

Chapter 8

Nash Equilibrium: Location, Segregation And Randomization

We first complete our discussion of the candidate-voter model showing, in particular, that, in equilibrium, two candidates cannot be too far apart. Then we play and analyze Schelling's location game. We discuss how segregation can occur in society even if no one desires it. We also learn that seemingly irrelevant details of a model can matter. We consider randomizations first by a central authority (such as in a bussing policy), and then decentralized randomization by the individuals themselves, "mixed strategies." Finally, we look at rock, paper, scissors to see an example of a mixed-strategy equilibrium to a game

Candidate Voter Model

- 1) Many Nash Equilibrium not all at center.
- 2) Entry can lead to a more distant candidate winning.
- 3) If the 2 candidates are too extreme, someone in the center will enter.
- 4) Guess and check effective.

Location Model

Strategies: Two towns East and West holds 100000.

Players: Two types of people Tall and Short 100000 of each.

Choice is simultaneous.

If there is no room, then randomize the ration.

Nash Equilibrium

- 1) Two segregated Nash equilibrium (Tall in East, Short in West and vice versa.) Stable, strict.
- 2) Integrated Nash equilibrium. (Half of each in each town.) Not stable, weak. Tipping point.
- 3) All choose the same town and get randomized.

Lesson: Seemingly irrelevant details of the game can matter.

Lesson: Having society randomize for you ended up better than active choice.

Lessons

- 1) Sociology. Seeing segregation does not imply preference for segregation.
- 2) Policy. Randomization, Bussing.
- 3) Individual randomization. Nash Equilibrium. Randomized or mixed strategies.

Rock Scissor Paper

Pure strategy = {R, P, S}

No Nash equilibrium in pure strategies.

Nash equilibrium, each player chooses $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

Expected payoff of Rock against $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = (\frac{1}{3})^*(0) + (\frac{1}{3})^*(1) + (\frac{1}{3})^*(-1) = 0$.

Expected payoff of Scissor against $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = (\frac{1}{3})^*(-1) + (\frac{1}{3})^*(0) + (\frac{1}{3})^*(1) = 0$.

Expected payoff of Paper against $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = (\frac{1}{3})^*(1) + (\frac{1}{3})^*(-1) + (\frac{1}{3})^*(0) = 0$.

Expected payoff of $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ against $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = (\frac{1}{3})^*(0) + (\frac{1}{3})^*(0) + (\frac{1}{3})^*(0) = 0$.

In Rock Scissor Paper playing $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ against $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is a best response so $[(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})]$ is a Nash Equilibrium.