HOMEWORK1

Read the data

We will read dataset that is about the house prices in a region in Europe together with the square footage and number of bedroom information denoted by the column (variable) with names sqft and bd respectively. There are n=10 million data points.

```
In [1]: options(warn=-1)
        houseData <- read.csv('house.csv')
```

Inspect the data read

R read the data as dataframe by default so there is no need to convert the "houseData" to datafram.

```
In [2]: print(paste("Is houseData a dataframe", is.data.frame(houseData),sep = " "),quote=FALSE)
        head(houseData)
```

[1] Is houseData a dataframe TRUE

	X	bd	sqft	price
1	1	5	1370	451000
2	2	4	2060	899000
3	3	4	1738	944000
4	4	3	1588	715000
5	5	4	2092	897000
6	6	3	1456	703000

```
In [3]: cat("Size of data frame",dim.data.frame(houseData))
```

Size of data frame 10000000 4

Question 1 - Rewrite the objective in the form

```
In [4]: y <- as.matrix(houseData$price)
        A <- as.matrix(houseData[,2:3])
        ones = matrix(rep(1,nrow(y)), nrow = nrow(y), ncol = 1)
        head(y)
        print("Price")
        head(y)
        print("Features")
        head(A)
```

[1] "Price"

451000

899000

944000

715000

897000

703000

[1] "Features"

bd	sqft
5	1370
4	2060
4	1738
3	1588
4	2092
3	1456

Normalise Data

```
In [5]: y <- (y - mean(y))/sd(y)
        A[,1] = (A[,1] - mean(A[,1]))/sd(A[,1])
        A[,2] = (A[,2] - mean(A[,2]))/sd(A[,2])

        print("After Normalization")
        print("Price")
        head(y)
        print("Features")
        head(A)
```

[1] "After Normalization"
[1] "Price"

-1.2490641

1.4362784

1.7060115

0.3333699

1.4242903

0.2614411

[1] "Features"

bd	sqft
-1.3411019	-0.6753744
1.3424687	1.9160003
1.3424687	0.7066921
0.4479452	0.1433498
1.3424687	2.0361800
0.4479452	-0.3523915

Complete Data matrix A with ones for intercept

```
In [6]: A = cbind(ones,A)
        colnames(A)[1] <- "intercept"
        head(A)
        ncol(A)
```

intercept	bd	sqft
1	-1.3411019	-0.6753744
1	1.3424687	1.9160003
1	1.3424687	0.7066921
1	0.4479452	0.1433498
1	1.3424687	2.0361800
1	0.4479452	-0.3523915

3

Objective Function in the Matric form with square of L2 Norm

```
In [7]: fx <- function(x,y,A){
            return ((norm((y-(A %*% x)), type = "2")**2)/(2*nrow(A)))
        }
        #This is sample x row matrix [x1,x2,x3] initialize as zerec
        x = matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
        dim(x)
```

1. 3
2. 1

Question 2 - Gradient descent

Defination of function for gradient descent and gradient calculation

```
In [28]: deltafx <- function(y1,A1,x1){
             yhat <- A1%*%x1
             diff <- y1 -yhat
             At <- t(A1)
             df <- -(At%*%diff)/nrow(A1))
             return(df)
         }

         gradientDescent <- function(y1,A1,x1,alpha,thrs, maxi){
             converged<-FALSE
             i=1
             x1<-x1
             gradientTrend <- c()
              while((!converged  && i <= maxi))
                     {
                         deltaf <- deltafx(y1,A1,x1)
                         if(is.infinite(deltaf)|| is.nan(norm(deltaf, type = "2"))){
                             break
                         }
                         x1 <- x1 - (alpha*deltaf)
                         gradientTrend <- append(gradientTrend,norm(deltaf, type = "2"))
                         converged <- (norm(deltaf, type = "2") <= thrs)
                         i <- i+1
                     }

             return (list("x1"= x1,"gradientTrend" =gradientTrend, "iteration" = i-1,"converged" = converged))
         }
```

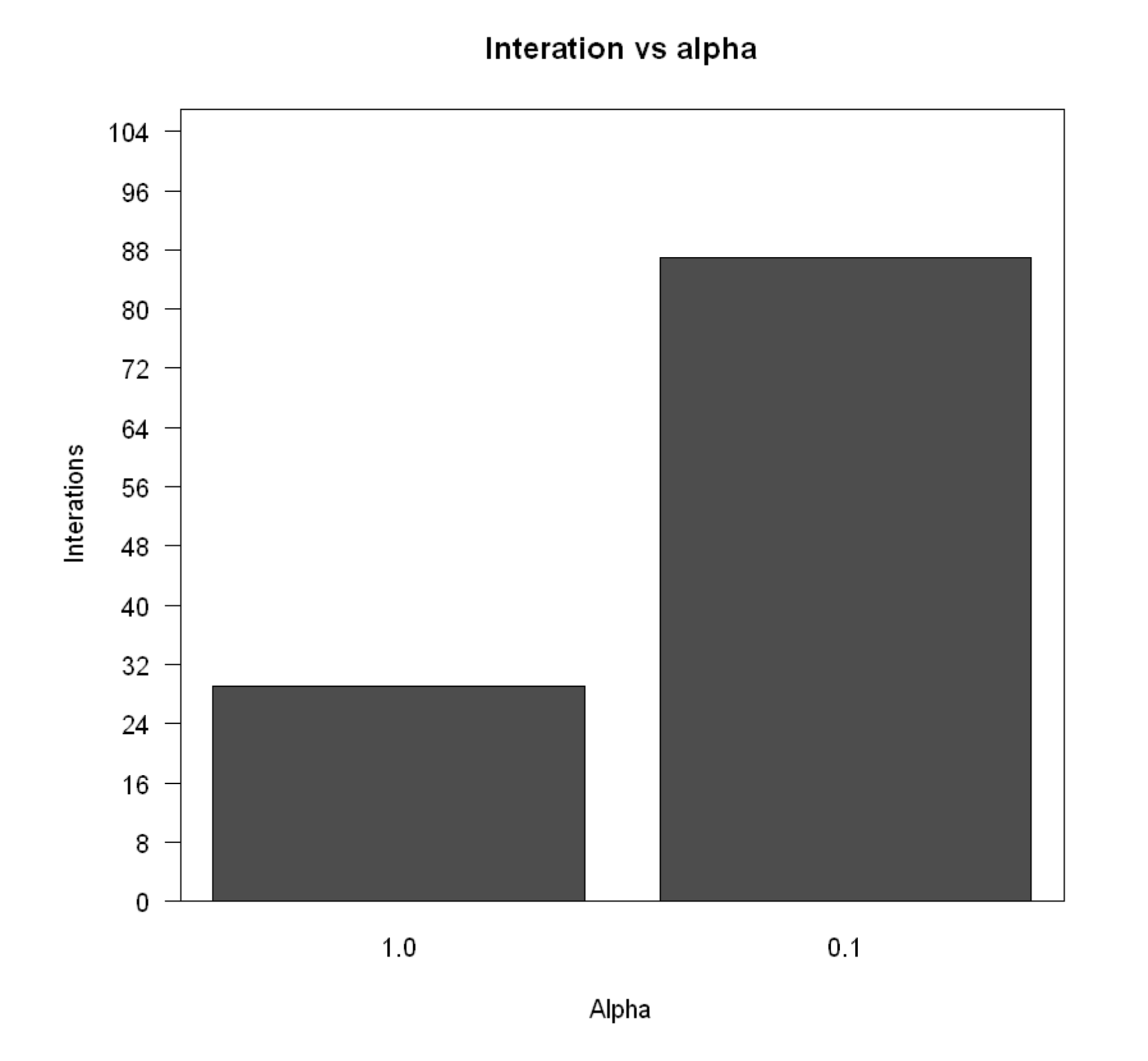
```
In [14]: #running gradient descent on complete data
         alphas <- c(1,0.1)
         thres <- 10**(-2)
         maxiter <- 10000
         A2 <- A
         y2 <- y
         successalphs <- c()
         iternations <- list()
         gradienttrend <- list()
         xsfull <- list()
         x = matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
         row = 1
         for (alpha in alphas) {
             answer <- gradientDescent(y,A,x,alpha,thres,maxiter)
             if (answer$converged){
             successalphs <- append(successalphs,alpha)
             xsfull[[row]] <- answer$x
             iternations[[row]] <- answer$iteration
             gradienttrend[[row]] <- answer$gradientTrend
             row = row+1
             }
         }
         successalphs <- format(successalphs, scientific = FALSE)
         xsfull <- do.call(cbind, xsfull)
         iternations <- do.call(cbind, iternations)

         colnames(xsfull)<- successalphs
         colnames(iternations)<- successalphs
         modifiedxs <- format(xsfull, scientific = FALSE)
         modifiedxs
         iternations
         successalphs
```

	1.0	0.1
intercept	0.00000000000000001107913	0.000000000000000853718
bd	0.672093809179243904026	0.627088865213385915537
sqft	0.323482542909781570870	0.362254585541362916785

1.0	0.1
29	87

1. '1.0'
2. '0.1'

```
In [15]: barplot(iternations, main="Interation vs alpha", names.arg = successalphs,
                 ylab = "Interations", ylim = c(0,max(iternations)+20), xlab = "Alpha", axes = FALSE)
         ylabel <- seq(0, max(iternations)+20, by = max(iternations)%/%10)
         axis(2, at = ylabel, las = 1)
         box()
```



Running gradient descent for varoius values of alpha with less no. number of rows

