① Strategic Information — Ch. 5 People's book

Introduction to Game theory

Take an example of McDonald's

↓

Neither a Monopolist Nor a Perfect Competitor

This is the Middle ground of oligopoly in which firms have visible rivals with whom Strategic Interaction is a fact of life.

→ Each firm is aware that its actions affect others & therefore prompt reactions.

→ Each firm must, ∴, take these interactions into account when making a decision about prices, or output, or other business actions.

→ Decisions in such an interactive setting is known as Strategic decisions.

→ Game theory is the branch of social science that formally analyses & models strategic decisions.

→ Game theory is divided into two branches

① Non Cooperative Game theory and ② Cooperative Game theory

The difference is ∵ of different nature of unit of analysis

→ In Non-co., the unit of analysis is single firm (Individual Players)

→ In Coope., the unit of analysis is Group of firms (Coalition of Players)

We will focus on Non-cooperative Game theory with unit of analysis as a firm

→ Rules of Game — It defines how competition b/w the diff. players, i.e., firms, take place.

→ Each player thinks about his/her self-interest

→ Two assumptions are there that underline the application of non-cooperative Game theory to oligopoly —

1. Firms are rational; → They pursue well defined goals, (Profit Max^{n})
2. firms apply their rationality to the process of reasoning strategically

→ In making decision, Each firm uses all the knowledge it has to form expectations regarding how other firms will behave.

Things to Remember! — There is no single, standard, oligopoly model.

There are No. of possible scenarios, Given the rules of the game, Information availability, timing of each players' ~~reacti~~ actions, among others

Understanding of broad Concepts is the main goal of this Course.

From Notes

Basics of Game theory

→ Started with 2nd world war.

→ Two fathers — John Von Neumann & Oskar Morgenstern

→ They talked about Zero-sum Game

→ Later on Nash (1951) wrote a paper & introduces the N.E. & its applicability in Non-Zero sum Game.

Start from Page (5) then come back to this on types of Games.

Zero Sum Game — A situation where one person's Gain is equal to another's loss so the net change in wealth or benefit is zero. e.g. — Poker & Gambling are the examples since the sum of amounts won by some players equals the combined losses of the others.

Game theoretic example — Matching pennies example

→ Two players — A & B

→ Simultaneously placing penny on the table [Simultaneous tossing a coin together]

Payoff depends whether the pennies match or not

If both pennies match (either Head or tail) Player A wins &
If both " don't " ———— Player B wins & keep other's penny.

These are actually bets — Casino & Gamblers

One participant gain at the expense of other participant's loss

In Zero-Sum Games - Optimal solution is always found - but it hardly represent the conflicts faced in the everyday world.

The Branch of GT that better represents the dynamics of ~~game~~ world we live in is called theory of Non-Zero-Sum games.

→ In this, there is no single optimal strategy that is preferable to all others, nor is there a predictable outcome.

→ Non-Zero Sum games are Non-strictly competitive i.e. it has elements of competition & Cooperation.

→ firms have complementary Interests & opposite Interests

e.g.

		P-2 Husband	
		Boxing Match	Ballet or Movie
P-1 Wife	Boxing Match	2, 3	1, 1
	Ballet (Movie) (Some performance) Dance	1, 1	3, 2

Assump:

Man & wife
Prefer to go outside & together
either Match or ballet

Opposing Interest area
→ Man prefers Match
→ Wife " ballet (Movie)

Common Interests
→ Both wants to go together

In short, Non-Zero Sum game is a situation where one's gain (or loss) does not necessarily result in the other decision maker's loss (or gain). In other words, where the winnings & losses of all players donot add upto zero and everyone can gain - a win-win game.

Basics of G.T. :->

① Player — $i = 1, 2, \ldots, N$ ← No. of players

② Actions — A_i ; $i = 1, 2, \ldots, N$ ← Action set

We can have games in which action set would be Continuous.

Continuum set contains real numbers.

③ Strategies — Each player's decision or plan of action is called a strategy.

S_i → Strategy set showing the list of strategies

Showing one particular strategy choice for each player is called a strategy combination.

