Also, $p(b \mid x, \rho^2, \sigma^2_{sc}, \sigma^2_{sp}) = \dfrac{p(b, x \mid \rho^2, \sigma^2_{sc}, \sigma^2_{sp})}{p(x \mid \rho^2, \sigma^2_{sc}, \sigma^2_{sp})}.$

$$= \frac{\mathcal{N}\left(x \mid 0, \rho^2 + (\sigma^2_{sp})^{1-b} (\sigma^2_{sc})^{b}\right) \times \frac{1}{2}}{\mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sp}) \times \frac{1}{2} + \mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sc}) \times \frac{1}{2}}$$

$$p(b=1 \mid x, \rho^2, \sigma^2_{sp}, \sigma^2_{sc}) = \frac{\mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sc})}{\mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sc}) + \mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sp})}$$

Plot of $p(b=1 \mid x, \rho^2, \sigma^2_{sp}, \sigma^2_{sc})$ with $\rho^2 = 0.01$, $\sigma^2_{sp} = 1$, $\sigma^2_{sc} = 100$

posterior distribution, $p(w \mid x, \rho^2, \sigma^2_{sp}, \sigma^2_{sc}) = \dfrac{p(x \mid w)\, p(w)}{p(x)}$ [Ignoring $\rho^2, \sigma^2_{sp}, \sigma^2_{sc}$]

observe that since $x = w + \varepsilon$, $\varepsilon \sim \mathcal{N}(\varepsilon \mid 0, \rho^2)$,

~~it follows that~~ $\cancel{w = x + \varepsilon}$, $\cancel{\varepsilon \sim \mathcal{N}(\varepsilon \mid -0, \rho^2) = \mathcal{N}(\varepsilon \mid 0, \rho^2)}$.

$\Rightarrow p(x \mid w) = \mathcal{N}(x \mid w, \rho^2)$

also, $p(w) = \frac{1}{2}\left[\mathcal{N}(w \mid 0, \sigma^2_{sp}) + \mathcal{N}(w \mid 0, \sigma^2_{sc})\right]$

and $p(x) = \frac{1}{2}\left[\mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sp}) + \mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sc})\right]$

$$\Rightarrow p(w \mid x, \rho^2, \sigma^2_{sp}, \sigma^2_{sc}) = \frac{\mathcal{N}(x \mid w, \rho^2)\left[\mathcal{N}(w \mid 0, \sigma^2_{sp}) + \mathcal{N}(w \mid 0, \sigma^2_{sc})\right]}{\mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sp}) + \mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sc})}$$

and, $p(x \mid w, \rho^2) \triangleq p(w \mid x, \rho^2)$.

$$\Rightarrow p(w \mid x, \rho^2, \sigma^2_{sp}, \sigma^2_{sc}) = \frac{\mathcal{N}(w \mid x, \rho^2)\left[\mathcal{N}(w \mid 0, \sigma^2_{sp}) + \mathcal{N}(w \mid 0, \sigma^2_{sc})\right]}{\mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sp}) + \mathcal{N}(x \mid 0, \rho^2 + \sigma^2_{sc})}$$