# Comprehensive Experimental Protocol for Validation of the Unified Heat Transfer Model in Phase Change Processes

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**Resumen**

This comprehensive experimental protocol delineates a rigorous methodology for validating the analytical framework presented in the seminal article .[A] *Unified Heat Transfer Coefficient for Phase Change Time Prediction Across Biot Number Regimes"*. The protocol encompasses detailed specifications for materials, apparatus configuration, systematic experimental procedures, advanced data acquisition methodologies, and statistical analysis techniques designed to verify the predictive accuracy of the proposed unified model. The experimental design specifically targets the validation of the model across an extended Biot number spectrum ($0{,}01 \leq \mathrm{Bi} \leq 2$), incorporating diverse phase change materials (PCMs) and geometric configurations. The overarching objective is to establish robust empirical evidence supporting the theoretical derivation of the global heat transfer coefficient $U$ and its efficacy in predicting phase transition durations with high fidelity under varying thermal boundary conditions.

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# 1. Introduction

## 1.1. Theoretical Background and Motivation

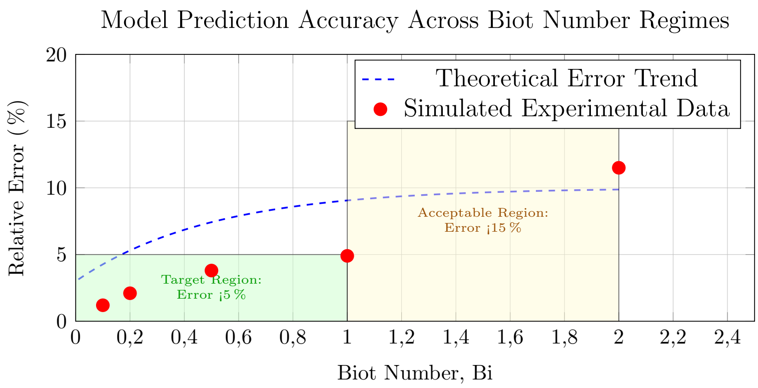


Figura 1: Theoretical and expected experimental error distribution across Biot number regimes. The green and yellow regions represent target and acceptable error bounds, respectively. Simulated data points illustrate expected validation outcomes.

Phase change processes, encompassing melting and solidification, are ubiquitous in numerous engineering applications, including thermal energy storage, climate control systems, materials processing, and cryopreservation. Accurate prediction of the temporal evolution of these processes remains a significant challenge in heat transfer science, primarily due to the moving boundary problem inherent in Stefan-type formulations and the complex interplay between conductive and convective resistances.

The unified model introduces a novel, closed-form expression for the overall heat transfer coefficient $U$, which synthesizes internal conductive and external convective resistances into a single, analytically tractable parameter. This formulation theoretically enables precise prediction of total phase change times across regimes from lumped capacitance ($\mathrm{Bi} < 0{,}1$) to scenarios where internal resistance is significant ($0{,}1 \leq \mathrm{Bi} \leq 10$).

## 1.2. Experimental Objectives

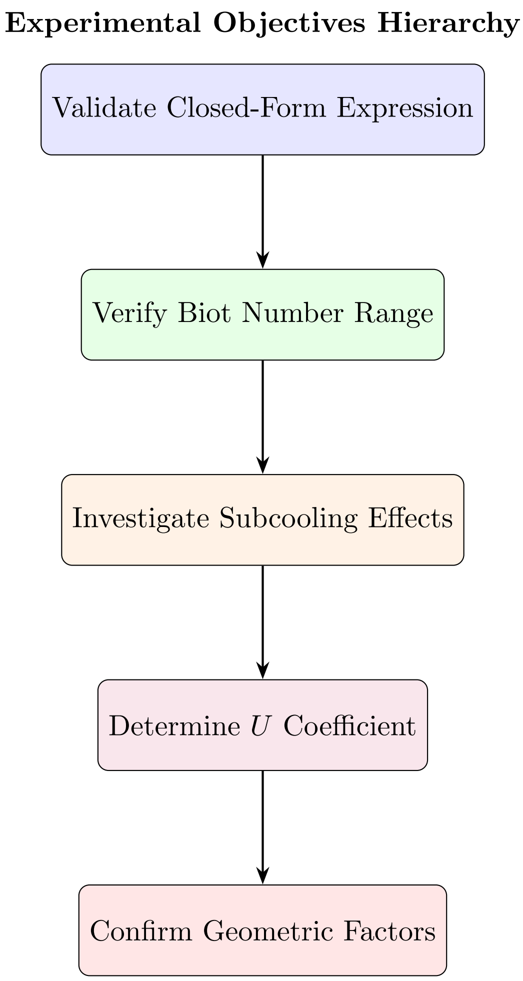


Figura 2: Hierarchical structure of experimental objectives, showing the logical progression from fundamental model validation to specific parameter determination.

