2. Now, we know $A = \{\vec{a}_1, \vec{a}_2, \dots \vec{a}_K\}$. This gives a linear transformation of $\vec{x}_n = A\vec{z}_n + \vec{\varepsilon}_n$; $\varepsilon_n \sim \mathcal{N}(0, \psi)$

with a prior of $p(\vec{z}_n) = \mathcal{N}(0, \gamma^{-1} I_K)$.

and likelihood $p(\vec{x}_n | \vec{z}_n, A) = \mathcal{N}(x_n | A\vec{z}_n + b, \psi)$

Using Baye's Theorem for gaussians,

$p(\vec{z}_n) = \mathcal{N}(z_n | 0, \Lambda^{-1})$ where $\Lambda = \gamma I_K$

and $p(\vec{x}_n | \vec{z}_n, A) = \mathcal{N}(\vec{x}_n | A\vec{z}_n, L^{-1})$ where $L = \psi^{-1}$

We know that

$$p(\vec{z}_n | \vec{x}_n, A) = \mathcal{N}(\vec{z}_n | \Sigma\{A^T L \vec{x}_n + \Lambda \cdot 0\}, \Sigma) \quad ; \Sigma = (\Lambda + A^T L A)^{-1}$$

$$= \mathcal{N}(\vec{z}_n | (\gamma I_K + A^T \psi^{-1} A)^{-1} A^T \psi^{-1} \vec{x}_n, (\gamma I_K + A^T \psi^{-1} A)^{-1})$$

since z_n only depends on $\vec{x}_n$ and <u>not</u> $\vec{x}_K$ for $K \neq n$, therefore,

$$p(\vec{z}_n | \cancel{x_n}, A) = \mathcal{N}(\vec{z}_n | ((\gamma I_K + A^T \psi^{-1} A)^{-1} A^T \psi^{-1} x_n), (\gamma I_K + A^T \psi^{-1} A)^{-1})$$