

Assignment - 4

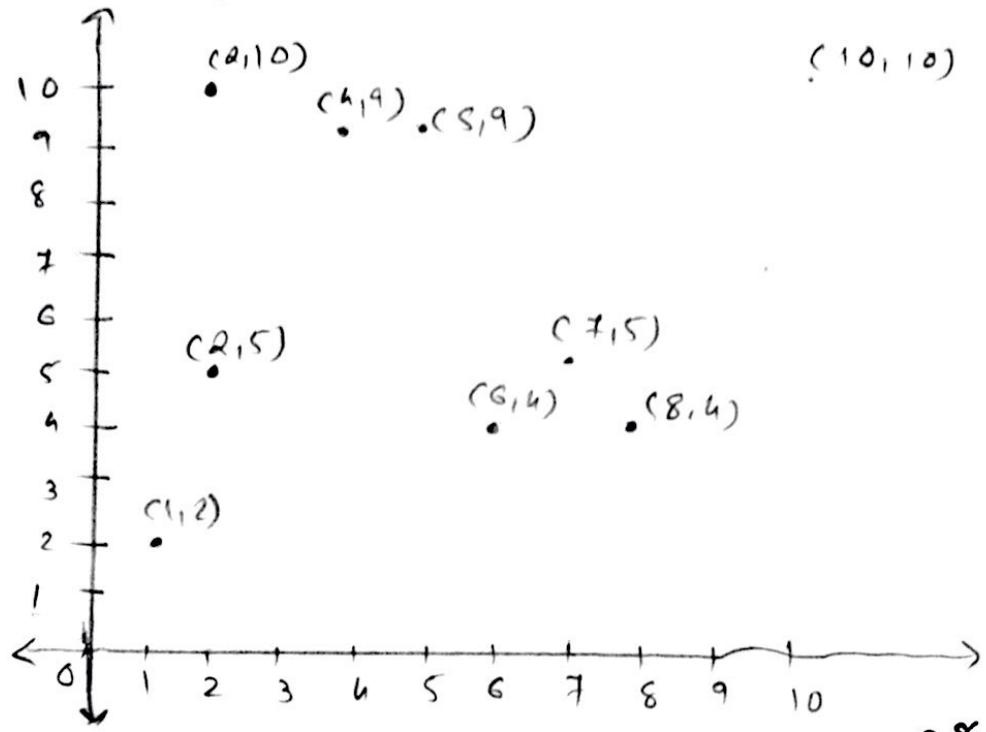
Question - A

k-means Algo : clustering

2D points:

$(2, 10), (2, 5), (8, 4), (5, 9), (7, 5), (6, 4)$
 $(1, 2), (4, 9), (10, 10)$

(i)



Initial cluster centers: $(2, 5), (5, 8), (4, 9)$

(ii) find Euclidean distance betⁿ points and cluster centers

	$(2, 10)$	$(2, 5)$	$(8, 4)$	$(5, 9)$	$(7, 5)$	$(6, 4)$	$(1, 2)$	$(4, 9)$	$(10, 10)$
$(2, 5)$	5	0	6.08	5	5	4.12	3.16	4.47	9.43
$(5, 8)$	3.60	4.24	5	1	3.60	6.12	7.21	1.41	5.38
$(4, 9)$	2.23	4.47	6.40	1	5	5.38	7.61	0	6.08

Initial clusters are:

$$C_1 : (3, 2), (2, 5)$$

$$C_2 : (8, 4), (7, 5), (10, 10), (6, 4)$$

$$C_3 : (2, 10), (5, 9), (4, 9)$$

(iii) New cluster centers:

$$\mu_1 = \left(\frac{2+1}{2}, \frac{5+2}{2} \right) = (1.5, 3.5)$$

$$\mu_2 = \left(\frac{8+7+10+6}{4}, \frac{4+5+10+4}{4} \right) = (7.75, 8.25)$$

$$\mu_3 = \left(\frac{4+2+5}{3}, \frac{9+10+9}{3} \right) = (3.67, 9.33)$$

New distances from given points to New cluster centroids

	$(2, 10)$	$(2, 5)$	$(8, 4)$	$(5, 9)$	$(7, 5)$	$(6, 4)$	$(1, 2)$	$(4, 9)$	$(10, 10)$
$(1.5, 3.5)$	6.51	1.58	6.57	6.51	5.7	4.52	1.78	6.09	10.70
$(7.75, 8.25)$	7.15	5.79	1.76	4.25	7.06	2.47	7.72	4.96	6.80
$(3.67, 9.33)$	1.79	4.64	6.87	1.37	5.46	5.82	7.80	0.467	6.36

$$\mu_1 : (2, 5), (1, 2)$$

$$\mu_2 : (8, 4), (7, 5), (6, 4), (10, 10)$$

$$\mu_3 : (2, 10), (5, 9), (4, 9)$$

New cluster centers:

$$\mu_1 : (1.5, 3.5)$$

$$\mu_2 : (7.75, 8.25)$$

$$\mu_3 : (3.67, 9.33)$$

Here cluster centers are not changing so these are final cluster assignments.

