

# **140 Machine Learning Formulas**

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**.PDF 17 pages**

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**NAÏVE BAYES**

$$P(a|c) = \frac{P(c|a).P(a)}{P(c)}$$

**MIXTURE MODELS**

$$P(B) = P(B|A).P(A)$$

**BAYES OPTIMAL CLASSIFIER**

$$\arg \max \sum P(x|T).P(T|D)$$

**MIXTURE OF GAUSSIANS  
ANOMALY DETECTION**

$$P(x|\bar{x}) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left[-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2\right]$$

**NAÏVE BAYES CLASSIFIER**

$$\arg \max P(Spo|Tot) \cdot \prod P(Soc|Spo)$$

$$Z_{CS} = \frac{N_A C_B + N_B C_A}{N_A + N_B}$$

$$P(Z_{cs}) \rightarrow 0.50$$

**BAYES MAP (maximum a posteriori)**

$$h_{MAP} = \arg \max P(c|a).P(a)$$

**EM ALGORITHM**

$$E \text{ step } P(\bar{x}|x) = \frac{P(\bar{x}).P(x|\bar{x})}{\sum P(x).P(\bar{x})}$$

**MAXIMUM LIKELIHOOD**

$$h_{ML} = \arg \max P(c|a)$$

$$M \text{ step } P(x') = \frac{\sum P(\bar{x}|x)}{n}$$

**TOTAL PROBABILITY**

$$TotalP(B) = P(B|A).P(A)$$

*E step*  $P(\bar{x}|x) = \text{Assign value}$

$$M \text{ step } P(x') = P(B = 1|A = 1, C = 0)$$

**LAPLACE ESTIMATE** (small samples)

$$P(A) = \frac{A + 0.5}{A + B + 1}$$

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) + f(x).g'(x)}{g(x)^2}$$

$$\frac{d}{dx} 2f(x) = 2 \frac{d}{dx} f(x)$$

**BAYESIAN NETWORKS**

tuples  $\neg$  for  $y = 0 \wedge y = 1$

**LIMITES**

$$\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$\frac{d}{dx} f(x) + g(x) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

$$\frac{d}{dx} f(x) + 2g(x) = \frac{d}{dx} f(x) + 2 \frac{d}{dx} g(x)$$

**CHAIN RULE**

$$h = \Delta x = x' - x$$

$$\frac{d}{dx} g(f(x)) = g'(f(x)).f'(x)$$

solve  $f(x)$  apply in  $g'(x)$

**DERIVADAS**

$$\frac{\partial}{\partial x} x^n = n \cdot x^{n-1}$$

**VARIANCE**

$$\frac{\partial}{\partial x} y^n = \frac{\partial y^n}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$Var = \frac{\sum (x - \bar{x})^2}{n - 1}$$

**PRODUCT RULE**

$$\frac{d}{dx} f(x) \cdot g(x) = f'(x)g(x) + f(x).g'(x)$$

**STANDARD DEVIATION**

$$\sqrt{Variance}$$

**COVARIANCE**

$$Cov = \frac{(x - \bar{x}).(y - \bar{y})}{n - 1}$$

**SUM OF SQUARED ERRORS**

$$E|\vec{w}| = \frac{\sum(\hat{y} - y)^2}{2}$$

**COST FUNCTION****CONFIDENCE INTERVAL**

$$x \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$J(\theta_j) := \theta_j - \eta \cdot \frac{\sum(\hat{y} - y)^2}{2}$$

**CHI SQUARED**

$$Chi = \frac{(\hat{y} - y)^2}{\sqrt{y}} = \frac{\delta^2}{\sqrt{y}}$$

**NUMBER OF EXAMPLES**

$$m \geq \frac{\log(N_H) + \log(\frac{1}{\delta})}{\epsilon}$$

$$\text{where } \epsilon = \frac{\hat{y}}{y} \wedge \delta = y - \hat{y}$$

**R SQUARED**

$$R^2 = \frac{n \sum xy - \sum x \cdot \sum y}{\sqrt{(n \sum x^2 - (\sum x)^2) \cdot (n \sum y^2 - (\sum y)^2)}}$$

