Formulário de Mecânica Quântica

Espaços vetoriais com coeficientes complexos

$$|\Psi\rangle + (|\Phi\rangle + |\Upsilon\rangle) = (|\Psi\rangle + |\Phi\rangle) + |\Upsilon\rangle \qquad |\Psi\rangle + |\Phi\rangle = |\Phi\rangle + |\Psi\rangle$$

$$|\Psi\rangle + |0\rangle = |\Psi\rangle \qquad |\Psi\rangle + |-\Psi\rangle = |0\rangle \qquad z(|\Psi\rangle + |\Phi\rangle) = z|\Psi\rangle + z|\Phi\rangle$$

$$z(w|\Psi\rangle) = zw|\Psi\rangle \qquad (z+w)|\Psi\rangle = z|\Psi\rangle + w|\Phi\rangle \qquad \langle\Psi|\Phi\rangle = \langle\Phi|\Psi\rangle^*$$

$$\langle\Psi|(|\Phi\rangle + |\Upsilon\rangle) = \langle\Psi|\Phi\rangle + \langle\Psi|\Upsilon\rangle \qquad z\langle\Psi|\Phi\rangle = \langle\Psi|(z|\Phi\rangle) \qquad \|\Psi\|^2 = \langle\Psi|\Psi\rangle$$

Bases e componentes

$$\langle \mathrm{e}_j|\mathrm{e}_k\rangle = \delta_{jk} \qquad |\Psi\rangle = \sum_{j=1}^{n} \mathit{\Psi}_j|\mathrm{e}_j\rangle \qquad \mathit{\Psi}_j = \langle \mathrm{e}_j|\Psi\rangle \qquad \langle\Psi| = \sum_{j=1}^{n} \mathit{\Psi}_j^*\langle \mathrm{e}_j|$$

$$\langle\Psi|\Phi\rangle = \sum_{j=1}^{n} \mathit{\Psi}_j^*\mathit{\Phi}_j \qquad \|\Psi\|^2 = \sum_{j=1}^{n} |\mathit{\Psi}_j|^2 \qquad \text{(igual a 1 nas próximas linhas)}$$

$$\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n} \mathit{\Omega}_{jk}\mathit{\Psi}_k|\mathrm{e}_j\rangle \qquad \mathit{\Omega}_{jk} = \langle \mathrm{e}_j|\hat{\Omega}|\mathrm{e}_k\rangle \qquad \langle\hat{\Omega}\rangle = \langle\Psi|\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n} \mathit{\Omega}_{jk}\mathit{\Psi}_j^*\mathit{\Psi}_k$$

Representação matricial

$$|\Psi\rangle = \begin{bmatrix} \mathit{\Psi}_1 \\ \mathit{\Psi}_2 \\ \vdots \\ \mathit{\Psi}_n \end{bmatrix} \qquad \langle\Psi| = \begin{bmatrix} \mathit{\Psi}_1^* & \mathit{\Psi}_2^* & \dots & \mathit{\Psi}_n^* \end{bmatrix} \qquad \hat{\Omega} = \begin{bmatrix} \mathit{\Omega}_{11} & \mathit{\Omega}_{12} & \dots & \mathit{\Omega}_{1n} \\ \mathit{\Omega}_{21} & \mathit{\Omega}_{22} & \dots & \mathit{\Omega}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathit{\Omega}_{n1} & \mathit{\Omega}_{n2} & \dots & \mathit{\Omega}_{nn} \end{bmatrix}$$

Valores e vetores próprios

$$\hat{\Omega}\,|\Omega_j\rangle = \mathit{\Omega}_j\,|\Omega_j\rangle \qquad \begin{vmatrix} \mathit{\Omega}_{11}-\lambda & \mathit{\Omega}_{12} & \dots & \mathit{\Omega}_{1n} \\ \mathit{\Omega}_{21} & \mathit{\Omega}_{22}-\lambda & \dots & \mathit{\Omega}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathit{\Omega}_{n1} & \mathit{\Omega}_{n2} & \dots & \mathit{\Omega}_{nn}-\lambda \end{vmatrix} = 0$$

