

# METHODS

## Self-adjoint ODEs (Slowly)

Periodic functions. Fourier series: definition and simple properties; Parseval's theorem. Equations of second order. Self-adjoint differential operators. The Sturm-Liouville equation; eigenfunctions and eigenvalues; reality of eigenvalues and orthogonality of eigenfunctions; eigenfunctions expansion (Fourier series as prototype), approximation in mean square, Statement of completeness.

## PDEs on bounded domains: Separation of variables

Physical basis of Laplace's equation, the wave equation and the diffusion equation. General method of separation of variables in Cartesian, cylindrical and spherical coordinates.

Legendre's equation: derivation, solutions including explicit forms of  $P_0$ ,  $P_1$  and  $P_2$ , orthogonality. Bessel's equations of integer order as an example of a self-adjoint eigenvalue problem with non-trivial weight.

Examples including potentials on rectangular and circular domains on a spherical domain (axisymmetric case only), waves on a finite string and heat flow down a semi-infinite rod.

## Inhomogeneous ODEs: Green functions

Properties of the Dirac delta function. Initial value problems and forced problems with two fixed end points; Solution using Green's functions. Eigenfunction expansions of the delta function and Green's function.

## Fourier transforms

Fourier transforms: definition and simple properties; inversion and convolution theorems. The discrete Fourier transform. Examples of application of to linear systems. Relationship of transfer function to Green function for initial value problems.

## PDEs on unbounded domains

Classification of PDEs in two independent variables. Well posedness. (not a very poised position huh!). Solution by the method of characteristics. Green's functions for PDEs in 1, 2 and 3 independent variables; fundamental solutions of the wave equation, Laplace's equation and the diffusion equation. The method of images. Application of the forced wave equation, Poisson's equation and forced diffusion equation. Transient solutions of diffusion problems; the error function ( $\text{erf}(x)$ ).



## Appropriate Books

G. B. Arfken, H. J. Weber & F. E. Harris Mathematical methods for Physicists. Elsevier 2013

M. L. Boas Mathematical methods in the physical sciences Wiley 2005

J. Matthews and R. L. Walker Mathematical Methods ~~and~~ of physics. Benjamin Cummings 1970

K. F. Riley, M. P. Hobson and S. J. Bence Mathematical methods for

Physics and engineering: a comprehensive guide. CUP 2002

Erwin Kreyszig Advanced Engineering Mathematics Wiley