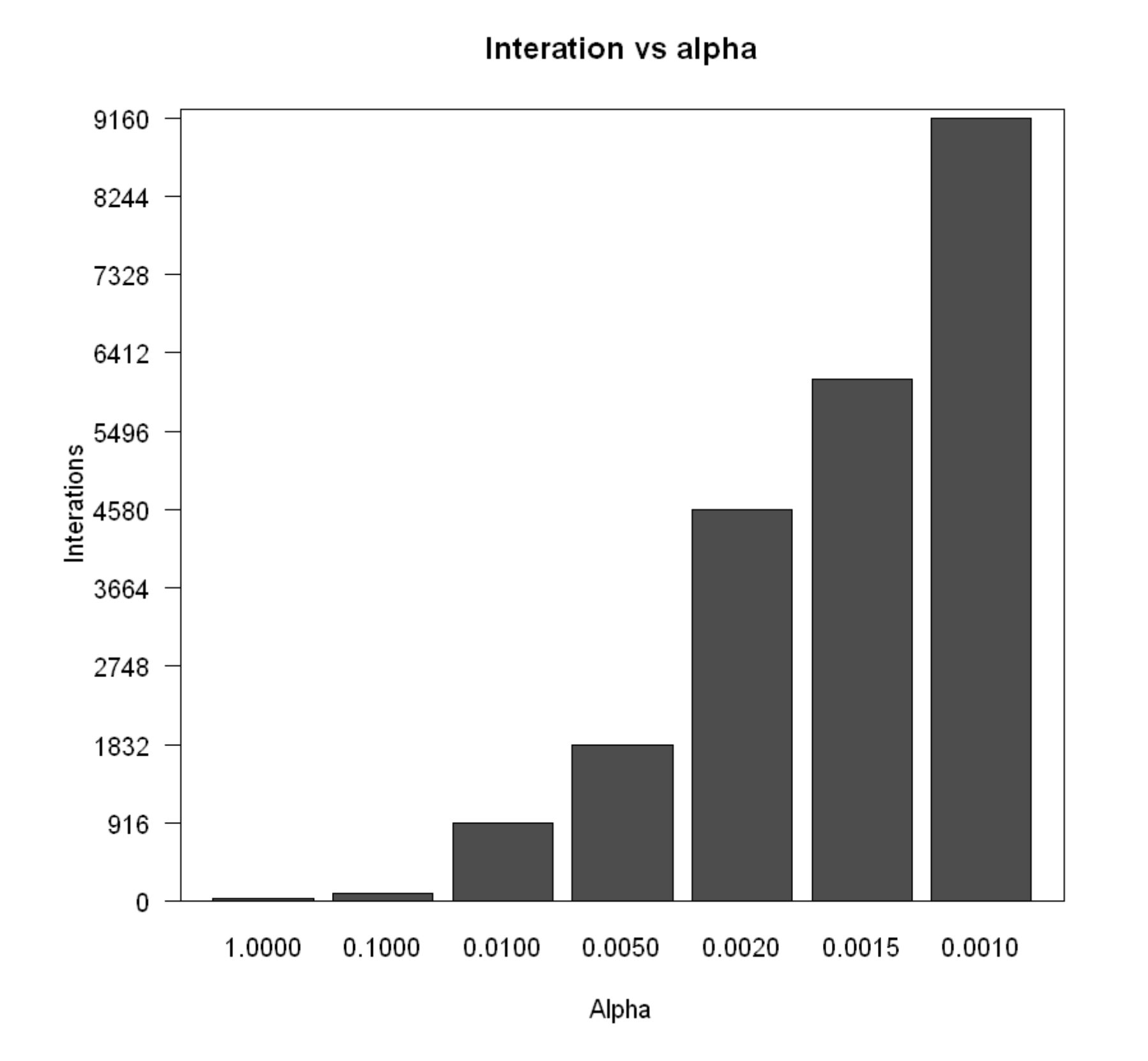
```
In [31]: alphas <- c(10,1,0.1,0.01,0.005,0.002,0.0015,0.001,0.0008,0.0006,0.0004,0.0002,0.0001)
         thres <- 10**(-2)
         maxiter <- 10000
         #considering first 1000 rows
         A2 <- A[1:1000,]
         y2 <- matrix(y[1:1000,],nrow=1000,byrow=TRUE)

         successalphs <- c()
         iternations <- list()
         gradienttrend <- list()
         xs <- list()
         row = 1

         for (alpha in alphas)
             {
                 x <- matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
                 answer <- gradientDescent(y2,A2,x,alpha,thres,maxiter)
                 if (answer$converged){
                     successalphs <- append(successalphs,alpha)
                     xs[[row]] <- answer$x
                     iternations[[row]] <- answer$iteration
                     gradienttrend[[row]] <- answer$gradientTrend
                     row = row+1
                 }
             }
         successalphs <- format(successalphs, scientific = FALSE)
         xs <- do.call(cbind, xs)
         iternations <- do.call(cbind, iternations)

         colnames(xs)<- successalphs
         colnames(iternations)<- successalphs
         xs
         iternations
         successalphs
```

	1.0000	0.1000	0.0100	0.0050	0.0020	0.0015	0.0010
intercept	-0.009551085	-0.01046902	-0.01048878	-0.01049056	-0.01049162	-0.0104917	-0.01049178
bd	0.678862653	0.63166109	0.63098822	0.63092720	0.63089058	0.6308880	0.63088548
sqft	0.306859609	0.34706206	0.34772673	0.34778701	0.34782319	0.3478257	0.34782822

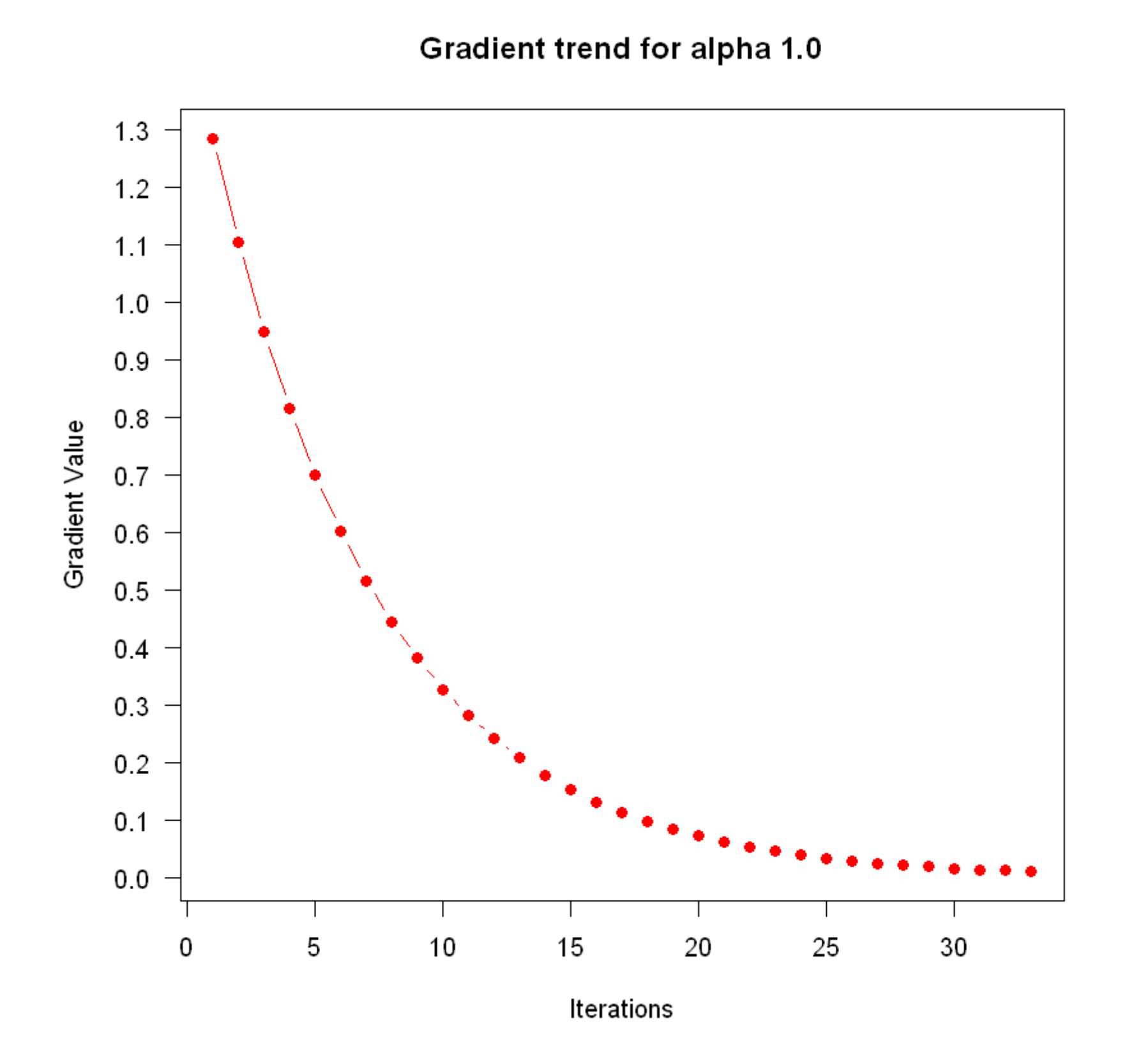
1.0000	0.1000	0.0100	0.0050	0.0020	0.0015	0.0010
33	92	917	1833	4581	6108	9162

1. '1.0000'
2. '0.1000'
3. '0.0100'
4. '0.0050'
5. '0.0020'
6. '0.0015'
7. '0.0010'

```
In [32]: barplot(iternations, main="Interation vs alpha", names.arg = successalphs,
                 ylab = "Interations", ylim = c(0,max(iternations)+100), xlab = "Alpha", axes = FALSE)
         ylabel <- seq(0, max(iternations)+100, by = max(iternations)%/%10)
         axis(2, at = ylabel, las = 1)
         box()
```




```
In [33]: ## Plot of gradient trend for best alpha
         plot(gradienttrend[[which("1.0000" == successalphs)]], type = "b", main="Gradient trend for alpha 1.0", pch
              col = "red", xlab = "Iterations", ylab = "Gradient Value", axes = FALSE)
         ylabel <- seq(0, 2, by = 0.1)
         xlabel <- seq(0, 70, by = 5)
         axis(1, at = xlabel, las = 1)
         axis(2, at = ylabel, las = 1)
         box()
```



Actual Solution by solving linear equations

```
In [76]: sol <- solve((t(A)%*%A))%*%(t(A)%*%y)
         modifiedsol <- format(sol, scientific = FALSE)
         modifiedsol
```

intercept	0.0000000000000001016019
bd	0.6700795745981675821668
sqft	0.3192638967577376973850

Calculating difference between actual solution and solutions obtained for various value of alphas for full data

```
In [77]: as <- apply(xsfull,2,function(x) sol -x)
         as <- format(as, scientific = FALSE)
         as
```

1.0	0.1
0.00000000000000002333068	0.00000000000000001682101
-0.00141772611840762507995	0.03061754198280486338746
-0.00297557917669799554261	-0.03061754169481095688710

Conclusion

For complete dataset algorithm converge for values of alpha = 1 with 31 iterations and alpha = 0.1 with 108 iterations

Multiple alphas Gradient descent was run for alphas = [1,0.1,0.01,0.005,0.002,0.0015,0.001,0.0008,0.0006,0.0004,0.0002,0.0001] There was no convergence for alpha greater than 1. Algorithm converged for alphas [1,0.1,0.01,0.005,0.002,0.0015] with in 10,000 internations, with minimum interations 36 for alpha = 1.

Also there is negligible difference between solutions using gradient descent and actual solutions.

