# Lec-2

## • Expected Utility of a Lottery:

$$U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i)$$

A utility $f^n$ exists for an agent, but they are not specific. An agent's behavior won't change if utility gets transferred:

$$U'(S) = aU(S) + b \; ; \quad a > 0, \; a \;\&\; b \text{ constant.}$$

In a game playing → agent needs preference ranking on states not a number → value $f^n$ or ordial utility $f^n$.

## Utility function:

- ✓ Function to map lotteries to real number.
- ✓ Agents should follow the axioms.
- ✓ An agent can have any preference it likes.

## Utility assessment & utility scale:

Preference Elicitation: Decision theoretic system needs to work on agent's utility $f^n$ to take decision of the agent.

Normalized utilities: Scale of preference -

best possible reward at $U(S) = u\top$

worst " " " $U(S) = u\bot$.

Scale it with $u\top = 1$ & $u\bot = 0$.

Standard lottery: Given the utility scale, the utility of a particular prize S can be assessed by asking the agent to chose betn S & lottery $[p, u_T; (1-p), u_\perp]$. p is adjusted untill the agent becomes indifferent betn S & the lottery:

Micromort → Value for life → million barreled revolver → Car purchase.
one in a million chance of death
(230 miles = 1 micromort
\$50/micromort pay).

You can't kill yoursel for \$50 million

Qualy → quality adjusted life.

## The utility of money

Monotonic Preference → More money over less money.

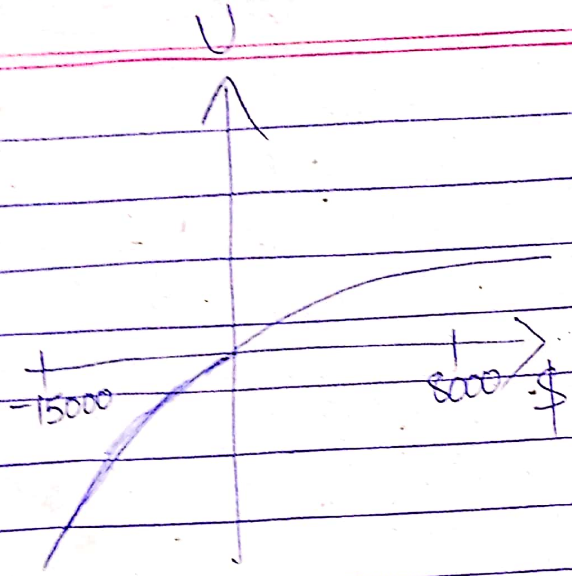
Expected Monetary Value (EMV)

Chose betn \$10000 & a lottery with 0 or \$25000 → Most peaple choses 1st one.
The EVM of the lottery $= \frac{1}{2}(0) + \frac{1}{2}(25000)$
$= 12500 > 10000$, but that does never mean 1st one is wrong.

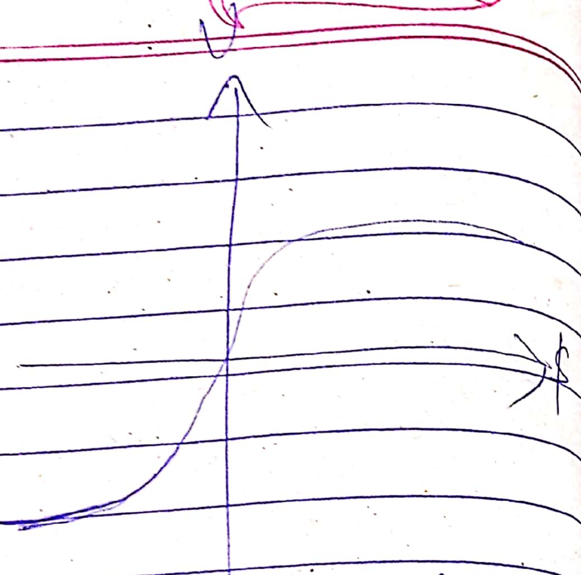
$$EU(Accept) = \frac{1}{2}U(S_k) + \frac{1}{2}U(S_{k+25000})$$

$$EU(Decline) = U(S_{k+10000}).$$

Assign utility 5 to $S_k$, 9 to $S_{k+25000}$ & 8 to $S_{k+10000}$ then EU(Decline) > EU(Accept).




Data over limited range
(People afraid to debt)
Risk average




A typical curve for full range
(Enough debt → less fear
to have more debt)
Risk Seeking

$$U(L) < U(S_{EMV(L)})$$

Value of lottery → Certainty equivalent → the value agent will accept in lieu of a lottery.