S_i : $i = 1, 2, 3, \ldots, N$ (Strategy set)

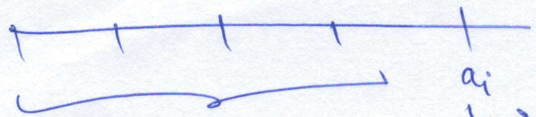
$$\boxed{S_i : A_i}$$

↓

Player i's strategy from strategy set. i's strategy depends on information available + then chooses one action from action set.

$$s_i \in S_i : \Omega_i \rightarrow A_i$$

Information set of player i (Contains all possible historical combinations)



(a_1, a_2) or past actions other players

a_i ↳ Present action] Based on Past Available info.

~~let $w_i \in \Omega_i$~~

(4) Pay-offs :- for any given Strategy Combination, the game will produce an out come describing the pay offs. or find net gains earned by each player.

Pay offs are the functions – like Utility functions & Profit functions.

Denoted by Π_i

For each player, there is a payoff function $\Pi_i \rightarrow R$

$i = 1, 2, \ldots, N$

[It assigns a Real number & this pay off function depend upon the strategy. [or It depends upon ~~which~~ action which would be taken and actions depends upon strategy.]

$$\Pi_i : \Omega_i \rightarrow A_i \rightarrow R$$

$$s_i \in S_i$$

[Its not only my actions which matters but it depend also upon the other player's strategy.

$$\Pi_i : \Omega_i \rightarrow A_i \rightarrow R$$
$$\Omega_j \rightarrow A_j \rightarrow R$$

e.g:- Two firms 1 & 2

Two decisions - ↑e P or N↑ to ↑e Price

Strategic form of the game is

		firm (2)	
		↑e Price	Do not change
firm (1)	↑e Price	140, 100	-20, 150
	Donot change	~~200~~ 60, -30	75, 50

This is a static game (Simultaneous Move)

→ Rewards are assigned to firm (1) for their particular actions) possible actions.

→ This game is playing once & it is known as static gam.

In static game:

$$S_1 = A_1$$

$$S_2 = A_2$$

$$S_3 = A_3$$

A_1 & A_2 → Two actions either to ↑e or not to ↑e

Solution later on :- on page - (8) next page

→ The traditional application of Game theory attempt to find equillibria in these games. In an equi., each player of the game has adopted a strategy that they are unlikely to change. On Page - 225 - L. People Book

Many equi. Concepts have been developed. - one of them is N.E.

→ Any time we have a situation with ~~a two~~ or more players that involves known payoffs or quantifiable consequences, we can use game theory to determine the most likely outcomes.

→ Game
Players
Strategy
Pay - off
Information set
Equillibrium

Basic Common Terms used in Game theory

Explain the Major 4 types of Games

Zero - Sum + Non - Zero Sum games

At Page 3-4

NASH Equilibrium (for a Non-Cooperative Game)

A set of strategies is a N.E. if each represents a best response to the other strategies. If all players are playing the strategies in a N.E., they have no incentive to deviate, since their strategy is the best they can do, given what others are doing. E.g.:- Duopoly Market
- Quantity Combination or
- Price Combination

		F-2 ↑P	F-2 ~P
Firm I (F1)	↑P	140, 100	-20, 40
	~P	60, -30	75, 50

N.E. would be (↑P, ↑P) and

N.E. payoffs are : $\left(\pi_1 (\uparrow P, \uparrow P), \pi_2 (\uparrow P, \uparrow P) \right)$

Explanation — Next Page

If firm 1 decide to ↑e the price the best strategy for firm II will be ↑e price. ∴ if firm 2 instead of ↑ing the price he sticks to his strategy of no ↑e in price then his profits will be 40 but if he ↑e the price then it will be 100. ∴ better to ↑e the price.

Same happens when firm II sticks on the strategy of ↑e in price then it would be better for firm 1 to ↑e the price.

∴ In this case, the equi. situation is ↑e price & ↑e in price by both.

But here if firm 1 sticks to his strategy of no price change then ~~to~~ on the same reasoning firm II best response will be no change in price.

∴ In this case, the N.E. will be No change in price by both.. It depends upon the other player's reaction.

> The conclusion is N.E. can exist but need not be unique

In a Mathematical sense, N.E. is a solution concept for non-competitive ~~Gap~~ Game.

In a two player game:

$\Pi_1 = f(\tilde{s}^*) \Rightarrow$ payoff of the function of strategies

$\tilde{s}^* = (s_1^*, s_2^*) \Rightarrow$ strategic combinations

$s_1^* =$ N.E. strategy of firm 1

$s_2^* =$ N.E. strategy of firm 2

If both firms / Players sticks to their strategies

$$\pi_1(s_1^*, s_2^*) \geq \pi_1(s_1, s_2^*)$$

If $s_1 = s_1^*$ then both would be equal.