**MARKOV CHAINS**

$$P^{t+1}(X = x) = \sum_x P^t \cdot (X = x) \cdot T(x \rightarrow x)$$

**LOSS**

$$Loss = Bias^2 + Variance^2 + Noise$$

**K NEAREST NEIGHBOR**

$$\hat{f}(x) \leftarrow \frac{\sum f(x)}{k}$$

$$DE(x_i, x_j) = \sqrt{(x_i - x_j)^2 + (y_{xi} - y_{xj})^2}$$

**WEIGHTED NEAREST NEIGHBOR**

$$f(x) = \sum \frac{f(x)}{D(x_1x_2)^2} \cdot \sum D(x_1x_2)^2$$

**LINEAR REGRESSION**

$$m_1 = \frac{\sum x_2^2 \sum x_1 y - \sum x_1 x_2 \sum x_2 y}{\sum x_1^2 \sum x_2^2 - (\sum x_1 x_2)^2}$$

$$b = \bar{y} - m_1 \bar{x}_1 - m_2 \bar{x}_2$$

$$f(x) = \sum_{i=1}^n m_i x_i + b$$

**LOGISTIC REGRESSION****PRINCIPAL COMPONENTS ANALYSIS**

$$x' = x - \bar{x}$$

$$Eigenvalue = [A] - \lambda I$$

$$Eigenvector = Engenvalue. [A]$$

$$f(x) = Eigenvector^T. [x_{i1} \dots x_{jn}]$$

$$Odds Ratio = \log \left( \frac{P}{1-P} \right) = mx + b$$

$$\left( \frac{P}{1-P} \right) = e^{mx+b}$$

$$J(\theta) = - \frac{\sum y \cdot \log (\hat{y}) + (1-y) \cdot \log (1-\hat{y})}{n}$$

$$where \hat{y} = \frac{1}{1 + e^{mx+b}}$$

$$for y = 0 \wedge y = 1$$

$$-2LL \rightarrow 0$$

**ENTROPY**

$$\bar{x}_1 \sim \bar{x}_2 \neq \bar{x}_1' \sim \bar{x}_2'$$

$$H(A) = - \sum P(A). \log P(A)$$

$$mx + b = \frac{p}{1-p}$$

**JOINT ENTROPY**

$$P(a|c) = \frac{mx + b}{mx + b + 1}$$

$$H(A, B) = - \sum P(A, B). \log P(A, B)$$

$$Logit = \frac{1}{100. \log(P(a|c))}$$

**CONDITIONAL ENTROPY****DECISION TREES**

$$Entropy = \sum_{v=0}^1 -P_v \cdot \log(P_v)$$

$$H(A|B) = - \sum P(A, B). \log P(A|B)$$

**MUTUAL INFORMATION**

$$InfoGain = P_+ \cdot [ -P_{+t} \cdot \log(P_{+t}) - P_{+(t-1)} \cdot \log(P_{+(t-1)}) ]$$

$$I(A, B) = H(A) - H(A|B)$$

**RULE INDUCTION**

$$Gain = P \cdot [(-P_{t-1} \cdot \log(P_{t-1})) - (-P_t \cdot \log(P_t))]$$

**EIGENVECTOR CENTRALITY = PAGE RANK**

$$PR(A) = \frac{1-d}{n} - d \left( \frac{PR(B)}{Out(B)} + \frac{PR(n)}{Out(n)} \right)$$

**RULE VOTE**

where  $d=1$  few connections

*Weight=accuracy . coverage*

**RATING**

$$\hat{R} = \bar{R}_i + \alpha \sum w_j \cdot (R_{jk} - \bar{R}_j)$$

**BATCH GRADIENT DESCENT**

$$J(\theta_j) := \theta_j \pm \eta \cdot \frac{\sum (\hat{y} - y)^2 \cdot x}{2n}$$

