Homework 2

Question 1 Plot some objective functions

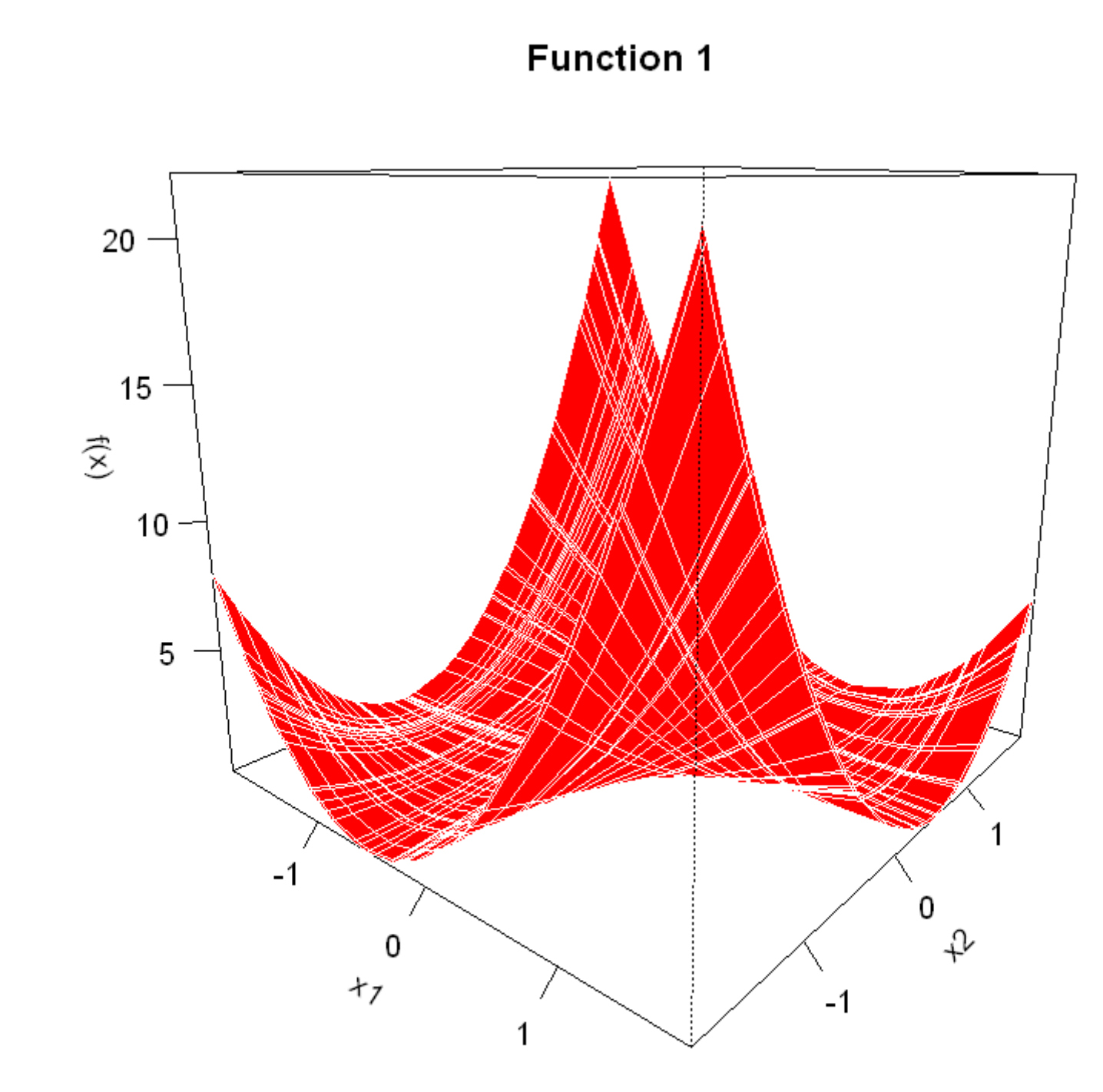
In [140…

```
##Generating random matrix with random values for x = [x1,x2]
xrandom1<- matrix(unique(runif(100,-2,2)),ncol=2)
xrandom1[,1] <- sort(xrandom1[,1],decreasing = FALSE)
xrandom1[,2] <- sort(xrandom1[,2],decreasing = FALSE)
xr1 = rep(xrandom1[,1],length(xrandom1[,1]))
xr2 = c()
for(i in xrandom1[,2]){
list = rep(i,length(xrandom1[,2]))
xr2 = append(xr2,list)
}
xrandom = cbind(xr1,xr2)

##Function to plot data

plotfunction <- function(x,y,z,name,theta = 40){
    persp(x,y,z, theta = theta,main = name,
      xlab = "x1", ylab = "x2",zlab = "f(x)" ,
      col="red", border="grey100", ticktype = "detailed" )
}
```

1) $f(x) = (1 - x_1x_2)^2$

In [141…

```
funct1 <- function(x){
    return ((1 - x[1]*x[2])^2)
}
fx <- matrix(apply(xrandom,1,funct1),nrow = length(xrandom1[,1]),ncol = length(xrandom1[,2]))
plotfunction(xrandom1[,1],xrandom1[,2],fx,"Function 1")
```



