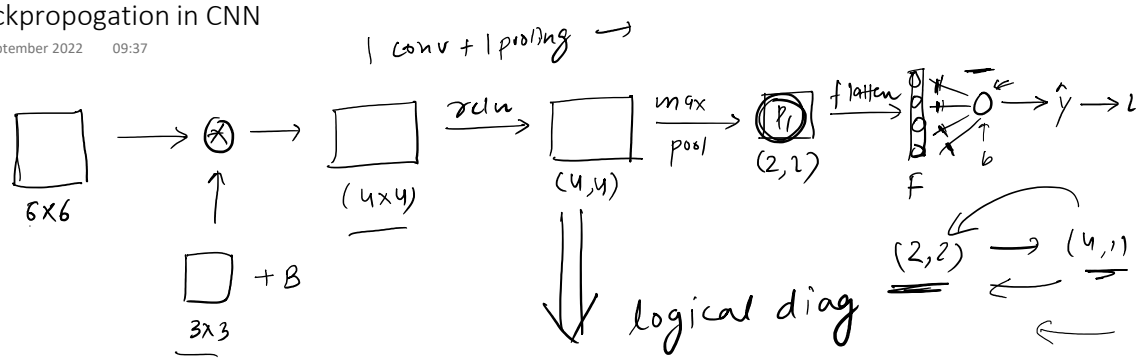
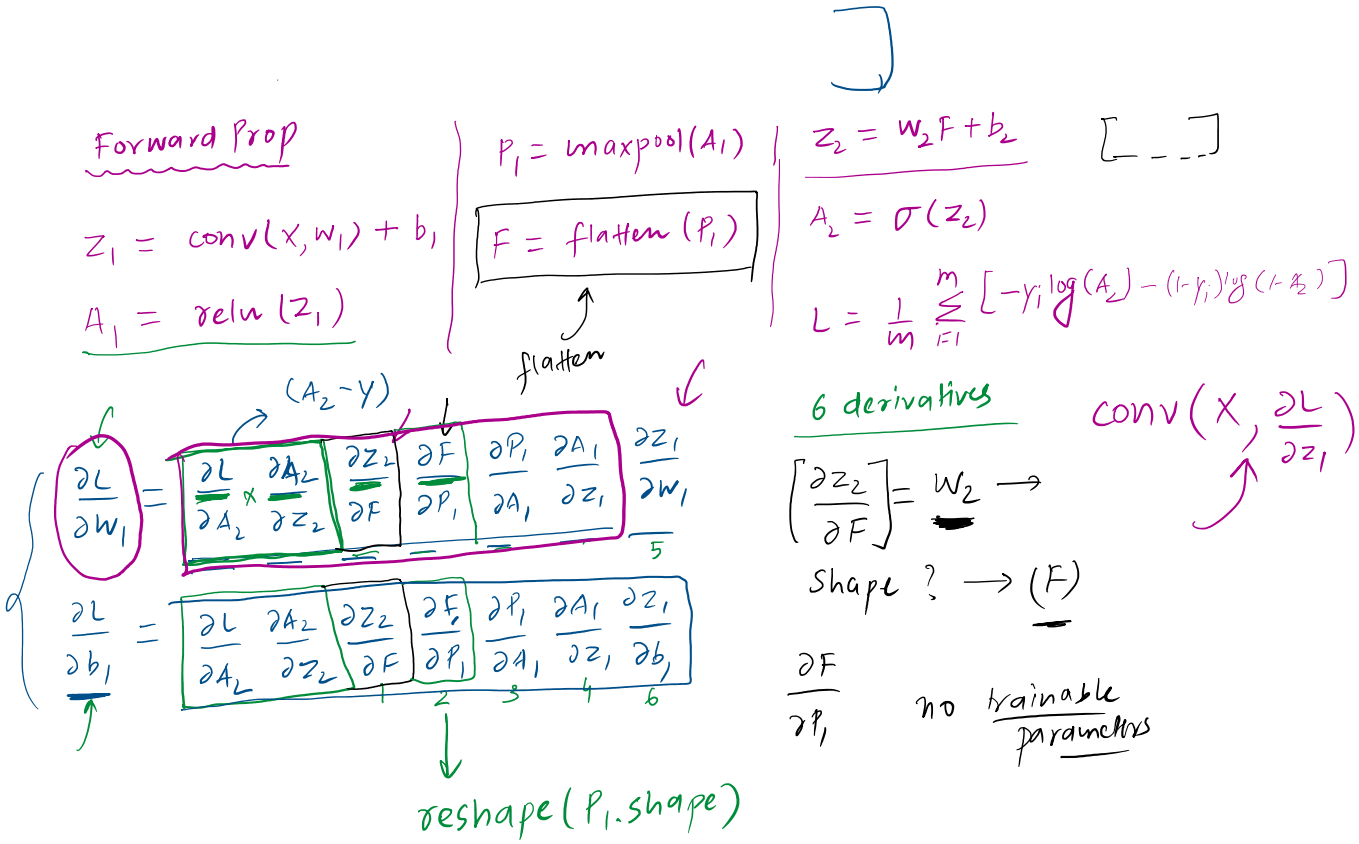
