O'DONNELL GROUP

Heterosynaptic plasticity rules induce small-world network topologies

James McAllister, John Wade, Cian O'Donnell

Intelligent Systems Research Centre, Ulster University

Introduction

Heterosynaptic plasticity is a form of 'off-target' synaptic plasticity where unstimulated synapses change strength. Here we propose that one purpose of heterosynaptic plasticity is to encourage small-world connectivity [5, 6]. We compare different plasticity rules in abstract weighted graphs, finding that they yield distinct network architectures.

Heterosynaptic plasticity

Heterosynaptic plasticity is where synapses that were not directly activated undergo weight changes.

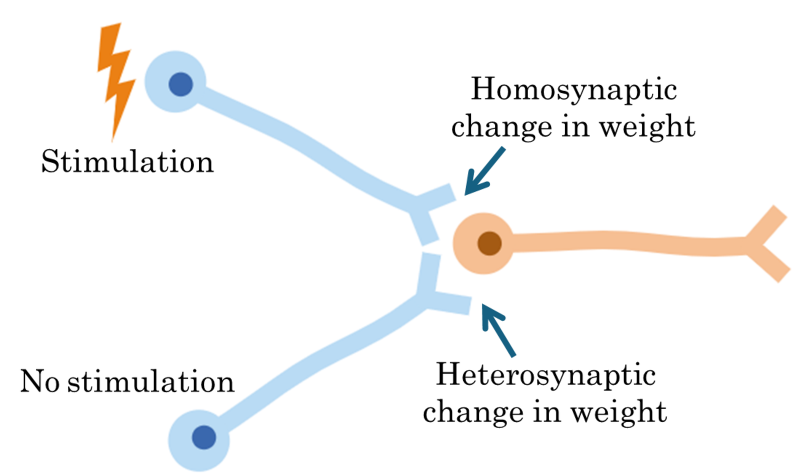


Figure 1. Illustration of homosynaptic and heterosynaptic plasticity

It can assume either a **cooperative** or **competitive** role in the alteration of synaptic weights (Figure 2).

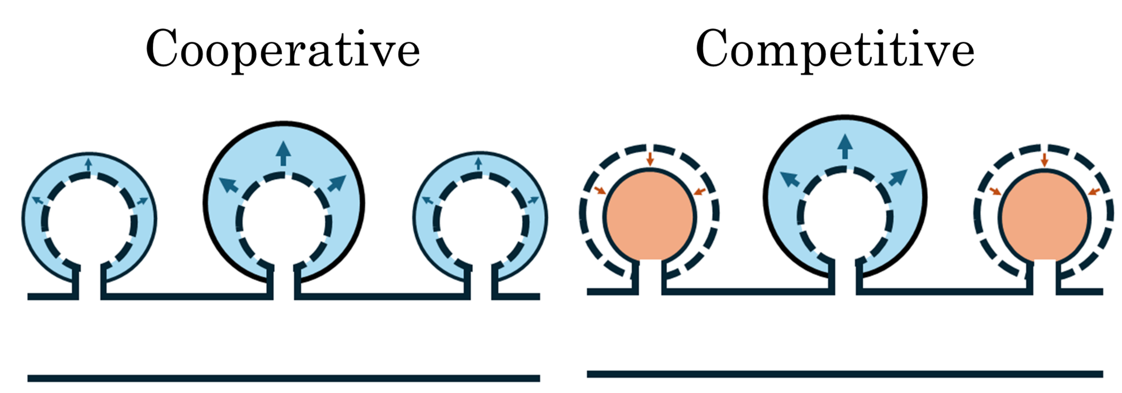


Figure 2. Cooperative (left) and competitive (right) directions of weight change in heterosynaptic plasticity

Heterosynaptic plasticity can operate locally on single dendrites at neighbouring spines, or across neurons and whole networks [3].