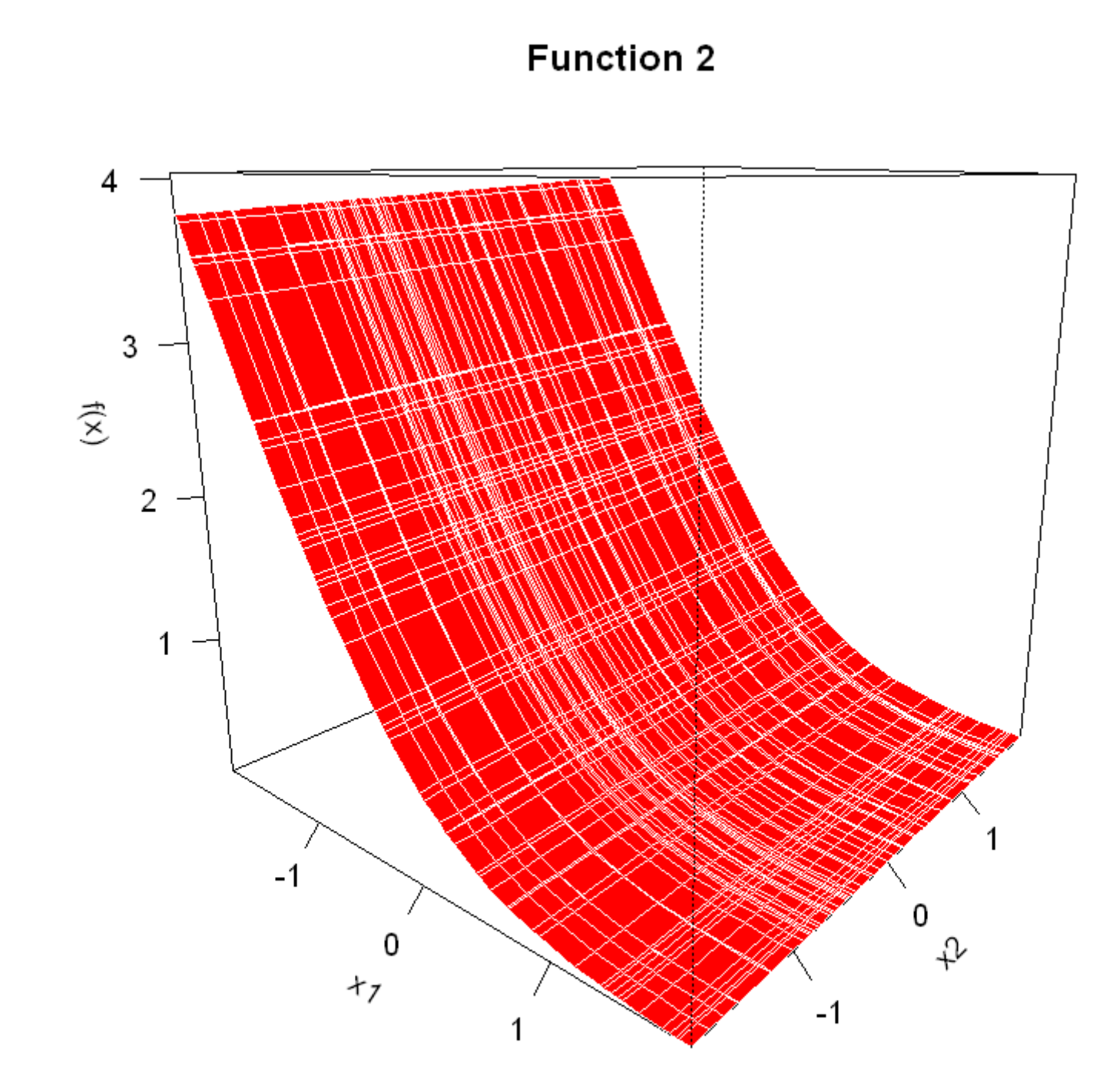
This function in not convex as there are multiple distinct minima

2) $f(x) = log(1 + exp(-w^Tx))$ where $w \in \mathbb{R}^2$

In [37]:

```
w<- matrix(unique(runif(2,-2,2)),ncol=2)
cat("value for W [",w,"]")
funct2 <- function(x){
    #converting x into row matrix
    x <- matrix(x,ncol=2,byrow = TRUE)
    return (log(1 + exp(-(x%*%t(w)))))
}
fx <- matrix(apply(xrandom,1,funct2),nrow = length(xrandom1[,1]),ncol = length(xrandom1[,2]))
plotfunction(xrandom1[,1],xrandom1[,2],fx,"Function 2")
```

```
value for W [ 1.94484 -0.06419089 ]
```



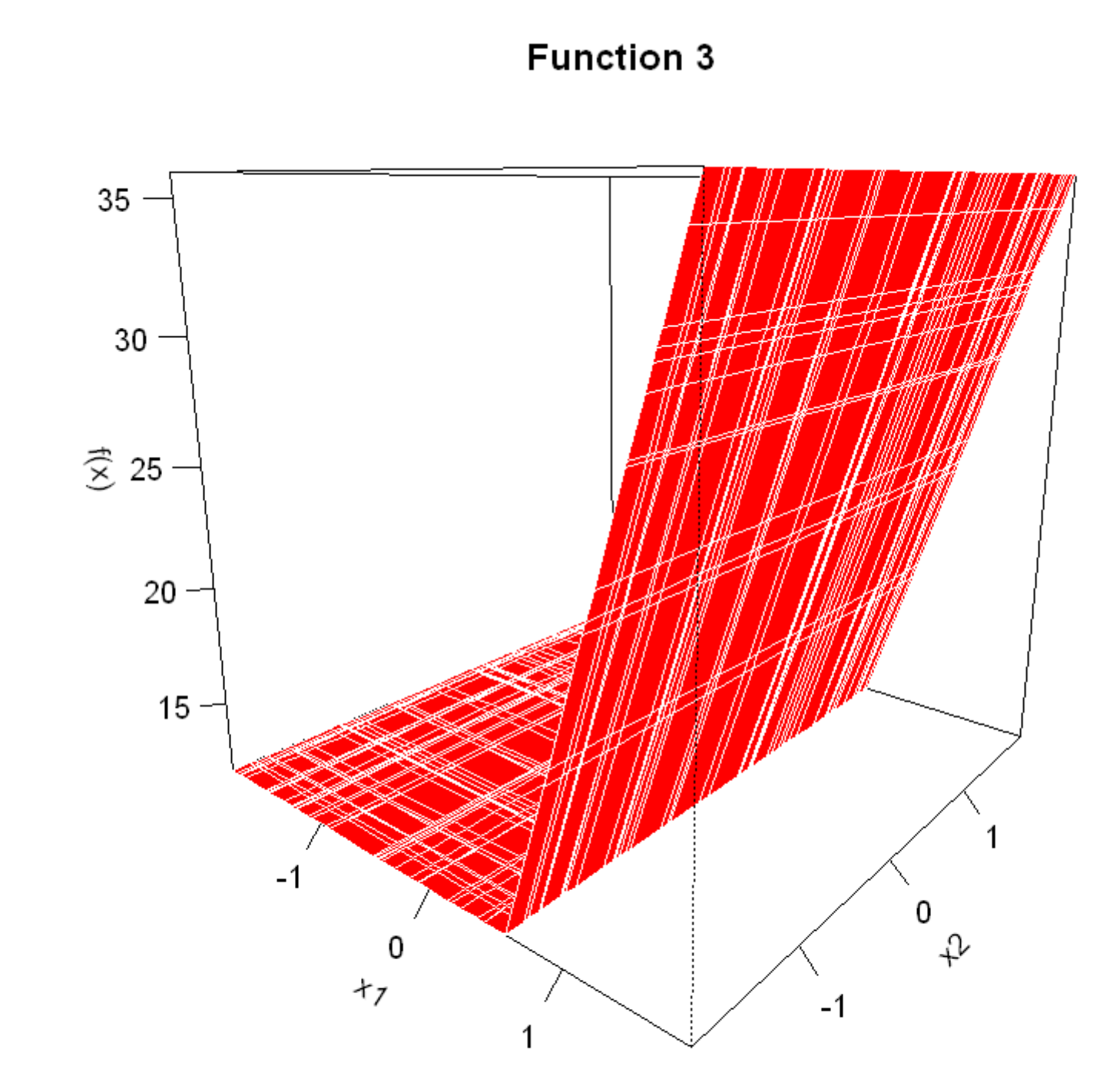
This function in convex, but it is not strongly convex

