Supplementary Material
Bayesian Performance Analysis for Algorithm Ranking Comparison

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1 Numerical results

In this section we provide the numerical results for the experiments conducted in the paper. For each posterior summary, we report the mean and standard deviation between parenthesis taken from the different posterior samples.

1.1 Synthetically generated scores

Table 1: Probability of each algorithm to be the top-ranked algorithm.

	A_1	A_2	A_3	A_4
PLD	9.10E-01 (9.10E-03)	8.30E-02 (8.48E-03)	6.79E-03 (9.14E-04)	5.47E-04 (9.43E-05)
PLG	9.11E-01 (8.67E-03)	8.17E-02 (8.03E-03)	6.66E-03 (9.31E-04)	5.26E-04 (9.11E-05)
BT	9.09E-01 (8.84E-03)	8.99E-02 (8.74E-03)	9.73E-04 (2.38E-04)	7.26E-07 (3.58E-07)
MM	1.00E+00 (8.75E-06)	3.86E-05 (8.75E-06)	1.56E-09 (7.30E-10)	6.66E-14 (4.90E-14)

Table 2: Probability of each algorithm to outperform others.

		A_1	A_2	A_3	A_4
PLD	A_1		9.16E-01 (8.59E-03)	9.93E-01 (1.04E-03)	9.99E-01 (1.07E-04)
	A_2	8.36E-02 (8.59E-03)		9.24E-01 (7.72E-03)	9.93E-01 (1.00E-03)
	A_3	7.41E-03 (1.04E-03)	7.57E-02 (7.72E-03)		9.26E-01 (7.48E-03)
	A_4	6.01E-04 (1.07E-04)	6.56E-03 (1.00E-03)	7.45E-02 (7.48E-03)	
PLG	A_1		9.18E-01 (8.14E-03)	9.93E-01 (1.06E-03)	9.99E-01 (1.03E-04)
	A_2	8.22E-02 (8.14E-03)		9.25E-01 (7.83E-03)	9.94E-01 (9.73E-04)
	A_3	7.26E-03 (1.06E-03)	7.54E-02 (7.83E-03)		9.27E-01 (8.25E-03)
	A_4	5.77E-04 (1.03E-04)	6.41E-03 (9.73E-04)	7.33E-02 (8.25E-03)	
BT	A_1		9.10E-01 (8.76E-03)	9.98E-01 (4.27E-04)	1.00E+00 (3.08E-06)
	A_2	9.00E-02 (8.76E-03)		9.20E-01 (8.29E-03)	9.99E-01 (3.53E-04)
	A_3	1.88E-03 (4.27E-04)	7.99E-02 (8.29E-03)		9.26E-01 (8.38E-03)
	A_4	8.75E-06 (3.08E-06)	1.37E-03 (3.53E-04)	7.39E-02 (8.38E-03)	
MM	A_1		1.00E+00 (8.75E-06)	1.00E+00 (1.46E-09)	1.00E+00 (1.47E-13)
	A_2	3.86E-05 (8.75E-06)		1.00E+00 (8.75E-06)	1.00E+00 (1.46E-09)
	A_3	3.13E-09 (1.46E-09)	3.86E-05 (8.75E-06)		1.00E+00 (8.75E-06)
	A_4	2.00E-13 (1.47E-13)	3.13E-09 (1.46E-09)	3.86E-05 (8.75E-06)	

Table 3: Probability of an algorithm to be in the top-k ranking.

		Top 1	Top 2	Top 3	Top 4
PLD	A_1		9.99E-01 (2.79E-04)	1.00E+00 (7.74E-07)	
	A_2		9.19E-01 (8.21E-03)	9.99E-01 (2.24E-04)	
	A_3		7.58E-02 (7.67E-03)	9.26E-01 (7.43E-03)	
	A_4		6.11E-03 (8.99E-04)	7.50E-02 (7.53E-03)	
PLG	A_1		9.99E-01 (2.73E-04)	1.00E+00 (7.20E-07)	
	A_2		9.20E-01 (8.23E-03)	9.99E-01 (2.13E-04)	
	A_3		7.56E-02 (7.80E-03)	9.27E-01 (8.21E-03)	
	A_4		5.97E-03 (8.76E-04)	7.37E-02 (8.30E-03)	
BT	A_1		9.99E-01 (2.40E-04)	1.00E+00 (4.59E-07)	
	A_2		9.20E-01 (8.33E-03)	9.99E-01 (2.04E-04)	
	A_3		8.08E-02 (8.36E-03)	9.26E-01 (8.36E-03)	
	A_4		6.68E-04 (1.87E-04)	7.46E-02 (8.46E-03)	
MM	A_1		1.00E+00 (7.30E-10)	1.00E+00 (4.91E-14)	
	A_2		1.00E+00 (8.75E-06)	1.00E+00 (7.30E-10)	
	A_3		3.86E-05 (8.75E-06)	1.00E+00 (8.75E-06)	
	A_4		1.56E-09 (7.30E-10)	3.86E-05 (8.75E-06)	