E.g.: \$400 instead of a lottery of 0 & \$1000 with equally probable. → EMV of the lottery = \$500

- EMV of a lottery − Certainty equivalent
  = Insurance premium → Insurance companies work on it.

Small amount can be invested for gambling causing almost no change in the linear curve of your wealth → risk-neutral.

Expected utility & Post-decision disappointments:

To chose optimal action:

$$a^* = \underset{a}{\arg\max}\, EU(a|e)$$

It will outcome the best possible action iff:

✓ EU is calculated correctly w.r.t. probability model
✓ Probability model correctly reflects underlying stochastic process,
✓ We will get the expected utility if whole process is repeated many times.

In reality → oversimplification → i) we don't know enough / ii) Computation of true utility is too difficult.

∴ Estimate $\widehat{EU}(a|e)$ of the true utility.

If estimate is unbiased: $E(\widehat{EU}(a|e) - EU(a|e)) = 0$.

But it's not the case in reality.

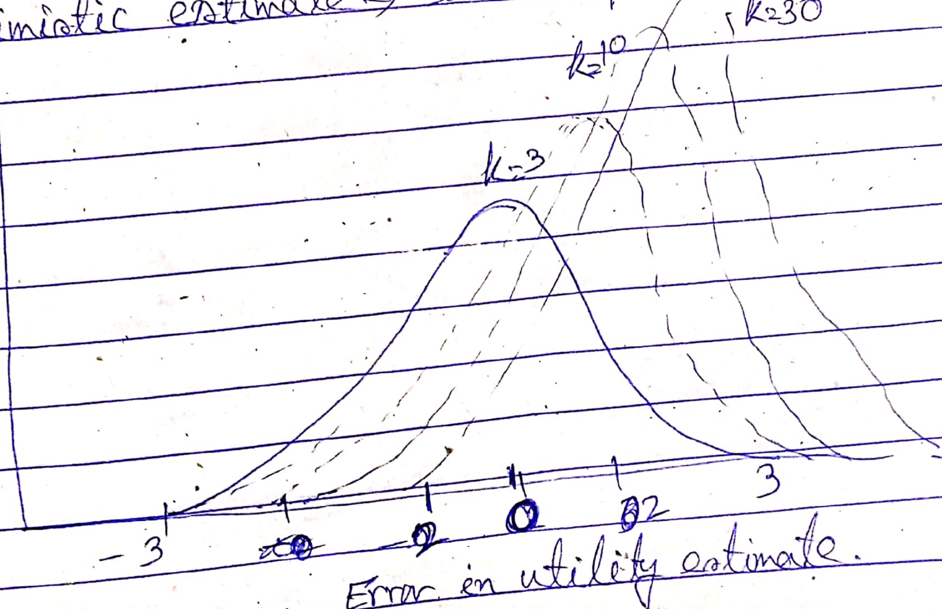
Ex: Decision problem with k choices. Error in each utility estimate → 0 mean & s.d → 1.

During generation of estimates → +ve error / → -ve error

∴ Selecting action with the highest utility ~~estimate~~ estimate → source of bias.
→ overly optimistic estimate → source of bias.

k=3, mean=0.85
∴ disappointment = 85% of s.d.

for k=30, disappointment = 2×s.d.



Optimizer's curse → drug that cured 9 out of 10 patients may probably be worse than the drug that cured 800 out of 1000 patients.

Human judgement & Irrationality:

Normative theory: Describes how rationally the agents should act.

Descriptive theory: How human actually acts.

A: 80% chance of \$4000
B: 100% " " \$3000

C: 20% chance of \$4000
D: 25% " " \$3000

A over B for sure,
C over D higher EMV:
Now, $C \succ D \Rightarrow U(4000\$) > U(\$3000)$ ✓
but, $A \succ B$ reflects the inverse.

Certainty effect: People are strongly attracted to the gain with certainty.

People with distrust will go with certainty to minimize the regret.

1/3 red balls, 2/3 either black or yellow.

A: \$100 for a red ball
B: \$100 for a black ball

C: \$100 for a red & yellow ball
D: \$100 " " black & yellow ball

$A \succ B$, $D \succeq C$
↓
People prefer known probability than unknown
→ Ambiguity aversion.

Scanned with CamScanner

Framing Effect: Same information → framed according to preferences → prefers treatments with 90% survival rate than 10% death rate.

Anchoring Effect: Sale → high estimate generation.

Human irrationality → field of evolutionary psychology → brain doesn't take decisions on probabilities & prizes with decimal number