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Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Winter Term 2015/16
Problem Set 2
21.10.2015

Algorithms for Programming Contests

Welcome to our practical course! This problem set is due by

Wednesday, 28.10.2015, 6:00 a.m.

Try to solve all the problems and submit them at

`https://judge.in.tum.de/`

This week's problems are:

The following amount of points will be awarded for solving the problems.

Problem	A	B	C	D	E
Difficulty	easy	easy	medium	medium	hard
Points	4	4	6	6	8

If the judge does not accept your solution but you are sure you solved it correctly, use the "request clarification" option. In your request, include:

- the name of the problem (by selecting it in the subject field)
- a verbose description of your approach to solve the problem
- the time you submitted the solution we should judge

We will check your submission and award you half the points if there is only a minor flaw in your code.

If you have any questions please ask by using the judge's clarification form.

A Marry Rich

Author: Stefan Toman

Lea has a new game to play: *The Fugger Family IV*. In this game, Lea plays Jakob Fugger "the Rich", one of the richest persons of all time, and tries to grow his family's wealth and influence. At the time that Jakob Fugger lived, 1459 - 1525, one of the main ways to achieve this was marrying into a family having both. In real life, Jakob Fugger married Sibylla Artzt, Grand Burgheress to Augsburg, but Lea wants to do better in that game.

Lea compiled a list of all notable people in the world, the amount of money they have, and a list of all relations and marriages between them. To increase her influence, she wants Jakob to marry somebody who is not already connected to his family via relations or marriages, also via several steps. For example, she does not want Jakob to marry relatives of his uncle's wife. Also, it is not possible to marry someone who is already married.

As the game's publisher Moonflowers does not want to be criticized by activists, Lea may make Jakob marry any person in the game, including males. Among the possible candidates, she wants Jakob to marry the one with the most money. But who is this?

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with a line containing three space-separated integers a, b and c, where a is the number of notable people in the world labelled from 1 to a with person a being Jakob Fugger, b is the number of family relations and c the number of marriages.

One line follows containing $a-1$ space-separated integers $m_0, \ldots, m_{a-1}$ where m_i denotes the amount of money person i owns.

b lines follow describing the family relations. Each of these lines contains two space-separated integers d and e meaning that d is related to e. Note that relations that are given as a transition of other relations may be omitted.

c lines follow describing the marriages. Each of these lines contains two space-separated integers f and g meaning that f is married to g. Everybody is married at most once.

Output

For each test case, output one line containing "Case #i: x" where i is its number, starting at 1, and x is the biggest amount of money Lea can get into the family by marrying or "impossible" if there is nobody to marry satisfying her constraints.

Constraints

- $1 \leq t \leq 20$
- $1 \leq a \leq 10^4$
- $0 \leq b, c \leq 10^4$
- $0 \leq m_i \leq 10^6$ for all $1 \leq i \leq a - 1$
- $1 \leq d, e \leq a$
- $1 \leq f, g \leq a - 1$

Sample Data

Input

```
2
7 1 1
1 2 3 4 5 6
6 7
3 5

7 2 1
1 2 3 4 5 6
6 7
5 4
4 6
```

Output

```
Case #1: 4
Case #2: 3
```

B Cable Car

Author: Christian Müller

Lea is a great fan of wintersports. She always follows the winter olympics on TV and just loves to go skiing herself. This year, she booked a room in an expensive hotel in a very exclusive ski resort called “Slippery Slopes and Hills”. One evening at the Après-Ski-Party, she met an interesting man - the architect who planned all the cable cars taking the tourists up the mountain. They talked for a bit and he described his latest problem to her.

The ski resort is trying to build a new cable car up a glacier. Through complicated computation, they even found out exactly how many posts are needed to support the cable car. Since glaciers move (albeit really slowly), the individual posts supporting the cable car have to be spaced as far apart from each other as possible. Additionally, the cable car should also span a canyon in the middle of the route. Now the architect is hard at work, trying to figure how to place the posts. Lea, who is always on the lookout for interesting problems, tells you about it. Can you help the architect?

Input

The first line of the input contains an integer t. t test cases follow.

