Chapter 20
Subgame Perfect Equilibrium: Wars of Attrition

We first play and then analyze wars of *attrition*; the games that afflict trench warfare, strikes, and businesses in some competitive settings. We find long and damaging fights can occur in class in these games even when the prizes are small in relation to the accumulated costs. These could be caused by irrationality or by players' having other goals like pride or reputation. But we argue that long, costly fights should be expected in these games even if everyone is rational and has standard goals. We show this first in a two-period version of the game and then in a potentially infinite version. There are equilibria in which the game ends fast without a fight, but there are also equilibria that can involve long fights. The only good news is that, the longer the fight and the higher the cost of fighting, the lower is the probability of such a fight.

Apply subgame perfect equilibrium
Solve Nash equilibrium for each subgame
Roll back position

Lesson: Strategic effects matter.

War of attrition
2 players.
In each period each chooses whether to fight or to quit.
Game ends as soon as someone quits.
Good news: If the other player quits first, you win a prize v.
Bad news: Each period in which both fights, each player pays a cost - c.
If both quit at once then they get 0.

Examples

- World war 1
- British Satellite Broadcasting versus Sky.
- Bribery contests.

2 Player game
Second subgame - There are two pure strategy Nash equilibrium in this subgame (fight, quit) and (quit, fight). The payoffs are (v, 0) and (0, v) respectively (Continuation Payoffs)
Pure strategy equilibrium: (Fight (1) Fight(2), Quit(1) Quit(2)) and (Quit(1) Quit(2), Fight (1) Fight(2)).
Mixed Equilibrium
If A fights then payoffs = -c*p + v*(1 - p).
If A quits then payoffs = 0*p + 0*(1 - p).
v*(1 - p) = p*c.
p = v/(v+c) => 1-p = c/(v + c).

Mixed Nash Equilibrium has both fight with probability $v/(v+c)$.
The payoffs in this mixed Nash equilibrium are (0, 0).
The the full game the mixed Nash equilibrium is to fight with probability $p^* = v/(v+c)$.
Mixed subgame perfect equilibrium = $[(p^*, p^*), (p^*, p^*)]$.
The expected payoff is 0.
The probability of fights occurring goes up as v goes up and down as c goes up.
Full game - If they mix in the future then the continuation value is (0, 0).