

Formulário de Mecânica Quântica

Espaços vetoriais com coeficientes complexos

$$\begin{array}{lll}
 |\Psi\rangle + (|\Phi\rangle + |\Upsilon\rangle) = (|\Psi\rangle + |\Phi\rangle) + |\Upsilon\rangle & |\Psi\rangle + |\Phi\rangle = |\Phi\rangle + |\Psi\rangle \\
 |\Psi\rangle + |0\rangle = |\Psi\rangle & |\Psi\rangle + |- \Psi\rangle = |0\rangle & z(|\Psi\rangle + |\Phi\rangle) = z|\Psi\rangle + z|\Phi\rangle \\
 z(w|\Psi\rangle) = zw|\Psi\rangle & (z+w)|\Psi\rangle = z|\Psi\rangle + w|\Phi\rangle & \langle\Psi|\Phi\rangle = \langle\Phi|\Psi\rangle^* \\
 \langle\Psi|(|\Phi\rangle + |\Upsilon\rangle) = \langle\Psi|\Phi\rangle + \langle\Psi|\Upsilon\rangle & z\langle\Psi|\Phi\rangle = \langle\Psi|(z|\Phi\rangle) & \|\Psi\|^2 = \langle\Psi|\Psi\rangle
 \end{array}$$

Bases e componentes

$$\begin{aligned}
 \langle e_j | e_k \rangle &= \delta_{jk} & |\Psi\rangle &= \sum_{j=1}^n \Psi_j |e_j\rangle & \Psi_j &= \langle e_j | \Psi \rangle & \langle \Psi | &= \sum_{j=1}^n \Psi_j^* \langle e_j |
 \end{aligned}$$

$$\langle \Psi | \Phi \rangle = \sum_{j=1}^n \Psi_j^* \Phi_j \quad \|\Psi\|^2 = \sum_{j=1}^n |\Psi_j|^2 \quad (\text{igual a 1 nas próximas linhas})$$

$$\hat{\Omega} |\Psi\rangle = \sum_{j=1}^n \sum_{k=1}^n \Omega_{jk} \Psi_k |e_j\rangle \quad \Omega_{jk} = \langle e_j | \hat{\Omega} | e_k \rangle \quad \langle \hat{\Omega} \rangle = \langle \Psi | \hat{\Omega} | \Psi \rangle = \sum_{j=1}^n \sum_{k=1}^n \Omega_{jk} \Psi_j^* \Psi_k$$

Representação matricial

$$|\Psi\rangle = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{bmatrix} \quad \langle \Psi | = [\Psi_1^* \ \Psi_2^* \ \dots \ \Psi_n^*] \quad \hat{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \dots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} & \dots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn} \end{bmatrix}$$

Valores e vetores próprios

$$\hat{\Omega} |\Omega_j\rangle = \Omega_j |\Omega_j\rangle \quad \begin{vmatrix} \Omega_{11} - \lambda & \Omega_{12} & \dots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} - \lambda & \dots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn} - \lambda \end{vmatrix} = 0$$

Síntese de mecânica analítica

$$\{F, G\} = \frac{\partial F}{\partial s} \frac{\partial G}{\partial p} - \frac{\partial F}{\partial p} \frac{\partial G}{\partial s} \quad \frac{dF}{dt} = \{F, H\}$$

Observáveis (operadores hermíticos)

$\Omega_{jk} = \Omega_{kj}^*$ Valores próprios reais: $\Omega_j = \Omega_j^*$ Base ortonormal: $\langle \Omega_j | \Omega_k \rangle = \delta_{jk}$

Possíveis valores medidos: Ω_k

Probabilidade de medir Ω_k : $P_k = |\langle \Omega_k | \Psi \rangle|^2$

Estado após medir Ω_k : $|\Omega_k\rangle$

Comutadores

$$\begin{array}{lll}
 [\hat{\Omega}, \hat{\Lambda}] = \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega} & [\hat{\Omega}, \hat{\Lambda}] = -[\hat{\Lambda}, \hat{\Omega}] & [\hat{\Omega}, \hat{\Lambda}, \hat{\Gamma}] = \hat{\Omega}[\hat{\Lambda}, \hat{\Gamma}] + [\hat{\Omega}, \hat{\Gamma}]\hat{\Lambda} \\
 [\hat{F}, \hat{G}] = i\hbar\{F, G\} & [\hat{x}, \hat{p}] = i\hbar & \hbar = 1.055 \times 10^{-34} \frac{\text{kg} \cdot \text{m}^2}{\text{s}}
 \end{array}$$

