ML-I

50×50

$50 \times 50 = 2500$

$x_i \in \mathbb{R}^{2500}$

$O(2^d)$

$Wx \in \mathbb{R}^{10 \times 1}$

$X = [x_1 \cdots x_n]$

$XX^T u_i = \lambda_i u_i$

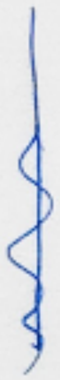
$$U = \begin{array}{c} u_1 \; u_2 \quad\quad u_{10} \\ \begin{bmatrix} | & | & | & | & | \end{bmatrix} \end{array}$$

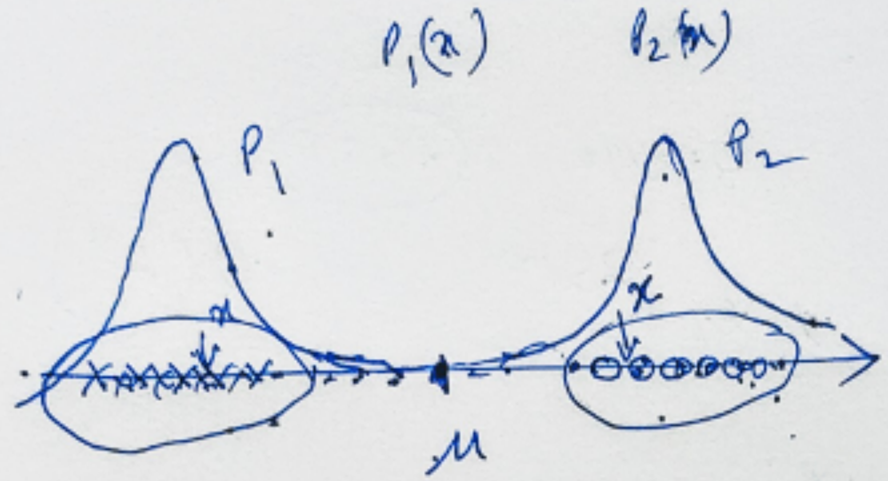
$n \times 10$

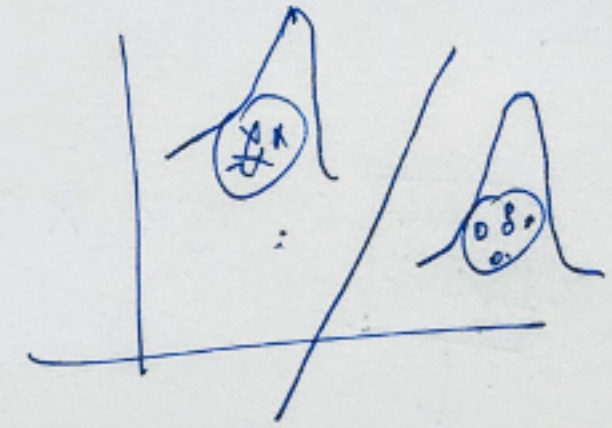
$W = U^T$

$\bar{x}_1 = W x_i$

$\hat{x}_i = U W x_i = U \bar{x}_i$







$$\sum_{i=1}^{n} (h(x_i) - y_i)^2 \Rightarrow$$

$$\mu = \frac{1}{m} \sum_{i=1}^{m} x_i$$

$S = \{x_1, x_2, x_3, x_4, x_5\}$

$= \{1, 0, 1, 1, 0\}$

Prob of servival $\theta = 3/5$

$S = \{x_1, x_2, \ldots, x_m\}$

$$\hat{\theta} = \frac{1}{m} \sum_{i=1}^{m} x_i$$

X and Y are two independent random variables then $P(x, y) = P(x) \cdot P(y)$.

$$P(\{x_1, x_2, x_3, \ldots, x_m\})$$

$$P(\{1, 0, 1, 1, \ldots, 1\}) = \underline{P(x_1)}\ P(x_2) \cdots P(x_m)$$

$$= \prod_{i=1}^{m} \theta^{x_i} (1-\theta)^{(1-x_i)}.$$

$$P(x_i) = \theta^{x_i} \cdot (1-\theta)^{1-x_i} \quad \Big\| \quad \begin{matrix} P(0) = 1-\theta \\ P(1) = \theta \end{matrix}$$