Question-B

	P ₁	P ₂	P ₃	P ₄	P ₅
P ₁	1.00	0.10	0.41	0.55	0.35
P ₂	0.10	1.00	0.64	0.47	0.98
P ₃	0.41	0.64	1.00	0.44	0.85
P ₄	0.55	0.47	0.44	1.00	0.76
P ₅	0.35	0.98	0.85	0.76	1.00

1. single link: maximum similarity betn any two points are selected first.

order of merge

1. maximum similarity is 0.98 betn P₂ & P₅ so, they are merged together.
so, (P₂, P₅)

	P ₁	P ₂ P ₅	P ₃	P ₄
P ₁	1.00	0.35	0.41	0.55
P ₂ P ₅		1.00	0.85	0.76
P ₃			1.00	0.85
P ₄				1.00

$$\max(P_1, P_2 P_5) = \max(0.10, 0.35) = 0.35$$

$$\max(P_3, P_2 P_5) = \max(0.64, 0.85) = 0.85$$

$$\max(P_4, P_2 P_5) = \max(0.47, 0.76) = 0.76$$

$$\max(0.35, 0.85, 0.76) = 0.85$$

so, P₃, P₂, P₅ merge together.

	P ₁	P ₂ P ₃ P ₅	P ₄
P ₁	1.0	0.41	0.55
P ₂ P ₃ P ₅		1.0	0.76
P ₄			1.0

$$\max(P_1, P_2P_3P_5, P_1) = \max(\max(P_1, P_2P_5), P_3) \\ = \max(0.35, 0.41) \\ = 0.41$$

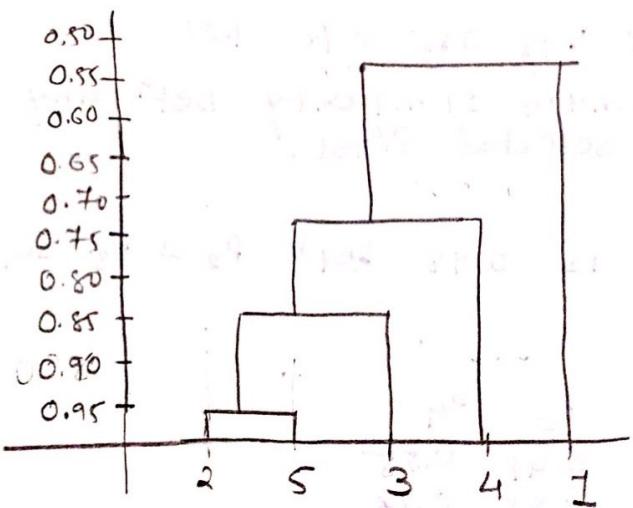
$$\max(P_4, P_2P_3P_5) = \max((P_4, P_2P_5), (P_4, P_3)) \\ = \max(0.76, 0.84) \\ = 0.84$$

$$\max(0.41, 0.84) = \underline{0.84} \quad \underline{0.76}$$

so, we will merge P_2, P_3, P_4, P_5 .

$$\max(P_1, P_2P_3P_4P_5) = \max((P_1, P_2P_3P_5), (P_1, P_4)) \\ = \max(0.41, 0.55) \\ = \underline{0.55}$$

Final dendrogram:



2. complete link

maximum similarity is 0.98 betw P₂ & P₅. so they are merged.