3) $f(x) = max(a_2max(a_1x_1, b_1), b_2)$ where a_1, a_2, b_1, b_2 are real scalars

In [14]:

```
#scalars a1,a2,b1,b2
s<- matrix(sample(seq(1,5),4,replace = FALSE),ncol=4)
cat("value for a1,a2,b1,b2 [",s,"]")

funct3 <- function(x){
    #converting x into row matrix
    x <- matrix(x,ncol=2,byrow = TRUE)
    return (max(s[2]*(max(s[1]*x[1],s[3])),s[4]))
}
#if majority of fx values are same function may give error just rerrun the function
fx <- matrix(apply(xrandom,1,funct3),nrow = length(xrandom1[,1]),ncol = length(xrandom1[,2]))
plotfunction(xrandom1[,1],xrandom1[,2],fx,"Function 3")
```

```
value for a1,a2,b1,b2 [ 5 4 3 1 ]
```



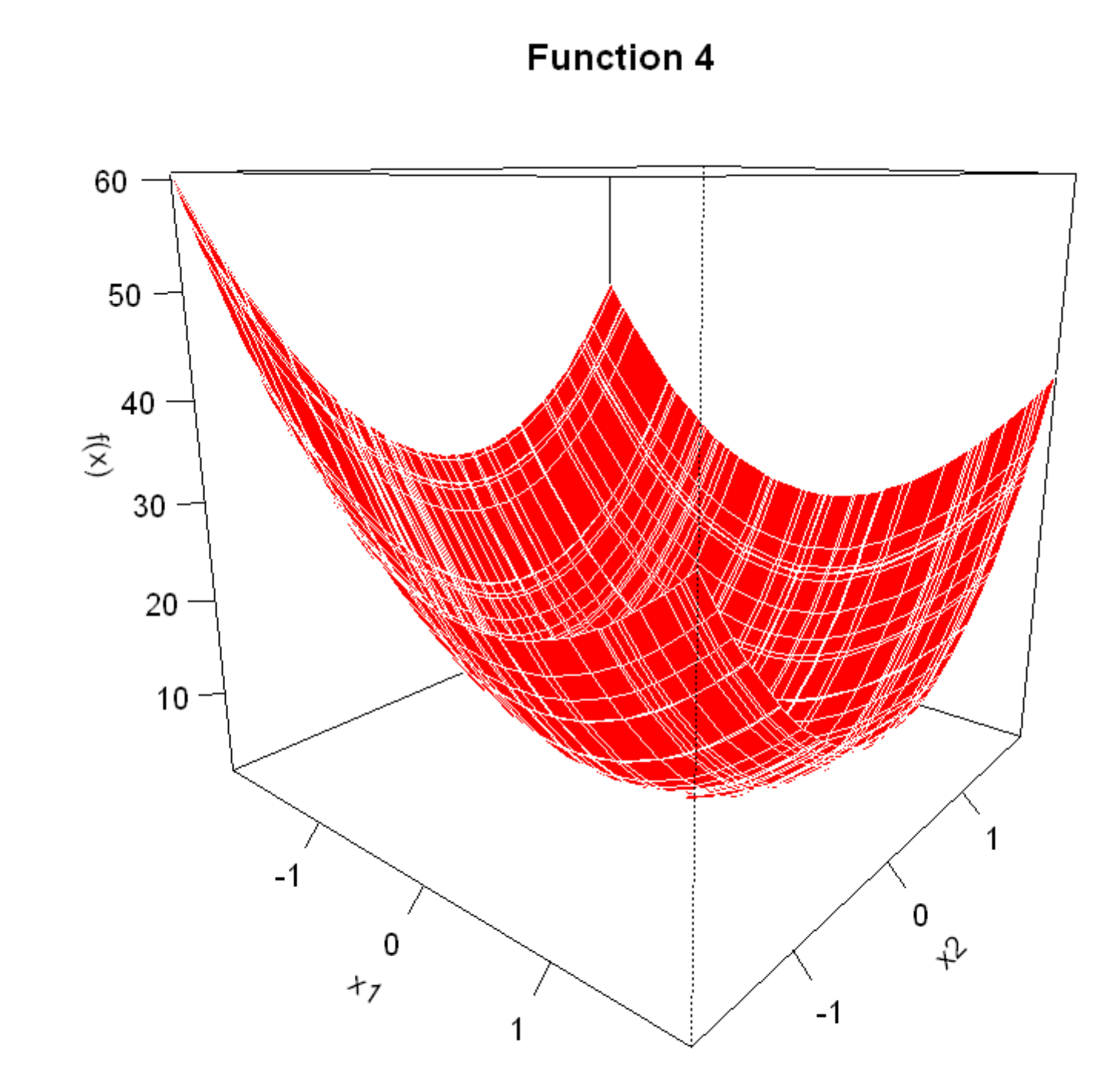
This function in convex, but it is not strongly convex

4) $f(x) = \|Ax - b\|^2 + \lambda L_\delta(x)$ where $\lambda > 0, \delta > 0$ and $L_\delta(x) = \begin{cases} \frac{1}{2}\|x\|^2 & \text{for } \|x\| \le \delta \\ \delta(\|x\| - \frac{1}{2}\delta), & \text{otherwise.} \end{cases}$

In [43]:

```
Ldelta  <- function(x,delta){
    if(norm(x, type = "2") <= delta)
        {
        return ((norm(x,type = "2")**2)/2)
    }
    return (delta * (norm(x,type = "2") - delta/2))
}

funct4 <- function(x,A,b,delta,lambda){
    #To statisfy Ax - b
    x = matrix(x,ncol=1)
    return ((norm(((A%*%x) - b), type = "2")**2) + lambda*Ldelta(x,delta))
}

#square matrix 2x2
A = matrix(unique(runif(4,-2,2)),ncol=2)
#column matrix matrix 2x1
b = matrix(unique(runif(2,-2,2)),ncol=1)
delta = runif(1,0,10)
lambda = runif(1,0,10)
cat("A = ",A," b = ",b," delta = ",delta," lambda = ",lambda)
fx <- matrix(apply(xrandom,1,funct4,A = A,b = b,delta = delta,lambda = lambda),nrow = length(xrandom1[,1]),n
plotfunction(xrandom1[,1],xrandom1[,2],fx,"Function 4",40)
```

```
A =  1.130383 0.3376974 0.01868903 1.722128  b =  1.339688 0.5110653  delta =  7.541152  lambda =  7.598412
```

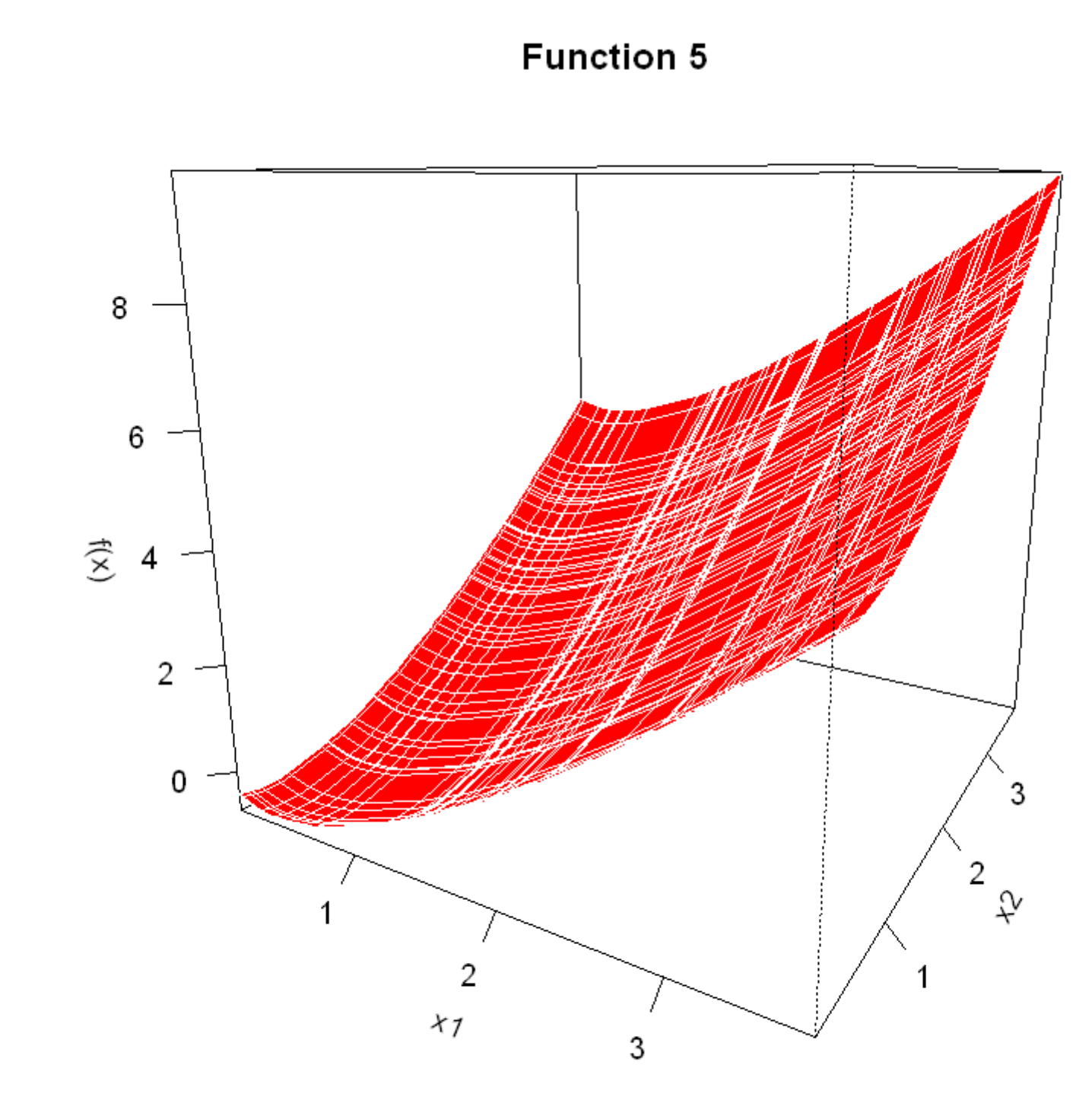


This function in strongly convex

5) $f(x) = x_1log(x_1) + x_2log(x_2)$ for $x_1 > 0$ and $x_2 > 0$

In [50]:

```
##Generating random matrix with random values for x = [x1,x2]
xrandom1<- matrix(unique(runif(100,0,4)),ncol=2)
xrandom1[,1] <- sort(xrandom1[,1],decreasing = FALSE)
xrandom1[,2] <- sort(xrandom1[,2],decreasing = FALSE)
xr1 = rep(xrandom1[,1],length(xrandom1[,1]))
xr2 = c()
for(i in xrandom1[,2]){
list = rep(i,length(xrandom1[,2]))
xr2 = append(xr2,list)
}
xrandom = cbind(xr1,xr2)

funct5 <- function(x){
    return ((x[1]*log(x[1])) + (x[2]*log(x[2])) )
}
fx <- matrix(apply(xrandom,1,funct5),nrow = length(xrandom1[,1]),ncol = length(xrandom1[,2]))
plotfunction(xrandom1[,1],xrandom1[,2],fx,"Function 5",30)
```

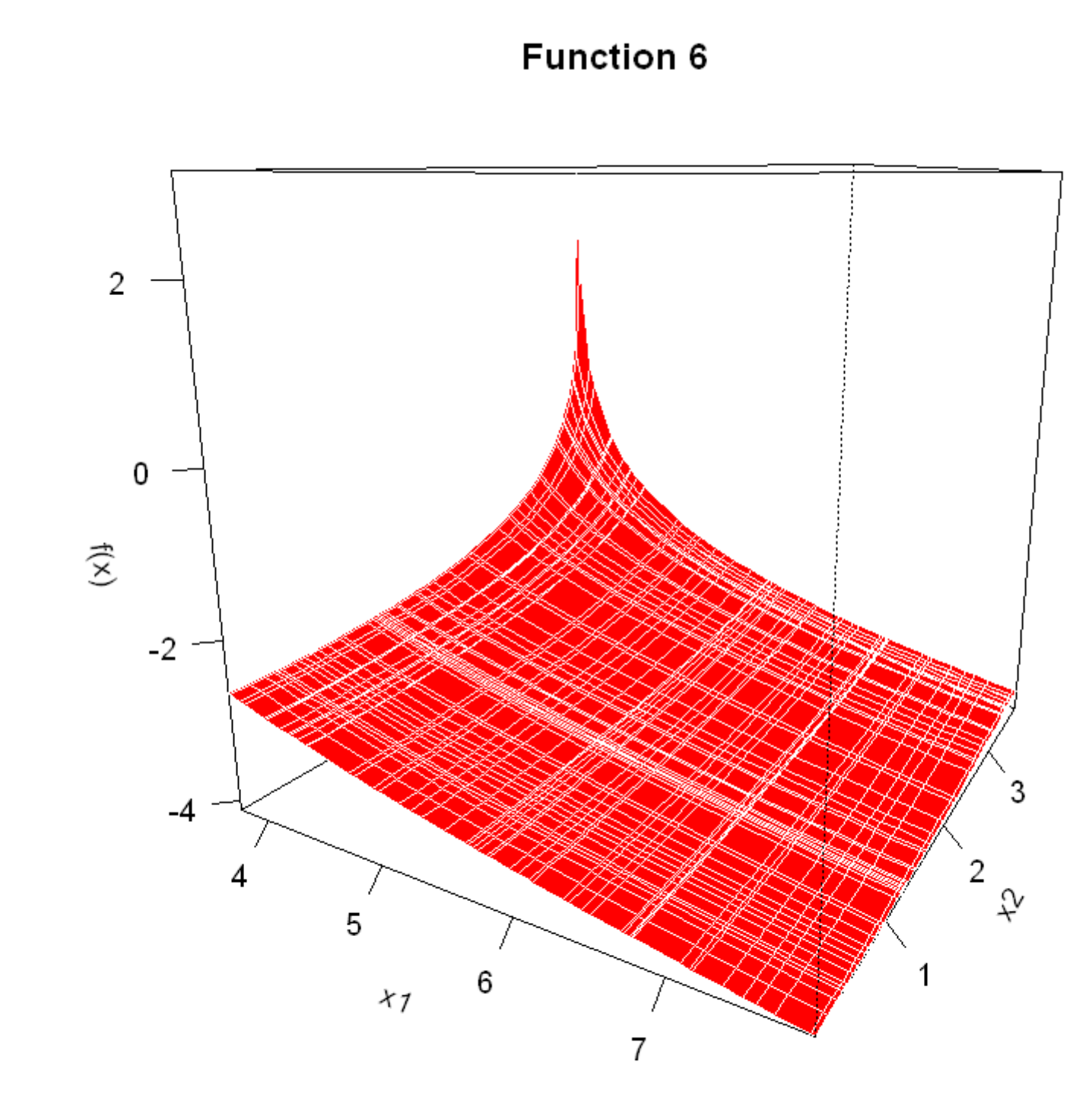


This function in convex

6) $f(x) = -log\det\left(\begin{bmatrix} x_1 & x_2 \\ x_2 & x_1 \end{bmatrix}\right)$ for (x_1, x_2) such that $x_1 > |x_2| \ge 0$

In [88]:

```
##Generating random matrix with random values for x = [x1,x2]
xrandom1<- matrix(rep(0,100),ncol=2)
xrandom1[,2]<- unique(runif(50,0,4))
m = max(xrandom1[,2])
for (i in seq(1,length(xrandom1[,2])))
{
    xrandom1[i,1] = runif(1,m,8)
}
xrandom1[,1] <- sort(xrandom1[,1],decreasing = FALSE)
xrandom1[,2] <- sort(xrandom1[,2],decreasing = FALSE)
xr1 = rep(xrandom1[,1],length(xrandom1[,1]))
xr2 = c()
for(i in xrandom1[,2]){
list = rep(i,length(xrandom1[,2]))
xr2 = append(xr2,list)
}
xrandom = cbind(xr1,xr2)
funct6 <- function(x){
    mat <- matrix(c(x[1],x[2],x[2],x[1]),byrow = TRUE, ncol =2)
    return ( - (log(det(mat))))
}
fx <- matrix(apply(xrandom,1,funct6),nrow = length(xrandom1[,1]),ncol = length(xrandom1[,2]))
plotfunction(xrandom1[,1],xrandom1[,2],fx,"Function 6",30)
```

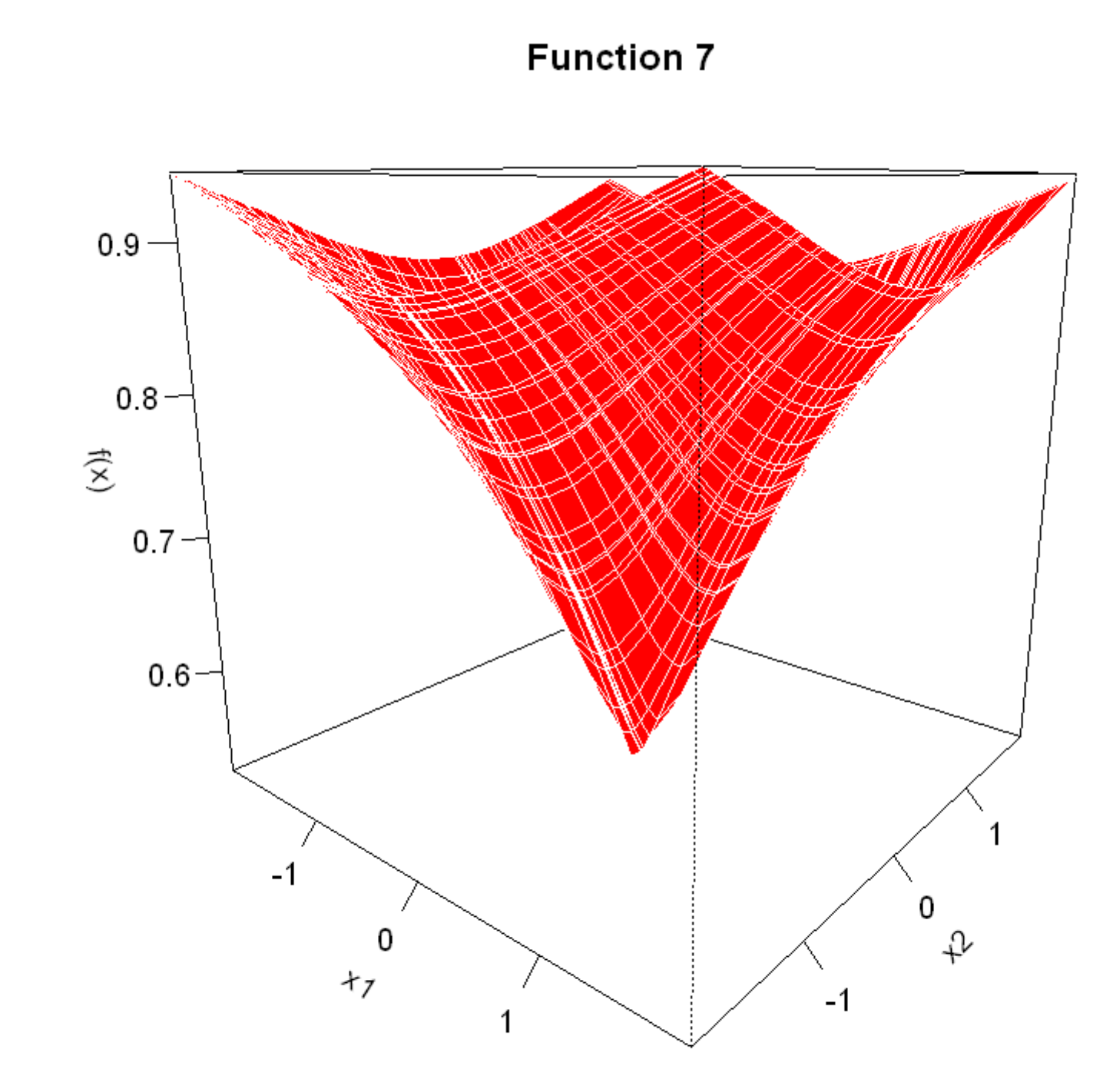


This function in convex

7) $\sigma(x) = \frac{1}{1+e^{-\|x\|}}$

In [137…

```
##Generating random matrix with random values for x = [x1,x2]
xrandom1<- matrix(unique(runif(100,-2,2)),ncol=2)
xrandom1[,1] <- sort(xrandom1[,1],decreasing = FALSE)
xrandom1[,2] <- sort(xrandom1[,2],decreasing = FALSE)
xr1 = rep(xrandom1[,1],length(xrandom1[,1]))
xr2 = c()
for(i in xrandom1[,2]){
list = rep(i,length(xrandom1[,2]))
xr2 = append(xr2,list)
}
xrandom = cbind(xr1,xr2)

funct7 <-function(x){
    x <- matrix(x,ncol=2,byrow = TRUE)
    return (1/(1 + exp(-(norm(x, type= "2")))))
}
fx <- matrix(apply(xrandom,1,funct7),nrow = length(xrandom1[,1]),ncol = length(xrandom1[,2]))
plotfunction(xrandom1[,1],xrandom1[,2],fx,"Function 7")
```



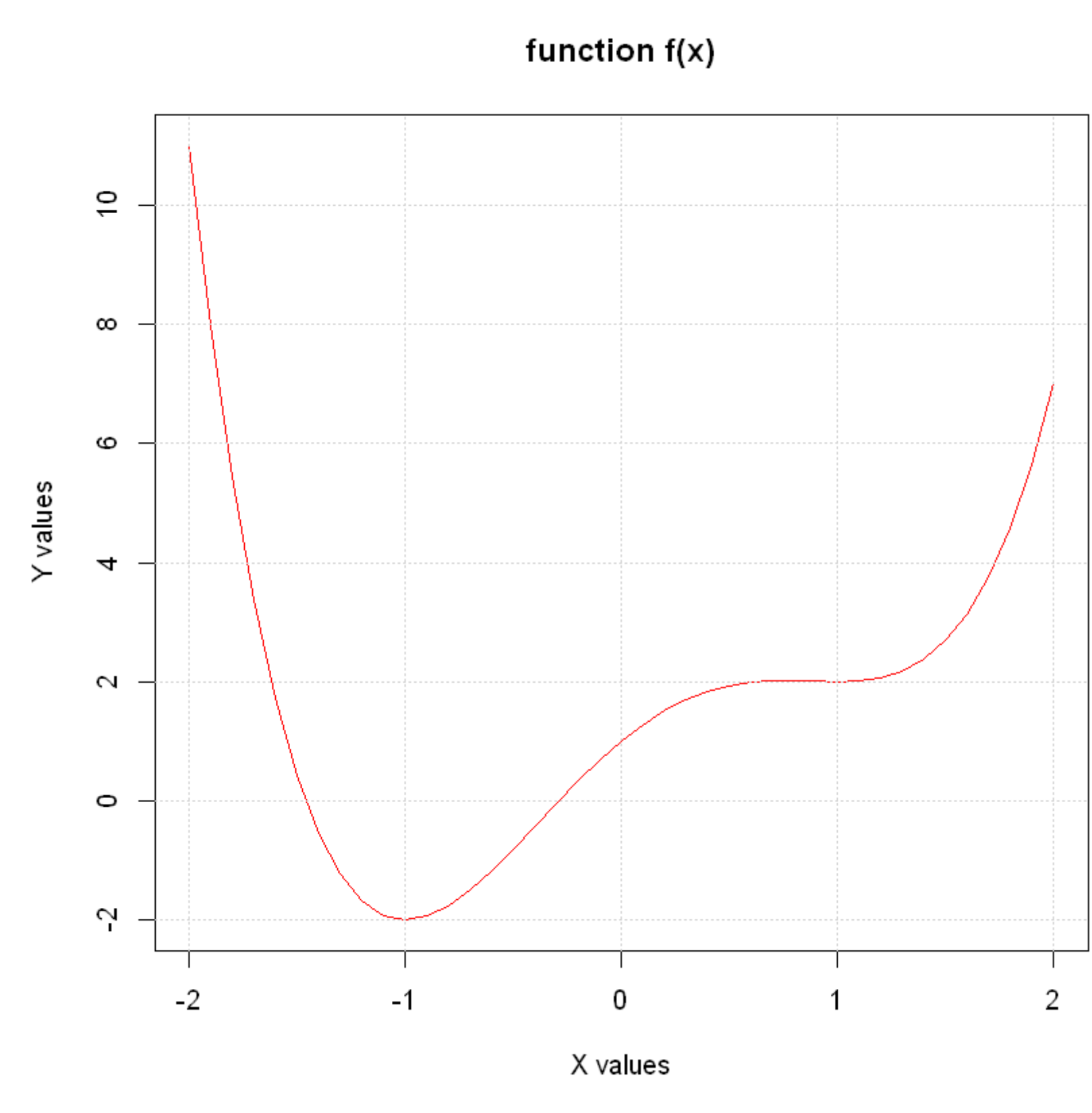
This function in stronly convex

Question 2 Newton-Raphson Method

Polynomial in dimension one $f(x) = x^4 - x^3 - 2x^2 + 3x + 1, \ \ x \in \mathbb{R}^2$

In [1]:

```
#Function f(x)
polyfunction <- function(x){
    return (x^4 - x^3 - 2*x^2 + 3*x + 1)
}

#Function f'(x)
dervpolyfunction <- function(x){
    return (4*x^3 - 3*x^2 - 4*x + 3)
}

#Function f''(x)
secdervpolyfunction <- function(x){
    return (12*x^2 - 6*x - 4)
}

#x range from -2 and 2
xseq <- seq(-2,2,0.1)

#function value x range from -2 and 2
y <- unlist(lapply(xseq,polyfunction))

#plot function
plot(xseq,y, type = "l", main="function f(x)", col = "red", xlab = "X values", ylab = "Y values", axes = TRU
grid()
```



In [21]:

```
#Implementation of Newton Raphson method
newtonRaphson <- function(x) {
    success = FALSE
    trendx = c(x)
    for (i in seq(1,1000)){
        newx = x - dervpolyfunction(x)/secdervpolyfunction(x)
        trendx <- append(trendx,newx)
        #Terminating condition
        if (newx == x) {
            success = TRUE
            break
        }
        x <- newx
    }
    return (list(success = success, trendx = trendx))
}

listtrenx = list()
listsuccess = list()
xvalues <- c( -50,-25,10,-3 ,-2 , 0 ,0.5, 2 , 3, 10,25,50)

#Running newtonRaphson method for various initial points
for (i in seq(1,length(xvalues)))
    {
    result <- newtonRaphson(xvalues[i])
    listsuccess[[i]] <- result$success
    listtrenx[[i]] <- result$trendx
}

iternation <- lapply(listtrenx,length)
answers <- lapply(listtrenx,function(x) x[length(x)])
iternation <- do.call(cbind, iternation)
answers <- do.call(cbind, answers)
listsuccess <- do.call(cbind, listsuccess)
colnames(iternation)<- xvalues
colnames(answers)<- xvalues
colnames(listsuccess)<- xvalues

print("Number of iteration for each starting points")
iternation
print("Success indicator for each starting points")
listsuccess
print("Optimum value of x as per Newton Raphson method for each starting points")
answers
```

```
[1] "Number of iteration for each starting points"
```

-50	-25	10	-3	-2	0	0.5	2	3	10	25	50
16	15	15	10	9	3	8	10	12	15	17	19

```
[1] "Success indicator for each starting points"
```