Method: Heterosynaptic plasticity network model

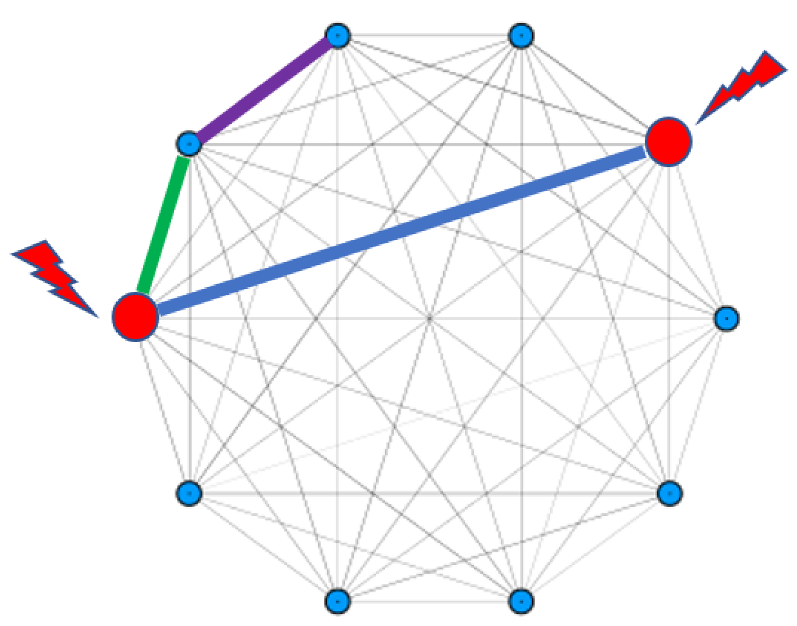


Figure 3. Cases for weight updates in model

Each edge is either

(1) between two active nodes ,
(2) between one active and one inactive node, or
(3) between two inactive nodes .

Based on these cases, we define the update rules:

$$R_1 = \begin{cases} w_{i,j_{n+1}} = w_{i,j_n} + \eta_1(1 - w_{i,j_n}) & \text{Case (1)} \\ w_{i,j_{n+1}} = \eta_2 w_{i,j_n} & \text{otherwise} \end{cases}$$

$$R_2 = \begin{cases} w_{i,j_{n+1}} = w_{i,j_n} + \gamma_1(1 - w_{i,j_n}) & \text{Case (1)} \\ w_{i,j_{n+1}} = \gamma_2 w_{i,j_n} & \text{Case (2)} \\ w_{i,j_{n+1}} = \gamma_3 w_{i,j_n}, & \text{Case (3)} \end{cases}$$

$$R_3 = \begin{cases} w_{i,j_{n+1}} = w_{i,j_n} + \kappa_1(1 - w_{i,j_n}) & \text{Case (1)} \\ w_{i,j_{n+1}} = w_{i,j_n} + \kappa_2(1 - w_{i,j_n}) & \text{Case (2)} \\ w_{i,j_{n+1}} = \kappa_3 w_{i,j_n}, & \text{Case (3)} \end{cases}$$

where R_1 is a homosynaptic rule, and R_2 and R_3 are versions of competitive/cooperative heterosynaptic rules.

η_i, γ_i and κ_i are learning parameters. We set

$\eta_1 = \gamma_1 = \kappa_1 = 0.2$

$\eta_2 = 1 - \eta_1 = 0.8$ $\qquad \gamma_2 = 1 - \gamma_1/2 = 0.9$

$\eta_2 = \gamma_3 = \kappa_3 = 0.8$ $\qquad \kappa_2 = \kappa_1/2 = 0.1$

Method: Activity patterns

Activity patterns based on different Beta distributions (with varying parameters α, β).

