Project 2 - Kinematics 1

Introduction to Robotics, MECE4602

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1 Objective

The objective of this document is to formalize the kinematic model by defining the constraints and reference systems according to the Denavit-Hartenberg convention. By deriving the table associated with the proposed model, the forward and inverse kinematics are obtained.

2 Kinematic model

A schematic diagram of the model is shown in Figure 1. The design uses four revolute joints that allow flexion and extension of the phalanges and abduction and adduction of the entire finger. Joint nomenclature follows standard anatomical terminology. The metacarpophalangeal (MCP) joint is modeled with an abduction-adduction degree of freedom, denoted MCP (AbAd), and an independent flexion-extension degree of freedom, denoted MCP (FlEx). The proximal interphalangeal joint is labeled PIP and corresponds to the primary flexion mechanism of the middle phalanx. The distal interphalangeal joint, labeled DIP, governs flexion of the distal phalanx.

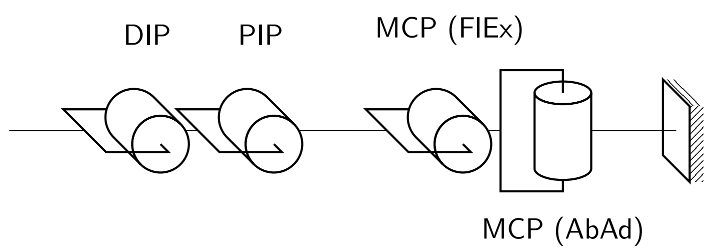

Figure 1: Kinematic model of the finger showing the serial arrangement of the MCP, PIP, and DIP.

3 Forward Kinematics

Below, Figure 2 shows the assignment of reference systems according to the Denavit-Hartenberg convention.

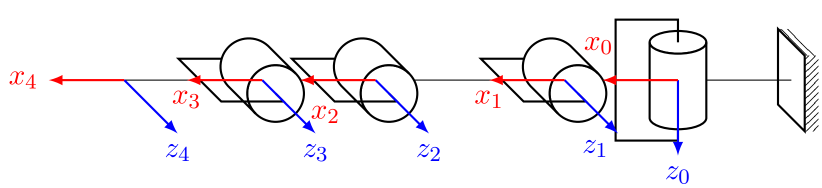

Figure 2: Finger model with assigned Denavit-Hartenberg frames.

Joint i	a_i (link length)	α_i (link twist)	d_i (link offset)	θ_i (joint angle)
1	0	$+90°$	0	θ_1^*
2	L_1	0	0	θ_2^*
3	L_2	0	0	θ_3^*
4	L_3	0	0	θ_4^*

Table 1: Denavit-Hartenberg parameters for the modeled manipulator.

According to the Denavit-Hartenberg formulation, the homogeneous transform associated with joint i is:

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Using the parameters in Table 1, the four elementary transforms A_1, A_2, A_3, and A_4 are obtained as follows.

$$A_1 = \begin{bmatrix} \cos\theta_1^* & 0 & \sin\theta_1^* & 0 \\ \sin\theta_1^* & 0 & -\cos\theta_1^* & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$A_2 = \begin{bmatrix} \cos\theta_2^* & -\sin\theta_2^* & 0 & L_1\cos\theta_2^* \\ \sin\theta_2^* & \cos\theta_2^* & 0 & L_1\sin\theta_2^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$A_3 = \begin{bmatrix} \cos\theta_3^* & -\sin\theta_3^* & 0 & L_2\cos\theta_3^* \\ \sin\theta_3^* & \cos\theta_3^* & 0 & L_2\sin\theta_3^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$A_4 = \begin{bmatrix} \cos\theta_4^* & -\sin\theta_4^* & 0 & L_3\cos\theta_4^* \\ \sin\theta_4^* & \cos\theta_4^* & 0 & L_3\sin\theta_4^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The forward kinematics is:

$$T_0^4 = A_1A_2A_3A_4 \tag{6}$$

Carrying out the multiplication yields:

$$T_0^4 = \begin{bmatrix} \cos\theta_1^*\cos(\theta_2^*+\theta_3^*+\theta_4^*) & -\sin(\theta_2^*+\theta_3^*+\theta_4^*)\cos\theta_1^* & \sin\theta_1^* & p_x \\ \sin\theta_1^*\cos(\theta_2^*+\theta_3^*+\theta_4^*) & -\sin(\theta_2^*+\theta_3^*+\theta_4^*)\sin\theta_1^* & -\cos\theta_1^* & p_y \\ \sin(\theta_2^*+\theta_3^*+\theta_4^*) & \cos(\theta_2^*+\theta_3^*+\theta_4^*) & 0 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where the position components are:

$$p_x = (L_1 \cos\theta_2^* + L_2 \cos(\theta_2^* + \theta_3^*) + L_3 \cos(\theta_2^* + \theta_3^* + \theta_4^*)) \cos\theta_1^* \tag{7}$$

$$p_y = (L_1 \cos\theta_2^* + L_2 \cos(\theta_2^* + \theta_3^*) + L_3 \cos(\theta_2^* + \theta_3^* + \theta_4^*)) \sin\theta_1^* \tag{8}$$

$$p_z = L_1 \sin\theta_2^* + L_2 \sin(\theta_2^* + \theta_3^*) + L_3 \sin(\theta_2^* + \theta_3^* + \theta_4^*) \tag{9}$$

