Algorithms for Programming Contests WS15/16 - Week 10

Chair for Efficient Algorithms (LEA), TU München

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This problem set is due by

Wednesday, 23.12.2015, 6:00 a.m.

Try to solve all the problems and submit them at

https://judge.in.tum.de/conpra/

This week's problems are:

Problems

A	Commander-in-Chief
B	Goldbach's Conjecture
C	N-athlon
D	Soft Skills
E	Cookies

The following amount of points will be awarded for solving the problems.

Problem	A	B	C	D	E
Difficulty	easy	easy	medium	medium	hard
Points	4	4	6	6	8

If the judge does not accept your solution but you are sure you solved it correctly, use the "request clarification" option. In your request, include:

- the name of the problem (by selecting it in the subject field)
- a verbose description of your approach to solve the problem
- the time you submitted the solution we should judge

We will check your submission and award you half the points if there is only a minor flaw in your code.

If you have any questions please ask by using the judge's clarification form.

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Problem A
Commander-in-Chief

A rainy Sunday afternoon. What should Lea and her friends do? They wanted to go tightrope walking between two mountain summits, but this is not going to happen with rain pouring down on them, and making the rope slippery and all. So, they decide to stay in and dust off an old warfare strategy board game they played a lot when they were younger. Epic battles between huge armies, invading continents, conquering new worlds. Though, all of this is only possible for the best players. And Lea really does want to be one of those players. Roughly explained, the game is about stationing armies on different regions on a map, and letting them fight against opposing armies from a neighbouring region in order to try to conquer that region.

As far as she can remember, the best strategy for her was to subdivide each army into smaller squadrons and letting each of them fight on a different battleground. However, the real strategic trick in this game was that each of these squadrons should be of the same size, even if they belong to different armies. To maximise Lea's probability to conquer the world, the squadrons should be as big as possible, and no troops should be left behind without a squadron. Luckily, Lea has time to calculate this squadron size during the other players' turns.

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with a line containing an integer n, the number of armies Lea commands. The next line contains n space-separated integers $a_1 \dots a_n$ denoting the sizes of the n armies.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is the biggest possible squadron size as described above. Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 500$
- $2 \leq n \leq 20$
- $1 \leq a_i \leq 10^9$ for all $1 \leq i \leq n$

Sample Input	Sample Output
2 3 12 6 21 4 24 7 12 18	Case #1: 3 Case #2: 1

Sample Input	Sample Output
6 5 35 19 37 11 8 2 48 48 3 44 41 26 3 33 44 33 5 9 18 45 36 36 3 29 29 29	Case #1: 1 Case #2: 48 Case #3: 1 Case #4: 11 Case #5: 9 Case #6: 29

Problem B
Goldbach's Conjecture

Goldbach's conjecture is a mathematical conjecture (that means, it is not yet proven but believed to be correct) that states the following:

Every even integer greater than 2 can be expressed as the sum of two primes.

Every odd integer greater than 5 can be expressed as the sum of three primes.

Lea is not quite convinced that this is true and asks you to prove it (and just like most computer scientists, this proof will be by example): She gives you a number and expects you to find two or three primes (depending on whether her number is even or odd) that sum up to her number.

Input

The first line of the input contains an integer t. t test cases follow.

Each test case consists of a single line containing one integer n, Lea's number.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is a space separated list of two or three primes that sum up to Lea's number. If there are multiple solutions, give the lexicographically smallest one. (I.e. if your solution is $x_1x_2x_3$ minimize x_1, then x_2. This in particular implies $x_1 \leq x_2 \leq x_3$.) Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 20$
- $n \leq 10^8$
- n is either even and greater than 2 or odd and greater than 5

Sample Input	Sample Output
3 6 18 41	Case #1: 3 3 Case #2: 5 13 Case #3: 2 2 37

Sample Input	Sample Output
7 77 98 55 42 48 76 11	Case #1: 2 2 73 Case #2: 19 79 Case #3: 3 5 47 Case #4: 5 37 Case #5: 5 43 Case #6: 3 73 Case #7: 2 2 7

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Problem C
N-athlon

Lea is a very active person. Whenever she is bored, she finds something new to do. This time, she invited all her friends over for a grand tournament. She devises many exciting (or sometimes just silly but fun to watch) team games, in which her friends have to compete and whoever is part of a winning team gets a point. Of course, most games need a different number of players on each team, so for every game, new teams are formed.

Nevertheless, the event is a big success and many of Lea's friends compete in suspense-packed matches of real competition sports like Soccer, Volleyball, First Person Shooters, Real Time Strategy, Hockey and some newly invented events like "Extreme Spaghetti Knitting" or "Plants vs Zombies - An Adaption" (where the contestants roleplay on Lea's front lawn and the audience votes on whoever gave the best impression).

In the evening, Lea is really tired but happy. But before she is able to sleep, she wants to determine who won the grand tournament. But in all the chaos, she realizes she doesn't even have any idea of how many people participated.