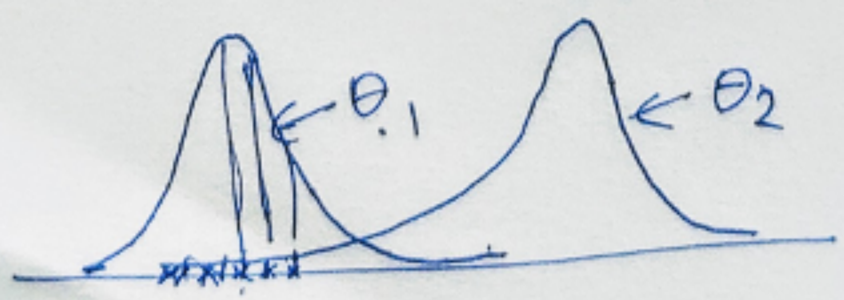
$\cancel{P(0)}$

$$P(0) = \theta^0 (1-\theta)^{1-0} = 1-\theta$$

$$P(1) = \theta^1 \cdot (1-\theta)^{1-1} = \theta$$

$$P(S) = \prod_{i=1}^{m} \theta^{x_i} (1-\theta)^{(1-x_i)}$$

$$\underline{P(S) = \theta^{\sum x_i} \cdot (1-\theta)^{\sum(1-x_i)}} \quad \Big\| \Rightarrow \arg\max_{\theta} P(S) = \hat{\theta}$$



$$P(S) = \theta^{\sum_i x_i} (1-\theta)^{\sum_i (1-x_i)}$$

$$\arg\max_{\theta} P(S) = \arg\max_{\theta} \log(P(S))$$

$$\max_{x} f(x)$$

$$\cancel{=}$$

$$\arg\max_{x} f(x) = \arg\max \log(f(x))$$

$$\log(P(S)) = \log\left[\theta^{\sum_i x_i}(1-\theta)^{\sum_i(1-x_i)}\right]$$

$$= \log\left(\theta^{\sum_i x_i}\right) + \log\left[(1-\theta)^{\sum_i(1-x_i)}\right]$$

$$L(S,\theta) = \sum_i x_i \log\theta + \sum_i (1-x_i)\log(1-\theta)$$

$$\frac{d}{d\theta} L(S,\theta) = \frac{\sum_i x_i}{\theta} - \frac{\sum_i (1-x_i)}{(1-\theta)} = 0$$

$P(x_i) = \theta$

IID

Independent $P(x_1, x_2, \ldots x_m) = P(x_1) \cdot P(x_2) \ldots P(x_n)$

Identical : $P(x_i) \sim \mathcal{N}(\mu, \sigma^2) \quad \forall \; i \in \{1, 2, \ldots m\}$.

$$L(s;\theta) = \left(\sum_{i=1}^{m} \log(P_\theta(x_i)) \right)$$

$$P_\theta(x_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$L(S;\theta) = \sum_{i=1}^{m} \log\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \right)$$

$$= \sum_{i=1}^{m} \log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) + \sum_{i=1}^{m} \log\left(e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}\right)$$

$$L(S,\mu,\sigma) = -\sum_{i=1}^{m} \log(\sigma\sqrt{2\pi}) - \frac{1}{2\sigma^2}\sum_{i=1}^{m} (x_i-\mu)^2$$

$(x_1, x_2) \quad x_i \in \{0, 1\} \quad i = 1, 2$

$P[y = 1 \mid x_1, x_2]$

x_1	x_2	$P[y = 1 \mid x_1, x_2]$
0	0	$P[y=1 \mid 0, 0] = 0.1$
0	1	$P[y=1 \mid 0, 1] = 0.5$
1	0	$P[y=1 \mid 1, 0] = 0.3$
1	1	$P[y=1 \mid 1, 1] = 0.1$

$P[y = 0 \mid x_1, x_2]$
$= 1 - P[y = 1 \mid x_1, x_2]$

2^d

$x = 0, 1$

$$\max_{y \in \{0,1\}} P[Y = y \mid 0, 1]$$

$P[Y = 0 \mid 0, 1] = 0.7$ and $P[Y = 1 \mid 0, 1] = 0.3$

$$P[X = x \mid Y = y]$$

$$= \prod_{i=1}^{m} P[x_i \mid Y = y]$$

A, B

$P(A|B) \cdot P(B) = P(B/A) \cdot P(A)$

$$h(x) = \underset{y \in \{0,1\}}{\arg\max}\; P[Y = y \mid X = x]$$

$$= \underset{y \in \{0,1\}}{\arg\max}\; \frac{P[X = x \mid Y = y] \cdot P[Y = y]}{P[X = x]}$$

$$= \underset{y \in \{0,1\}}{\arg\max}\; P[Y = y]\, \underbrace{P[X = x \mid Y = y]}$$

$$= \underset{y \in \{0,1\}}{\arg\max}\; P[Y = y] \prod_{i=1}^{m} P[x_i \mid y = y]$$

$$P[X_i = x_i \mid y = y]$$

$$d \left\{ \begin{array}{ll} P[X_1 \mid y = 0] & P[X_1 \mid y = 1] \\ P[X_2 \mid y = 0] & P[X_2 \mid y = 1] \\ \vdots & \\ P[X_d \mid y = 0] & P[X_d \mid y = 1] \end{array} \right.$$

$$\underbrace{\qquad\qquad\qquad}_{2}$$

$$2d + 1 \leftarrow P[y = 0] = 1 - P[y = 1].$$