$$\min(P_1, P_2P_5) = \min(0.10, 0.35) = 0.10$$

$$\min(P_3, P_2P_5) = \min(0.64, 0.84) = 0.64$$

$$\min(P_4, P_2P_5) = \min(0.47, 0.76) = 0.47$$

	P ₁	P ₂	P ₅	P ₃	P ₄
P ₁	1.0	0.10	0.35		
P _{2P5}		1.0	0.64	0.47	
P ₃			1.0	0.44	
P ₄				1.0	

$$\max_{\text{min}}(0.10, 0.64, 0.47) = \underline{0.64} \quad 0.64$$

so, P₂, P₃, P₅ are merged. → 0.64 //

$$\min(p_1, p_3, p_2, p_5) = \min(0.10, 0.44) = 0.10$$

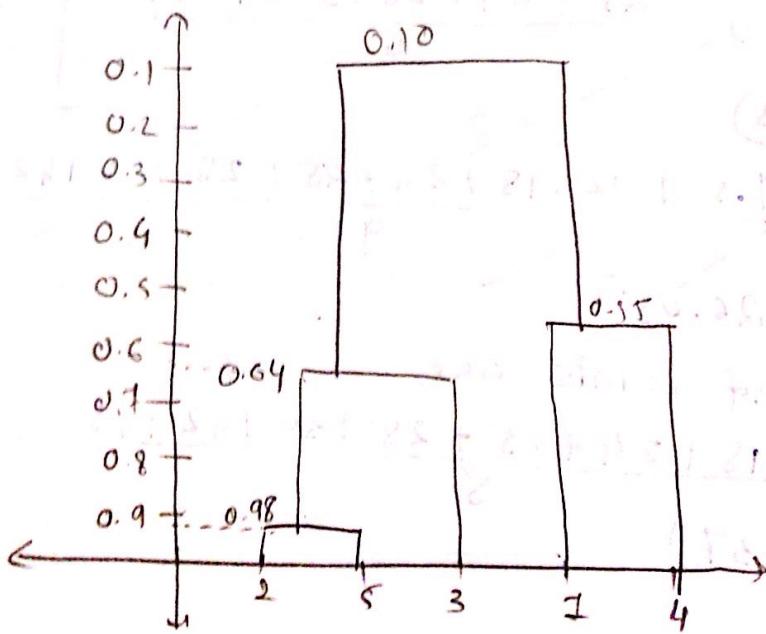
$$\min(p_4, p_1, p_2, p_3, p_5) = \min(0.64, 0.44) = 0.44$$
~~$$\min(p_1, p_4) = 0.55$$~~

$$\max(0.10, 0.44, 0.55) = 0.55$$

so, p_1 & p_4 will be merged.

	p_2, p_3, p_5	p_1, p_4
p_2, p_3, p_5	1.0	0.10
p_1, p_4		1.0

final dendrogram:



Question - c

{ 6, 12, 18, 24, 25, 28, 30, 42, 48 }

9.	①	6	12	18	24	25	28	30	42	48
5	1	7	13	19	20	23	25	37	43	
7.5	0.5	4.5	10.5	16.5	17.5	20.5	23.5	24.5	40.5	

cluster 1 : 5, 6

cluster 2 : 7.5, 12, 18, 24, 25, 28, 30, 42, 48

sum of squared error for C1: $0^2 + 1^2 = 1$

$$\begin{aligned} C_2 &= 1.5^2 + 4.5^2 + 10.5^2 + 16.5^2 + \\ &\quad 17.5^2 + 20.5^2 + 22.5^2 + \\ &\quad 34.5^2 + 40.5^2 \\ &= 1879.35 \end{aligned}$$

② (15, 25)

	C	12	18	24	25	28	30	42	48
15	9	3	3	9	10	13	15	27	33
25	19	13	7	1	0	3	5	17	23

C1 : 15, 6, 18, 12

C2 : 25, 7.5, 24, 28, 30, 42, 48

sum of squared error for C1: $0^2 + 9^2 + 3^2 = 90 + 9 = 99$

C2: $(0^2 + 1^2 + 3^2 + 5^2 + 17^2) = 853$

New centroid points are:

$$\frac{15+6+18+12}{4}$$

$$\frac{25+24+28+30+42+48}{6}$$

$$\therefore (12.75, 32.83)$$

$$\frac{5+6}{2}$$

$$\frac{7.5+12+18+24+25+28+30+42+48}{9}$$

$$(5.5, 26.05)$$

for (15, 25)

for

(5, 7.5)

b. New centroid points are

$$① \quad \frac{6+12+18+24+25+28+30+42+48}{8} \quad \text{for } (5, 7.5) \text{ set}$$

$$\therefore (6, 28.37)$$

$$\textcircled{2} \quad \frac{6+18+12}{3}, \frac{24+28+30+42+48}{5} \quad \text{for } (15, 25)$$

$\therefore (12, 32.8)$

	6	12	18	24	25	28	30	42	48
6	0	6	12	18	19	22	24	36	42
28.37	22.37	16.37	10.37	4.37	3.37	0.37	1.63	13.63	20.37

$C_1 : 6, 12,$

$C_2 : 28.37, 18, 24, 25, 30, 42, 48, 28$

Here 12 is included in cluster 1 which was previously not stable

	6	12	18	24	25	28	30	42	48
12	6	0	6	12	13	16	18	30	36
32.83	26.8	20.83	14.13	8.83	7.83	4.83	2.83	9.17	15.17

$C_1 : 12, 6, 18,$

$C_2 : 34.4, 24, 25, 28, 30, 42, 48$

Here points are in the same clusters, they were previously in.

so No change

They are stable points

→ New points: (9, 30.74) centroid

To be continued for first set of Points (5, 7.5)

(d) MIN produces "most natural" clustering in this situation.