Síntese de mecânica analítica

$$\{F,G\} = \frac{\partial F}{\partial s}\frac{\partial G}{\partial p} - \frac{\partial F}{\partial p}\frac{\partial G}{\partial s} \qquad \frac{\mathrm{d}\,F}{\mathrm{d}\,t} = \{F,H\}$$

Observáveis (operadores hermíticos)

$\mathit{\Omega}_{jk} = \mathit{\Omega}_{kj}^*$ Valores próprios reais: $\mathit{\Omega}_j = \mathit{\Omega}_j^*$ Base ortonormal: $\langle \mathit{\Omega}_j|\mathit{\Omega}_k\rangle = \delta_{jk}$

Possíveis valores medidos: $\mathit{\Omega}_k$

Probabilidade de medir $\mathit{\Omega}_k$: $\mathrm{P}_k = |\langle\Omega_k|\Psi\rangle|^2$

Estado após medir $\mathit{\Omega}_k$: $|\Omega_k\rangle$

Comutadores

$$[\hat{\Omega},\hat{\Lambda}] = \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega} \qquad [\hat{\Omega},\hat{\Lambda}] = -[\hat{\Lambda},\hat{\Omega}] \qquad [\hat{\Omega}\,\hat{\Lambda},\hat{\Gamma}] = \hat{\Omega}[\hat{\Lambda},\hat{\Gamma}] + [\hat{\Omega},\hat{\Gamma}]\hat{\Lambda}$$

$$[\hat{\mathrm{F}},\hat{\mathrm{G}}] = \mathrm{i}\,\hbar\{F,G\} \qquad [\hat{\mathrm{x}},\hat{\mathrm{p}}] = \mathrm{i}\,\hbar \qquad \hbar = 1.055\times 10^{-34}\ \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}}$$

Oscilador harmónico

$$\hat{\mathrm{H}} = \frac{1}{2\,m}\hat{\mathrm{p}}^2 + \frac{m\,\omega^2}{2}\hat{\mathrm{x}}^2 \qquad \hat{\mathrm{a}} = \sqrt{\frac{m\,\omega}{2\,\hbar}}\left(\hat{\mathrm{x}} + \frac{\mathrm{i}}{m\,\omega}\hat{\mathrm{p}}\right) \qquad \hat{\mathrm{a}}^\dagger = \sqrt{\frac{m\,\omega}{2\,\hbar}}\left(\hat{\mathrm{x}} - \frac{\mathrm{i}}{m\,\omega}\hat{\mathrm{p}}\right)$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad (n = 0, 1, \ldots) \quad \hat{\mathrm{a}}\,|\mathrm{E}_n\rangle = \sqrt{\frac{E_n}{\hbar\,\omega} - \frac{1}{2}}\;|\mathrm{E}_{n-1}\rangle \quad \hat{\mathrm{a}}^\dagger|\mathrm{E}_n\rangle = \sqrt{\frac{E_n}{\hbar\,\omega} + \frac{1}{2}}\;|\mathrm{E}_{n+1}\rangle$$

Matrizes de Pauli

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \hat{\sigma}_y = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \qquad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\langle \mathrm{z}_+|\mathrm{z}_-\rangle = 0 \qquad \langle \mathrm{z}_+|\mathrm{z}_+\rangle = \langle \mathrm{z}_-|\mathrm{z}_-\rangle = 1 \qquad \hat{\sigma}_z|\mathrm{z}_\pm\rangle = \pm|\mathrm{z}_\pm\rangle$$

$$\hat{\sigma}_x|\mathrm{z}_k\rangle = |\mathrm{z}_{-k}\rangle \qquad \hat{\sigma}_y|\mathrm{z}_k\rangle = -k\,\mathrm{i}|\mathrm{z}_{-k}\rangle \qquad (k = +, -)$$

$$\hat{\sigma}_j\hat{\sigma}_k = -\hat{\sigma}_k\hat{\sigma}_j \qquad \hat{\sigma}_x\hat{\sigma}_y = \mathrm{i}\hat{\sigma}_z \qquad \hat{\sigma}_y\hat{\sigma}_z = \mathrm{i}\hat{\sigma}_x \qquad \hat{\sigma}_z\hat{\sigma}_x = \mathrm{i}\hat{\sigma}_y$$

