18.600 Recitation 7

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Partial solutions available at math.mit.edu/~visheshj
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Problem 1. Two students Alice and Bob are meeting to work on their 18.600 homework. Alice arrives at a uniform time between 11:15 and 11:45, while Bob arrives independently at a uniform time between 11:00 and 12:00.

(a) What is the probability that the one who arrives first waits for less than 5 minutes on the other person?

(b) What is the probability that Bob arrives first?

Problem 2. Alice and Bob bring their car to service. The number of days is takes to service Bob's car is an exponential random variable of parameter $1/2$, while the number of days it takes to service Alice's car (which is older) is an independent exponential random variable of parameter $1/5$.

(a) Alice and Bob bring their car in for service at the same time. What is the chance that Alice's car is ready before Bob's car?

(b) Ten days have passed, and the service of Bob's car is still not finished. What is the expected number of further days he'll have to wait until it gets finished?

(c) Assume now that there is only one person at work at the service station, and that she first works on Bob's car and then works on Alice's car. Let T be the time it takes until both cars are ready. What is the probability density function of T?

Problem 3. The joint probability density function of X and Y is the following for some constant a:
$$f(x, y) = a(x^2 + xy/2), \qquad 0 < x < 1,\ 0 < y < 2.$$
(a) Find a.
(b) Find the density function of X.
(c) Find $E[X]$.
(d) Are X and Y independent?
(e) Find $P[Y > 1/2; X < 1/2]$.
(f) Find $P[Y > 1/2 \mid X < 1/2]$.

Problem 4. Let X and Y be random variables with joint distribution $f_{X,Y}(x, y) = \frac{1}{2}$ for (x, y) in the triangle formed by the line $y = 2 - x$ and the x and y axes, and $f_{X,Y}(x, y) = 0$ outside this triangle. Find the distribution of the sum, $f_{X+Y}(a)$.