

Chapter 5

Nash Equilibrium: Dating And Cournot

We apply the notion of Nash Equilibrium, first, to some more coordination games; in particular, the Battle of the Sexes. Then we analyze the classic Cournot model of imperfect competition between firms. We consider the difficulties in colluding in such settings, and we discuss the welfare consequences of the Cournot equilibrium as compared to monopoly and perfect competition.

Communication can help in a coordinated game where there is a scope for leadership.

Cournot Duopoly

Players: 2 Firms

Strategies: Quantities they produce of identical products (q_1 , q_2).

Cost of Production: cq (Constant marginal costs).

Prices: $p = a - b(q_1 + q_2)$.

Payoffs: Firms aim to maximize profits $U_1(q_1, q_2) = [p]q_1 - c \cdot q_1$ (revenues - total cost) = $a \cdot q_1 - b \cdot q_1^2 - b \cdot q_1 \cdot q_2 - c \cdot q_1$

Differentiate with respect to q_1 and set it equal to 0.

$$a - 2 \cdot b \cdot q_1 - b \cdot q_2 - c = 0.$$

$$BR_1(q_2) = (a-c)/(2 \cdot b) - q_2/2.$$

$$BR_2(q_1) = (a-c)/(2 \cdot b) - q_1/2.$$

$$BR_1(q_2) = BR_2(q_1)$$

$$q_1^* = (a-c)/(2 \cdot b) - q_1^*/2$$

$$2 \cdot q_1^* = (a-c)/b - q_1^*$$

$$3 \cdot q_1^* = (a-c)/b$$

$$q_1^* = (a-c)/(3 \cdot b) = q_2^* \text{ (Cournot quantity).}$$

$$\text{Nash Equilibrium} = q_1^* = q_2^* = (a-c)/(3 \cdot b).$$

A game of strategic substitutes.

$$\text{Total quantity: } 2 \cdot (a-c)/(3 \cdot b)$$

$$\text{Monopoly quantity: } (a-c)/(2 \cdot b)$$

$$\text{Competitive quantity: } (a-c)/b$$

$$\text{Competitive quantity} > \text{Total quantity} > \text{Monopoly quantity}.$$