Problem 2.1 : In The Problem we are given The Ratio of The Masses of The Plent and The star.

$$\frac{M_P}{M_* + M_P} = 10^{-3} \quad ; \quad M = M_* + M_P = 0.6\,M_\odot$$

$$M_P = (0.6\,M_\odot \times 10^{-3}) \quad ; \quad a = 1.32\text{ AU} \text{ and}$$

$e = 0.68$ Also we have an expression for The Radius of The orbit (elliptical) in Terms of f and E. f is The True Anomaly Measured from The Pericenter and E called The eccentric anomaly.

$$r(f) = \frac{a(1-e^2)}{1+e\cos(f)}$$

$$r(E) = a(1-e\cos(E))$$

$f = E = \pi$ at Apocenter

We need to find The equation for The radial component of The Planet's velocity About its orbit

$$\dot{r} = \frac{dr}{df}\frac{df}{dt} \quad \text{Now} \quad r(f) = \frac{a(1-e^2)}{1+e\cos(f)}$$

$$\frac{dr(f)}{df} = \frac{d}{df}\left(\frac{a(1-e^2)}{1+e\cos(f)}\right) = \frac{d}{df}\left(a(1-e^2)(1+e\cos(f))^{-1}\right)$$

$$\frac{dM}{df} = -\left(a(1-e^2)\right)(1+e\cos(f))^{-2}(-e\sin(f)) = \frac{e\sin(f)\left(a(1-e^2)\right)}{(1+e\cos(f))^2} = \frac{dM}{df}$$

Also from the question we know that

$$L = M^2 \frac{df}{dt} \quad \text{so} \quad \frac{df}{dt} = L/M^2$$

so,

$$\frac{dM}{dt} = \frac{dM}{df}\frac{df}{dt} \Rightarrow \frac{e\sin(f)\left(a(1-e^2)\right)}{(1+e\cos(f))^2} L/M^2$$

Also $L = \sqrt{GMa(1-e^2)}$ so

$$\frac{dM}{dt} = \dot{M} = \frac{e\sin(f)\left(a(1-e^2)\right)}{(1+e\cos(f))^2} \frac{\sqrt{GMa(1-e^2)}}{M^2}$$

Also $M(f) = \dfrac{a(1-e^2)}{1+e\cos(f)}$ so

$$\frac{dM}{dt} = \dot{M} = \frac{e\sin(f)}{(1+e\cos(f))} \cdot \frac{a(1-e^2)}{1+e\cos(f)} \frac{\sqrt{GMa(1-e^2)}}{M^2} \Rightarrow$$

$$\dot{M} = \frac{e\sin(f)}{1+e\cos(f)} \cdot M \frac{\sqrt{GMa(1-e^2)}}{M^2} = \frac{e\sin(f)}{1+e\cos(f)} \frac{\sqrt{GMa(1-e^2)}}{M} \Rightarrow$$

$$\frac{e\sin(f)\,a(1-e^2)}{(1+e\cos(f))} \sqrt{\frac{GM}{a(1-e^2)}} \frac{1}{M} \Rightarrow e\sin(f)\sqrt{\frac{GM}{a(1-e^2)}}$$

so

$$\boxed{\frac{dM}{dt} = \dot{M} = e\sin(f)\sqrt{\frac{GM}{a(1-e^2)}}}$$

Also The Perpendicular velocity is given by:

$$V_\perp = r\dot{f} = r\frac{df}{dt} \Rightarrow r \times L/r^2 \Rightarrow V_\perp = L/r$$