1.2 Permutation Flowshop Scheduling Problem

Table 4: Probability of each algorithm to be the top-ranked.

	GM-EDA	HGM-EDA	AGA	VNS	NVNS
PLD	1.67E-02 (1.29E-03)	3.45E-01 (1.02E-02)	4.39E-01 (1.28E-02)	1.29E-01 (5.49E-03)	7.05E-02 (3.85E-03)
PLG	1.67E-02 (1.26E-03)	3.45E-01 (1.07E-02)	4.40E-01 (1.20E-02)	1.28E-01 (5.82E-03)	7.00E-02 (3.65E-03)
BT	1.03E-03 (2.04E-04)	3.77E-01 (1.27E-02)	4.62E-01 (1.33E-02)	1.08E-01 (6.53E-03)	5.19E-02 (4.16E-03)
MM	5.49E-05 (1.76E-05)	9.13E-01 (7.06E-03)	7.96E-02 (5.82E-03)	6.99E-03 (1.08E-03)	6.18E-04 (1.46E-04)

Table 5: Probability of each algorithm to outperform others.

		GM-EDA	HGM-EDA	AGA	VNS	NVNS
PLD	GM-EDA		4.62E-02 (3.60E-03)	3.67E-02 (3.26E-03)	1.15E-01 (7.73E-03)	1.92E-01 (1.18E-02)
	HGM-EDA	9.54E-01 (3.60E-03)		4.40E-01 (1.36E-02)	7.28E-01 (1.05E-02)	8.30E-01 (8.80E-03)
	AGA	9.63E-01 (3.26E-03)	5.60E-01 (1.36E-02)		7.73E-01 (1.10E-02)	8.62E-01 (8.65E-03)
	VNS	8.85E-01 (7.73E-03)	2.72E-01 (1.05E-02)	2.27E-01 (1.10E-02)		6.46E-01 (1.34E-02)
	NVNS	8.08E-01 (1.18E-02)	1.70E-01 (8.80E-03)	1.38E-01 (8.65E-03)	3.54E-01 (1.34E-02)	
PLG	GM-EDA		4.60E-02 (3.67E-03)	3.65E-02 (3.02E-03)	1.15E-01 (8.37E-03)	1.92E-01 (1.19E-02)
	HGM-EDA	9.54E-01 (3.67E-03)		4.40E-01 (1.36E-02)	7.29E-01 (1.19E-02)	8.31E-01 (8.70E-03)
	AGA	9.63E-01 (3.02E-03)	5.60E-01 (1.36E-02)		7.74E-01 (1.07E-02)	8.63E-01 (7.97E-03)
	VNS	8.85E-01 (8.37E-03)	2.71E-01 (1.19E-02)	2.26E-01 (1.07E-02)		6.47E-01 (1.46E-02)
	NVNS	8.08E-01 (1.19E-02)	1.69E-01 (8.70E-03)	1.37E-01 (7.97E-03)	3.53E-01 (1.46E-02)	
BT	GM-EDA		2.38E-02 (2.65E-03)	1.62E-02 (1.93E-03)	1.07E-01 (7.75E-03)	1.74E-01 (9.88E-03)
	HGM-EDA	9.76E-01 (2.65E-03)		4.50E-01 (1.44E-02)	7.39E-01 (1.21E-02)	8.26E-01 (1.00E-02)
	AGA	9.84E-01 (1.93E-03)	5.50E-01 (1.44E-02)		7.80E-01 (1.11E-02)	8.59E-01 (9.15E-03)
	VNS	8.93E-01 (7.75E-03)	2.61E-01 (1.21E-02)	2.20E-01 (1.11E-02)		6.11E-01 (1.39E-02)
	NVNS	8.26E-01 (9.88E-03)	1.74E-01 (1.00E-02)	1.41E-01 (9.15E-03)	3.89E-01 (1.39E-02)	
MM	GM-EDA		2.14E-04 (6.80E-05)	1.79E-03 (4.18E-04)	1.34E-02 (2.03E-03)	8.02E-02 (5.97E-03)
	HGM-EDA	1.00E+00 (6.80E-05)		9.20E-01 (5.97E-03)	9.87E-01 (2.03E-03)	9.98E-01 (4.18E-04)
	AGA	9.98E-01 (4.18E-04)	8.02E-02 (5.97E-03)		9.20E-01 (5.97E-03)	9.87E-01 (2.03E-03)
	VNS	9.87E-01 (2.03E-03)	1.34E-02 (2.03E-03)	8.02E-02 (5.97E-03)		9.20E-01 (5.97E-03)
	NVNS	9.20E-01 (5.97E-03)	1.79E-03 (4.18E-04)	1.34E-02 (2.03E-03)	8.02E-02 (5.97E-03)	

