WiDS

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December 2022

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2.1 Postulates of Quantum Mechanics

Quantum mechanics on its own is definitely not sufficient for solving physical systems rather, it only provides the framework for solving these systems, these quantum mechanics postulates would help us in building the bridge between mathematical tools of quantum mechanics and the physical systems that we need to solve. These postulates were developed over years of trial and errors and more often that not, we can not guess the motivation behind these postulates but it is important to understand what each postulate has to convey.

2.1.1 Postulate 1 - State space

Associated to any physical isolated system is a complex vector space with inner product (that is, a Hilbert space) known as the state space of the system . The system is defined completely by its state vector which is a unit vector in the system's state space. Like I mentioned earlier, Quantum mechanics is not sufficient to define a state of a particle for example: Consider a photon, to define the state of a photon, you need the knowledge of Quantum mechanics coupled with quantum electrodynamics. In the qubit system, any state vector can be defined as $\alpha |0\rangle + \beta |1\rangle$ where α and β are any two complex numbers, for this state vector to be normalised that is $\langle \phi | \phi \rangle = 1$, $|\alpha|^2 + |\beta|^2 = 1$

2.1.2 Postulate 2 - Evolution

Evolution of any closed system (that is, a system which is left undisturbed) is defined by any unitary operation, in other words, if the state of a system at any time 't' is $|\psi\rangle$, and at any later time t_1 $|\psi_1\rangle$, then there exists a unitary matrix U such that $|\psi_1\rangle = U |\psi\rangle$

The above postulate is only helpful in discrete time intervals, for the continous case, we define the schrodinger equation which states that :

$i\hbar \frac{d|\psi\rangle}{dt} = H |\psi\rangle$ where H is the unitary operator known as the hamiltonian operator.

2.1.3 Postulate 3 - Measurement

Quantum measurements are described by the set of measurement operators M_m where the index 'm' refers to the different possible outcomes of measurement. If the system is in a state $|\psi\rangle$ just before the measurement, then the probability of the measurement 'm' occurring is given by $\langle\psi| M^{\dagger}M |\psi\rangle$ and the state immediately after the measurement is $\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M^{\dagger}M|\psi\rangle}}$

The set of measurement operators should satisfy the completeness relation that is $\sum_m M^{\dagger}M = I$ The completeness relation ensures that the sum of all probabilities of the possible outcomes is 1.

An important example of the measurement operators is in the qubit system where $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$. It can be easily verified that they satisfy the completeness relation.

2.1.4 Distinguishing quantum states

An important application of Postulate 3 is to distinguish quantum states, physical states can easily be distinguished , for example: we can always figure whether a coin lands on heads or a tails but distinguishing quantum states is much more complex in quantum mechanics. We figure out that states which are not orthonormal to each other can't be measured. If the states are orthonormal then, we can define measurement operators as $|\psi_i\rangle\langle\psi_i|$, these need to satisfy the completeness relation, and if the state is $|\psi_i\rangle$ at that time, then probability of measuring 'i' at that time comes out to be one using our measurement operator.Hence, we conclude that states can be measured if they are orthonormal to each other an can't be measured if they are not.

2.1.5 Projective measurements

A projective measurement is described by an observable, M, a Hermitian operator on the state space of the system being observed. The observable has a spectral decomposition : M = $\sum mP_m$ where P_m is the projector onto the eigen space of M with the corresponding eigen value m, the probability of getting result 'm' is given by : p(m) = $\langle\psi| P_m |\psi\rangle$, given that the output of the measurement was 'm' then, the state of the quantum system immediately after the measurement is $\frac{P_m|\psi\rangle}{\sqrt{p(m)}}$

Projective measurements are just a special case of measurement operators that we discussed in postulate 2 , the only difference being that here, in addition to satisfying the completeness relation, they also satisfy that the set of M_m are orthogonal projectors that is, the M_m are hermitian and satisfy $M_m M'_m = \delta_m m' M_m$.

2.1.6 POVM measurements

The postulate 3 that we discussed earlier has 2 aspects to it. First , it tells us about the respective probabilities of different outcomes of a particular measurement and second, it tells us about the state of the system after the measurement has taken place. However for a lot of cases, the state of the system post measurement is of little interest while the main interest is the probabilities of the measurements. In these systems where we measure the system only once, we use the mathematical tool of POVM which stands for 'Positive Operator Valued Measure'.