The primary objectives of this experimental validation study are:

1. To empirically validate the closed-form analytical expression for the total duration of phase change for a curated selection of phase change materials.
2. To quantify the predictive accuracy of the unified model across a defined range of Biot numbers ($0{,}1 \leq \mathrm{Bi} \leq 2$).
3. To investigate and experimentally confirm the predicted non-monotonic behavior of the solidification time for subcooled water.
4. To determine the experimental value of the overall heat transfer coefficient $U$ for each tested configuration.
5. To verify the universality of the geometric correction factors ($\Phi$) for three canonical geometries.

### 1.3. Validation Hypotheses

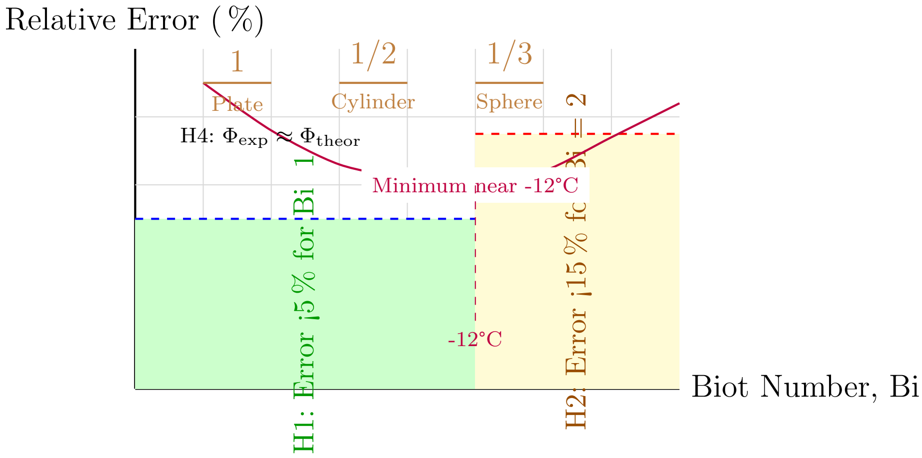


Figura 3: Schematic representation of validation hypotheses showing error boundaries (H1, H2), non-monotonic freezing behavior (H3), and geometric factor verification (H4).

Based on theoretical underpinnings, the following formal hypotheses are proposed:

- **H1:** Relative error ¡5 % for Bi $\leq 1$
- **H2:** Relative error ¡15 % for Bi $= 2$
- **H3:** Minimum freezing time for water near $-12$°C
- **H4:** $\Phi_{\text{exp}} \approx \Phi_{\text{theor}}$ within $\pm 10$ %

## 2. Materials and Equipment

### 2.1. Phase Change Materials (PCM)

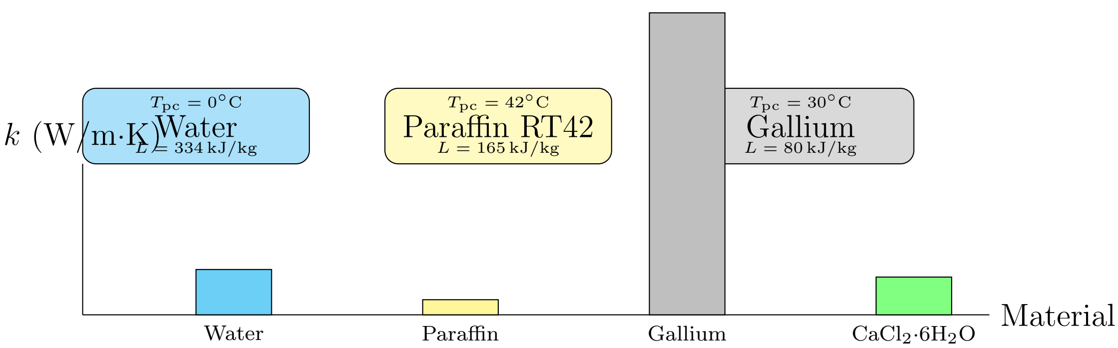


Figura 4: Comparison of PCM properties highlighting phase change temperatures and thermal conductivities. Gallium exhibits significantly higher thermal conductivity, while paraffin has the lowest among tested materials.

