Tutorial - 2

$X \sim N(10, 36)$

(1) (a) $P(X > 5) = P(X-10 > 5-10)$

$= P\left(\frac{X-10}{6} > \frac{-5}{6}\right) = 1 - P\left(Z < -\frac{5}{6}\right)$

$= 1 - \left\{1 - P\left(Z < \frac{5}{6}\right)\right.$

$= P\left(Z < \frac{5}{6}\right)$

$= P(Z < .83)$

$= 0.79673$

(b) $P(4 < X < 16)$

$= P\left(\frac{4-10}{6} < Z < \frac{16-10}{6}\right) = P(-1 < Z < 1)$

$= \phi(1) - \phi(-1)$

$= \phi(1) - (1 - \phi(1))$

$= 2 \times \phi(1) - 1$

$= 2 \times 0.84134 - 1$

$= 0.68268$

(c) $P(X < 20)$

$= P\left(Z < \frac{10}{6}\right) = \phi(1.67) = 0.95254$

(2) (a) $\cancel{P(X \sim N(1500, (100)^2))}$ $X \sim N(25, 5^2)$

$P(25 < X < 35)$

$= P\left(\frac{0}{5} < Z < \frac{10}{5}\right) = \phi(2) - \phi(0)$

$= 0.97725 - 0.5$

$= 0.47725.$

(b) $P(26 < X < 28) = P\left(\frac{1}{5} < X < \frac{3}{5}\right)$

$= \phi(0.6) - \phi(0.2)$

$= 0.72575 - 0.57926$

$= 0.14649$

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(c) $P(20 < X < 25)$

$= P\left(-\frac{5}{5} < Z < \frac{0}{5}\right) = \Phi(0) - \Phi(-1)$

$= 0.5 - \{1 - .84134\}$

$= 0.5 - 0.15866 = 0.34134$

(d) $P(18 < X < 24)$

$= P\left(-\frac{7}{5} < Z < \frac{-1}{5}\right) = \Phi(-0.2) - \Phi(-1.4)$

$= \Phi(1.4) - \Phi(0.2)$

$= 0.91924 - 0.57926$

$= 0.33998$

(e) $P(19 < X < 30)$

$= P\left(-\frac{6}{5} < Z < \frac{5}{5}\right) = \Phi(1) - \Phi(-1.2)$

$= 0.84134 - (1 - 0.88493)$

$= 0.84134 - 0.11507$

$= 0.72627$

(f) $P(24.5 < X < 25.7)$

$= P\left(\frac{-0.5}{5} < Z < \frac{.7}{5}\right)$

$= \Phi(0.14) - \Phi(-0.1)$

$= .55567 - (1 - .53983)$

$= .55567 - 0.46017$

$= 0.0955$

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(3) $X \sim N(5000, (1000)^2)$

(a) $P(X > 6500)$

$\Rightarrow P\left(Z > \frac{1500}{1000}\right) = 1 - P(Z < 1.5)$

$= 1 - \phi(1.5)$

$= 1 - 0.93319$

$= 0.06681$

(b) $P(5500 < X < 6500)$

$\Rightarrow P\left(\frac{500}{1000} < Z < \frac{1500}{1000}\right)$

$\Rightarrow \phi(1.5) - \phi(0.5) = 0.93319 - 0.69146$

$= 0.24123$

(c) $P(X > 5000) = P(Z > 0) = \frac{1}{2}$

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Ans 4. let X = actual weight in box.

Given - $X \sim N(41.2, (0.8)^2)$

(a.) $Pr(40 < X < 42) = Pr\left(\frac{40-41.2}{0.8} < \frac{X-41.2}{0.8} < \frac{42-41.2}{0.8}\right)$

$= Pr\left(\frac{-1.2}{0.8} < Z < \frac{0.8}{0.8}\right)$, where ~~Z follow~~ $Z \sim N(0,1)$

$= Pr(-1.5 < Z < 1)$

$= \phi(1) - \phi(-1.5)$, where $\phi(z) = Pr(Z \leq z)$

$= \phi(1) - [1 - \phi(1.5)]$ $\{ \because \phi(-z) = 1 - \phi(z) \}$.

