Chapter 15

Backward induction: Chess Strategies And Credible Threats

We first discuss Zermelo's theorem: that games like tic-tac-toe or chess have a solution. That is, either there is a way for player 1 to force a win, or there is a way for player 1 to force a tie, or there is a way for player 2 to force a win. The proof is by induction. Then we formally define and informally discuss both perfect information and strategies in such games. This allows us to find Nash equilibria in sequential games. But we find that some Nash equilibria are inconsistent with backward induction. In particular, we discuss an example that involves a threat that is believed in an equilibrium but does not seem credible.

Zermelo Theorem

2 Players
Perfect information
Finite nodes
Three outcomes Win, Loss, Tie
Either 1 can force a win for 1 or 1 can force a tie or 2 can force a win on 1.
Example: In nim, if piles are unequal then 1 can force a win or if they are equal then 2 can force a win.
Tic Tac Toe is a game that leads to a tie.

Proof

By induction on maximum length of the game n.
If n = 1 then it is true.
Suppose the claim is true for all games of length <= n.
We claim therefore it will be true for games of length n+1.
There is a subgame of length n which has a solution which reduces the game size.
Hence proved.

Definition: A game of perfect information is one in which at each node of the game the player whose turn it is to move knows which node she is at (know how you got there).

Definition: A pure strategy for player i in a game of perfect information is a complete plan of action. It specifies which action I will take at each of i’s decision nodes.