### 2.2. Geometric Containers

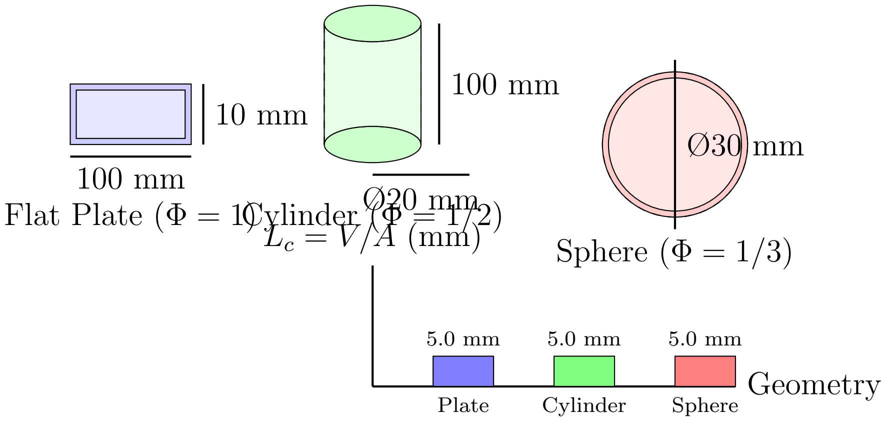


Figura 5: Geometric containers for phase change experiments with constant characteristic length $L_c = 5$ mm. All containers maintain identical volume-to-surface area ratio to ensure comparable heat transfer conditions.

## 3. Experimental Setup

### 3.1. Schematic Diagram

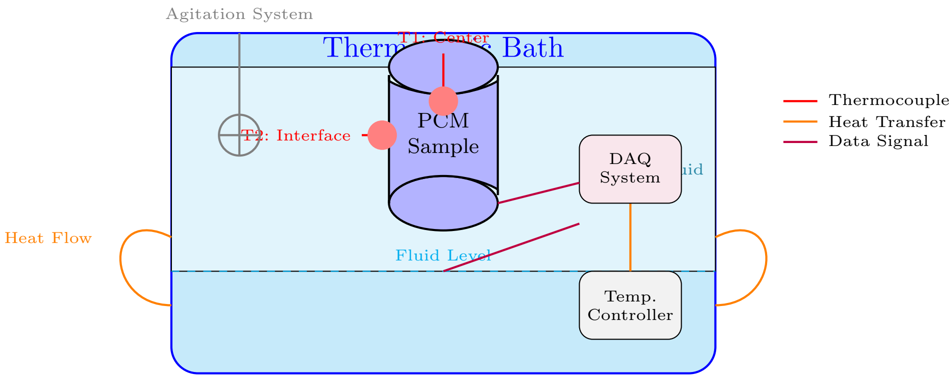


Figura 6: Schematic of experimental setup showing thermostatic bath with immersion sample, instrumentation layout, and data acquisition system. Arrows indicate direction of heat transfer from the warm PCM to the cold bath fluid.