$L = \sqrt{GMa(1-e^2)}$ and $r(f) = \dfrac{a(1-e^2)}{1+e\cos f}$

$$V_\perp = \frac{\sqrt{GMa(1-e^2)}\,(1+e\cos f)}{a(1-e^2)} \Rightarrow$$

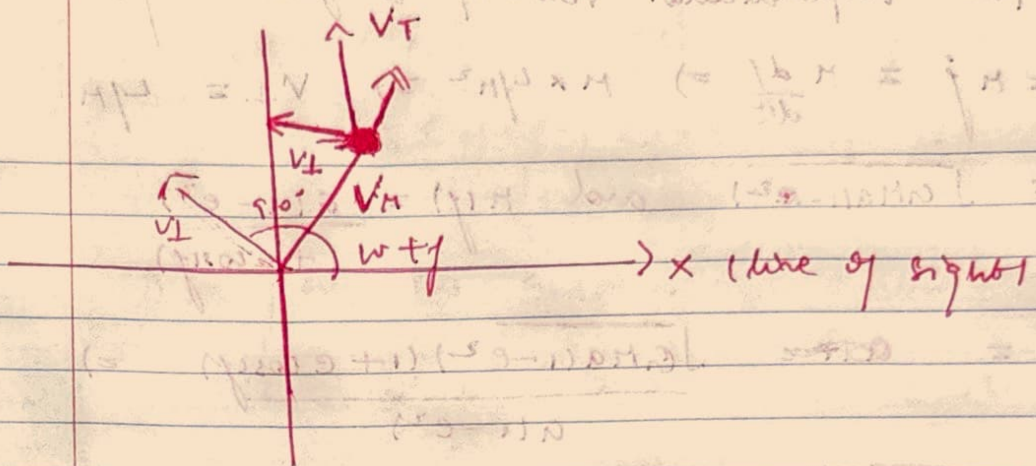
$$\sqrt{\frac{GM}{a(1-e^2)}}\,(1+e\cos f) = V_\perp$$

if The line of sight is Aligned with The line of Apses when $w=0$ so when we change The inclination Angle we get That That The Radius vector is At an Angle of $w+f$ with The X-Axis so we need to Project The star's velocity onto The x-Axis The line of sight Also from The question.

The star moves in The opposite direction to The Planet about Their center of Mass and get Multiplied by $\dfrac{-m_p}{M} = \dfrac{-m_p}{m_p+m_*}$

So

$V_M^{star} = \dfrac{-m_p}{M} \cdot V_x$ Now The velocity of The star in The x-direction is given by



Now the components of V_M in the cartesian frame (frame of the observer) is

$$V_{Mx} = V_M \cos(wt+f)$$

$$V_{My} = V_M \sin(wt+f)$$

components of $V_\perp$ are

$180 - w - f - 90$

$\Rightarrow \left(\frac{\pi}{2} - w - f\right)$

$$V_{\perp x} = V_\perp \cos(wt+f+\pi/2) = -V_\perp \sin(wt+f)$$

$$V_{\perp y} = V_\perp \sin(wt+f+\pi/2) = V_\perp \cos(wt+f)$$

so

$$V_x = V_M \cos(wt+f) - V_\perp \sin(wt+f)$$

$$V_y = V_M \sin(wt+f) + V_\perp \cos(wt+f)$$

so

$$V_M^{star} = -\frac{m_P}{M} V_x \Rightarrow$$

$$-\frac{m_P}{M}\left[e\sin(f)\sqrt{\frac{GM}{a(1-e^2)}}\cos(wt+f) - (1+e\cos(f))\sqrt{\frac{GM}{a(1-e^2)}}\sin(wt+f)\right]$$

$$V_H^{star} = -\frac{m_p}{M}\left[\sqrt{\frac{GM}{a(1-e^2)}}\left[e\sin(f)\cos(w+f) - (1+e\cos(f))\sin(w+f)\right]\right]$$

The reason why none of the curves for $w=0,1,2$ we don't observe a sine function because of the non-uniform angular motion. The 3 curves are different because for different w values the orbital oretiation differs. The non-linearity of the term f explains the non-sine function behaviour of the graph.

The value of the amplitude of the radial velocity of the star is independent of w because the sine and cosine components have a range of -1 to 1 and each term depends on f as well that changes fromm 0 to 2π for each fixed value of w so the amplitude is given by the constants $-\frac{m_p}{M}\sqrt{\frac{GM}{a(1-e^2)}}$.

