Q. Consider a short run production function of firm as $Q = 50L + 6L^2 - 0.5L^3$ where Q is the quantity of output and L is the number of labours. With the help of the above information, Calculate the range of values for labour and output over which Stage I, II and III of production occurs.

Solution: $Q = 50L + 6L^2 - 0.5L^3$

1) Stage I: corresponds to the stage of increasing returns. (0 till MP_L is maximum)

MP of labour = $dQ / dL = 50 + 12L - 1.5L^2$

For MP_L to be maximum, $dMP_L / dL = 0$ (First order condition for any function to attain maximum)

$12 - 3L = 0$ or $L = 4$

For MP_L to be maximum, $d^2MP_L / dL^2 < 0$, (Second order condition for any function to attain maximum)

or $-3 < 0$

Thus, MP_L attains maximum at L = 4.

Putting the value of L = 4, in the TP function for getting the value of total product, we get Q =264

	Stage I
Labour (range)	$0 < L \leq 4$
OutPut (range)	$0 < Q \leq 264$

2) Stage II: corresponds to the stage of decreasing or diminishing returns. (where MP_L is maximum to MP_L is zero (or TP_L is maximum)

MP of labour = $dQ / dL = 50 + 12L - 1.5L^2$

For MP of labour = 0, $50 + 12L - 1.5L^2 = 0$

L = 11 or L will be negative

For TP_L to be maximum, $d^2TP_L / dL^2 < 0$, (Second order condition for any function to attain maximum)

$12 - 33 < 0$

Thus, TP_L attains maximum at L = 11.

Putting the value of L = 11, in the TP function for getting the value of total product, we get Q =611

	Stage II
Labour (range)	$4 < L \leq 11$
OutPut (range)	$264 < Q \leq 611$

3) Stage III: corresponds to the stage of negative returns.(where MP_L is negative or TP_L starts to decline)

	Stage III
Labour (range)	$11 \leq L$ or $L \geq 11$
OutPut (range)	$611 \leq Q$ or $Q \geq 611$