

Chapter 7

Nash Equilibrium: Shopping, Standing And Voting On A Line

We first consider the alternative "Bertrand" model of imperfect competition between two firms in which the firms set prices rather than setting quantities. Then we consider a richer model in which firms still set prices but in which the goods they produce are not identical. We model the firms as stores that are on either end of a long road or line. Customers live along this line. Then we return to models of strategic politics in which it is voters that are spread along a line. This time, however, we do not allow candidates to choose positions: they can only choose whether or not to enter the election. We play this "candidate-voter game" in the class, and we start to analyze both as a lesson about the notion of equilibrium and a lesson about politics.

Bertrand Competition

Players: Two firms.

Costs: Constant marginal costs = c .

Strategy set: Prices ($0 \leq p \leq 1$).

Quantity produced $Q(p) = 1 - p$ where p is the lower of the two prices.

Demand for firm 1

$q_1 = 1 - p_1$ if $p_1 < p_2$

$q_1 = 0$ if $p_1 > p_2$.

$q_1 = (1 - p_1)/2$ if $p_1 = p_2$.

Payoffs: $q_1 \cdot p_1 - q_1 \cdot c = q_1 \cdot (p_1 - c)$ (revenue - cost).

$BR_1(p_2) = p_1 > p_2$ if $p_2 < c$.

$BR_1(p_2) = p_1 = p_2 - \epsilon$ if $c < p_2 \leq p$ monopoly.

$BR_1(p_2) = p_1 = p$ monopoly if $p_2 > p$ monopoly.

$BR_1(p_2) = p_1 \geq c$ if $p_2 = c$.

Nash Equilibrium = ($p_1 = c, p_2 = c$).

$p = c$. Profit = 0.

Outcome is like perfect competition even though only 2 firms.

Same setting like Cournot, but with a different strategy set, led to a very different outcome.

Linear City Model

Firms set prices.

Each consumer chooses the product whose total cost to her is smaller.

For example, if a consumer buys from firm 1, it pays $p_1 + t \cdot y$.

If a consumer buys from firm 2, it pays $p_2 + t \cdot (1 - y)$.

Candidate Voter Model

Even distribution of voters.

Voters vote for the closest candidate.

The number of candidates is not fixed (endogenous).

Candidates cannot choose their position.

Each voter is a potential candidate.

Players: Voters/Candidates.

Strategy: To run or to not run.

Voters vote for the closest running candidate.

Win if get plurality. Flip is tie.

Payoffs

Price of win = B . $B \geq 2c$

Cost of running = C .

And if you are at X and the winner of the election is at Y then you pay a cost of $-|X - Y|$.

If X enters and wins then his payoff is $B - C$.

If X enters but Y wins then his payoff is $-C - |X - Y|$.

If X stays out but Y wins then his payoff is $-|X - Y|$.