

# Title

## Tools for Quantum State Distinguishability and Exclusion

### Abstract

This project aims to develop a comprehensive module within the `toqito` library to analyze quantum state distinguishability and exclusion. The module will take an ensemble of quantum states as input and perform various "figure-of-merit" analyses, utilizing semidefinite programming (SDP) solvers. The core of this development will involve applying SDP to address various figures of merit relevant to these tasks, including minimum-error discrimination, unambiguous discrimination, and state exclusion. The new module will provide a general framework capable of handling diverse sets of quantum states and measurement constraints. It will be designed to integrate seamlessly with popular Python SDP solvers, such as `cvxpy` and `picos`, offering users flexibility in their choice of optimization backend. Furthermore, the project includes a commitment to creating thorough documentation, complete with illustrative examples, to ensure the module is accessible and user-friendly. This enhancement is significant, as it will enhance `toqito`'s capabilities in a fundamental area of quantum information theory, providing researchers and students with a powerful tool to explore the theoretical limits of extracting information from quantum systems. The expected outcome is a well-documented and rigorously tested Python module that substantially expands the functionality of the `toqito` library in the domain of quantum state distinguishability and exclusion.

### Technical Details

The proposed module will be built within or on the `toqito.state_opt` module and will:

- **Input:** Accept an ensemble of quantum states as density matrices or state vectors and their probability distribution.
- **Processing:** Formulate the corresponding optimization problems as SDPs:
  - *Minimum-Error Discrimination:* Maximize the probability of correct state identification.
  - *Unambiguous Discrimination:* Minimize the probability of inconclusive outcomes while ensuring error-free decisions.
  - *State Exclusion:* Determine the minimum error probability for excluding states, including the verification of anti-distinguishability.
- **Output:** Return the computed figures-of-merit and, where applicable, the optimal measurement operators (POVMs).

The module will be implemented primarily using the `cvxpy` solver for SDP, with an exploratory stretch goal of integrating the `picos` solver to provide users with additional flexibility.

## Code Architecture

- **Modular Design:** The codebase is structured into distinct modules for various discrimination strategies, facilitating easy integration and extension.
- **Comprehensive Documentation and Tutorials:** We provide detailed documentation and practical tutorials, enabling users to understand and utilize the functionalities effectively. The module will be primarily implemented using the cvxpy solver for SDP, with an exploratory stretch goal of integrating the picos solver for added flexibility. Additionally, the codebase will feature a modular design, structured into distinct modules for various discrimination strategies, facilitating easy integration and extension. Comprehensive documentation and tutorials will also be provided and also if possible few research experiment examples would be nice, enabling users to understand and utilize the functionalities effectively.

## Schedule of Deliverables

### Community Bonding Period

- Establish the development environment and install required libraries (toqito, cvxpy, picos).
- Deep dive into the existing toqito codebase (modules such as ppt\_distinguishability.py, state\_distinguishability.py, etc.) and review relevant SDP literature.
- Refine project goals and create a detailed work plan.
- Set up a blog and share the link.
- Wrap up existing code related to state properties like anti-distinguishability and its dependent code.

### Phase 1 (June 2 - July 14) (Continued)

- Design and implement the SDP formulation for minimum-error discrimination.
- Develop functions to compute the maximum success probability and optionally retrieve the optimal POVM.
- Write unit tests to validate the implementation against known cases.
- Implement the SDP formulation for unambiguous state discrimination.
- Focus on minimizing inconclusive outcomes while ensuring error-free discrimination.
- Develop corresponding unit tests and validate the approach for linearly independent state ensembles.
- Formulate and implement the SDP for quantum state exclusion.
- Incorporate functions to verify antidistinguishability.
- Ensure robust error handling and numerical stability, accompanied by comprehensive testing.
- Finalize Phase 1 deliverables and submit code for review.
- Write a blog post summarizing Phase 1 achievements.

## **Phase 2 (July 15 - August 25)**

- Work on advanced features and enhancements (additional figure-of-merit implementations).
- Unit test writing and code quality maintenance.
- Seamlessly integrate the new module with the rest of the Toqito library.
- Develop detailed documentation with clear function descriptions, usage examples, and tutorials.
- Perform extensive testing and incorporate feedback from mentors and the community.
- (Stretch Goal) Explore integration with the picos solver as an alternative optimization backend.
- Work on examples and demos showcasing real research workflows and core functionality of the module.
- Address performance and scalability issues.
- Write a blog post highlighting Phase 2 progress.
- (Stretch Goal) Research conducting experiments using this new module.
- Expand documentation and user guide.
- Finalize Phase 2 deliverables and submit code for review.
- Write a blog post summarizing Phase 2 achievements.