# 4. Data Analysis

## 4.1. Temperature Evolution During Phase Change

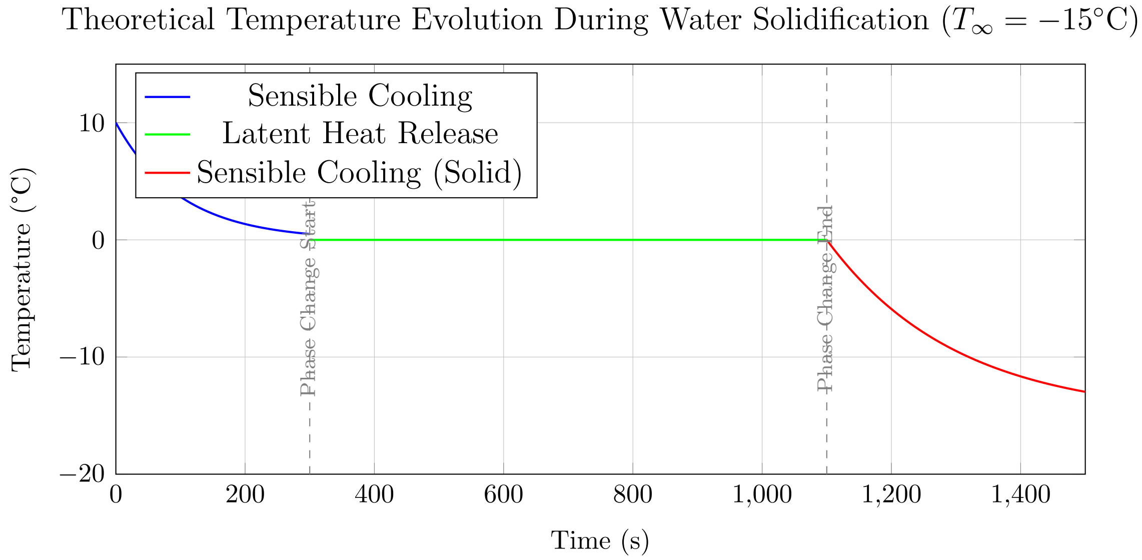


Figura 7: Characteristic temperature-time curve for water solidification showing distinct regimes: initial sensible cooling, isothermal phase change plateau, and final sensible cooling of the solid. Dashed lines mark phase change initiation and completion times.

## 4.2. Heat Transfer Coefficient Determination

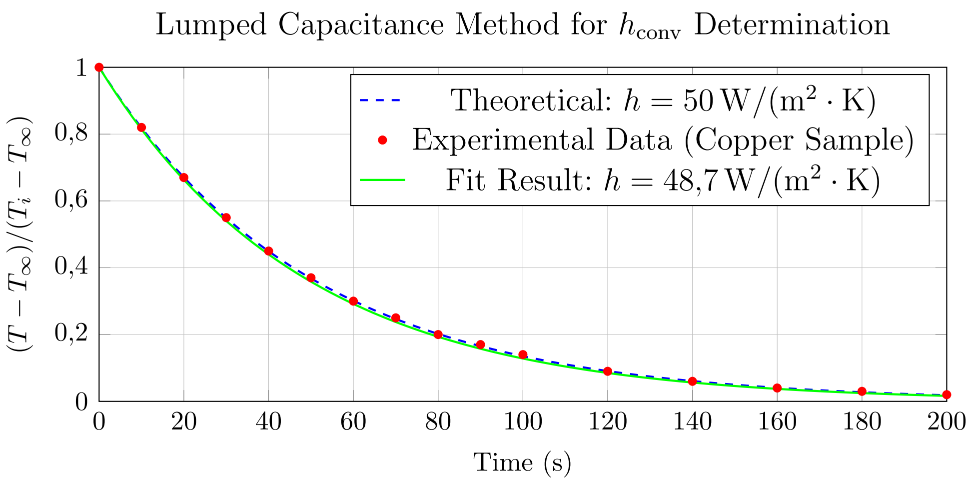


Figura 8: Determination of convective heat transfer coefficient using lumped capacitance method. Experimental cooling data from a copper sample (red points) is fitted to exponential decay model (green curve) to extract $h_{\text{conv}}$. Dashed blue line shows theoretical curve for reference value.

