Winter Term 2015/16 Problem Set 5 11.11.2015

# Algorithms for Programming Contests

This problem set is due by

Wednesday, 18.11.2015, 6:00 a.m.

Try to solve all the problems and submit them at

https://judge.in.tum.de/

This week's problems are:

| $\mathbf{A}$ | Chocolate          |
|--------------|--------------------|
| В            | Bank Robbery       |
| $\mathbf{C}$ | Christmas Presents |
| D            | Football Champion  |
| $\mathbf{E}$ | Spell Casting      |

The following amount of points will be awarded for solving the problems.

| Problem    | A    | В    | С      | D      | <u>E</u> |
|------------|------|------|--------|--------|----------|
| Difficulty | easy | easy | medium | medium | hard     |
| Points     | 4    | 4    | 6      | 6      | 8        |

If the judge does not accept your solution but you are sure you solved it correctly, use the "request clarification" option. In your request, include:

- the name of the problem (by selecting it in the subject field)
- a verbose description of your approach to solve the problem
- the time you submitted the solution we should judge

We will check your submission and award you half the points if there is only a minor flaw in your code.

If you have any questions please ask by using the judge's clarification form.

### A Chocolate

Author: Philipp Hoffmann

When thinking about chocolate, Lea usually gets happy. But not today! She has no more chocolate at home and all supermarkets near her home have "delivery problems". Of course she immediately calls the chocolate factory to find out what caused the problem. On the phone she is told that the lead organizer of chocolate transportation systems, Mr. D. I. Abetis is sick. Because no one else knows how to properly manage the conveyor belts that transport the chocolate from one production step to another, the whole production is on hold.

Lea has only one choice: She will go to the factory and solve the problem! At the factory, she is presented with the following situation:

There are multiple chocolate fountains from which warm chocolate springs in unlimited quantity. At these fountains, chocolate is cooled down and then transported to the wrapping station. This transport happens via a network of conveyor belts. Each conveyor belt has a maximum amount of chocolate it can transport per hour. Conveyor belts can end either at one of the fountains, at one of the wrapping stations, or at a distribution station. At a distribution station, all chocolate that arrives via an incoming conveyor belt has to be distributed onto outgoing conveyor belts.

The running direction of each conveyor belt can be switched, so a belt between a and b can transport either from a to b or the other way. For simplicity, assume that the wrapping stations have no limit to the amount of chocolate they can wrap.

Given the network of conveyor belts, Lea has to figure out how much chocolate can be wrapped per hour without violating the maximum load on the conveyor belts.

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with four integers n k m l, the number of chocolate fountains n, distribution stations k, wrapping stations m and conveyor belts l. l lines follow describing conveyor belts, each containing three integers  $v_i$   $w_i$   $c_i$  where  $v_i$  and  $w_i$  are locations and  $c_i$  is the capacity of the conveyor belt connecting  $v_i$  and  $w_i$ . The locations will be described by integers where 1 to n are chocolate fountains, n+1 to n+k are distribution stations, n+k+1 to n+k+m are wrapping stations.

## Output

For each test case, output one line containing "Case #i: d" where i is its number, starting at 1, and d is the maximum amount of chocolate that can be wrapped per hour.

Each line of the output should end with a line break.

- $1 \le t \le 20$
- $1 \le n, m \le 100$
- $1 \le k \le 500$
- $1 \le l \le 50000$
- $1 \le v_i, w_i \le m+n+k$  for all  $1 \le i \le l$
- $1 \le c_i \le 20$  for all  $1 \le i \le l$
- $\bullet\,$  The graph of locations is connected.

### Input

```
8
2 2 2 5
1 3 1
2 3 1
 2
 3
 4
    3 4 1
4 5 1
 5
 6
 7
     4 6 1
 8
     2 2 3 7
 9
10
    1 3 1
     1 4 1
11
    2 4 1 2 5 1
12
13
    3 6 1
14
    4 7 2
5 7 1
15
16
17
    1 3 1 7
1 4 2
18
19
20
    3 4 4
    3 5 1
2 3 2
2 4 1
21
22
23
     1 2 4
2 5 3
24
25
26
27
     1 2 1 5
28
     3 4 4
    1 2 5
2 3 5
2 4 2
1 3 2
29
30
31
32
33
    1 2 1 6
1 4 2
34
35
     3 4 5
36
    2 3 3 1 3 3
37
38
    2 4 3 1 2 1
39
40
41
42
    1 2 1 5
    4 3 2
2 4 5
3 2 4
1 2 2
43
44
45
46
47
     1 4 4
48
    1 3 1 7
49
    3 4 3
4 5 1
2 4 3
50
51
52
    1 2 4
2 5 5
53
54
    5 3 4
56
     1 5 3
57
58
    1 3 1 6
59
    4 5 4
60
     1 3 1
    3 5 2
2 4 3
61
62
    3 2 3 1 5 4
63
64
```

```
1 Case #1: 1
2 Case #2: 4
3 Case #3: 4
4 Case #4: 6
5 Case #5: 6
6 Case #6: 6
7 Case #7: 7
8 Case #8: 5
```

