Given spike and slab prior $p(w|b, \sigma^2_{sp}, \sigma^2_{sl}) = \begin{cases} \mathcal{N}(w|0, \sigma^2_{sp}) \; ; b=0 \\ \mathcal{N}(w|0, \sigma^2_{sl}) \; ; b=1. \end{cases}$

This is modelled by bernoulli b; s.t. $b=1$ with probability $\pi = \frac{1}{2}$
and $b=0$ with probability $(1-\pi) = \frac{1}{2}$

Then, $p(w|b, \sigma^2_{sp}, \sigma^2_{sl}) = \mathcal{N}(w|0, (\sigma^2_{sp})^{1-b}(\sigma^2_{sl})^{b})$

Marginal prior $p(w|\sigma^2_{sl}, \sigma^2_{sp}) = \sum_{b\in\{0,1\}} p(w|b, \sigma^2_{sl}, \sigma^2_{sp})\, p(b)$

$$= \frac{1}{2}\mathcal{N}(w|0, \sigma^2_{sp}) + \frac{1}{2}\mathcal{N}(w|0, \sigma^2_{sl})$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi\sigma^2_{sp}}} \exp\left\{\frac{-w^2}{2\sigma^2_{sp}}\right\} + \frac{1}{\sqrt{2\pi\sigma^2_{sl}}} \exp\left\{\frac{-w^2}{2\sigma^2_{sl}}\right\}\right]$$

A mixture of 2 gaussians

plot for $p(w|\sigma^2_{sl}=100, \sigma^2_{sp}=1)$

Now, considering a noisy version of w, modelled by $x = w + \varepsilon$, $\varepsilon \sim \mathcal{N}(\varepsilon|0, \rho^2)$

$\Rightarrow$ since w and ε are independent variables,

and $w \sim \mathcal{N}(w|0, (\sigma^2_{sp})^{1-b}(\sigma^2_{sl})^{b})$, $\varepsilon \sim \mathcal{N}(\varepsilon|0, \rho^2)$

we can write x as $x \sim \mathcal{N}(x|0, \rho^2 + (\sigma^2_{sp})^{1-b}(\sigma^2_{sl})^{b})$

[using the results that $x = w + \varepsilon \Rightarrow \mu_x = \mu_w + \mu_\varepsilon$ and $\Sigma_x = \Sigma_w + \Sigma_\varepsilon$]

Thus,

$$p(b, x|\rho^2, \sigma^2_{sp}, \sigma^2_{sl}) = \mathcal{N}(x|0, \rho^2 + (\sigma^2_{sp})^{1-b}(\sigma^2_{sl})^{b})\, p(b)$$