**SIMILARITY**

$$w_{ij} = \frac{\sum_k (R_{ix} - \bar{R}_i) \cdot (R_{jk} - \bar{R}_j)}{\sqrt{\sum_k (R_{ix} - \bar{R}_i)^2 \cdot (R_{jk} - \bar{R}_j)^2}}$$

**STOCHASTIC GRADIENT DESCENT**

$$J(\theta_j) := \theta_j \pm \eta \cdot (\hat{y} - y)^2 \cdot x$$

**CONTENT-BASED RECOMMENDATION**

$$Rating = \sum_{i=1}^{class} \sum_{j=1}^m x_i y_j$$

**NEURAL NETWORKS**

$$f(x) = o = w_0 + \sum_{i=1}^n w_i x_i$$

**COLLABORATIVE FILTERING**

$$\begin{aligned} \hat{R}_{ik} &= \bar{R}_i + \alpha \cdot \\ &\sum \left( (R_{jk} - \bar{R}_j) \cdot \frac{\sum_k (R_{ix} - \bar{R}_i) \cdot (R_{jk} - \bar{R}_j)}{\sqrt{\sum_k (R_{ix} - \bar{R}_i)^2 \cdot (R_{jk} - \bar{R}_j)^2}} \right) \end{aligned}$$

**LOGIT**

$$\log(odds) = wx + b = \log \left( \frac{p}{1-p} \right)$$

**SOFTMAX NORMALIZATION**

$$S(f(x)) = \frac{e^{wx+b}}{\sum e^{wx+b}}$$

**CROSS ENTROPY**

$$H(S(f(x), f(x))) = - \sum f(x). \log S(f(x))$$

**LOSS**

$$\text{Loss} = \frac{\sum H(S(f(x), f(x)))}{N}$$

**PERCEPTRON**

$$f(x) = \text{sign} \left[ \sum_{i=1}^n w_j x_{ij} \right]$$

**PERCEPTRON TRAINING**

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta \cdot (t - o) \cdot x$$

**L2 REGULARIZATION**

$$w \leftarrow w - \left( \eta \cdot \delta \cdot x + \frac{\lambda \cdot w^2}{2} \right)$$

**ERROR FOR A SIGMOID**

$$\epsilon = \sum (t - o) \cdot o \cdot (1 - o) \cdot x$$

**SIGMOID**

$$\frac{1}{1 + e^{-(wx+b)}}$$

**AVOID OVERFIT NEURAL NETWORKS L2**

$$\sum \vec{w} = \frac{\sum_{d \in D} \sum_{k \in y} (t - o)^2}{2} + F \cdot \sum w_{ij}^2$$

**RADIAL BASIS FUNCTION**

$$h(x) = e^{\left(-\frac{(x-c)^2}{r^2}\right)}$$

where  $F = \text{penalty}$

**BACKPROPAGATION**

$$\delta_k = o_k \cdot (1 - o_k) \cdot (t - o_k)$$

$$\delta_h = o_h \cdot (1 - o_h) \cdot \sum w_{jk} \delta_k$$

$$w_{ij} \leftarrow w_{ij} + \eta_{xi} \cdot \delta_h \cdot x_{ij}$$

$$w_1 = 1 + (t - o_j)$$

$$\Delta w_{ij}(n) = \eta \cdot \delta_k \cdot x_{ij} + M \cdot \Delta w_{ij}(n-1)$$

where  $M$ =momentum

$$J_\theta = \frac{\sum_{i=1}^n \sum_{k=1}^k t_k \cdot \log(o) + (1-t) \cdot \log(1-o)}{N} + \frac{\lambda \sum_{l=1}^{ii} \sum_{i=1}^{jj} \sum_{j=1}^{jj+1} \theta_{ji}^2}{2N}$$

