Number Link Puzzle

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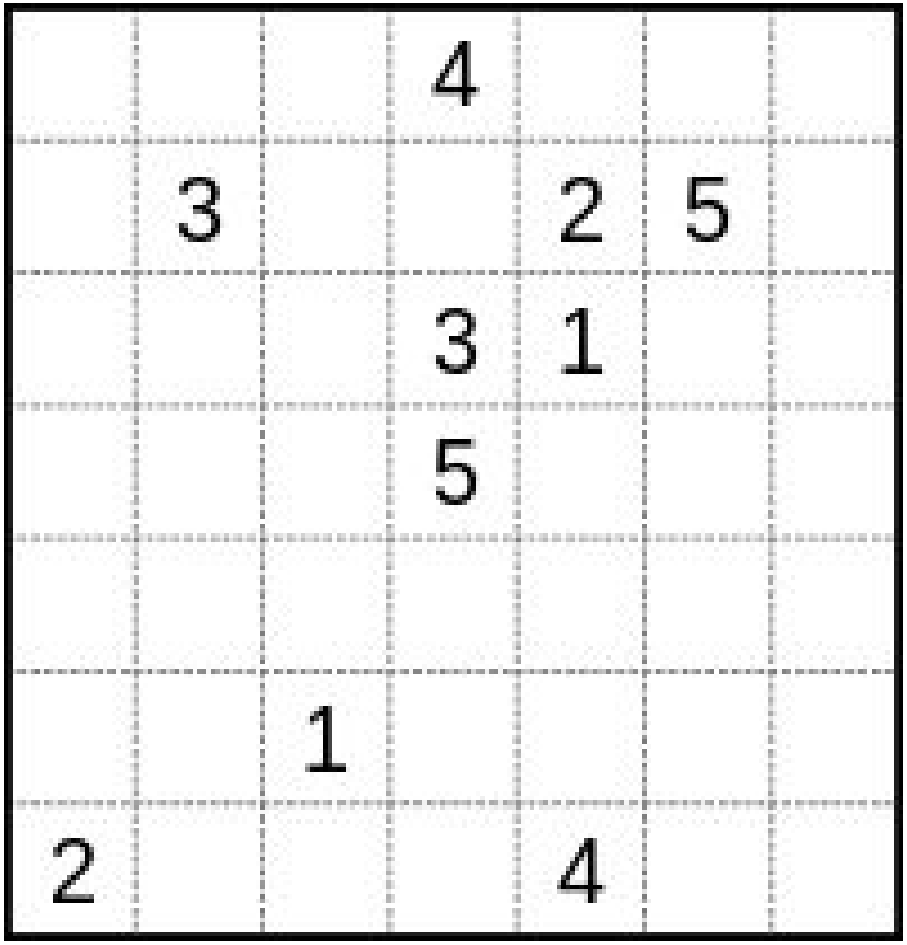
Solving the NUmber LInk Problem using LogiC Programming

The Problem

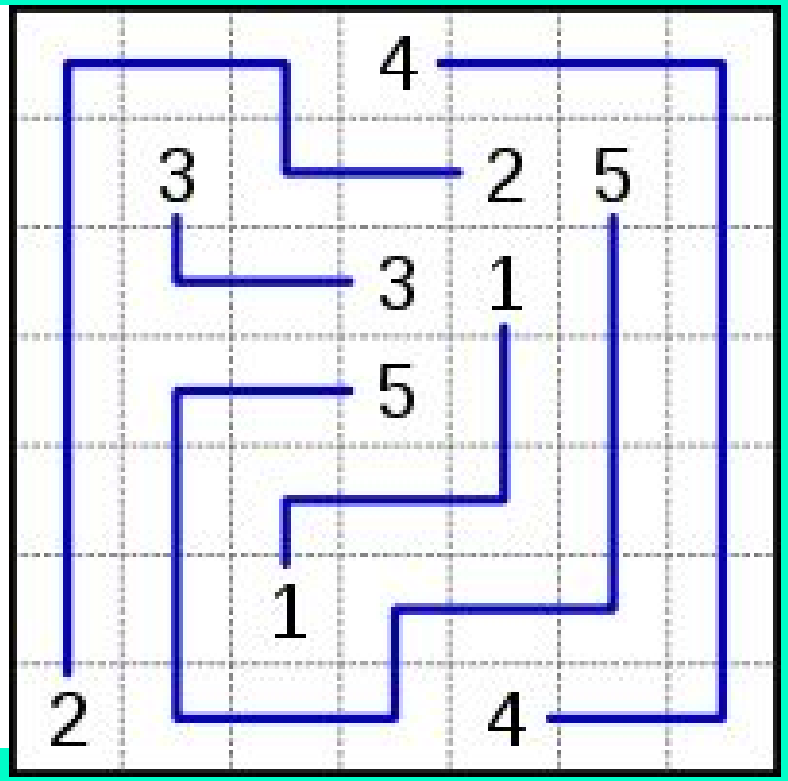
Rules for the game

1. Connect pairs of the same numbers with a continuous line.
2. Lines go through the center of the cells, horizontally, vertically, or changing direction, and never twice through the same cell.
3. Lines cannot cross, branch off, or go through the cells with numbers.



