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PARAMETER ESTIMATION ASSIGNMENT

Q1. Normal distribution

Given mean $\equiv \theta_1$

variance $\equiv \theta_2$

$$\text{PDF} \equiv f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Joint density for $(X_1, X_2 \text{ -- } X_n) =$

$$J(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Taking log

$$\log_e(J(\theta_1, \theta_2)) = \log_e\left((2\pi\theta_2)^{-\frac{n}{2}} \cdot e^{\frac{-\Sigma(x_i-\theta_1)^2}{2\theta_2}}\right)$$

$$= -\frac{n}{2}\log_e(2\pi\theta_2) - \frac{1}{2\theta_2}\Sigma(x_i-\theta_1)^2$$

Differentiate with respect to θ_1

$$\frac{\partial \log_e J}{\partial \theta_1} = \frac{1}{\theta_2}\sum_1^n (x_i - \theta_1) = 0$$

$$\boxed{\theta_1 = \frac{\Sigma x_i}{n}}$$ (sample mean)

Differentiate with respect to θ_2

$$\frac{\partial \log_e J}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2}\Sigma(x_i-\theta_1)^2 = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2}\Sigma(x_i-\theta_1)^2$$

$$\boxed{\theta_2 = \frac{1}{n}\Sigma(x_i-\theta_1)^2} = \text{variance}$$

Maximum likelihood estimation of θ_1 is sample mean and θ_2 is sample variance

Q2. Binomial distribution (given)

$m \rightarrow$ no. of trails

$\theta \rightarrow (0,1)$ $\quad B(m, \theta)$

PDF of binomial distribution $= {}^{m}C_x \, p^x (1-p)^{m-x}$

Joint density distribution for 'm' trails

$$J(\theta; x_1 \dots x_m) = \prod_{i=1}^{m} P(x_i \mid m, p)$$

$$J(\theta) = \prod_{i=1}^{n} \left({}^{n}C_{x} \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i} \right)$$

Taking $\log_e$

$$\ln(J(\theta)) = \sum_{i=1}^{n} \log({}^{n}C_{x_i}) + \sum_{i=1}^{n} x_i \log\theta + \sum_{i=1}^{n} (m-x_i) \log(1-\theta)$$

Differentiate with θ

$$\frac{\partial \log J(\theta)}{\partial(\theta)} = \frac{1}{\theta} \sum_{i=1}^{n} x_i + \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i)(-1) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i)$$

$$(1-\theta) \Sigma x_i = \theta \Sigma (m-x_i)$$

$$\Sigma x_i = \theta \Sigma (m)$$

$$\boxed{\theta = \frac{\Sigma x_i}{m}} \leftarrow \text{mean}$$

Maximum likelihood estimation of θ is mean.