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Design And Analysis Of Algorithms (Tutorial-4)

Q1 $T(n) = 3T(n/2) + n^2$

$\Rightarrow a = 3, \; b = 2, \; f(n) = n^2$

$\Rightarrow n^{\log_b a} = n^{\log_2 3}$

$n^{\log_2 3} < n^2 \Rightarrow \underline{T(n) = O(n^2)}$

Q2 $T(n) = 4T(n/2) + n^2$

$\Rightarrow a = 4, \; b = 2, \; f(n) = n^2$

$n^{\log_b a} = n^{\log_2 4} \Rightarrow n^{2\log_2 2} = n^2$

$= n^{\log_b a} = f(n)$

$\Rightarrow \underline{T(n) = O(n^2 \log n)}$

Q3 $T(n) = T(n/2) + 2^n$

$\Rightarrow a = 1, \; b = 2, \; f(n) = 2^n$

$\Rightarrow n^{\log_b a} = n^{\log_2 1} \Rightarrow n^{\log_2 2^0} = n^0$

$= 1$

$\Rightarrow$ Since $1 < f(n)$

$\Rightarrow \underline{T(n) = O(2^n)}$.

Q4 $T(n) = 2^n T(n/2) + n^2$

$\Rightarrow$ Master's Theorem is not applicable since 'a' is a function.

Q5 $T(n) = 16T(n/4) + n$

$\Rightarrow a = 16, \; b = 4, \; f(n) = n$

$\Rightarrow n^{\log_b a} = n^{\log_4 16} \Rightarrow n^{\log_4 4^2} = n^{2\log_4 4}$

$= n^2$

Since $n^2 > n$

$\Rightarrow$ $T(n) = O(n^2)$

Q6. $T(n) = 2T(n/2) + n\log n$

$\Rightarrow$ $a = 2$, $b = 2$, $f(n) = n\log n$

$\Rightarrow$ $n^{\log_b a} = n^{\log_2 2} \Rightarrow n$

Since $n^{\log_b a} < f(n)$

$\Rightarrow$ $T(n) = O(n\log n)$

Q7. $T(n) = 2T(n/2) + n/\log n$

$\Rightarrow$ $a = 2$, $b = 2$, $f(n) = n/\log n$

$\Rightarrow$ ~~$n^{\log_b a} > f(n)$~~ $n^{\log_b a} = n^{\log_2 2} \Rightarrow n$

Since, $n^{\log_b a} > f(n)$

$\Rightarrow$ $T(n) = O(n)$

Q8. $T(n) = 2T(n/4) + n^{0.51}$

$\Rightarrow$ $a = 2$, $b = 4$, $f(n) = n^{0.51}$

$\Rightarrow$ $n^{\log_b a} = n^{\log_4 2} \Rightarrow n^{0.5}$

Since $n^{\log_b a} < f(n)$

$\Rightarrow$ $T(n) = O(n^{0.51})$

Q9. $T(n) = 0.5T(n/2) + 1/n$

$\Rightarrow$ Master's Theorem is not applicable since $a < 1$

Q10. $T(n) = 16T(n/4) + n!$

$\Rightarrow$ $a = 16$, $b = 4$, $f(n) = n!$

$\Rightarrow$ $n^{\log_b a} = n^{\log_4 16} \Rightarrow n^{\log_4 4^2} = n^2$

Since $n^{\log_b a} < n!$

$\Rightarrow$ $T(n) = O(n!)$

Q11. $T(n) = 4T(n/2) + \log n$

$\Rightarrow a = 4 \quad , b = 2 \quad , f(n) = \log n$

$\Rightarrow n^{\log_b a} = n^{\log_2 4} \Rightarrow n^2$

Since, $n^{\log_b a} > f(n)$

$\Rightarrow \underline{T(n) = O(n^2)}$

Q12. $T(n) = \text{Sqrt}(n) + (n/2) + \log n$

$\Rightarrow$ Master's Theorem is not applicable, since, a is not constant.

Q13. $T(n) = 3T(n/2) + n$

$\Rightarrow a = 3 \quad , b = 2 \quad , f(n) = n$

$\Rightarrow n^{\log_b a} = n^{\log_2 3} \Rightarrow n^{1.58}$

Since $n^{\log_b a} > f(n)$

$\Rightarrow \underline{T(n) = O(n^{1.58})}$

Q14. $T(n) = 3T(n/3) + \sqrt{n}$

$\Rightarrow a = 3 \quad , b = 3 \quad , f(n) = \sqrt{n}$

$\Rightarrow n^{\log_b a} = n^{\log_3 3} \Rightarrow n$

Since, $n^{\log_b a} > f(n)$

$\Rightarrow \underline{T(n) = O(n)}$

Q15. $T(n) = 4T(n/2) + cn$

$\Rightarrow a = 4, \quad b = 2 \quad , f(n) = n$

$\Rightarrow n^{\log_b a} = n^{\log_2 4} \Rightarrow n^2$

Since, $n^{\log_b a} > f(n)$

$\Rightarrow \underline{T(n) = O(n^2)}$

Q16 $T(n) = 3T(n/4) + n\log n$

$\Rightarrow a = 3, \; b = 4; \; f(n) = n\log n$

$\Rightarrow n^{\log_b a} = n^{\log_4 3} \Rightarrow n^{0.787}$

Since $n^{\log_b a} < f(n)$

$\Rightarrow T(n) = O(n\log n)$

Q17. $T(n) = 3T(n/3) + n/2$

$\Rightarrow a = 3, \; b = 3, \; f(n) = n/2$

$\Rightarrow n^{\log_b a} = n^{\log_3 3} \Rightarrow n$

Since, $n^{\log_b a} > f(n)$

$\Rightarrow T(n) = O(n)$

Q18 $T(n) = 6T(n/3) + n^2\log n$

$\Rightarrow a = 6, \; b = 3, \; f(n) = n^2\log n$

$\Rightarrow n^{\log_b a} = n^{\log_3 6} \Rightarrow n^{1.63}$

Since, $n^{\log_b a} < n^2\log n$

$\Rightarrow T(n) = O(n^2\log n)$

Q19 $T(n) = 4T(n/2) + n/\log n$

$\Rightarrow a = 4, \; b = 2, \; f(n) = n/\log n$

$\Rightarrow n^{\log_b a} = n^{\log_2 4} \Rightarrow n^2$

Since, $n^{\log_b a} \geq f(n)$

$\Rightarrow T(n) = O(n^2)$

$T(n) = 64T(n/8) - n^2 \log n$

⇒ Master's Theorem is not applicable, Since, 'f(n)' is not an increasing function.

Q21. $T(n) = 7T(n/3) + n^2$

⇒ $a = 7$, $b = 3$, $f(n) = n^2$

⇒ $n^{\log_b a} = n^{\log_3 7}$

⇒ $n^{1.7}$

Since, $n^{\log_b a} < f(n)$

⇒ $T(n) = O(n^2)$

Q22. $T(n) = T(n/2) + n(2 - \cos n)$

⇒ Master's Theorem is not applicable, ~~Since~~, due to violation of regularity condition.