Type I and Type II Errors

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

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	H_0 True ($\mu = 66.7$)	H_0 False ($\mu > 66.7$)
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What is the **Type II error probability** if the true value is $\mu = 70$?

$\mathbf{P}\left(\text{Do not reject } H_0 \,|\, \mu = 70\right) \longrightarrow \mathbf{P}(\overline{X} < 68.26 \,|\, \mu = 70)$

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$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $n = 10$ $\sigma = 3$

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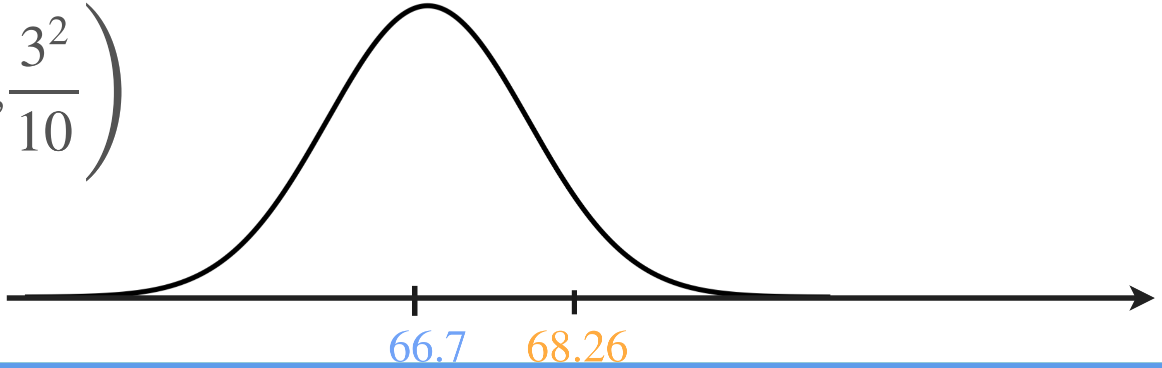

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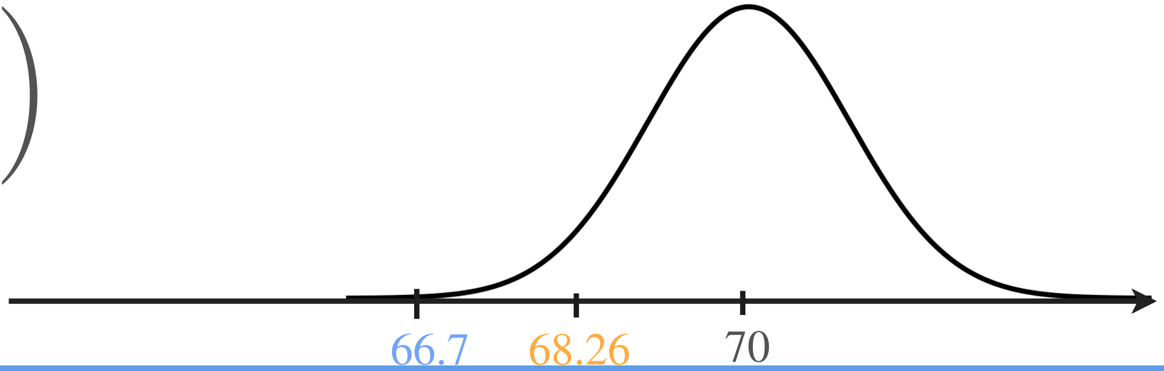

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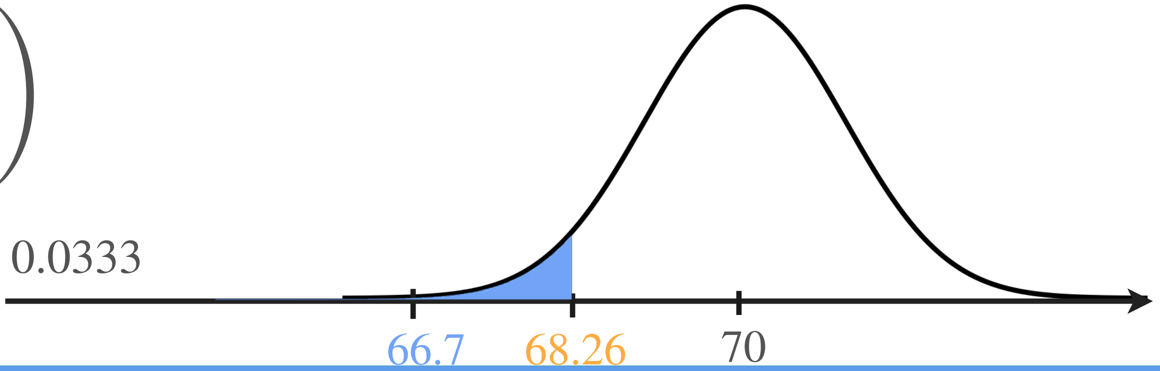

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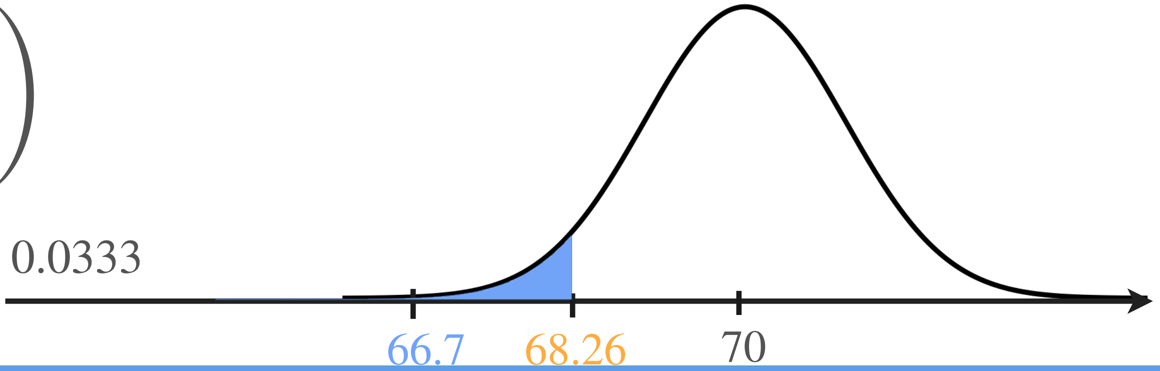

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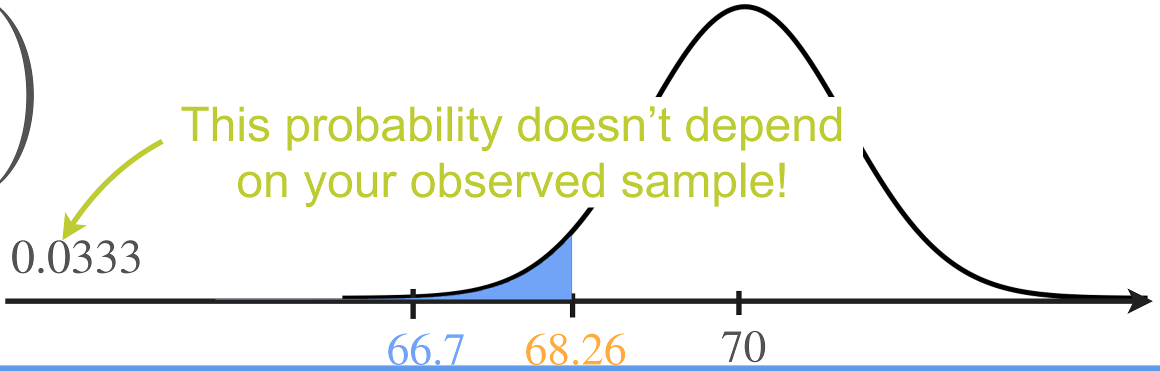
This probability doesn't depend on your observed sample!



Power of the Test

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$$\mathbf{P}\left(\text{Reject } H_0 \,|\, \mu \in H_1\right)$$

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Complementary probabilities

Power of the Test

$$\left.\begin{array}{r}\text{Type II error: } \mathbf{P}\overbrace{\left(\text{Do not reject } H_0 \mid \mu \in H_1\right)}^{\beta} \\[2em] \text{Power of the test: } \mathbf{P}\underbrace{\left(\text{Reject } H_0 \mid \mu \in H_1\right)}_{1-\beta}\end{array}\right\}\ \text{Complementary probabilities}$$

Power of the Test

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Power of the test: $\underbrace{\mathbf{P}\left(\text{Reject } H_0 \mid \mu \in H_1\right)}_{1-\beta}$

Complementary probabilities

$$\text{Power of the test} = 1 - \text{Type II error probability}$$

$$= 1 - \mathbf{P}\left(\text{Do not reject } H_0 \mid \mu \in H_1\right)$$

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$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

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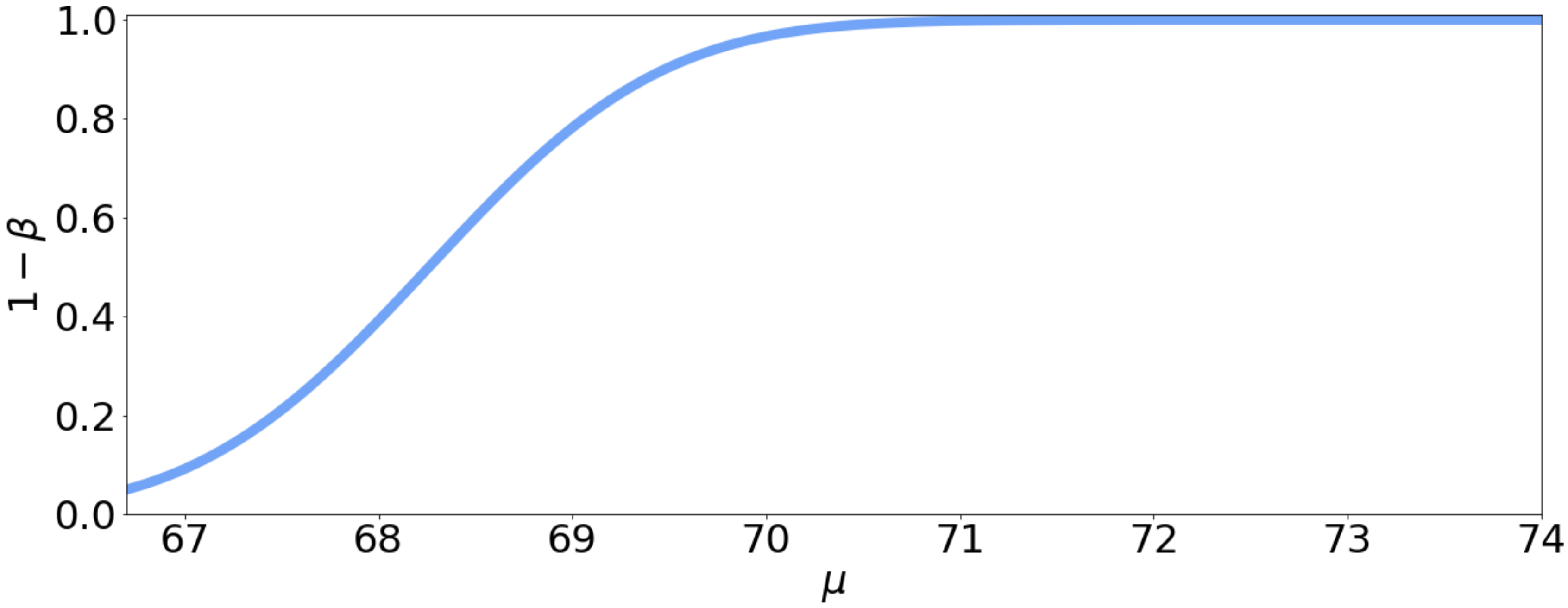

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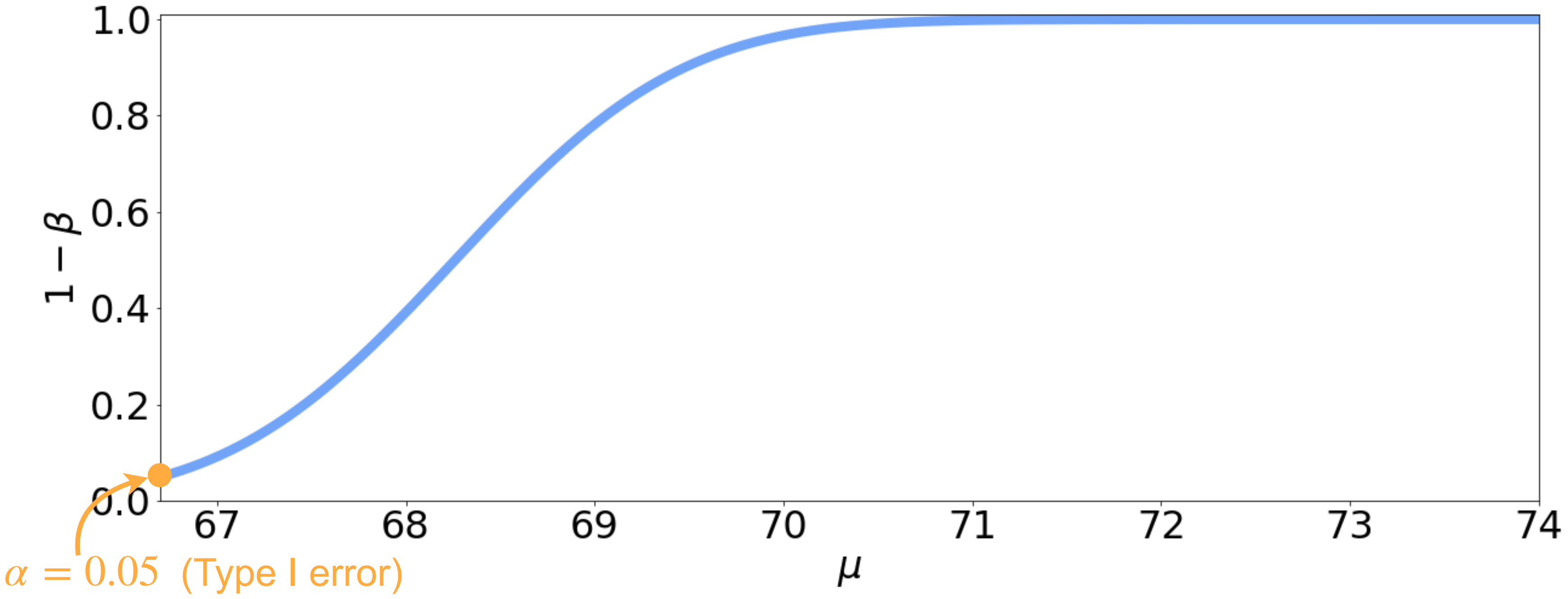

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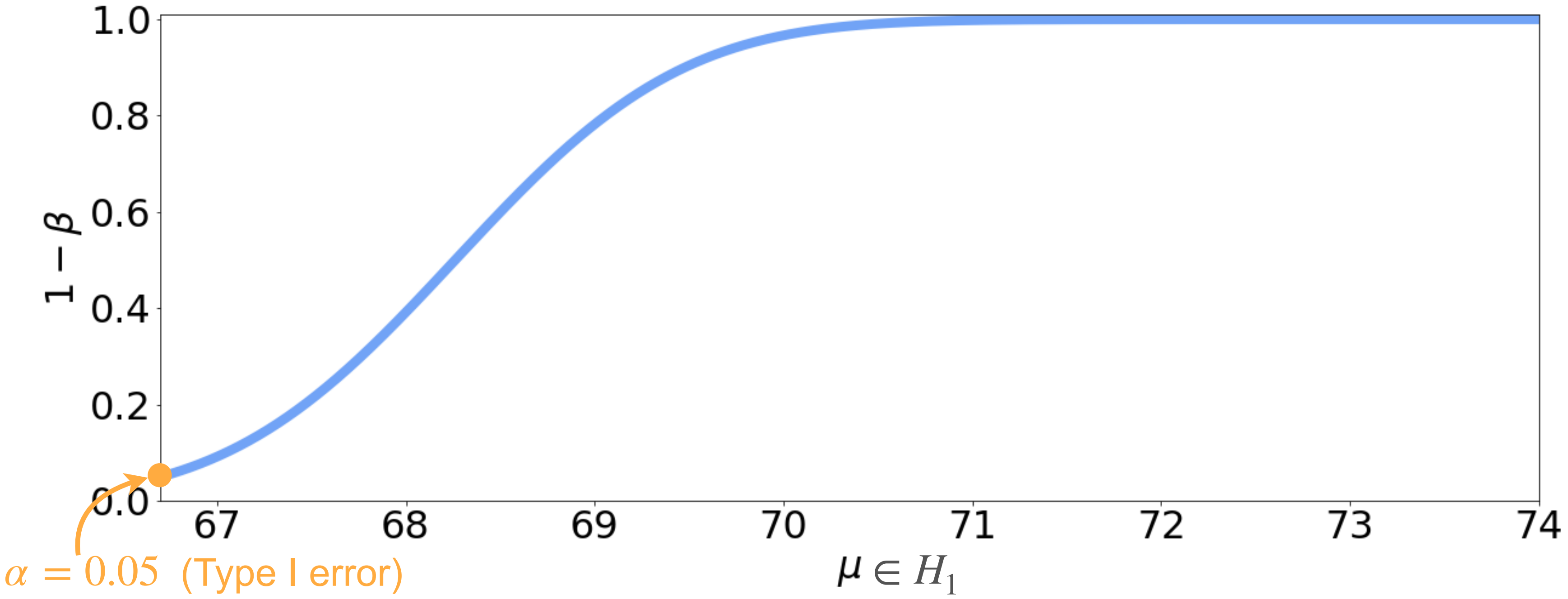

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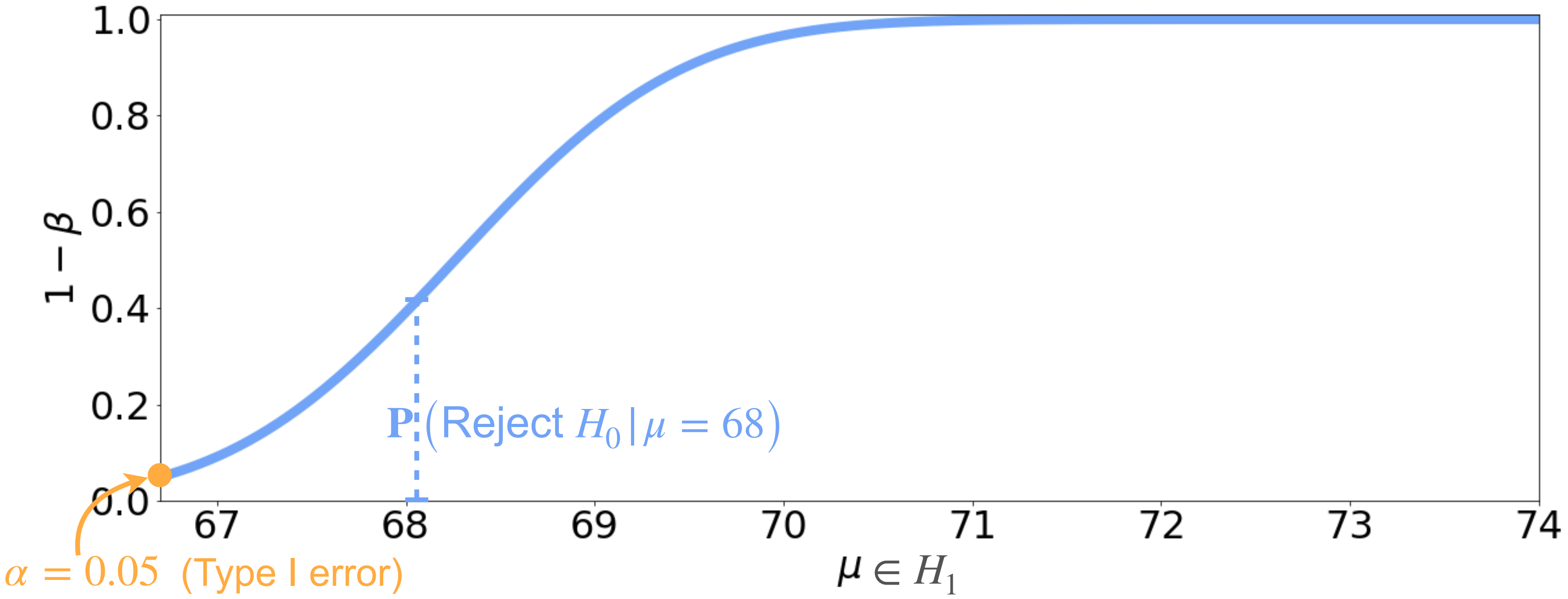

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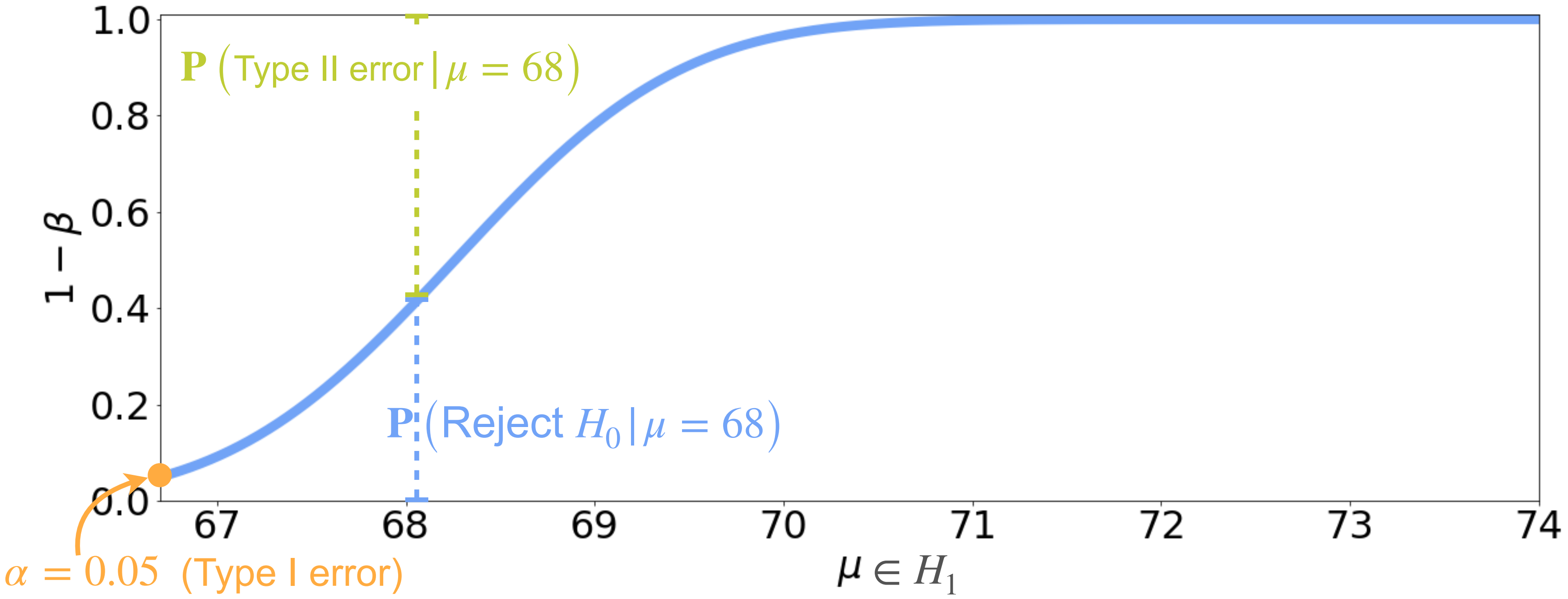