	6	12	18	24	25	28	30	42	48
9	3	3	9	15	16	19	21	33	39
30.71	24.71	18.71	12.71	6.71	5.71	2.71	0.71	11.29	17.29

C₁ : 9, 6, 12, 18

C₂ : 30.71, 24, 25, 28, 30, 42, 48

New centroid points: (12, 32.83)

	6	12	18	24	25	28	30	42	48
12	6	0	6	12	13	15	18	30	36
32.83	26.83	10.83	14.83	8.83	7.83	4.83	2.83	9.17	15.17

C₁ : 12, 6, 18

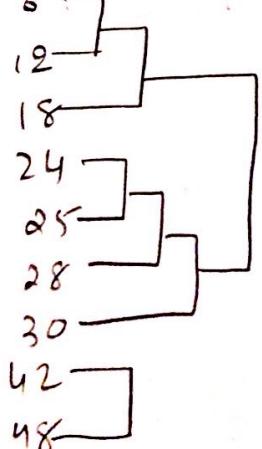
C₂ : 32.83, 24, 25, 28, 30, 42, 48

Here points are in the same clusters as they were in the previous iteration, but points 12 and 18 are in the second cluster in first iteration so, the points (5, 7.5) is not stable solution:

(e)

6	12	18	24	25	28	30	42	48
6	12	18	(24 25)	28	30	42	48	
6	12	18	(24 25 28)	30	42	48		
6	12	18	(24 25 28 30)	42	48			
6	(12 18)	(24 25 28 30)		42	48			
(6 12 18)	(24 25 28 30)			42	48			
(6 12 18)	(24 25 28 30)				42 48			
(6 12 18 24 25 28 30)						(42 48)		

TWO clusters: (6 12 18 24 25 28 30)
(42 48)



(d) mLN produces "most Natural" clustering in this situation.

(e) K-means is not good for finding clusters of diff. size when they are not clearly separated.

Reason: objective function is to minimize SSE
lets break it down large cluster into small ones

so K-means ~~solution~~ solution does not give natural clustering in this solution

part-II

(ii) Naive Bayes:

$$① 1. P(A|+) = P(A=1|+) = \frac{3}{5}$$
$$P(A=0|+) = \frac{2}{5}$$

$$2. P(B|+) = P(B=1|+) = \frac{1}{5}$$
$$P(B=0|+) = \frac{4}{5}$$

$$3. P(C|+) = P(C=1|+) = \frac{4}{5}$$
$$P(C=0|+) = \frac{1}{5}$$

$$4. P(A|-) = P(A=1|-) = \frac{2}{5}$$
$$P(A=0|-) = \frac{3}{5}$$

$$5. P(B|-) = P(B=1|-) = \frac{2}{5}$$
$$P(B=0|-) = \frac{3}{5}$$

$$6. P(C|-) = P(C=1|-) = 1$$
$$P(C=0|-) = 0$$

$$\begin{aligned}
 \textcircled{2} \quad P(+ | A=1, B=\Phi, C=0) &= \frac{P(A=1, B=\Phi, C=0 | +) \cdot P(+)}{P(A=1, B=\Phi, C=0)} \\
 &= \frac{P(+)}{P(A=1, B=\Phi, C=0)} \cdot \frac{P(A=1 | +) \cdot P(B=\Phi | +) \cdot P(C=0 | +)}{P(A=1, B=\Phi, C=0)} \\
 &= \frac{\frac{5}{10} \cdot \frac{3}{5} \cdot \frac{4}{5} + \frac{4}{5}}{P(A=1, B=\Phi, C=0)} = \frac{\cancel{4} \cancel{5} \cancel{10} \cancel{3} \cancel{5} \cancel{4} / 250}{\cancel{P(A=1, B=\Phi, C=0)}} \\
 &\quad \times (-1)^{1+2+3} = (-1)^6 = 1
 \end{aligned}$$

$$\begin{aligned}
 P(-|A=1, B=1, C=0) &= \frac{P(A=1, B=1, C=0) - P(-, B=1)}{P(A=1, B=1, C=0)} \\
 &= \frac{P(A=1|-) \cdot P(B=1|-) \cdot P(C=0|-) \cdot P(-)}{P(A=1, B=1, C=0)} \\
 &= \frac{2/5 \cdot 2/5 \cdot 0}{P(A=1, B=1, C=0)} \\
 &= 0
 \end{aligned}$$

