

# 12th CBSE MATHEMATICS

2012-13

## 1 SECTION A

1.1. If the binary operation  $*$  on the set  $Z$  of integers is defined by  $a * b = a + b - 5$ , then write the identity element for the operation  $*$  in  $Z$ .

1.2. Write the values of  $\cot(\tan^{-1} a + \cos^{-1} a)$

1.3. If  $A$  is a square matrix such that  $A^2 = A$ , then write the value of  $(I + A)^2 - 3A$

1.4. If  $x \begin{pmatrix} 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ , write the value of  $x$ .

1.5. Write the following determinant :

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

1.6. If

$$\int \left( \frac{x-1}{x^2} \right) e^x dx = f(x)e^x + c$$

then write the value of  $f(x)$

1.7. If  $\int_a^0 3x^2 dx = 8$ , write the value of  $'a'$ .

1.8. Write the value of

$$(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i}$$

1.9. Write the value of the area of the parallelogram determined by the vectors  $2\hat{i}$  and  $3\hat{j}$

1.10. Write the direction cosines of a line parallel to z-axis

## 2 SECTION B

2.1. If

$$f(x) = \frac{4x+3}{6x-4}$$

$x \neq \frac{2}{3}$ , show that  $f \circ f(x) = x$  all  $x \neq \frac{2}{3}$ . What is the inverse of  $f$ ?

2.2. a) Prove that:

$$\sin^{-1} \left( \frac{63}{65} \right) = \sin^{-1} \left( \frac{5}{13} \right) + \cos^{-1} \left( \frac{3}{5} \right)$$

b) Solve for  $x$ :

$$2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x), x \neq \frac{\pi}{2}$$

2.3. Using properties of determinants, prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

2.4. If  $x^m y^n = (x+y)^{m+n}$ , prove that

$$\frac{dy}{dx} = \frac{y}{x}$$

2.5. a) If

$$y = e^{a \cos^{-1} x}, -1 \leq x < 1$$

show that :

$$(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$$

b) If

$$x\sqrt{1+y} + y\sqrt{1+x} = 0, 1 < x < 1, x \neq y$$

then prove that

$$\frac{dy}{dx} = -\frac{1}{(1+x^2)}$$

2.6. a) Show that

$$y = \log(1+x) - \frac{2x}{2+x}, x > -1$$

is an increasing function of  $x$  throughout its domain.

- b) Find the equation of the normal at the point  $(am^2, am^3)$  for the curve  $ay^2 = x^3$

2.7. a) Evaluate

$$\int x^2 \tan^{-1} x \, dx$$

b) Evaluate

$$\int \frac{3x-1}{(x+2)^2} dx$$

2.8. Solve the following differential equation :

$$\left( \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right) \frac{dx}{dy} = 1, \quad x \neq 0$$

2.9. Solve the following differential equation :

$$3e^x \tan y \, dx + (2 - e^x) \sec^2 y \, dy = 0$$

given that when  $x = 0$ ,  $y = \frac{\pi}{4}$

- 2.10. If  $\vec{\alpha} = 3\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$ , then express  $\vec{\beta}$  in the form  $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ , where  $\vec{\beta}_1$  is parallel of  $\vec{\alpha}$  and  $\vec{\beta}_2$  is perpendicular to  $\vec{\alpha}$ .

- 2.11. Find the vector and cartesian equations of the line passing through the point  $P(1, 2, 3)$  and parallel to the planes  $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$ .

- 2.12. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

### 3 SECTION C

- 3.1. a) Using matrices, solve the following system of equations:

$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

b) If  $\mathbf{A}^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & -6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ , find  $(\mathbf{AB})^{-1}$ .

- 3.2. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius is  $\frac{4R}{3}$ .

- 3.3. Find the area of the region in the first quadrant enclosed by x-axis, the line  $x = \sqrt{3}y$  and the circle  $x^2 + y^2 = 4$ .

3.4. a) Evaluate:

$$\int_1^3 (x^2 + x) dx$$

as a limit of a sum.

b) Evaluate:

$$\int_0^{\pi/4} \frac{\cos^2}{\cos^2 x + 4 \sin^2 x} dx$$

- 3.5. Find the vector equation of the plane passing through the points  $(2, 1, -1)$  and  $(-1, 3, 4)$  and perpendicular to the plane  $x - 2y + 4z = 10$ . Also show that the plane thus obtained contains the lines  $\vec{r} = \hat{i} + 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} - 2\hat{j} - 5\hat{k})$ .

- 3.6. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs |10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs |4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimize cost ? Make a LPP and solve graphically.

- 3.7. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college

and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.