4 Inverse Kinematics

The MCP abduction angle determines the orientation of the flexion plane. Its value is obtained directly from the horizontal projection of the fingertip position:

$$\theta_1^* = \operatorname{atan2}(p_y, p_x) \tag{10}$$

The fingertip coordinates in the flexion plane are therefore obtained by projecting the global position (p_x, p_y, p_z) onto the local flexion-frame defined by θ_1^*. Since the flexion axis is rotated by θ_1^* about the global z-axis, the in-plane fingertip coordinates are:

$$p'_x = \sqrt{p_x^2 + p_y^2} \tag{11}$$

$$p'_z = p_z \tag{12}$$

For the numerical implementation, because $\theta_1^* = \operatorname{atan2}(p_y, p_x)$, the in-plane horizontal coordinate can be expressed either as $p'_x = \sqrt{p_x^2 + p_y^2}$ or, equivalently, $p'_x = p_x \cos\theta_1^* + p_y \sin\theta_1^*$ (see Appendix A for a proof). In this rotated frame, the remaining joints $(\theta_2^*, \theta_3^*, \theta_4^*)$ form a planar 3R chain. The anatomical coupling between the proximal and distal interphalangeal joints is modeled as [1]

$$\theta_4^* = \tfrac{1}{2}\theta_3^* \tag{13}$$

Using this relation, the forward kinematics in the flexion plane becomes

$$p'_x = L_1 \cos\theta_2^* + L_2 \cos(\theta_2^* + \theta_3^*) + L_3 \cos(\theta_2^* + \tfrac{3}{2}\theta_3^*) \tag{14}$$

$$p'_z = L_1 \sin\theta_2^* + L_2 \sin(\theta_2^* + \theta_3^*) + L_3 \sin(\theta_2^* + \tfrac{3}{2}\theta_3^*) \tag{15}$$

The contribution of the first link cannot be eliminated algebraically, and θ_2^* and θ_3^* cannot be decoupled in closed form. The inverse kinematics therefore requires solving the following nonlinear system:

$$\begin{cases} p'_x(\theta_2^*, \theta_3^*) = \sqrt{p_x^2 + p_y^2} \\ p'_z(\theta_2^*, \theta_3^*) = p_z \end{cases} \tag{16}$$

with $\theta_4^* = \frac{1}{2}\theta_3^*$. The solution is obtained numerically by applying a Newton-type method to the above system. Once θ_3^* and θ_2^* have been computed, the distal joint follows from the coupling constraint defined in (13).

References

[1] J. N. A. L. Leijnse, P. M. Quesada, and C. W. Spoor. "Kinematic evaluation of the finger's interphalangeal joints coupling mechanism—variability, flexion-extension differences, triggers, locking swanneck deformities, anthropometric correlations". In: *Journal of Biomechanics* 43.12 (2010), pp. 2381–2393. ISSN: 0021-9290. DOI: 10.1016/j.jbiomech.2010.04.021.

Appendix A: Equivalence of the Flexion-Plane Horizontal Coordinate Expressions

This appendix provides a short proof of the identity

$$p'_x = p_x \cos\theta_1^* + p_y \sin\theta_1^* \qquad \Longleftrightarrow \qquad p'_x = \sqrt{p_x^2 + p_y^2} \tag{17}$$

under the definition

$$\theta_1^* = \operatorname{atan2}(p_y, p_x) \tag{18}$$

By definition of the two-argument arctangent,

$$\cos\theta_1^* = \frac{p_x}{\sqrt{p_x^2 + p_y^2}}, \qquad \sin\theta_1^* = \frac{p_y}{\sqrt{p_x^2 + p_y^2}} \tag{19}$$

Substituting these into the projection formula gives

$$p'_x = p_x \cos\theta_1^* + p_y \sin\theta_1^* \tag{20}$$

$$= p_x \frac{p_x}{\sqrt{p_x^2 + p_y^2}} + p_y \frac{p_y}{\sqrt{p_x^2 + p_y^2}} \tag{21}$$

$$= \frac{p_x^2 + p_y^2}{\sqrt{p_x^2 + p_y^2}} \tag{22}$$

$$= \sqrt{p_x^2 + p_y^2} \tag{23}$$

Hence, the two expressions for the coordinate p'_x in the flexion plane are mathematically equivalent and interchangeable.

To Do

With the proposed content, we have a solid foundation for implementing the truly innovative part of our project. We need to arrange four fingers on a plane, using the middle finger frame as a global reference frame. The other three fingers will have a translation on x and y and a rotation around z (hand completely open), which we can obtain from a simple photograph. After one of us volunteers, we will then measure the length of each phalanx and build the model. From there, we need to define a line parallel to the global y-axis at a given x and z, then solve the inverse kinematics and plot the final joint angles. The parallelism between the y-axis of the end-effector and the direction of the line studied will be entered as a constraint. The goal is to apply a control strategy that allows all end effectors to touch the line at the same time.