Winter Term 2015/16 Problem Set 7 25.11.2015

Algorithms for Programming Contests

This problem set is due by

Wednesday, 02.12.2015, 6:00 a.m.

Try to solve all the problems and submit them at

https://judge.in.tum.de/

This week's problems are:

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D	Woodchucking	. 1
${f E}$	Mice	13

The following amount of points will be awarded for solving the problems.

Problem	A	В	С	D	E
Difficulty	easy	easy	medium	medium	hard
Points	4	4	6	6	8

If the judge does not accept your solution but you are sure you solved it correctly, use the "request clarification" option. In your request, include:

- the name of the problem (by selecting it in the subject field)
- a verbose description of your approach to solve the problem
- the time you submitted the solution we should judge

We will check your submission and award you half the points if there is only a minor flaw in your code.

If you have any questions please ask by using the judge's clarification form.

A Exam Preparation

Author: Stefan Toman

Lea heard that at TUM (Thomas Underwood University Markistan - Thomas Underwood was a famous scientist, but he never got well-known beyond the borders of Markistan) students are not allowed to take any material to their exams except a hand-written single-sided sheet of paper. Therefore, all students try to write as much as possible on their cheat sheets. There is obviously a limit in font size which means they have to decide which information helps the most. To make things more complicated, each professor allows different sizes of the cheat sheets: Some of them are very generous, whereas others only allow for stamp-sized sheets.

Lea is unsure whether she wants to apply at TUM, but if she does, she wants to master the cheat sheet system. For each lecture she knows all topics covered and the number of characters that fit on a cheat sheet of the allowed size. For each topic, she also knows the number of characters one piece of information needs and how useful they are (Lea has an integer score system where a higher score means more useful). For each topic Lea will find an unlimited supply of information units and each of them will have the same length and score. It would be sufficient to add many pieces of information for only a few topics, but Lea wants the overall score of her cheat sheet to be as big as possible.

After all, Lea knows that understanding the lectures is much more useful than a perfect cheat sheet. That is why she does not want to spend too much time creating the cheat sheets. Lea is happy if she has information with at least half of the best possible score on her cheat sheet. Help her to choose what to add to it.

Input

The first line of the input contains an integer t, the number of lectures. t lectures follow, each of them separated by a blank line.

Each lecture starts with a line containing two integers: m, the number of characters that fit on the allowed cheat sheet, and n, the number of topics covered. n lines describing the topics follow. The i-th line contains two integers l_i and s_i where l_i is the length of a piece of information for this topic and s_i is its score.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is a space-separated list of topics to be added (topics may appear several times in this list). The sum of their lengths should be at most m and the sum of their scores should be at least half of the optimal value.

Constraints

• $1 \le t \le 20$

- $1 \le n \le 100$
- $1 \le m \le 10000$
- $1 \le l_i \le 10000$ for all $1 \le i \le n$
- $1 \le s_i \le 10000$ for all $1 \le i \le n$

Input

```
8
10 2
2 3
    3 5
1 1
6
    10 3
    3 7
    2 8
9
    7 5
10
    17 4
11
    2 4
3 3
12
13
    5 2
14
15
    4 7
    8 2
17
19
20
    25 1
21
22
    2 1
23
    15 5
   2 1
4 5
6 7
25
26
27
28
    8 9
29
    10 11
30
   80 3
2 1
40 20
31
32
33
34
    20 11
35
36
    10 3
    100 10000
37
38
    50 100
    20 300
```

B Street Lights

Author: Chris Pinkau

During another of Lea's visits to her uncle in Chaosville, she realizes that the streets of Chaosville are in horrible condition. Driving at night is really dangerous: you never know when you will hit a pothole, because most street lights are not working and you see close to nothing. One day, Lea decides to visit the mayor's office to talk about the bad lighting on the streets. The mayor explains the city's problem to Lea: indeed, there are many street lights, but erected in an irregular fashion at the side of every street. And most of them are switched off due to severe budget cuts. Not willing to give up, Lea offers to come up with a plan that specifies how many of the street lights should be switched on to illuminate the main street, at least. The mayor agrees, but only if she can provide him with a solution that needs to switch on only as few street lights as possible.