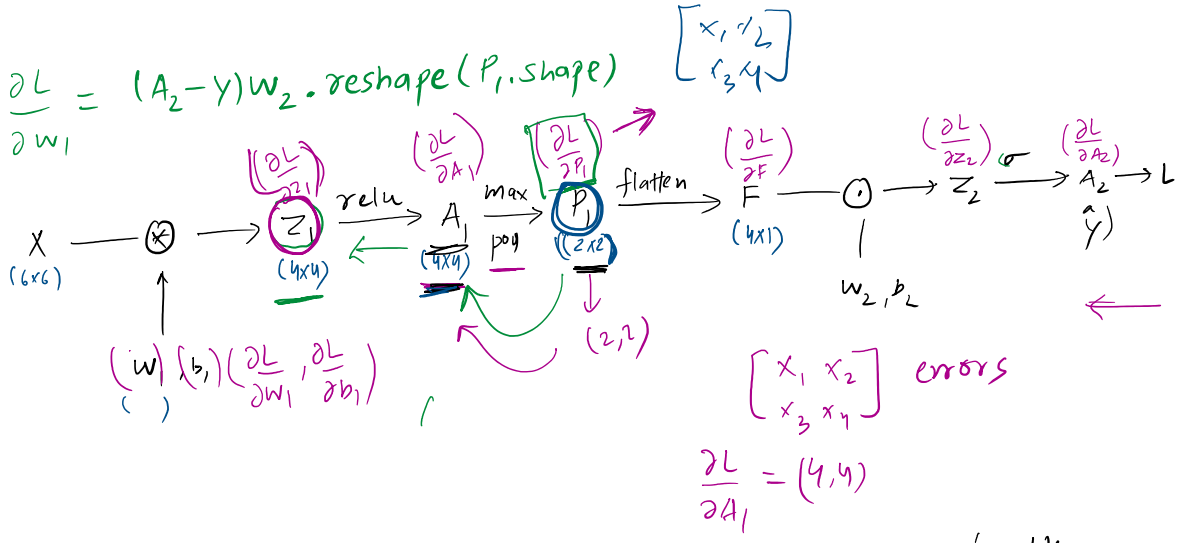
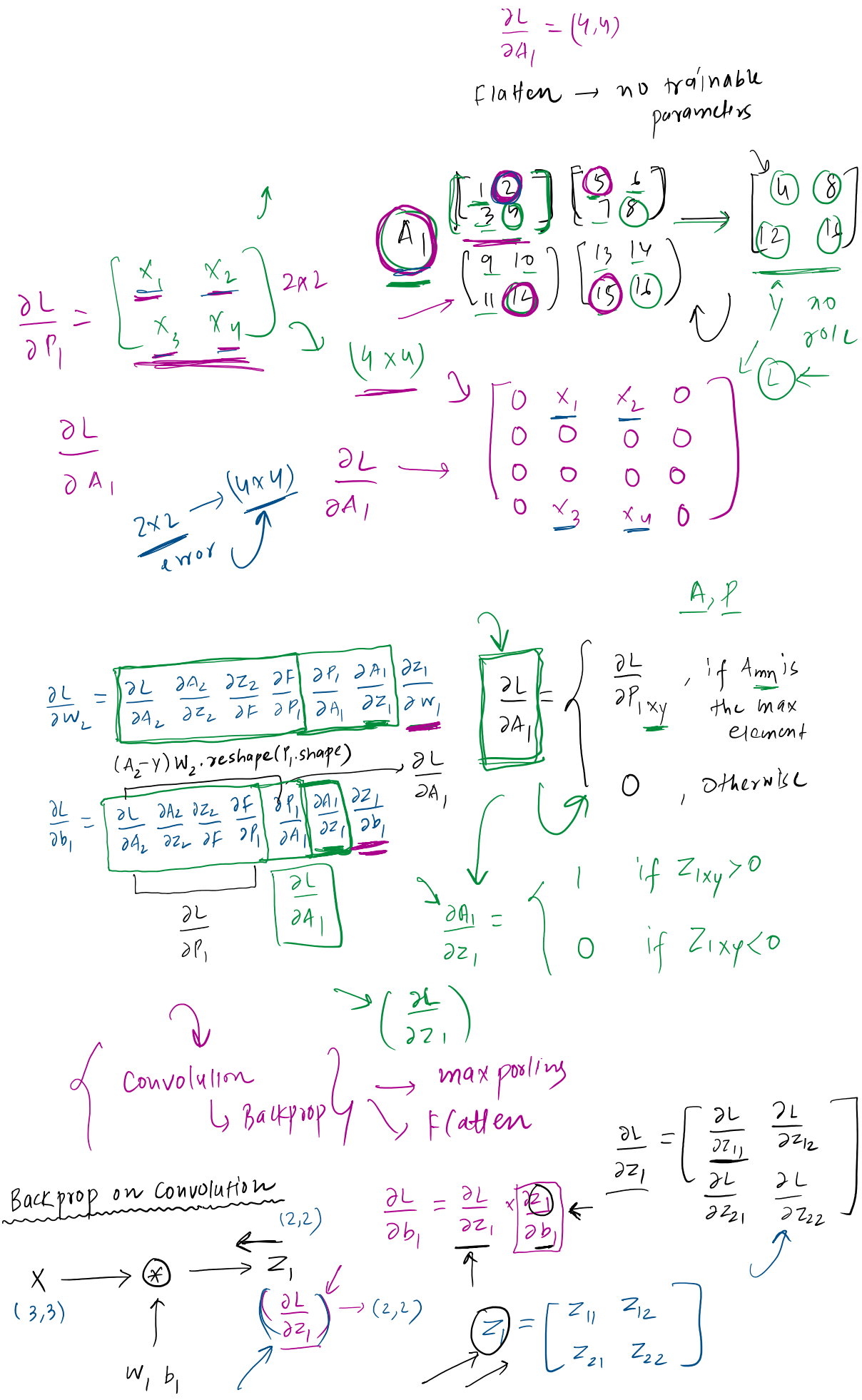
Backpropogation in CNN