Question 3 - Plot Contour plot with movement of gradient descent and cost function for two features with 3rd kept as constant

Generating random values for x2 and x3 while keeping x1 to optimum value

```
In [78]: #random values are generated and arranged in a matrix to feed to cost function

         xrange2 <- seq(from = sol[2]*(0.5),to = sol[2]*(1.5), length.out = 50)
         xrange3 <- seq(from = sol[3]*(0.5),to = sol[3]*(1.5), length.out = 50)
         xr2 = rep(xrange2,length(xrange2))
         xr1 <- rep(sol[1],length(xr2))
         xr3 = c()
         for(i in xrange3){
         list = rep(i,length(xrange3))
         xr3 = append(xr3,list)
         }
         xrandom = cbind(xr1,xr2,xr3)
         head(xrandom)
```

xr1	xr2	xr3
1.016019e-16	0.3350398	0.1596319
1.016019e-16	0.3487149	0.1596319
1.016019e-16	0.3623900	0.1596319
1.016019e-16	0.3760651	0.1596319
1.016019e-16	0.3897402	0.1596319
1.016019e-16	0.4034153	0.1596319

Solving fx (cost function) for xrandom

```
In [81]: fxValues <- c()
         for (i in seq(1,nrow(xrandom))){
             xi <- as.matrix(xrandom[i,], nrow = 3)
             fxValues[i]<-fx(xi,y,A)
         }
         fxValues <- matrix(fxValues,nrow = length(xrange2),ncol = length(xrange3) )
         dim(fxValues)
         head(fxValues)
```

1. 50
2. 50

0.1586738	0.1558219	0.1530125	0.1502456	0.1475211	0.1448390	0.1421994	0.1396023	0.1370476	0.1345353	...	0.07771249	0.07655874	0.0
0.1523526	0.1495756	0.1468410	0.1441488	0.1414991	0.1388919	0.1363271	0.1338048	0.1313249	0.1288875	...	0.07438395	0.07330501	0.0
0.1462184	0.1435162	0.1408564	0.1382391	0.1356642	0.1331318	0.1306418	0.1281943	0.1257893	0.1234267	...	0.07124241	0.07023829	0.0
0.1402713	0.1376438	0.1350589	0.1325164	0.1300163	0.1275587	0.1251436	0.1227709	0.1204406	0.1181528	...	0.06828788	0.06735858	0.0
0.1345111	0.1319585	0.1294484	0.1269807	0.1245554	0.1221726	0.1198323	0.1175344	0.1152790	0.1130660	...	0.06552036	0.06466587	0.0
0.1289379	0.1264602	0.1240248	0.1216319	0.1192815	0.1169735	0.1147080	0.1124850	0.1103043	0.1081662	...	0.06293984	0.06216018	0.0

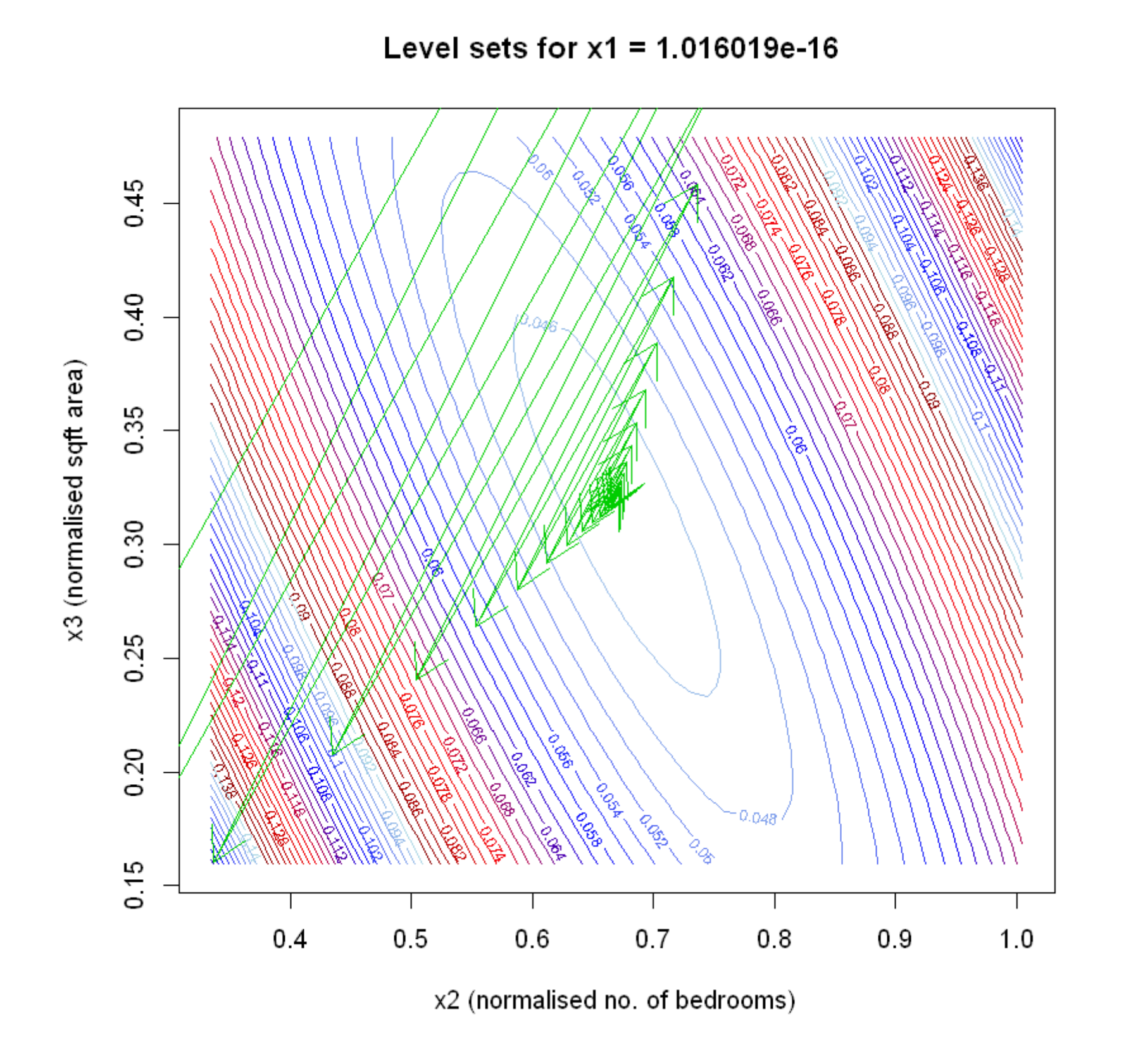
Gradient descent is for with x1 kept at its optimum values.

```
In [22]: #storing xs for ploting
         gradX <- list()

         x2 <- matrix(c(sol[1],0,0),nrow =ncol(A), byrow = TRUE)
         thres <- 10**(-2)
         converged<-FALSE
         maxi <- 100
         gradX[[1]] <- x2
         i=2
         alpha = 1
         gradientTrend <- c()
          while((!converged  && i <= maxi))
                 {
                     deltaf <- deltafx(y,A,x2)
                     if(is.infinite(deltaf)|| is.nan(norm(deltaf, type = "2"))){
                         break
                     }
                     x2 <- x2 - (alpha*deltaf)
                     # x1(inside column matrix x2) is changed to optimum value
                     x2[1] = sol[1]
                     gradX[[i]] <- x2
                     gradientTrend <- append(gradientTrend,norm(deltaf, type = "2"))
                     converged <- (norm(deltaf, type = "2") <= thres)
                     i <- i+1
                 }
         #creating matrix of solution of xs to plot direction of gradient
         mgradX <- do.call(cbind, gradX)
         tmgradX <- t(mgradX)
         head(tmgradX)

         xe2 <- tmgradX[,2]
         xe3 <- tmgradX[,3]
```

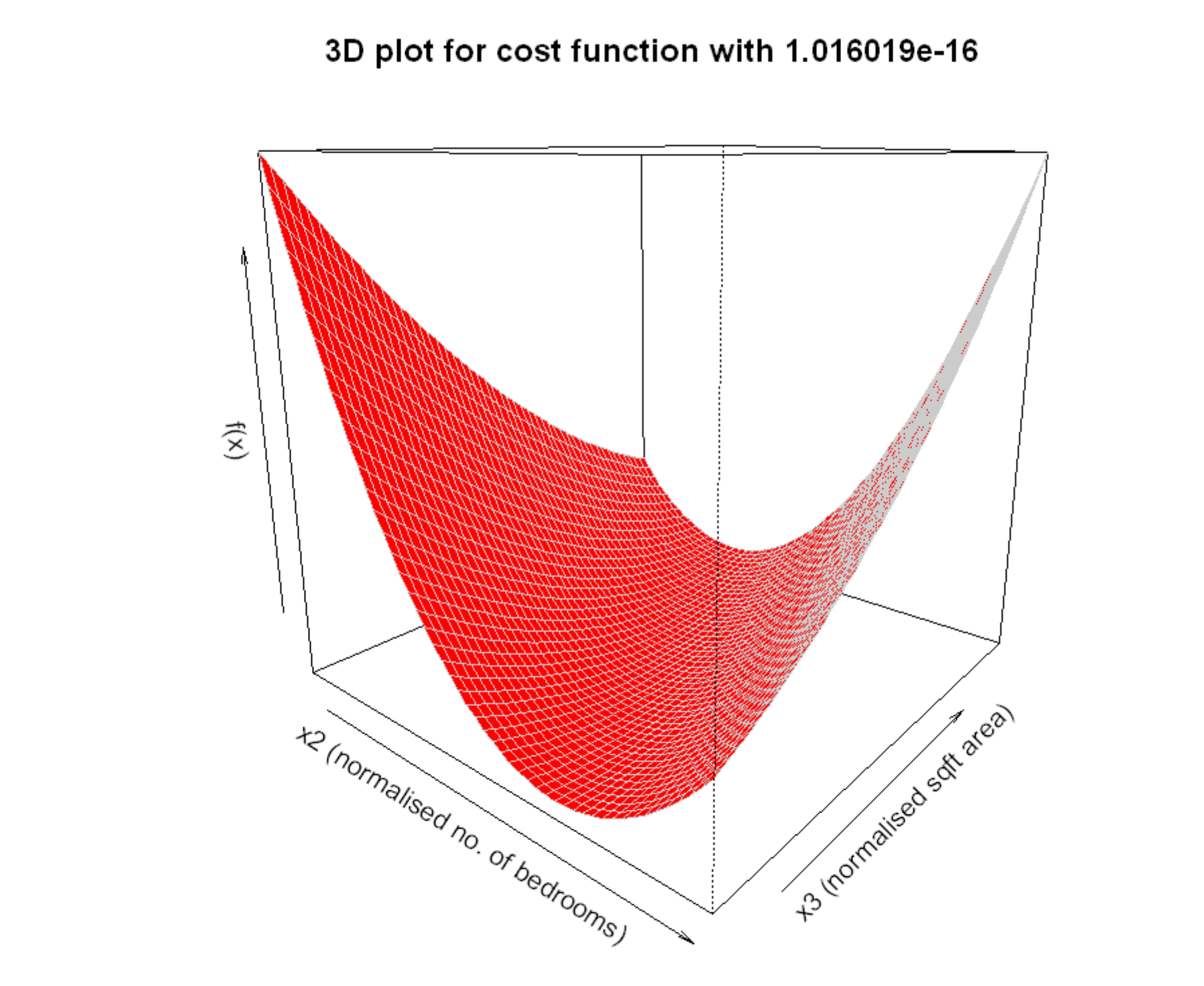
intercept	bd	sqft
1.016019e-16	0.0000000	0.00000000
1.016019e-16	0.9381574	0.88191285
1.016019e-16	0.1976374	0.09416576
1.016019e-16	0.8590888	0.71596169
1.016019e-16	0.3369824	0.16055765
1.016019e-16	0.8033412	0.59895719