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with three integers ℓ n d, the length of the main street ℓ , the number of street lights on main street n, and the radius of the light cone of each street light d, which indicates how far each light shines. Then, another line follows, consisting of n integers p_1 p_2 ... p_n , describing the positions of the street lights.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is either the smallest number of street lights that are needed to illuminate the whole main street (which goes in a straight line from 0 to ℓ), or "impossible" if there is no way to illuminate the whole street.

To illuminate the whole main street, there should be light from at least one street light at every point on the main street between 0 and ℓ . The boundary of each light cone is considered to be illuminated as well. In particular, this means that a point on the street is illuminated if two light cones touch there, they do not need to intersect. See the sample data explanation.

Constraints

- $1 \le t \le 20$
- $1 < \ell < 50000$
- $0 \le n \le \min(1000, \ell + 1)$
- $0 \le d \le 1000$

- $0 \le p_i \le \ell$ for all $1 \le i \le n$
- $p_i \neq p_j$ for all $1 \leq i, j \leq n$

Input

```
2
    15 6 3
3
    2 4 9 7 6 12
4
    15 4 3
5
    10 4 6 2
    15 2 4
8
    2 11
10
11
    18 3 3
12
    3 15 9
13
14
    14 8 10 7 20 21 3 6 18 15 16 12 9 5
15
16
17
   127380964
18
19
20
    14 12 5
    2 3 12 10 8 1 7 0 5 13 14 11
21
22
23
    2 9 7 11 5 19 8 0 23 14 15 6 21 10
24
25
26
    14 8 4
    14 8 0 6 13 9 10 1
```

Output

```
Case #1: 3
Case #2: impossible
Case #3: impossible
Case #4: 3
Case #5: 3
Case #6: 2
Case #7: 2
Case #8: 5
Case #8: 5
Case #9: 3
```

Sample Data Explanation

In the first sample case, three street lights need to be switched on. Note that these may also be three different street lights, for example switching on the first, third, and sixth street light works as well. The second and third sample cases are impossible as there is always at least one section of the street that cannot be illuminated. In the fourth sample case, the street can be illuminated on its entire length by switching on all street lights. The points at which the light cones meet between the street lights are illuminated as well.

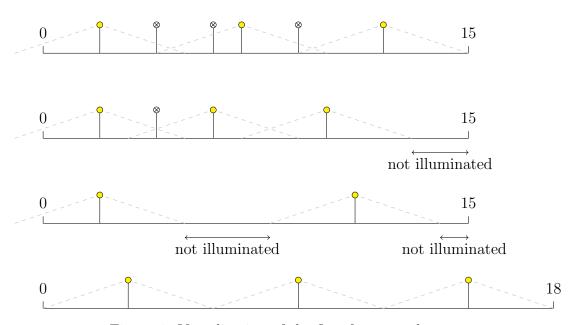


Figure 1: Visualization of the first four sample cases.

C Customs

Author: Philipp Hoffmann

Making money is hard, not just for people but also for countries. It is so hard that an especially broke country, Poorland, has come up with a completely new idea: They will split their country into two countries. This way, they can collect customs duties each time someone passes from one country to another.

As they want to maximize the amount of money they collect, the number of roads that lead from one of the new countries to the other should be maximal. Finding an optimal solution is very costly, thus they asked Lea to find a solution that differs from the optimal solution by at most a factor of $\frac{1}{2}$. As streets start and end at cities, the task will be to partition the cities into two sets.

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with a number n, the number of cities in Poorland, n lines follow. The i-th line describes city i and contains an integer k_i followed by k_i integers $c_1, ..., c_{k_i}$. k_i is the number of streets exiting city i, $c_1, ..., c_{k_i}$ are the cities directly connected to city i by streets. Cities are indexed from 1 to n.

You can assume that streets are undirected: If city i is directly connected to city j, then city j will be directly connected to city i.