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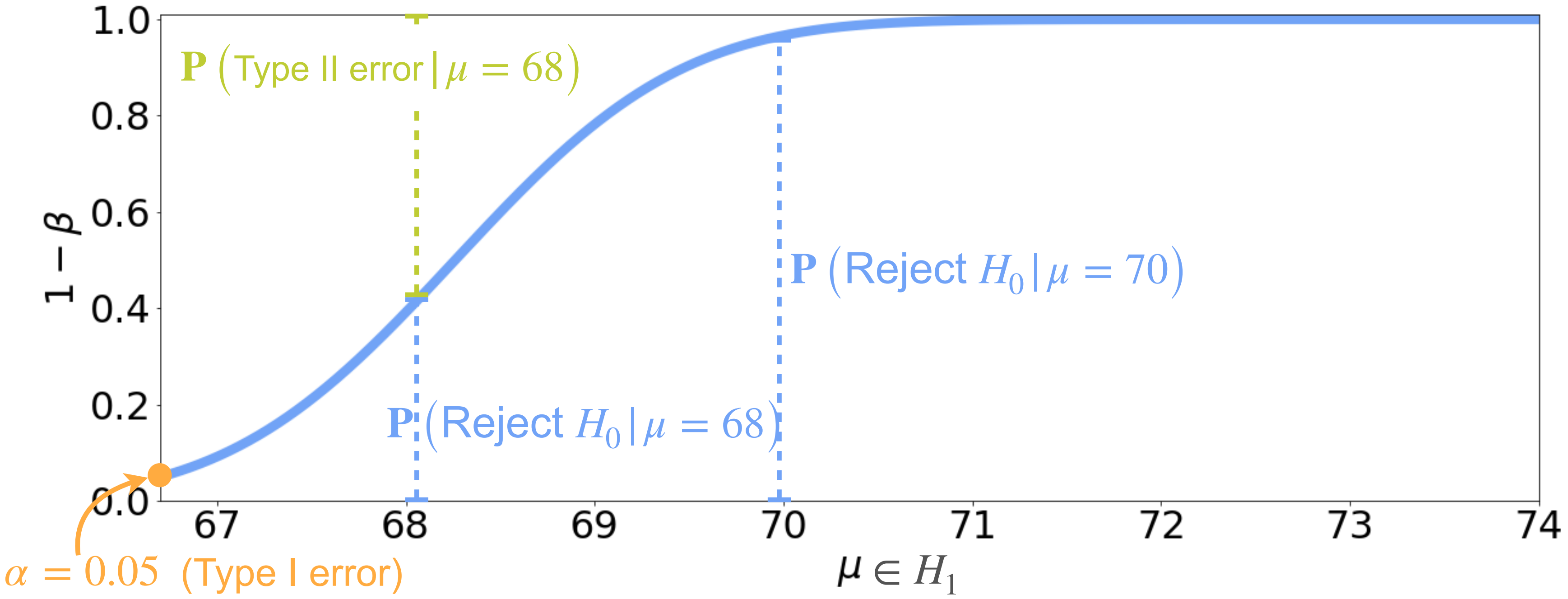

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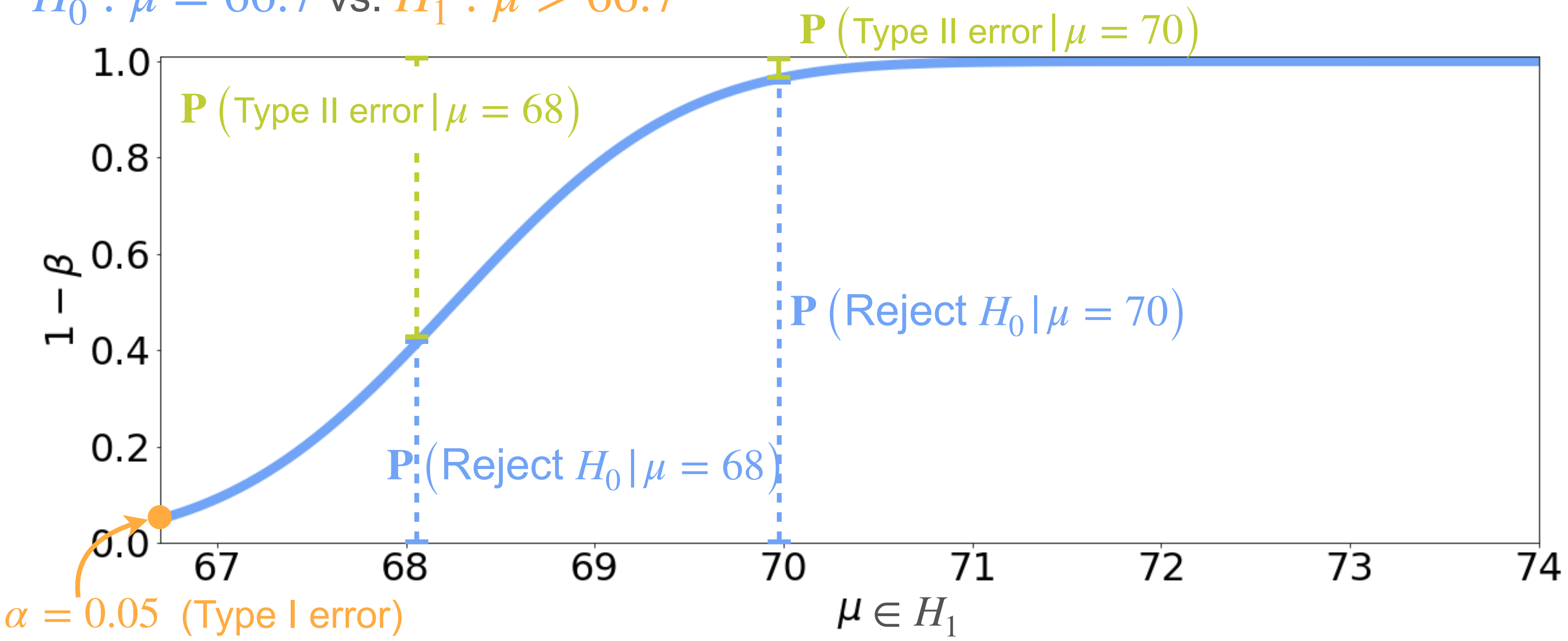

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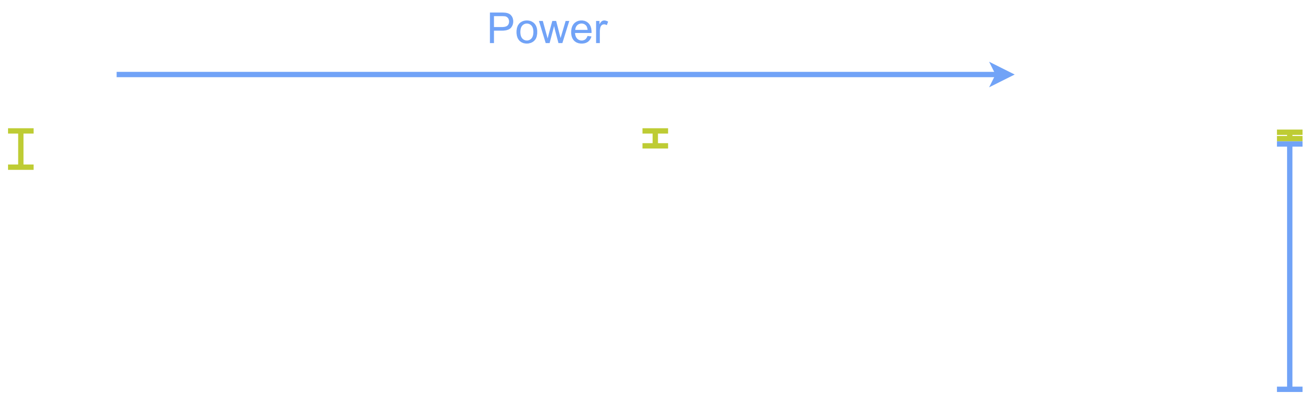

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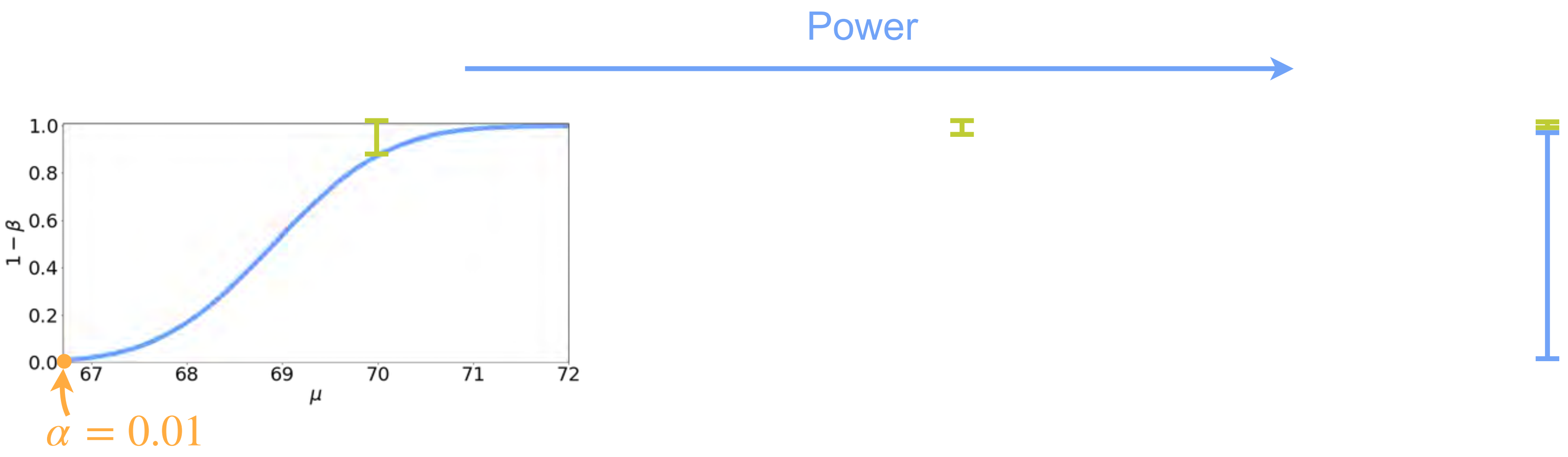
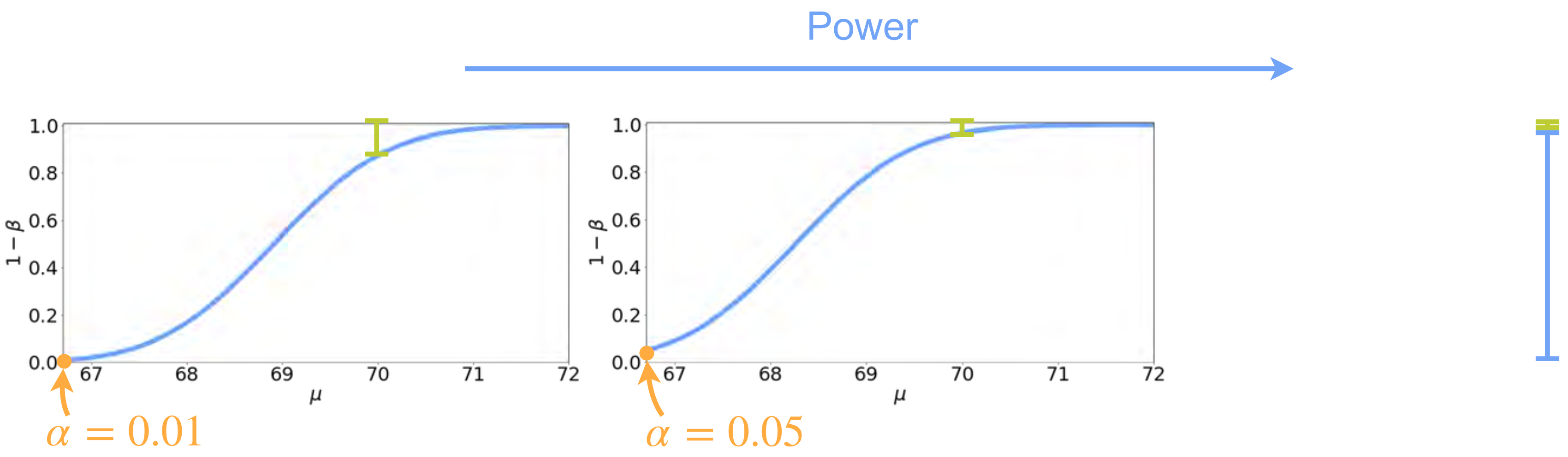


DeepLearning.AI

Power of the Test



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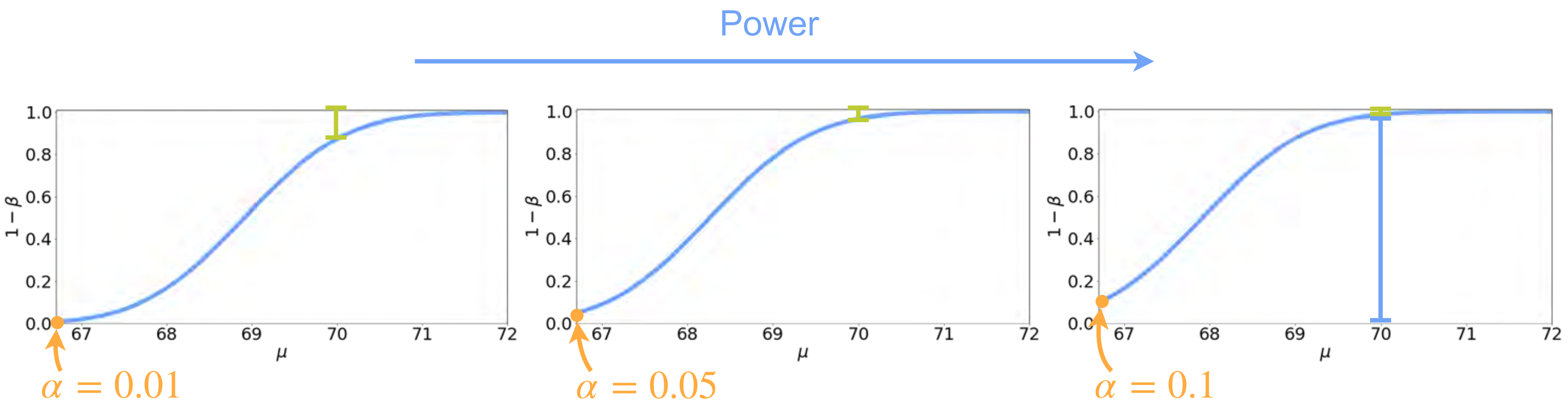

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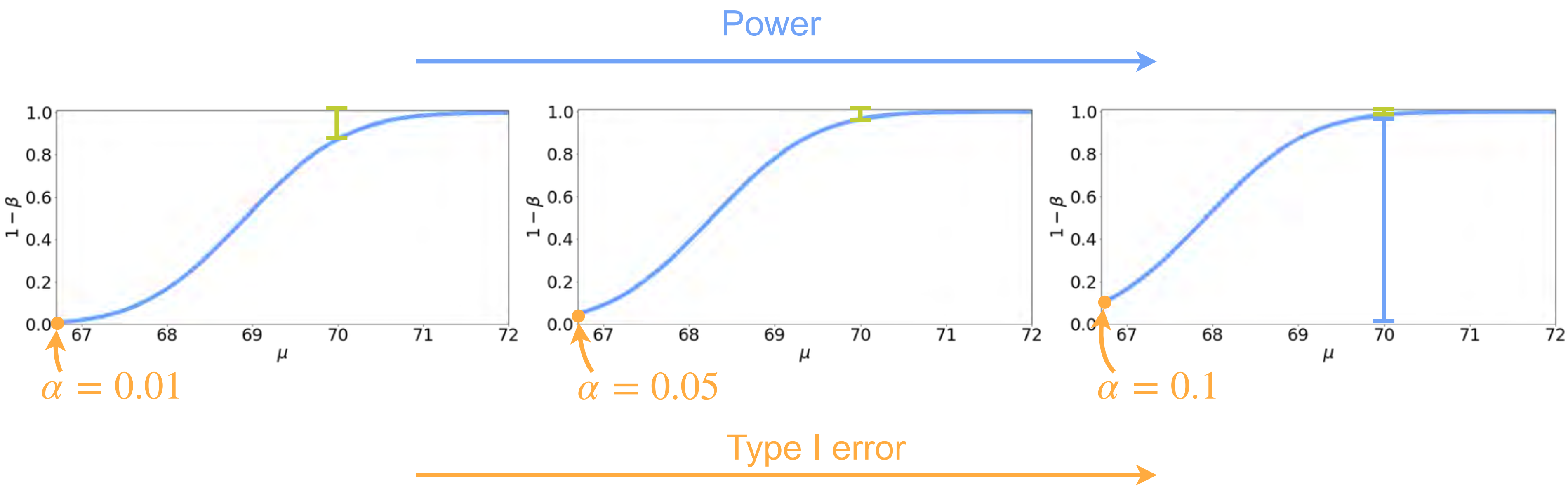
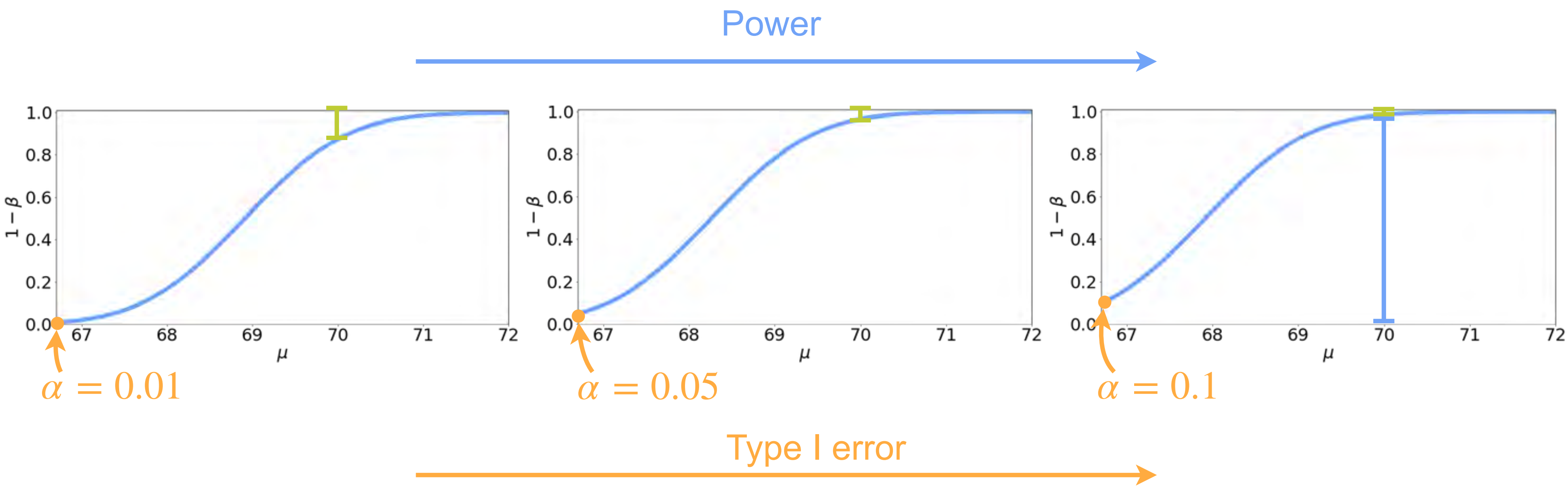


Power of the Test

$\mu = 70$



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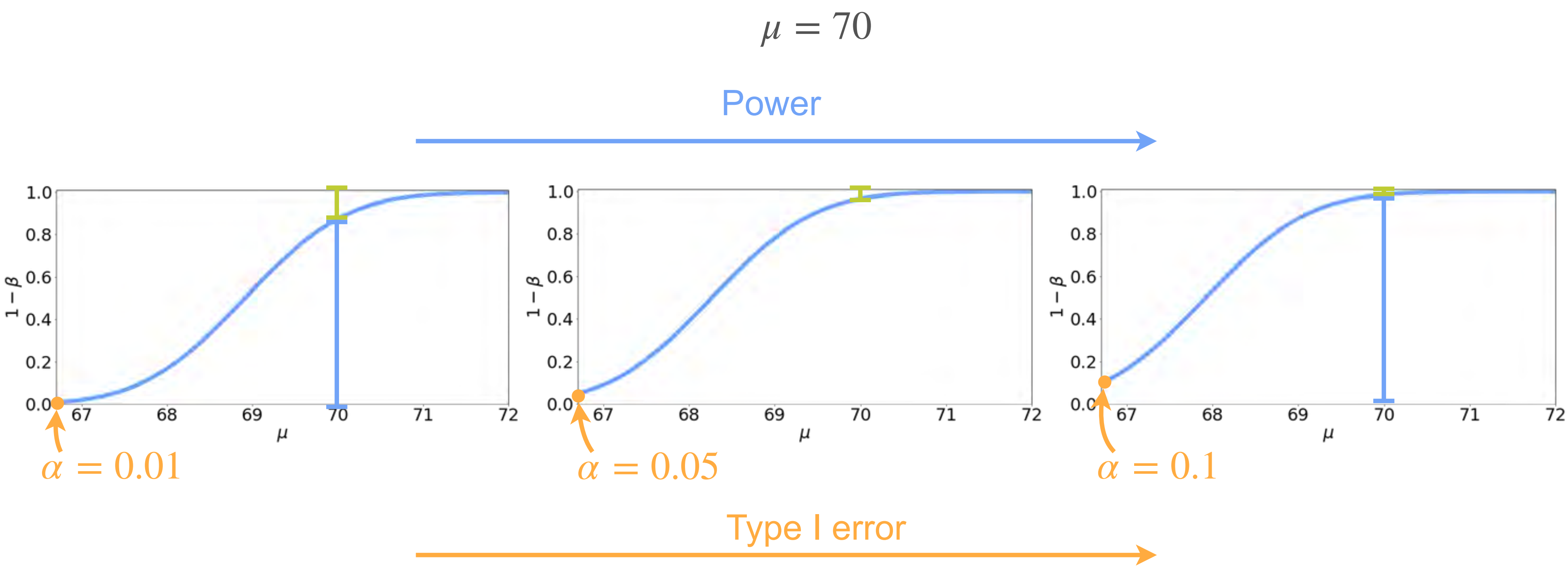

