Variational principles

Stationary points for functions on $\mathbb{R}^n$. Necessary and sufficient conditions for minima and maxima. Impoltance of convexity. Variational problems with constraints; method of Lagrange multipliers, the Legendre Transform; need for convexity to ensure invertibility; illustrations from thermodynamics.

The idea of a functional and a functional derivatives. First variation for functionals, Euler-Lagrange equations, for both ordinary and partial differential equations. Use of Lagrange multipliers and multiplier functions

Fermat's principle; geodesics; least action principles; Lagrange's and Hamilton's equations for particles and fields. Noether theorems and first integrals, including two forms of Noether's theorem for ordinary differential equations (energy, momentum, for example). Interpretation in terms of conservation laws.

Second variation for functionals; associated eigenvalue problem.

Appropiate books

D.S Lemons Perfect form. Princeton University Press 1997

C Lanczos The variational principles of Mechanics Dover 1986

R Weinstock Calculus of Variations with Applications to Physics and engineering. Dover 1974

I.M Gelfand and S.V. Fomin Calculus of Variations Dover 2000

W Yourgrau and S Mandelstam Variational Principles in Dynamics and Quantum Theory. Dover 2007

S. Hildebrandt and A Tromba Mathematics and Optimal form. Scientific American Library 1985