**MOMENTUM Y**

$$\theta = \theta - (\gamma v_{t-1} + \eta \cdot \nabla J(\theta))$$

**NESTEROV**

$$\theta = \theta - (\gamma v_{t-1} + \eta \cdot \nabla J(\theta - \gamma v_{t-1}))$$

**ADAGRAD**

$$\theta = \theta - \frac{\eta}{\sqrt{SSG_{diag} + \epsilon}} \cdot \nabla J(\theta)$$

**ADADELTA**

$$\theta = \theta - \frac{RMS[\Delta\theta]_{t-1}}{RMS \nabla J(\theta)}$$

**NEURAL NETWORKS COST FUNCTION**

$$RMS[\Delta\theta] = \sqrt{E[\Delta\theta^2] + \epsilon}$$

**RMSprop**

$$\theta = \theta - \frac{\eta}{\sqrt{E[g^2] + \epsilon}} \cdot \nabla J(\theta)$$

**ADAM**

$$\theta = \theta - \frac{\eta}{\sqrt{\hat{v}} + \epsilon} \cdot \hat{m}$$

$$\hat{m} = \frac{\beta_1 m_{t-1} + (1 - \beta_1) \cdot \nabla J(\theta)}{1 - \beta_1}$$

$$(x_1 \cdot x_2) = \sqrt{(x_1^2 + y_1^2) \cdot \left(1 - \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{x_2^2 + y_2^2}\right)}$$

$$\hat{v} = \frac{\beta_2 v_{t-1} + (1 - \beta_2) \cdot \nabla J(\theta)^2}{1 - \beta_2}$$

## SUPPORT VECTOR REGRESSION

$$\hat{Y} = \lambda \cdot K(x_i \cdot x_j) + b$$

## SUPPORT VECTOR MACHINES

$$f(x) = sign[\lambda \cdot y \cdot K(x_i \cdot x_j)]$$

$$K(x_i \cdot x_j) = exp\left[-\frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{width_{hist}}\right]$$

$$\lambda \rightarrow \nabla L = 0$$

$$K(x_i \cdot x_j) = exp\left[-\frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{width_{hist}}\right]$$

$$\lambda \rightarrow \arg \min \nabla L$$

## RIDGE REGRESSION - REGULARIZATION

$$y = 1 \wedge y = -1$$

$$m := m - \frac{\sum(\hat{y} - y)^2}{N} - \frac{\lambda \cdot m}{N}$$

$$DotProduct = \overrightarrow{x_1} \cdot \cos\theta$$

$$y = \lambda \cdot mx + b - \frac{\lambda}{N}$$

$$\cos^2\theta + \sin^2\theta = 1$$

## LASSO REGRESSION - REGULARIZATION

$$\sin\theta = \frac{\sqrt{(x_i - x_j)^2 + (y_{xi} - y_{xj})^2}}{\overrightarrow{x_2}}$$

