Chapter 4

Best Responses In Soccer And Business Partnerships

We continue the idea (from last time) of playing a best response to what we believe others will do. More particularly, we develop the idea that you should not play a strategy that is not a best response for any belief about others' choices. We use this idea to analyze taking a penalty kick in soccer. Then we use it to analyze a profit-sharing partnership. Toward the end, we introduce a new notion: Nash Equilibrium.

Best response in soccer

1. Do not shoot in the middle.
2. Do not choose a strategy that is never a best response to any belief.

Definition: Player i's strategy s'_i is a best response to the strategy s_{-i} of the other players if $u_i(s'_i,s_{-i}) >= u_i(s_i, s_{-i})$ for all s_i in S_i OR s'_i solves max $_{Si}$ $Eu_i(s_i, s_{-i})$.

Definition: Player i's strategy s'_i is a best response to the belief P about other player's choices, if $Eu_i(s'_I, p) >= Eu_i(s_i, p)$ for all s_i in S_i OR s'_i solves max $_{Si}$ $u_i(s_i, P)$.

Partnership Game

Two agents own firm jointly, share 50% of the profits each. Each agent is going to choose her effort level to put into this firm. Firm profit is given by 4*(s1 + s2 + b*s1*s2).

Payoff U_1 (s1, s2) = (½)*4*(s1 + s2 + b*s1*s2) – $s1^2$.

U_2 (s1, s2) = (½)*4*(s1 + s2 + b*s1*s2) – $s2^2$.

Max_{s1} 2*(s1 + s2 + b*s1*s2) – $s1^2$

Differentiate First Order 2(1 + b*s2) – 2*s1 = 0. <=> s1 = 1 + b*s2. (Player 1's best response.)

Differentiate Second Order -2 < 0.

Similarly, s2 = 1 + b*s1. (Player 2's best response.)

Solving the game s1* = s2* = 1/(1-b).

Nash Equilibrium: The players are playing at a best response to each other.