DeepLearning.AI

Probability and Statistics for Machine Learning and Data Science

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Week 3: Sampling and Point Estimates

W3 Lesson 1

DeepLearning.AI

Sample and Population

Population and Sample

Population and Sample

Population and Sample

Population and Sample

Population and Sample

Find the **average height** of the people living on Statistopia

Population and Sample

Find the **average height** of the people living on Statistopia

Population and Sample

Population and Sample

- Ask everyone on the island for their height.
- Divide by the total number

Population and Sample

- Ask everyone on the island for their height.
- Divide by the total number

Population and Sample

- Ask everyone on the island for their height.
- Divide by the total number

Population and Sample

- Ask everyone on the island for their height.
- Divide by the total number

Population and Sample

Population and Sample

- Only ask a subset of the group to estimate the average height

Population and Sample

- Only ask a subset of the group to estimate the average height

Population and Sample

Population and Sample

Population:
the entire group of individuals or elements you want to study which share a common behaviour

Population and Sample

The people of statistopia

Population:
the entire group of individuals or elements you want to study which share a common behaviour

Population and Sample

Population and Sample

Sample:
subset of the population you use to draw conclusions about the population as a whole

Population and Sample

The people you select for your study

Sample:
subset of the population you use to draw conclusions about the population as a whole

Population and Sample

The people you select for your study

Sample:
subset of the population you use to draw conclusions about the population as a whole

Population and Sample

Population and Sample

Population Size (N)

10,000

Population and Sample

Population Size (N)

10,000

Population and Sample

Population Size (N)

10,000

Sample Size (n)

Population and Sample

Population Size (N)

10,000

Sample Size (n)

1 - 9,999

Population and Sample

Population and Sample

Population and Sample

N = 10

Random Sampling

Random Sampling

Random Sampling

A

Random Sampling

A

n = 4

Random Sampling

A

Random Sampling

A

B

Random Sampling

A

Which is the better sample to estimate the population mean height?

B

Random Sampling

A

Which is the better sample to estimate the population mean height?

B

Independent Sample

Independent Sample

Example 1

Independent Sample

Example 1

Independent Sample

Example 1

Independent Sample

Example 1

Independent Sample

Example 1

Independent Sample

Example 1

1st sample set

Independent Sample

Example 1

1st sample set

2nd sample set

Independent Sample

Example 1

1st sample set

2nd sample set

Independent Sample

Example 1

1st sample set

Why is sample set two not a good sample?

2nd sample set

Independent Sample

Example 1

1st sample set

Why is sample set two not a good sample?

2nd sample set

Independent Sample

Example 2

1st sample set

2nd sample set

Independent Sample

Example 2

1st sample set

2nd sample set

Identically Distributed Samples

Identically Distributed Samples

Example 1

Identically Distributed Samples

Example 1

Identically Distributed Samples

Example 1

Identically Distributed Samples

Example 1

n = 4

Identically Distributed Samples

Example 1

Identically Distributed Samples

Example 1

A

Identically Distributed Samples

Example 1

A

B

Identically Distributed Samples

Example 1

A

Which of the following samples are identically distributed?

B

Identically Distributed Samples

Example 1

A

Which of the following samples are identically distributed?

B

Identically Distributed Samples

Example 2

A

B

Identically Distributed Samples

Example 2

The Avocado Toast Trend

The Avocado Toast Trend

The Avocado Toast Trend

Study the price of avocados in the United States

The Avocado Toast Trend

Study the price of avocados
in the United States

The Avocado Toast Trend

Study the price of avocados
in the United States

What is the population of your study?

The Avocado Toast Trend

All avocados sold in the US

The Avocado Toast Trend

Study the price of avocados
in the United States

The Avocado Toast Trend

Study the price of avocados in the United States

What is the sample of your study?

The Avocado Toast Trend

Study the price of avocados in the United States

What is the sample of your study?

The Avocado Toast Trend

Study the price of avocados in the United States

What is the sample of your study?

