

## Chapter 18

### Imperfect Information: Information Sets And Sub-Game Perfection

We consider games that have both simultaneous and sequential components, combining ideas from before and after the midterm. We represent what a player does not know within a game using an information set: a collection of nodes among which the player cannot distinguish. This lets us define games of imperfect information; and also lets us formally define subgames. We then extend our definition of a strategy to imperfect information games, and use this to construct the normal form (the payoff matrix) of such games. A key idea here is that it is information, not time *per se*, that matters. We show that not all Nash equilibria of such games are equally plausible: some are inconsistent with backward induction; some involve non-Nash behavior in some (unreached) subgames. To deal with this, we introduce a more refined equilibrium notion, called subgame perfection.

**Formal Definition:** An information set of player  $i$  is a collection of player  $i$ 's nodes among which  $i$  cannot distinguish.

**Perfect Information:** All information sets in the tree contain just one node..

**Imperfect Information:** Not perfect information.

**Definition:** A pure strategy of player  $i$  is a complete plan of action. It specifies what  $i$  will do at each of  $i$ 's information set.

Information matters and not time.

**Definition:** A subgame is a part of the game that looks like a game within the tree. It satisfies the following 3 properties.

1. It starts from a single node.
2. It comprises all successors to that node.
3. It does not break up any information sets.

**Definition:** A Nash equilibrium  $(s_1^*, s_2^*, \dots, s_n^*)$  is a subgame perfect equilibrium if it induces a Nash equilibrium in every subgame of a game.