Thats why we observe very similar amplitudes for all the 3 graphs of $w=0,1,2$.

Problem 2-2: So if A piece of Parallelized code can be optimised by using Multiple Processors

we can Reduce The time it Takes to run The entire code by increasing The number of Processors

So let $T(N)$ Be a function That models The time it Takes to Process A Piece of code with N Processors ; let P Be The fraction of The code That is parallel And $1-P$ is The serial Part

$T(N) = (1-P) + P/N$ → This is Actually what Amdahl's law says from CSC371.

Also The speed up will be given by The Ratio of Time it Takes to run The code on 1 Processor and N Processors

$$S(N) = \frac{T(1)}{T(N)}$$

$T(1) = 0.2 + \frac{0.8}{1} = 1$ So with 1 Processor it Takes 1 unit of Time and This will be our Benchmark.

$S(1) = \frac{T(1)}{T(1)} = 1$ → Speedup is 1.

for $N = 2$

$$T(2) = 0.2 + \frac{0.8}{2} = 0.6$$

$$S(2) = 1/0.6 = 10/6 = 5/3$$

for $N = 16$

$$T(16) = 0.2 + \frac{0.8}{16} = 0.2 + \frac{8 \times 10^{-1}}{162} = 0.2 + 0.05 = 0.25$$

$$S(16) = 1/0.25 = 4$$

for $N = 128$

$$T(128) = 0.2 + \frac{0.8}{128} = 0.2 + 0.00625 = 0.20625$$

$$S(128) = \frac{1}{0.20625} \approx 4.85$$

for $N = 1024$

$$T(1024) = 0.2 + \frac{0.8}{1024} = 0.2 + 0.00078 = 0.20078$$

$$S(1024) = \frac{1}{0.20078} \approx 4.98$$

Now every hour the computations cost 3.6 \$\$ and 0.1 \$/hr per Processor

So the cost function is given by

$$C(n) = [3.6 + 0.1(n)]\, T(n)$$

Now for $T(n) = 0.2 + \frac{0.8}{n}$ we get

$$C(n) = [3.6 + 0.1(n)]\left[0.2 + \frac{0.8}{n}\right] \Rightarrow$$

$$C(n) = 0.72 + \frac{2.88}{n} + 0.02n + 0.08 \Rightarrow$$

$$C(n) = 0.8 + \frac{2.88}{n} + 0.02n$$

$C(n) = 0.8 + \frac{2.88}{n} + 0.02n$

Not

$\frac{dC(n)}{dn} = \frac{d}{dn}(2.88/n) + \frac{d}{dn}(0.02n) \Rightarrow$

$-\frac{2.88}{n^2} + 0.02 = 0 \Rightarrow$

$0.02 = \frac{2.88}{n^2} \Rightarrow$

$n^2 = \frac{2.88}{0.02} = 144$

$\boxed{n=12}$ So using 12 Processors

Minimises cost with giving a speedup of

$T(12) = 0.2 + \frac{0.8}{12} \Rightarrow 0.266$

$S(12) = \frac{1}{0.266} = 3.75$ Times.

Now if we have an unlimited Budget we can use All The Processors Available

So $T(1024) = 0.20078$

$S(1024) \approx 4.98$ Times

So The Processing will be 1.23 Times Faster

But The cost of using $N = 1024$ Processors is:

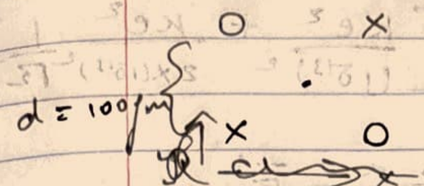
$C(1024) = [3.6 + 0.1(1024)][0.20078] = 21.28\$$

$C(12) = 1.28\$$

So The Added costs is 20 \$!!