So label of $(A=1, B=1, C=0)$ is positive ($\underline{+}$)

(3) $m=4, p=1/2$

$$1. P(A=1|+) = \frac{3 + 4 \cdot 1/2}{5 + 4} = \frac{5}{9} \quad \left(\because P(x_j=q_{jk}|C=c_i) = \frac{n_c + mp}{n + m} \right)$$

$\therefore n = \text{Total Number of Positive class}$
 $n_c = \text{Total Number of } A=1 \text{ values in Positive class.}$

$$2. P(A=1|-) = 4/9$$

$$3. P(A=0|+) = 4/9$$

$$4. P(A=0|-) = 5/9$$

$$5. P(B=1|+) = 3/9, \quad P(B=0|+) = 6/9$$

$$6. P(B=1|-) = 4/9, \quad P(B=0|-) = 5/9$$

$$7. P(C=1|+) = 6/9, \quad P(C=0|+) = 3/9$$

$$8. P(C=1|-) = 3/4, \quad P(C=0|-) = 1/4$$

$$4. P(+ | A=1, B=1, C=0) = \frac{P(A=1, B=1, C=0 | +) \cdot P(+)}{P(A=1, B=1, C=0)}$$

$$\Rightarrow \frac{P(+). P(A=1 | +). P(B=1 | +). P(C=0 | +)}{P(A=1, B=1, C=0)}$$

$$\Rightarrow \frac{\frac{5}{10} \cdot \frac{5}{9} \cdot \frac{3}{9} \cdot \frac{3}{9}}{P(A=1, B=1, C=0)} = \frac{\cancel{5}}{\cancel{216}} = \frac{0.0308}{0.0308}$$

$$P(- | A=1, B=1, C=0) = \frac{P(A=1, B=1, C=0 | -) \cdot P(-)}{P(A=1, B=1, C=0)}$$

$$\Rightarrow \frac{P(-) \cdot P(A=1 | -) \cdot P(B=1 | -) \cdot P(C=0 | -)}{P(A=1, B=1, C=0)}$$

$$\Rightarrow \frac{\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{4}{9} \cdot \frac{11}{4}}{P(A=1, B=1, C=0)} = \frac{\cancel{9}}{\cancel{81}} = 0.02469$$

So here $0.0308 > 0.02469$. So, (+) ~~Negative~~^{positive} class has high probability. So, $(A=1, B=1, C=0)$ has ~~negative~~^{positive} class.

(S) when one of the probabilities is zero, the estimate for conditional probabilities using m-estimate is better.

(ii) Ada Boosting

$$x_1 = (-1, 0, +), \quad x_2 = (-0.5, 0.5, +), \quad x_3 = (0, 1, -), \\ x_4 = (0.5, 1, -), \quad x_5 = (1, 0, +), \quad x_6 = (1, -1, +), \\ x_7 = (0, -1, -), \quad x_8 = (0, 0, -).$$

Weight for all points = $\frac{1}{\text{no. of points}}$
= $\frac{1}{8}$

Round-1

If $z > 0.75$ class - positive

$z \leq 0.75$ class - negative x_1, x_2 are misclassified

Training error 2 positives : $\frac{1}{8} + \frac{1}{8} = 0.25$
 $\varepsilon_1 = \frac{0.25}{1} = 0.25$

$$\alpha_1 = \frac{1}{2} \ln \left(\frac{1 - \varepsilon_1}{\varepsilon_1} \right) = \frac{1}{2} \ln \left(\frac{1 - 0.25}{0.25} \right) = 0.549$$

correct classified : $D_2(i) = \frac{0.125 \times \exp(-0.549)}{z_1}$
= $\frac{0.125 \times 0.577}{z_1}$
= $\frac{0.072125}{z_1}$

Incorrect classified $D_2(i) = \frac{0.125 \times \exp(\alpha_1)}{z_1}$ ($\because 0.125$ is initial weight)
= $\frac{0.125 \times \exp(0.549)}{z_1}$
= $\frac{0.125 \times 1.731}{z_1}$
= $\frac{0.216375}{z_1}$

No of correct classified $\times \frac{0.072}{z_1}$ + incorrect classified $\times \frac{0.216}{z_1}$

$$5 \times \frac{0.072}{z_1} + 2 \times \frac{0.216}{z_1}$$

$$= \frac{0.864}{z_1}$$

Total weight = 1 $\therefore z_1 = 0.864$

$$\text{Correct} = \frac{0.072}{0.864} = 0.083$$

$$\text{incorrect} = \frac{0.216}{0.864} = 0.25$$

Round 1

Hypothesis $\alpha_1 \vdash (x > 0.75) \Rightarrow 0.549 \vdash (x > 0.75)$

Round 2:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
-1, 0	-0.5, 0.5	0, 1	0.5, 1	1, 0	1, -1	0, -1	0, 0
+	+	-	-	+	+	-	-
0.25	0.25	0.08	0.08	0.08	0.08	0.08	0.08