Oscilador harmónico

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{m\omega^2}{2}\hat{x}^2 \quad \hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right) \quad \hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad (n = 0, 1, \dots) \quad \hat{a}|E_n\rangle = \sqrt{\frac{E_n}{\hbar\omega} - \frac{1}{2}}|E_{n-1}\rangle \quad \hat{a}^\dagger|E_n\rangle = \sqrt{\frac{E_n}{\hbar\omega} + \frac{1}{2}}|E_{n+1}\rangle$$

Matrizes de Pauli

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\langle z_+ | z_- \rangle = 0 \quad \langle z_+ | z_+ \rangle = \langle z_- | z_- \rangle = 1 \quad \hat{\sigma}_z | z_\pm \rangle = \pm | z_\pm \rangle$$

$$\hat{\sigma}_x | z_k \rangle = | z_{-k} \rangle \quad \hat{\sigma}_y | z_k \rangle = -k i | z_{-k} \rangle \quad (k = +, -)$$

$$\hat{\sigma}_j \hat{\sigma}_k = -\hat{\sigma}_k \hat{\sigma}_j \quad \hat{\sigma}_x \hat{\sigma}_y = i \hat{\sigma}_z \quad \hat{\sigma}_y \hat{\sigma}_z = i \hat{\sigma}_x \quad \hat{\sigma}_z \hat{\sigma}_x = i \hat{\sigma}_y$$

Sistemas de dois spins

Base ortonormal:

$$\{|z_+, z_+\rangle, |z_+, z_-\rangle, |z_-, z_+\rangle, |z_-, z_-\rangle\} \quad \langle z_j, z_k | z_l, z_m \rangle = \delta_{jl}\delta_{km} \quad (j, k, l, m = +, -)$$

$$\hat{\sigma}_{1z}\hat{\sigma}_{2z}|z_j, z_k\rangle = jk|z_j, z_k\rangle \quad \hat{\sigma}_{1x}\hat{\sigma}_{2x}|z_j, z_k\rangle = |z_{-j}, z_{-k}\rangle \quad \text{etc...}$$

Forma geral dos estados: $|\Upsilon\rangle = \Upsilon_{++}|z_+, z_+\rangle + \Upsilon_{+-}|z_+, z_-\rangle + \Upsilon_{-+}|z_-, z_+\rangle + \Upsilon_{--}|z_-, z_-\rangle$

Dois estados independentes: $|\Psi\rangle = \Psi_+|z_+\rangle + \Psi_-|z_-\rangle \quad |\Phi\rangle = \Phi_+|z_+\rangle + \Phi_-|z_-\rangle$

$$|\Psi, \Phi\rangle = \Psi_+\Phi_+|z_+, z_+\rangle + \Psi_+\Phi_-|z_+, z_-\rangle + \Psi_-\Phi_+|z_-, z_+\rangle + \Psi_-\Phi_-|z_-, z_-\rangle$$

Estados entrelaçados: $\frac{\Upsilon_{++}}{\Upsilon_{+-}} \neq \frac{\Upsilon_{-+}}{\Upsilon_{--}}$ ou: $\frac{\Upsilon_{++}}{\Upsilon_{-+}} \neq \frac{\Upsilon_{+-}}{\Upsilon_{--}}$

Matrizes: $|\Upsilon\rangle = \begin{bmatrix} \Upsilon_{++} \\ \Upsilon_{+-} \\ \Upsilon_{-+} \\ \Upsilon_{--} \end{bmatrix} \quad \langle \Upsilon | = [\Upsilon_{++}^* \quad \Upsilon_{+-}^* \quad \Upsilon_{-+}^* \quad \Upsilon_{--}^*]$

$$\hat{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \quad \hat{\Lambda} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \quad \hat{\Omega}_1 \hat{\Lambda}_2 = \begin{bmatrix} \Omega_{11}\Lambda_{11} & \Omega_{11}\Lambda_{12} & \Omega_{12}\Lambda_{11} & \Omega_{12}\Lambda_{12} \\ \Omega_{11}\Lambda_{21} & \Omega_{11}\Lambda_{22} & \Omega_{12}\Lambda_{21} & \Omega_{12}\Lambda_{22} \\ \Omega_{21}\Lambda_{11} & \Omega_{21}\Lambda_{12} & \Omega_{22}\Lambda_{11} & \Omega_{22}\Lambda_{12} \\ \Omega_{21}\Lambda_{21} & \Omega_{21}\Lambda_{22} & \Omega_{22}\Lambda_{21} & \Omega_{22}\Lambda_{22} \end{bmatrix}$$