# B Bank Robbery

Author: Christian Müller

You just got word that a bank robber is robbing the NBEAM (National Bank with Enormous Amounts of Money). The police of course wants to catch him, but they can not storm the building since he has taken hostages. Their plan is to catch him on his way out of the country.

Unfortunately, the police only has a very limited amount of personnel, but they want to be absolutely sure to catch that bank robber.

To catch him, they can build roadblocks on a few roads and check every person using those roads. Since some roads are large and some small, they all need different amounts of policemen to be blocked.

At dinner, Lea's father complains that he has been appointed "Head of Strategic Positioning" but he has no idea how to distribute the policemen. Can you tell them if they have enough policemen to cover every possible route out of the country?

### Input

The first line of the input contains an integer t. t test cases follow.

Each test case begins with a line containing three integers l, the amount of available policemen, n, the amount of intersections and m, the amount of roads. m lines follow, each consisting of three integers i, j, k, specifying a road from intersection i to j with k being the amount of policemen it takes to construct a roadblock on it. All roads are useable in both directions. The robber always starts at intersection 1 and wants to get to the border (intersection n). Every test case ends with a blank line.

## Output

For each test case, print a line containing "Case #i: t" with i being the number of the test case, starting at 1, and t being "yes" if the minimal amount of policemen it takes to construct at least one road block on every path from intersection 1 to n is smaller or equal to l, "no" otherwise.

- $1 \le t \le 20$
- $1 \le l \le 10000$
- 2 < n < 400
- $1 \le m \le 100000$

- $1 \le i, j \le n$
- $\bullet \ 1 \le k \le 10000$
- $\bullet$  There will always be at least one possible path from 1 to n.

### Input

```
10
    10 2 3
2 2 3
2
3
    1 2 4
5
    1 1 3
6
7
8
    1 3 5
9
    1 2 5
    3 3 5
10
11
    8 2 3
12
13
   2 2 4
   1 2 5
14
15
    1 1 4
16
    2 5 5
17
   1 5 1
5 4 1
18
19
20
    4 2 1
21
    5 3 1
22
    1 2 1
23
   1 9 11
1 4 1
24
25
26
    4 8 1
27
    8 2 1
   2 7 1
2 3 1
7 5 1
28
29
30
   1 6 1
6 9 1
31
32
33
    2 8 1
    2 9 1
5 7 1
34
35
36
    2 7 9
1 6 1
37
38
39
    1 5 1
40
    1 4 1
41
    4 3 1
42
    1 2 1
43
    3 7 1
    2 6 1
44
    4 5 1 6 7 1
45
46
47
    1 3 3
48
49
   1 2 1
    1 3 1
1 3 1
50
51
52
    2 3 2
1 3 1
53
54
    3 2 1
56
    4 2 1
57
58
   1 2 1
59
    5 3 3
60
61
    1 2 1
   2 3 1
62
63
```

```
Case #1: yes
Case #2: no
Case #3: yes
Case #4: yes
Case #4: yes
Case #5: yes
Case #6: yes
Case #7: no
Case #8: yes
Case #9: yes
Case #10: yes
```

### C Christmas Presents

Author: Stefan Toman

Approximately 25% of the people who buy christmas presents begin to search for presents as late as one week before christmas. Lea has a different approach to this problem: She buys all the presents long before and uses December to decide which presents will be assigned to which of her friends. Lea has some constraints when she creates the assignment:

- All of Lea's friends should get a present.
- To be fair, each of her friends gets at most one present.
- None of the presents will be divided into multiple parts.
- Presents should only be assigned to a friend if he or she will like the present.

Luckily, Lea knows exactly which of her friends will like each present. She wants to prepare the algorithm to distribute the presents among her friends early before christmas, too. Can you help her writing it?