$= \phi(1) + \phi(1.5) - 1$

$= 0.84134 + 0.93319 - 1$

$= 0.77453$.

(b.) We want to evaluate value of x such that

$Pr(X < x) = 0.20$.

i.e. $Pr\left(\frac{X-41.2}{0.8} < \frac{x-41.2}{0.8}\right) = 0.20$.

i.e. $Pr\left(Z < \frac{x-41.2}{0.8}\right) = 0.2$, where $Z \sim N(0,1)$ — (1)

By standard normal distribution table,

$Pr(Z < 0.85) \simeq 0.8$

$\Rightarrow Pr(Z < -0.85) \simeq 0.2$ — (2)

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By (1) and (2)

$$\frac{x - 41.2}{0.8} = -0.85$$

$$\Rightarrow \quad x = 41.2 + (0.8)(-0.85)$$

$$= 41.2 - 0.68 = 40.52.$$

(c.) If $x < 40$ then there is re-scrapped cost of Rs. 1.

Now,

$$Pr(X < 40) = Pr\left(\frac{X - 41.2}{0.8} < \frac{40 - 41.2}{0.8}\right)$$

$$= Pr(z < -1.5)$$

$$= 1 - Pr(z < 1.5) = 0.06681$$

# of boxes Containing less than 40 g. out of 100 boxes

$$= 0.06681 \times 100$$

$$= 6.681$$

$$\simeq 7$$

So Rs. 7 is scrapping cost associated with the sale of 100 boxes.

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n-1

Ans. 5. Let $X \sim N(\mu, \sigma^2)$

PDF of r.v. X is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

By definition of pdf,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1.$$

$$\Rightarrow \frac{1}{\sigma}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \sqrt{2\pi}$$

Put $z = \frac{x-\mu}{\sigma}$ | $x \Rightarrow -\infty$ then $z \Rightarrow -\infty$

$\Rightarrow dz = \frac{dx}{\sigma}$ | x tends to ∞ then z tends to ∞

$\therefore$

$$\frac{1}{\sigma}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dz.$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dz = \sqrt{2\pi}}$$

Ans.

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Ans. 6. By chi-square table,

$$P(\chi^2_{25} > 40.646) \leq P(\chi^2_{25} > 38.5) \leq P(\chi^2_{25} > 37.652)$$

$$\Rightarrow \quad 0.025 \leq P(\chi^2_{25} > 38.5) \leq 0.05$$

Ans. 7. By chi-square table

$$a = 9.236.$$

Ans. 8.

(a.) By t distribution table. –

$$P(X > 1.108) \leq Pr(X > 1) \leq Pr(X > 0.889)$$

$$\Rightarrow \quad 0.15 \leq Pr(X > 1) \leq 0.20$$

(b) $Pr(X \leq 2) = 1 - Pr(X > 2)$

$$Pr(X > 2.306) \leq Pr(X > 2) \leq Pr(X > 1.860)$$

$$\Rightarrow \quad 0.025 \leq Pr(X > 2) \leq 0.05$$

$$\Rightarrow \quad \boxed{0.95 \leq Pr(X \leq 2) \leq 0.975}$$

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(c) $Pr(-1 < X < 1) = Pr(X < 1) - Pr(X \le -1)$

$= Pr(X < 1) - [Pr(X \geq 1)]$ { $\because$ t-distribution is Symmetric }

$= Pr(X < 1) - [1 - Pr(X < 1)]$

$= 2\,Pr(X < 1) - 1$

Since $0.15 \le Pr(X \geq 1) \le 0.20$ (By part a)

$\Rightarrow$ $1.6 \le 2\,Pr(X < 1) \le 1.7$

$\Rightarrow$ $\boxed{0.6 \le Pr(-1 < X < 1) \le 0.7}$

Ans.

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