Sistemas de dois spins

Base ortonormal:

$$\{|z_+, z_+\rangle, |z_+, z_-\rangle, |z_-, z_+\rangle, |z_-, z_-\rangle\} \qquad \langle \mathrm{z}_j, \mathrm{z}_k|\mathrm{z}_l, \mathrm{z}_m\rangle = \delta_{jl}\delta_{km} \qquad (j, k, l, m = +, -)$$

$$\hat{\sigma}_{1z}\hat{\sigma}_{2z}|\mathrm{z}_j, \mathrm{z}_k\rangle = jk|\mathrm{z}_j, \mathrm{z}_k\rangle \qquad \hat{\sigma}_{1x}\hat{\sigma}_{2x}|\mathrm{z}_j, \mathrm{z}_k\rangle = |\mathrm{z}_{-j}, \mathrm{z}_{-k}\rangle \qquad \text{etc} \ldots$$

Forma geral dos estados: $|\Upsilon\rangle = \varUpsilon_{++}|\mathrm{z}_+, \mathrm{z}_+\rangle + \varUpsilon_{+-}|\mathrm{z}_+, \mathrm{z}_-\rangle + \varUpsilon_{-+}|\mathrm{z}_-, \mathrm{z}_+\rangle + \varUpsilon_{--}|\mathrm{z}_-, \mathrm{z}_-\rangle$

Dois estados independentes: $|\Psi\rangle = \Psi_+|\mathrm{z}_+\rangle + \Psi_-|\mathrm{z}_+\rangle \qquad |\Phi\rangle = \Phi_+|\mathrm{z}_+\rangle + \Phi_-|\mathrm{z}_+\rangle$

$$|\Psi, \Phi\rangle = \Psi_+\Phi_+|\mathrm{z}_+, \mathrm{z}_+\rangle + \Psi_+\Phi_-|\mathrm{z}_+, \mathrm{z}_-\rangle + \Psi_-\Phi_+|\mathrm{z}_-, \mathrm{z}_+\rangle + \Psi_-\Phi_-|\mathrm{z}_-, \mathrm{z}_-\rangle$$

Estados entrelaçados: $\dfrac{\varUpsilon_{++}}{\varUpsilon_{+-}} \neq \dfrac{\varUpsilon_{-+}}{\varUpsilon_{--}}$ ou: $\dfrac{\varUpsilon_{++}}{\varUpsilon_{-+}} \neq \dfrac{\varUpsilon_{+-}}{\varUpsilon_{--}}$

Matrizes: $|\Upsilon\rangle = \begin{bmatrix} \varUpsilon_{++} \\ \varUpsilon_{+-} \\ \varUpsilon_{-+} \\ \varUpsilon_{--} \end{bmatrix} \qquad \langle\Upsilon| = \begin{bmatrix} \varUpsilon^*_{++} & \varUpsilon^*_{+-} & \varUpsilon^*_{-+} & \varUpsilon^*_{+-} \end{bmatrix}$

$$\hat{\Omega} = \begin{bmatrix} \varOmega_{++} & \varOmega_{+-} \\ \varOmega_{-+} & \varOmega_{--} \end{bmatrix} \quad \hat{\Lambda} = \begin{bmatrix} \varLambda_{++} & \varLambda_{+-} \\ \varLambda_{-+} & \varLambda_{--} \end{bmatrix} \quad \hat{\Omega}_1\hat{\Lambda}_2 = \begin{bmatrix} \varOmega_{++}\varLambda_{++} & \varOmega_{++}\varLambda_{+-} & \varOmega_{+-}\varLambda_{++} & \varOmega_{+-}\varLambda_{+-} \\ \varOmega_{++}\varLambda_{-+} & \varOmega_{++}\varLambda_{--} & \varOmega_{+-}\varLambda_{-+} & \varOmega_{+-}\varLambda_{--} \\ \varOmega_{-+}\varLambda_{++} & \varOmega_{-+}\varLambda_{+-} & \varOmega_{--}\varLambda_{++} & \varOmega_{--}\varLambda_{+-} \\ \varOmega_{-+}\varLambda_{-+} & \varOmega_{-+}\varLambda_{--} & \varOmega_{--}\varLambda_{-+} & \varOmega_{--}\varLambda_{--} \end{bmatrix}$$