Contour plot with Gradient Flow

```
In [23]: cols = rev(colorRampPalette(c('darkred','red','blue','lightblue'))(24))
         contour(xrange2,xrange3,z = fxValues,nlevels = 80, main = "Level sets for x1 = 1.016019e-16 ", xlab = "x2 (n
         s <- seq(length(xe2)-1)
         arrows(xe2[s], xe3[s], xe2[s+1], xe3[s+1], col = 3)
```




```
In [24]: persp(xrange2,xrange3,fxValues, theta = 40,main = "3D plot for cost function with 1.016019e-16", xlab = "x2
               col="red", border="grey80" )
```



As graph is not perfect cereal bowl shape to inspect the possible reason coorelation is calculated b/w "bd" and "sqft". Thus same thing will be present in their solution and range aroung it

```
In [123…: cor(c(A[,2]), c(A[,3]), use="complete.obs", method="pearson")
```

0.839674958214801

Conclusion

Level set is plotted for a range of x2 and x3 with x1 kept at optimum it is observed that gradient to flow perpendicular to level sets and as visibile in the plot gradient jumped around a lot before converging to minima. For even though not shown in code 3 iterartion were done one with x = [0,0,0], one with x started with = [x1,*0,0] and final one shown in code where x1 is kept x1* for complete gradient process

3D plot is not perfect cereal bowl shape possible reason for this high coorelation between two features of the dataset as indicated indicated by correlation factor 0.839674958214801

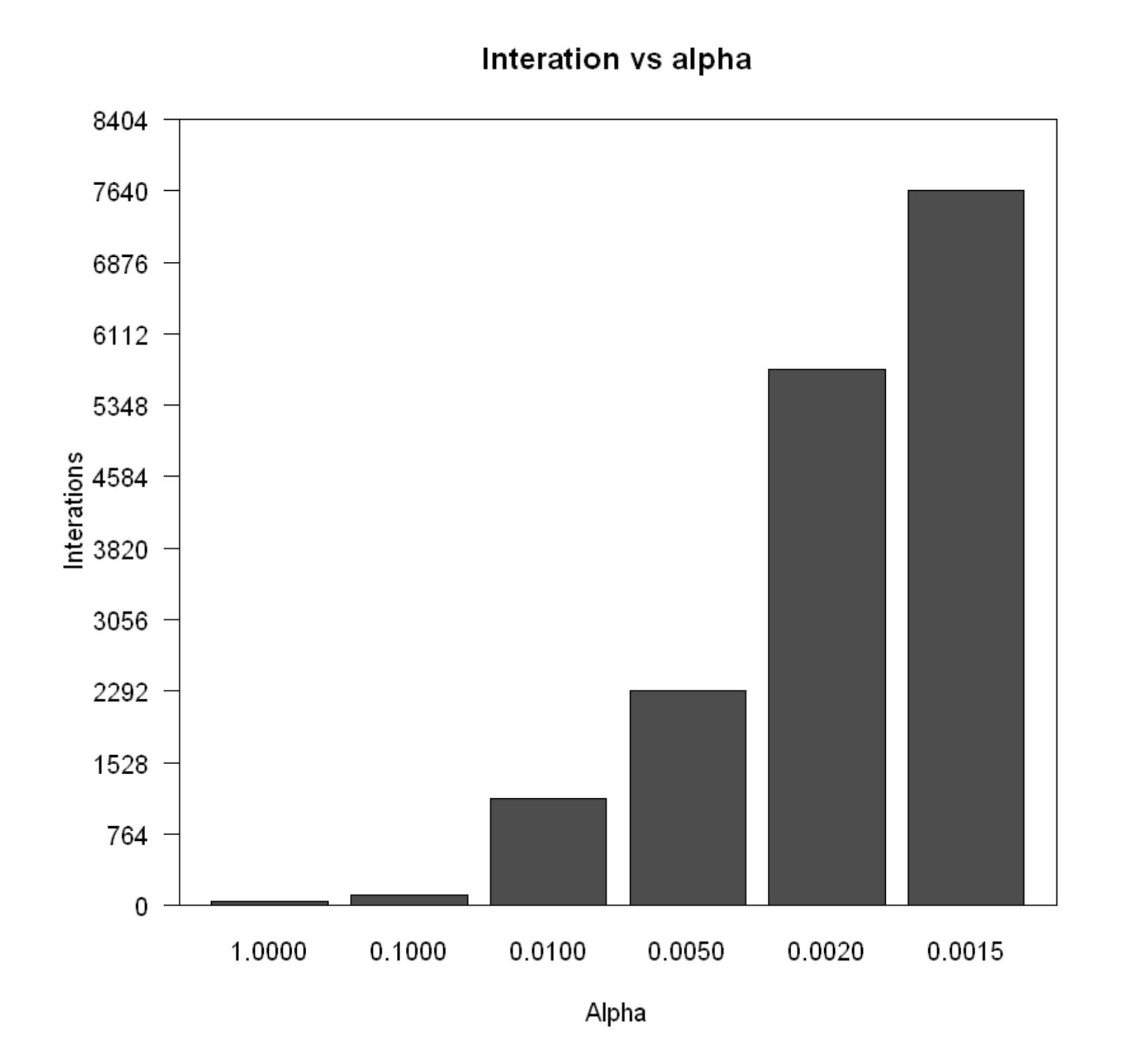
Question 4 -Repeat the previous question with different choise of the stepsize alpha in your implementation

Conclusion for part a)

Expanding on the analysis done for Question 2, starting with alpha = 10 we moved to alpha = 0.0001 and for higher alpha there was no convergence. Convergence started with alpha = 1 with minimum no. of iterations for alpha = 1.

For this problem any alpha greater than 1 will lead to divergence.

```
In [38]: barplot(iternations, main="Interation vs alpha", names.arg = successalphs,
                 ylab = "Interations", ylim = c(0,max(iternations)+max(iternations)%/%10), xlab = "Alpha", axes = FAL
         ylabel <- seq(0, max(iternations)+max(iternations)%/%10, by = max(iternations)%/%10)
         axis(2, at = ylabel, las = 1)
         box()
```



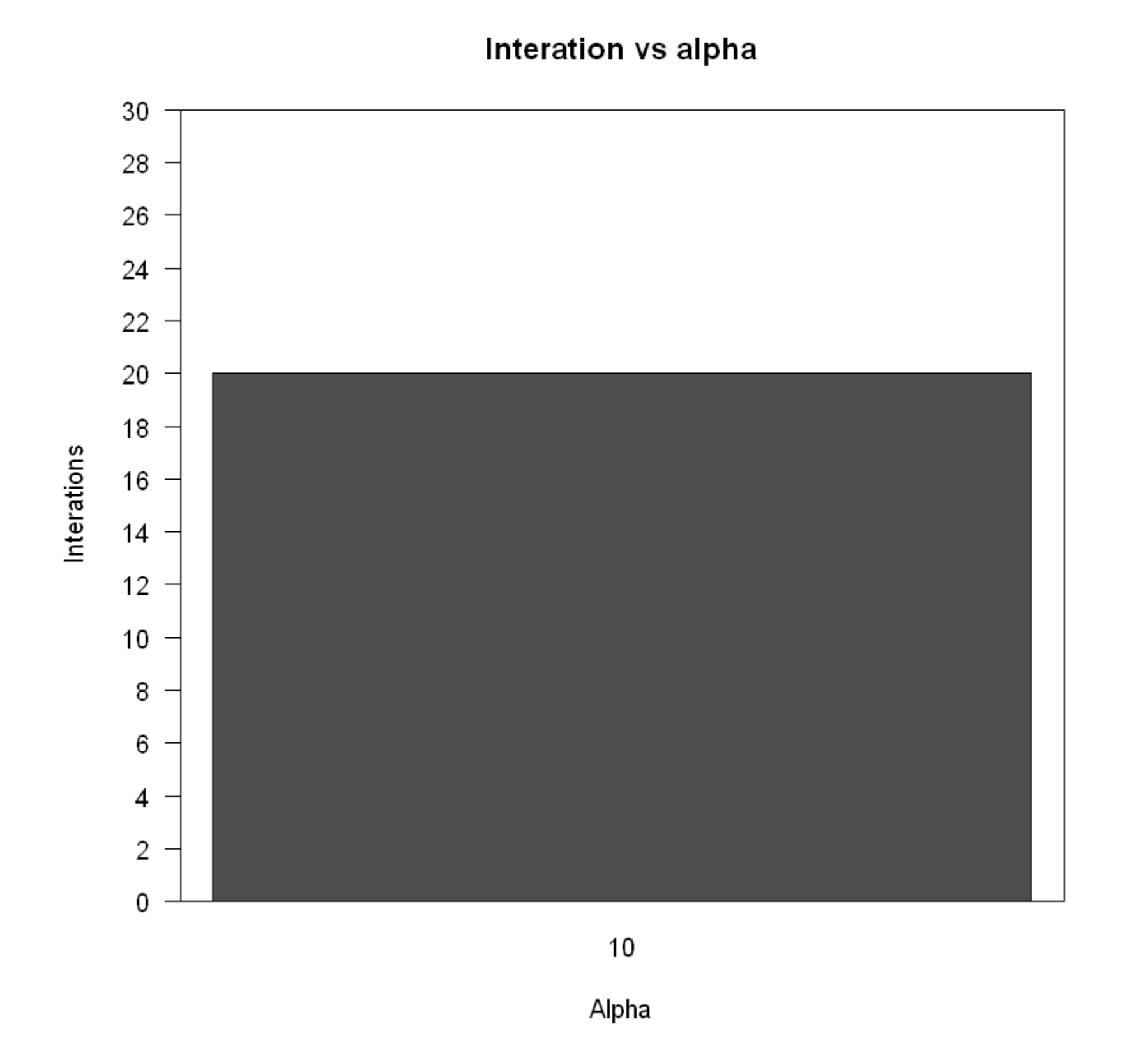

```
In [39]: gradientDescent2 <- function(y1,A1,x1,advalpha,thrs, maxi, option){
             converged<-FALSE
             i=1
             x1<-x1
             gradientTrend <- c()
             alpha <- advalpha
              while((!converged  && i <= maxi))
                     {
                         deltaf <- deltafx(y1,A1,x1)
                         if(is.infinite(deltaf)|| is.nan(norm(deltaf, type = "2"))){
                             break
                         }

                         #option for alpha
                         if (option == 1)
                             {
                             alpha <- advalpha/i
                         } else if (option == 2)
                             {
                             alpha <- advalpha/sqrt(i)
                         }
                         x1 <- x1 - (alpha*deltaf)
                         gradientTrend <- append(gradientTrend,norm(deltaf, type = "2"))
                         converged <- (norm(deltaf,type = "2") <= thrs)
                         i <- i+1
                     }

             return (list("x1"= x1,"gradientTrend" =gradientTrend, "iteration" = i-1,"converged" = converged, "alpha"
         }
```

