

MARKOV CHAINS

Discrete-time chains

Definition and basic properties, the transition matrix. Calculation of n -step transition probabilities. Communicating classes, closed classes, absorption, irreducibility. Calculation of hitting probabilities and mean hitting times; survival probability of birth and death chains. Stopping times and statement of the strong Markov property.

Recurrence and transience; equivalence of ~~transience~~ transience and summability of n -step transition probabilities; equivalence of recurrence and certainty of returns.

Recurrence as a class property, relation with closed classes. Simple random walks in dimensions one, two and three

Invariant distributions, statement of existence and uniqueness. Mean return time, positive recurrence; equivalence of positive recurrence and the existence of an invariant distribution. Convergence to equilibrium, positive recurrent, aperiodic chains and proof by coupling. *Long-run proportion of time spent in given state*.

Appropriate Books

- G.R. Grimmett and D.R. Stirzaker Probability and Random Processes (OUP 2001)
- G.R. Grimmett and D. Welsh Probability, An Introduction OUP, 2nd edition, 2014
- J.R. Norris Markov chains CUP 1997