Assignment 2:

Problem 2.3:



x = Positron
o = Proton

Using Columb's Law to identity the forces on each particle so

On Particle 1 The Proton on The Top Left

Particle 2 is The Positron on Top Right

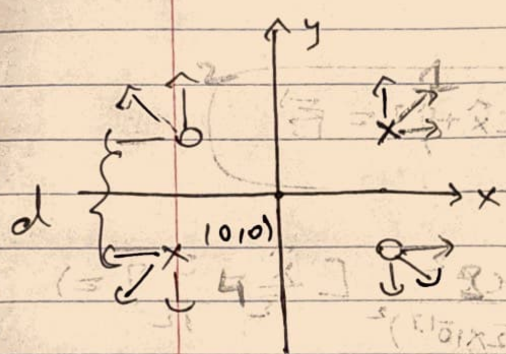
$$F_e = \frac{k e_1 e_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{e_1 e_2}{r^2}\hat{r}$$

$k = 9\times10^{9}$ Both The Proton and Positron have The same charge $1.6\times10^{-19}\,C$

$100\sqrt{2}$, 100, 100×10^{-15}

So $$\vec{F}_1 = \frac{ke^2}{(100\times10^{-15})^2}\hat{y} + \frac{ke^2}{(100\times10^{-15})^2}\hat{x} + \frac{ke^2}{(100\times10^{-15}\times\sqrt{2})^2}\hat{r}_1$$

$$\hat{r}_1 = \frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}} \quad \text{so}$$



$$\vec{F}_1 = \frac{ke^2}{(10^{-13})^2}\hat{y} + \frac{ke^2}{(10^{-13})^2}\hat{x} + \frac{ke^2}{(\sqrt{2}\times10^{-13})^2}\left(\frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}}\right)$$

$$\Rightarrow \left[\frac{ke^2}{(10^{-13})^2} + \frac{ke^2}{(10^{-13})^2(2\sqrt{2})}\right]\hat{y} + \hat{x}\left[\frac{ke^2}{(10^{-13})^2} + \frac{ke^2}{2\sqrt{2}(10^{-13})^2}\right]$$

$$\Rightarrow \frac{ke^2}{(10^{-13})^2 2\sqrt{2}}(2\sqrt{2}+1)\hat{y} + \frac{(1+2\sqrt{2})ke^2}{2\sqrt{2}(10^{-13})^2}\hat{x} = \vec{F}_R$$

$\frac{ke^2\hat{x}}{(10^{-13})^2} +$
$\frac{ke^2\hat{y}}{(10^{-13})}$

$$\vec{F}_2 = -\frac{ke^2}{(10^{-13})^2}\hat{x} + \frac{ke^2}{(10^{-13})^2}\hat{y} + \frac{ke^2}{(10^{-13})^2(2)}\hat{r}_2$$

$$\hat{r}_2 = \frac{\hat{y}}{\sqrt{2}} - \frac{\hat{x}}{\sqrt{2}} \quad \text{so}$$

$$\vec{F_2} = \frac{-ke^2}{(10^{-13})^2}\hat{x} + \frac{ke^2}{(10^{-13})^2}\hat{y} + \frac{ke^2}{(\sqrt{2}\times10^{-13})^2}\left[\frac{\hat{y}}{\sqrt{2}} - \frac{\hat{x}}{\sqrt{2}}\right] \Rightarrow$$

$$\vec{F_2} = \left[\frac{ke^2}{(10^{-13})^2} + \frac{ke^2}{2\times(10^{-13})^2}\frac{1}{\sqrt{2}}\right]\hat{y} - \left[\frac{ke^2}{(10^{-13})^2} + \frac{ke^2}{2\times(10^{-13})^2}\frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow \frac{(2\sqrt{2}+1)ke^2}{2\sqrt{2}(10^{-13})^2}\hat{y} - \frac{(2\sqrt{2}+1)ke^2}{2\sqrt{2}(10^{-13})^2}\hat{x} = \vec{F_2}$$

$$\boxed{\frac{(2\sqrt{2}+1)ke^2}{2\sqrt{2}(10^{-13})^2}(\hat{y}-\hat{x}) = \vec{F_2}}$$