$$b := \frac{\sum(\hat{y} - y)^2}{N} + \frac{\lambda \cdot b}{N}$$

$$m \rightarrow 0$$

## MÉDIA GEOMÉTRICA

$$\{1,2,4\} = \sqrt[3]{1.2.4}$$

$$y = mx + \lambda \cdot b + \frac{\lambda}{N}$$

## MEDIANA

### SKEWNESS

$$\frac{Max - Min}{2}$$

Skewness < 1

### KOLMOGOROV SMIRNOV

### TESTE t

Normal sig > .005

$$t = \frac{(x_1 - x_2) - (\mu_1 - \mu_2)}{(x_1 - x_2)}$$

Diferença significante sig < .05

### NÃO PARAMÉTRICOS

### TESTE t 2 AMOSTRAS

T test = Normal Teste U Mann Whitney sig < .05

Levene Variância

### CRONBACH

> .60 .70

### ANOVA + 3

### MÉDIA ARITMÉTICA

$$\frac{\sum x}{N}$$

$$F = \frac{Variância\ entre\ grupos}{Variância\ dentro\ do\ grupo}$$

Sig < .05

### TOLERÂNCIA

## ANÁLISE DISCRIMINANTE

Tolerância > .1

$$Tolerância = \frac{1}{VIF}$$

Box M sig < .05 rejeita H0

Wilk's Lambda sig < .05

## VARIANCE INFLATION FACTOR

VIF < 10

$$\bar{x}_1 \sim \bar{x}_2 \neq \bar{x}'_1 \sim \bar{x}'_2$$
$$P(x|\bar{x}) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left[-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2\right]$$

## ENTER METHOD

+ 15 cases / Variable

$$Z_{CS} = \frac{N_A C_B + N_B C_A}{N_A + N_B}$$

## STEPWISE METHOD

+ 50 cases / Variable

## ERROR MARGIN

$$1.96 \frac{\sigma}{\sqrt{N}}$$

## VARIABLE SELECTION

F Test = 47 sig < .05

## ACCURACY

Confidence Interval ~ P value

## MISSING DATA

Delete if > 15%

## HYPOTHESES TESTING

P value < .05

**TRANSFORMATION OK**

$$\frac{\bar{x}}{\sigma} < 4$$

**MULTICOLLINEARITY**

Correlation &gt; .90

VIF &lt; 10

Tolerance &gt; .1

**MAHALANOBIS DISTANCE**

same variable

$$M = \sqrt{\frac{(x_1 - \bar{x}_1)^2}{\sigma^2}}$$

**MANHATTAN DISTANCE L**

$$Manh = |x_1 - x_2| + |y_1 - y_2|$$

**NET PRESENT VALUE**

$$P_t = P_0 \cdot \theta^t$$

$$P_0 = P_t \cdot \theta^{-t}$$

**MARKOV DECISION PROCESS**

$$U_s = R_s + \delta \max_a \sum_s T(s, a, s'). U(s')$$

**STANDARD ERROR ESTIMATE (SEE)**

$$SEE = \sqrt{\frac{SumSquaredErrors}{n - 2}}$$

$$SEE = \sqrt{\frac{\sum(\hat{y} - y)^2}{n - 2}}$$

$$\hat{Q}_{s,a} \leftarrow_{\eta} R_s + \delta \max_a Q(s', a')$$

$$Q_{s,a} = R_s + \delta \max_{s'} \sum_s T(s, a, s'). \max_{a'} Q(s', a')$$

## PROBABILIDADE (coins)

$$P(a) = \frac{P(a)}{P(A)}$$

$$\begin{aligned} P(A \cup B \cup C)_{\text{NÃO EXCLUDENTES}} \\ = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C) \end{aligned}$$

## EVENTO COMPLEMENTAR

$$P(\tilde{A}) = 1 - P(A)$$

## FREQUENTISTA

$$\lim_{n \rightarrow \infty} \frac{m}{n} = \frac{\text{sucessos}}{\text{todas possibilidades}} = \frac{\text{eventos}}{\text{espaço amostral}}$$

## PROBABILIDADE MARGINAL

$$P(a) = \frac{P(A = a)}{\sum P(A)}$$

## AXIOMÁTICA

$$P(A) \geq 0$$

$$\sum P(A, B, C) = 1$$

## PROBABILIDADE A e B

$$P(A \text{ e } B) = \frac{P(A \cap B)}{P(B)}$$

## TEOREMAS DE PROBABILIDADE

### UNIÃO = A ou B

$$P(A \cup B)_{\text{EXCLUDENTES}} = P(A) + P(B)$$

### PROBABILIDADE CONDICIONAL

$$P(A|B)_{\text{INDEPENDENTES}} = P(A)$$

$$P(A \cup B)_{\text{NÃO EXCLUDENTES}} = P(A) + P(B) - P(A \cap B)$$

**BAYES (52 cartas , cancer)**

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A).P(A)}{P(B)}$$

**BINOMIAL DISTRIBUTION (0,1 sucesso)**

$$P(D) = \binom{\text{espaço amostral}}{\text{sucesso}} \cdot P(s)^s \cdot (1 - P(s))^{s^c}$$