Avocados sold in the 4 stores you selected

Population and Sample in Machine Learning

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

NOT the population

Cats

Not cats

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

NOT the population

Cats

Not cats

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

NOT the population

Cats

Not cats

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

NOT the population

Cats

Not cats

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

NOT the population

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

NOT the population

Recap

Population

the entire group of individuals or elements you want to study which share a common behaviour

Sample

subset of the population you use to draw conclusions about the population as a whole

Recap

Population

the entire group of individuals or elements you want to study which share a common behaviour

Sample

subset of the population you use to draw conclusions about the population as a whole

Population Size: N

Sample Size:

Recap

Population

the entire group of individuals or elements you want to study which share a common behaviour

Sample

subset of the population you use to draw conclusions about the population as a whole

Population Size: N

Sample Size: n

DeepLearning.AI

Sample and Population

Sample Mean, Proportion, and Variance

Population and Sample Mean

Population and Sample Mean

Population and Sample Mean

Population and Sample Mean

What is the average height in statistopia?

Population and Sample Mean

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10}$$

Population and Sample Mean

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10}$$

$$= \frac{1600}{10} = 160cm$$

Population and Sample Mean

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10}$$

$$= \frac{1600}{10} = 160cm$$

Population mean μ

Population and Sample Mean

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10}$$

$$= \frac{1600}{10} = 160cm$$

Population mean μ

Population and Sample Mean

Population and Sample Mean

What is the average height in statistopia?

Population and Sample Mean

$n = 6$

μ

What is the average height in statistopia?

Population and Sample Mean

$n = 6$

μ

What is the average height in statistopia?

$$\frac{153 + 155 + 159 + 163 + 165 + 169}{6}$$

Population and Sample Mean

$n = 6$

μ

What is the average height in statistopia?

$$\frac{153 + 155 + 159 + 163 + 165 + 169}{6} = \frac{964}{6} = 160.97$$

Population and Sample Mean

Sample mean

$n = 6$

$\mu \quad \bar{x}_1$

What is the average height in statistopia?

$$\frac{153 + 155 + 159 + 163 + 165 + 169}{6} = \frac{964}{6} = 160.97$$

$\bar{x}$

Population and Sample Mean

$n = 6$

What is the average height in statistopia?

$$\frac{153 + 155 + 159 + 163 + 165 + 169}{6} = \frac{964}{6} = \boxed{160.97}$$

$\bar{x}$

Population and Sample Mean

$n = 6$

μ $\bar{x}_1$

Population and Sample Mean

$n = 6$

What is the average height in statistopia?

Population and Sample Mean

$n = 6$

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6}$$

Population and Sample Mean

$$n = 6$$

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156cm$$

Population and Sample Mean

Another sample estimate for the population average height

$$n = 6$$

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156cm$$

Population and Sample Mean

Another sample estimate for the population average height

$$n = 6$$

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156cm$$

Population and Sample Mean

Another sample estimate for the population average height

$n = 6$

What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156cm$$

Population and Sample Mean

Population and Sample Mean

Population and Sample Mean

Population and Sample Mean

Population and Sample Mean

$n = 6$

Population and Sample Mean

$n = 6$

Population and Sample Mean

$n = 6$

μ $\bar{x}_1$

Population and Sample Mean

$n = 6$

$n = 2$

Population and Sample Mean

$n = 6$

$n = 2$

Population and Sample Mean

$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

Population and Sample Mean

$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2}$$

Population and Sample Mean

$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = 158cm$$

Population and Sample Mean

$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = \boxed{158cm}$$

Population and Sample Mean

$n = 6$

$n = 2$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = \boxed{158cm}$$

Population and Sample Mean

$n = 6$

$n = 2$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = \boxed{158cm}$$

Proportion

Proportion

Population size: 10

Proportion

Population size: 10

Proportion

Population size: 10

What proportion of people own a bicycle?

Proportion

Population size: 10

What proportion of people own a bicycle?

Proportion

Population size: 10

What proportion of people own a bicycle?

Proportion

Population size: 10

What proportion of people own a bicycle?

Proportion

Population size: 10

What proportion of people own a bicycle?