The 3rd Particle is The Bottom left Particle

$$\vec{F_3} = \frac{-ke^2}{(10^{-13})^2}(\hat{x}) - \frac{ke^2}{(10^{-13})^2}\hat{y} - \frac{ke^2}{(\sqrt{2}\times10^{-13})^2}\left(\frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}}\right) \Rightarrow$$

$$\boxed{\frac{-2ke^2(\sqrt{2}) - ke^2}{\sqrt{2}(\sqrt{2}\times10^{-13})^2}[\hat{x}+\hat{y}]} \Rightarrow$$

$$\cancel{(2+\sqrt{2})} \quad \boxed{-\frac{(2\sqrt{2}+1)ke^2}{2\sqrt{2}(10^{-13})^2}[\hat{x}+\hat{y}] = \vec{F_3}}$$

$$\vec{F_4} = \frac{ke^2}{(10^{-13})^2}\hat{x} - \frac{ke^2}{(10^{-13})^2}\hat{y} + \frac{ke^2}{(\sqrt{2}\times10^{-13})^2}\left[\frac{\hat{x}}{\sqrt{2}} - \frac{\hat{y}}{\sqrt{2}}\right] \Rightarrow$$

$$\boxed{\frac{(2\sqrt{2}+1)ke^2}{2\sqrt{2}(10^{-13})^2}(\hat{x}-\hat{y}) = \vec{F_4}}$$

All The four Particle Move to infinity along The 2 Diagonals.

~~Problem 2.4~~

One way we can Model The 2 Positron and 2 Proton system is to consider That $m_p >> m_e$ so $v_e >> v_p$ so from The Perspective of The 2 Positrons The 2 Protons are Basically at Rest and for The 2 Protons. The 2 Positrons are instantaneously very far apart.

So to find The Ratios of The velocities of Positrons and Protons, The Ratio of Temperatures and Ratio of Kinetic energies we can Apply energy conservation Separately on Both The Particle sub-systems.

So for The 2 Positrons The initial energy is entirely Potential

$$E_i = U_i = Ke^2 +$$

The Potential energy is basically The energy to Bring The 4 Particles from infinity to The given configuration.

$$U_i = U_1 + U_2 + U_3 + U_4$$

$$U_1 = 0 ; \quad U_2 = \frac{Ke^2}{(10^{-13})} ; \quad U_3 = \frac{Ke^2}{(10^{-13})} + \frac{Ke^2}{\sqrt{2}(10^{-13})}$$

$$U_4 = \frac{Ke^2}{10^{-13}} + \frac{Ke^2}{10^{-13}} + \frac{Ke^2}{\sqrt{2}(10^{-13})}$$

So $U_i = \frac{4Ke^2}{10^{-13}} + \frac{2Ke^2}{\sqrt{2}\,10^{-13}} \Rightarrow \frac{Ke^2}{10^{-13}}(4+\sqrt{2})$

for The 2 Positrons at $t \to \infty$ The Total energy is entirely Kinetic so

$$\frac{1}{2} m_e v_e^2 + \frac{1}{2} m_e v_e^2 = \frac{ke^2}{10^{-13}}(4+\sqrt{2}) \Rightarrow$$

$$m_e v_e^2 = \frac{ke^2}{10^{-13}}(4+\sqrt{2}) \Rightarrow$$

$$\sqrt{\frac{ke^2(4+\sqrt{2})}{m_e(10^{-13})}} = v_e$$

Now lets Apply energy conservation to The 2 Proton system

for The Proton system at The instant The 4 Particles are Brought together The 2 Positrons go off to infinity so The system initially is The 2 Protons

$$E_i = U_i = \frac{ke^2}{\sqrt{2}(10^{-13})}$$

$$E_f = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_p v_p^2 = m_e v_p^2$$

so

$$m_p v_p^2 = \frac{ke^2}{\sqrt{2}(10^{-13})} \Rightarrow$$

$$v_p = \sqrt{\frac{ke^2}{m_p(\sqrt{2})(10^{-13})}}$$

so $$\frac{v_p}{v_e} = \frac{\sqrt{\frac{ke^2}{m_p(\sqrt{2})(10^{-13})}}}{\sqrt{\frac{ke^2(4+\sqrt{2})}{m_e(10^{-13})}}}$$