## **Final Week**

- **Remaining task completion and code submission/merging.**
- **Documentation polishing and final project report preparation.**
- **Final blog post reflecting on GSoC experience and future plans.**

## **Development Experience**

- Contributed to related issues and feature requests like quantum common overlap and intro tutorials for quantum channels and noisy channels.
- Developed an open-source high-performance quantum simulator during the QOSF program under the mentorship of Parasa Vamsi (Intel Senior R&D Engineer) using Julia. The project leveraged hardware accelerators like TPUs and achieved state-of-the-art performance (20 x Grover's circuit on 30 qubits in under 32 seconds with 8 GB RAM and 4 threads).
- Implemented Hybrid Quantum Neural Networks for conspicuity detection in the TIG Aluminium dataset
- Worked on utilizing the HHL algorithm for computational physics, specifically atmosphere modeling, and demonstrated quantum advantage when working with implicit finite difference methods for simulation.
- Developed and trained a vision transformer model on an annotated Canadian vehicle dataset for object detection as a term project for a data science course at IITR.

## Other Experiences

- Completed university courses in quantum mechanics - I, introduction to quantum information theory, scientific computing with Python and currently pursuing optimization techniques course
- Organized IBM's Qiskit fall fest 2024 at IIT Roorkee
- Leadership experience as a senior member at the university's quantum computing club, where I conducted workshops on quantum machine learning and demonstrated motorized radio telescopes developed at IIT Roorkee.
- Participated in multiple hackathons, including placing second in a quantum computing hackathon during Cognizance. These experiences have equipped me with collaborative and problem-solving skills essential for a successful GSOC project.
- Experience working in distributed teams and using agile methodologies, which will be valuable in meeting project deadlines and milestones. Completed university courses in quantum mechanics I, introduction to quantum information theory, and scientific computing with Python and am currently pursuing a course in optimization techniques
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## Why this project?

My deep interest in quantum information and computation drives my desire to explore the potential of quantum systems in computing and information processing. Active participation in this project would not only allow me to enhance crucial skills like linear algebra, convex optimization, and quantum information theory but also contribute to the quantum computing community. By contributing to the toqito library, I could aid researchers and developers, potentially leading to new discoveries or practical applications in quantum information theory.

## Appendix

### Reference materials

- **Quantum State Discrimination and Its Applications**  
Bae, J., & Kwek, L.-C. (2015). *Quantum state discrimination and its applications*. Journal of Physics A: Mathematical and Theoretical, 48(8), 083001.  
<https://doi.org/10.1088/1751-8113/48/8/083001>
- **How to Differentiate Between Non-Orthogonal States (Ivanovic)**  
Ivanovic, I. D. (1987). *How to differentiate between non-orthogonal states*. Physics Letters A, 123(5), 257–259.  
[https://doi.org/10.1016/0375-9601\(87\)90222-2](https://doi.org/10.1016/0375-9601(87)90222-2)
- **Overlap and Distinguishability of Quantum States (Dieks)**  
Dieks, D. (1988). *Overlap and distinguishability of quantum states*. Physics Letters A, 126(5), 303–306.  
[https://doi.org/10.1016/0375-9601\(88\)90840-7](https://doi.org/10.1016/0375-9601(88)90840-7)
- **How to Differentiate Between Non-Orthogonal States (Peres)**  
Peres, A. (1988). *How to differentiate between non-orthogonal states*. Physics Letters A, 128, 19.  
[https://doi.org/10.1016/0375-9601\(88\)91034-1](https://doi.org/10.1016/0375-9601(88)91034-1)
- **Optimal Discrimination of Quantum States with a Fixed Rate of Inconclusive Outcomes**  
Bagan, E., Muñoz-Tapia, R., Olivares-Rentería, G. A., & Bergou, J. A. (2012). *Optimal discrimination of quantum states with a fixed rate of inconclusive outcomes*. Physical Review A, 86(4), 040303.  
<https://doi.org/10.1103/PhysRevA.86.040303>
- **Optimal State Discrimination with a Fixed Rate of Inconclusive Results**  
Herzog, U. (2012). *Optimal state discrimination with a fixed rate of inconclusive results: Analytical solutions and relation to state discrimination with a fixed error rate*. Physical Review A, 86(3), 032314.  
<https://doi.org/10.1103/PhysRevA.86.032314>
- **CVXPY: A Python-Embedded Modeling Language for Convex Optimization**  
<https://www.cvxpy.org/>
- **Picos Documentation**  
<https://picos-api.gitlab.io/picos/>
- **Improvements in Quantum SDP-Solving with Applications**  
van Apeldoorn, J., & Gilyén, A. (2018). *Improvements in Quantum SDP-Solving with Applications*. arXiv:1804.05058.  
<https://doi.org/10.48550/arXiv.1804.05058>