Table 6: Probability of an algorithm to be in the top-k ranking.

		Top 1	Top 2	Top 3	Top 4	Top 5
PLD	GM-EDA		4.23E-02 (3.10E-03)	9.65E-02 (6.33E-03)	2.34E-01 (1.34E-02)	
	HGM-EDA		6.98E-01 (1.11E-02)	9.17E-01 (5.88E-03)	9.92E-01 (9.45E-04)	
	AGA		7.76E-01 (1.23E-02)	9.47E-01 (5.25E-03)	9.96E-01 (6.72E-04)	
	VNS		3.09E-01 (1.07E-02)	6.53E-01 (1.26E-02)	9.38E-01 (5.11E-03)	
	NVNS		1.74E-01 (8.43E-03)	3.86E-01 (1.37E-02)	8.40E-01 (1.06E-02)	
PLG	GM-EDA		4.22E-02 (3.07E-03)	9.66E-02 (6.48E-03)	2.35E-01 (1.37E-02)	
	HGM-EDA		6.99E-01 (1.20E-02)	9.18E-01 (6.14E-03)	9.92E-01 (9.72E-04)	
	AGA		7.77E-01 (1.12E-02)	9.48E-01 (4.69E-03)	9.96E-01 (5.90E-04)	
	VNS		3.08E-01 (1.18E-02)	6.53E-01 (1.45E-02)	9.38E-01 (5.76E-03)	
	NVNS		1.73E-01 (8.13E-03)	3.84E-01 (1.42E-02)	8.39E-01 (1.07E-02)	
BT	GM-EDA		8.39E-03 (1.23E-03)	5.41E-02 (5.12E-03)	2.57E-01 (1.21E-02)	
	HGM-EDA		7.19E-01 (1.22E-02)	9.09E-01 (6.33E-03)	9.87E-01 (1.46E-03)	
	AGA		7.82E-01 (1.11E-02)	9.36E-01 (5.10E-03)	9.92E-01 (9.96E-04)	
	VNS		3.12E-01 (1.22E-02)	6.47E-01 (1.27E-02)	9.18E-01 (5.86E-03)	
	NVNS		1.79E-01 (9.82E-03)	4.54E-01 (1.35E-02)	8.47E-01 (8.45E-03)	
MM	GM-EDA		6.73E-04 (1.63E-04)	7.66E-03 (1.24E-03)	8.73E-02 (7.06E-03)	
	HGM-EDA		9.92E-01 (1.24E-03)	9.99E-01 (1.63E-04)	1.00E+00 (1.76E-05)	
	AGA		9.13E-01 (6.90E-03)	9.92E-01 (1.22E-03)	9.99E-01 (1.46E-04)	
	VNS		8.60E-02 (6.75E-03)	9.14E-01 (6.75E-03)	9.93E-01 (1.08E-03)	
	NVNS		7.60E-03 (1.22E-03)	8.66E-02 (6.90E-03)	9.20E-01 (5.82E-03)	