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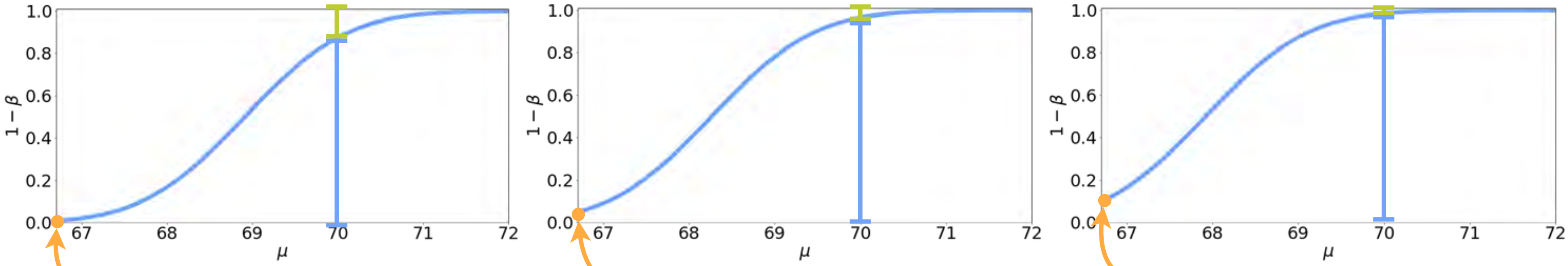
Power



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Type I error

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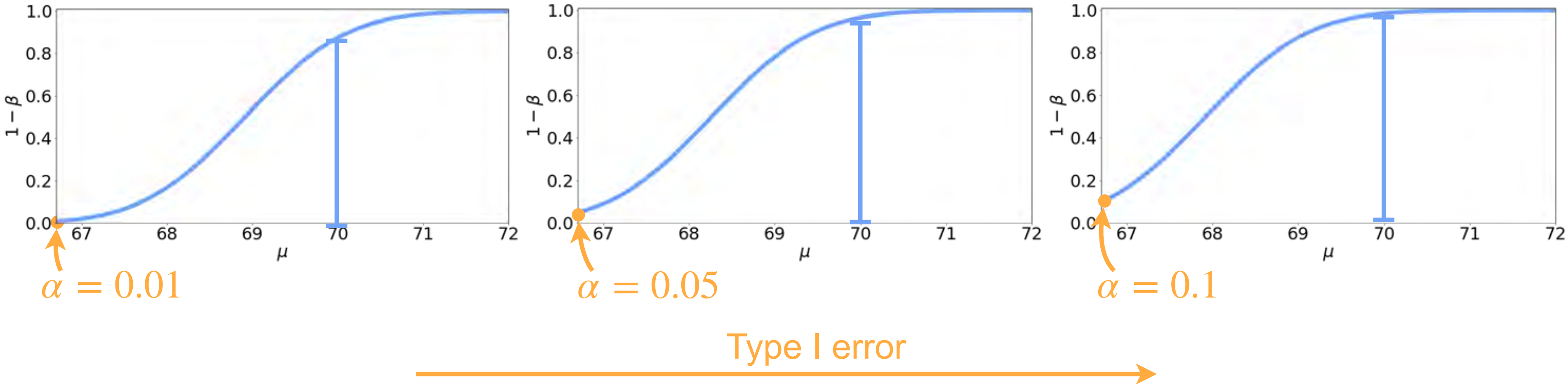

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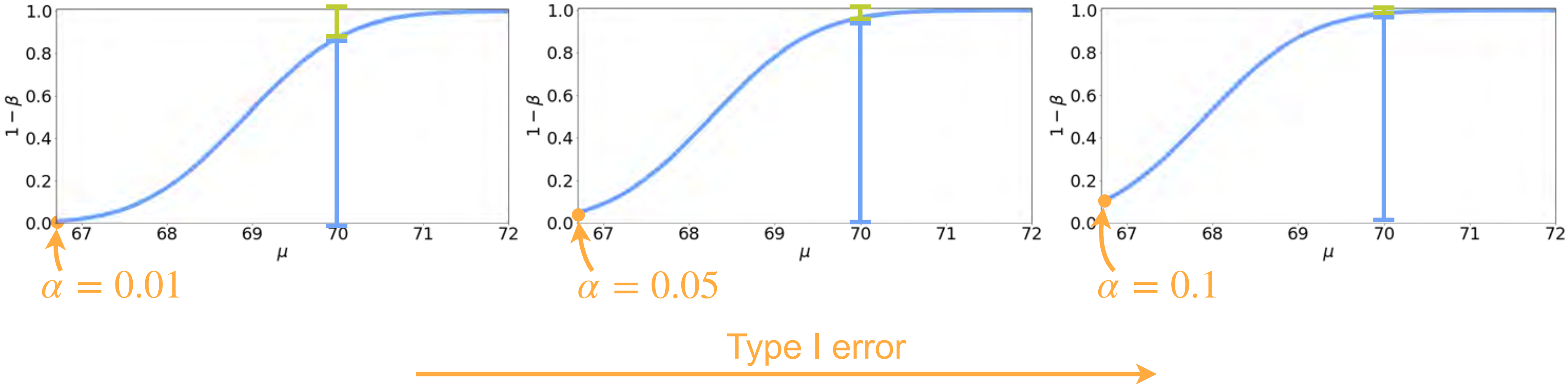

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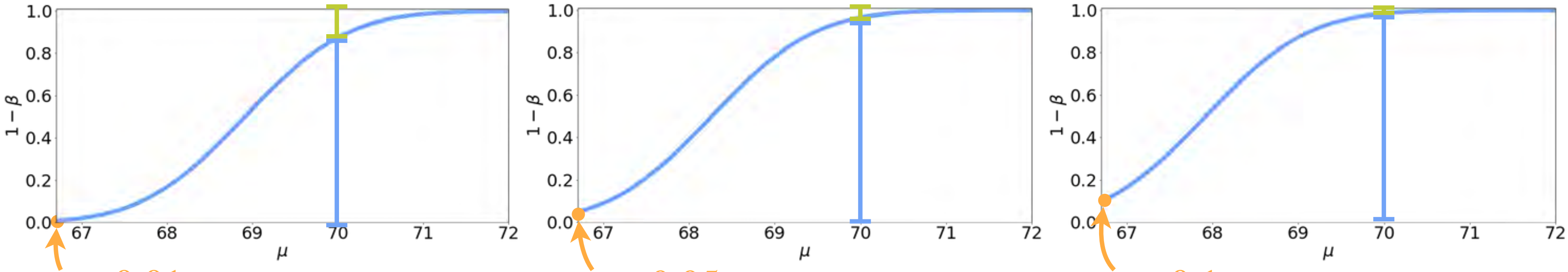
Type II error



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Type I error

Hypothesis Testing

Interpreting results

Steps for Performing Hypothesis Testing

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4. **Reach a conclusion:**
   - If the *p*-value is less than the significance level reject H_0

$$\longrightarrow \mathbf{P}\left(\overline{X} > 68.442 \,|\, \mu = 66.7\right) \overset{?}{>} 0.05$$

Important Remarks - Interpreting Tests

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- Type I error:
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Important Remarks - Interpreting Tests

- *p*-values:
  - If $\mathbf{P}(\text{Reject } H_0 \mid H_0) < \alpha \longrightarrow$ Reject H_0 and then accept H_1
  - A small *p*-value indicates that the probability of seeing the observed data by chance is small
- Test conclusions
  - Reject $H_0 \rightarrow H_1$ true
  - Do not reject H_0 $\cancel{\rightarrow H_0}$ ~~true~~

You can only say that there is not enough evidence

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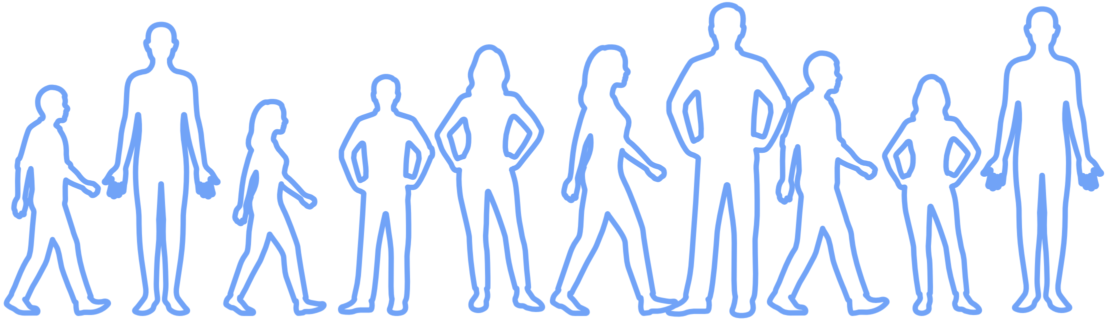
Hypothesis Testing

t-Distribution

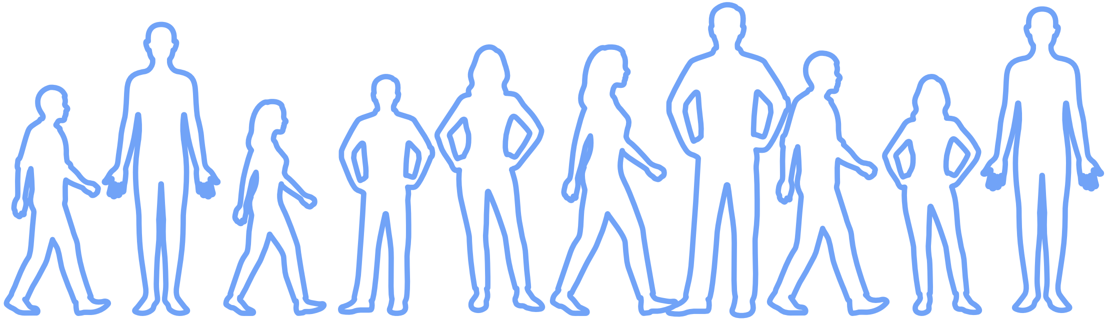
t-Distribution: Motivation

t-Distribution: Motivation



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

t-Distribution: Motivation



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$$X_i \overset{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

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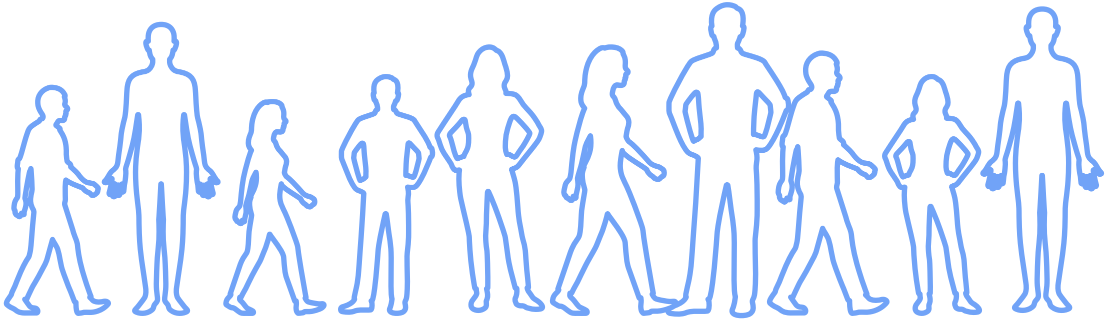

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$$X_i \overset{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$

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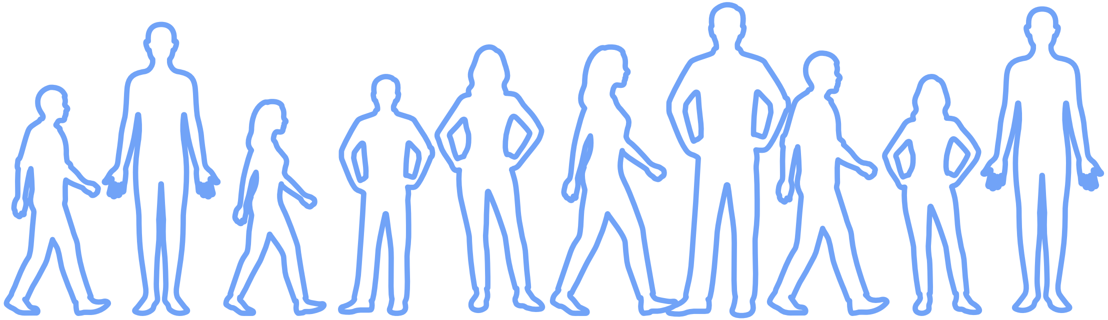

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This is fine if you know μ and σ

What if σ is unknown?

t-Distribution: Motivation

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If μ, σ are known

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

t-Distribution: Motivation

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If μ, σ are known

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{10}} \sim \mathcal{N}(0,1^2)$$ (Standardization)

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Z statistic

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What if σ is unknown?

Replace σ with its estimate $\dfrac{\bar{X}-\mu}{S/\sqrt{10}}$

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T statistic

What if σ is unknown?

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$$S = \sqrt{\frac{1}{10-1}\sum_{i=1}^{10}(X_i - \bar{X})^2}$$

If μ, σ are known

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T statistic

Replace σ with its estimate

$$\boxed{\frac{\bar{X}-\mu}{S/\sqrt{10}}} \sim \mathcal{N}(0,1^2)$$

t-Distribution

t-Distribution

$\dfrac{\overline{X} - \mu}{S/\sqrt{10}}$ follows a t distribution

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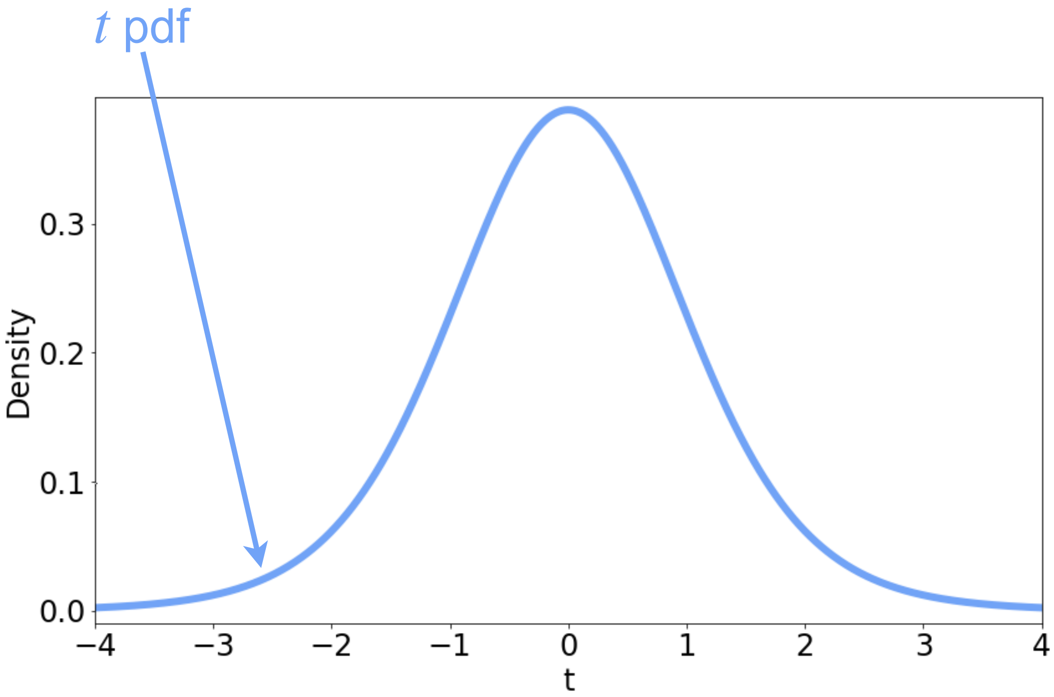
$\frac{\overline{X} - \mu}{S/\sqrt{10}}$ follows a t distribution

What does it look like?

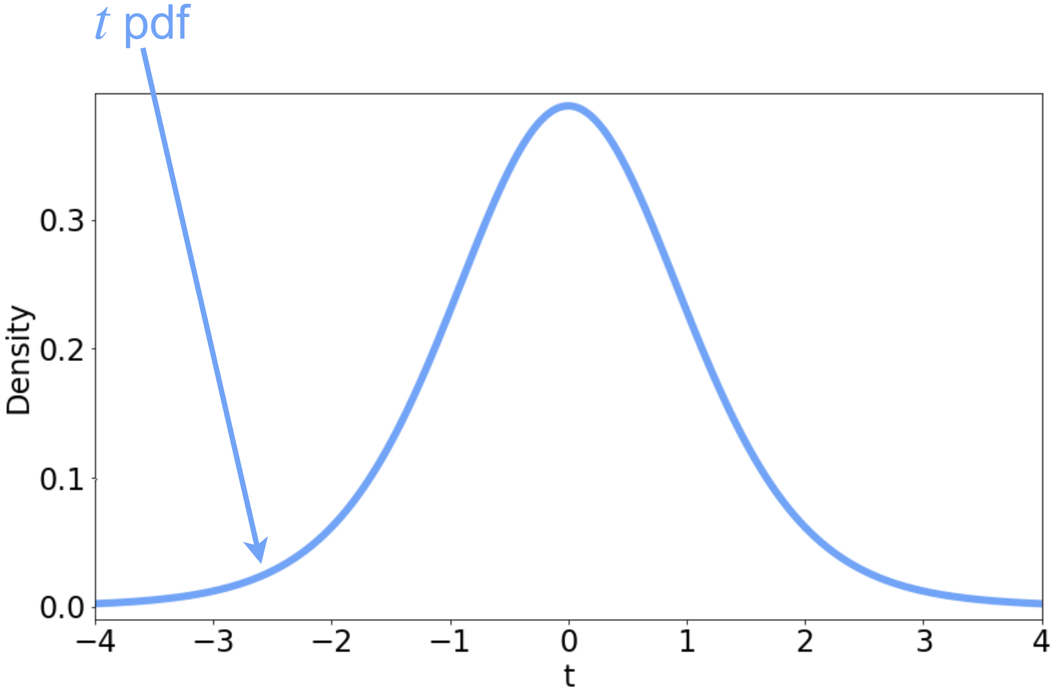


t-Distribution

$\dfrac{\overline{X} - \mu}{S/\sqrt{10}}$ follows a t distribution

What does it look like?

Still bell-shaped



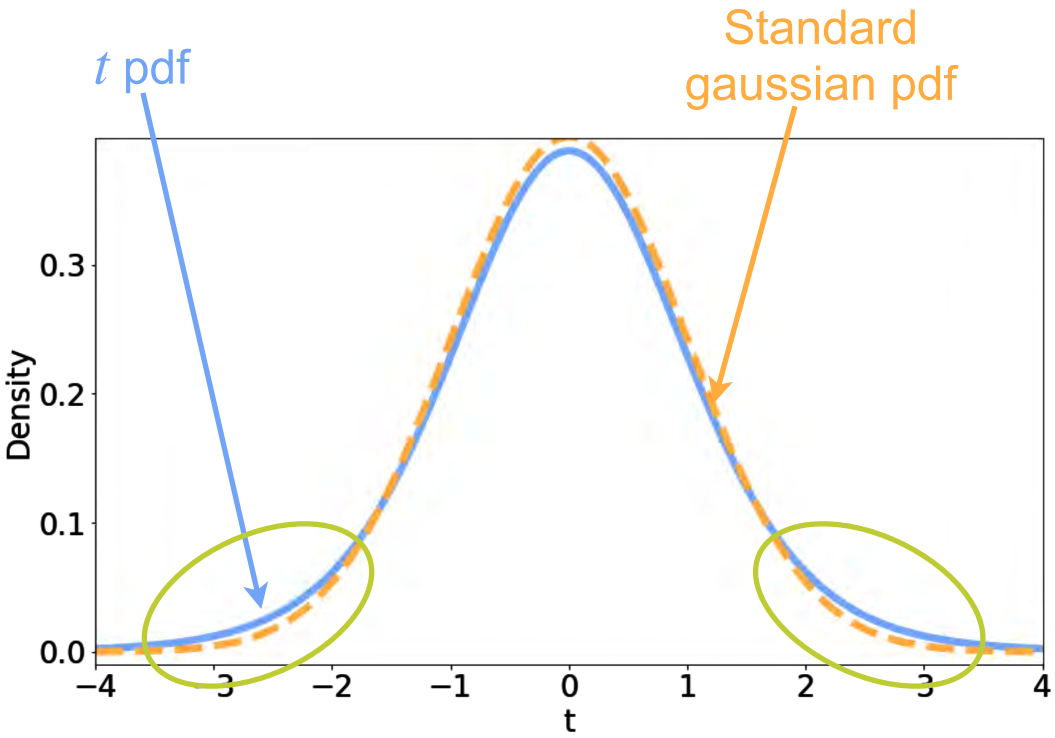
t-Distribution

$$\frac{\overline{X} - \mu}{S/\sqrt{10}}$$ follows a t distribution

What does it look like?

