

Metric and topological spaces

Metrics

Definition and examples. Limits and continuity. Open sets and neighbourhoods. Characterizing limits and continuity using neighbourhoods and open sets

Topology

Definition of a topology. Metric topologies. Further examples. Neighbourhoods, closed sets, convergence and continuity. ~~Hand~~ Hausdorff spaces. Homeomorphisms. Topological and non-topological properties. Completeness, Subspace, quotient and product topologies

Connectedness

Definition using open sets and integer-valued functions. Examples, including intervals. Components. The continuous image of a connected. Path-Connectedness. Path-connected spaces. are connected but not conversely. Connected open sets in Euclidean space are path-connected.

Compactness

Definition using open covers. Examples: finite sets and $[0,1]$. Closed subsets of compact spaces are compact. Compact subsets of a Hausdorff space must be closed. The compact subsets of the real line. Continuous images of compact sets are compact. Quotient spaces. Continuous real-valued functions on a compact space are bounded and attain their bounds. The product of two compact spaces is compact. The compact subsets of Euclidean space. Sequential compactness.

Appropriate books

- + W.A Sutherland Introduction to metric and Topological Spaces. Clarendon 1975
- D.J.H Garling A Course in Mathematical Analysis (Vol 2) CUP 2013
- A.J White Real analysis: an introduction Addison-Wesley 1968
- B Mendelson Introduction to Topology Dover 1990.