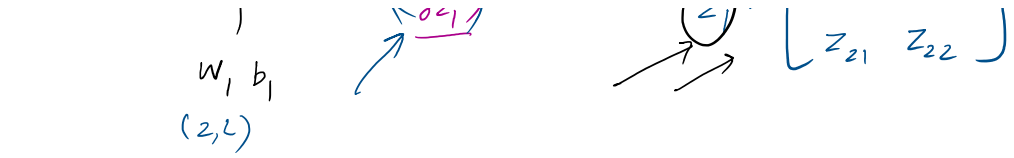
15 September 2022 09:37








$\frac{\partial L}{\partial A_1} = (4,4)$
Flatten → no trainable parameters
A_1
$\hat{y}$ no of L
$\frac{\partial L}{\partial P_1} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ 2×2
(4×4)
$\begin{bmatrix} 0 & x_1 & x_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & x_3 & x_4 & 0 \end{bmatrix}$
2×2 → (4×4) error
A, P
$\frac{\partial L}{\partial A_1} = \begin{cases} \frac{\partial L}{\partial P_{1xy}}, & \text{if } A_{mn} \text{ is the max element} \\ 0, & \text{otherwise} \end{cases}$
$\frac{\partial A_1}{\partial Z_1} = \begin{cases} 1 & \text{if } Z_{1xy} > 0 \\ 0 & \text{if } Z_{1xy} < 0 \end{cases}$
$(A_2 - Y) W_2 \cdot \text{reshape}(P_1.\text{shape})$
$\frac{\partial L}{\partial W_2} = \frac{\partial L}{\partial A_2} \frac{\partial A_2}{\partial Z_2} \frac{\partial Z_2}{\partial F} \frac{\partial F}{\partial P_1} \frac{\partial P_1}{\partial A_1} \frac{\partial A_1}{\partial Z_1} \frac{\partial Z_1}{\partial W_1}$
$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial A_2} \frac{\partial A_2}{\partial Z_2} \frac{\partial Z_2}{\partial F} \frac{\partial F}{\partial P_1} \frac{\partial P_1}{\partial A_1} \frac{\partial A_1}{\partial Z_1} \frac{\partial Z_1}{\partial b_1}$
$\frac{\partial L}{\partial P_1}$
$\frac{\partial L}{\partial A_1}$
$\frac{\partial L}{\partial Z_1}$
Convolution → Backprop → max pooling → Flatten
$\frac{\partial L}{\partial Z_1} = \begin{bmatrix} \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} \\ \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} \end{bmatrix}$
Backprop on Convolution
X (3,3) → (*) → Z_1 (2,2)
W_1 b_1
$\frac{\partial L}{\partial Z_1}$ → (2,2)
$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial Z_1} \times \frac{\partial Z_1}{\partial b_1}$
$Z_1 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$