Still bell-shaped

It has heavier tails that account for the uncertainty introduced with the std estimation



t-Distribution

t-Distribution

Parameters:
- Degrees of freedom

t-Distribution

Parameters:

- Degrees of freedom (ν)

t-Distribution

Parameters:

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Controls how heavy the tails are

t-Distribution

Parameters:

- Degrees of freedom (ν)

Controls how heavy the tails are

$$X \sim t_\nu$$

t-Distribution

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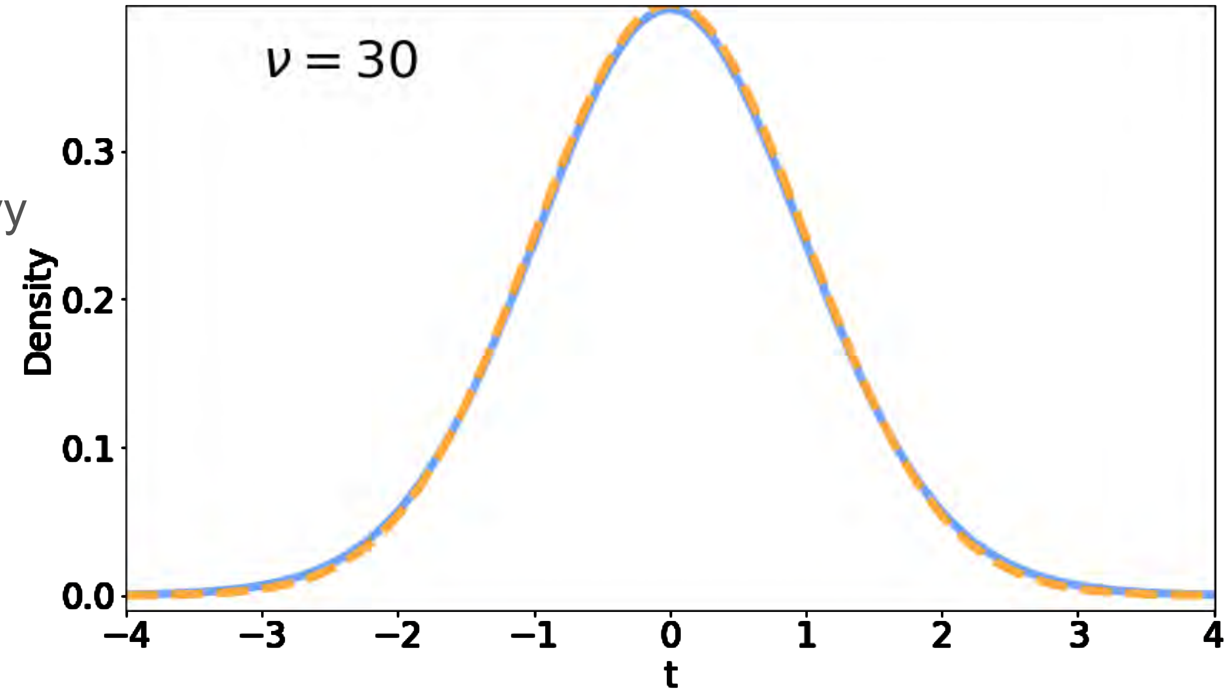
- Degrees of freedom (ν)

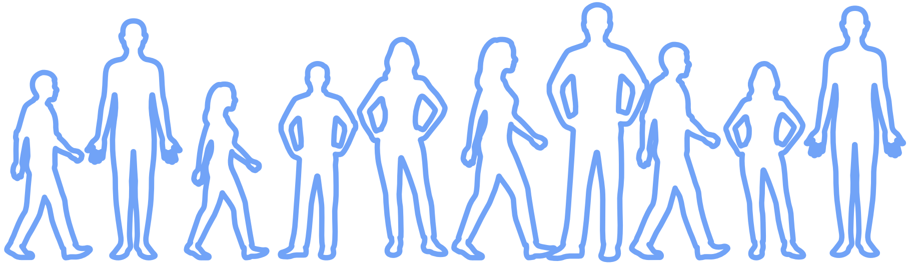
Controls how heavy the tails are

$X \sim t_\nu$



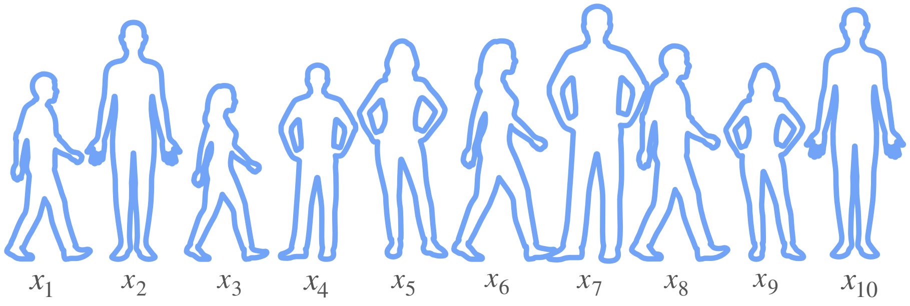
t-Distribution and T-Statistic

t-Distribution and T-Statistic



$n = 10$

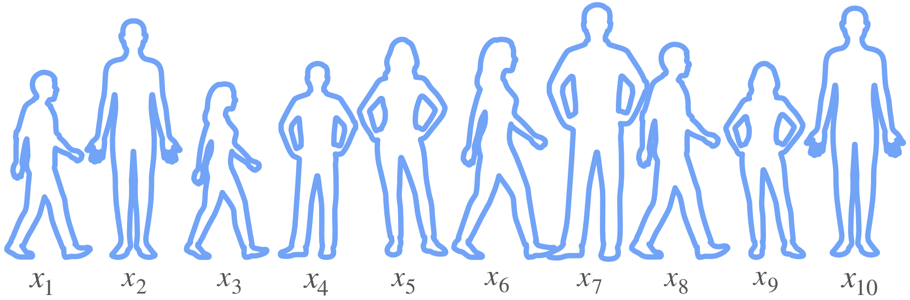
t-Distribution and T-Statistic



$n = 10$

$$T = \frac{\overline{X} - \mu}{S/\sqrt{10}} \sim t_\nu$$

t-Distribution and T-Statistic

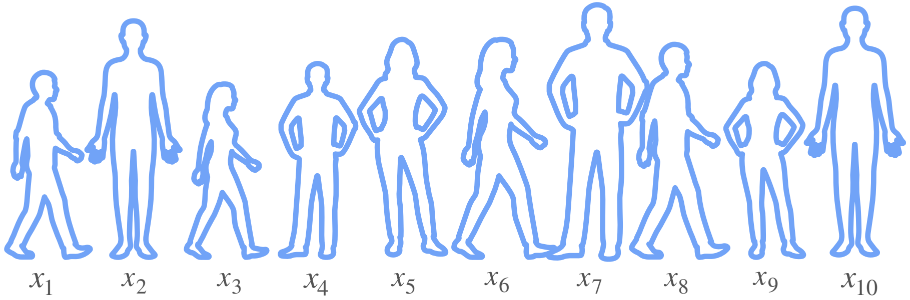


$$n = 10$$

$$\nu = 10 - 1$$

$$T = \frac{\overline{X} - \mu}{S/\sqrt{10}} \sim t_9$$

t-Distribution and T-Statistic



$n = 10$

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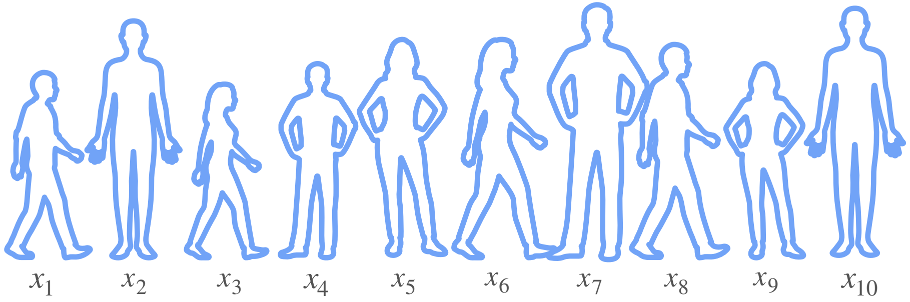
$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_9$$

Degrees of freedom (ν) = sample size -1

= $(n - 1)$

t-Distribution and T-Statistic



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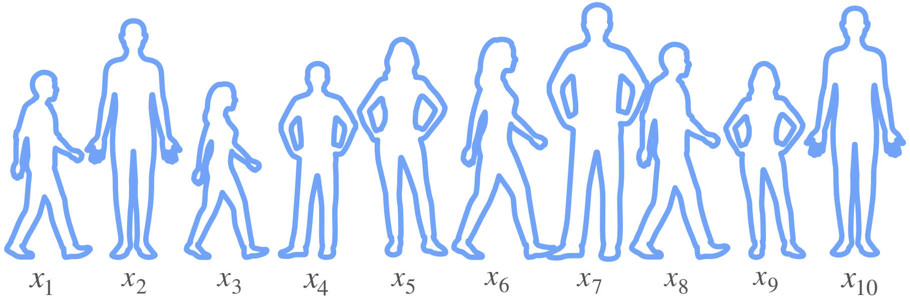
As n increases, this looks more like a $\mathcal{N}(0,1^2)$

t-Distribution and T-Statistic



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T-statistic is used when

- The population has a Gaussian distribution
- But you don't know the variance

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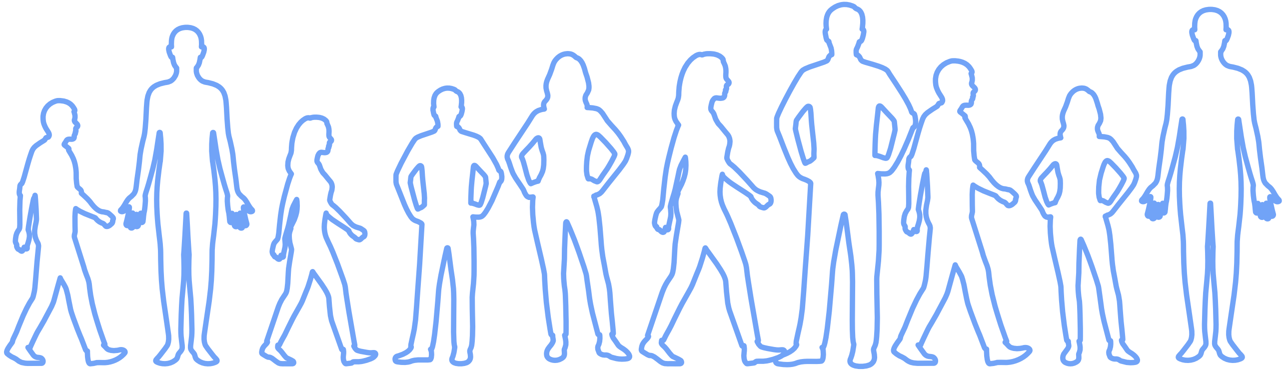
Hypothesis Testing

t-Tests

Example: Heights

Example: Heights



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$$\bar{x} = 68.442$$

Example: Heights

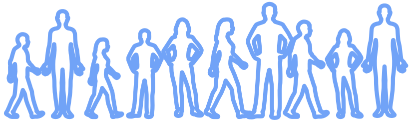
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

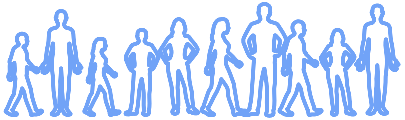
3 questions

3 sets of hypothesis

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

Example: Heights

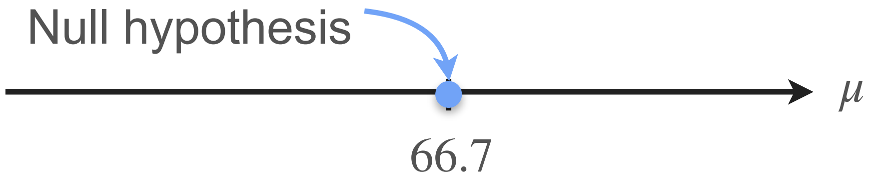
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$

$H_0 : \mu = 66.7$

If H_0 is true: $\overline{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$



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Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\cancel{\sigma = 3}$

$n = 10$

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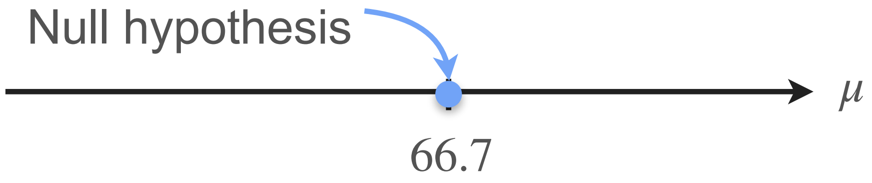

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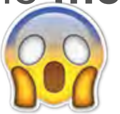

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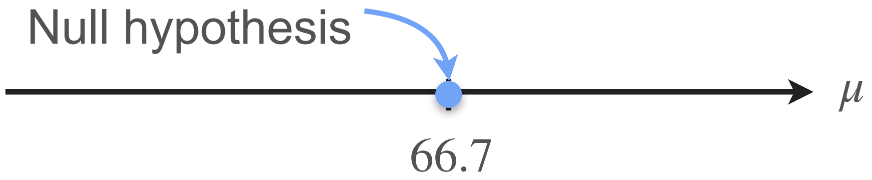

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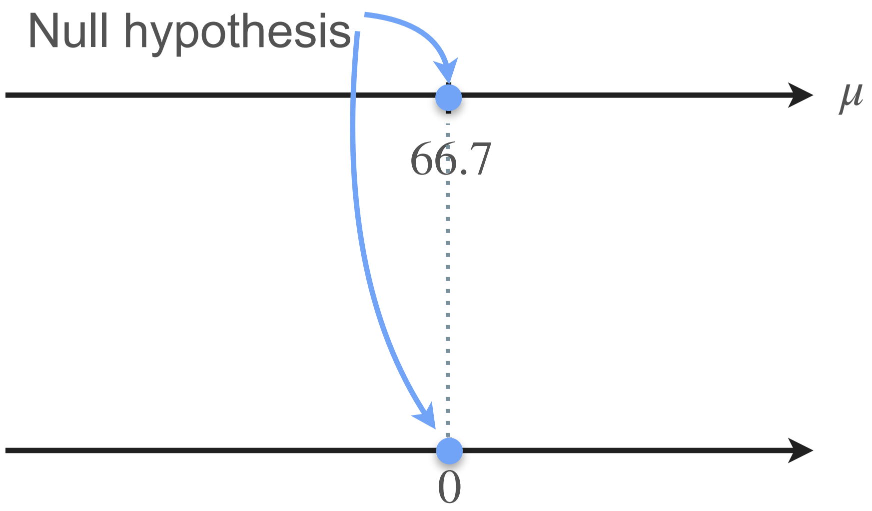
If H_0 is true: $\bar{X} \sim$



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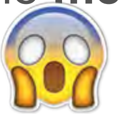
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **~~66.7 in~~**.

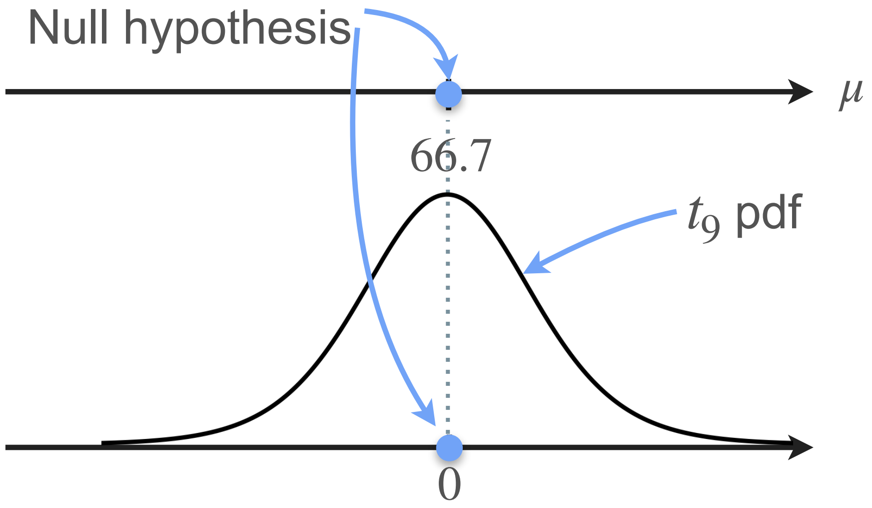


$\cancel{\sigma = 3}$

$n = 10$

$H_0 : \mu = 66.7$

If H_0 is true: $\bar{X} \sim$ [illegible]

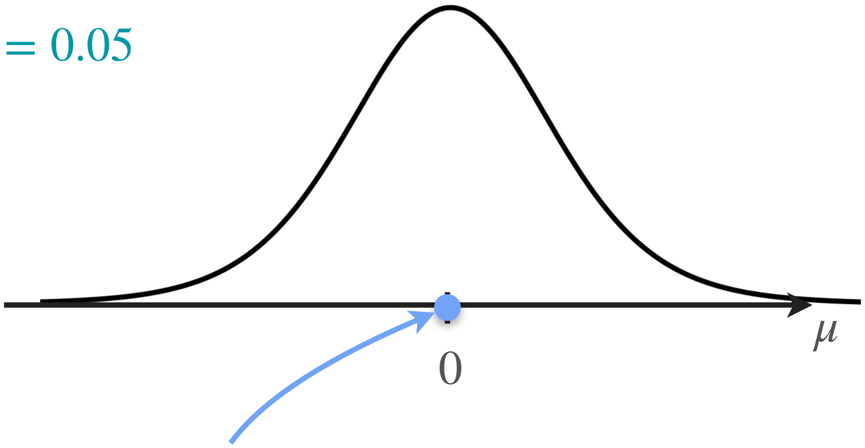


$$\text{If } H_0 \text{ is true: } T = \frac{\bar{X} - 66.7}{S/\sqrt{10}} \sim t_9$$

Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\alpha = 0.05$



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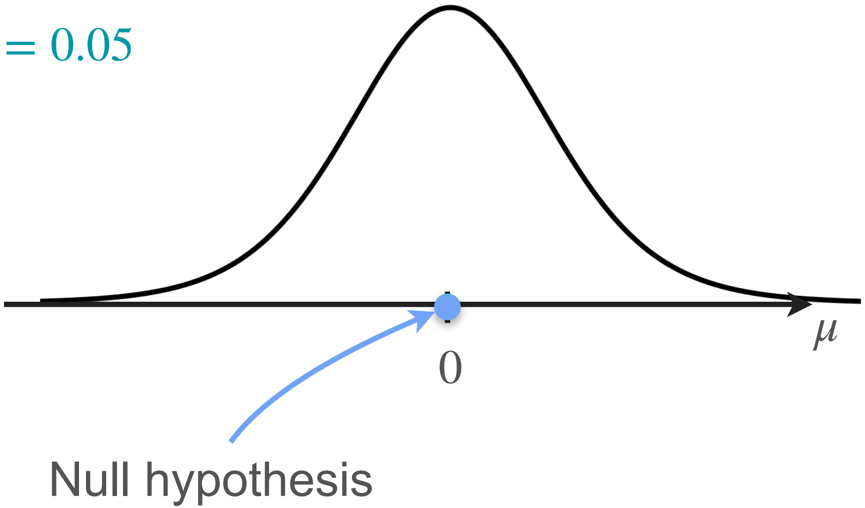
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

$\alpha = 0.05$



Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

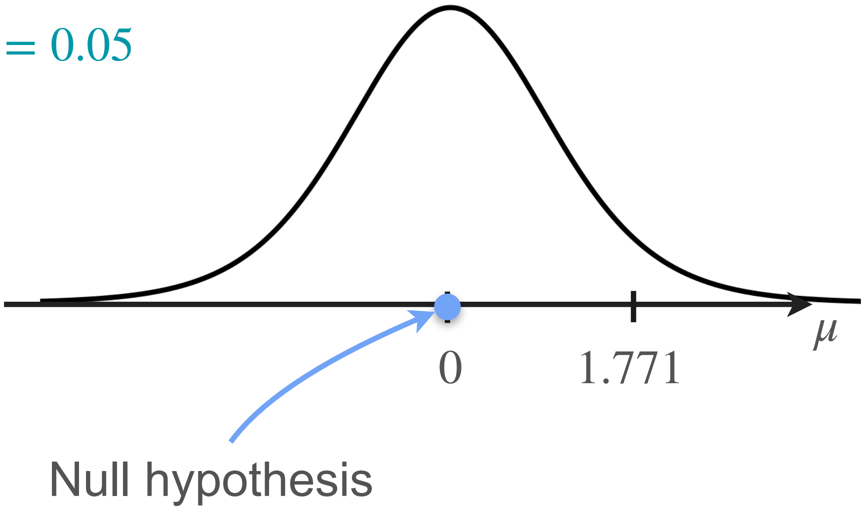
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $\quad n = 10$

$\bar{x} = 68.442$

$s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



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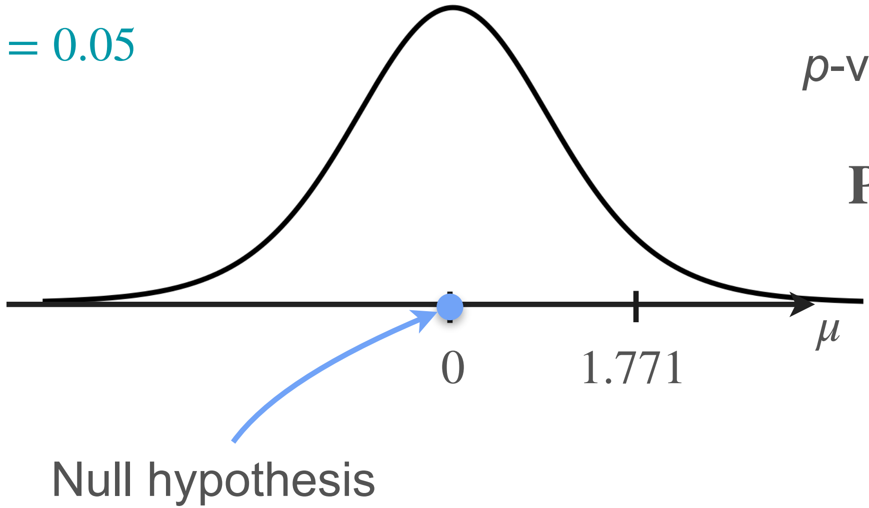

p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \,\middle|\, \mu = 66.7\right)?$$

Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $\quad n = 10$

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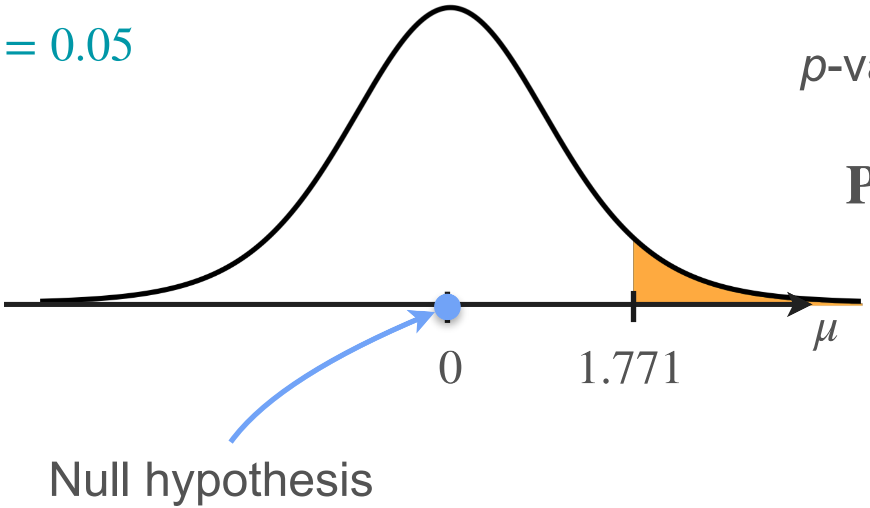


p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \,\middle|\, \mu = 66.7 \right)$$

$$= 0.0552$$

Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

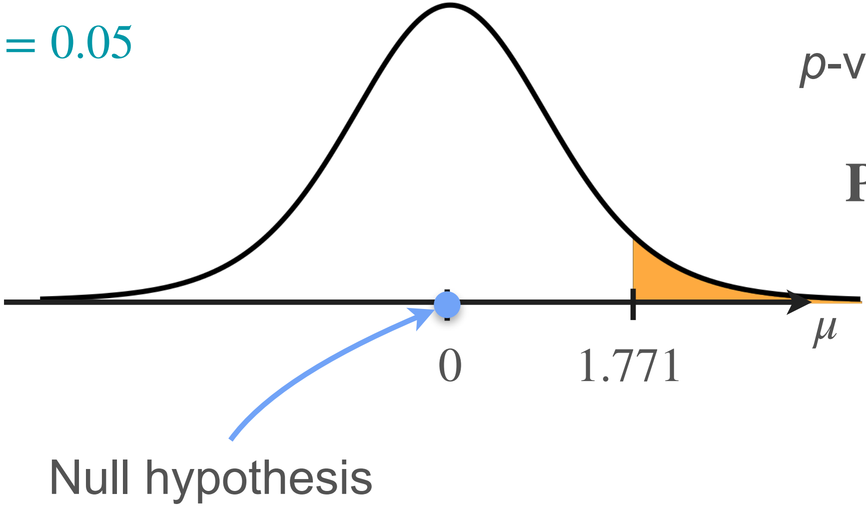
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $\quad n = 10$