Suppose we define $E_m = M^\dagger M$, then from postulate 3 and elementary linear algebra, E_m is a positive operator such that $\sum_m E_m = I$ and $p_m = \langle\psi| E_m |\psi\rangle$.

Thus, the set of operators E_m are sufficient to measure the probabilities of the different measurement outcomes. The complete set of E_m is known as a POVM.

2.1.7 Phase

Phase is an important aspect of quantum mechanics. Consider a vector :$|\psi\rangle = e^{i^\theta} |\psi\rangle$, then $|\psi\rangle$ is said to be equal upto a global phase factor $'\theta'$ with respect to $|\psi\rangle$. However it can be shown that this phase factor has no impact on the statistics of measurement as for any particular outcome 'm' $\langle\psi| e^{-i\theta} M^\dagger M e^{i\theta} |\psi\rangle = \langle\psi| M^\dagger M |\psi\rangle$ .The other kind of phase is the relative phase consider $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$, these two have the same amplitudes for the corresponding amplitudes but the qubit $|1\rangle$ has a relative phase factor of '-1'

2.1.8 Postulate 4 - Composite systems

The composite of many physical systems can be described as the tensor of several states , for example, consider states $|\psi_1\rangle, |\psi_2\rangle \ldots |\psi_n\rangle$ then the composite system can be defined as $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots |\psi_n\rangle$

3 Quantum Circuits

3.1 Quantum Algorithms

Many intriguing problems are practically hard to solve on a traditional computer due to the astronomical resources needed to do so, not because they are intractable in theory.

real-world examples of the issue The amazing potential of quantum computers is the creation of new algorithms that make it possible to solve issues that would otherwise require enormous amounts of computing power to solve.

3.2 Single qubit operations

Any single qubit is a vector of the form $|\psi\rangle$ = a$|0\rangle$ + b$|1\rangle$ where $|a|^2 + |b|^2 =$ 1.........to be continued

4 Introduction to Quantum Machine learning

Machine learning algorithms learn a desired input-output relation from examples in order to interpret new inputs. This is important for tasks such as image and speech recognition or strategy optimisation, with growing applications in the IT industry. In the last couple of years, researchers investigated if quantum computing can help to improve classical machine learning algorithms. Ideas range from running computationally costly algorithms or their subroutines efficiently on a quantum computer to the translation of stochastic methods into the language of quantum theory. This contribution gives a systematic overview of the emerging field of quantum machine learning.

4.1 Classical Machine Learning

In the theory of machine learning, the term learning can be categorized as supervised, unsupervised and reinforcement learning.

1. Supervised Learning - In supervised learning, a computer is given examples of correct input-output relations and has to infer a mapping therefrom. Probably the most important task is pattern classification, where vectors of input data have to be assigned to different classes. This is something that we do in our everyday lives where we identify faces from different angles and light conditions as well.

2. Unsupervised Learning - For a long time, machine learning did not include unsupervised learning, which refers to the process of identifying patterns in data without the use of examples or past knowledge. Data clustering, or creating subgroups from a dataset, is a common job used to condense a big quantity of information into a small number of stereotypes.

3. Reinforcement Learning - This is a form of learning in which an agent ie usually a computer program that acts as a player in a game is either rewarded or punished based on the strategy it uses to win, a reward leads to reinforcement of the strategy while a punishment leads to adaptation of the strategy

4.2 Quantum Machine learning

The ability of quantum states to be in superposition of several states can lead to much faster computations since operations on many states can be done in the same time, Quantum gates are usually expressed as unitary matrices.

Quantum algorithms are created in quantum machine learning in order to utilise the effectiveness of quantum computing to tackle common machine learning challenges.

Typically, this is accomplished by making expensive classical algorithms or their time-consuming subroutines compatible with a hypothetical quantum computer. It is anticipated that such devices would soon be widely accessible for applications and be able to assist in processing the expanding amounts of global information. The new discipline also contains methodologies that work the other way around, notably well-proven machine learning techniques that can aid in extending and enhancing quantum information theory.

4.3 Quantum versions of machine learning algorithms

4.3.1 Quantum versions of k-nearest neighbour methods

The goal is to select the class c^x for the new input that appears more frequently among its k nearest neighbours given a training set T of feature vectors with their associated classification and an unclassified input vector x.