Output

For each test case, output one line containing "Case #i:" where i is its number, starting at 1. Output one more line containing integers $a_1...a_l$ in ascending order such that when the country is partitioned into the cities $a_1, ..., a_l$ and the remaining cities, the number of roads between the parts is at least half as big as in the optimal solution. If there are multiple solutions, any of them will be accepted. Each line of the output should end with a line break.

Constraints

- $1 \le t \le 20$
- $1 \le n \le 300$
- $0 \le k_i \le n$ for all $1 \le i \le n$
- $1 \le c_i \le n$ for all $1 \le i \le k_i$

Input

```
8
2
1 2
 2
 4
    1 1
 5
6
     4
 7
    1 3
    1 4
2 1 4
2 2 3
 8
 9
10
11
12
    3 2 4 5
3 1 3 5
13
14
    3 2 5 6
3 1 5 6
15
16
    4 1 2 3 4
2 3 4
17
18
19
20
    0 2 3 4
21
22
    1 2 1 2
23
24
25
    5
1 4
0
26
27
28
29
    0
30
    1 1
31
     0
32
    3
33
    1 2
1 1
34
35
36
37
   4
2 2 3
2 1 4
1 1
38
39
40
41
42
    1 2
43
44
    1 4
2 4 5
2 4 6
5 1 2 3 5 6
45
46
47
48
    2 2 4 2 3 4
49
```

```
Case #1:
    1
Case #2:
 2
 3
    1 2 4
     Case #3:
1 3 5 6
5
6
7
     Case #4:
     1 2
8
     Case #5:
10
    1 2 3 5
11
     Case #6:
    1 3
Case #7:
12
13
14
     1 4
    Case #8:
1 2 3 5 6
15
```

D Woodchucking

Author: Christian Müller

You may remember Lea's friend Nick, the biologist. Recently, he decided to do a study at a saw mill, Jack Lumber Inc., a business producing lumber. He decided to do his study on the possible productivity increase of local industries by using rodents (woodchucks, mostly). So for a few weeks, Nick got in touch with his inner lumberjack and hacked away at everything that even remotely resembled a tree (the lumberjacks quickly learned not to come too close to Nick when he was armed with an axe). Once a month, the lumberjacks had to heave all the trees onto large conveyor belts at the saw mill, where they were cut up into small pieces by huge disk saws, ready to be shipped to whoever wanted to build barstools, wooden swords, toothpicks or whatever else. There were a lot of disk saws and conveyor belts and every tree took sawing time according to its height. This meant nobody knew on which conveyor the next tree had to be put to minimize the total sawing time. So these days were mostly ending in chaos and the buzzing and sawing took quite a while.

As a scientist, Nick could not resist to try to help them and quickly devised a computer program that computed the optimal distribution of trees to the saws. It took ages to compute the value, but now the lumberjacks could chuck down the trees, then sit around for a day and then Nick guided them to cut the trees up much faster.

Unfortunately, Handsome Jack, the boss at *Jack Lumber Inc.*, really did not like the fact that his employees sat around, not chucking wood, waiting for Nick's program to spurt out some numbers. Nick tried to reason with him, but it was no use. They had to go back to the old system until Nick came up with a better idea.

Can you help him devise a faster computation that cuts up the trees in at most 50% more time than his previous solution?

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line. Each test case consists of a line containting two integers n and m. n is the amount of trees the lumberjacks felled during the course of the month, m is the amount of disk saws available. n lines follow, each containing a single integer l_i , the time it takes to cut up tree i.

Output

For each test case, output one line containing "Case #i: s" where i is its number, starting at 1, and s is at most 50% more than the minimal amount of time it takes to cut up every tree. Each line of the output should end with a line break.