$\alpha = 0.05$

$\bar{x} = 68.442$

$s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \,\middle|\, \mu = 66.7 \right)$$

$$= 0.0552 > \alpha$$

Conclusion: do not reject H_0
(with a 5% significance level)

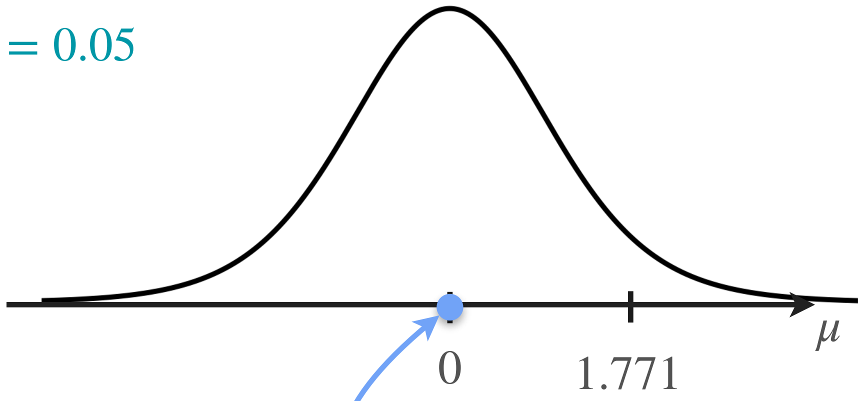
Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$$n = 10 \qquad \bar{x} = 68.442 \qquad s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



Null hypothesis

Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

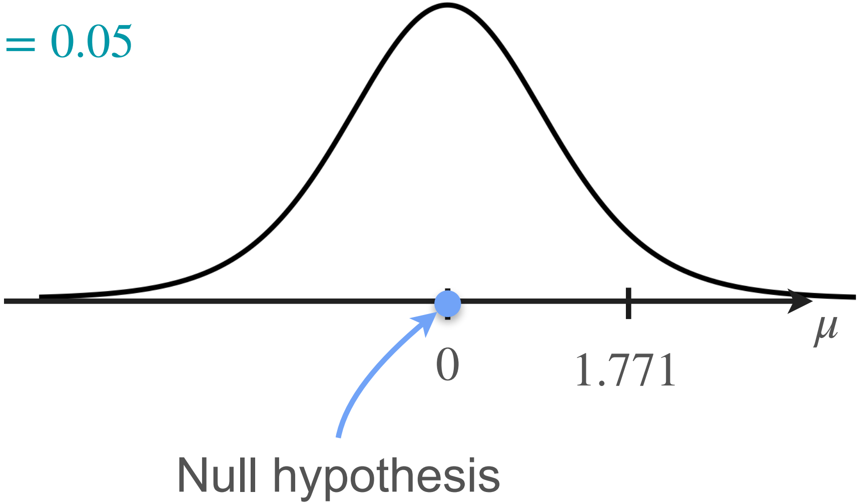
$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$

$\bar{x} = 68.442$

$s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



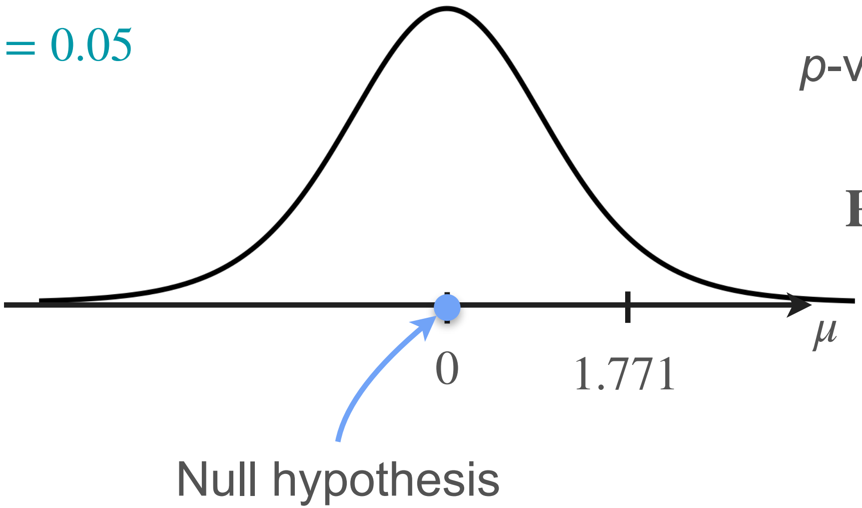
Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$ $\quad \bar{x} = 68.442$ $\quad s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > 1.771 \,\middle|\, \mu = 66.7\right)?$$

Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

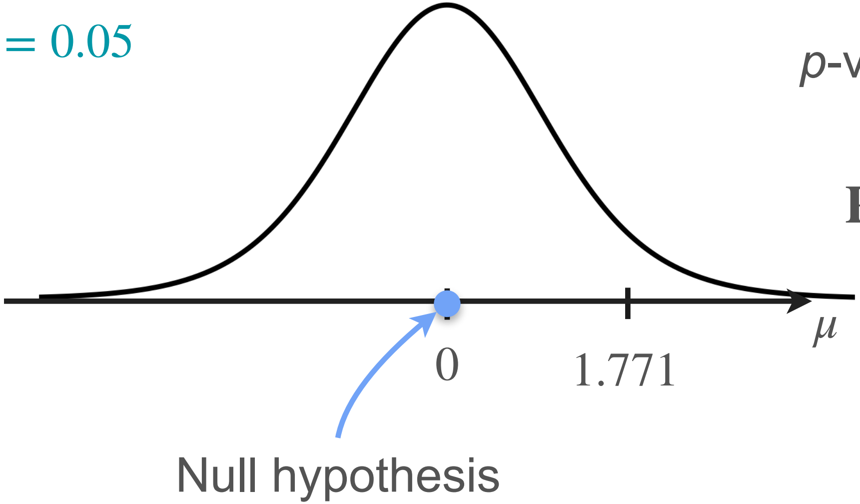
$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$

$\bar{x} = 68.442$

$s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\left| \frac{\bar{X} - 66.7}{S/\sqrt{10}} \right| > |1.771| \,\middle|\, \mu = 66.7 \right) ?$$

Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$ $\quad \bar{x} = 68.442$ $\quad s = 3.113$

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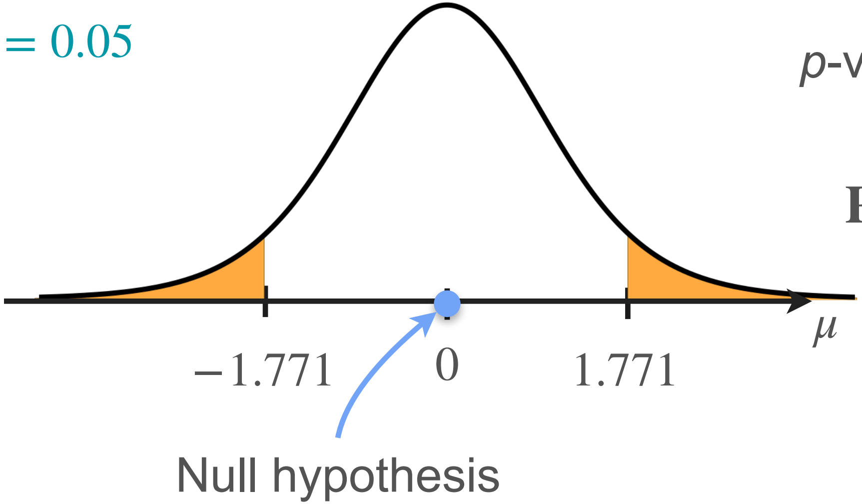


p-value:

$$\mathbf{P}\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \,\middle|\, \mu = 66.7\right) = 0.1103$$

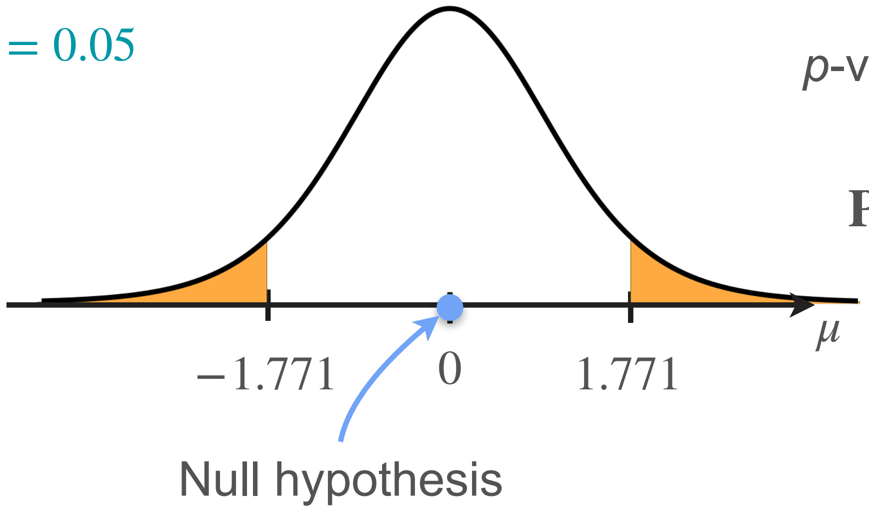
Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0: \mu = 66.7$ vs. $H_1: \mu \neq 66.7$ $\quad n = 10$ $\quad \bar{x} = 68.442$ $\quad s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \,\middle|\, \mu = 66.7\right) = 0.1103 > \alpha$$

Conclusion: do not reject H_0

(with a 5% significance level)

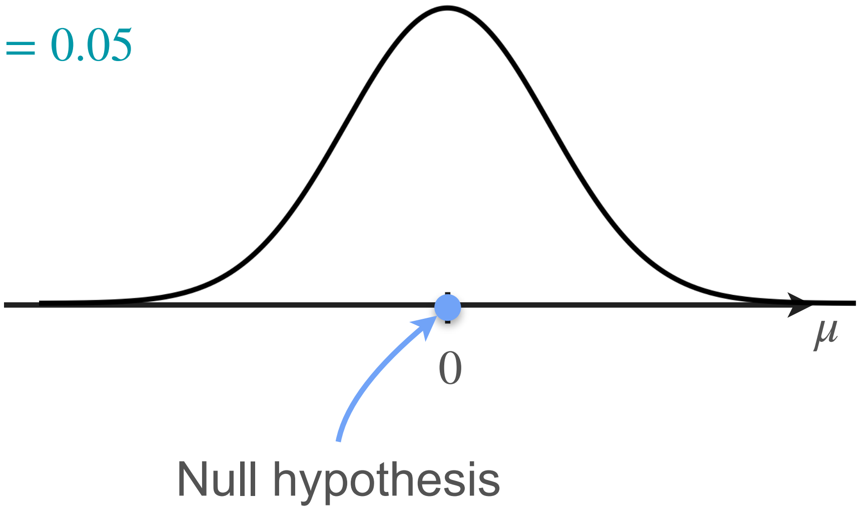
Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$$n = 10 \qquad \bar{x} = 64.252 \qquad s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$

$\alpha = 0.05$



Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

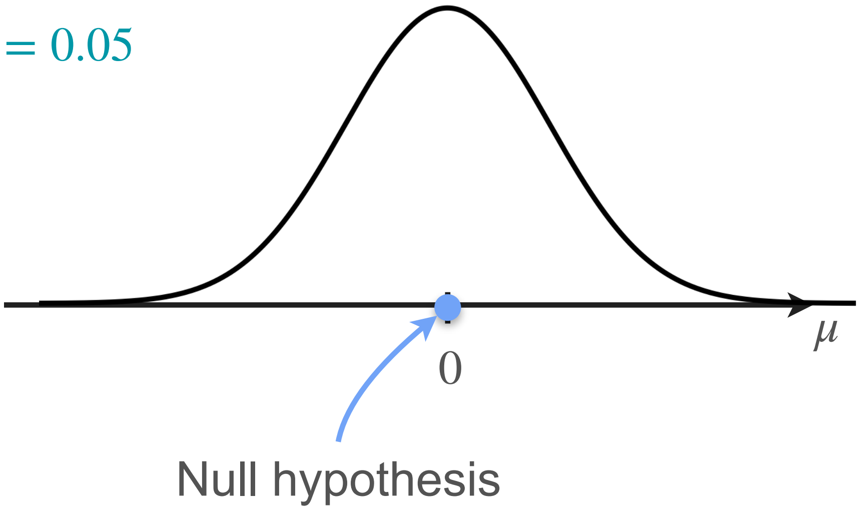
$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$

$\bar{x} = 64.252$

$s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$

$\alpha = 0.05$



Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

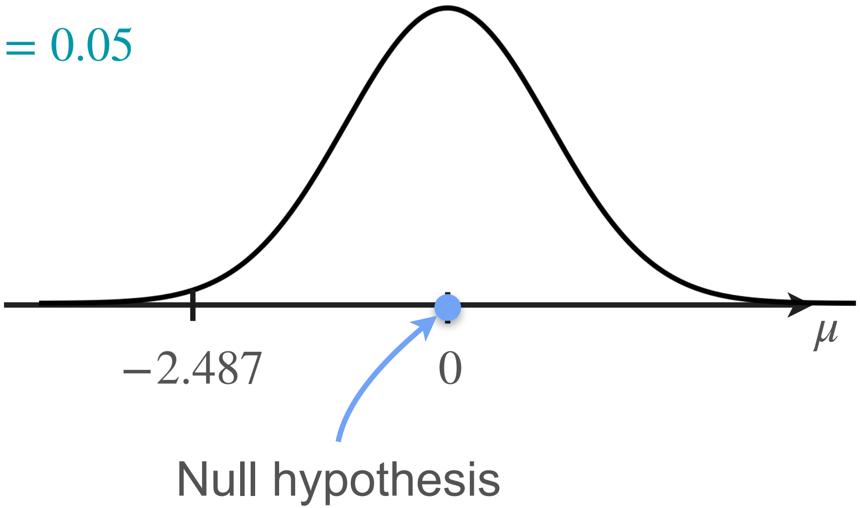
$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$

$\bar{x} = 64.252$

$s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



Left-Tailed Test for Gaussian Data (Unknown σ)

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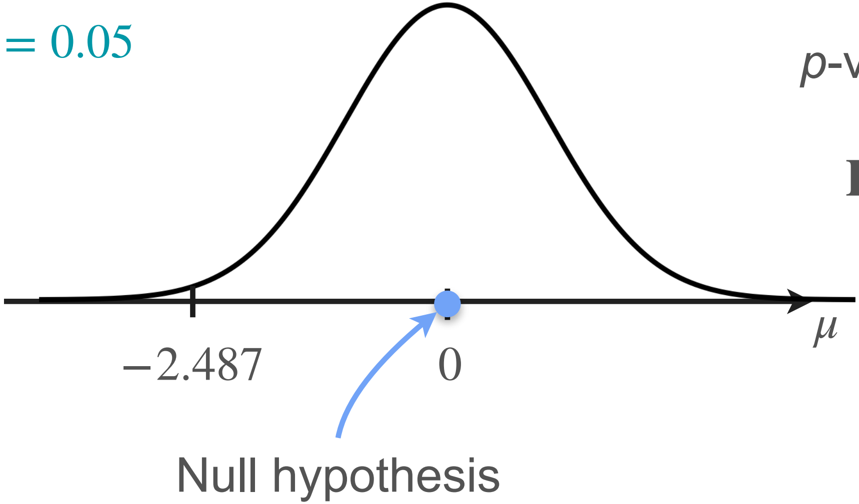
$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \,\middle|\, \mu = 66.7\right)?$$

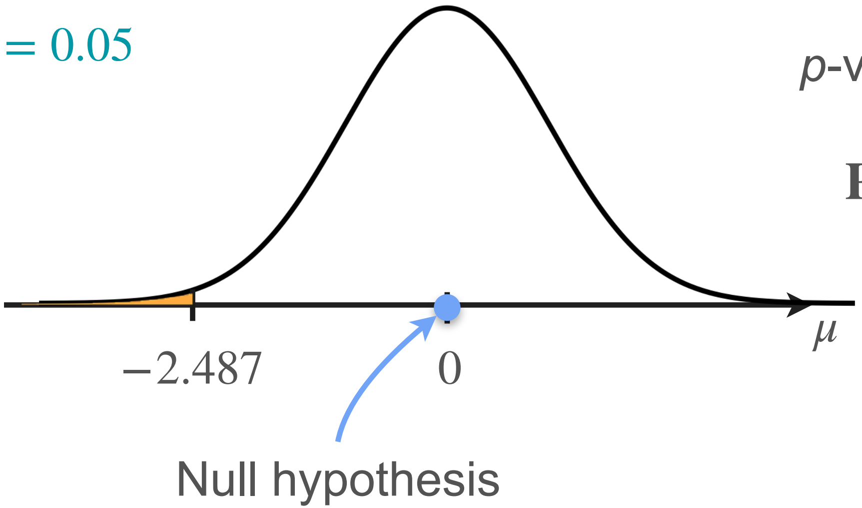
Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$ $\quad \bar{x} = 64.252$ $\quad s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \,\middle|\, \mu = 66.7\right)$$

$$= 0.0173$$

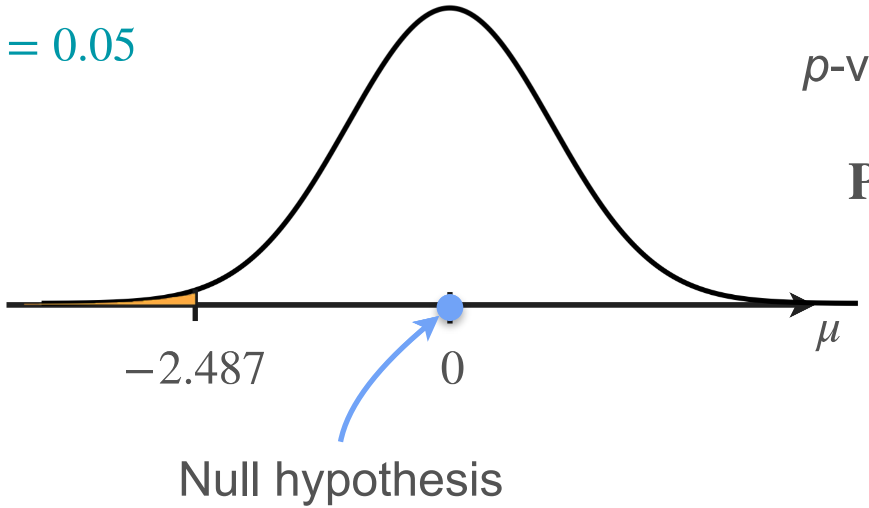
Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$ $\quad \bar{x} = 64.252$ $\quad s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \,\middle|\, \mu = 66.7\right)$$

$$= 0.0173 < \alpha$$

Conclusion: reject H_0
(with a 5% significance level)

Hypothesis Testing

Two sample t-test

Independent Two-Sample t-Test

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

Height of 18 y/o in the US

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

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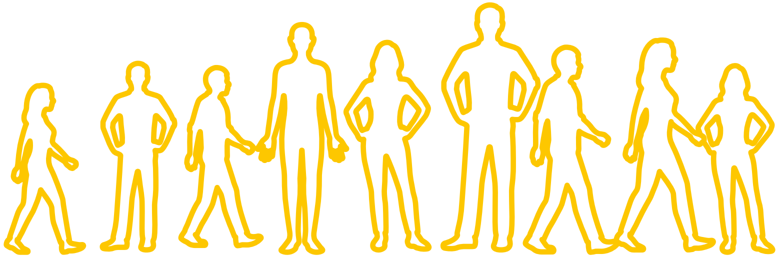
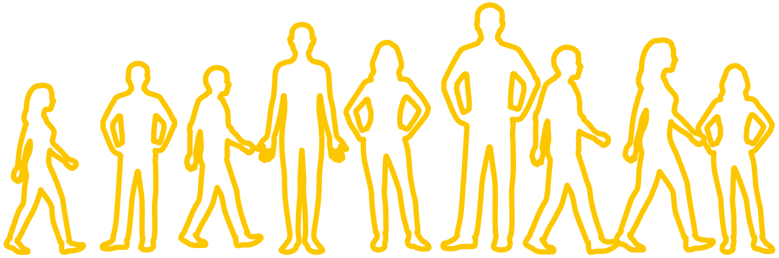


y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

Height of 18 y/o in Argentina

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74



y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

$n_X = 10$

$\bar{x} = 68.442$

$s_X = 3.113$

Height of 18 y/o in the US

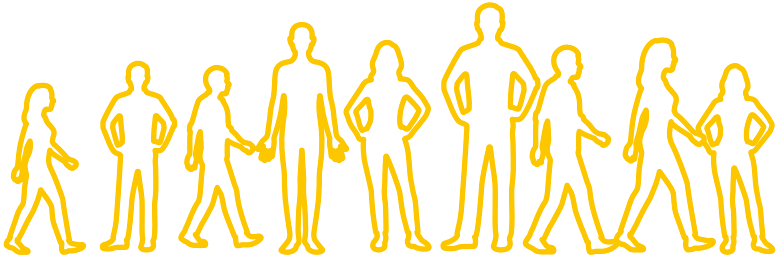
Height of 18 y/o in Argentina

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

$n_X = 10$ $\quad \bar{x} = 68.442$ $\quad s_X = 3.113$

Height of 18 y/o in the US



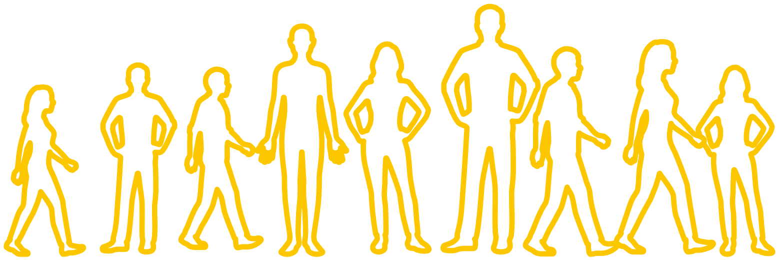
y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