Proportion

Population size: 10

What proportion of people own a bicycle?

$$= \frac{4}{10} = 0.4 = 40\,\%$$

Proportion

Population size: 10

What proportion of people own a bicycle?

$$\text{population proportion} = \frac{4}{10} = 0.4 = 40\,\%$$

Proportion

Population size: 10

What proportion of people own a bicycle?

p population proportion $= \frac{4}{10} = 0.4 = 40\,\%$

Proportion

$$p = \frac{\text{number of items with a given characteristic } (x)}{\text{population } (n)}$$

Proportion

population proportion

$$p = \frac{\text{number of items with a given characteristic } (x)}{\text{population } (n)}$$

Proportion

Population size: 10

Proportion

Sample Proportion

Sample Proportion

Sample size: 6

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

$$= \frac{2}{6} = 0.333 = 33.3\,\%$$

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

$$\text{sample proportion} = \frac{2}{6} = 0.333 = 33.3\,\%$$

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

$\hat{p}$ sample proportion $= \frac{2}{6} = 0.333 = 33.3\,\%$

Sample Proportion

Sample size: 6

What proportion of people own a bicycle?

$\hat{p}$ sample proportion $= \frac{2}{6} = 0.333 = 33.3\,\%$

Sample Proportion

Sample size: 6

estimate of the population proportion

What proportion of people own a bicycle?

$\hat{p}$ sample proportion $= \frac{2}{6} = 0.333 = 33.3\,\%$

Sample Variance

Sample Variance

Sample Variance

Sample Variance

Sample Variance

Sample Variance

Sample Variance

Sample Variance

Sample Variance

Sample Variance

σ^2

Sample Variance

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Sample Variance

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

population size

Sample Variance

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

population mean

population size

Sample Variance

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

population mean

population size

How to estimate population variance with the sample?

Variance Estimation

Variance Estimation

Variance Estimation

μ

Variance Estimation

1	2	3

$$\mu = \frac{1+2+3}{3}$$

Variance Estimation

1	2	3

$$\mu = \frac{1+2+3}{3} = \frac{6}{3}$$

Variance Estimation

1	2	3

$$\mu = \frac{1+2+3}{3} = \frac{6}{3} = 2$$

Variance Estimation

1	2	3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation

1	2	3

$\mu = 2$

x	$x - \mu$	$(x - \mu)^2$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1		
2		
3		

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

x
1
2
3

$x - 2$

$x - \mu$ $\quad (x-\mu)^2$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2$

x	$x - \mu$	$(x-\mu)^2$
1	-1	
2	0	
3	1	

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$x - 2$

x	$x-\mu$	$(x-\mu)^2$
1	-1	1
2	0	0
3	1	1

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2$

x	$x-\mu$	$(x-\mu)^2$
1	-1	1
2	0	0
3	1	1

$$\sum (x-\mu)^2$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2$

x	$x-\mu$	$(x-\mu)^2$
1	-1	1
2	0	0
3	1	1

$$\sum (x-\mu)^2 = 2$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2$

x	$x-\mu$	$(x-\mu)^2$
1	-1	1
2	0	0
3	1	1

$$\frac{\sum (x-\mu)^2}{N} = 2$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2$

x	$x-\mu$	$(x-\mu)^2$
1	-1	1
2	0	0
3	1	1

$$\frac{\sum (x-\mu)^2}{N} = \frac{2}{3}$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2$

x	$x-\mu$	$(x-\mu)^2$
1	-1	1
2	0	0
3	1	1

$$\frac{\sum (x-\mu)^2}{N} = \frac{2}{3}$$

σ^2

Population variance

Variance Estimation

1	2	3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

Variance Estimation

$n = 2$

Samples

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	
1	2	
1	3	
2	1	
2	2	
2	3	
3	1	
3	2	
3	3	

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$	
1	1	1	0
1	2	1,5	0,25
1	3	2	1
2	1	1,5	0,25
2	2	2	0
2	3	2,5	0,25
3	1	2	1
3	2	2,5	0,25
3	3	3	0

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$	
1	1	1	0
1	2	1,5	0,25
1	3	2	1
2	1	1,5	0,25
2	2	2	0
2	3	2,5	0,25
3	1	2	1
3	2	2,5	0,25
3	3	3	0

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$	
1	1	1	0
1	2	1,5	0,25
1	3	2	1
2	1	1,5	0,25
2	2	2	0
2	3	2,5	0,25
3	1	2	1
3	2	2,5	0,25
3	3	3	0