The problem was first published in 1897, and recently popularized by Nikoli , a Japanese puzzle magazine

- Numberlink is known to be NP-complete
- studied as a model of VLSI layout designTable salt
- Considered a good puzzle, if it has exactly 1 solution
- Several SAT based constraint solvers are proposed


4
3
2
5
3
1
5
1
2
4

How To solve

Plan for Project

Use IDP to create a Solver for NumberLink

Use zero-suppressed binary decision diagram to create a logic based algorithm

Comparison between IDP and Picat based solution

- a zero-suppressed binary decision diagram (ZDD) ,is a compressed data structure for representing and manipulating families of sets.

Picat SOLUTION

https://github.com/hakank/hakank/blob/master/picat/numberlink.pi

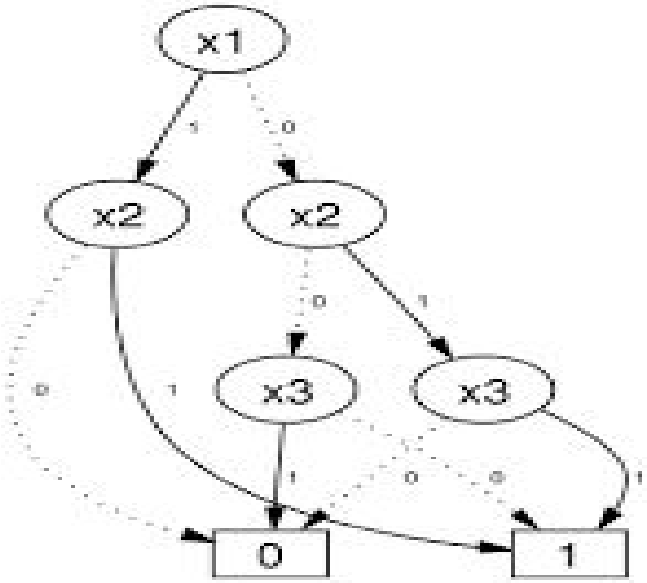
To solve essentially we fill the boxes in our “path ” with the number we are connecting.

"each end node has one connected neighbor and each interior node has two connected neighbors"

Zero Suppressed Decision diagrams

any directed acyclic graph such that it has :

1) A root node with 0 indegree
2) 2 terminal nodes, 1 and 0 with 0 outdegree
3) Each node has at most 2 children, and 2 edges marked 0 and 1
4) Each node is labeled ,1 each from the universal set
5) Parent’s label is strongly less than children



Zero Suppressed Decision diagrams

ZDD can be used to efficiently compute the basic operations on sets.

Set of operations to reach a node is called Path.

We represent Numberlink Puzzle as an undirected graph G = (V,E) where G is the grid, v are boxes and E are connected boxes.

Path Matching - A path that can be generated by adding simple paths(no duplicated nodes) at each step.

i.e no cycles, and only 2 outdegree along the way

Algorithm

```
mate n (v) = v ; if v has degree 0

mate n (v) = 0 ; if v has degree 2
```

Algorithm

Algorithm 2: Numberlink solver

Data: A graph $G = (V, E)$ and a pair matching h over V

begin
  create a root node and two terminal nodes **0** and **1**;
  let $N_1 \leftarrow \{n_{\text{root}}\}$ and $N_i \leftarrow \emptyset$ for $i = 2, \ldots, |E|$;
  foreach $i = 1, \ldots, |E|$ **do**
    foreach $n \in N_i$ **do**
      if $(\text{mate}_n, i, 0)$ *is incompatible with* h **then** let the 0-child of n be **0**;
      else let the 0-child of n be $\text{GN}(i+1, \text{mate}_n|_{V \geq e_{i+1}})$;
      if $(\text{mate}_n, i, 1)$ *is incompatible with* h **then** let the 1-child of n be **0**;
      else let the 1-child of n be $\text{GN}(i+1, \text{MU}(\text{mate}_n, e_i)|_{V \geq e_{i+1}})$;
    end
  end
  return *the constructed diagram*;
end

References

ww.mdpi.com/1999-4893/5/2/176/pdf

https://en.wikipedia.org/wiki/Numberlink

http://www.geeksforgeeks.org/a-number-link-game/

http://www.nikoli.co.jp/en/puzzles/numberlink.html