$n_Y = 9$ $\quad \bar{y} = 65.949$ $\quad s_Y = 3.106$

Height of 18 y/o in Argentina

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74



y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

$n_X = 10$ $\quad \bar{x} = 68.442$ $\quad s_X = 3.113$

Height of 18 y/o in the US

$n_Y = 9$ $\quad \bar{y} = 65.949$ $\quad s_Y = 3.106$

Height of 18 y/o in Argentina

$$\mu_{US} \neq \mu_{Ar}$$

Independent Two-Sample t-Test: Hypothesis

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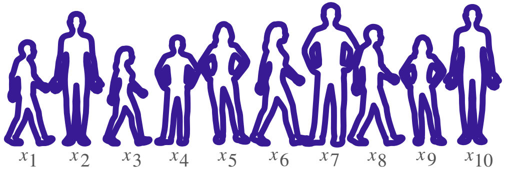
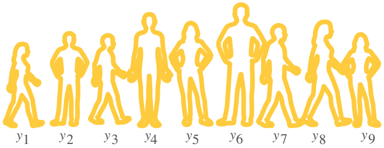
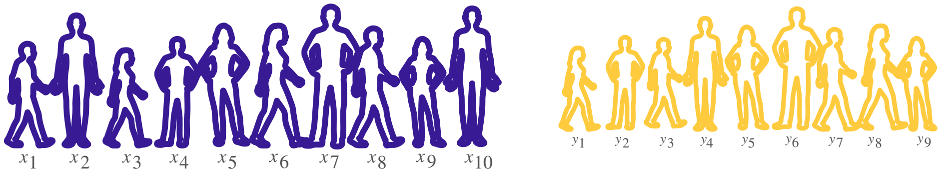


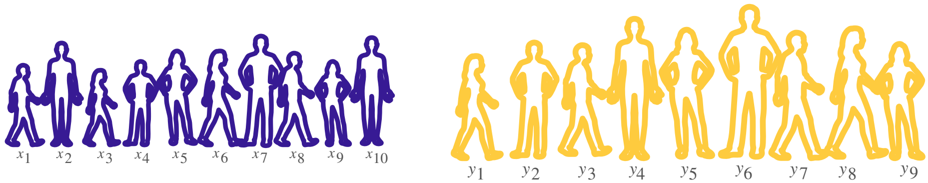


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} > \mu_{Ar}$

Independent Two-Sample t-Test: Hypothesis

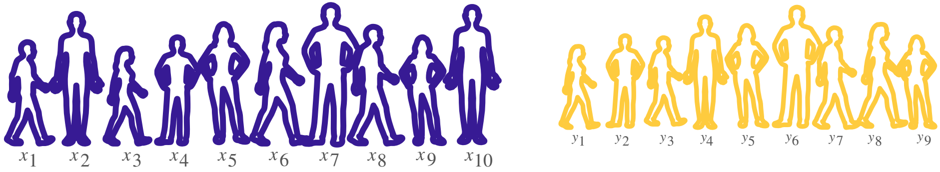


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} > \mu_{Ar}$

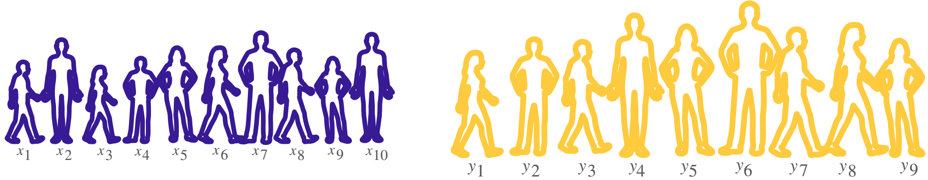


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} < \mu_{Ar}$

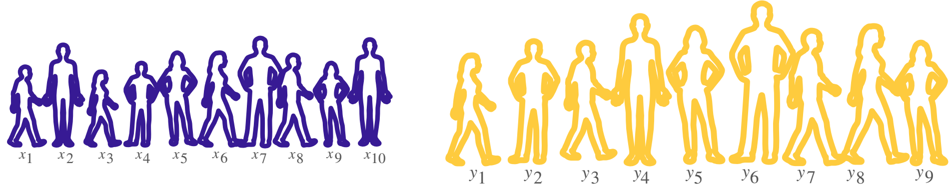
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} > \mu_{Ar}$

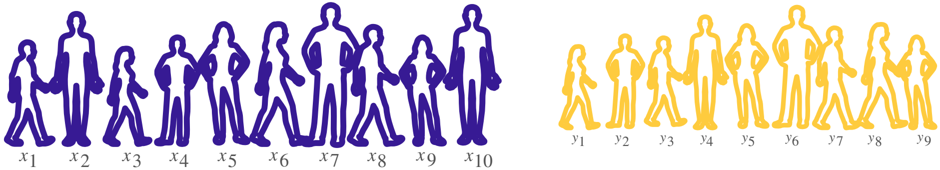


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} < \mu_{Ar}$

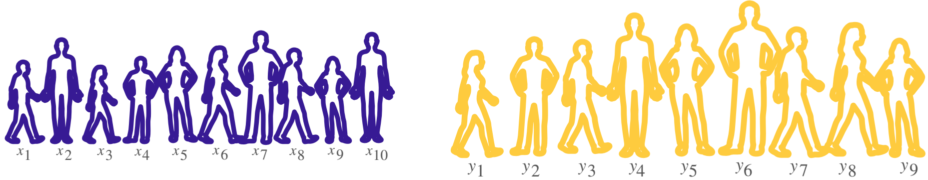


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} \neq \mu_{Ar}$

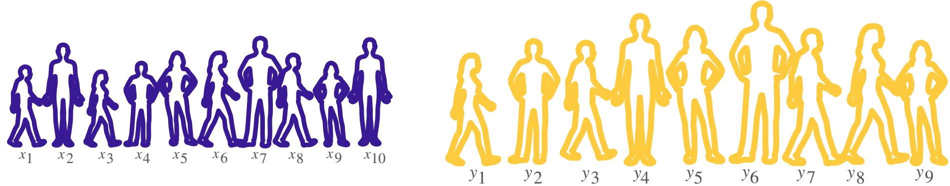
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$



$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} < \mu_{Ar}$



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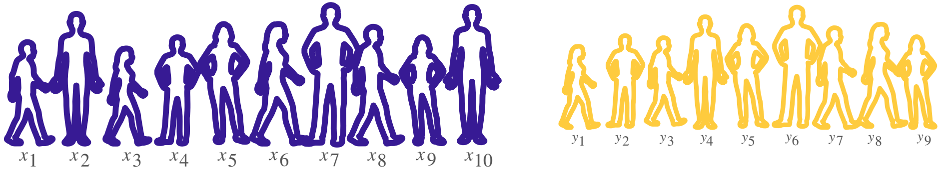
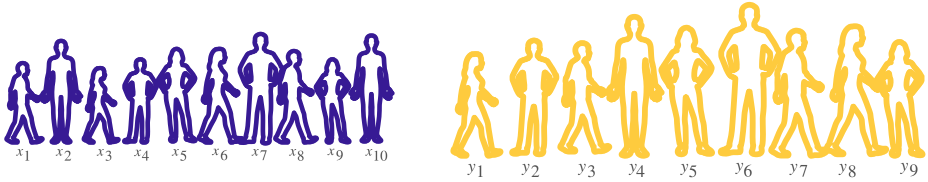
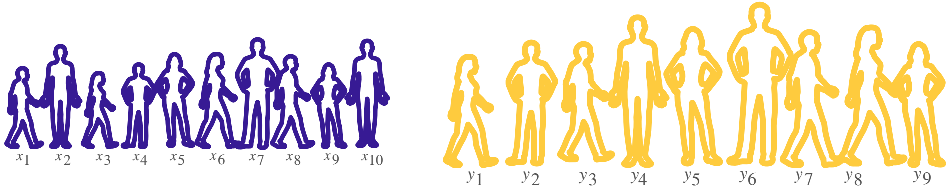
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

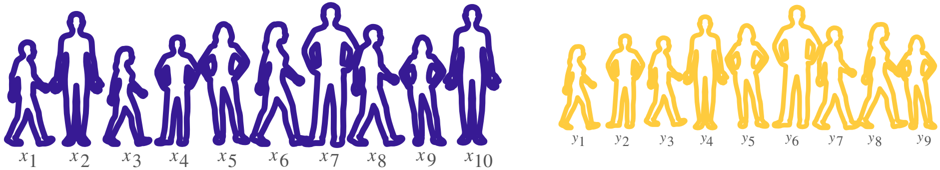


$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} < 0$

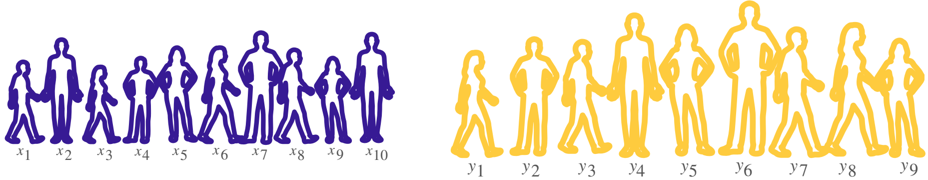


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} \neq \mu_{Ar}$

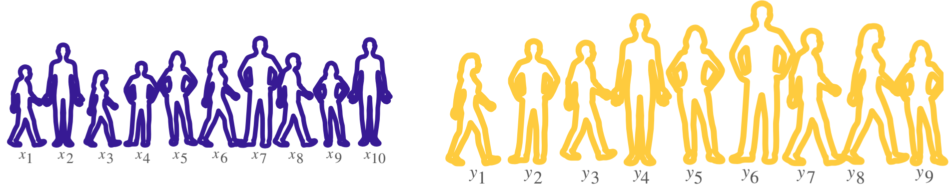
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} < 0$



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

Independent Two-Sample t-Test: Assumptions

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- All people in the sample from the two groups are different

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$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\bar{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

Independent Two-Sample t-Test: Assumptions

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$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\overline{X} - \overline{Y} \sim ?$$

$$\overline{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

Independent Two-Sample t-Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
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$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}^2\right)$$

$$Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}^2\right)$$

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\bar{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\quad , \quad \right)$$

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$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\overline{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \qquad \right)$$

Independent Two-Sample t-Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}^2\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}^2\right)$$

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right)$$

$$\overline{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

Independent Two-Sample t-Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right)$$

$$Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\bar{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}\right)$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right)$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \rightarrow \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \qquad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \rightarrow \frac{(\overline{X} - \overline{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know σ_{US}, σ_{Ar} 😱

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \longrightarrow \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don't know σ_{US}, σ_{Ar} 😱 $\longrightarrow$ Replace it with the sample standard deviation

Independent Two-Sample t-Test: Statistic

$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right)$ $\quad$ $Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \longrightarrow \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don't know σ_{US}, σ_{Ar}  $\longrightarrow$ Replace it with the sample standard deviation

$$\frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}}$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \rightarrow \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don’t know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

$$T = \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{\nu}$$

Independent Two-Sample t-Test: Statistic

$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$ $\quad$ $Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$

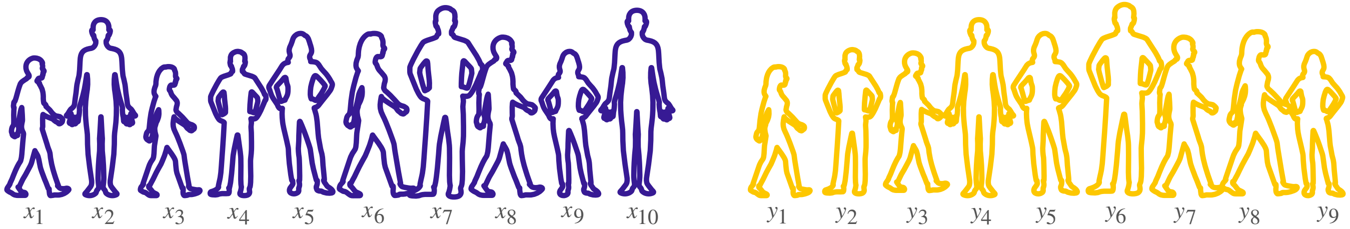
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don't know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_\nu$$

Degrees of freedom $= \dfrac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

Independent Two-Sample t-Test

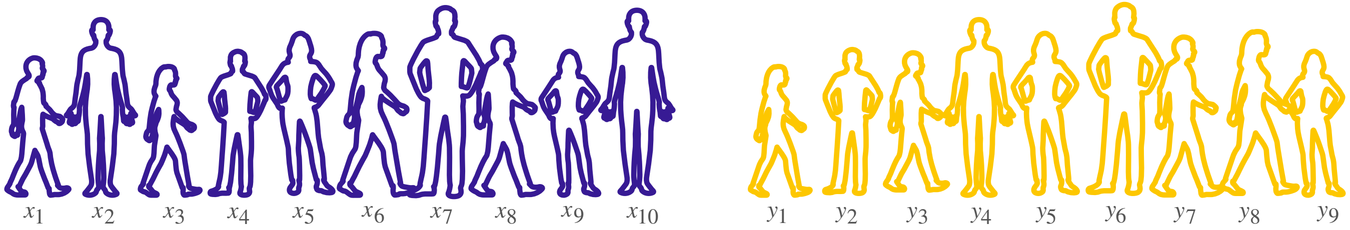


$n_X = 10$ $\quad \bar{x} = 68.442$ $\quad s_X = 3.113$

$n_Y = 9$ $\quad \bar{y} = 65.949$ $\quad s_Y = 3.106$

Degrees of freedom $= \dfrac{\left(\frac{\;^2}{\;} + \frac{\;^2}{\;}\right)^2}{\frac{\left(\frac{\;^2}{\;}\right)^2}{\;-1} + \frac{\left(\frac{\;^2}{\;}\right)^2}{\;-1}}$

Independent Two-Sample t-Test



$n_X = 10$

$\bar{x} = 68.442$

$s_X = 3.113$

$n_Y = 9$

$\bar{y} = 65.949$

$s_Y = 3.106$

$$\text{Degrees of freedom} = \frac{\overbrace{\left(\frac{3.113^2}{10} + \frac{3.106^2}{9}\right)^2}^{16.8}}{\frac{\left(\frac{3.113^2}{10}\right)^2}{10-1} + \frac{\left(\frac{3.106^2}{9}\right)^2}{9-1}}$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}\right) \rightarrow \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1)$$

You don't know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

$$T = \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom $= \dfrac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1)$$

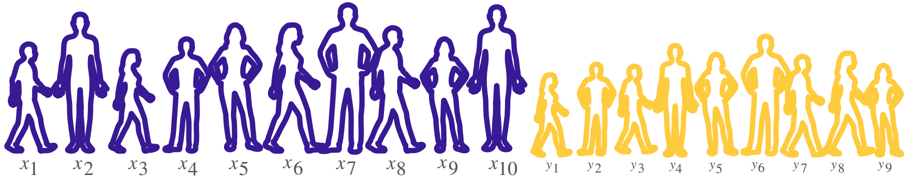
You don't know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

$$T = \frac{(\bar{X} - \bar{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom $= \dfrac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

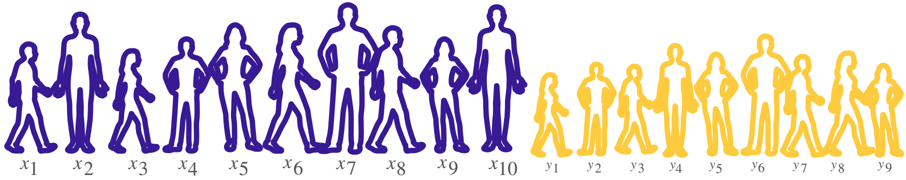
Independent Two-Sample t-Test: Right Tailed Test

Independent Two-Sample t-Test: Right Tailed Test



$n_X = 10$ $\qquad$ $n_Y = 9$

Independent Two-Sample t-Test: Right Tailed Test



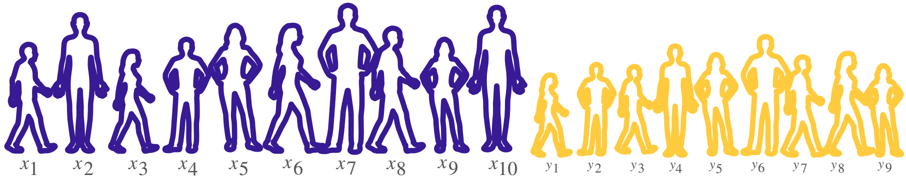
$\bar{x} = 68.442$

$s_X = 3.113$

$n_X = 10$

$n_Y = 9$

Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$

$s_X = 3.113$

$n_X = 10$

$\bar{y} = 65.949$

$s_Y = 3.106$

$n_Y = 9$

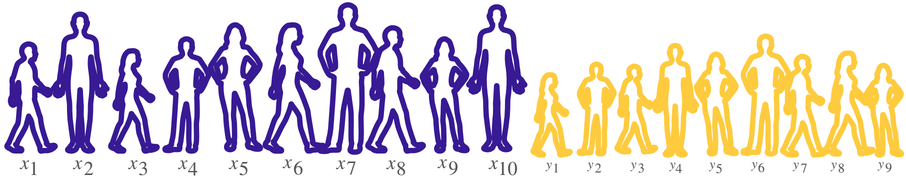
Independent Two-Sample t-Test: Right Tailed Test

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$

$s_X = 3.113$

$n_X = 10$

$\bar{y} = 65.949$

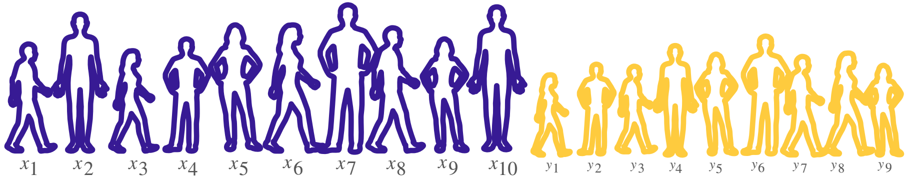
$s_Y = 3.106$

$n_Y = 9$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

Independent Two-Sample t-Test: Right Tailed Test



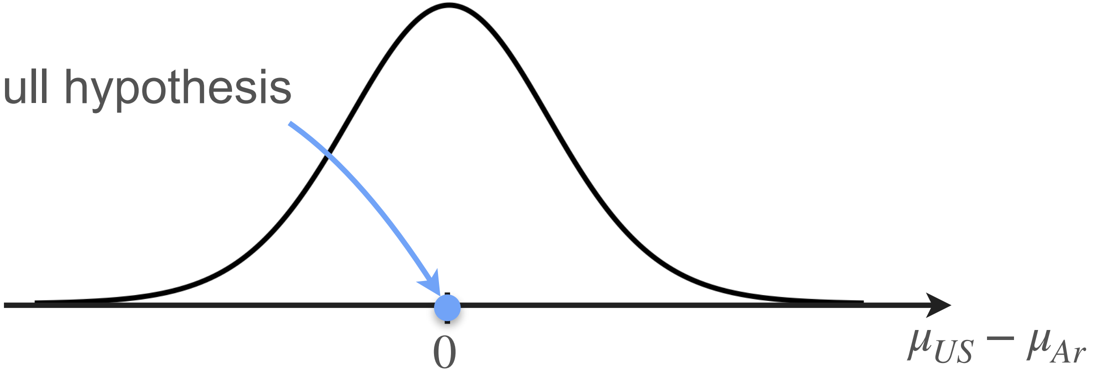
$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

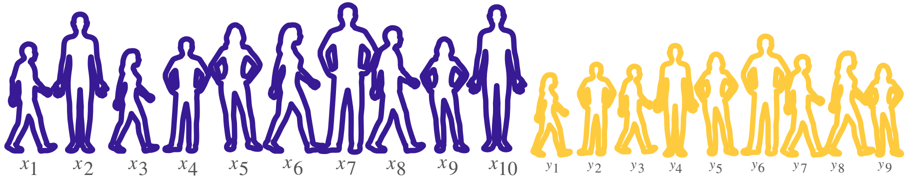
$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



Independent Two-Sample t-Test: Right Tailed Test

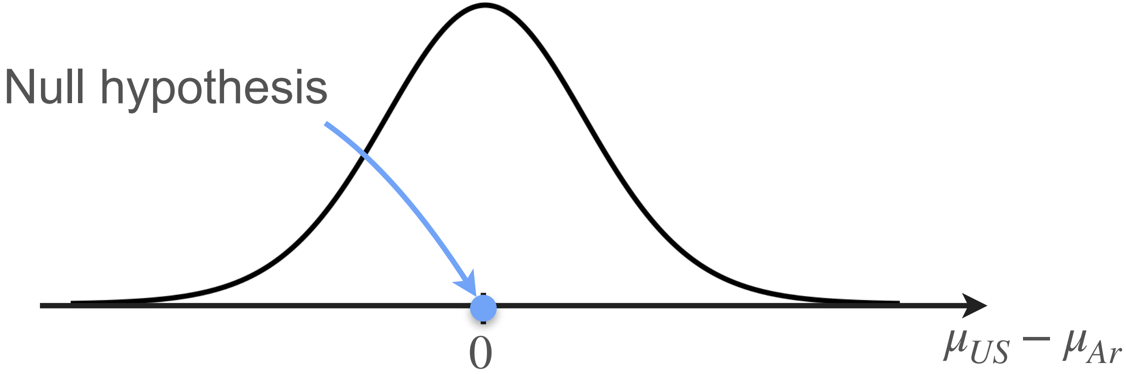


$$\underbrace{\bar{x} = 68.442 \quad s_X = 3.113 \quad n_X = 10 \quad \bar{y} = 65.949 \quad s_Y = 3.106 \quad n_Y = 9}_{t = 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