Now take $x > -0.25$ - Negative
 $x \leq -0.25$ + Positive

Incorrectly classified: x_5 & x_6

$$\varepsilon_2 = 0.08 + 0.08 = 0.16$$

$$\alpha_2 = \frac{1}{2} \ln\left(\frac{1-\varepsilon_2}{\varepsilon_2}\right) = \frac{1}{2} \ln\left(\frac{1-0.16}{0.16}\right) = 0.829$$

$$\text{Correct classified } (x_1, x_2) = D_3(i) = \frac{0.25 \times 8(-0.829)}{z_2}$$
$$= \frac{0.109}{z_2}$$

$$\text{incorrect classified } (x_5, x_6) = D_3(i) = \frac{0.08 \times 8(0.829)}{z_2}$$
$$= \frac{0.183}{z_2}$$

$$\text{Correct classified } (x_3, x_4, x_8, x_7) = D_3(i) = \frac{0.08 \times 8(-0.829)}{z_2}$$
$$= \frac{0.034}{z_2}$$

Sum of weights should be 1

$$8 \times \frac{0.109}{z_2} + 2 \times \frac{0.183}{z_2} + 4 \times \frac{0.034}{z_2} = 1 \Rightarrow z_2 = 0.72$$

$$x_1, x_2 \rightarrow 0.109 / 0.72 = 0.151$$

$$x_5, x_6 \rightarrow 0.183 / 0.72 = 0.254$$

$$x_3, x_4, x_8, x_7 \rightarrow 0.034 / 0.72 = 0.047$$

Hypo: $0.829 \vdash (x \leq -0.25)$

Round-3

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
+	+	-	-	+	+	-	-
0.151	0.151	0.047	0.047	0.254	0.254	0.047	0.047

take $y > 0.75$ -
 $y \leq 0.75$ +

incorrect x_8, x_7

$$\varepsilon_3 = 0.047 + 0.047 = 0.094$$

$$z_3 = \frac{1}{2} \ln \left(\frac{1-0.094}{0.094} \right) = 1.132$$

$$\text{correct classified } = D_{\text{uci}} = \frac{0.047 \times \exp(-1.132)}{z_3} = \frac{0.015}{z_3}$$

x_3, x_4

$$\text{correct classified } = D_{\text{uci}} = \frac{0.151 \times \exp(-1.132)}{z_3} = \frac{0.048}{z_3}$$

$$\text{correct classified } D_{\text{uci}} = \frac{0.254 \times \exp(-1.132)}{z_3} = \frac{0.081}{z_3}$$

$$\text{incorrect classified } D_{\text{uci}} = \frac{0.047 \times \exp(1.132)}{z_3} = \frac{0.145}{z_3}$$

$$\frac{1 \times 0.048}{z_3} + \frac{2 \times 0.015}{z_3} + \frac{3 \times 0.081}{z_3} + \frac{2 \times 0.145}{z_3} = 1 \Rightarrow z_3 = 0.611$$

weights

$$x_3, x_4 \rightarrow 0.024$$

$$x_1 \rightarrow 0.078$$

$$x_2, x_3, x_6 \rightarrow 0.132$$

$$x_2, x_8 \rightarrow 0.237$$

$$\text{Hypo: } 1.132 \cdot I(y \leq 0.75)$$

$$\text{Hypo: } 0.569 \cdot I(x > 0.25) + 0.829 \cdot I(x \leq -0.25)$$

$$+ 1.132 \cdot I(y \leq 0.75)$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
original class	+	+	-	-	+	+	-	-
predicted class	+	+	-	-	+	+	-	-