$$\frac{KE_e}{KE_p} = \frac{\dfrac{ke^2(4+\sqrt{2})}{2(10^{-13})}}{\dfrac{ke^2}{2\sqrt{2}(10^{-13})}} \Rightarrow$$

$$\frac{\cancel{ke^2}(4+\sqrt{2})}{\cancel{2}(\cancel{10^{-13}})} \cdot \frac{\cancel{2}\sqrt{2}(\cancel{10^{-13}})}{\cancel{ke^2}} \Rightarrow 4\sqrt{2}+2$$

So $\dfrac{KE_e}{KE_p} = 2+4\sqrt{2}$

$$\frac{v_e}{v_p} = \sqrt{\frac{ke^2(4+\sqrt{2})}{m_e(10^{-13})}} \sqrt{\frac{m_p(\sqrt{2})(10^{-13})}{ke^2}} \Rightarrow$$

$$\sqrt{\frac{(4\sqrt{2}+2)m_p}{m_e}} = \frac{v_e}{v_p}$$

$$\frac{v_e}{v_p} = \sqrt{\frac{(4\sqrt{2}+2)(1.672\times10^{-27})}{(9.10\times10^{-31})}} \Rightarrow$$

$$\sqrt{\frac{(4\sqrt{2}+2)(1.672)\times10^{4}}{9.1}} = 10^{2}\sqrt{\frac{(4\sqrt{2}+2)(1.672)}{9.1}} \Rightarrow$$

$$\frac{v_e}{v_p} = 140.68$$

Now The mean kinetic energy Per Particle is The

Average kinetic energy of The Positron is

$$\frac{1}{2} m_e v_e^2 = KT_e$$

$$\frac{1}{2} m_p v_p^2 = KT_p$$

so $$\frac{T_e}{T_p} = \frac{m_e v_e^2}{m_p v_p^2} = \frac{KE_e}{KE_p} = 4\sqrt{2}+2$$

Problem 2.4

A) Let us Take t_1 to be The Time at which Particle 1 is detected and t_2 be The time to Detect Particle 2.

Also we Take Both t_1 and t_2 be a Datapoint Out of The Uniform Distribution so

$t_1, t_2 \in [0,1]$

Now The Probability That 2 arrivals are seperated by more Than $1/3$ days is given by