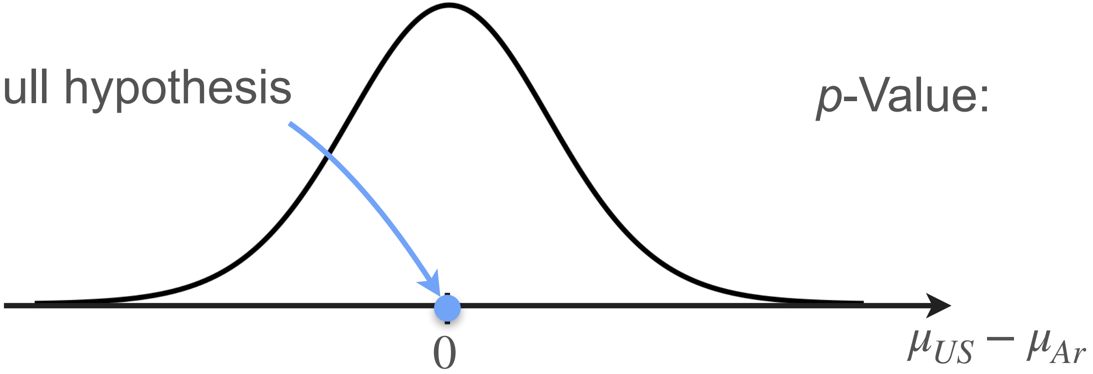
Independent Two-Sample *t*-Test: Right Tailed Test

$$\underbrace{\bar{x} = 68.442 \quad s_X = 3.113 \quad n_X = 10 \quad \bar{y} = 65.949 \quad s_Y = 3.106 \quad n_Y = 9}_{t\, =\, 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

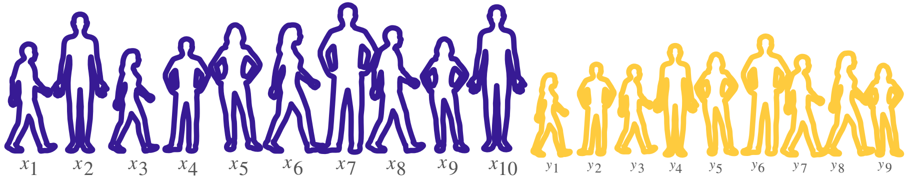
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value:

Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

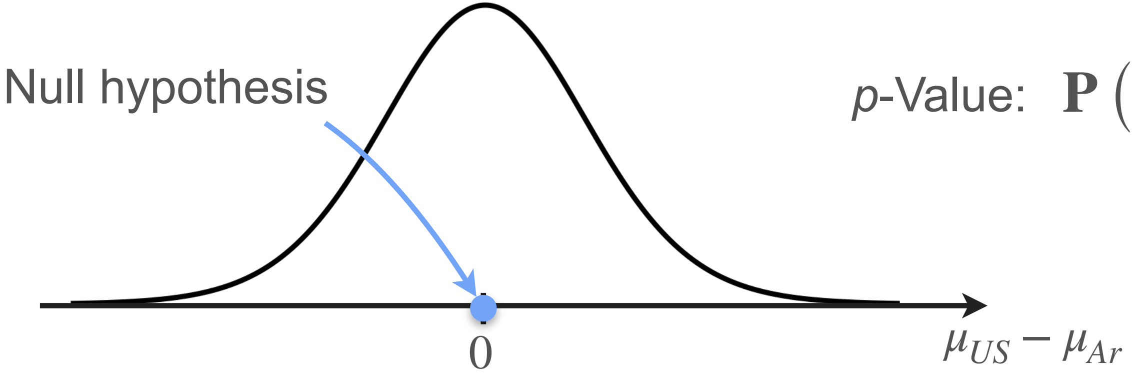
$$\underbrace{\qquad\qquad\qquad\qquad}_{t\,=\,1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

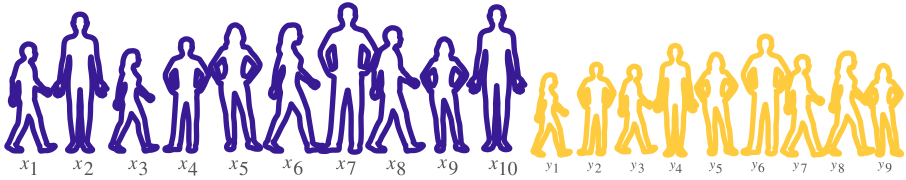
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

p-Value: $\mathbf{P}\left(T > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$



Independent Two-Sample t-Test: Right Tailed Test

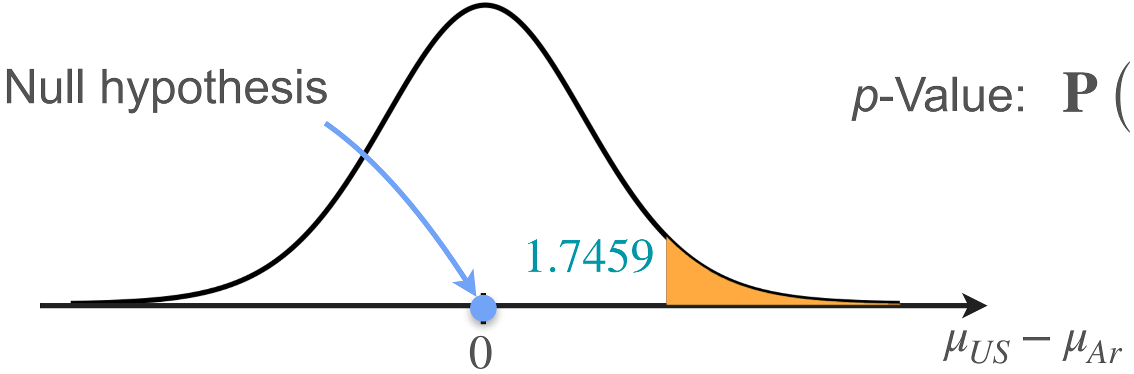


$$\underbrace{\bar{x} = 68.442 \quad s_X = 3.113 \quad n_X = 10 \quad \bar{y} = 65.949 \quad s_Y = 3.106 \quad n_Y = 9}_{t\, = 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

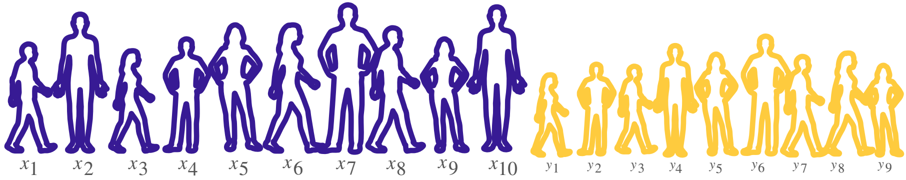
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\bar{X} - \bar{Y}\right) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(T > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$

Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10 \quad$ $\bar{y} = 65.949$ $\quad n_Y = 9$

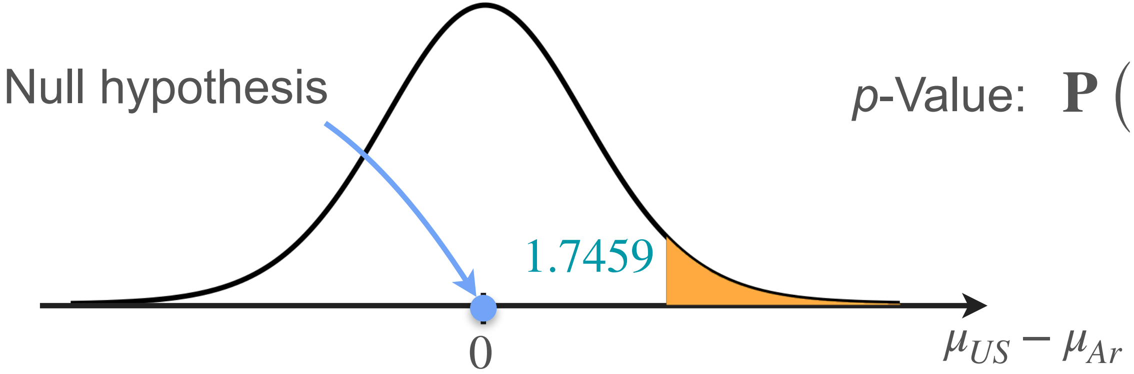
$s_X = 3.113$ $\quad$ $s_Y = 3.106$

$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(T > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$

$= 0.0495$

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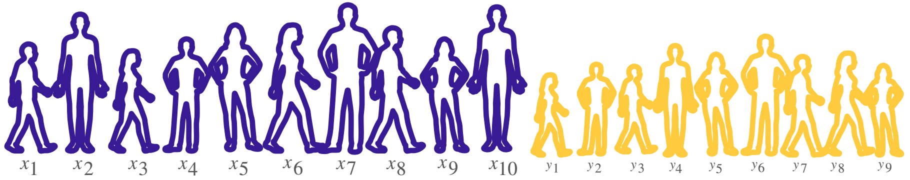
Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

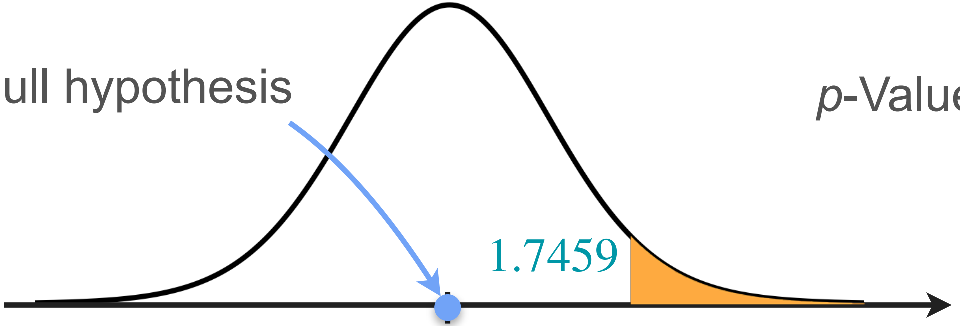
$s_X = 3.113$ $\quad s_Y = 3.106$

$$\underbrace{\hphantom{s_X = 3.113 \quad s_Y = 3.106}}_{t\, =\, 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

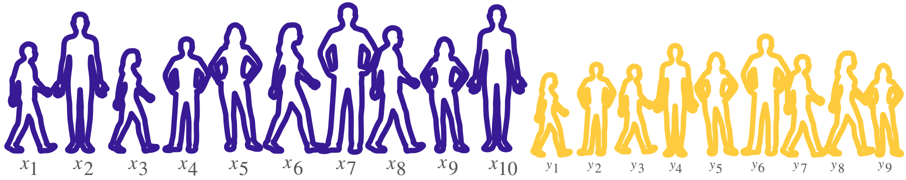


p-Value: $\mathbf{P}\left(T > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$

$= 0.0495 < 0.05$

$\Rightarrow$ Reject H_0 (and accept H_1)
(with a 5% significance level)

Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10 \quad$ $\bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

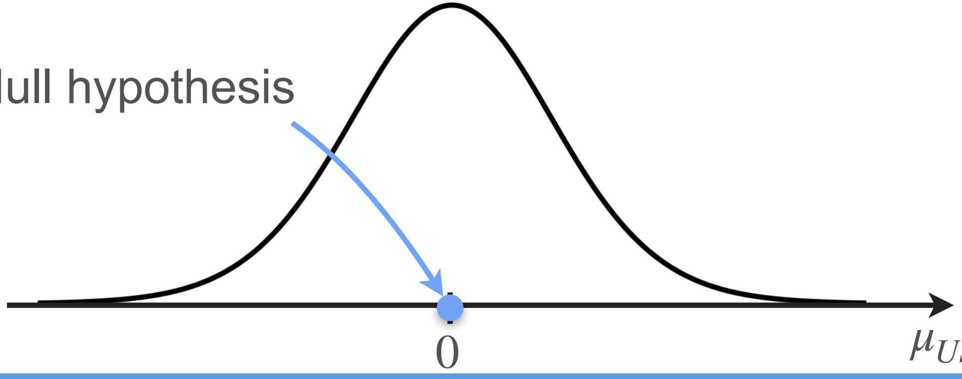
$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

Null hypothesis



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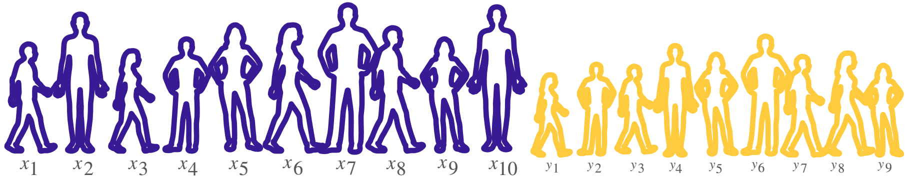
Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10 \quad$ $\bar{y} = 65.949$ $\quad n_Y = 9$

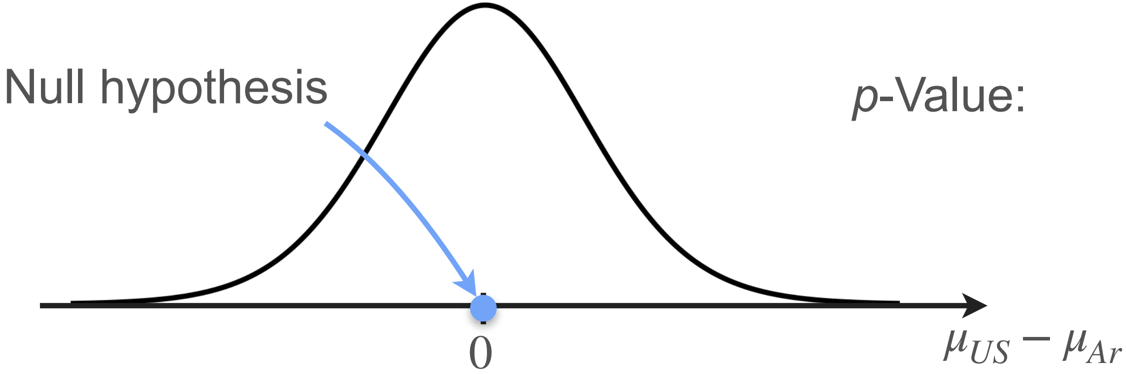
$s_X = 3.113$ $\quad s_Y = 3.106$

$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

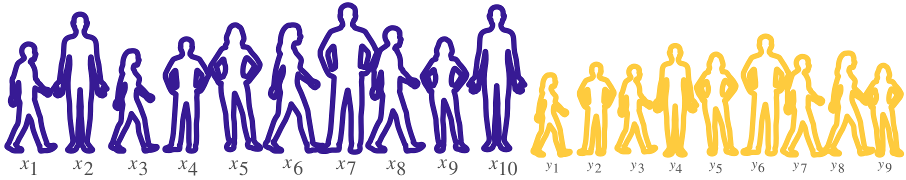
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value:

Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

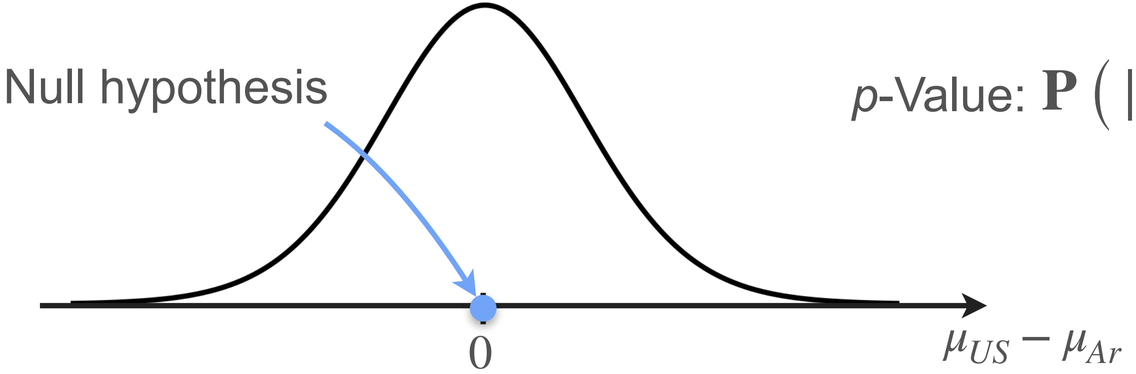
$s_X = 3.113$ $\quad s_Y = 3.106$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{t\, = 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

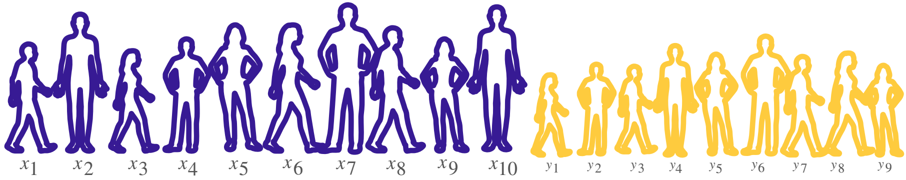
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(|T| > 1.7459 \;|\, \mu_{US} - \mu_{Ar} = 0\right)$

Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

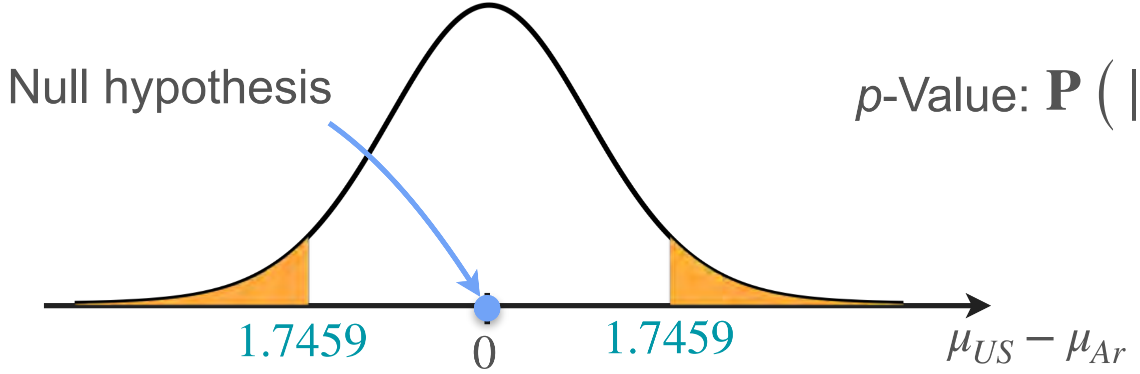
$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

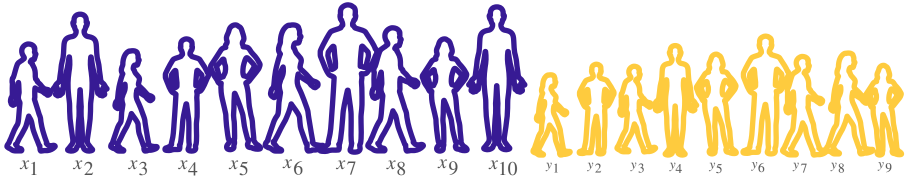
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

p-Value: $\mathbf{P}\left(|T| > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0\right)$



Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

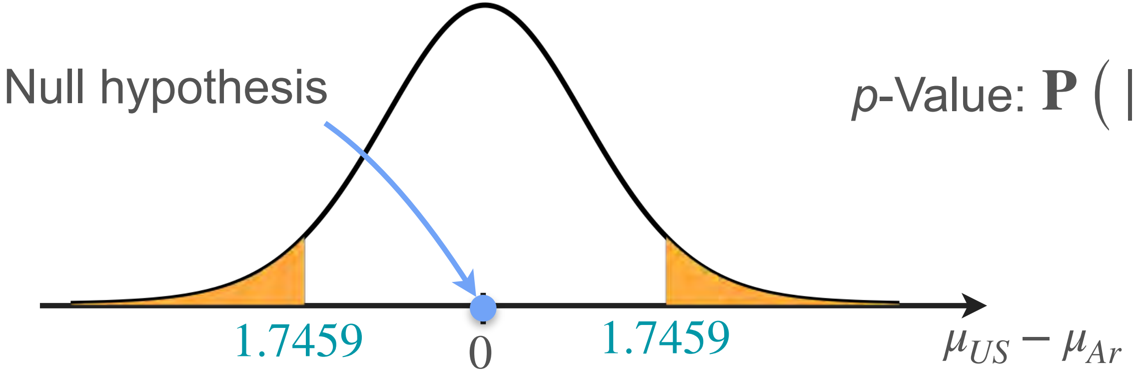
$s_X = 3.113$ $\quad s_Y = 3.106$

$$\underbrace{\qquad\qquad\qquad\qquad}_{t\ =\ 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$

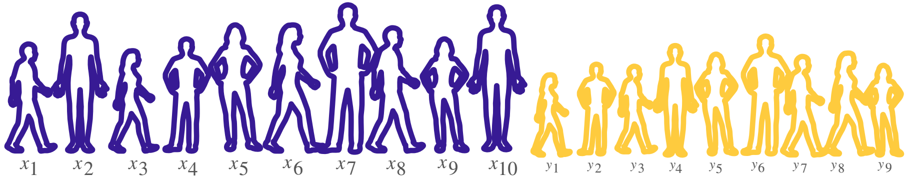
If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(\,|T| > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$

$= 0.0991$

Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

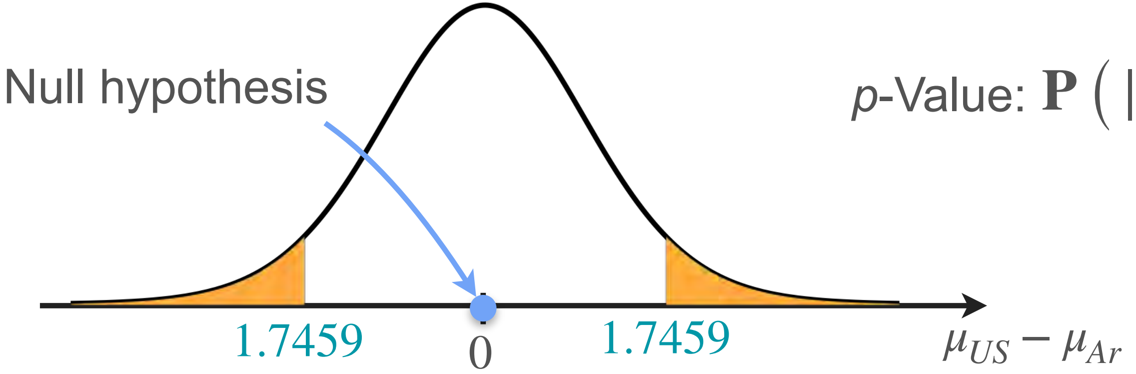
$s_X = 3.113$ $\quad s_Y = 3.106$

$$\underbrace{\qquad\qquad\qquad\qquad}_{t\, = 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(|T| > 1.7459 \;|\, \mu_{US} - \mu_{Ar} = 0\right)$

