

Q1. Given a random sample  $(X_1, X_2, \dots, X_n)$ .

Mean  $\rightarrow \theta_1$

Variance  $\rightarrow \theta_2$

$$L(\theta_1, \theta_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

take log.

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^n (x_i - \theta_1)^2$$

for  $\theta_1$ , diff  $\log(L(\theta_1, \theta_2))$  w.r.t.  $\theta_1$ , & set it to zero

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^n (x_i - \theta_1) = 0$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^n x_i$$

M.L.E of  $S_1$  is sample mean.

for  $\theta_2$ , diff. w.r.t.  $\theta_2$  & put zero.

$$\theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \theta_1)^2$$

$\Rightarrow$  Binomial distribution -

$m$  = no. of trials

$\theta = (0, 1)$  prob. of success

$$L_0 = \prod_{i=1}^n f(x_i, n, \theta)$$

$$\text{P.M.F } f(x, n, \theta) = {}^n C_x \cdot \theta^x \cdot (1-\theta)^{m-n}$$

$$L(\theta) = \prod_{i=1}^n ({}^n C_{x_i}) \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i}$$

Take log.



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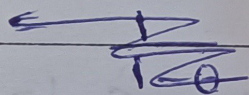
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$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^n x_i - \frac{1}{1-\theta} \sum_{i=1}^n (n - x_i) = 0$$

$$= \frac{1}{\theta} \sum_{i=1}^n x_i - \frac{1}{1-\theta} \sum_{i=1}^n (n - x_i)$$



Multiply by  $\theta(1-\theta)$ .

$$\Rightarrow (1-\theta) \sum_{i=1}^n x_i = \theta \sum_{i=1}^n (n - x_i)$$

$$\theta = \frac{\sum_{i=1}^n x_i}{n}$$