Cases:-

1. $\boxed{\pi_1(s_1^*, s_2^*) \geq \pi_1(s_1, s_2^*)} \rightarrow N.E. \rightarrow 1$

$$\forall s_1 \in S_1$$

$s_1^* \rightarrow$ Best response of Player 1 to s_2^*

Note:- There can be More than one response of s_1 to s_2^*

2. $\boxed{\pi_2(s_1^*, s_2^*) \geq \pi_2(s_1^*, s_2)} \rightarrow 2$

$$\forall s_2 \in S_2$$

$s_2^* \rightarrow$ best response to s_1^*

$s^*(s_1^*, s_2^*)$ is a N.E. if 1 & 2 holds

Explanation of 1 $\rightarrow$ If any other strategy is adopted by firm 1 given that strategy of firm 2 is s_2^* then it would provide less profit (pay off)

Explanation of 2 $\rightarrow$ If any other strategy is adopted by firm 2 in response of s_1^* then profit would be lesser.

Generalisation :→

for N-number of Players :

① $\Pi_1(s_1^*, s_2^*, ---, s_N^*) \geq \Pi_1(s_1, s_2^*, ---, s_N^*)$

$\forall s_1 \in S_1$

② $\Pi_2(s_1^*, s_2^*, ---, s_N^*) \geq \Pi_2(s_1^*, s_2, ---, s_N^*)$

$\forall s_2 \in S_2$

$i = 1, 2, ---, N$

for Player i :- $\tilde{s}^* \rightarrow$ vector of N.E. strategies

$$\tilde{s}^* = (s_1^*, s_2^*, ---, s_i^*, ---, s_N^*)$$

$$= (s_i^*, \tilde{s}_{-i}^*)$$

Where $\tilde{s}_{-i}^* = (s_1^*, s_2^*, ---, s_{i-1}^*, s_{i+1}^*, s_N^*)$

∵ s_i^* is already excluded & written.

$$\boxed{\Pi_i(s_i^*, \tilde{s}_{-i}^*) \geq \Pi_i(s_i, \tilde{s}_{-i}^*) \quad \forall s_i \in S_i}$$

→ True for $\forall i = 1, 2, ---, N$

let's take another Example

	↑P	~P
↑P	140, 100	−20, 150
~P	200, −30	75, 50

Calculate N.E.

Sol:- In this game, N.E will be (~P, ~P) not (↑P, ↑P) ∵ by Tiny Price by firm ① firm ② will gain.

N.E. ! (~P, ~P) and

$\boxed{\text{NOT } (\uparrow P, \uparrow P)}$ ∵ both firms Cannot give their best Response to the others.

Note:- In this game, irrespective of other players, Best Strategy is not to change the Price.

In such Case, uniform best Strategy is not to ↑ the Price regardless of what other Player do. & this Strategy is known as dominant Strategy

Note:- In a NE, we are looking at Best Response ~~Cany~~ and my best strategy is my best response and it is unique ∵ both players playing their dominant Strategy.

Definition $\bar{s}_i$ → Dominant Strategy for firm i

$$\bar{s}_i \cdot \pi_i(\bar{s}_i, \tilde{s}_{-i}) > \pi_i(s_i, \tilde{s}_{-i})$$

$$\forall\, \tilde{s}_{-i} \quad \forall\, s_i \in S_i, \quad s_i \neq \bar{s}_i$$

Criticism of N.E.

It is appropriate in some cases to play non-equi. strategy if one expects others to play non-equi. strategies as well.

E.g.:- Prisoner's Dilemma

In this case, each player pursuing his own self-interest leads both players worse off than had they not pursued their own self-interest.

Prisoner's Dilemma

- Two suspected – arrested by police
- Police has not sufficient evidence & separated both.
- Police offer ~~them~~ each a deal (separately)

(1) If both remains silent, both are sentenced to only six months jail.

(2) If each betrays the other, each receives a five-year sentence

(3) If one testifies for the prosecution against the other (defects) and other remain silent (cooperate), then the ~~person~~ defector ~~who who defects~~ goes free and the silent accomplice receives the full 10-yr. sentence.