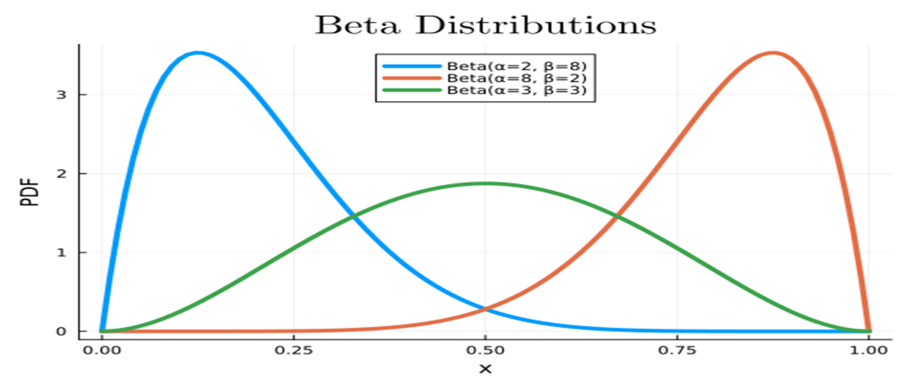


Figure 4. Beta distributions

Method: Graph theory measures

- Weighted clustering coefficient:

$$\tilde{C} = \int_0^1 C_t \, dt$$

where $C_t = C(A_t)$ for $A^t_{ij} = 1$ if $w_{ij} \geq t$ and 0 otherwise.

$$C_i = \frac{|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}$$

- Average shortest path length (Dijkstra's algorithm)

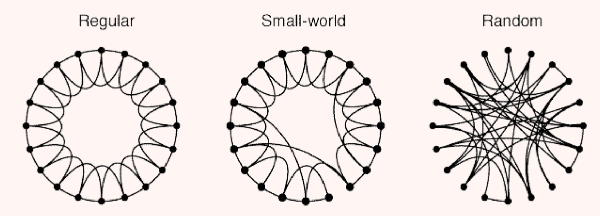
$$L = \frac{1}{n(n-1)} \sum_{\substack{i,j \in V \\ i \neq j}} d_{ij}$$

- Small-world measure ↓

Small-world topologies

A small-world network [6] has high degrees of clustering & low average shortest path length.

$$\sigma = \frac{C/C_{rand}}{L/L_{rand}} > 1$$



Results

Below is a simulation of how the network characteristics evolve under the homosynaptic (R1) and heterosynaptic (R2 & R3) plasticity rules over 100 timesteps.