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$	
1	1	1	0
1	2	1,5	0,25
1	3	2	1
2	1	1,5	0,25
2	2	2	0
2	3	2,5	0,25
3	1	2	1
3	2	2,5	0,25
3	3	3	0

estimated variance

$= 0.333$

$= \frac{1}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$	
1	1	1	0
1	2	1,5	0,25
1	3	2	1
2	1	1,5	0,25
2	2	2	0
2	3	2,5	0,25
3	1	2	1
3	2	2,5	0,25
3	3	3	0

estimated variance

$= 0.333$

$= \frac{1}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$	
1	1	1	0
1	2	1,5	0,25
1	3	2	1
2	1	1,5	0,25
2	2	2	0
2	3	2,5	0,25
3	1	2	1
3	2	2,5	0,25
3	3	3	0

estimated variance

$= 0.333$

$= \frac{1}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$	$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$
1	1	1	0
1	2	1,5	0,25
1	3	2	1
2	1	1,5	0,25
2	2	2	0
2	3	2,5	0,25
3	1	2	1
3	2	2,5	0,25
3	3	3	0

estimated variance

$= 0.333$

$= \frac{1}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$		
1	1	1	0	
1	2	1,5	0,25	
1	3	2	1	
2	1	1,5	0,25	
2	2	2	0	
2	3	2,5	0,25	
3	1	2	1	
3	2	2,5	0,25	
3	3	3	0	

estimated variance

$= 0.333$

$= \frac{1}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Samples		$\bar{x}$		
1	1	1	0	
1	2	1,5	0,25	
1	3	2	1	
2	1	1,5	0,25	
2	2	2	0	
2	3	2,5	0,25	
3	1	2	1	
3	2	2,5	0,25	
3	3	3	0	

estimated variance

$= 0.333$

$= \frac{1}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$
1	1	1
1	2	1,5
1	3	2
2	1	1,5
2	2	2
2	3	2,5
3	1	2
3	2	2,5
3	3	3

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$	$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$
1	1	1	0
1	2	1,5	0,5
1	3	2	2
2	1	1,5	0,5
2	2	2	0
2	3	2,5	0,5
3	1	2	2
3	2	2,5	0,5
3	3	3	0

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$	$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$
1	1	1	0
1	2	1,5	0,5
1	3	2	2
2	1	1,5	0,5
2	2	2	0
2	3	2,5	0,5
3	1	2	2
3	2	2,5	0,5
3	3	3	0

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$	$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$
1	1	1	0
1	2	1,5	0,5
1	3	2	2
2	1	1,5	0,5
2	2	2	0
2	3	2,5	0,5
3	1	2	2
3	2	2,5	0,5
3	3	3	0

estimated variance

$= 0.667$

$= \frac{2}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$	$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$
1	1	1	0
1	2	1,5	0,5
1	3	2	2
2	1	1,5	0,5
2	2	2	0
2	3	2,5	0,5
3	1	2	2
3	2	2,5	0,5
3	3	3	0

estimated variance

$= 0.667$

$= \frac{2}{3}$

Variance Estimation

1	2	3

$\mu = 2$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$

Samples		$\bar{x}$	$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$
1	1	1	0
1	2	1,5	0,5
1	3	2	2
2	1	1,5	0,5
2	2	2	0
2	3	2,5	0,5
3	1	2	2
3	2	2,5	0,5
3	3	3	0

estimated variance

$= 0.667$

$= \frac{2}{3}$

Variance Estimation

Population Variance Formula

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Sample Variance Formula

$$Var(x) = \frac{1}{n-1} \sum (x - \bar{x})^2$$

Variance Estimation

Variance Estimation

Variance Estimation

Variance Estimation

1	2	3	4

$$\mu = \frac{1 + 2 + 3 + 4}{4}$$

Variance Estimation

$$\mu = \frac{1+2+3+4}{4} = 2.5$$

Variance Estimation

1	2	3	4

$$\mu = 2.5$$

Variance Estimation

1	2	3	4

$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation

1	2	3	4

$\mu = 2.5$

x	$x - \mu$	$(x - \mu)^2$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

x	$x-\mu$	$(x-\mu)^2$
1		
2		
3		
4		

Variance Estimation

1	2	3	4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2.5$

x	$x-\mu$	$(x-\mu)^2$
1		
2		
3		
4		

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2.5$

x	$x - \mu$	$(x-\mu)^2$
1	-1,5	
2	-0,5	
3	0,5	
4	1,5	

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2.5$

x	$x-\mu$	$(x-\mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$x - 2.5$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$\sum (x - \mu)^2$$