~~In such cases,~~ Each player must choose to betray the other or to remain silent. Each one is assured that other would not know about the betrayal before the end of investigation. How should the prisoners' act?

Sol:- If each player choose his own-self interest then it forms a non-zero sum game in which two players may each either cooperate with or defect from the other player.

The unique equi. for this game is a Pareto – Suboptimal solution. i.e rational choice leads the two players to both play defect even though each player's individual reward would be greater if both played cooperatively.

B

A	Silent	Betrays
Silent	1/2 , 1/2	10 , 0
Betrays	0 , 10	5 , 5

Acting independently, they are in dilemma & if they will send in one room ~~their~~ then they resolve their dilemma & ~~search~~ search at optimal solution.

In casual usage, it applies in those situations in which two entities could gain important benefits from cooperating or suffer from the failure to do so, but find it merely difficult or expensive, not necessarily impossible, to coordinate their activities to achieve cooperation.

Types of Strategies

1. Maximax
2. Maximin
3. Dominant Strategy ✓
4. Dominated Strategy :→ Some of firm's possible strategies may be dominated.

In case of two firms, A & B, one of the A's strategies is such that it is never a profit - maximising strategy regardless of the choice made by B.— that is, there is an alternative strategy for firm A that yields higher profits than does the strategy in question. This strategy in question is known as dominated strategy. and it will never be chosen.

In short, this means that ~~one can think that~~ in determining the game's equilibrium, we don't have to worry about any strategy combinations that include the dominated strategy.

~~Dominated~~

e.g.:- ~~Three~~ Player Game — a Zero Sum Game.
with three

P-2

P-1	Left	Middle	Right
Top	2, -2	7, -7	10, -10
Middle	6, -6	4, -4	12, -12
Bottom	8, -8	6, -6	14, -14

find out dominated strategies

for Player 1 :- Middle is the dominated St. ∵ he has more profitable St (bottom)

for Player 2: Right is the dominated St. ∵ P-2 doesn't want to play it. ∵ loss is higher than others

~~Dominated Strategies~~

Note:- Dominated strategies can be eliminated one by one. Once the dominated strategies for one firm have been eliminated, we turn to the other firms to see if any of their strategies have the same feature in light of the strategies still remaining for the first firm examined.

We then proceed one by one, eliminating all dominated untill only non-dominated remains.

Reduce the Game :-

P-2

P-1	Left	Middle
Top	2, -2	7, -7
Bottom	8, -8	6, -6

No N.E. Exist.

Other Solution Concepts

Take another Example

P-2

P-1	New P.	Not New P.
New Product	4, 4	3, 6
Not New Product	6, 3	2, 2

→ Each firm has two strategies to introduce a product or not to introduce a product.

→ Both players move simultaneously. If they both introduce they both share common market.

Two Equili:

(NNP, NP) : NE(1) → Payoff (3,6)

(NP, NNP) : NE(2) → Payoff (6,3)

This creates a Confusion i.e. of undesirable aspect of dualism or dilemma.

Another way to solve the game is Maximin Rule

		2 NP	NNP
1	NP	4,4	3,6
	NNP	6,3	2,2

for firm 1 → In case of NP — Min payoff is 3 ; In case of NNP → Min. " " is 2] Maximin — 3, NNP is Maximin Strategy

for firm 2 → In case of NP → Min. payoff is 3 ; In case of NNP → Min. " " 2] Maximin — 3, NNP is maximin St.

Choose Max. from Minimum

So. equi. solution is: (NNP, NNP) & Payoff will be (2,2)

But this is not a N.E., This is a solution of the Game but with diff. method.

More Discussion on Dominant & Dominated Strategies

	L	R
T	8	10
M	8	7
B	6	5

1. Strategy Top is strictly better than Bottom
2. " M is weekly dominated (not strongly dominated)
3. Strategy Top is weekly dominant Strategy
4. Strategy B is strongly dominated Strategy

In case of weekly dominated St., one can not cut simply by using domination method.

Reduced Game becomes:

	L	R
T	8, 1	10, 2
M	8, 5	7, 3

Apply N.E. & Solve

Sol:- Two N.E.

1. N.E. (T, R)
2. N.E. (M, L)

→ If one Player sticks to the middle then this is the only N.E. Other Player don't have any incentive to change.

→ Player 1 is indiff. b/w T & M

→ If Player 2 sticks to L then the best choice will be Middle.