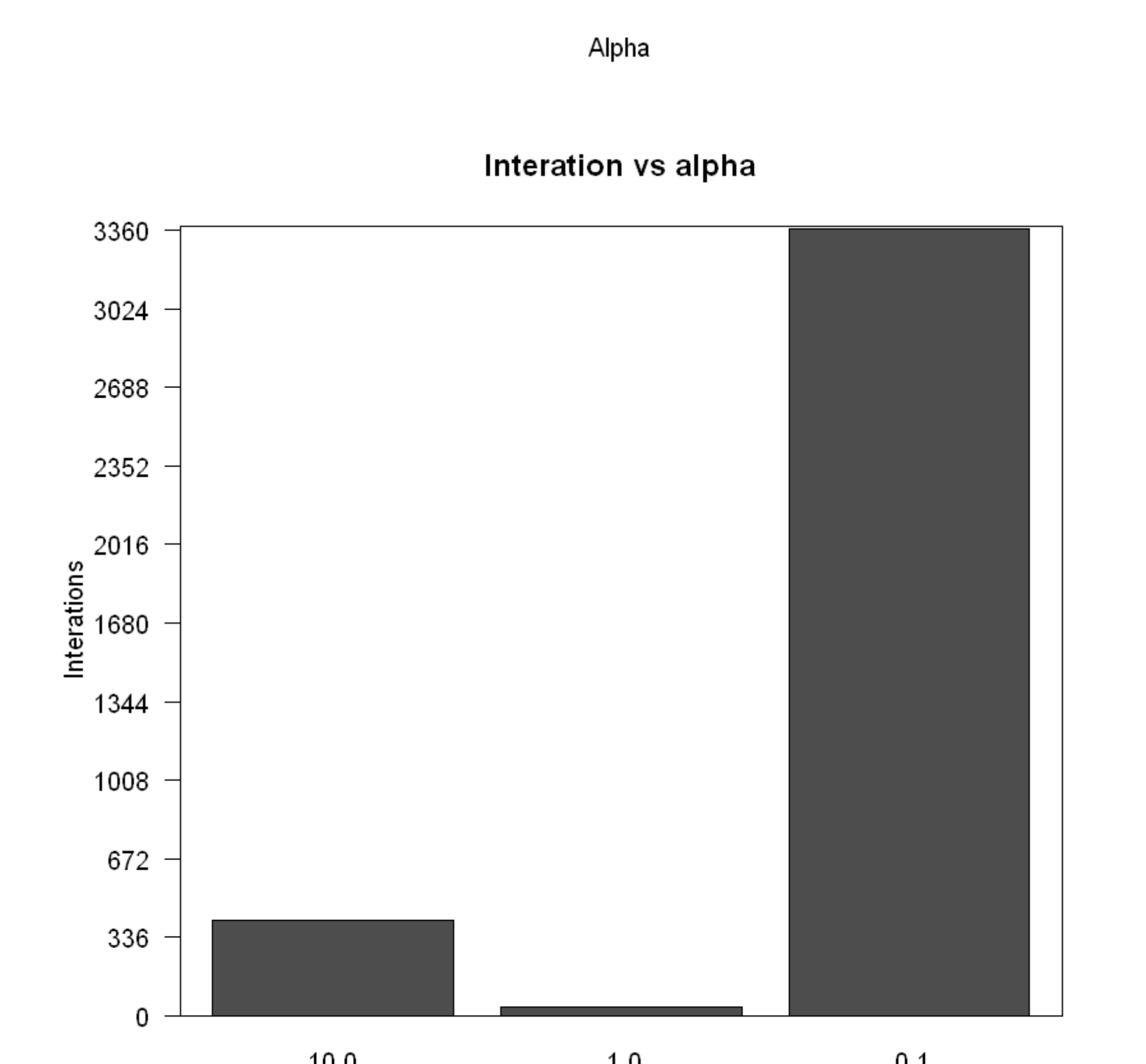
```
In [43]: gdwithadpstep <- function(alphas,opt){
             thres <- 10**(-2)
             maxiter <- 10000
             #considering first 1000 rows as complete data set takes lot of time this many iterations
             A3 <- A[1:1000,]
             y3 <- matrix(y[1:1000,],nrow=1000,byrow=TRUE)
             successalphs <- c()
             finalalphs <- list()
             iternations <- list()
             gradienttrend <- list()
             xs <- list()
             row = 1

             for (alpha in alphas)
                 {
                     x <- matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
                     answer <- gradientDescent2(y3,A3,x,alpha,thres,maxiter,opt)
                     if (answer$converged){
                         successalphs <- append(successalphs,alpha)
                         finalalphs[[row]] <- answer$alpha
                         xs[[row]] <- answer$x
                         iternations[[row]] <- answer$iteration
                         gradienttrend[[row]] <- answer$gradientTrend
                         }
                         row = row+1
                 }

             if (is.null(successalphs))
                 {
                 print("Convergence not achived")
             }else {
                 successalphs <- format(successalphs, scientific = FALSE)
                 xs <- do.call(cbind, xs)
                 iternations <- do.call(cbind, iternations)
```

```
        finalalphs <- do.call(cbind, finalalphs)
        colnames(xs)<- successalphs
        colnames(iternations)<- successalphs
        colnames(finalalphs)<- successalphs
        as <- apply(xs,2,function(x) sol -x)
        as <- format(as, scientific = FALSE)
        print(as)
        barplot(iternations, main="Interation vs alpha", names.arg = successalphs,
        ylab = "Interations", ylim = c(0,max(iternations)+10), xlab = "Alpha", axes = FALSE)
        ylabel <- seq(0, max(iternations)+10, by = max(iternations)%/%10)
        axis(2, at = ylabel, las = 1)
        box()
    }
}
```

In [45]:
```
alphas <- c(10,1,0.1,0.01,0.0010)
options <- c(1,2)
for (opt in options)
{
    gdwithadpstep(alphas,opt)
}
```

```
     10
[1,] " 0.009205546"
[2,] "-0.006607395"
[3,] " 0.016984225"
     10.0          1.0           0.1
[1,] " 0.01001283" " 0.01006655" " 0.01010885"
[2,] " 0.02197309" " 0.02391818" " 0.02543807"
[3,] "-0.01154990" "-0.01347169" "-0.01497336"
```





Conclusion for part b) and c)

Use both the method of adaptive step size for data(limited to 1000 rows) it can be concluded on the basis of graphs above (first is for alpha = a/i and second is for alpha = a/sqrt(i)) first method with work well for high starting learning rate and it will converge in less iterations but it won't provide convergence for lower starting learning rate as alpha will drop to zero very quickly.

Second method provide convergence for smaller learning rates giving chance to better solutions . Though both the approches have same similar difference from solution by solving linear equations.

Question 5 - Stochastic Gradient Method

In [16]:
```
stochasticGradientDes <- function(y1,A1,x1,advalpha,thrs, maxi, option){
    converged<-FALSE
    i=1
    x1<-x1
    gradientTrend <- c()
    alpha <- advalpha
     while((!converged  && i <= maxi))
            {
                index <- sample(nrow(A1),1)
                yi <- matrix(y1[index],nrow = 1,byrow = TRUE)
                Ai <- matrix(A1[index,],nrow = 1,byrow = TRUE)
                deltaf <- deltafx(yi,Ai,x1)
                if(is.infinite(deltaf)|| is.nan(norm(deltaf, type = "2"))){
                    break
                }

                #option for alpha
                if (option == 1)
                    {
                    alpha <- advalpha
                } else if (option == 2)
                    {
                    alpha <- advalpha/i
                } else if (option == 3)
                    {
                    alpha <- advalpha/sqrt(i)
                }

                x1 <- x1 - (alpha*deltaf)
                gradientTrend <- append(gradientTrend,norm(deltaf, type = "2"))
                converged <- (norm(deltaf, type = "2") <= thrs)
                i <- i+1
            }

    return (list("x1"= x1,"gradientTrend" =gradientTrend, "iteration" = i-1,"converged" = converged, "alpha"
}
```

In [23]:
```
# Stochastic Gradient on complete data for alpha = 1 and 0.1
alphas <- c(1,0.1,0.01)
thres <- 10**(-2)
maxiter <- 1000
A2 <- A
y2 <- y
successalphs <- c()
iternations <- list()
gradienttrend <- list()
xsfull <- list()
x <- matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
row = 1
for (alpha in alphas) {
    answer <- stochasticGradientDes(y,A,x,alpha,thres,maxiter,1)
        if (answer$converged){
        successalphs <- append(successalphs,alpha)
        xsfull[[row]] <- answer$x
        iternations[[row]] <- answer$iteration
        gradienttrend[[row]] <- answer$gradientTrend
        row = row+1
    }
    }
successalphs <- format(successalphs, scientific = FALSE)
xsfull <- do.call(cbind, xsfull)
iternations <- do.call(cbind, iternations)

colnames(xsfull)<- successalphs
colnames(iternations)<- successalphs
modifiedxs <- format(xsfull, scientific = FALSE)
modifiedxs
iternations
successalphs
```

0.10	0.01
0.026916195	0.004764189
0.538005611	0.521706534
0.415810632	0.405792914