$P(|t_1-t_2| > 1/3) = 1 - P(|t_1-t_2| \leq 1/3)$

$|t_1-t_2| \leq 1/3 \Rightarrow$

$t_1 - t_2 \leq 1/3$ or

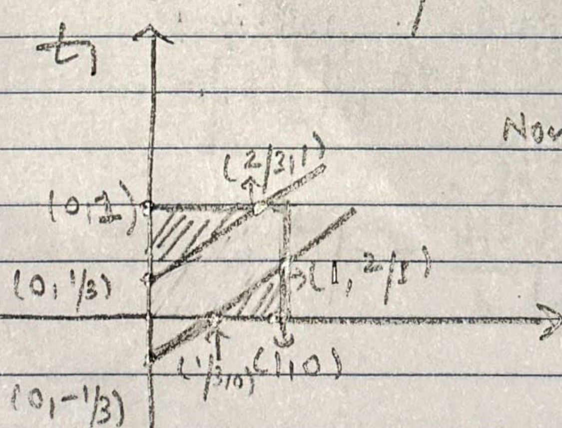
$t_2 - t_1 \leq 1/3$

$t_1 \leq t_2 + 1/3$

$t_1 \geq t_2 - 1/3$

$1 = t_2 - 1/3$

$2/3$



Now our Region of interest is The shaded region

So

Basically The Area of 2 Triangles

$P(|t_1-t_2| > 1/3) = \frac{1}{2}(1-1/3)(2/3-0) + 1/2(1-1/3)(1-2/3)$

$P(|t_1 - t_2| > 1/3) = 2\left(\frac{1}{2}(2/3)(2/3)\right) \Rightarrow 4/9$

so $\boxed{P(|t_1 - t_2| > 1/3) = 4/9}$

B) (i) The Average of a Random Variable is The expectation value of The RV

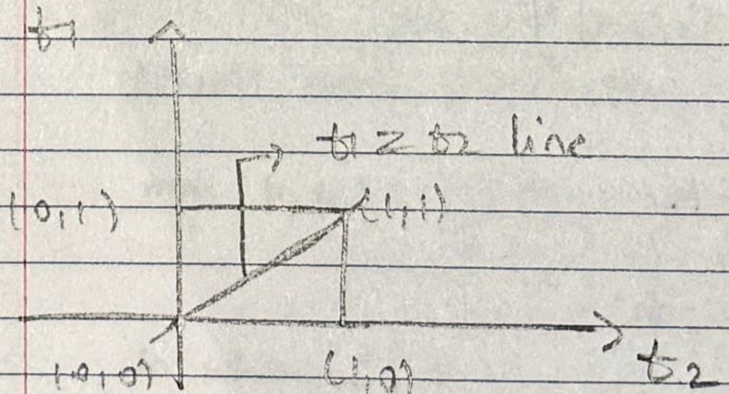
$$\langle x \rangle = \iint x(t_1, t_2)\, dt_1\, dt_2$$

Now The Time before The Arrival of the first Particle is Basically The Expected value of The Arrival of The 1st Particle

Now The first Particle would be either Particle 1 or 2 so we need to find

$P(\min(t_1, t_2))$ we have 2 Possibilities for this

$t_1 > t_2$ so $\min(t_1, t_2) = t_2$
and $t_2 > t_1$ so $\min(t_1, t_2) = t_1$



$$E[(\min(t_1, t_2))] = \iint \min(t_1, t_2)\, dt_1\, dt_2$$

when $\min(t_1, t_2) = t_1$ we get

$0 \le t_1 \le t_2$ and $0 \le t_2 \le 1$ so

$$\int_{t_2=0}^{t_2=1}\int_{t_1=0}^{t_1=t_2} t_1 \, dt_1 \, dt_2 = \int_0^1 \left| \frac{t_1^2}{2} \right|_0^{t_2} dt_2 \Rightarrow$$

$$\int_0^1 \frac{t_2^2}{2} dt_2 = \left| \frac{t_2^3}{6} \right|_0^1 = 1/6$$

when $\min(t_1, t_2) = t_2$ we get

$0 \le t_2 \le t_1$ and $0 \le t_1 \le 1$

$$\int_{t_1=0}^{t_1=1}\int_{t_2=0}^{t_2=t_1} t_2 \, dt_2 \, dt_1 = \int_0^1 \left\| \frac{t_2^2}{2} \right|_0^{t_1} dt_1 = 1/6$$

So $\boxed{E[(\min(t_1, t_2))] = 1/6 + 1/6 = 1/3}$

(ii) Now The Average Time Between The Arrivals of The 1st and 2nd Particle is given by

$$E(|t_1 - t_2|) = \iint |t_1 - t_2| \, dt_1 \, dt_2$$

when $t_1 \ge t_2$

$|t_1 - t_2| = t_1 - t_2$

when $t_2 \ge t_1$

$|t_1 - t_2| = t_2 - t_1$

Case 1: when $t_1 \ge t_2$ we get

$0 \le t_2 \le t_1$ and $0 \le t_1 \le 1$

$$\int_{t_1=0}^{t_1=1}\int_{t_2=0}^{t_2=t_1}(t_1-t_2)\,dt_2\,dt_1 \Rightarrow$$

$$\int_0^1\int_0^{t_1}(t_1-t_2)\,dt_2\,dt_1 =$$

$$\left| t_1 t_2 - \frac{t_2^2}{2}\right|_0^{t_1} = t_1(t_1) - \frac{t_1^2}{2} = t_1^2/2$$