$$P(D) = \binom{\text{espaço amostral}}{\text{sucesso}} \cdot P(s)^s \cdot (P(\bar{s}))^{s^c}$$

$$P(\text{Sucesso}) = \binom{S}{s} \prod_{s \in S} P(s) \cdot \prod_{\bar{s} \in S} P(\bar{s})$$

$$P(D) = \frac{c!}{a! (c-a)!} \cdot P(a)^a \cdot (1 - P(a))^{c-a}$$

**PROBABILIDADE TOTAL (urnas)**

$$P(B) = \sum P(A \cap B) = \sum P(A) \cdot P(B|A)$$

**PROBABILITY k SUCCESS in n TRIALS**

$$P(k \text{ in } n) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

**INTEGRAIS**

$$\int_a^b F(b) - F(a)$$

$$\int_1^2 x^2 dx = \frac{1}{3}x^3 = \frac{1}{3}2^3 - \frac{1}{3}1^3$$

**PRODUCT RULE**

$$\int c \cdot f'(x) \cdot dx = c \int f'(x) \cdot dx$$

**CHAIN RULE**

$$\int f(x) + g(x) \cdot dx = \int f(x) \cdot dx + \int g(x) \cdot d(x)$$

**INTEGRATION**

$$\sum f'(x) \cdot \Delta x \underset{N \rightarrow \infty}{=} 0$$

**DIFFERENTIATION**

$$\lim_{n \rightarrow \infty} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

## LINEAR ALGEBRA

### ADDITION

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 9 & 6 \end{bmatrix}$$

### SCALAR MULTIPLY

$$3 * \begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 15 & 9 \end{bmatrix}$$

### MATRIX VECTOR MULTIPLICATION

Linhos x Colunas

**x Vetor: Colunas A = Linhas B**

$$A_{i,j} * B_{j,i} = C_{i,i}$$

$$\begin{bmatrix} 0 & 3 \\ 1 & 3 \\ 2 & 4 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ 0 & 3 & 2 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 6 \end{bmatrix}$$

OU

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ 0 & 3 & 2 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = 1 * \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 2 * \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} + 0 * \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 6 \end{bmatrix}$$

**x Matrix: Colunas A = Linhas B**

**Linhos A = Colunas B**

*A<sub>2,1</sub> = 2a linha x 1a coluna*

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 2 & 1 & 3 \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 14 \\ 37 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = [12 \quad 30 \quad 0]$$

### IMPORTANT

*A<sub>2,3</sub> = 2a linha x 3a coluna*

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

## PERMUTATION

**LEFT=exchange rows**

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

**RIGHT=exchange columns**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

## IDENTIDADE

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## DIAGONAL

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

## TRANSPOSE

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

## PROPRIEDADES

Not commutative

$$A * B \neq B * A$$

Associative

$$A * B * C = A * (B * C)$$

Inverse (only squared)

$$A^{-1} \neq \frac{1}{A}$$

$$A^{-1} \cdot A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

## DETERMINANTE

$$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} = 1.2 - 3.4 = -10$$

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{array}{l} 1 \quad 4 \\ 2 \quad 5 \\ 3 \quad 6 \end{array} = 1.5.9 + 4.8.3 + 7.2.6 - 7.5.3 - 1.8.6 - 4.2.9$$

## ELASTICIDADE DE DEMANDA

$$\rho = \frac{(Q_1 - Q_0)}{(Q_1 + Q_0)} \cdot \frac{(P_1 + P_0)}{(P_1 - P_0)}$$