MODULE *beans*

EXTENDS *Integers*
CONSTANTS R, G, B

ASSUME $\land R \in 0 \,..\, 200$
$\land G \in 0 \,..\, 200$
$\land B \in 0 \,..\, 200$

```
--fair algorithm beansAlgo{
   variable red = R, blue = B, green = G ;
   { S: while ( TRUE )
   { either
A1blue: { await (blue > 1);     same color and blue
           blue := blue - 2; green := green + 1; red := red + 1;
 } ;
        or
A1red: { await (red > 1);     same color and red
           red := red - 2; green := green + 1; blue := blue + 1;
 } ;
        or
A1green: { await (green > 1);     same color and green
           green := green - 2; blue := blue + 1; red := red + 1;
 } ;
        or
A2rg: { await (red > 0 ∧ green > 0);   different color red and green
          red := red - 1; green := green - 1; blue := blue + 1;
 } ;
        or
A2gb: { await (blue > 0 ∧ green > 0);   different color blue and green
          blue := blue - 1; green := green - 1; red := red + 1;
 } ;
        or
A2br: { await (red > 0 ∧ blue > 0);   different color red and blue
          red := red - 1; blue := blue - 1; green := green + 1;
 } ;

     }  end while
    }  end algo
 }  \*end algo
```

BEGIN TRANSLATION

VARIABLES *red*, *blue*, *green*, *pc*

$vars \triangleq \langle red, blue, green, pc \rangle$

$Init \triangleq$ Global variables
$\land red = R$

$$
\begin{array}{l}
\quad \land\ blue = B \\
\quad \land\ green = G \\
\quad \land\ pc = \text{"S"}
\end{array}
$$

$$
\begin{array}{lcl}
S & \triangleq & \land\ pc = \text{"S"} \\
& & \land\ \lor\ \land\ pc' = \text{"A1blue"} \\
& & \quad\ \lor\ \land\ pc' = \text{"A1red"} \\
& & \quad\ \lor\ \land\ pc' = \text{"A1green"} \\
& & \quad\ \lor\ \land\ pc' = \text{"A2rg"} \\
& & \quad\ \lor\ \land\ pc' = \text{"A2gb"} \\
& & \quad\ \lor\ \land\ pc' = \text{"A2br"} \\
& & \land\ \text{UNCHANGED}\ \langle red,\ blue,\ green \rangle
\end{array}
$$

$$
\begin{array}{lcl}
A1blue & \triangleq & \land\ pc = \text{"A1blue"} \\
& & \land\ (blue > 1) \\
& & \land\ blue' = blue - 2 \\
& & \land\ green' = green + 1 \\
& & \land\ red' = red + 1 \\
& & \land\ pc' = \text{"S"}
\end{array}
$$

$$
\begin{array}{lcl}
A1red & \triangleq & \land\ pc = \text{"A1red"} \\
& & \land\ (red > 1) \\
& & \land\ red' = red - 2 \\
& & \land\ green' = green + 1 \\
& & \land\ blue' = blue + 1 \\
& & \land\ pc' = \text{"S"}
\end{array}
$$

$$
\begin{array}{lcl}
A1green & \triangleq & \land\ pc = \text{"A1green"} \\
& & \land\ (green > 1) \\
& & \land\ green' = green - 2 \\
& & \land\ blue' = blue + 1 \\
& & \land\ red' = red + 1 \\
& & \land\ pc' = \text{"S"}
\end{array}
$$

$$
\begin{array}{lcl}
A2rg & \triangleq & \land\ pc = \text{"A2rg"} \\
& & \land\ (red > 0 \land green > 0) \\
& & \land\ red' = red - 1 \\
& & \land\ green' = green - 1 \\
& & \land\ blue' = blue + 1 \\
& & \land\ pc' = \text{"S"}
\end{array}
$$

$$
\begin{array}{lcl}
A2gb & \triangleq & \land\ pc = \text{"A2gb"} \\
& & \land\ (blue > 0 \land green > 0) \\
& & \land\ blue' = blue - 1 \\
& & \land\ green' = green - 1 \\
& & \land\ red' = red + 1 \\
& & \land\ pc' = \text{"S"}
\end{array}
$$

$$A2br \triangleq \begin{array}{l} \land pc = \text{"A2br"} \\ \land (red > 0 \land blue > 0) \\ \land red' = red - 1 \\ \land blue' = blue - 1 \\ \land green' = green + 1 \\ \land pc' = \text{"S"} \end{array}$$

$$Next \triangleq S \lor A1blue \lor A1red \lor A1green \lor A2rg \lor A2gb \lor A2br$$

$$Spec \triangleq \begin{array}{l} \land Init \land \Box[Next]_{vars} \\ \land \mathrm{WF}_{vars}(Next) \end{array}$$

END TRANSLATION

$$Termination \triangleq \Diamond(red + green + blue = 1)$$

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*Consider a coffee can containing an arbitrary (but finite) number of beans. The beans come
*in 3 different colors: red, green, and blue.
*Now consider the following program:
*Choose two beans from the can;
*if they are the same color, toss them out and add two beans for the other two colors
*if they are different colors, toss them out and add a bean of the third color
*Repeat.

* ANSWER 1:
* 3 Invariant Conditions
* $red + green + blue > 0$
* $red + blue \leq 400 \lor blue + green \leq 400 \lor green + red \leq 400$
* $red + green > 0 \lor green + blue > 0$

* ANSWER 2:
* Fixed Point of the Program:
* Since invariant is $red + green + blue > 0$ *i.e.* this condition is always satisfied for the program. Now the fixed point can be reached
* when the quantity of beans for any two colors falls below 1 in the can and only 1 third bean is present so that
* that they don't get picked up in the next iteration (either for same color or different color)
* $(green \leq 1 \land (red \leq 0 \land blue \leq 0)) \lor (blue \leq 1 \land (red \leq 0 \lor green \leq 0)) \lor (red \leq 1 \land (blue \leq 0 \land green \leq 0))$

* ANSWER 3:
* Variant Function or metric function
* $red + green + blue$
* As a metric, we choose $red + green + blue$. This value is bounded below, as evident by the invariant.
* Also, it never increases since the rate of decrement in number of beans is greater than their rate of increment.

* (As, when the beans tossed out are of same color, the count of beans in the box remains the same. If the beans tossed
* out are of different colors, then the rate of decrement is greater than the increment)
* Therefore, this program terminates.
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