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3C08

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Q1. Given a random sample $(X_1, X_2 \dots X_n)$.
Mean $\to \theta_1$ Variance $\to \theta_2$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-(x_i-\theta_1)^2/2\theta_2}$$

take log.

$$\log L(\theta_1, \theta_2) = -\frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_1}\sum_{i=1}^{n}(x_i-\theta_i)^2$$

for θ_i, diff $\log(L(\theta_1, \theta_2))$ w.r.t. θ_1 & set it to zero.

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_i) = 0$$

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

M.L.E of S_1 is sample mean.
for θ_2, diff. w.r.t. θ_2 & put zero.

$$\theta_2 = \frac{1}{n}\sum_{i=0}^{n}(x_i - \theta_1)^2$$

$\Rightarrow$ Binomial distribution -
m = no. of trails
$\theta = (0,1)$ prob. of success.

$$L_0 = \prod_{i=1}^{n} f(x_i, n, \theta).$$

P.M.F $f(x, n, \theta) = {}^{n}C_{x} \cdot \theta^{x} \cdot (1-\theta)^{m-n}$.

$$L(\theta) = \prod_{i=1}^{n} \left({}^{n}C_{x_i}\right) \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i}.$$

Take log.

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$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{m} (n - x_i) = 0$$

$$= \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i)$$

$$\cancel{= \frac{1}{1-\theta}}$$

Multiply by $\theta(1-\theta)$.

$$\Rightarrow (1-\theta) \sum_{i=1}^{n} x_i = \theta \sum_{i=1}^{m} (m - x_i)$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m}$$