Probabilistic Robotics Hw 2

1.) $bel(x_t) = p(x_t \mid z_{1..t}, u_{1..t})$

$\overline{bel}(x_t) = p(x_{t+1} \mid z_{1..t}, u_{1...t+1})$

↳ Prediction model

Also no new information to update model

coming in no reason to think door is open or closed

Prior

$bel(open) = 0.5$

$bel(closed) = 0.5$

→ Now

Predictions is no change

$p(x_{t+1} = open \mid x_t = open, u_t = none) = 1$

$p(x_{t+1} = closed \mid x_t = open, u_t = none) = 0$

$p(x_{t+1} = open \mid x_t = closed, u_t = none) = 0$

$p(x_{t+1} = closed \mid x_t = closed, u_t = none) = 1$

0 just means u = none

$\overline{bel}(open) = bel(open \mid open, 0)\, bel(open) + bel(open \mid closed, 0)\, bel(closed)$

$\overline{bel}(open) = (1)(0.5) + (0)(0.5) = \boxed{0.5}$

$\overline{bel}(closed) = bel(closed \mid open, 0)\, bel(closed) + bel(closed \mid closed, 0)\, bel(closed)$

$= (0)(0.5) + (1)(0.5) = \boxed{0.5}$

After computation it is evident that the assumption of no model change holds through!

2.)

Robot Action Policy

$bel(open) < 0.9$	u_t = push then measure
$bel(open) > 0.9$	only measure
$bel(open) > 0.99$	proceed through

COMPUTATIONS ON NEXT PAGE

step	bel (open)	bel (closed)	Action	$\overline{bel}$ (open)	$\overline{bel}$ (closed)	bel^+ (open)	bel^+ (closed)
0	0.5	0.5	push	0.8	0.2	0.878	0.122
1	0.878	0.122	push	0.9512	0.0498	0.9723	0.0277
2	0.9723	0.0277	measure	0.97229	0.0277	0.9844	0.01558
3	0.9844	0.01558	measure	0.9844	0.01558	$\boxed{0.99128}$	$\boxed{0.008716}$

Robot needs 4 steps before the Robot actually walks through the door. 2 push attempts, each with a measurement and then 2 more measurements. Hence 4 measurements for Robot to move
2 Actions

This has to be done only with state
no measurement selection

Computations

Step 1 :

$\overline{bel}(open) = P(open \mid push, open)\, bel(open) + P(open \mid push, closed)\, bel(closed)$

$= (1)(0.878) + (0.6)(0.122) = 0.9512$

$\overline{bel}(closed) = P(closed \mid push, open)\, bel(open) + P(closed \mid push, closed)\, bel(closed)$

$= (0)(0.878) + (0.4)(0.122) = 0.0488$

Apply measurement update

$\rightarrow$ $bel(x_t) = P(x_t \mid z_{1,\dots,t}) = \eta\, P(z \mid x)\, P(x)$

↳ normalization factor

sensor model

$P(x \mid z) = \eta\, P(z \mid x)\, P(x)$

$P(x=open \mid z=open) = \eta\, P(z=open \mid x=open)\, bel(open) = \eta\,(0.9)(0.9512) = \eta\,(0.85608)$

$P(x=closed \mid z=open) = \eta\, P(z=open \mid x=closed)\, bel(closed) = \eta\,(0.5)(0.0488) = \eta\,(0.0244)$

$P(z=open \mid x=open) = 0.9$

$P(z=open \mid x=closed) = 0.5$

$\eta\,(0.85608 + 0.0244) = 1$

$\eta = 1.1357$

$bel^{+}(open) = 0.9723$

$bel^{+}(closed) = 0.0277$

Step 2 :

Since $bel(open) > 0.90$ hence $u =$ measure only

Hence prediction step does NOTHING

Measurement step

↳ $P(open \mid open) = \eta\, P(z=open \mid x=open)\, bel(open) = \eta\,(0.90)(0.9723)$

$P(closed \mid open) = \eta\, P(z=open \mid x=closed)\, bel(closed) = \eta\,(0.50)(0.0277)$

measure open

remember this is using state to update measurement.

$$\eta = \frac{1}{(0.90)(0.9723) + (0.50)\,0.0277}$$

$\eta = 1.1249$

hence

$bel^{+}(open) = 0.9844$

$bel^{+}(closed) = 0.01558$

Step 3 :

$bel(open) > 0.9$ hence still measurement only

Same update as step 2

$P(open \mid open) = \eta\, P(open \mid open)\, bel(open) = \eta\,(0.885)$

$P(closed \mid open) = \eta\, P(open \mid closed)\, bel(closed) = \eta\,(0.00779)$

$\eta = 1.1260$

hence $bel^{+}(open) = 0.9913$

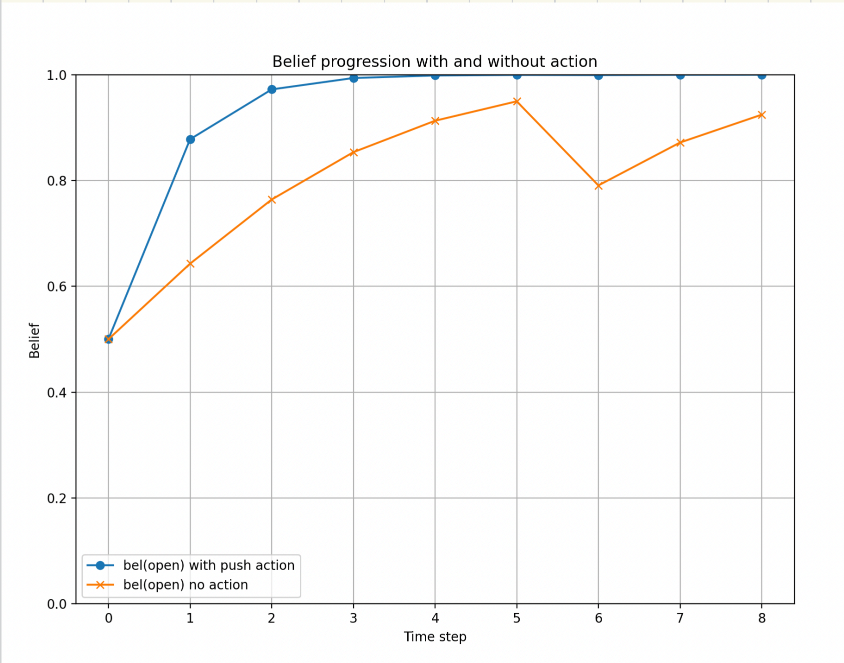
$bel^{+}(closed) = 0.0087$

3.) Prediction model

$$M = \begin{bmatrix} P(\text{open} \mid \text{open}) & P(\text{open} \mid \text{closed}) \\ P(\text{closed} \mid \text{open}) & P(\text{closed} \mid \text{closed}) \end{bmatrix}$$

$$\begin{bmatrix} \overline{bel}(\text{open}) \\ \overline{bel}(\text{closed}) \end{bmatrix} = M \begin{bmatrix} bel(\text{open}) \\ bel(\text{closed}) \end{bmatrix}$$

Push $M = \begin{bmatrix} 1 & 0.6 \\ 0 & 0.4 \end{bmatrix}$ No action $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



GitHub: https://github.com/jaiselsingh1/Probabilistic-Robotics/blob/main/hw2_pred.py