

Problem 3: Runtime Analysis

a) $O(\log_2(\log_2 n))$

b)

If statement is true for all multiples of \sqrt{n} .
There are \sqrt{n} multiples of \sqrt{n} that are less than n .

$$\Rightarrow \sum_{i=1}^n \sqrt{n} \sum_{k=0}^{i^3} O(1)$$

$$= \sqrt{n} \sum_{i=1}^n i^3$$

$$= \sqrt{n} \cdot O(n^4)$$

$$\Rightarrow O(n^4)$$

c) $\sum_{i=1}^n \sum_{k=1}^i O(1) \cdot O(\sqrt{n})$

$$\rightarrow O(n \cdot n \cdot \sqrt{n})$$

$$= O(n^{5/2})$$

d) $\sum_{i=0}^n \left(O(3) + \sum_{j=0}^{\lfloor 3i/2 \rfloor} O(1) \right) + O(1)$

$$= O(n) + O\left(\frac{3n^2}{2}\right) + O(n)$$

$$\Rightarrow O(n^2)$$