$= 0.0991 > 0.05$

$\Rightarrow$ Do not reject H_0

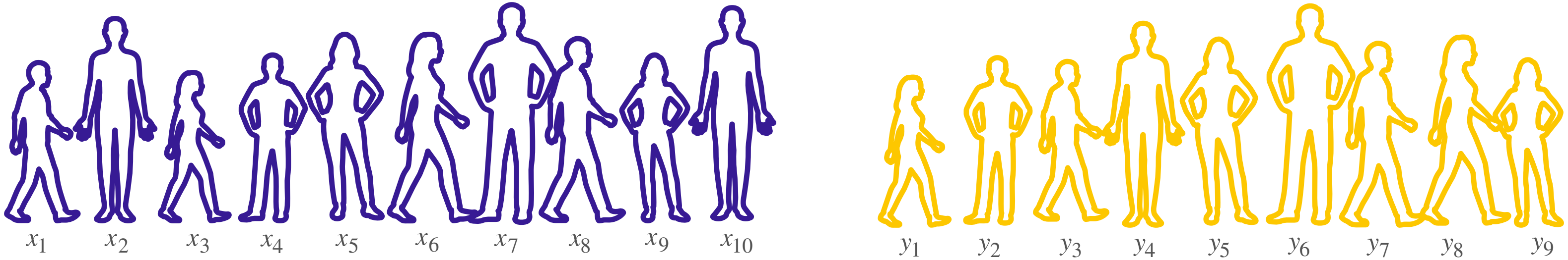
(with a 5% significance level)

Hypothesis Testing

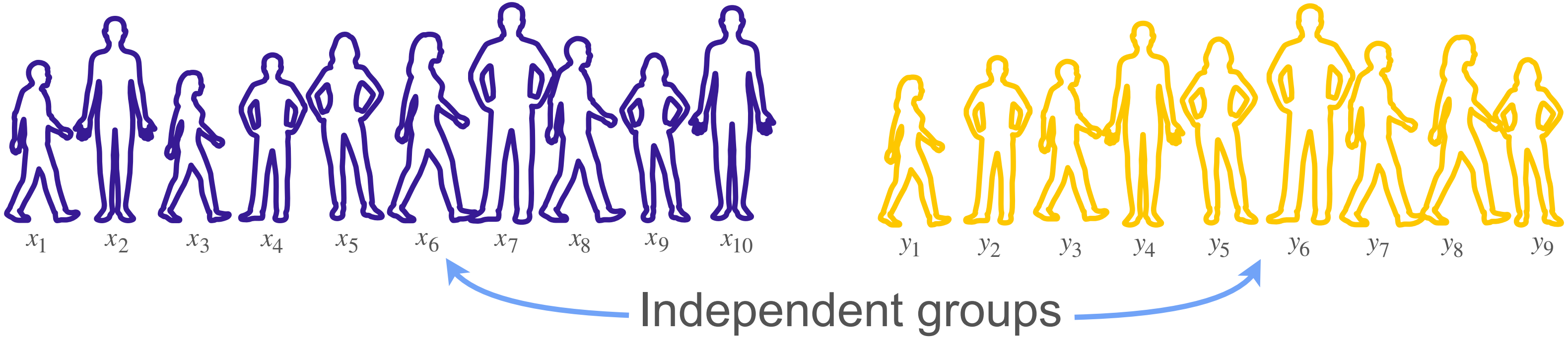
Paired t-test

Paired t-Test and Two-Sample t-Test

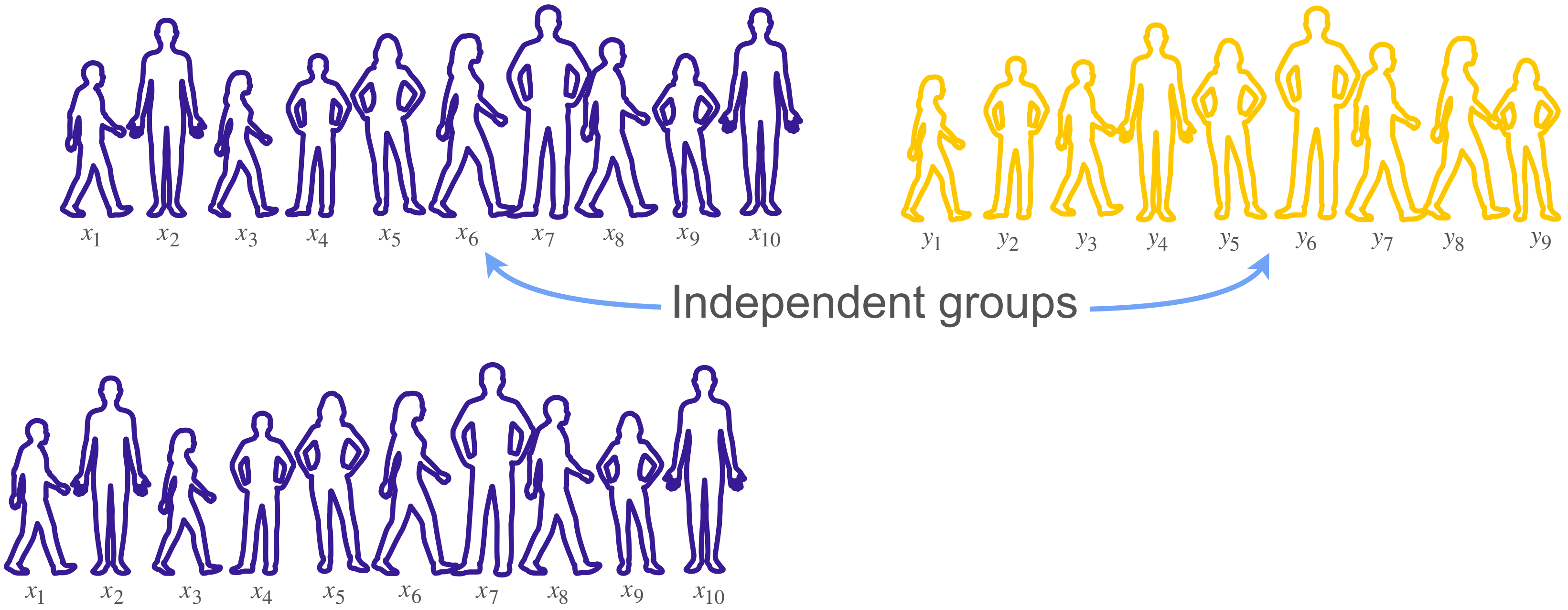
Paired t-Test and Two-Sample t-Test



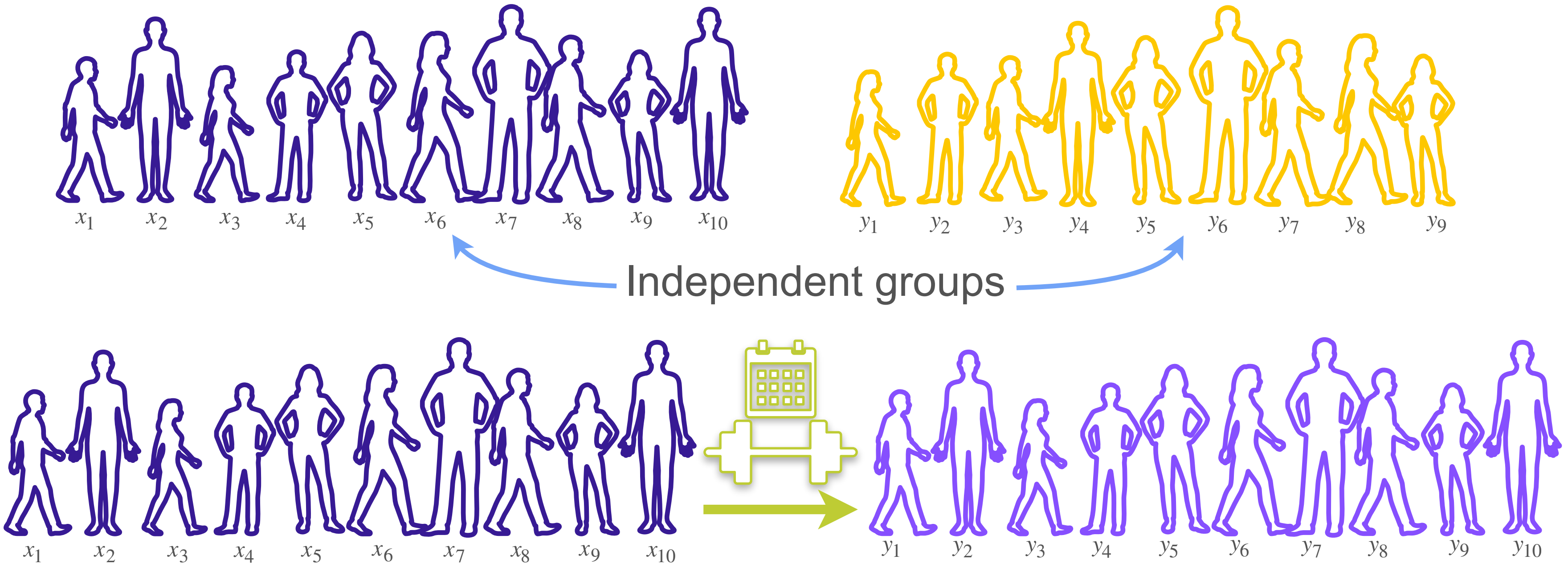
Paired t-Test and Two-Sample t-Test



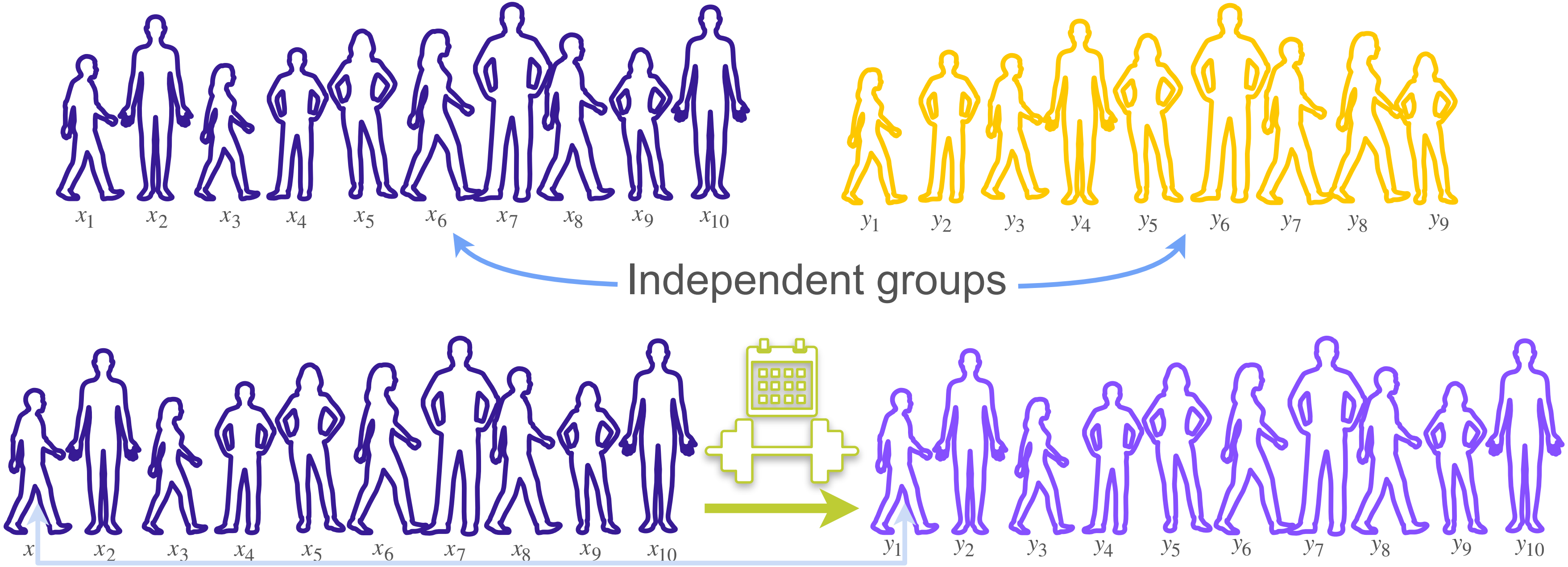
Paired t-Test and Two-Sample t-Test



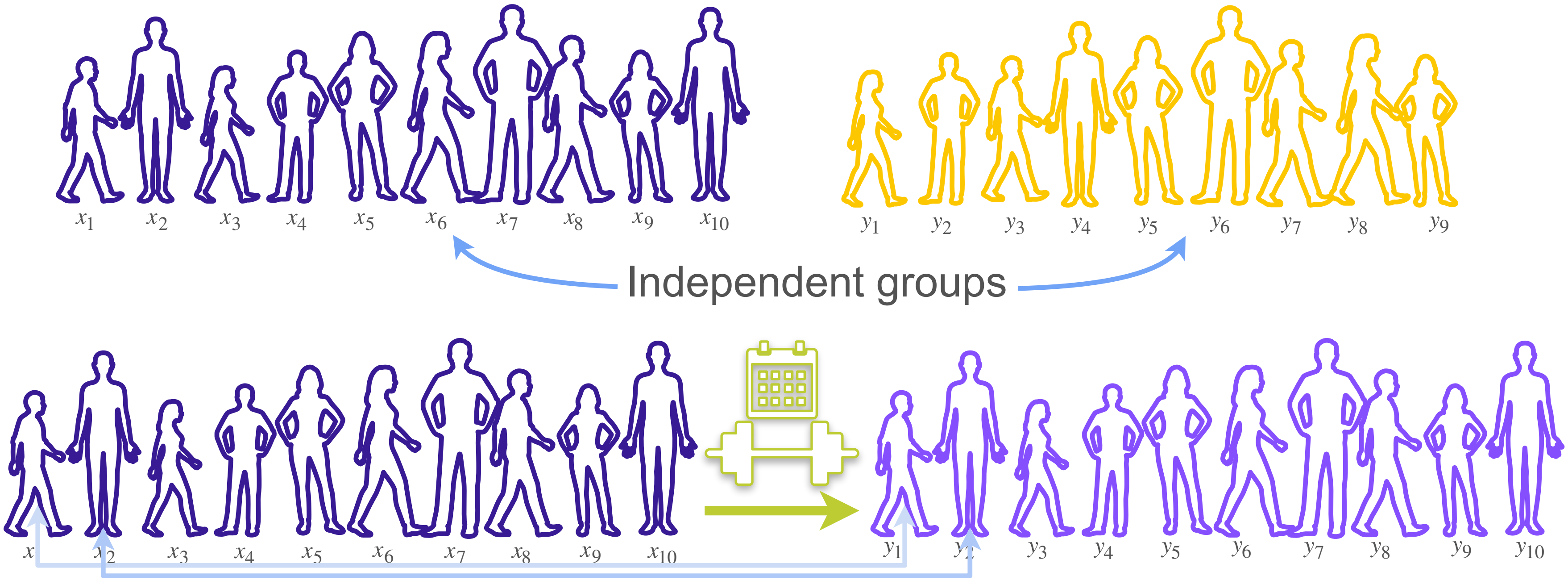
Paired t-Test and Two-Sample t-Test



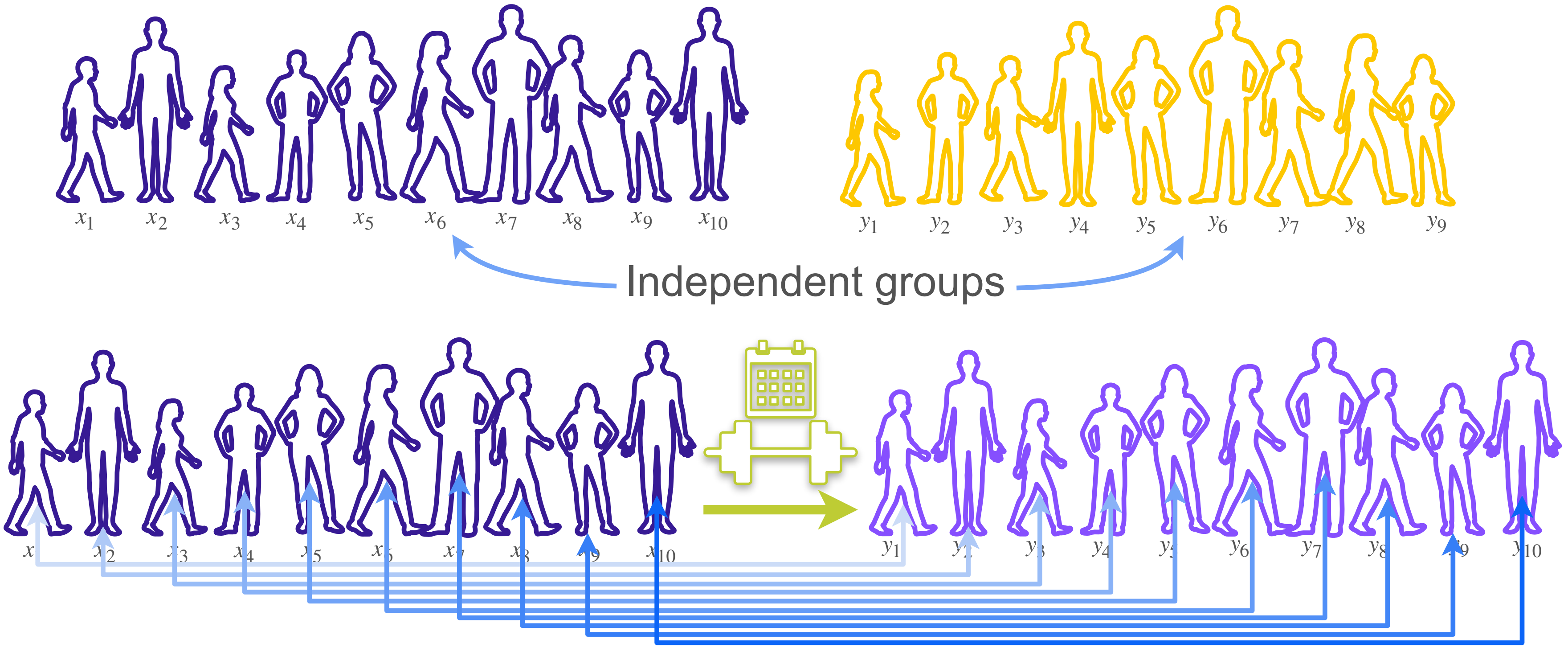
Paired t-Test and Two-Sample t-Test



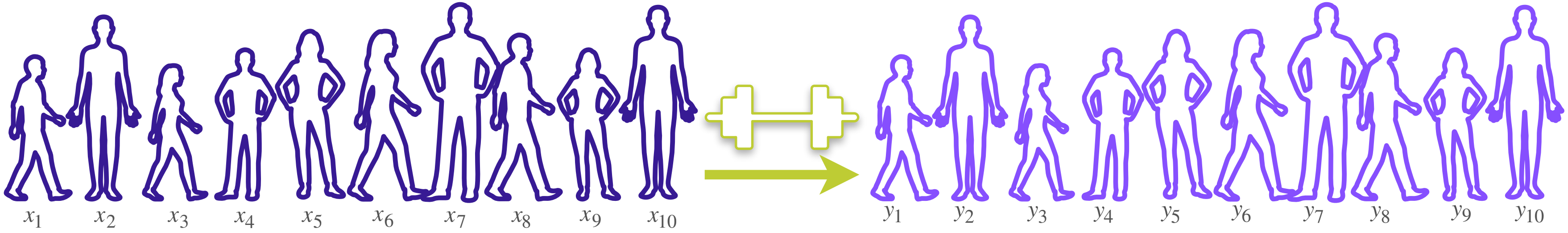
Paired t-Test and Two-Sample t-Test



Paired t-Test and Two-Sample t-Test

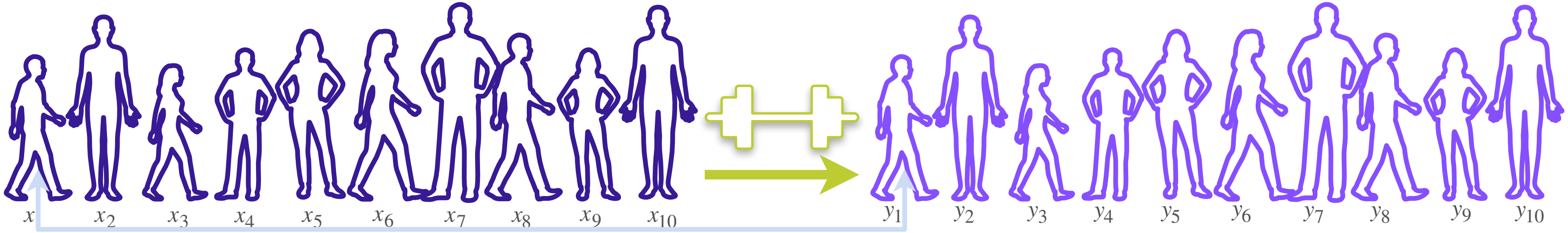


Paired t-Test: Statistic



Now you're interested in the difference between pair of samples

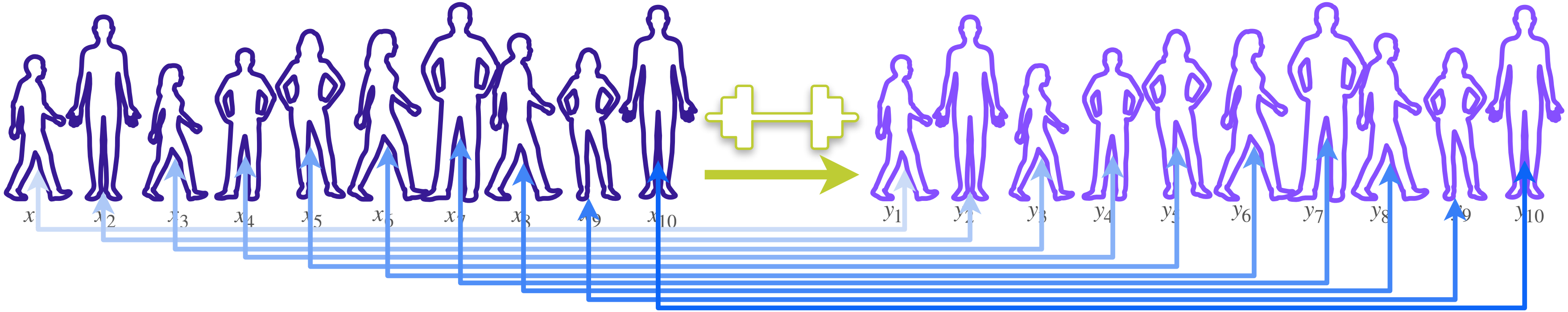
Paired *t*-Test: Statistic



Now you're interested in the difference between pair of samples

$(X_1 - Y_1)$

Paired t-Test: Statistic



Now you're interested in the difference between pair of samples

$(X_1 - Y_1)$ $(X_2 - Y_2)$ $(X_3 - Y_3)$ $(X_4 - Y_4)$ $(X_5 - Y_5)$ $(X_6 - Y_6)$ $(X_7 - Y_7)$ $(X_8 - Y_8)$ $(X_9 - Y_9)$ $(X_{10} - Y_{10})$