$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \circledast \quad W_1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

b_1

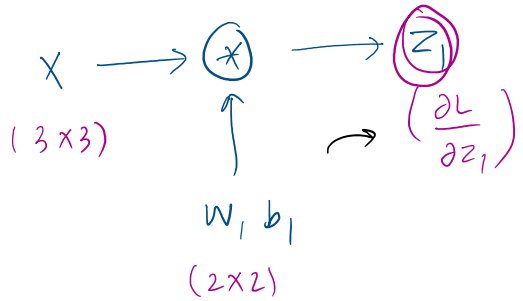
$$\boxed{\frac{\partial L}{\partial b_1}} = \frac{\partial L}{\partial z_1} \times \frac{\partial z_1}{\partial b_1} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial b_1} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial b_1} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial b_1} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial b_1}$$

$$= \left(\frac{\partial L}{\partial z_{11}} + \frac{\partial L}{\partial z_{12}} + \frac{\partial L}{\partial z_{21}} + \frac{\partial L}{\partial z_{22}} \right) = \text{sum}\left(\frac{\partial L}{\partial z_1} \right)$$

$$\boxed{\frac{\partial L}{\partial b_1}} = \text{sum}\left(\frac{\partial L}{\partial z_1} \right) \rightarrow \text{scalar}$$

bias

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \quad W_1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} + b$$



$$\frac{\partial L}{\partial W_1} = \begin{bmatrix} \frac{\partial L}{\partial w_{11}} & \frac{\partial L}{\partial w_{12}} \\ \frac{\partial L}{\partial w_{21}} & \frac{\partial L}{\partial w_{22}} \end{bmatrix} \qquad \frac{\partial L}{\partial z_1} = \begin{bmatrix} \frac{\partial L}{\partial z_{11}} & \frac{\partial L}{\partial z_{12}} \\ \frac{\partial L}{\partial z_{21}} & \frac{\partial L}{\partial z_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial z_1} X \frac{\partial z_1}{\partial W_1}$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{11}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{11}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{11}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{11}}$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{12}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{12}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{12}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{12}}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{21}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{21}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{21}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{21}}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{21}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{21}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{21}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{21}}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{22}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{22}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{22}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{22}}$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_{11}} x_{11} + \frac{\partial L}{\partial z_{12}} x_{12} + \frac{\partial L}{\partial z_{21}} x_{21} + \frac{\partial L}{\partial z_{22}} x_{22}$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_{11}} x_{12} + \frac{\partial L}{\partial z_{12}} x_{13} + \frac{\partial L}{\partial z_{21}} x_{22} + \frac{\partial L}{\partial z_{22}} x_{23}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} x_{21} + \frac{\partial L}{\partial z_{12}} x_{22} + \frac{\partial L}{\partial z_{21}} x_{31} + \frac{\partial L}{\partial z_{22}} x_{32}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_{11}} x_{22} + \frac{\partial L}{\partial z_{12}} x_{23} + \frac{\partial L}{\partial z_{21}} x_{32} + \frac{\partial L}{\partial z_{22}} x_{33}$$

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \qquad \frac{\partial L}{\partial z_1} = \begin{bmatrix} \frac{\partial L}{\partial z_{11}} & \frac{\partial L}{\partial z_{12}} \\ \frac{\partial L}{\partial z_{21}} & \frac{\partial L}{\partial z_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial w_1} = \text{conv}\left(X, \frac{\partial L}{\partial z_1}\right)$$

$$\frac{\partial L}{\partial w_1} = \text{conv}\left(X, \frac{\partial L}{\partial z_1}\right)$$

$$\frac{\partial L}{\partial b_1} = \text{sum}\left(\frac{\partial L}{\partial z_1}\right)$$