
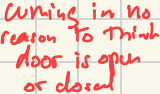




Probabilistic Robotics ttw 2

1.) $bel(x_t) = P(x_t | z_{1:t}, u_{1:t})$
 $bel(x_{t+1}) = P(x_{t+1} | z_{1:t}, u_{1:t+1})$
 ↳ Prediction model

Also no new
information to
update model

Prior  
 $bel(open) = 0.5$
 $bel(closed) = 0.5$ → Now

↳ just
measure $u = none$

$P(x_{t+1} = open | x_t = open, u_t = none) = 1$
 $P(x_{t+1} = closed | x_t = open, u_t = none) = 0$
 $P(x_{t+1} = open | x_t = closed, u_t = none) = 0$
 $P(x_{t+1} = closed | x_t = closed, u_t = none) = 1$

Predictions is no
change

$bel(open) = bel(open | open, 0) bel(open) + bel(open | closed, 0) bel(closed)$
 $bel(open) = (1)(0.5) + (0)(0.5) = 0.5$
 $bel(closed) = bel(closed | open, 0) bel(closed) + bel(closed | closed, 0) bel(closed)$
 $= (0)(0.5) + (1)(0.5) = 0.5$

After computation it is evident
that the assumption of
no model change
holds through!

2.)

Robot Action Policy

$bel(open) < 0.9$ $u_t = \text{push then measure}$
 $bel(open) > 0.9$ only measure
 $bel(open) > 0.99$ proceed through

COMPUTATIONS ON NEXT PAGE

step	$bel(open)$	$bel(closed)$	Action	$bel(open)$	$bel(closed)$	$bel^+(open)$	$bel^+(closed)$
0	0.5	0.5	push	0.8	0.2	0.978	0.122
1	0.978	0.122	push	0.9512	0.0498	0.9723	0.0277
2	0.9723	0.0277	measure	0.97229	0.0277	0.9844	0.01558
3	0.9844	0.01558	measure	0.9844	0.01558	0.99128	0.008716

Robot needs 4 steps before the Robot actually walks through the door.

2 push attempts, each with a measurement and then 2 more measurements.

Hence 4 measurements for Robot to move
2 Actions

Computations

Step 1:

$$\begin{aligned} bel(open) &= P(open | push, open) bel(open) + P(open | push, closed) bel(closed) \\ &= (1)(0.878) + (0.6)(0.122) = 0.9512 \\ bel(closed) &= P(closed | push, open) bel(open) + P(closed | push, closed) bel(closed) \\ &= (0)(0.878) + (0.4)(0.122) = 0.0488 \end{aligned}$$

Apply measurement update

$$bel(x_t) = P(x_t | z_{1:t}) = \frac{P(z_t | x_t) P(x_t)}{\int P(z_t | x) P(x) dx}$$

sensor model \leftarrow $P(x | z) = \int P(z | x) P(x) dx$

$$\begin{aligned} P(x=open | z=open) &= \eta P(z=open | x=open) bel(open) = \eta (0.9)(0.9512) = \eta (0.85608) \\ P(x=closed | z=open) &= \eta P(z=open | x=closed) bel(closed) = \eta (0.5)(0.0488) = \eta (0.0244) \\ \eta (0.85608 + 0.0244) &= 1 \\ \eta &= 1.1357 \\ bel^+(open) &= 0.9723 \\ bel^+(closed) &= 0.0277 \end{aligned}$$

Step 2:

Since $bel(open) > 0.9$ hence $u = \text{measure only}$

Hence prediction step does NOTHING

Measurement step

$$\begin{aligned} P(open | open) &= \eta P(z=open | x=open) bel(open) = \eta (0.9)(0.9723) \\ P(closed | open) &= \eta P(z=open | x=closed) bel(closed) = \eta (0.5)(0.0277) \end{aligned}$$

remember this is using state to update measurement.

$$\begin{aligned} \eta &= \frac{1}{(0.9)(0.9723) + (0.5)(0.0277)} \\ \eta &= 1.1249 \end{aligned}$$

Step 3:

$bel(open) > 0.9$ hence still measurement only

Same updates as step 2

$$\begin{aligned} P(open | open) &= \eta P(open | open) bel(open) = \eta (0.995) \\ P(closed | open) &= \eta P(open | closed) bel(closed) = \eta (0.00779) \\ \eta &= 1.1260 \end{aligned}$$

$$\begin{aligned} \text{hence } bel^+(open) &= 0.9913 \\ bel^+(closed) &= 0.0087 \end{aligned}$$

hence

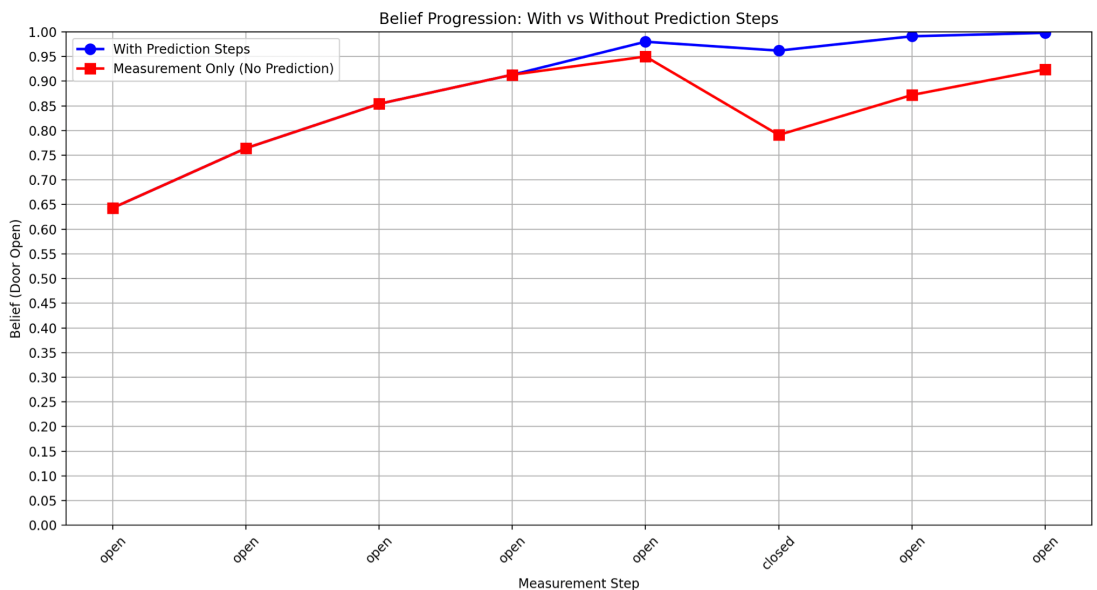
$$\begin{aligned} bel^+(open) &= 0.9944 \\ bel^+(closed) &= 0.01558 \end{aligned}$$

3.) Prediction model

$$M = \begin{bmatrix} P(\text{open}|\text{open}) & P(\text{open}|\text{closed}) \\ P(\text{closed}|\text{open}) & P(\text{closed}|\text{closed}) \end{bmatrix}$$

$$\begin{bmatrix} \text{bel}(\text{open}) \\ \text{bel}(\text{closed}) \end{bmatrix} = M \begin{bmatrix} \text{bel}(\text{open}) \\ \text{bel}(\text{closed}) \end{bmatrix}$$

Push $M = \begin{bmatrix} 1 & 0.6 \\ 0 & 0.4 \end{bmatrix}$ No action $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



GitHub: https://github.com/jaiselsingh1/Probabilistic-Robotics/blob/main/hw2_pred.py