

# Linear Algebra

Definition of a vector space (over  $\mathbb{R}$  or  $\mathbb{C}$ ), subspaces, the space spanned by a subset. Linear independence, bases, dimension, Direct sums and complementary subspaces. Quotient spaces

Linear maps, isomorphisms. Relation between rank and nullity. The space of linear maps from  $U$  to  $V$ , Representation by matrices. Change of Basis. Row Rank and Column Rank

Determinant and trace of a square matrix. Determinant of the product of two matrices and of the inverse matrix. Determinant of an endomorphism. The adjugate matrix.

Eigenvalues and eigenvectors. Diagonals and ~~tri~~ triangular forms. Characteristic and minimal polynomials. ~~Cayley~~ Cayley-Hamilton theorem over  $\mathbb{C}$ . Algebraic and geometric multiplicity of eigenvalues. Statement and illustration of Jordan normal form.

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Dual of a finite-dimensional vector space, dual bases and maps. Matrix representation, rank and determinant of dual map

Bilinear forms. Matrix representation, change of basis. Symmetric forms and their link with quadratic forms. Diagonalisation of quadratic forms. Law of inertia, classification by rank and signature. Complex Hermitian forms

Inner product spaces, orthonormal sets, orthogonal projection,  $V = W \oplus W^\perp$ . Gram-Schmidt orthogonalisation. Adjoints. Diagonalisation of Hermitian matrices. Orthogonality of eigenvectors and properties of eigenvectors.

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## Appropriate Books

- E.W. Curtis Linear Algebra: an introductory approach Springer 1984
- P.R. Halmos Finite-dimensional vector spaces Springer 1974
- K. Hoffman and R. Kunze Linear Algebra (Pearson 1971)