

# Heterosynaptic plasticity rules induce small-world topologies

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**Abstract:** Heterosynaptic plasticity is a form of ‘off-target’ synaptic plasticity where unstimulated synapses change strength. Although some theoretical work has explored its implications [2], its function for brain learning remains unknown. Here we propose that one purpose of heterosynaptic plasticity is to encourage small-world connectivity [4, 5]. We used numerical and mathematical analysis to compare different plasticity rules in fully-connected abstract weighted graphs, and found that they yield distinct network architectures. Simple heterosynaptic plasticity-based learning – in contrast to Hebbian-style homosynaptic learning – not only reduces the over-saturation of synaptic weights, but causes the networks to rapidly converge to small-world topologies.

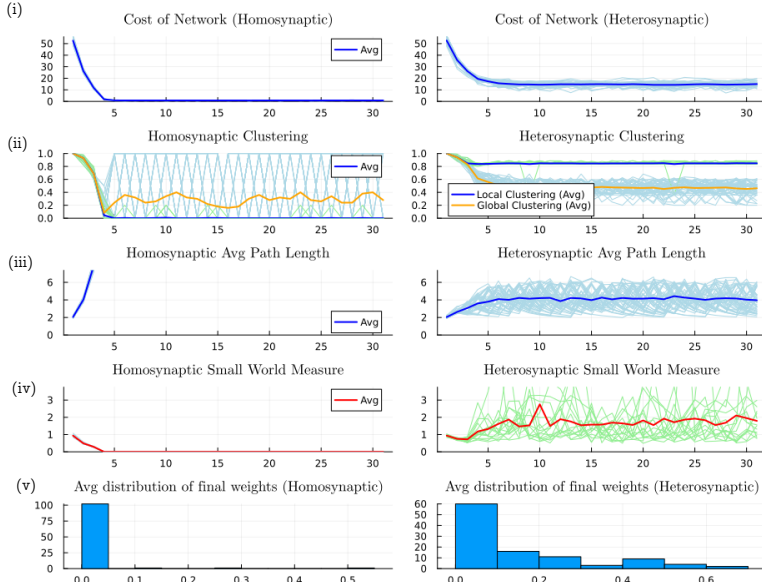
The simplest version of the model is constructed as follows: We initialise a fully connected, weighted, and symmetric mathematical graph, consisting of  $N$  nodes (abstracted to represent single neurons, or groups of neurons) and weighted relations  $w_{i,j} \in (0,1)$  between the  $i^{th}$  and  $j^{th}$  nodes (interpreted as interneural synaptic weights). In this simplest model, the nodes have no self connections. This may be represented by a hollow symmetric weight matrix (e.g. Figure 1, top).

At every iterative step, a subset  $S$  of nodes is sampled (where nodes in  $S$  may be taken to be neurons/populations which are “active”). We explore different constructions of the subset  $S$ , from consisting of sets of constant members and constant size, to sets of varying size with probabilities dependent on the weights associated with each node. (The probability of a node  $i$  being in  $S$  in this final case is defined as  $\frac{\sum_j w_{i,j}}{N-1}$ . This preferentially selects nodes with higher accumulations of weights). With this setup of  $S$ , there are three cases: each weight is either (1)

between two active nodes, (2) between one active and one inactive node, or (3) between two inactive nodes. Based on these cases, 2 plasticity rules are defined:

$$R_1 = \begin{cases} w_{i,j_{n+1}} = w_{i,j_n} + \eta_1(1 - w_{i,j_n}) & \text{if } i \text{ and } j \in S \\ w_{i,j_{n+1}} = \eta_2 w_{i,j_n}, & \text{otherwise} \end{cases} \quad (1) \quad R_2 = \begin{cases} w_{i,j_{n+1}} = w_{i,j_n} + \gamma_1(1 - w_{i,j_n}) & \text{if } i \text{ and } j \in S \\ w_{i,j_{n+1}} = w_{i,j_n} + \gamma_2(1 - w_{i,j_n}) & \text{if } i \in S \text{ and } j \notin S \\ w_{i,j_{n+1}} = \gamma_3 w_{i,j_n}, & \text{otherwise} \end{cases} \quad (2)$$

$R_1$  updates the weights in the network by a simple rule which strengthens weights between co-active nodes (“those that fire together, wire together”), and weakens all others.  $\eta_1$  and  $\eta_2$  are constants determining the strengthening and weakening of weights, respectively.  $R_2$ , the second, heterosynaptic rule is defined so as to strengthen weights between sets of co-active nodes, partially strengthen weights between partially active sets of nodes, and weaken others.  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  denote the constants of heterosynaptic strengthening ( $\gamma_1, \gamma_2$ ) and weakening ( $\gamma_3$ ). Without loss of generality, the formulation of these above learning rules ensures that the weights are bounded between the values of 0 and 1. When a weight becomes sufficiently small (i.e.  $w_{i,j} < 0.1$ ), it is set to equal zero, denoting the total loss of a synaptic connection.



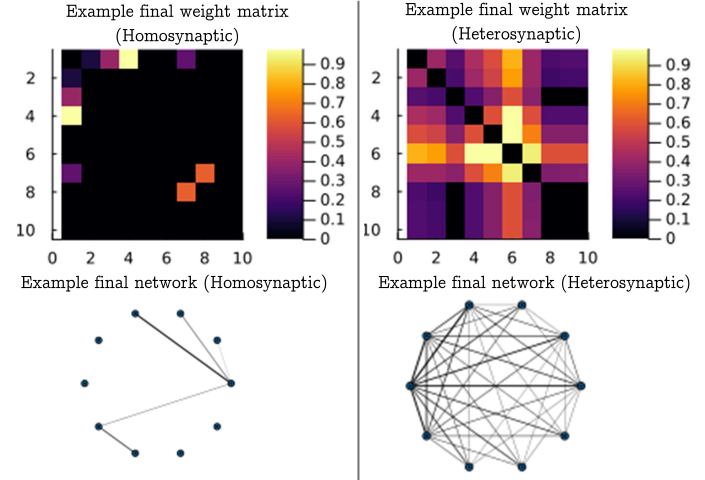
**Figure 2:** Small-world network measures across 100 trials; Homosynaptic plasticity rule (left), Heterosynaptic plasticity rule (right)

On the other hand, the implementation of  $R_2$  vastly reduces synaptic over-saturation, preserving a greater distribution of weights (Figure 1, bottom right & Figure 2, (v) right). This lends evidence to the suggestion that heterosynaptic plasticity may maintain a degree of homeostasis. The “cost” of the network decreases significantly from the randomly initialised graph (Figure 2, (i) right), suggesting that heterosynaptic plasticity optimises efficiency and economy. Finally, this research demonstrates that heterosynaptic plasticity rules tend to stabilise network clustering and average shortest path length (Figure 2, (ii) & (iii)), therefore producing graphs with  $\sigma$  values greater than 1 – indicating small-world topologies (Figure 2, (iv) right).

These findings may have implications for artificial neural networks where there is a need for ever-increasing computational power, while mitigating the constraints of cost and energy consumption. It furthermore may offer insight into the role of heterosynaptic plasticity in various neurological disorders in which brain connectivity studies have demonstrated atypical topologies (e.g. Schizophrenia, Autism Spectrum Disorder, Alzheimer’s disease [1, 6, 3]).

## References

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**Figure 1:** Example final weight matrices and networks