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with a line containing two integers n, the number of Lea's friends, and m, the number of presents Lea bought. The friends and presents are numbered from 1 to n or m, respectively. n lines follow describing the preferences of her friends: The i-th line contains a string with the numbers of the presents friend i likes. The numbers are comma-separated and may be given as sections where the first and last present are separated by a dash. For instance the string "1,10,3,5-8" represents presents 1, 3, 5, 6, 7, 8 and 10. Note that this string may also be empty.

## Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is "yes" if there is an assignment satisfying all constraints or "no" if there is no such assignment. Each line of the output should end with a line break.

- $1 \le t \le 20$
- $1 \le n \le 250$

- $1 \le m \le 250$
- $\bullet\,$  No presents will be mentioned several times per line of input.
- $\bullet$  Sections of presents will always be given with the smaller index first.

### Input

```
14
2
    3 3
   1
   1-2
5
    2,3
6
    2 2
7
8
    1
9
10
   2 5
11
12
   2-4
13
14
   2 1
1
15
16
17
18
   3 5
19
20
   3-5
21
    3-5
22
    1
23
    4 5
24
25
   1,5,2-3
26
   3-5
27
   4-5
28
    2-3
29
30
    3 5
   1,2,4-5
3-5
31
32
33
   1,2-3
34
35
   1 3
36
37
   3 5
38
39
   5,2-3,1
40
   1-5
41
    1-4
42
    3 1
43
44
    1
45
   1
46
    1
47
   1 3
48
49
   2,3
50
   3 3
51
52
   2-3
53
   2-3,1
54
55
56
    4 4
57
   1-2
58
   1-2
59
   3-4,2
60
    1-4
61
62
   1 3
    2-3,1
```

```
Case #1: yes
    Case #2: no
    Case #3: yes
    Case #4: no
    Case #5: yes
    Case #6: yes
    Case #7: yes
    Case #8: yes
    Case #9: yes
10
    Case #10: no
11
    Case #11: yes
12
    Case #12: yes
13
    Case #13: yes
    Case #14: yes
```

# D Football Champion

Author: Philipp Hoffmann

Lea likes to chat a lot with her colleagues and friends. A good topic to talk about is usually the latest football match. So this Sunday she sat down and looked at the past and scheduled matches to decide which upcoming matches she wants to watch. To keep it interesting, she only wants to watch matches of teams that still have a chance to win the current tournament. Help her to find out which teams could still win and which cannot.

The tournament consists of a series of matches some of which have already occured. Each match has a winner, there is no draw. In the end, there is either a single team that has the most wins or there will be playoffs between all teams with the (identical) maximal number of wins. Lea considers it possible for a team i to win the tournament if it is possible to assign a winner to each match such that team i has at least as many wins as any other team at the end of the tournament (thus this team reaches the playoffs, if any, or is directly declared winner).

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with two integers, n and m, the number of teams n indexed from 1 to n and the number of matches m that are scheduled to be played. The next line contains n space separated integers  $w_i$ , where  $w_i$  is the number of wins team i already has. m lines follow describing the matches that are to be played. Line i contains two distinct integers  $a_i$  and  $b_i$ , the two teams that play each other in match i.

## Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is a space separated list that contains n items, each either "yes" if team i can still win or "no" if not. Each line of the output should end with a line break.

Note: This tournament is not like any real life tournament. In particular there may be teams that play a lot more matches than others.

- $1 \le t \le 20$
- $2 \le n \le 30$
- $0 \le m \le 300$
- $0 \le w_i \le 1000$  for all  $1 \le i \le n$

- $1 \le a_i, b_i \le n$  for all  $1 \le i \le n$
- $a_i \neq b_i$  for all  $1 \leq i \leq n$

#### Input

```
12
2
    3 1
3
   1 1 1
   1 2
5
   2 2
6
7
   2 1
8
    1 2
9
    1 2
10
    4 1
11
   0 0 0 0
12
   3 2
13
14
15
   1 0 1
16
   3 2
17
   3 2 1 3
18
19
20
   4 2
1 0 0 1
21
22
23
   2 1
24
   4 2
25
26
   2 1
27
   1 1
28
    2 1
29
   6 3
30
31 0 1 0 1 0 1
32 3 4
33
   6 3
34
    1 3
35
    3 4
36
    0 1 3
37
   1 3
38
39
   1 2
   1 3
2 3
40
41
   3 0
0 0 0
43
44
45
    3 4
46
   2 0 2 2 1
47
48
49
   1 2
   3 1
2 1
50
51
52
   4 3
1 0 2 2
53
54
   4 2 4 1
56
57
58
59
    4 2
60
    1 0 0 0
61
   1 4
    4 3
62
```

```
1 Case #1: yes yes no
2 Case #2: yes yes
3 Case #3: no yes yes no
4 Case #4: yes yes yes
5 Case #5: yes yes no yes
6 Case #6: yes yes
7 Case #7: no no yes yes no yes
8 Case #8: yes yes yes
9 Case #9: yes yes yes
10 Case #10: yes yes yes
11 Case #11: yes yes yes
12 Case #12: yes no yes yes
```