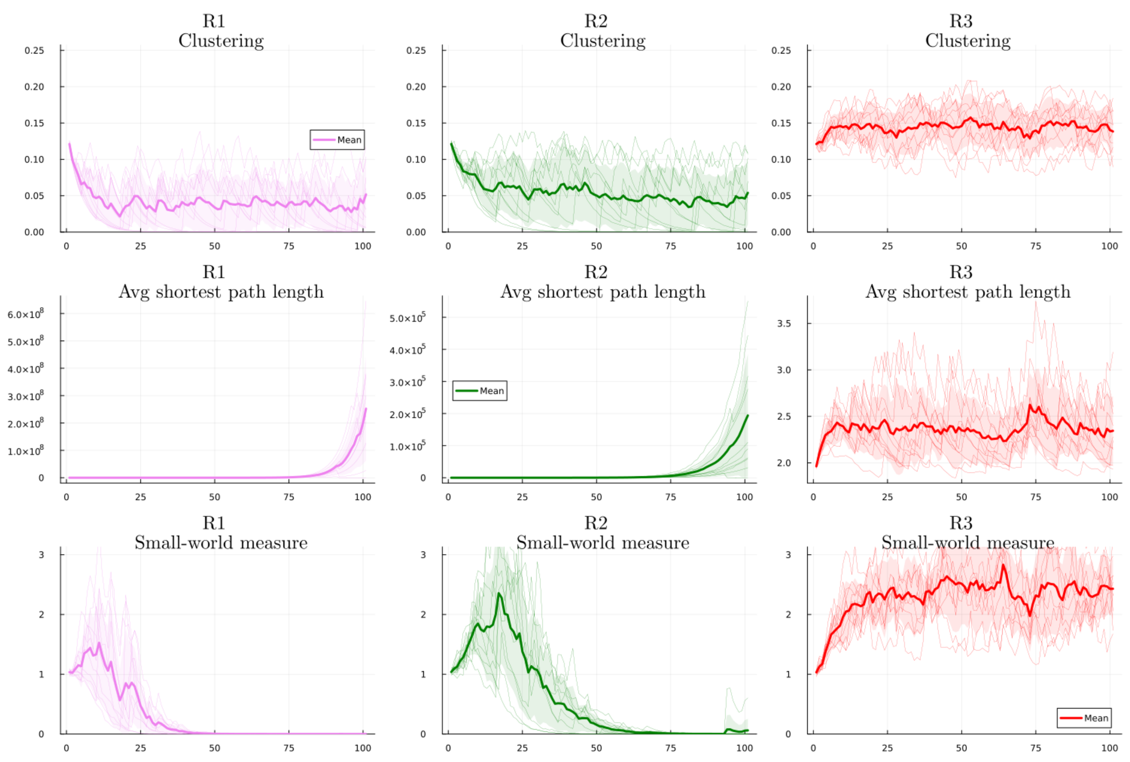


Figure 5. Graph theory measures for the three rules (N = 50 nodes)

Given the probability distribution of activity, we can find a closed form solution for the weight matrix, which saves us having to do numerical simulations:

$$\text{R1: } w^{\infty}_{i,j} \approx \frac{\eta_1 p_{ij}}{1 - (p_{ij} - p_{ij}\eta_1 - p_{ij}\eta_2 + \eta_2)}$$

$$\text{R2: } w^{\infty}_{i,j} \approx \frac{p_{ij}\gamma_1}{1 - (p_{ij}(1 - \gamma_1 - 2\gamma_2 + \gamma_3) + q_{ij}(\gamma_2 - \gamma_3) + \gamma_3)}$$

$$\text{R3: } w^{\infty}_{i,j} \approx \frac{(p_{ij}(\kappa_1 - 2\kappa_2) + q_{ij}\kappa_2)}{1 - (p_{ij}(-1 - \kappa_1 + 2\kappa_2 + \kappa_3) + q_{ij}(1 - \kappa_2 - \kappa_3) + \kappa_3)}$$

where $p_{i,j}$ is the probability of node i and node j being active, and $q_{i,j}$ is the probability of node i or node j being active.

