Q. The short run total cost function of a firm is given as

$$TC = \frac{1}{3}Q^3 - 8.5Q^2 + 60Q + 27$$

A) Find the level of output at which AVC is minimum and the value of AVC.
B) Prove that at the minimum point of AVC, MC = AVC.
C) Prove that the minimum of AVC lies to the right side of the minimum of MC.

A)

$$TVC = \frac{1}{3}Q^3 - 8.5Q^2 + 60Q$$

$$AVC = \frac{1}{3}Q^2 - 8.5Q + 60$$

Min of AVC is when d(AVC)/dQ = 0 or 2/3Q – 8.5 = 0
Or Q = 12.75
$d^2(AVC) / dQ^2 = 2/3 > 0$.
Thus, AVC is minimum at Q= 12.75.
Value of AVC at Q = 12.75 is = 5.81

B) $MC = d(TVC)/dQ = Q^2 - 17Q + 60$

At Q = 12.75, value of MC = 5.81 = value of AVC at its minimum (Hence proved)

C) $MC = d(TVC)/dQ = Q^2 - 17Q + 60$

d(MC) / dQ = 2Q -17 = 0 or Q = 8.5
Again $d^2MC / dQ^2 = 2 > 0$, Thus MC is minimum at Q = 8.5
So, the minimum of AVC at Q = 12.75 is to the right of minimum of MC at Q = 8.5 (Hence proved).