0.10	0.01
64	217

1. '0.10'
2. '0.01'

Observation on SGD

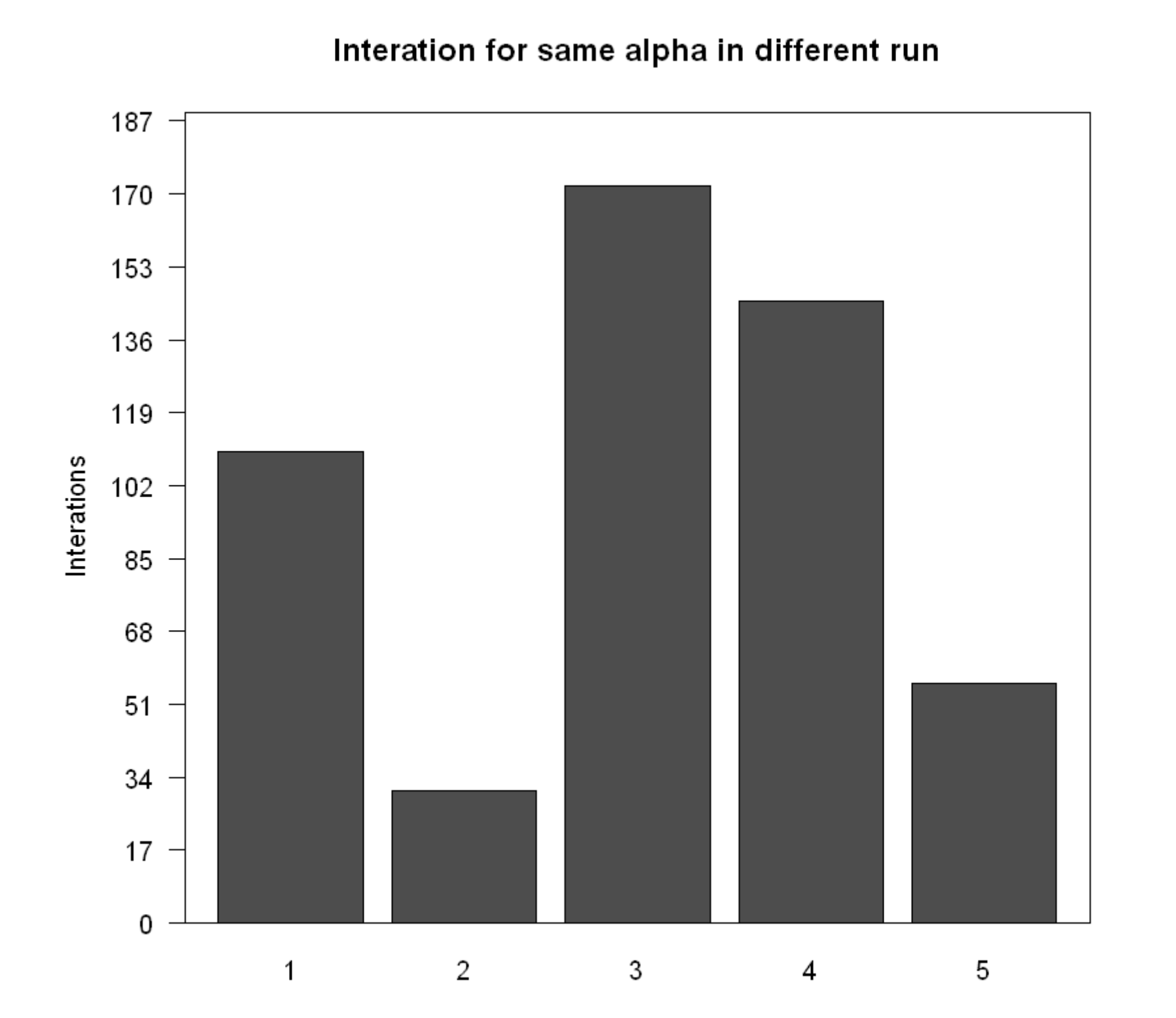
It is fast compare to normal GD, but it doesn't guarantee same solution in each iteration or as a matter of fact best solution. Above I ran SGD for three alphas out of which it converged for two but the interation and solution is different in each run, this will be shown in the next section.

In [38]:
```
alpha <- 0.1
thres <- 10**(-2)
maxiter <- 1000
A2 <- A
y2 <- y
Siternations <- list()
Sgradienttrend <- list()
Sxsfull <- list()
x <- matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
row = 1
for (i in seq(1,5,1)) {
    answer <- stochasticGradientDes(y,A,x,alpha,thres,maxiter,1)
        if (answer$converged){
        Sxsfull[[row]] <- answer$x
        Siternations[[row]] <- answer$iteration
        Sgradienttrend[[row]] <- answer$gradientTrend
        row = row+1
    }
    }
Sxsfull <- do.call(cbind, Sxsfull)
Siternations <- do.call(cbind, Siternations)

colnames(Sxsfull)<- seq(1,5,1)
colnames(Siternations)<- seq(1,5,1)
modifiedxs <- format(Sxsfull, scientific = FALSE)
modifiedxs
Siternations
```

1	2	3	4	5
-0.017651936	0.006640093	0.075477447	0.005380836	0.044466205
0.810941528	0.486160311	0.707340578	0.650536353	0.556324112
0.462324871	0.388821118	0.330708252	0.305325255	0.348899485

1	2	3	4	5
110	31	172	145	56

In [39]:
```
barplot(Siternations, main="Interation for same alpha in different run", names.arg = seq(1,5,1),
        ylab = "Interations", ylim = c(0,max(Siternations)+max(Siternations)%/%10), xlab = "", axes = FALSE)
ylabel <- seq(0, max(Siternations)+max(Siternations)%/%10, by = max(Siternations)%/%10)
axis(2, at = ylabel, las = 1)
box()
```

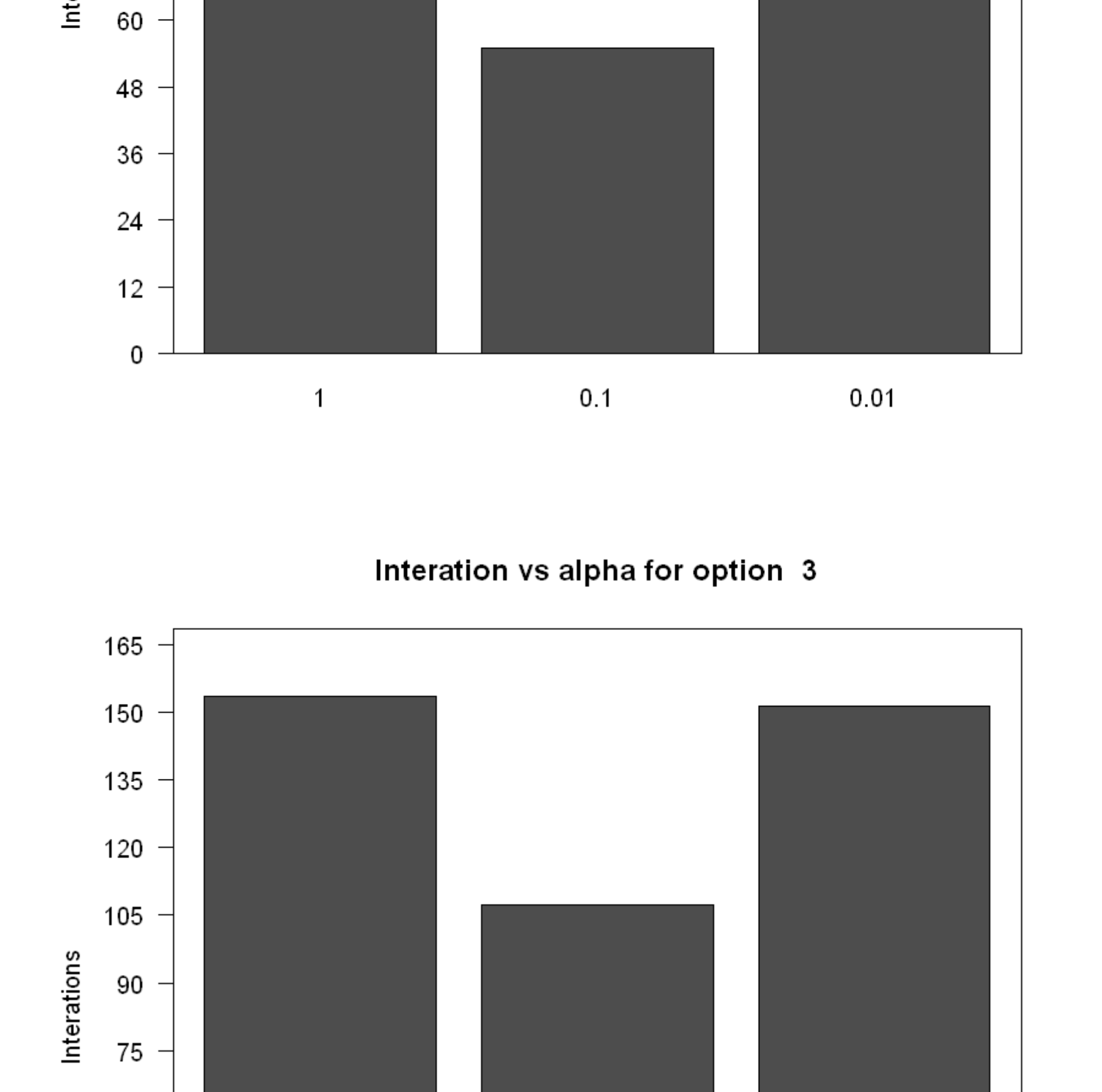


In [40]:
```
as <- apply(Sxsfull,2,function(x) sol -x)
as <- format(as, scientific = FALSE)
as
```

1	2	3	4	5
0.017651936	-0.006640093	-0.075477447	-0.005380836	-0.044466205
-0.140861953	0.183919264	-0.037261003	0.019543222	0.113755462
-0.143060974	-0.069557222	-0.011444355	0.013938642	-0.029635588

SGD for Adaptive Step Size

In [53]:
```
options <- c(2,3)
for (opt in options){
    alphas <- c(1,0.1,0.01)
    thres <- 10**(-2)
    Siternations <- list()
    Ssuccessalphs <- c()
    x <- matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
    row = 1
    for (alpha in alphas) {
        iterationcount <- c()
        for (i in seq(1,5,1))
            {
                answer <- stochasticGradientDes(y,A,x,alpha,thres,maxiter,opt)
                if (answer$converged){

                iterationcount = append(iterationcount,answer$iteration)

                }

        }
            Ssuccessalphs <- append(Ssuccessalphs,alpha)
            Siternations[[row]] <- mean(iterationcount)
            row = row+1
    }
    Siternations <- do.call(cbind, Siternations)
    colnames(Siternations)<- Ssuccessalphs

    barplot(Siternations, main= paste("Interation vs alpha for option ", opt,sep = " "), names.arg = Ssucces
        ylab = "Interations", ylim = c(0,max(Siternations)+max(Siternations)%/%10), xlab = "", axes = FALSE)
ylabel <- seq(0, max(Siternations)+max(Siternations)%/%10, by = max(Siternations)%/%10)
axis(2, at = ylabel, las = 1)
box()
}
```