$$\int_0^1 \frac{t_1^2}{2}\,dt_1 = \left| t_1^3/6 \right|_0^1 = 1/6$$

for $t_2 \geq t_1$

$0 \leq t_1 \leq t_2$ and $0 \leq t_2 \leq 1$

$$\int_{t_2=0}^{t_2=1}\int_{t_1=0}^{t_1=t_2}(t_2-t_1)\,dt_1\,dt_2 = 1/6$$

So

$$\boxed{E(|t_1-t_2|) = 1/6 + 1/6 = 1/3}$$

(iii) The time after the second Arrival is given by the time it takes the second particle to get detected subtracted from the unit day

Time after 2nd arrival $= 1 - \max(t_1, t_2)$
Because our Distribution upper Bound is 1 day.

$$\max(t_1, t_2) = \begin{cases} t_1 & \text{if } t_1 > t_2 \\ t_2 & \text{if } t_2 > t_1 \end{cases}$$

So when $t_1 > t_2$

when $0 \leq t_2 \leq t_1$, $0 \leq t_1 \leq 1$

$$\int_0^1\int_0^{t_1} t_1\,dt_2\,dt_1 = \int_0^1 t_1(t_1)\,dt_1 \Rightarrow \left| t_1^3/3 \right|_0^1 = 1/3$$

when $t_2 > t_1$

$0 \le t_1 \le t_2 ; \quad 0 \le t_2 \le 1$

$$\int_0^1 \int_0^{t_2} t_2 \, dt_1 \, dt_2 = 1/3$$

so

$\mathbb{E}(\max(t_1, t_2)) = 2/3$

So $1 - \mathbb{E}(\max(t_1, t_2)) = 1/3 = \mathbb{E}($after the second arrival$)$

Part c:

Monte Carlo Methods are used to estimate The expected value of a Random Variable by Averaging The observed Values of The RV when sampled Multiple Time from its underlying Distribution.

$$\int \int f(t_1, t_2) \, dt_1 \, dt_2 \approx \frac{1}{N} \sum_{i=1}^{N} f(t_1^{(i)}, t_2^{(i)})$$

Using This Method we estimate The values for Part A and B of The Question.

The estimated Probability of

→ $P(|t_1 - t_2| > 1/3) = 0.44431$ when Taken $N = 10^8$ samples

→ for $\mathbb{E}(\min(t_1, t_2)) = 0.33376$ for $N = 10^8$

→ for $\mathbb{E}(|t_1 - t_2|) = 0.33316$ for $N = 10^8$

→ for $\mathbb{E}(1 - \max(t_1, t_2)) = 0.33311$ for $N = 10^8$

→ The value of σ for $\mathbb{E}(\min(t_1, t_2)) = 0.235712$

→ The value of σ for $\mathbb{E}(|t_1 - t_2|) = 0.235703$

→ The value of σ for $\mathbb{E}(1 - \min(t_1, t_2)) = 0.235$

From The Theory of Monte Carlo simulations

As $N \to \infty$ $\iint f(t_1, t_2)\,dt_1\,dt_2 = \frac{1}{N}\sum_{i=1}^{N} f(t_1^{(i)}, t_2^{(i)})$

So

$$\lim_{N \to \infty} \frac{1}{N}\sum_{i=1}^{N} f(t_1^{(i)}, t_2^{(i)}) = \iint f(t_1, t_2)\,dt_1\,dt_2$$

So larger and larger values converge to The True mean. From The ~~Plot~~ Analysis we can notice That The relationship of The Relative Error is Proportional to N^{-1}. Error Decreases As N increases.

$E = |\text{True-val} - \text{Pred-val}|$

$$\left| \iint f(t_1, t_2)\,dt_1\,dt_2 - \frac{1}{N}\sum (f(t_1^{(i)}, t_2^{(i)})) \right| = E$$

$$\boxed{E \propto N^{-1}}$$

The Term σ Means The deviation of The values from The ~~Populat~~ sample mean → Basically The Average of The Dataset.