Each test case consists of a single line of four integers d, p, u, and v, where d is the length of the route (going from 0 to d) of the cable car, p is the amount of posts that should be placed, u is the beginning point of the canyon and v the end point. Posts may be placed anywhere between 0 and d, i.e. exactly on 0, d, u, and v, but not in between u and v.

Output

For each test case, output one line containing “Case #i: x” where i is its number, starting at 1, and x is the maximal minimum distance between two posts that can be achieved with an absolute error of up to 10^{-4}. This means the maximum x such that the architect can place all the posts and no two posts are less than x apart. Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 20$
- $1 \leq d \leq 1000000$
- $2 \leq p \leq 2000000$
- $0 \leq u \leq v \leq d$

Sample Data

Input

```
4
2 3 1 2
3 3 0 1
9 10 5 6
9 10 5 7
```

Output

```
Case #1: 1.0000000009
Case #2: 1.5000000007
Case #3: 1.0000000001
Case #4: 0.8333333338
```

C Bank Loans

Author: Philipp Hoffmann

Recently, Lea has been thinking quite a lot about buying a house for herself. Since she wants a modern design, a large garden for real life Plants vs. Zombies, a pool and a garage, this will be a quite costly endeavour. Thus she is looking into bank loans. Her bank consultant told her lots of stuff about the loaning system and now she needs to adapt that to her situation. Since the math is a bit complicated, she asks you to help her.

Here is how the loaning system works: In the beginning, you borrow money at a fixed interest rate r. Every month you pay a fixed amount of money back. After the payment, the remaining loan is increased by $r\%$ and then rounded down to the next integer.

Lea has multiple questions and that should help her answer: For a fixed loan, interest rate and time span, what is the minimal monthly payment to finish in that timespan? For a fixed loan, monthly payment and time span, what is the maximal interest rate at which the payments will be finished in that timespan? And finally, for a fixed interest rate, montly payment and time span, what is the maximal loan that will be payed back in that time span? In each case, it may happen that the last payment is less than all previous payments.

Input

The first line of the input contains an integer t. t test cases follow.

Each test case consists of a single line containing four integers l, r, p and y, where l is amount of the loan, r is the monthly interest rate, p it the monthly payment and y is a number of years.

Output

For each test case, output one line containing "Case #i: l_{max} r_{max} p_{min}" where i is its number, starting at 1, and

- l_{max} is the maximal integral loan that will be payed off after y years with interest rate r and monthly payments p.
- r_{max} is the maximal integral positive interest rate at which the loan l will be payed off after y years with monthly payments of p. In the case that any interest rate works, output "infinity". In the case that no positive interest rate works, output "impossible".
- p_{min} is the minimal integral amount that must be payed each month so that the loan l will be payed off after y years with interest rate r.

Note that l_{max} does not depend on l, r_{max} does not depend on r (and in particular may be greater than 100), and p_{min} does not depend on p.
Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 20$
- $1 \leq l \leq 10^7$
- $0 \leq r \leq 100$
- $1 \leq p \leq 5 * 10^6$
- $1 \leq y \leq 30$

Sample Data

Input

```
2
100 5 10 1
50 10 50 2
```

Output

```
Case #1: 96 4 11
Case #2: 498 infinity 5
```

D Diplomacy

Author: Christian Müller

International relations are hard. In the light of some recent international crisis, Lea's school decided that graduates should at least have a basic understanding of why that is. So, they set up a simulation of an international meeting for their graduates where every person represents a different country and has their own agenda. Lea decided to participate and so, she got her own agenda: As a representative of one of the larger countries, she wants to sway the course of history in her favour by rallying an alliance of countries that are able to overcome the resistance of all the other countries. For that, she gathered allies by making friends through diplomatic relations.

However, there are some countries that just hate one another because one country killed the king of the other country some hundreds of years ago. So, Lea has to pick up on the small, intricate signs of mutual hate or friendliness between the other countries' representatives. For example, she might pick up a small, intimate grin shared between the representatives of Templonia and Poorland and concludes that the two may be allied and plotting her downfall. Or she might see the representative of Beachistan slip some ominous powder into the drink of the representative of another country and infer that these two will probably not form an alliance anytime soon.