Assignment of sign based on hypo is

$$+ \text{ for } x_1 x_2 x_5 x_6$$

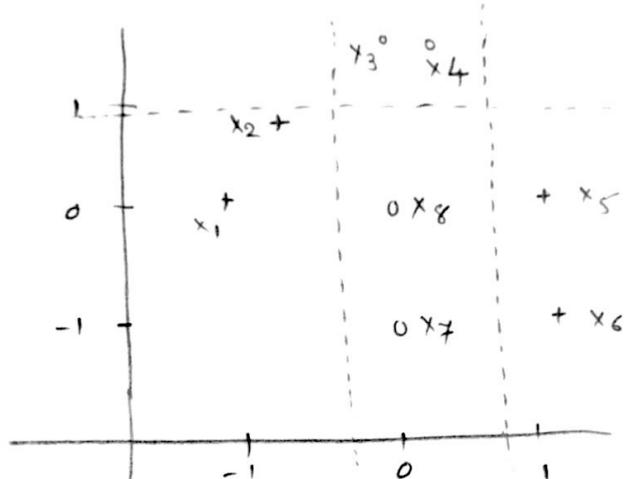
$$- \text{ for } x_3 x_4 x_7 x_8$$

Table ①

t	ϵ_t	α_t	z_t	$w(1)$	$w(2)$	$w(3)$	$w(4)$	$w(5)$	$w(6)$	$w(7)$	$w(8)$
1	0.25	0.549	0.864	$\frac{1}{8}$							
2	0.16	0.829	0.708	0.25	0.25	0.08	0.08	0.08	0.08	0.08	0.08
3	0.094	1.132	0.611	0.151	0.254	0.047	0.047	0.254	0.254	0.047	0.047

(2) Training Error is 0, all predictions are same as actual value

Decision stump



Ada Boost combines several weak classifiers (linear classifiers) together to produce nonlinear decision boundary unlike a single decision stump which creates linear boundary

Question-4

9. total 6 negative class,
5 positive class.

$$\text{so, Infogain}(D) = -\frac{6}{11} \log_2 \frac{6}{11} - \frac{5}{11} \log_2 \frac{5}{11}$$

$$= 0.9940$$

$$\begin{aligned} \text{Infogain}_{x_1}(D) &= \frac{5}{11} \left(-\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} \right) \\ &\quad + \frac{6}{11} \left(-\frac{3}{6} \log_2 \frac{3}{6} - \frac{3}{6} \log_2 \frac{3}{6} \right) \\ &= 0.9867 \end{aligned} \quad \left(\begin{array}{l} \because b=3 \\ a=2 \\ \text{for positive} \\ b=3 \\ a=3 \\ \text{for Negative} \end{array} \right)$$

$$\begin{aligned} \text{Infogain}_{x_2}(D) &= \frac{5}{11} \left(-\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} \right) \\ &\quad + \frac{4}{11} \left(-\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} \right) \\ &\quad + \frac{1}{11} \left(-\frac{1}{1} \log_2 \frac{1}{1} - \frac{0}{1} \log_2 \frac{0}{1} \right) \\ &\quad + \frac{1}{11} \left(-\frac{1}{1} \log_2 \frac{1}{1} - \frac{0}{1} \log_2 \frac{0}{1} \right) \\ &= 0.8049 \end{aligned} \quad \left(\begin{array}{l} \because b=3 \\ a=2 \\ \text{for positive} \\ a=2 \\ \text{for Negative} \\ c=2 \\ \text{for Negative} \\ 4=1 \\ \text{for Negative} \\ 9=1 \\ \text{for Negative} \end{array} \right)$$

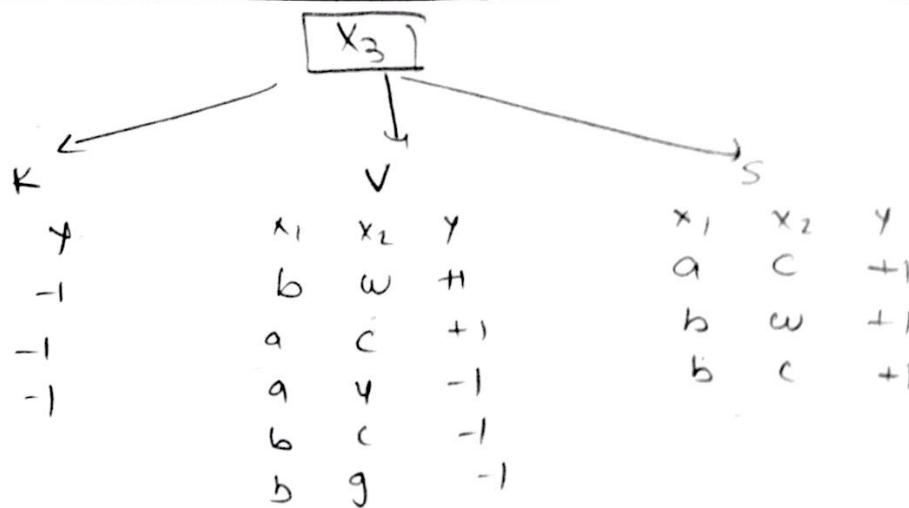
$$\begin{aligned} \text{Infogain}_{x_3}(D) &= \frac{3}{11} \left(-\frac{3}{3} \log_2 \frac{3}{3} \right) \\ &\quad + \frac{5}{11} \left(-\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} \right) \\ &\quad + \frac{3}{11} \left(-\frac{3}{3} \log_2 \frac{3}{3} \right) \\ &= 0.4413 \end{aligned}$$