Constraints

- $1 \le t \le 20$
- $\bullet \ 1 \le n \le 2 \cdot 10^5$
- $1 \le m \le 10^4$
- $1 \le l_i \le 10^9$

Sample Data

Input

```
7
3 2
2
8
5 4
 1
2
3
4
5
6
7
8
9
     4 3
1
1
2
4
10
11
12
       4 5
13
15
16
17
18
19
        4 2
        4
9
5
5
21
22
23
24
25
      3 2
10
9
26
27
28
29
30
      4 4
6
4
8
7
31
32
33
34
35
      3 4
8
4
10
36
37
38
```

```
1 Case #1: 8
2 Case #2: 4
3 Case #3: 7
4 Case #4: 13
5 Case #5: 18
6 Case #6: 8
7 Case #7: 10
```

E Mice

Author: Philipp Hoffmann

Recently Lea read a book about, among other things, the creation of the earth. In this scientifically accurate work it was detailed that the earth was a construction ordered by mice. This left Lea baffled: could it really be that humans are inferior to mice? Only one way to find out!

Lea wants to make sure that humans are superior by comparing their DNA to the DNA of an obviously superior being, dolphins. Should she find out that they are closely related, it is clear that mice are no match for humans.

Comparing DNA is done by scoring: Lea has some DNA sequences from humans and some from dolphins. A sequence is a string consisting only of the letters A,C,T and G. Two equal length sequences are scored with a 4×4 scoring matrix whose rows and column are labelled with A,C,T and G. An entry in row A and column T is then called score(A,T). The sequences are scored by computing the score of the first letters of each sequence, the second letters, and so on, and summing up the result. For example, the sequences ATA and CGT would be scored as score(A,C) + score(T,G) + score(A,T).

The matrix must have the following properties:

- All entries must be integers between -10 and 10, inclusively.
- It must be symmetric $(score(x, y) = score(y, x) \text{ for all } x, y \in \{A, C, T, G\}).$
- Diagonal entries must be positive (score(x, x) > 0 for all $x \in \{A, C, T, G\}$).
- The sum of all 16 entries must be 0.

Of course Lea wants to choose the matrix such that the sum of the scores she gets when comparing every human DNA sequence to every dolphin DNA sequence is maximal. Can you compute the maximal score that she can achieve?

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

The first line of each test case contains two integers n m, the number of human and dolphin DNA sequences. The next n lines contain one human DNA sequence each. The last m lines contain a dolphin DNA sequence each.

Output

For each test case, print a line containing "Case #i: x" where i is its number, starting at 1, and x is the maximal score that can be reached. Each line of the output should end with a line break.

Constraints

- $1 \le t \le 20$
- $1 \le m, n \le 200$
- \bullet A sequence consists of at most 100 letters of A,C,T and G.
- \bullet For each test case, all sequences have equal length.

Input

```
2 3
    1 1
    ACAC
    ATGC
5
    2 2
6
7
    AAA
    CTG
8
9
    TCC
10
    GGT
11
12
    ACACACTGGTCTATACTTCG
13
    ACTCGCTGAAGTTAATTACC
14
15
    ACTGTGCGTCCAAGGGTAAT
16
    ATCGGACAGATCACGCCCCT
    ATGGTGATATTCAATGCTGT
17
18
    CATATCGTACTAGGGTAAAG
19
20
    2 5
21
    CTATTTAGTT
    AAACTGTAAT
22
    GATTGATTTG
24
    CTAGAGACCA
    ATTGCTCCCG
25
26
    GAGTACTAAA
27
    TGCCGTTCTT
28
29
    3 2
    GGACGCG
30
31
    ATTCGAT
    ATAGCGA
32
33
    TCACCCG
34
    CCCACAG
35
    4 5
36
37
    TACG
    GGAG
38
39
    ACCC
40
    GTCA
    GCAC
41
42
    GCGG
43
    GAAG
44
    GGTA
45
    ATCT
46
47
    CGTATGTCCCGCGA
48
    GCAAGTGCGCTTTG
49
50
    CAGGTTGGGGTCAA
    GGTAAGGGGAAAAC
51
    ATAAGCTTGCTCTC
```

```
1 Case #1: 40
2 Case #2: -4
3 Case #3: 322
4 Case #4: 363
5 Case #5: 147
6 Case #6: 236
7 Case #7: 149
```