# E Spell Casting

Author: Philipp Hoffmann

One of Lea's passions is playing computer games. In her favourite game, many players fight together against bosses of incredible strength. Some of those players are sorcerers, some are warriors. The sorcerers can enhance the power of a warrior by casting spells. To do so, they need one magic powder per spell, a reagent that is quite rare.

Each warrior can only benefit by some spells (but by those possibly multiple times) and also has a maximum amount of spells that can be cast on him. Each sorcerer can only cast certain spells, of those he can cast as many as he likes provided he has enough magic powder. There is only a fixed amount of magic powder available. Your task is to find out the maximum amount of spells that can be cast in total.

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with four integers, n m p s, the number of warriors n and sorcerers m, the amount of magic powder available p and the number of different spells s. Spells are numbered from 1 to s.

The next n lines describe the warriors, each contains two integers  $c_i$   $k_i$  followed by  $k_i$  more integers  $s_{i,1}, ..., s_{i,k_i}$ .  $c_i$  is the maximum number of spells that can be cast on warrior i,  $k_i$  the number of different spells that can be cast on warrior i,  $s_{i,1}, ..., s_{i,k_i}$  are the spells that can be cast on warrior i.

The next m lines describe the sorcerers, each contains an integer  $a_j$  followed by  $a_j$  integers  $t_{j,i},...,t_{j,a_j},a_j$  is the number of different spells sorcerer j can cast,  $t_{j,i},...,t_{j,a_j}$  are the spells that he can cast.

## Output

For each test case, output one line containing "Case #i: d" where i is its number, starting at 1, and d is the maximum amount of spells that can be cast. Each line of the output should end with a line break.

- $1 \le t \le 20$
- $1 \le m, n, p, s \le 100$
- $1 < c_i < 100$  for all 1 < i < n
- $1 \le k_i \le s$  for all  $1 \le i \le n$

- $1 \le a_j \le s$  for all  $1 \le j \le m$
- $1 \le s_{i,k} \le s$  for all  $1 \le i \le n, 1 \le k \le k_i$
- $1 \le t_{j,a} \le s$  for all  $1 \le j \le m, 1 \le a \le a_j$
- The elements  $s_{i,k}$  resp.  $t_{j,a}$  will be distinct elements for each i resp. j.

#### Input

```
2 2 1 2 2
3 1 1 1
   1 1 2
 5
    2 1 2
 6
 7
    1 1 2 1
 8
    2 1 1
 9
    1 1
10
   2 3 1 3
11
12 | 2 2 1 3
13 | 3 2 1 2
   2 2 3
14
    2 1 2
1 1
15
16
17
   1 2 3 2
3 1 2
18
19
20
   2 1 2
21
    1 1
22
23 | 1 3 2 1
24 | 3 1 1
25
    1 1
26
   1 1
27
    1 1
28
29
   1 3 2 3
30
   1 2 1 3
31 3 1 2 3
32 1 2
33
   1 1
34
   3 1 3 2
35
36
    1 1 2
    2 1 1
1 2 1 2
37
38
39
    1 2
40
    3 2 3 2
41
42
   3 2 1 2
    2 2 1 2
43
   3 1 1
2 1 2
2 1 2
44
45
46
47
   2 2 1 3
48
49
   2 1 1
   2 1 1 1 1 1
50
51
52 3 1 2 3
53
    3 2 3 3
54
   1 2 1 3
    1 3 1 2 3
56
    2 3 1 2 3
57
58
   3 1 2 3
59
    1 1
60
61
    3 1 1 2
62 3 1 2
63
    1 1 2
64
   1 2 1 2
65
   2 1 2
```

```
1 Case #1: 2
2 Case #2: 2
3 Case #3: 1
4 Case #4: 3
5 Case #5: 2
6 Case #6: 1
7 Case #7: 2
8 Case #8: 3
9 Case #9: 1
10 Case #10: 3
11 Case #11: 1
```