ANALYSIS II

Uniform Convergence

The general principle of uniform convergence. A uniform limit of continuous functions is continuous. Uniform convergence and termwise integration and differentiation of series of real-valued functions. Local uniform convergence of power series.

Uniform continuity and integration

Continuous functions on closed bounded intervals are uniform continuous. Review of basic facts on Riemann integration (from Analysis I). Informal discussion of integration of complex-valued and $\mathbb{R}^n$-valued functions of one variable; proof that

$$\left|\left| \int_a^b f(x)\,dx \right|\right| \leq \int_a^b ||f(x)||\,dx$$

$\mathbb{R}^n$ as a normed space

Definiton of a normed space. Examples ,including the Euclidean norm on $\mathbb{R}^n$ and the uniform norm on $C[a,b]$. Lipschitz mapping and Lipschitz equivalence norms. The Bolzano-Weierstrass theorem in $\mathbb{R}^n$. Completeness. Open and closed sets. Continuity for functions between normed spaces. A continuous function on a closed bounnded set in $\mathbb{R}^n$ is uniformly continuous and has closed bounded image. All norms on a finite-dimensional are Lipschitz equivalent.

Differentation from $\mathbb{R}^m$ to $\mathbb{R}^n$

Definition of derivative as a linear map; elementary propteries, the chain rule. Partial derivatives; continuous partial derivatives imply differentiability. Higher-order derivatives; Symmetry of mixed derivatives(assumed continuous). Taylor theorem. The mean value inequality. Path-connectedness for subsets of $\mathbb{R}^n$; a function having zero derivative on a path-connected open Subjet is constant.

Metric Spaces

Definiton and examples, * Metrics used in Geometry*. Limit, continuity, balls, neighbourhoods, open and closed sets.

The Contradiction Mapping Theorem

The contradiction mapping theorem. Applications including the inverse function theorem (proof of continuity of inverse function, statement of differentiability). Picard's solution of differential equations.

Appropiate books

† J.C Burkill and H Burkill A Second Course in Mathematical Analysis CUP 2002

A.F Beardon Limits: A New approach to Real Analysis Springer 1997

D.J.H Garling A Course in Mathematical Analysis (Vol 3) CUP 2014

†W. Rudin Principles of Mathematical Analysis McGraw Hill 1976

W.A Sutherland Introduction to Metric and Topological Spaces Clarendon

A.J White Real Analysis: An Introduction (Addison-Wesley 1968) 1975

T.W Körner A Companion to analysis AMS, 2004