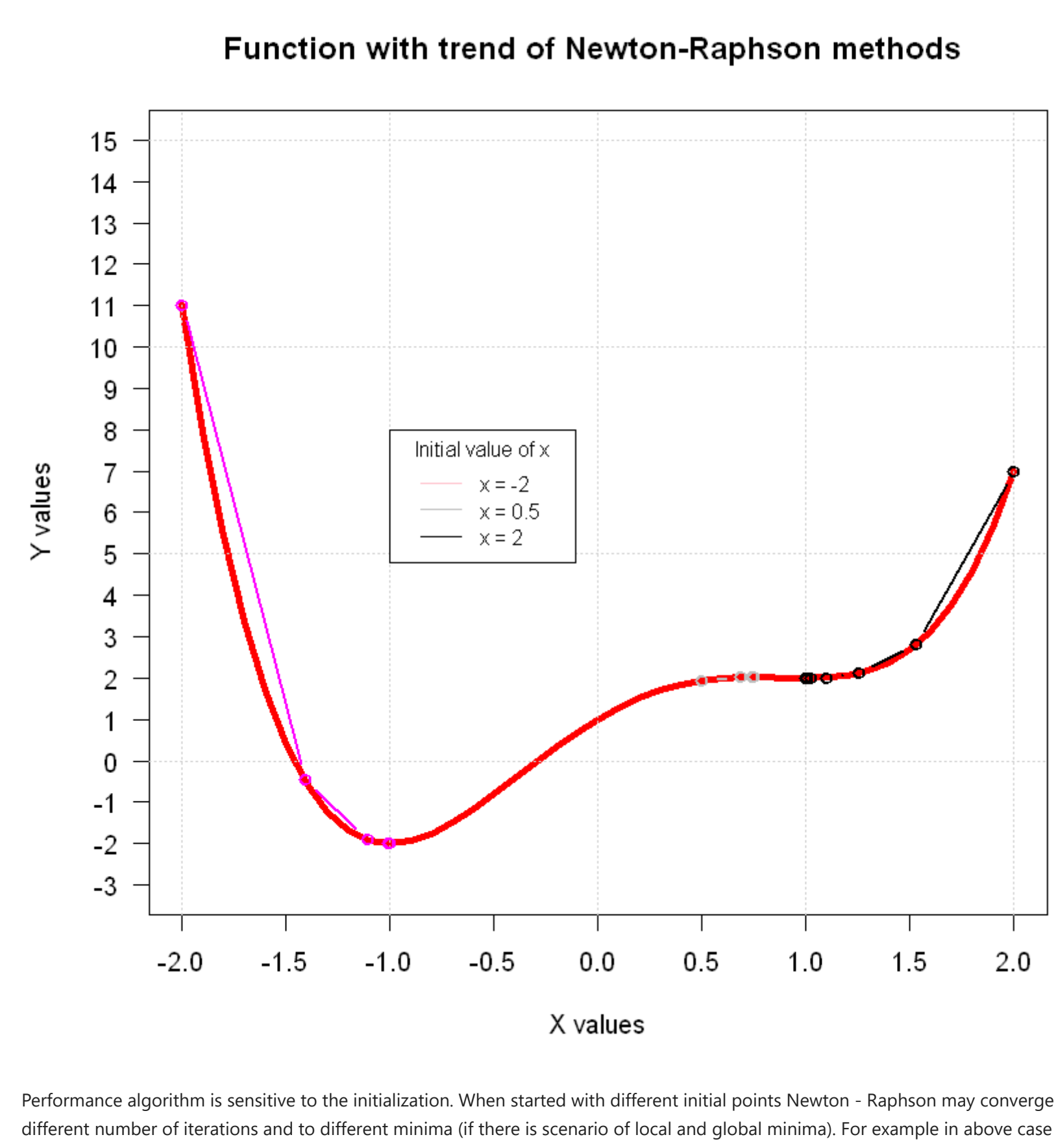
-50	-25	10	-3	-2	0	0.5	2	3	10	25	50
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

```
[1] "Optimum value of x as per Newton Raphson method for each starting points"
```

-50	-25	10	-3	-2	0	0.5	2	3	10	25	50
-1	-1	1	-1	-1	0.75	0.75	1	1	1	1	1

In [23]:

```
y <- unlist(lapply(xseq,polyfunction))
plot(xseq,y, type = "l", main="Function with trend of Newton-Raphson methods ", col = "red", xlab = "X value
xlabel <- seq(-4, 4, by = 0.5)
ylabel <- seq(-3, 15, by = 1)
axis(1, at = xlabel, las = 1)
axis(2, at = ylabel, las = 1)
box()
grid()
index <- c(5,7,8)
for (i in index)
    {
    yl <- unlist(lapply(listtrenx[[i]],polyfunction))
    lines(listtrenx[[i]],yl,type="b", col = i+1, lwd = 2)
}
legend(-1,8 , legend=c("x = -2", "x = 0.5","x = 2"),title="Initial value of x",
       col=c("pink", "grey", "black"), lty=1:1, cex=0.8)
```



Performance algorithm is sensitive to the initialization. When started with different initial points Newton - Raphson may converge in different number of iterations and to different minima (if there is scenario of local and global minima). For example in above case x = -2 converges in 9 iterations, x = 0.5 converges in 8 iterations and x = 2 coverges in 10 iterations. Also for x = -2 algorithm converges with x = -1 *while for x = 2 algorithm converges with x* = 1 and 0.5 algorithm converges with x* = 0.75.

Newton Raphson method may fail becuase of overshooting. This is the reason that with initial value of $x \ge 0$ Newton - Raphson fails to converge at global minima point (x* = -1) and converges at local minima x = 0.75 or x = 1.

For this function I haven't observed an initial point where Newton - Raphson method does not converge at all.

Question 3 Newton's Method on HW1 question

In [168…

```
#reading House data
houseData <- read.csv('house.csv')
```

In [169…

```
#creating variable A, y and x
y <- as.matrix(houseData$price)
A <- as.matrix(houseData[,2:3])
ones = matrix(rep(1,nrow(y)), nrow = nrow(y), ncol = 1)
y <- (y - mean(y))/sd(y)
A[,1] = (A[,1] - mean(A[,1]))/sd(A[,1])
A[,2] = (A[,2] - mean(A[,2]))/sd(A[,2])
A = cbind(ones,A)
x = matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
colnames(A)[1] <- "intercept"
colnames(y)[1] <- "price"
head(A)
head(y)
```

intercept	bd	sqft
1	-1.3411019	-0.6753744
1	1.3424687	1.9160003
1	1.3424687	0.7066921
1	0.4479452	0.1433498
1	1.3424687	2.0361800
1	0.4479452	-0.3523915

price
-1.2490641
1.4362784
1.7060115
0.3333699
1.4242903
0.2614411

In [173…

```
#delta f(x)
deltafx <- function(x){
    return (-t(A)%*%(y - A%*%x))
    }

#Hessian matrix
secdeltafx <- t(A)%*%A
```

In [171…

```
# Calculating x* using Newton method
newx = x - solve(secdeltafx)%*%deltafx(x)

#Using Newton method we find solution just in one iteration
newx
```

	price
intercept	1.016019e-16
bd	6.700796e-01
sqft	3.192639e-01

In [172…

```
newx <- format(newx, scientific = FALSE)
newx
```

	price
intercept	0.00000000000000001016019
bd	0.6700795745981675821668
sqft	0.3192638967577376973850

Newton method provided optimal solution just in one iteration as compared to Gradient descent or Stochastic Gradient descent which took multiple iterations.

Github link for the code

https://github.com/jainsanyam786/OptimizationForMachineLearning.git