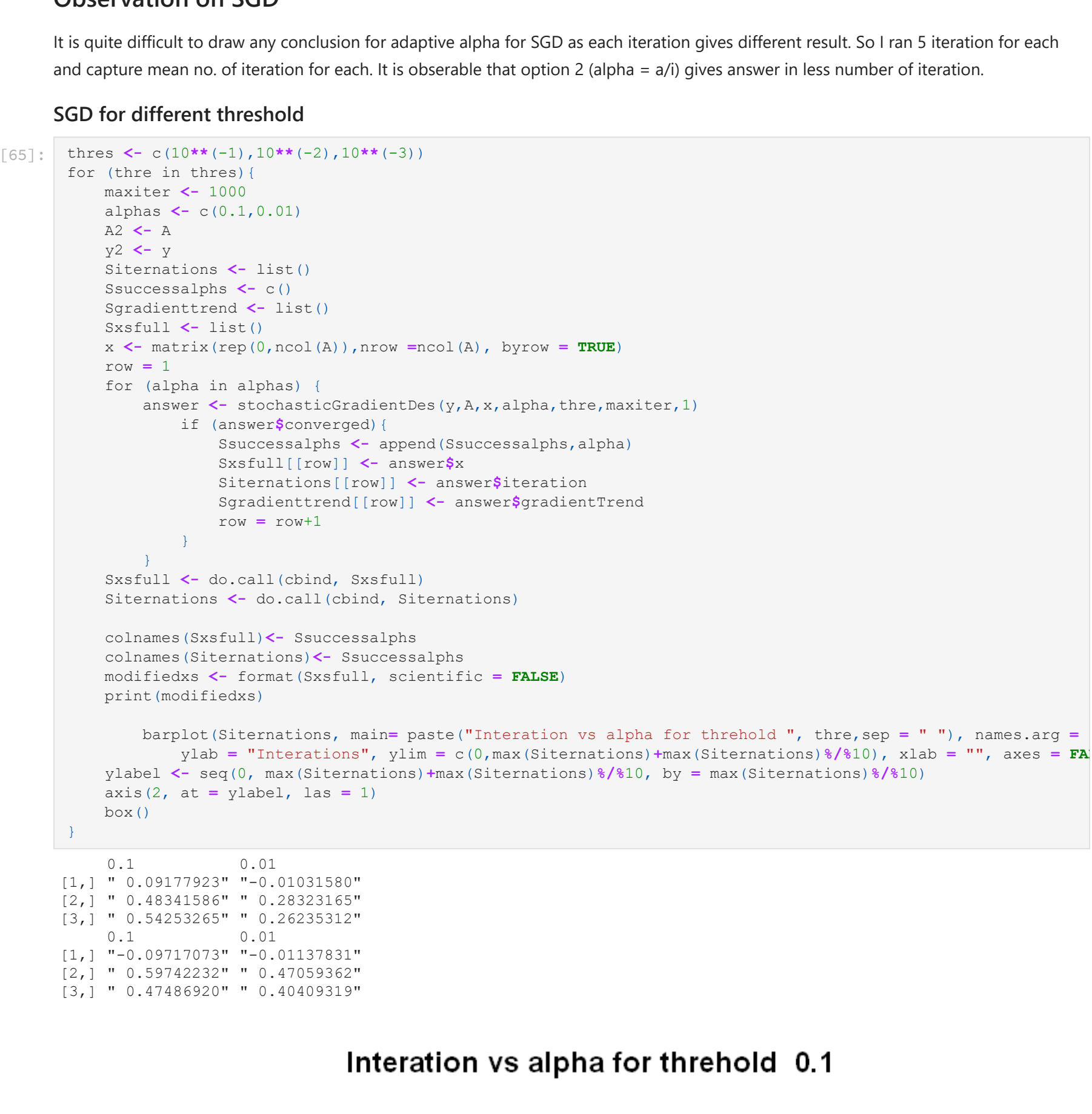
Observation on SGD

It is quite difficult to draw any conclusion for adaptive alpha for SGD as each iteration gives different result. So I ran 5 iteration for each and capture mean no. of iteration for each. It is obserable that option 2 (alpha = a/i) gives answer in less number of iteration.