She also uses some basic properties of friendship and animosity:

- The friends of my friends are my friends as well.
  If x and y are allied and y and z, then x and z are allied as well.
- Friendship is mutual.
  If x is allied with y, then y is allied with x.
- Hatred is mutual.
  If x hates y, then y hates x.
- A common cause unites people.
  If x hates z and y hates z, then x and y form an alliance.
- An alliance has common enemies.
  If x is allied with y and x hates z, then y hates z as well.

After picking up on all these hints, can you tell her if her alliance gathered more than half of all the countries? This would establish a very powerful position for Lea (and her country, of course) from which she can use diplomacy to batter the rest of her enemies into submission (trade sanctions, for example).

Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case begins with a line consisting of two integers n, the number of countries, and m, the number of interactions between countries. The countries are numbered from 1 to n, with Lea representing country number 1. m lines follow, signaling a sign for either friendship or antipathy between the representatives of two countries. The i-th line is either `F` x y, signaling friendship or `A` x y, signaling antipathy between countries x and y.

Output

For each test case, output one line containing "Case #i: r" where i is its number, starting at 1, and r is "yes", if Lea's alliance has gathered more than half of all countries and "no" otherwise. Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 20$
- $1 \leq n \leq 20000$
- $0 \leq m \leq 20000$
- $1 \leq x, y \leq n, x \neq y$
- The given relations will not be inconsistent (no two countries will be allies and enemies at the same time).

Sample Data

Input

```
2
5 3
F 1 2
A 2 3
A 3 4

5 3
F 1 2
A 2 3
F 4 5
```

Output

```
Case #1: yes
Case #2: no
```

E Sharing

Author: Philipp Hoffmann

Lea is a cookie enthusiast. She loves every flavour, form and variety of cookies, and has quite a few at home. Every time her nephew Tom visits her, he wants to eat as many cookies as possible. Because Lea does not want him to eat all of her cookies, she has invented a game to decide which cookies Tom gets.

She arranges the cookies in different bowls, in a long line. Each bowl may contain an arbitrary amount of cookies. She then chooses two bowls, say a and b (1-based), where a is to the left of or equal to b. Tom may then choose to either eat all cookies in bowls 1 to $a-1$, or all cookies in bowls a to b , or all cookies in bowls $b+1$ to n where n is the number of bowls.

Today Tom will visit again and she has already made the arrangement of the bowls and cookies. She only has to decide on the points a and b. Of course she wants to keep as many cookies as possible for herself. With the optimal choice, and assuming Tom will choose the part with the most cookies for himself, how many cookies can she keep?

Input

The first line of the input contains an integer t. t test cases follow.

Each test case consists of a single line containing 5 integers n p q r s describing the bowls filled with cookies. There are n bowls and the i-th of them (1-based) contains $((i*p+q)MODr)+s$ cookies. Note that this format was chosen to keep the input small, it is not necessary to somehow exploit this formula.

Output

For each test case, print a line containing "Case #i: x" where i is its number, starting at 1, and x is the maximal number of cookies that Lea can keep for herself. Each line of the output should end with a line break.

Constraints

- $1 \leq t \leq 50$
- $1 \leq n \leq 10^6$
- $1 \leq p, q, r, s \leq 10^6$

Sample Data

Input

```
4
1 1 1 1 1
3 2 2 3 1
10 3 4 11 2
999999 999997 999995 1000000 999999
```

Output

```
Case #1: 0
Case #2: 3
Case #3: 46
Case #4: 999997333337
```

Sample Case Explanation

In the first case there is only one bowl with one cookie in it, no matter which a and b Lea chooses Tom will pick the range with the cookie.

In the second case there are three bowls with 2,1 and 3 cookies. Lea chooses $a = b = 2$, Tom chooses bowl 1 and Lea keeps three cookies.

In the third case, there are 10 bowls with 9,12,4,7,10,2,5,8,11,3 cookies. Lea chooses $a = 4$, $b = 7$. The three parts contain 25, 24 and 22 cookies. 25 for Tom, Lea keeps 46 for herself.