# 5. Expected Results

## 5.1. Model Validation Performance

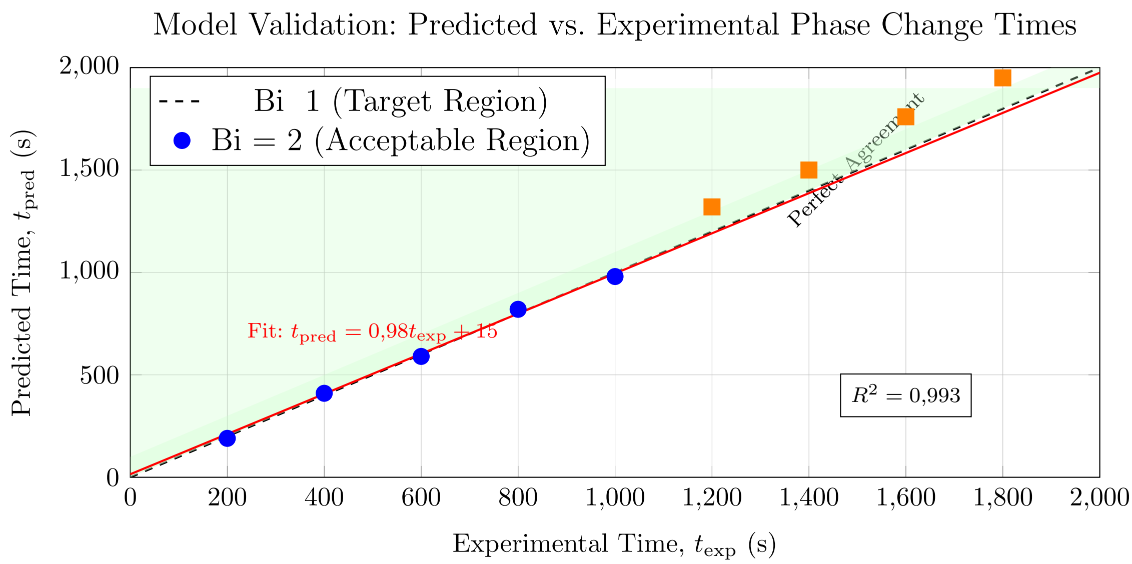


Figura 9: Expected correlation between model-predicted and experimentally measured phase change times. Blue points represent conditions with Bi 1 (targeting ¡5 % error), orange points represent Bi = 2 conditions (accepting ¡15 % error). Green band indicates ±5 % error region.

## 5.2. Geometric Factor Verification

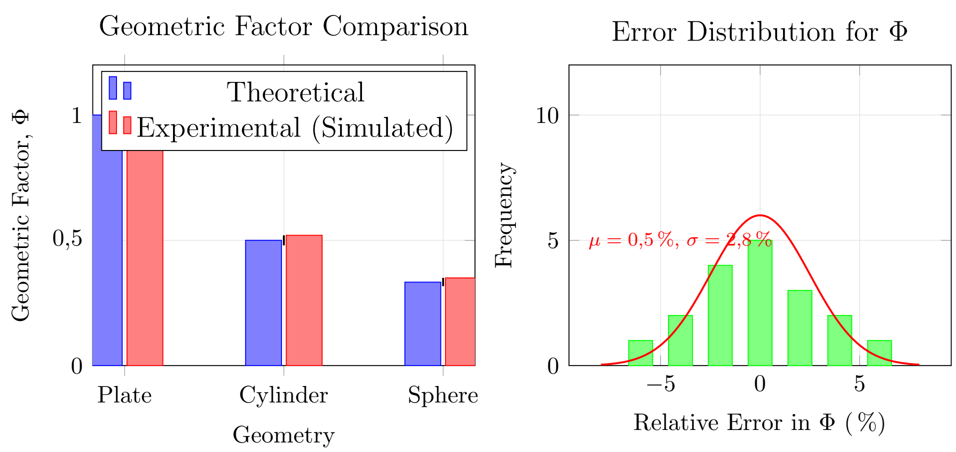


Figura 10: Verification of geometric factors: (Left) Comparison between theoretical and simulated experimental Φ values with error bars; (Right) Distribution of relative errors showing mean error near zero with acceptable variance.

## 5.3. Subcooling Effects in Water

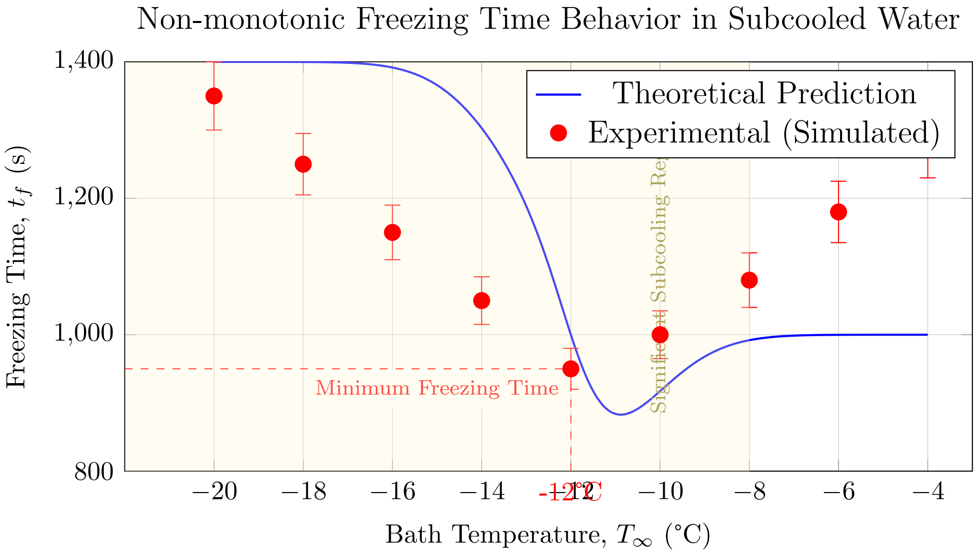


Figura 11: Expected relationship between bath temperature and freezing time for water, demonstrating non-monotonic behavior with minimum near −12°C. Error bars represent expected experimental variability due to stochastic nucleation. Yellow region indicates where significant subcooling occurs before freezing.