At least, she recalls the following facts: For every event she organized, she divided all the participants into teams of equal size. Since she is a math enthusiast, she thought it would be a cool idea that every team size would be a prime number, but not many people even noticed. And since she oversaw the building of those teams, for each event she thinks she remembers the amount of people that were left over and had to watch from the sidelines. She also knows how many people she invited and that almost everyone she invited was there. Can you tell her how many people participated? (Please also tell her if her memories are inconsistent and no number of friends satisfy her memorized numbers)

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case begins with a line containing two integers n, k, where n is the number of different games Lea organized and k is the number of friends that were invited. n lines follow, each consisting of two integers $size_i$ $rest_i$, where $size_i$ is the team size for game i and $rest_i$ is the amount of people that remained after dividing the amount of participants into teams of size $size_i$.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x being the people that participated in Lea's contest satisfying the constraints on team sizes and being the largest such number less or equal to k or "impossible" if Lea has made a counting error and there is no x that satisfies these constraints. Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 20$
- $0 \leq n \leq 15$
- $0 \leq k < 10^{12}$
- $0 \leq rest_i < size_i < 10^8$
- $size_i$ is prime for all i
- The number of total participants x does not change between games
- $\prod^{i} size_i < 10^{18}$

Sample Input	Sample Output
3 2 20 3 1 5 2 1 5 2 1 2 20 11 1 17 2	Case #1: 7 Case #2: 5 Case #3: impossible

Sample Input	Sample Output
6 3 29 19 5 43 8 47 13 1 29 43 22 1 20 11 9 2 13 5 1 3 1 4 43499412 11 2 73 53 17 12 79 32 3 10497640 29 24 17 11 37 35	Case #1: impossible Case #2: 22 Case #3: 20 Case #4: 1 Case #5: 43457537 Case #6: 10492717

Problem D
Soft Skills

Lea wants to get a better job and is therefore looking for some soft skill courses as she knows that every recruiter loves to read the courses' inspiring titles on a CV. She found the list of courses at IAH, the Institute for Amazing Headlines, which will look great on her CV. The people at IAH also award certificates if the participants take enough of their courses. There is not much information about the contents of the courses, but this is not what Lea is looking for, anyway.

The number of courses at IAH is overwhelming, so Lea begins to wonder how many possibilities there are to combine them for the certificate. For the certificate, only the number of courses matters. It is not important whether Lea participates in "Handling Extremely Complex Systems" or "How to Get Promoted During the Coffee Break", for instance. As the number of possible combinations for the certificate may be very big, Lea is already satisfied to find the number modulo $223092870 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$.

Input

The first line of the input contains an integer t. t test cases follow.

Each test case consists of a single line containing two integers n, the number of courses available at IAH, and m, the number of courses needed for the certificate.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is the number of possibilities to pick exactly m courses modulo 223092870. Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 20$
- $1 \leq n \leq 10000000$
- $1 \leq m \leq n$

Sample Input	Sample Output
3 5 2 7 4 100 12	Case #1: 10 Case #2: 35 Case #3: 96633810

Sample Input	Sample Output
5 7 6 11 6 6 5 10 4 11 8	Case #1: 7 Case #2: 462 Case #3: 6 Case #4: 210 Case #5: 165

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Problem E
Cookies

Most people are quite happy to invite friends. Lea is too, and of course she strives to make the invitees as happy as possible. Sometimes, this proves to be quite difficult. This time she plans to buy cookies for everyone. While this sounds like a simple task, the eating habits of her friends complicate things.

When eating cookies, all friends sit in a big circle. All cookies are poured into a big bowl that is passed around. Each friend has a specific number of cookies that he eats every time he gets the bowl. The bowl starts full at Lea, is passed around each time in the same order, and Lea always eats exactly one cookie whenever she gets the bowl back.

She now wants to buy a number of cookies such that, no matter which of her friends show up, the bowl will end up empty after Lea takes a cookie (Then the bowl is passed around no more). It may be passed around a couple of times, but it should not happen that a friend cannot take his number of cookies or that it returns to Lea empty.

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with an integer n, the number of Lea's friends. The next line contains a space separated list of n integers $c_1, \ldots, c_n$, c_i is the number of cookies her i-th friend eats each time he gets the bowl.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is the minimal number of cookies Lea has to buy to satisfy the constraints above modulo $2147483647 = 2^{31} - 1$. Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 20$
- $1 \leq n \leq 500$
- $1 \leq c_i \leq 50$

Sample Input	Sample Output
3 2 3 5 3 1 1 1 6 45 46 47 48 49 50	Case #1: 36 Case #2: 12 Case #3: 1929057266

Sample Input	Sample Output
7 2 2 3 3 2 2 2 10 4 5 7 2 5 4 4 5 3 3 3 4 2 2 1 2 10 7 1 7 5 5 6 7 3 3 4 15 2 1 3 7 1 4 5 5 6 5 2 7 5 3 2	Case #1: 12 Case #2: 105 Case #3: 2102860397 Case #4: 315 Case #5: 3 Case #6: 1835120897 Case #7: 1264400816