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Making Simple Decisions

(Russel & Norvig, 3rd Edn, Ch-16)

Agent's decision → belief
→ want

Combining belief & desire under uncertainty

Decision theory: Chosing action based on their desirability of their immediate outcome.

Results (s_0, a) → the outcome state from s_0 after taking action a.

- Dealing with partially observable env. → agent may know ct. state ⇒ defining only Result(a) as a random variable.

∴ Probability of outcome s', given evidence e:

$$P(Results(a) = s' | a, e)$$

event that action a executed.

To make S_0 implicit:

$$P(Results(a) = s' | a, e) = \sum_s P(Results(s, a) = s' | a) \, P(S_0 = s | e)$$

Agents preference → quantified with a single number → Utility function $U(s)$.

Expected utility of an action given evidence, $EU(a|e)$ → average utility value of the outcomes, weighted by the probability of occurance of the outcome.

$$\therefore \; EU(a|e) = \sum_{s'} P(Results(a) = s' \mid a, e)\, U(s')$$

Maximum expected utility principle → Agent should chose the action that maximises agent's expected utility.

$$action = \arg\max_{a} EU(a|e).$$

MEU formalizes → agent should do the right thing but partially executes it.

If an agent acts to maximize a utility f^n that correctly reflects the performance measure → agent will achieve highest possible performance score.

Basis of Utility Theory:

MEU → one resonable way but not the only one.

Issues:

- Why maximizing average so special?
- What's wrong with weighted sum maximization or loss minimization?
- Can only preferrence without quantification be allowed?
- Why should such a utility f^n exist at all?

Constraints on rational preferrence:

$A \succ B$ → agent prefers A over B.

$A \sim B$ → agent is indifferent bet^n A & B

$A \succsim B$ → agent prefers A over B or is indifferent.

Agent does have no initial idea of A & B, or how the things going to be: e.g.: Chicken & pasta

∴ Lottery → each option is a ticket.

Lottery L with possible outcomes $S_1, \ldots, S_n$ that occurs with probabilities $p_1, \ldots, p_n$:

$$L = [p_1, S_1; p_2, S_2; \ldots, p_n, S_n].$$

Outcome of a lottery → Another lottery / → Atomic state

Preference to complex lotteries ⇔ Preference bet^n underlying states of those lotteries.

Six constraints:

✓ Orderability: Exactly one of: $(A \succ B)$, $(B \succ A)$ or $(A \sim B)$ holds.

✓ Transitivity: Given 3 lotteries A, B & C: if $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$.

✓ Continuity: If B is in bet^n A & C in preference then there will be some probability p for which

$$A \succ B \succ C \Rightarrow \exists p \, [p, A; 1-p, C] \sim B.$$

✓ Substitutibility: If agent is indifferent bet^n 2 lotteries A & B, then they'll be indifferent bet^n two complex lotteries:

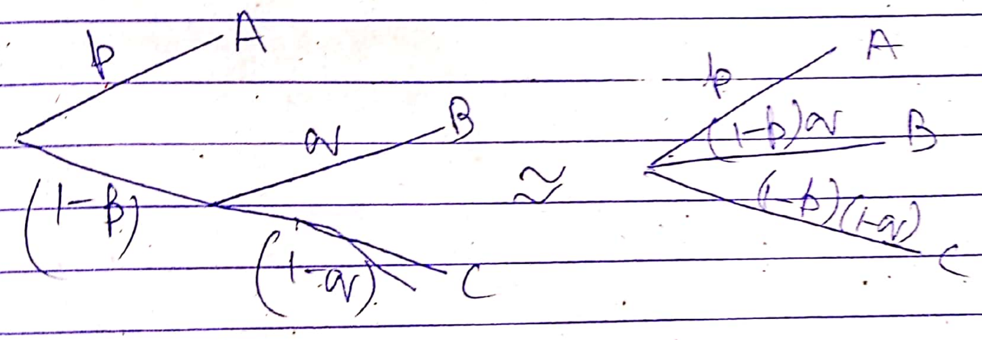
$$A \sim B \Rightarrow [p, A; 1-p, C] \sim [p, B; 1-p, C].$$

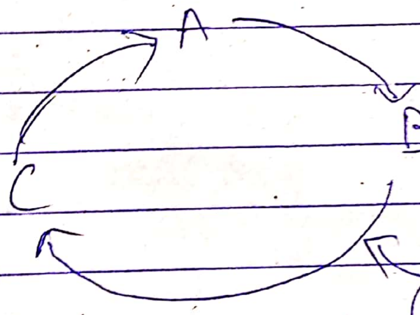
Monotonicity: Lottery has two possible outcome A & B, if agent prefers A, it will then prefer the lottery with higher possibility of A.

$$A \succ B \Rightarrow (p > q \Leftrightarrow [p, A; 1-p, B] \succ [q, A; 1-q, B]).$$

Decomposibility: Compound lotteries can be reduced to simpler ones using laws of probability → No fun gambling.

$$[p, A; 1-p, [q, B; 1-q, C]] \sim [p, A; (1-p)q, B; (1-p)(1-q), C].$$





⇒ ~~Example~~: Non transitive ~~behavior~~ preference results in irrational behavior.

$A \succ B \succ C \succ A$

Preferences lead to utility:

Axioms of utility = axioms of preferrence

- Existence of utility f^n:

$$U(A) > U(B) \Leftrightarrow A \succ B$$
$$U(A) = U(B) \Leftrightarrow A \sim B.$$