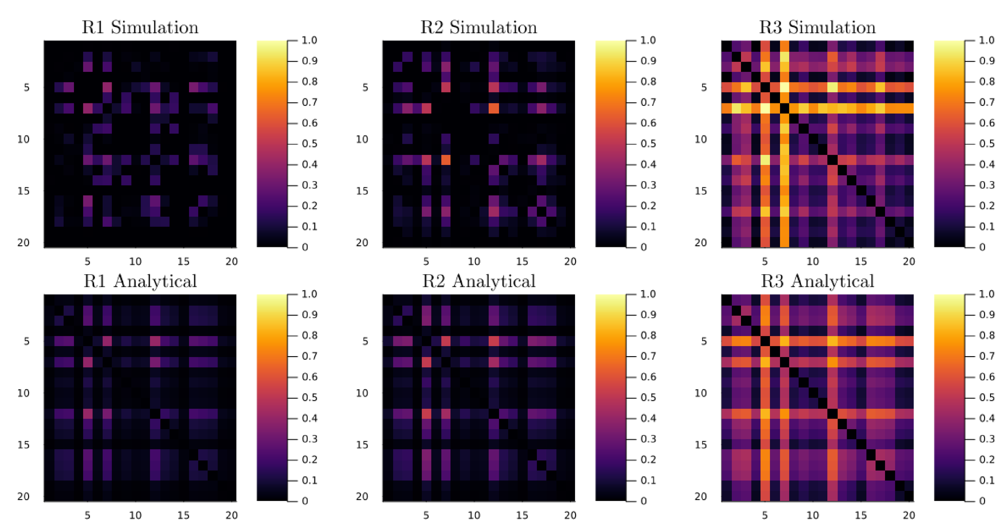


Figure 6. Weight matrices for example network of 20 nodes

Results continued

Small-world measures across network sizes

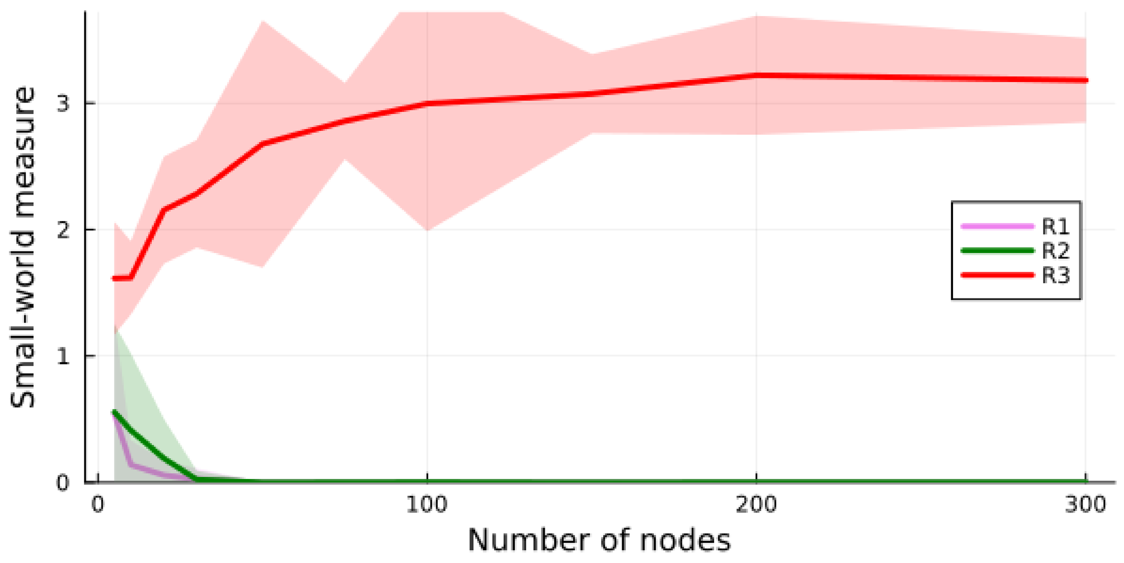


Figure 7. Small-world measures across different sizes of networks

Beta distributions activity & small-world measures

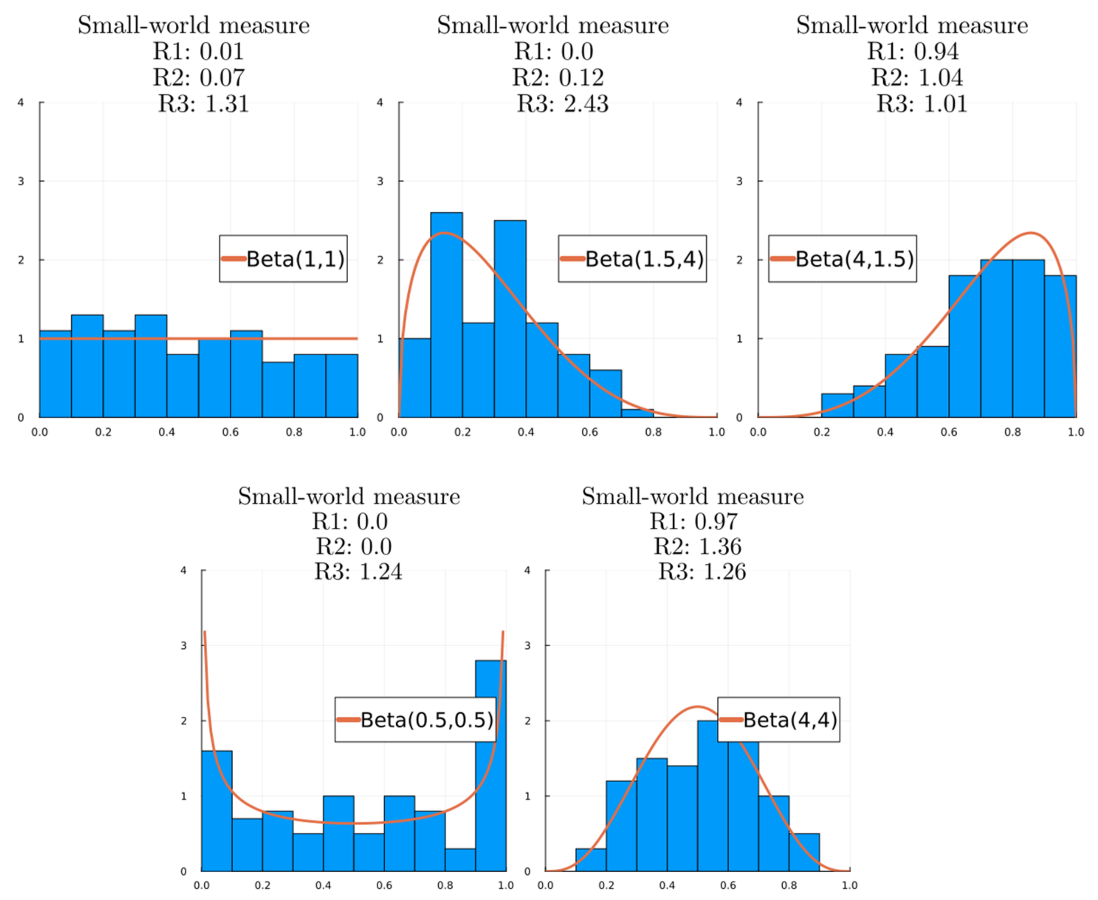


Figure 8. Small-world measures across different activity distributions (N=100)

Distribution of weights weighted node degrees in the three plasticity rules under the "log normal" activity:

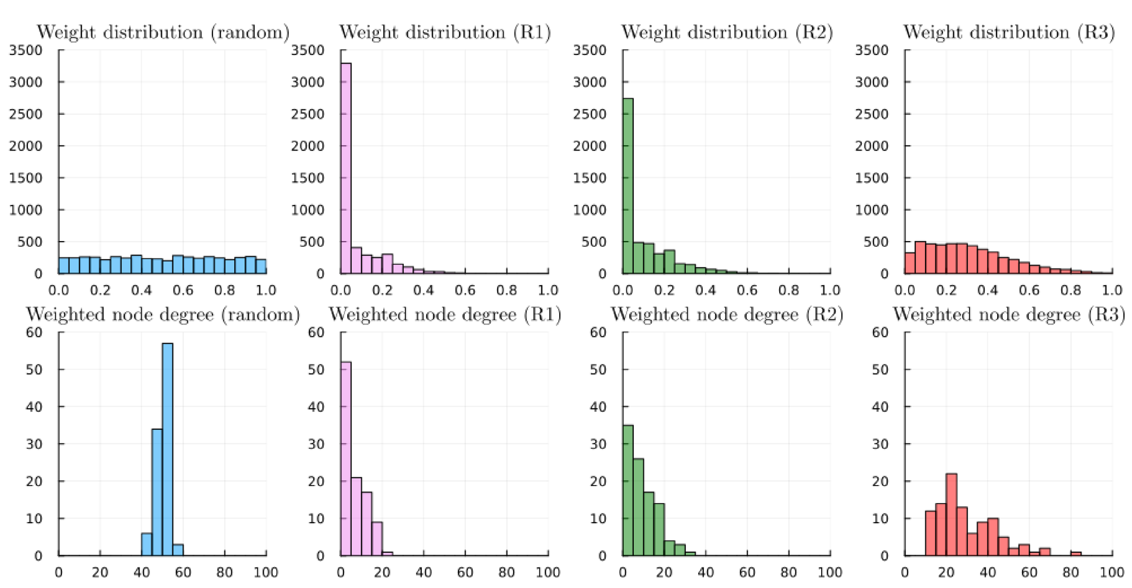


Figure 9. Weight and weighted node degree distributions (N=100)

Future ideas

This work can be extended with greater complexity, e.g. directed graphs, more complicated activity dynamics, use of spatial proximity and correlations in setting individualised learning rates. Other ideas include implementing some of the resultant networks in reservoirs, using heterosynaptic plasticity in Hopfield models, and studying brain connectome data to find if similar structural signatures exist as in these networks.

Conclusion

Simple plasticity rules make a big difference to weighted network architectures. This work shows that heterosynaptic plasticity – in certain neural activity patterns – encourages small-world characteristics. This may have implications for optimised computational capacity and robustness.

References

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