so $\text{Infogain}_{x_3} < \text{Infogain}_{x_2} < \text{Infogain}_{x_1}$

$$\text{Infogain}(x_3) = 0.4413 - 0.4413 = 0.8527$$

First splitting Attribute is x_3

(i)



$$x_3 = K \quad y = -1$$

$$\text{Info}(D_K) = -\frac{2}{3} \log_2 \frac{2}{3} = 0$$

$$\text{Ent} x_1(D_K) = \frac{2}{3} \left(-\frac{1}{2} \log_2 \frac{1}{2} \right) + \frac{1}{3} \left(-\frac{1}{1} \log_2 \frac{1}{1} \right) = 0$$

$$\text{Ent} x_2(D_K) = \frac{2}{3} \left(-\frac{1}{2} \log_2 \frac{1}{2} \right) + \frac{1}{3} \left(-\frac{1}{1} \log_2 \frac{1}{1} \right) = 0$$

$$x_3 = V \quad D_V$$

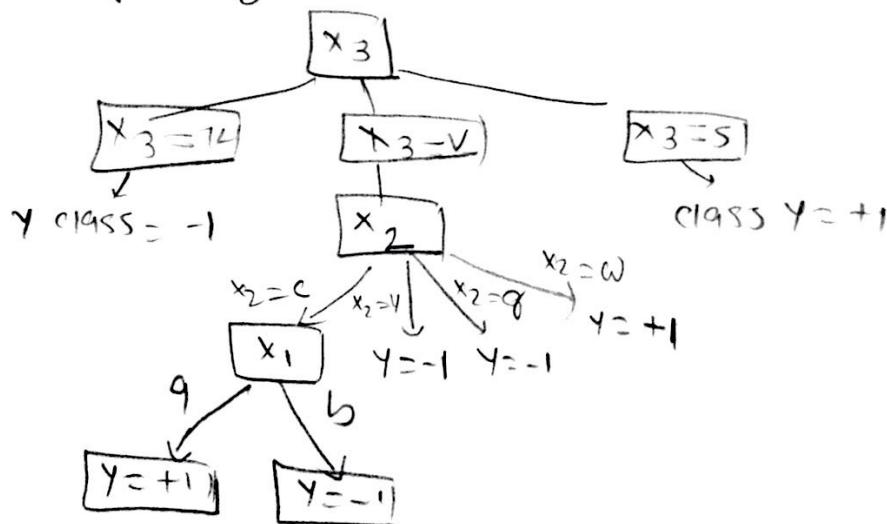
$$\text{Ent} x_1(D_V) = \frac{3}{5} \left(-\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} \right) + \frac{2}{5} \left(-\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \right) \\ = 0.9509$$

$$\text{Info} x_2(D_V) = \frac{3}{5} \left(-\frac{1}{3} \log_2 \frac{1}{3} - \frac{1}{3} \log_2 \frac{1}{3} - \frac{1}{3} \log_2 \frac{1}{3} \right) + \frac{2}{5} \left(-\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \right) \\ = 0.4$$

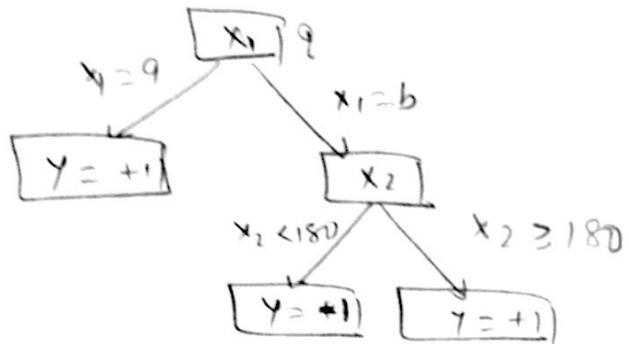
$$\text{Ent gain}(x_2) = 0 - 0.4 = -0.40$$

$$\text{Info gain}(x_1) = 0 - 0.9509 = -0.9509$$

x_2 is splitting attribute



b.



Decision Tree with x_1 & x_2 as asked attributes and 3 leaf class nodes which gives 100% training accuracy on the given training data.

(ii) given test data

$x_1 = b \quad x_2 = 170 \quad x_3 = f \quad x_4 = d \quad y = -1$ and

$x_1 = 9 \quad x_2 = 150 \quad x_3 = f \quad x_4 = d \quad y = +1$ are correctly classified

1st sample is not correctly classified

so Accuracy is $2/3 = 0.67$ (67%)