12th CBSE MATHEMATICS

2012-13

1 Section A

1.1. If the binary operation $*$ on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation $*$ in Z.

1.2. Write the values of $\cot(\tan^{-1} a + \cos^{-1} a)$

1.3. If $\mathbf{A}$ is a square matrix such that $\mathbf{A}^2 = \mathbf{A}$, then write the value of $(\mathbf{I} + \mathbf{A})^2 - 3\mathbf{A}$

1.4. If

$$x \begin{pmatrix} 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \tag{1.4.1}$$

write the value of x.

1.5. Write the following determinant :

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

1.6. If

$$\int \left(\frac{x-1}{x^2}\right) e^x dx = f(x)e^x + c$$

then write the value of $f(x)$

1.7. If

$$\int_a^0 3x^2 dx = 8 \tag{1.7.1}$$

write the value of $'a'$.

1.8. Write the value of

$$(\hat{i} \ \times \ \hat{j}) \cdot \hat{k} + (\hat{j} \ \times \ \hat{k}) \cdot \hat{i}$$

1.9. Write the value of the area of the parallelogram determined by the vectors $2\hat{i}$ and $3\hat{j}$

1.10. Write the direction cosines of a line parallel to z-axis

2 Section B

2.1. If

$$f(x) = \frac{4x+3}{6x-4}$$

x $\neq \frac{2}{3}$, show that $fof(x) = x$ all $x \neq \frac{2}{3}$. What is the inverse of f?

2.2. a) Prove that:

$$\sin^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$$

b) Solve for x:

$$2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$$

2.3. Using properties of determinants, prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

2.4. If $x^m \ y^n = (x+y)^{m+n}$, prove that

$$\frac{dy}{dx} = \frac{y}{x}$$

2.5. a) If

$$y = e^{a\cos^{-1} x}, -1 \leq x < 1$$

show that :

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$$

b) If

$$x\sqrt{1+y} + y\sqrt{1+x} = 0 \ , \\ -1 < x < 1 \ , \ x \neq y$$

then prove that

$$\frac{dy}{dx} = -\frac{1}{(1+x^2)}$$

2.6. a) Show that

$$y = \log(1+x) - \frac{2x}{2+x}\ ,\ \ x > -1$$

is an increasing function of x throughout its domain.

b) Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$

2.7. a) Evaluate

$$\int x^2 \tan^{-1} x\ dx$$

b) Evaluate

$$\int \frac{3x-1}{(x+2)^2} dx$$

2.8. Solve the following differential equation :

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1\ ,\ \ x \neq 0$$

2.9. Solve the following differential equation :

$$3\ e^x \tan y\ dx + (2 - e^x)\sec^2 y\ dy = 0$$

given that when $x = 0$, $y = \frac{\pi}{4}$

2.10. If $\overrightarrow{\alpha} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\overrightarrow{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$, then express $\overrightarrow{\beta}$ in the form $\overrightarrow{\beta} = \overrightarrow{\beta_1} + \overrightarrow{\beta_2}$, where $\overrightarrow{\beta_1}$ is parallel of $\overrightarrow{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\overrightarrow{\alpha}$.

2.11. Find the vector and cartesian equations of the line passing through the point $P(1,\ 2,\ 3)$ and parallel to the planes $\overrightarrow{r} \cdot (\hat{i} - \hat{j} + 2\hat{k} = 5$ and $\overrightarrow{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

2.12. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

3 Section C

3.1. a) Using matrices, solve the following system of equations:

$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

b) If

$$\mathbf{A}^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & -6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

and

$$\mathbf{B} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

find $(\mathbf{AB})^{-1}$.

3.2. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius is $\frac{4R}{3}$.

3.3. Find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}\ y$ and the circle $x^2 + y^2 = 4$.

3.4. a) Evaluate:

$$\int_1^3 (x^2 + x)dx$$

as a limit of a sum.

b) Evaluate:

$$\int_0^{\pi/4} \frac{\cos^2}{\cos^2 x + 4\sin^2 x} dx$$

3.5. Find the vector equation of the plane passing through the points $\left(2, 1, -1\right)$ and $\left(-1, 3, 4\right)$ and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtained contains the lines $\overrightarrow{r} = \hat{i} + 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} - 2\hat{j} - 5\hat{k})$.

3.6. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink.

The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs | 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs | 4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimize cost ? Make a LPP and solve graphically.

3.7. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.