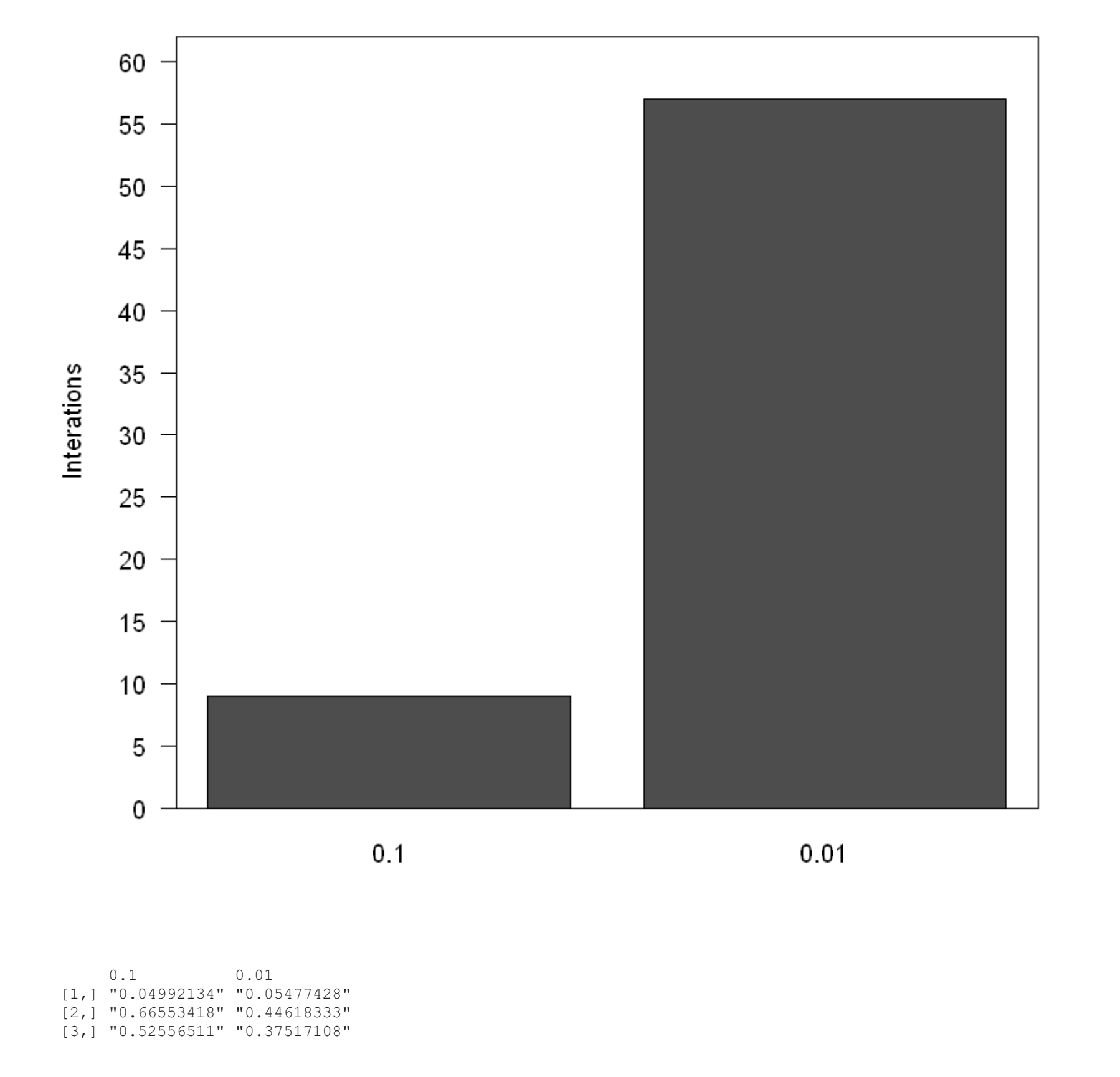
SGD for different threshold

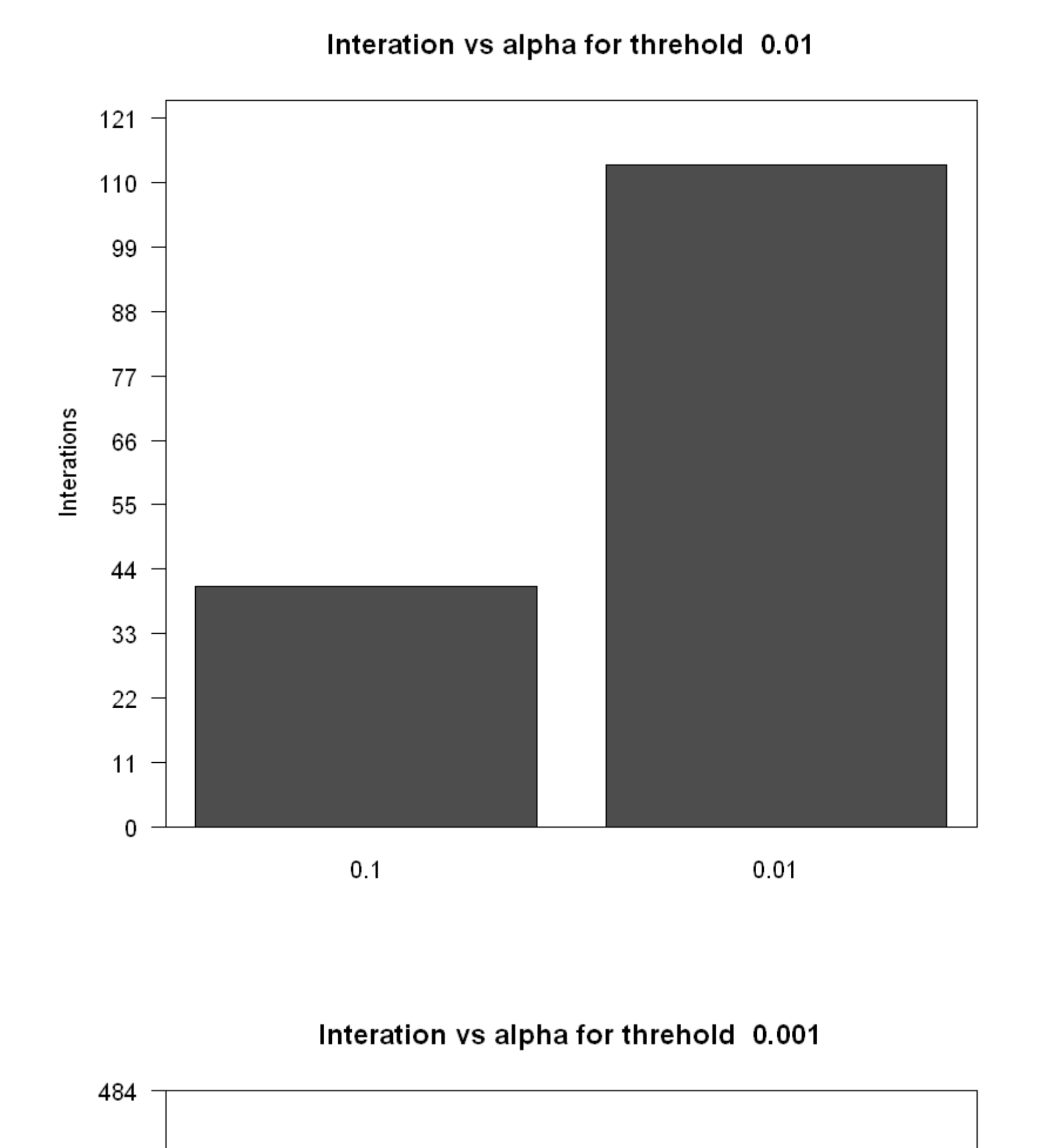
In [65]:
```
thres <- c(10**(-1),10**(-2),10**(-3))
for (thre in thres){
    maxiter <- 1000
    alphas <- c(0.1,0.01)
    A2 <- A
    y2 <- y
    Siternations <- list()
    Ssuccessalphs <- c()
    Sgradienttrend <- list()
    Sxsfull <- list()
    x <- matrix(rep(0,ncol(A)),nrow =ncol(A), byrow = TRUE)
    row = 1
    for (alpha in alphas) {
        answer <- stochasticGradientDes(y,A,x,alpha,thre,maxiter,1)
            if (answer$converged){
                Ssuccessalphs <- append(Ssuccessalphs,alpha)
                Sxsfull[[row]] <- answer$x
                Siternations[[row]] <- answer$iteration
                Sgradienttrend[[row]] <- answer$gradientTrend
                row = row+1
            }
        }
    Sxsfull <- do.call(cbind, Sxsfull)
    Siternations <- do.call(cbind, Siternations)

    colnames(Sxsfull)<- Ssuccessalphs
    colnames(Siternations)<- Ssuccessalphs
    modifiedxs <- format(Sxsfull, scientific = FALSE)
    print(modifiedxs)

        barplot(Siternations, main= paste("Interation vs alpha for threhold ", thre,sep = " "), names.arg =
            ylab = "Interations", ylim = c(0,max(Siternations)+max(Siternations)%/%10), xlab = "", axes = FA
    ylabel <- seq(0, max(Siternations)+max(Siternations)%/%10, by = max(Siternations)%/%10)
    axis(2, at = ylabel, las = 1)
    box()
}
```

```
     0.1           0.01
[1,] " 0.09177923" "-0.01031580"
[2,] " 0.48341586" " 0.28323165"
[3,] " 0.54253265" " 0.26235312"
     0.1           0.01
[1,] "-0.09717073" "-0.01137831"
[2,] " 0.59742232" " 0.47059362"
[3,] " 0.47486920" " 0.40409319"
```




```
     0.1          0.01
[1,] "0.04992134" "0.05477428"
[2,] "0.66553418" "0.44618333"
[3,] "0.52556511" "0.37517108"
```





Observation on SGD

as threshold decreases no. of iteration increases which is resonable as are aiming for more accurate solution

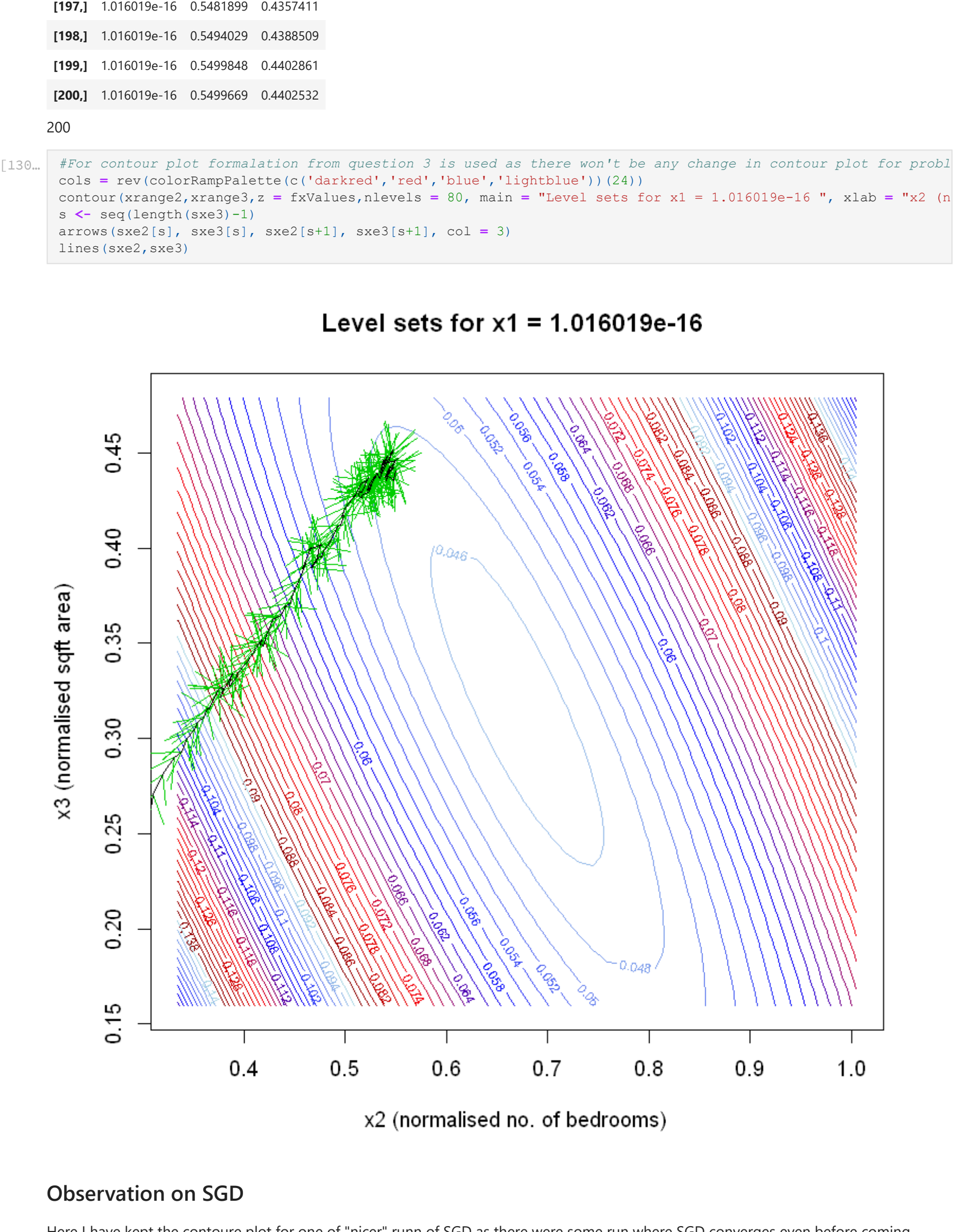
Contour Plot for the SGD

In [129…
```
#storing xs for ploting stochastic gradient flow
SgradX <- list()

sx <- matrix(c(sol[1],0,0),nrow =ncol(A), byrow = TRUE)
thres <- 10**(-2)
converged<-FALSE
maxi <- 1000
SgradX[[1]] <- sx
i=2
alpha = 0.01
gradientTrend <- c()
 while((!converged  && i <= maxi))
        {
            index <- sample(nrow(A),1)
            yi <- matrix(y[index],nrow = 1,byrow = TRUE)
            Ai <- matrix(A[index,],nrow = 1,byrow = TRUE)
            deltaf <- deltafx(yi,Ai,sx)
            if(is.infinite(deltaf)|| is.nan(norm(deltaf, type = "2"))){
                break
            }
            sx <- sx - (alpha*deltaf)
            # x1(inside column matrix x2) is changed to optimum value
            sx[1] = sol[1]
            SgradX[[i]] <- sx
            gradientTrend <- append(gradientTrend,norm(deltaf, type = "2"))
            converged <- (norm(deltaf, type = "2") <= thres)
            i <- i+1
        }
#creating matrix of solution of xs to plot direction of gradient
mSgradX <- do.call(cbind, SgradX)
tmSgradX <- t(mSgradX)
tail(tmSgradX)
nrow(tmSgradX)

sxe2 <- tmSgradX[,2]
sxe3 <- tmSgradX[,3]
```

[195,]	1.016019e-16	0.5448748	0.4371755
[196,]	1.016019e-16	0.5478437	0.4353875
[197,]	1.016019e-16	0.5481899	0.4357411
[198,]	1.016019e-16	0.5494029	0.4388509
[199,]	1.016019e-16	0.5499848	0.4402861
[200,]	1.016019e-16	0.5499669	0.4402532

200

In [130…
```
#For contour plot formalation from question 3 is used as there won't be any change in contour plot for probl
cols = rev(colorRampPalette(c('darkred','red','blue','lightblue'))(24))
contour(xrange2,xrange3,z = fxValues,nlevels = 80, main = "Level sets for x1 = 1.016019e-16 ", xlab = "x2 (n
s <- seq(length(sxe3)-1)
arrows(sxe2[s], sxe3[s], sxe2[s+1], sxe3[s+1], col = 3)
lines(sxe2,sxe3)
```



Observation on SGD

Here I have kept the contoure plot for one of "nicer" runn of SGD as there were some run where SGD converges even before coming anywhere near the optimal solution.

Gradient flow for SGD is random, though it has aim for value less than thresold but since with every itreration we are using differernt value of gradient it may converges to some what sub optimal solution. This may be good for problems where even a sub optimal solution will be good enough

In [ ]:

Github link for the code

https://github.com/jainsanyam786/OptimizationForMachineLearning.git

In [ ]: