Q Suppose we are about to randomly sample 16 values for a ND population with $\sigma = 8$. Consider the following hypotheses

$$H_0 : \mu = 75$$

$$H_1 : \mu \neq 75$$

At $\alpha = 0.05$, what is the power of test if true mean is 76.

→ for $\alpha = 0.05$, power $= 0.079$ and P(Type II error) $= 0.921 \sim 92\%$.

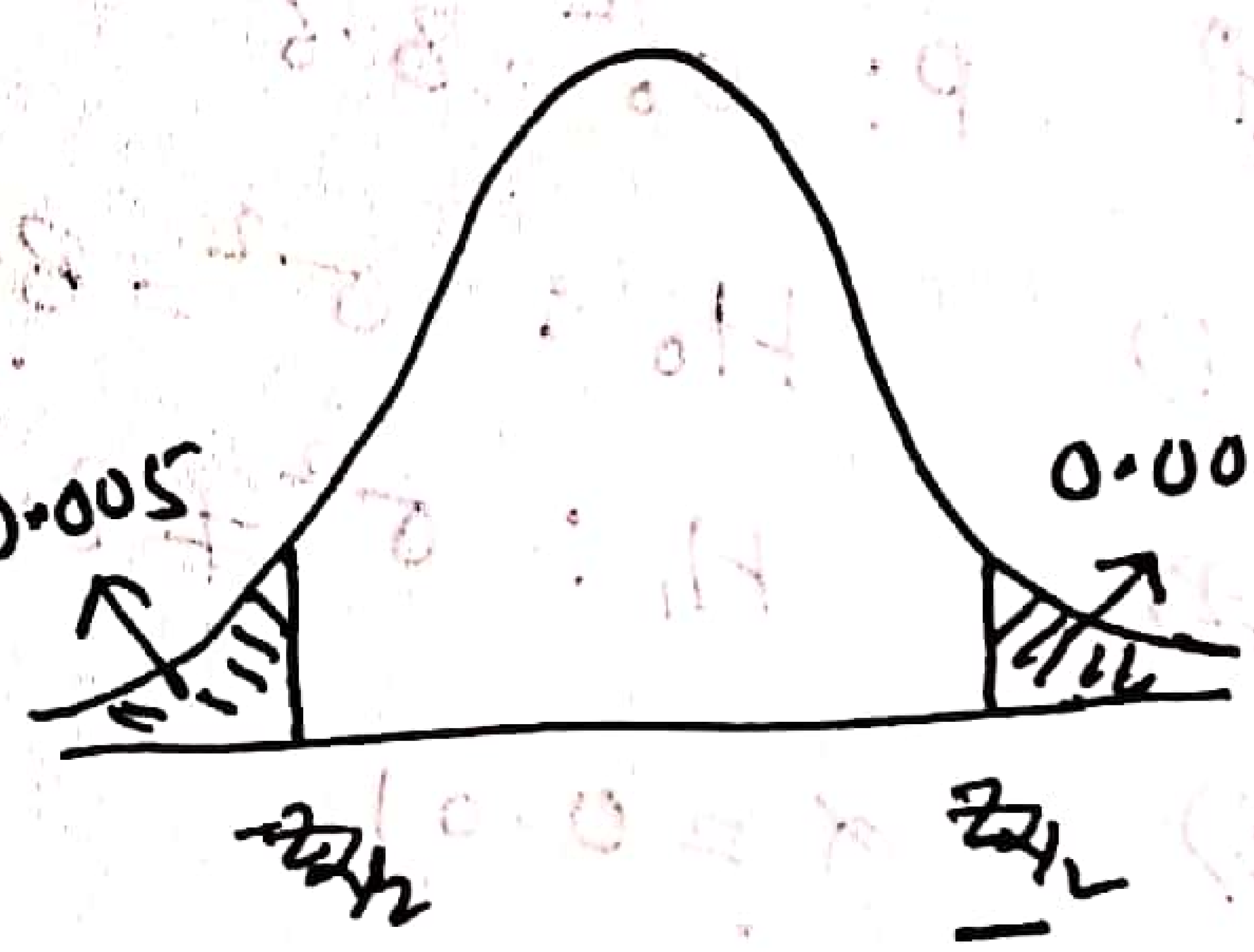
$\alpha = 0.01$ power $= 0.011 \Rightarrow$ P(Type II error) $\approx 0.989 \approx 98\%$.

→ LOS $\equiv \alpha = 0.01$

Test statistic: Z: when Reject H_0

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \quad ; \quad \sigma = 8 \quad \sqrt{n} = \sqrt{16} = 4 \Rightarrow \sigma/\sqrt{n} = 2$$

$$\mu = 75$$



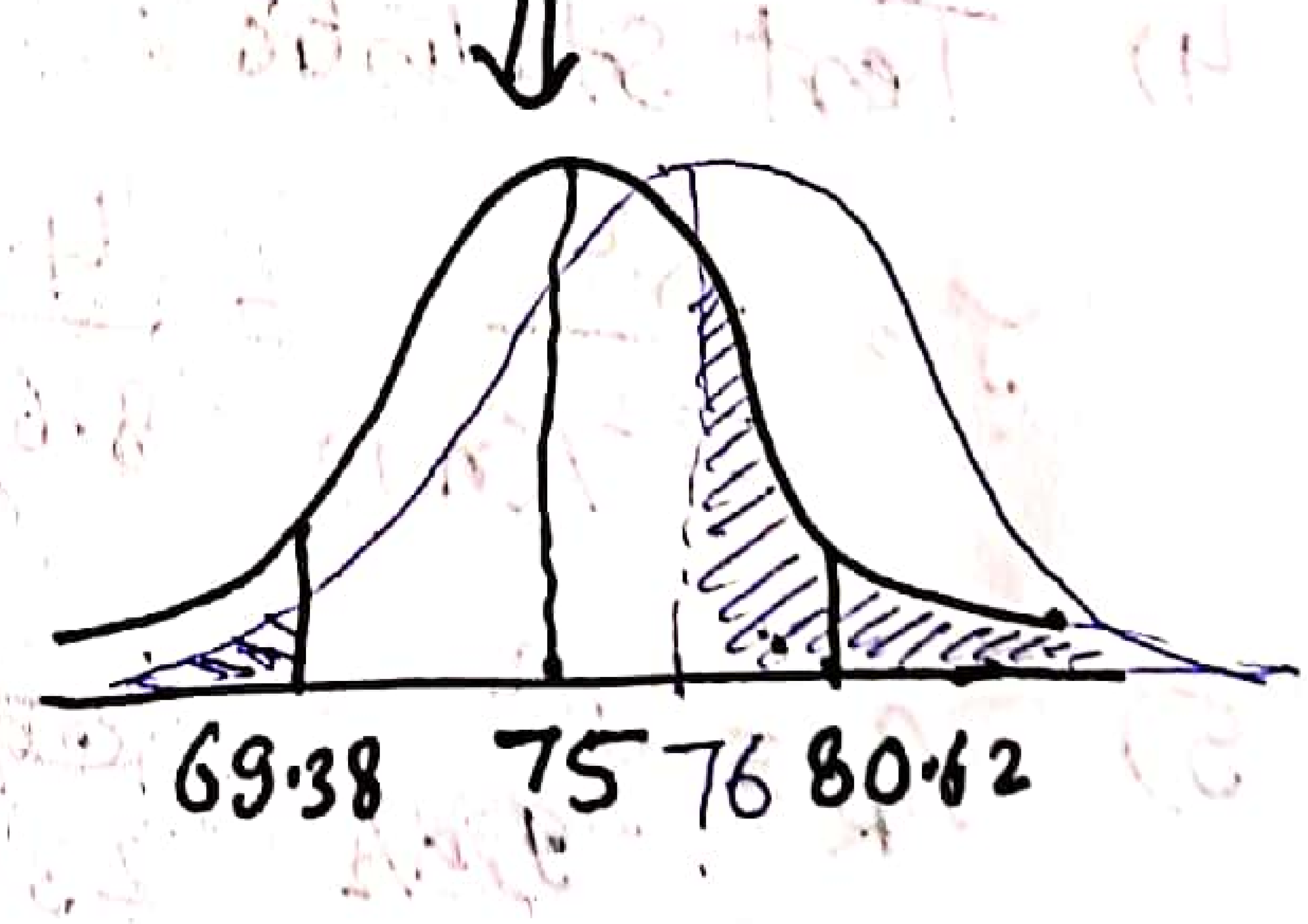
Reject H_0 when

$Z \geq z_{0.005}$ $\qquad Z \leq -z_{0.005}$

$Z \geq 2.81$ $\qquad Z \leq -2.81$

$\frac{\bar{X} - 75}{2} \geq 2.81$ $\qquad \frac{\bar{X} - 75}{2} \leq -2.81$

$\Rightarrow \bar{X} \geq 80.62$ $\qquad \Rightarrow \bar{X} \leq 69.38$



$$\text{power} = P(\bar{X} \leq 69.38 \mid \mu = 76) + P(\bar{X} \geq 80.62 \mid \mu = 76)$$

$$= 0.0109 \approx 0.011$$

* $\boxed{\text{Lower LOS} \Rightarrow \text{Lower Power of the Test} \Rightarrow \text{Higher Prob. of Type II error.}}$

HT for variance:

1) Null Hypothesis
2) Alternative Hypothesis.
3) LOS (α)
4) Test of Statistic ($\mathcal{T}$)
5) $\mathcal{T}$ and critical points to accept or reject the H_0:

(1) Population is ND.

$X_i : i = 1, 2, \ldots, n$

$X_i \sim N(\underbrace{\mu, \sigma^2}_{\text{Unknown}})$

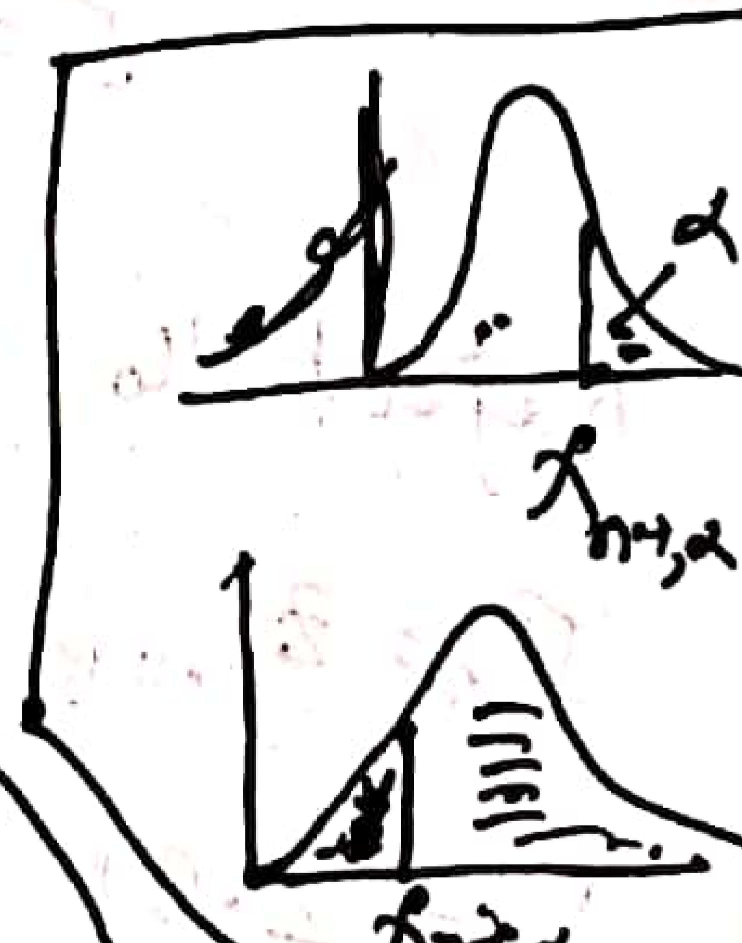
$$\mathcal{T} = \frac{s^2}{\sigma_0^2/(n-1)} \sim \chi^2_{n-1}$$

Q. Coat company claims that the variance of the diameter in certain car tyres is 8.6. A random sample of 10 tyres has a variance of 4.3. At $\alpha = 0.01$, is there enough evidences to reject the company claim.

Sol. P: $\sigma_0^2 = 8.6$ S: $s^2 = 4.3$ $\alpha = 0.01$, $n = 10$

1) $H_0 : \sigma^2 = 8.6$.

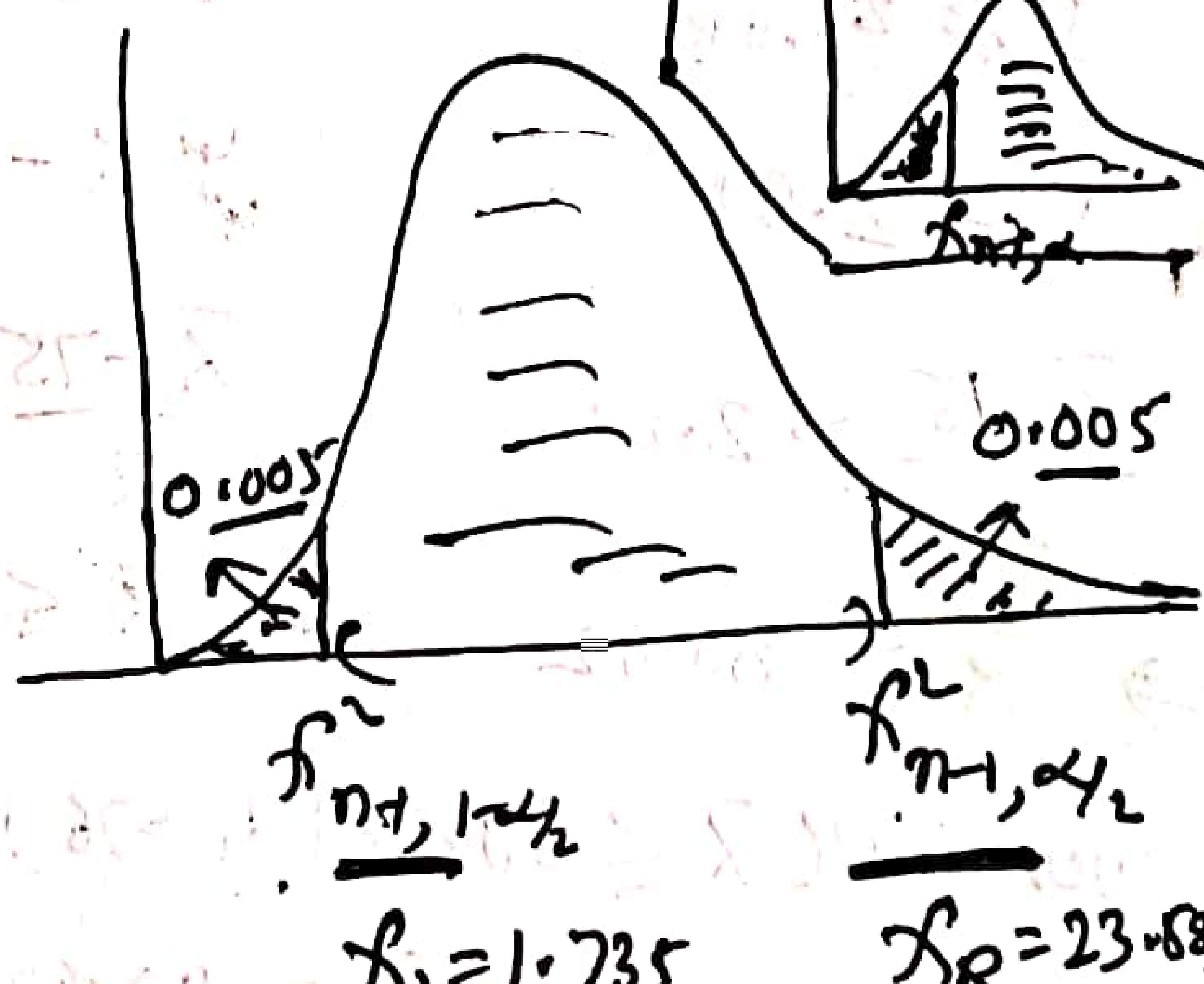
2) $H_1 : \sigma^2 \neq 8.6$ (Two tail analysis)



3) $\alpha = 0.01$

4) Test Statistic:

$$\mathcal{T} = \frac{s^2}{\sigma_0^2/(n-1)} = \frac{4.3}{8.6/9} = 4.5$$



5) $\chi_R \equiv \chi^2_{9, \alpha/2} = \cancel{1.735}$ 23.589

$\chi_L \equiv \chi^2_{9, 1-\alpha/2} = 1.735$

$\Rightarrow \mathcal{T} \in [1.735, 23.589] \Rightarrow$ Accept $H_0 \Rightarrow$ Yes, we have enough evidences to accept the Company claim.

Q The nine items of a sample have the following values

45, 47, 50, 52, 48, 47, 49, 53, 51

Does the mean of these differ significantly from the assumed mean of 47.5?

Sol^n P: $\mu = 47.5$

S: 45, 47, 50, 52, 48, 47, 49, 53, 51

$$\bar{x} = \frac{\Sigma x_i}{n} = 49.1$$

$$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n} = (2.47)^2 \Rightarrow s = 2.47$$

1) $H_0: \mu = 47.5$

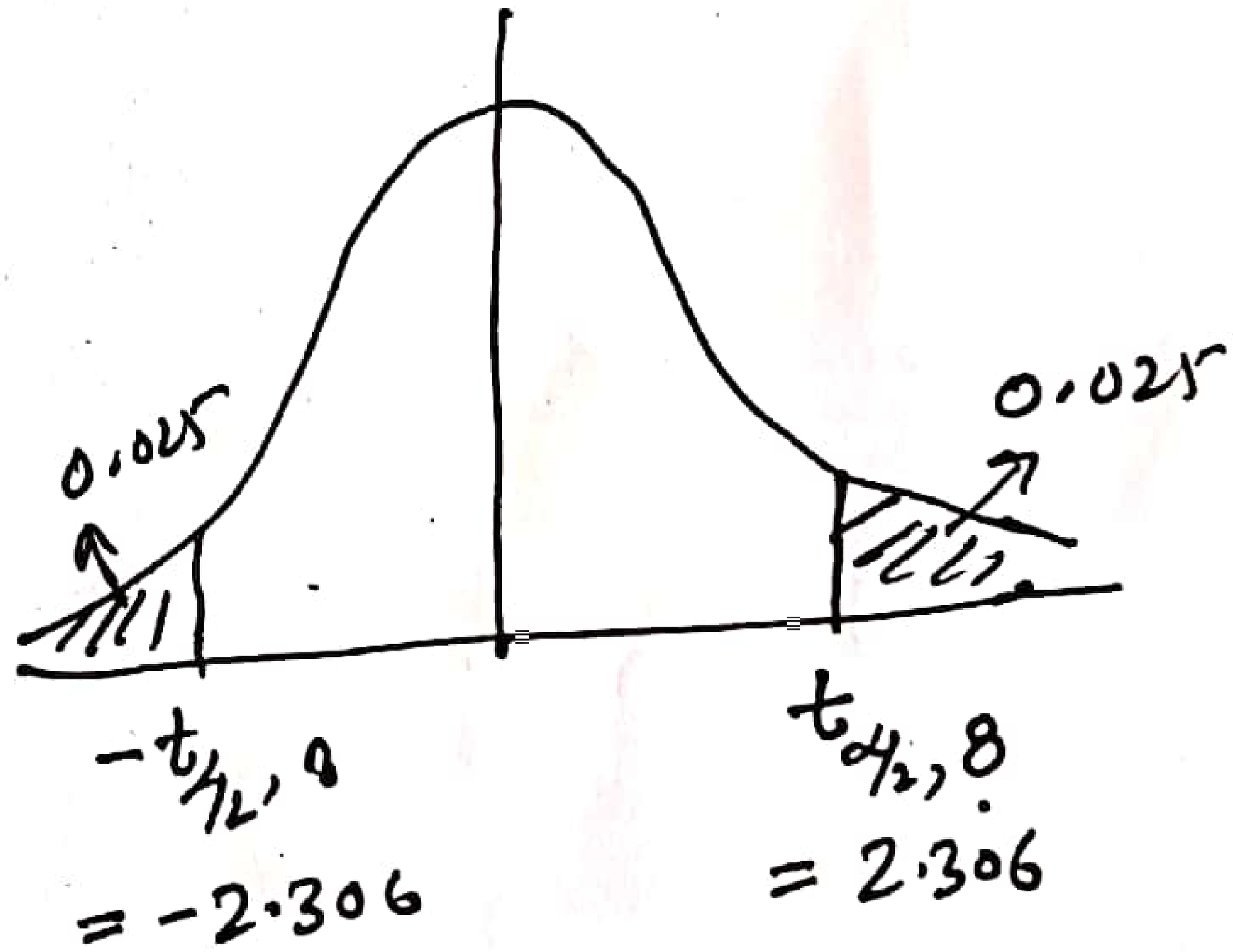
2) $H_1: \mu \neq 47.5$ (Two tail analysis)

3) $\alpha = 0.05$

4) Test Statistic:

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{49.1 - 47.5}{2.47/\sqrt{8}}$$

$$= 1.83$$



5) Critical points:

$$t_{\alpha/2, 8} \equiv t_{0.025, 8} = 2.306$$

$$-t_{\alpha/2, 8} \equiv -t_{0.025, 8} = -2.306$$

$T \in$ Region of Acceptance $\Rightarrow$ Accept H_0

$\Rightarrow$ There is no significant difference in $\bar{x}$ and μ.

Random / Stochastic Processes (SP)

Def A SP is a rule for assigning a function to every $\xi \in S$.

Motivation:

$X : S \to \mathbb{R}$

Tossing of a coin: $S = \{H, T\}$

$$X = \begin{cases} 0 & \text{if } H \\ 1 & \text{if } T. \end{cases}$$

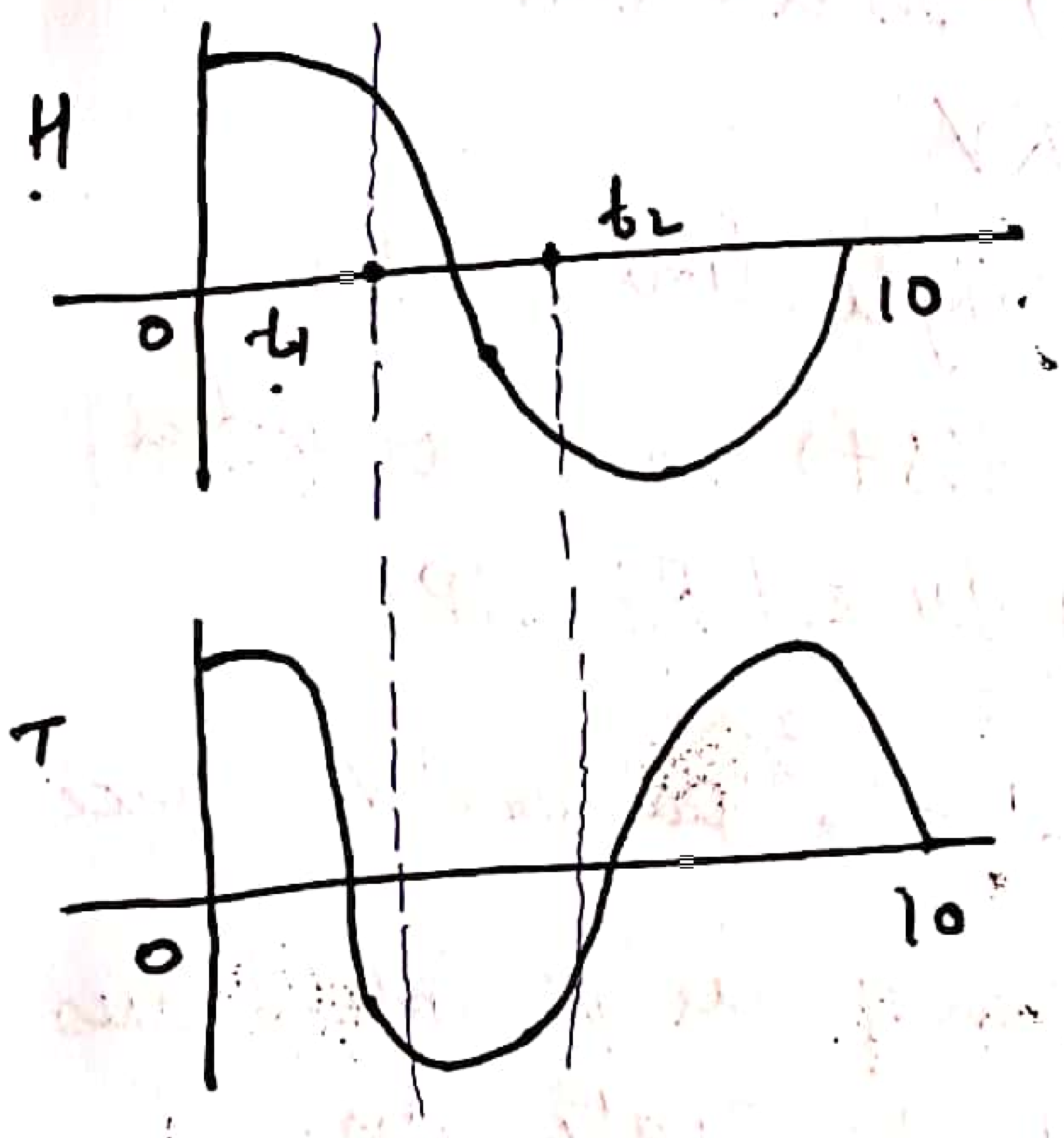
$S = \{H, T\}$

H ⓪ — $\cos \omega_1 t$ — x

T ① — $\cos \omega_2 t$ — x^2

$$\Rightarrow \quad X(t) = \begin{cases} \cos \omega_1 t & H \\ \cos \omega_2 t & T. \end{cases}$$

$t \in [0, 10]$



at time t_1:

$H \Rightarrow \cos \omega_1 t_1 = a$.

$T \Rightarrow \cos \omega_2 t_1 = b$

$$X = \begin{cases} a & \text{if } H. \\ b & \text{if } T \end{cases}$$

$X(t_1) = X_1$

at time t_2:

$H \Rightarrow \cos \omega_1 t_2 = c$

$T \Rightarrow \cos \omega_2 t_2 = d$

$$X(t_2) = X_2 = \begin{cases} c & \text{if } H \\ d & \text{if } T. \end{cases}$$

$X_1, X_2, X_3 - , X_{10}$

$X_1, X_2, X_3, \text{-------}$

eg Rolling of a dice:

$S = \{1, 2, 3, 4, 5, 6\}$

T

$t \in [0, 5]$

OES	1	2	3	4	5	6
Function	$x_1(t) = -4$	$x_2(t) = -2$	$x_3(t) = 2$	$x_4(t) = 4$	$x_5(t) = -t/2$	$x_6(t)$ $t/2$

Def A RP/SP is a collection of RVs $\{X(s,t)\}$ that are function of a real variable (t) where $s \in S$. Consider the domain of $t \in T$ is called the index set.

→ $X(s,t)$

State

Parameter
- time
- length
- Area

Notes:
(1) If s and t are fixed, $X(s,t)$ is a number (real).
2) If t is fixed, $X(s,t)$ is a RV
3) If s is fixed, $X(s,t)$ is a single time function.
4) If s and t both are variable, $X(s,t)$ is a collection of RV's that are the function of time. / RP/SP.

* Parameter space: Set T is called the parameter space.

* State space: The set S (Collection of all possible values of $X(t)$) is called the state space

$S = \{X(t_1), X(t_2) \dots\}$

Representation of SP/RP

$X(t) \approx \{ X(t,s) \mid s \in S,\ t \in T \}$

→ $X(t) \approx \{ X(t, \cdot) \mid t \in T \}$

⇒ it is called the family of RV's.

→ $X(t) \approx \{ X(\cdot, s) \mid s \in S \}$

⇒ Set of functions defined on T.

⇒ Trajectory of function w.r.t t.

Dimension of RP/SP:

The dimension of a RP/SP can be 1, 2, or n depending on the different RV's used.

eg Temperature of Jodhpur

⇒ $X(t): \{ X(t,s) \mid s \in S,\ t \in T \}$

1D RP: Maximum temp of Jodhpur in a day

Day 1	Day 2	Day 3	...
X_1	X_2	X_3	

2D RP: Maximum and Minimum temp of Jodhpur in a day

Day 1.	Day 2	...
$\{X_1^M, X_1^m\}$	$\{X_2^M, X_2^m\}$	...

24D RP Temp of Jodhpur every hour in a day

Day 1 = t_1	Day 2 = t_2	
$\{X_1^1\ X_1^2\ X_1^3 \ldots X_1^{24}\}$	$\{X_2^1\ X_2^2\ X_2^3, \ldots X_2^{24}\}$	...

Classification of RP:

$X(s,t)$ → State, Parameter

Case 1: State Space

finite

$f: S \to N$

S	$X(s,t) \approx X(t)$, $t \in T$
Countable	Discrete state RP
Uncountable	Continuous state RP

Case 2: Parameter Space

T	$X(s,t) \approx X(s)$, $s \in \Omega / S$
Countable	Discrete Parameter RP
Uncountable	Continuous Parameter RP

Case 3: $\{X(s,t) \mid s \in S, t \in T\}$

→ State

↓ Parameter

Parameter \ State	Discrete	Continuous
Discrete	Discrete State Discrete Parameter RP or Discrete Random Sequence	CSDP RP or Continuous Random Sequence
Continuous	DSCP RP OR Discrete Random Process	CSCP RR or Continuous Random Process.

eg If X_n represent the outcome of nth toss of a fair dice. Then $X(s,t)$ is a discrete random sequence.

$$S = \{1, 2, 3, 4, 5, 6\} \quad T = \{1, 2, 3, 4, --, n, --\}$$

$f: N \to N$

eg let X_n represents the temp at the end of the n^{th} hour of a day. $X(s,t)$.

$S = \{ -\infty, \ldots, -1, 0, 1, \overset{1.2}{}2, \ldots \} \Rightarrow$ Any Real value

$S = \mathbb{R} \rightarrow$ uncountable $\Rightarrow$ Continuum

$T = \{1, 2, 3, \ldots, 24\} \Rightarrow$ countable $\Rightarrow$ Discrete

$\Rightarrow$ Continuous Random Sequence.

eg let $X(t)$ represents the # of telephone calls received in the interval $(0, t)$. Then classify the RP $X(s,t)$.