = 1.055\times10^{-34}\ \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}}$$

Oscilador harmónico

$$\hat{\mathrm{H}} = \frac{1}{2\,m}\hat{\mathrm{p}}^2 + \frac{m\,\omega^2}{2}\hat{\mathrm{x}}^2 \qquad \hat{\mathrm{a}} = \sqrt{\frac{m\,\omega}{2\,\hbar}}\left(\hat{\mathrm{x}} + \frac{\mathrm{i}}{m\,\omega}\hat{\mathrm{p}}\right) \qquad \hat{\mathrm{a}}^\dagger = \sqrt{\frac{m\,\omega}{2\,\hbar}}\left(\hat{\mathrm{x}} - \frac{\mathrm{i}}{m\,\omega}\hat{\mathrm{p}}\right)$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad (n = 0, 1, \ldots) \quad \hat{\mathrm{a}}\,|\mathrm{E}_n\rangle = \sqrt{\frac{E_n}{\hbar\,\omega} - \frac{1}{2}}\,|\mathrm{E}_{n-1}\rangle \quad \hat{\mathrm{a}}^\dagger|\mathrm{E}_n\rangle = \sqrt{\frac{E_n}{\hbar\,\omega} + \frac{1}{2}}\,|\mathrm{E}_{n+1}\rangle$$

Matrizes de Pauli

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \hat{\sigma}_y = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \qquad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{\sigma}_j\hat{\sigma}_k = -\hat{\sigma}_k\hat{\sigma}_j \qquad \hat{\sigma}_x\hat{\sigma}_y = \mathrm{i}\hat{\sigma}_z \qquad \hat{\sigma}_y\hat{\sigma}_z = \mathrm{i}\hat{\sigma}_x \qquad \hat{\sigma}_z\hat{\sigma}_x = \mathrm{i}\hat{\sigma}_y$$

Combinação de dois sistemas

$$|\Psi, \Phi\rangle = |\Psi\rangle \otimes |\Phi\rangle \qquad \langle \mathrm{e}_j, \mathrm{e}_k|\mathrm{e}_r, \mathrm{e}_s\rangle = \delta_{jr}\delta_{ks} \qquad |\Psi, \Phi\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \varPsi_j \varPhi_r |\mathrm{e}_j, \mathrm{e}_r\rangle$$

$$\left(\hat{\Omega} \otimes \hat{\Lambda}\right)|\Psi, \Phi\rangle = \hat{\Omega}|\Psi\rangle \otimes \hat{\Lambda}|\Phi\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \varOmega_{jk}\varLambda_{rs}\varPsi_k\varPhi_s|\mathrm{e}_j, \mathrm{e}_r\rangle$$

Estados entrelaçados: $|\Upsilon\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \varUpsilon_{jr}|\mathrm{e}_j, \mathrm{e}_r\rangle$

$$\left(\hat{\Omega} \otimes \hat{\Lambda}\right)|\Upsilon\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \varOmega_{jk}\varLambda_{rs}\varUpsilon_{ks}|\mathrm{e}_j, \mathrm{e}_r\rangle$$