

Sumário de Mecânica Quântica

Espaços vetoriais com coeficientes complexos

$$\begin{aligned} |\Psi\rangle + (|\Phi\rangle + |\Upsilon\rangle) &= (|\Psi\rangle + |\Phi\rangle) + |\Upsilon\rangle & |\Psi\rangle + |\Phi\rangle &= |\Phi\rangle + |\Psi\rangle \\ |\Psi\rangle + |0\rangle &= |\Psi\rangle & |\Psi\rangle + |-\Psi\rangle &= |0\rangle & z(|\Psi\rangle + |\Phi\rangle) &= z|\Psi\rangle + z|\Phi\rangle \\ z(w|\Psi\rangle) &= zw|\Psi\rangle & (z+w)|\Psi\rangle &= z|\Psi\rangle + w|\Psi\rangle & \langle\Psi|\Phi\rangle &= \langle\Phi|\Psi\rangle^* \\ \langle\Psi|(|\Phi\rangle + |\Upsilon\rangle) &= \langle\Psi|\Phi\rangle + \langle\Psi|\Upsilon\rangle & z\langle\Psi|\Phi\rangle &= \langle\Psi|(z|\Phi\rangle) & \langle\Psi|\Psi\rangle &= ||\Psi\rangle|^2 \geq 0 \end{aligned}$$

Bases e componentes

$$\begin{aligned} \langle e_j | e_k \rangle &= \delta_{jk} & |\Psi\rangle &= \sum_{j=1}^n \Psi_j |e_j\rangle & \Psi_j &= \langle e_j | \Psi \rangle & \langle \Psi | &= \sum_{j=1}^n \Psi_j^* \langle e_j | \\ \langle \Psi | \Phi \rangle &= \sum_{j=1}^n \Psi_j^* \Phi_j & \langle \Psi | \Psi \rangle &= \sum_{j=1}^n |\Psi_j|^2 = ||\Psi\rangle|^2 & & \text{(igual a 1 nas próximas linhas)} \\ \hat{\Omega}|\Psi\rangle &= \sum_{j=1}^n \sum_{k=1}^n \Omega_{jk} \Psi_k |e_j\rangle & \Omega_{jk} &= \langle e_j | \hat{\Omega} | e_k \rangle & \langle \hat{\Omega} \rangle &= \langle \Psi | \hat{\Omega} | \Psi \rangle = \sum_{j=1}^n \sum_{k=1}^n \Omega_{jk} \Psi_j^* \Psi_k \end{aligned}$$

Representação matricial

$$|\Psi\rangle = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{bmatrix} \quad \langle \Psi | = [\Psi_1^* \ \Psi_2^* \ \dots \ \Psi_n^*] \quad \hat{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \dots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} & \dots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn} \end{bmatrix}$$

Valores e vetores próprios

$$\hat{\Omega}|\lambda\rangle = \lambda |\lambda\rangle \quad \begin{vmatrix} \Omega_{11} - \lambda & \Omega_{12} & \dots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} - \lambda & \dots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn} - \lambda \end{vmatrix} = 0$$

Observáveis (operadores hermíticos)

$$\begin{aligned} \Omega_{jk} &= \Omega_{kj}^* & \text{Valores próprios reais: } \lambda &= \lambda^* & \text{Base ortonormal: } \langle \lambda_j | \lambda_k \rangle &= \delta_{jk} \\ \text{Possíveis valores medidos: } & \lambda_k \\ \text{Probabilidade de medir } \lambda_k: & P_k = \langle \Psi | \lambda_k \rangle \langle \lambda_k | \Psi \rangle \\ \text{Estado após medir } \lambda_k: & |\lambda_k\rangle \end{aligned}$$

Comutadores

$$\begin{aligned} [\hat{\Omega}, \hat{\Lambda}] &= \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega} & [\hat{x}, \hat{p}] &= i\hbar & \hbar &= 1.055 \times 10^{-34} \frac{\text{kg} \cdot \text{m}^2}{\text{s}} \\ [\hat{\Omega}, \hat{\Lambda}] &= -[\hat{\Lambda}, \hat{\Omega}] & [\hat{\Omega}, \hat{\Omega}] &= 0 & \hat{\Omega}\hat{\Lambda} &= \hat{\Lambda}\hat{\Omega} + [\hat{\Omega}, \hat{\Lambda}] \end{aligned}$$

Matrizes de Pauli

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{\sigma}_j \hat{\sigma}_k = -\hat{\sigma}_k \hat{\sigma}_j$$

$$\hat{\sigma}_x \hat{\sigma}_y = i \hat{\sigma}_z$$

$$\hat{\sigma}_y \hat{\sigma}_z = i \hat{\sigma}_x$$

$$\hat{\sigma}_z \hat{\sigma}_x = i \hat{\sigma}_y$$

Equação de Schrödinger

$$i \hbar \frac{d|\Psi\rangle}{dt} = \hat{H}|\Psi\rangle$$

$$\hat{H}|E_j\rangle = E_j|E_j\rangle$$

$$|\Psi(0)\rangle = \sum_{j=1}^n \Psi_j |E_j\rangle$$

Oscilador harmónico

$$\hat{H} = \frac{1}{2m} \hat{p}^2 + \frac{k}{2} \hat{x}^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$E_j = \hbar \omega \left(j + \frac{1}{2} \right) \quad (j = 0, 1, 2, \dots)$$

Combinação de dois sistemas

$$|\Psi, \Phi\rangle = |\Psi\rangle \otimes |\Phi\rangle$$

$$\langle e_j, e_k | e_r, e_s \rangle = \delta_{jr} \delta_{ks}$$

$$|\Psi, \Phi\rangle = \sum_{j=1}^n \sum_{r=1}^m \Psi_j \Phi_r |e_j, e_r\rangle$$

$$(\hat{\Omega} \otimes \hat{\Lambda}) |\Psi, \Phi\rangle = \hat{\Omega} |\Psi\rangle \otimes \hat{\Lambda} |\Phi\rangle = \sum_{j=1}^n \sum_{r=1}^m \Omega_{jk} \Lambda_{rs} \Psi_j \Phi_r |e_j, e_r\rangle$$

$$\text{Estados entrelaçados: } |\Upsilon\rangle = \sum_{j=1}^n \sum_{r=1}^m \Upsilon_{jr} |e_j, e_r\rangle$$

$$(\hat{\Omega} \otimes \hat{\Lambda}) |\Upsilon\rangle = \sum_{j=1}^n \sum_{r=1}^m \Omega_{jk} \Lambda_{rs} \Upsilon_{rs} |e_j, e_r\rangle$$