12th CBSE MATHEMATICS

2012-13

1 SECTION A

1.1. If the binary operation $*$ on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation $*$ in Z.

1.2. Write the values of $\cot(tan^{-1}a + \cos^{-1} a)$

1.3. If $\mathbf{A}$ is a square matrix such that $\mathbf{A}^2 = \mathbf{A}$, then write the value of $(\mathbf{I} + \mathbf{A})^2 - 3\mathbf{A}$

1.4. If $x \begin{pmatrix} 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, write the value of x.

1.5. Write the following determinant : $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

1.6. If

$$\int (\frac{x-1}{x^2})e^x dx = f(x)e^x + c$$

then write the value of $f(x)$

1.7. If $\int_a^0 3x^2 dx = 8$, write the value of $'a'$.

1.8. Write the value of $(\hat{i} \times \hat{j}).\hat{k} + (\hat{j} \times \hat{k}).\hat{i}$

1.9. Write the value of the area of the parallelogram determined by the vectors $2\hat{i}$ and $3\hat{j}$

1.10. Write the direction cosines of a line parallel to z-axis

2 SECTION B

2.1. If $f(x) = \frac{4x+3}{6x-4}$, x $\neq \frac{2}{3}$, show that $fof(x) = x$ all $x \neq \frac{2}{3}$. What is the inverse of f?

2.2. Prove that:

$$\sin^{-1}(\frac{63}{65}) = \sin^{-1}(\frac{5}{13}) + \cos^{-1}(\frac{3}{5})$$

OR

Solve for x:

$$2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$$

2.3. Using properties of determinants, prove that
$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

2.4. If $x^m \ y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

2.5. If $y = e^{a\cos^{-1}x}, -1 \leq x < 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

OR

If $x\sqrt{1+y}+y\sqrt{1+x} = 0 \, , 1 < x < 1 \, , \ x \neq y$ then prove that

$$\frac{dy}{dx} = -\frac{1}{(1+x^2)}$$

2.6. Show that $y = \log(1+x) - \frac{2x}{2+x}$,$x > -1$, is an increasing function of x throughout its domain.

OR

2.7. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$

2.8. Evaluate

$$\int x^2 \tan^{-1} x \ dx$$

OR

Evaluate

$$\int \frac{3x-1}{(x+2)^2}dx$$

2.9. Solve the following differential equation :

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1\ ,\ x\neq 0$$

2.10. Solve the following differential equation :

$$3\ e^x\tan y\ dx+(2-e^x)\sec^2 y\ dy=0$$

given that when $x=0$, $y=\frac{\pi}{4}$

2.11. If $\overrightarrow{\alpha}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\overrightarrow{\beta}=2\hat{i}+\hat{j}-4\hat{k}$, then express $\overrightarrow{\beta}$ in the form $\overrightarrow{\beta}=\overrightarrow{\beta_1}+\overrightarrow{\beta_2}$, where $\overrightarrow{\beta_1}$ is parallel of $\overrightarrow{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\overrightarrow{\alpha}$.

2.12. Find the vector and cartesian equations of the line passing through the point $P(1,\ 2,\ 3)$ and parallel to the planes $\overrightarrow{r}.\ (\hat{i}-\hat{j}+2\hat{k}=5$ and $\overrightarrow{r}\ .\ (3\hat{i}+\hat{j}+\hat{k})=6$.

2.13. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

3 Section C

3.1. Using matrices, solve the following system of equations:

$$x-y+z=4;\ 2x+y-3z=0;\ x+y+z=2$$

OR

If $A^{-1}=\begin{bmatrix}3 & -1 & 1\\ -15 & -6 & -5\\ 5 & -2 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & -1\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$, find $(AB)^{-1}$.

3.2. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius is $\frac{4R}{3}$.

3.3. Find the area of the region in the first quadrant enclosed by x-axis, the line $x=\sqrt{3}\ y$ and the circle $x^2+y^2=4$.

3.4. Evaluate:

$$\int_1^3 (x^2+x)dx$$

as a limit of a sum.

OR

Evaluate:

$$\int_0^{\pi/4}\frac{\cos^2}{\cos^2 x+4\sin^2 x}dx$$

3.5. Find the vector equation of the plane passing through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane $x-2y+4z=10$. Also show that the plane thus obtained contains the lines $\overrightarrow{r}=\hat{i}+3\hat{j}+4\hat{k}+\lambda(3\hat{i}-2\hat{j}-5\hat{k})$.

3.6. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs |10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs |4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimize cost ? Make a LPP and solve graphically.

3.7. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.