

Geometry

Euclidean and spherical geometry; length, lines and groups of isometries; Möbius maps and stereographic projection.

Triangulations of the sphere and the torus.* Informal discussion of abstract smooth surfaces, orientability and statement of the classification of compact smooth surfaces*

Riemannian metrics on open subsets of the plane. The hyperbolic plane. Poincaré models and their metrics. The isometry group. Hyperbolic triangles and the Gauss-Bonnet theorem. The hyperboloid model.

Embedded surfaces in \mathbb{R}^3 . The first fundamental form. Length and area. Examples.

~~Light~~ Length and energy. Geodesics for general Riemannian metrics as stationary points of the energy. First variation of the energy and geodesics as solutions of the corresponding Euler-Lagrange equations. Geodesic polar coordinates (informal proof of existence). Surfaces of revolution.

The second fundamental form and Gaussian curvature.

For metrics of the form $du^2 + G(u,v)dv^2$, expression of the curvature as $-(\sqrt{G})_{uu}/\sqrt{G}$. Abstract smooth surfaces and isometries, with examples. Euler numbers and Statement of Gauss-Bonnet theorem, examples and applications.

Appropriate books

P-M-H Wilson Curved Spaces (CUP January 2008)

M Do Carmo Differential Geometry of Curves and Surfaces. Pearson, Englewood Cliffs, N.J 1976

A Pressley Elementary Differential Geometry. Springer Undergraduate Series. Springer-Verlag London, 2001

E Rees Notes on Geometry Springer, 1983

M Reid and B Szendrői Geometry and Topology CUP 2005