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$x - 2.5$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$\sum (x - \mu)^2 = 5$$

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2.5$

x	$x-\mu$	$(x-\mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$\frac{\sum (x-\mu)^2}{N} = 5$$

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2.5$

x	$x-\mu$	$(x-\mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$\frac{\sum (x-\mu)^2}{N} = \frac{5}{4}$$

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$x - 2.5$

x	$x-\mu$	$(x-\mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$\frac{\sum (x-\mu)^2}{N} = \frac{5}{4}$$

Population variance σ^2

Variance Estimation

1	2	3	4

$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

Variance Estimation

1	2	3	4

$\mu = 2.5$

$n = 2$

Samples

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

Variance Estimation

1	2	3	4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x

Variance Estimation

1	2	3	4

$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$$n = 2$$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4

Variance Estimation

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$																

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

Variance Estimation

1	2	3	4

$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$$n = 2$$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

Variance Estimation

1	2	3	4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

$var(x)$

Variance Estimation

$n = 2$

Samples

1	2	3	4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,25	1	2,25	0,25	0	0,25	1	1	0,25	0	0,25	2,25	1	0,25	0

Variance Estimation

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$

$\mu = 2.5$

$$\boxed{\sigma^2 = \frac{1}{N}\sum(x-\mu)^2}$$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$$var(x) = \frac{5}{8}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,25	1	2,25	0,25	0	0,25	1	1	0,25	0	0,25	2,25	1	0,25	0

Variance Estimation

1	2	3	4

$\mu = 2.5$

$n = 2$

Samples

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$$var(x) = \frac{5}{8}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,25	1	2,25	0,25	0	0,25	1	1	0,25	0	0,25	2,25	1	0,25	0

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2 \qquad Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

Variance Estimation

1 2 3 4

$\mu = 2.5$

$n = 2$

Samples

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2$$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$$Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2 \qquad Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2 \qquad Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$

Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4

$var(x)$

Variance Estimation

1 2 3 4

$\mu = 2.5$

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2 \qquad Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$
Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation

1 2 3 4

$\mu = 2.5$

$n = 2$

Samples

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2 \qquad Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation

1 2 3 4

$\mu = 2.5$

$n = 2$

Samples

$$\sigma^2 = \frac{1}{N}\sum(x-\mu)^2 \qquad Var(x) = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{5}{4} \qquad Var(x) = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation

1 2 3 4

$\mu = 2.5$

$n = 2$
Samples

$$\sigma^2 = \frac{1}{N}\sum (x-\mu)^2$$

$$Var(x) = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{5}{4}$$

$$Var(x) = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
$\bar{x}$	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation

Population Variance Formula

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Sample Variance Formula

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

DeepLearning.AI

Sample and Population

Law of Large Numbers

Law of Large Numbers

Law of Large Numbers

Law of Large Numbers

1 2 3 4

Law of Large Numbers

1 2 3 4

$\mu = 2.5$

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

Law of Large Numbers

1 2 3 4

$$\mu = 2.5$$

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Law of Large Numbers

1 2 3 4

$$\mu = 2.5$$

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

	1	2	3	4
1	1	1.5	2	2.5
2	1.5	2	2.5	3
3	2	2.5	3	3.5
4	2.5	3	3.5	4

Law of Large Numbers

1 2 3 4

$\mu = 2.5$

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

	1	2	3	4
1	1	1.5	2	2.5
2	1.5	2	2.5	3
3	2	2.5	3	3.5
4	2.5	3	3.5	4

	1	2	3	4
1	μ			
2	**2.5**			
3				
4				

Law of Large Numbers

1 2 3 4

$\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
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Law of Large Numbers

1 2 3 4

$\mu = 2.5$

1 trial

	1	2	3	4
1	1,1	1,2	1,3	1,4
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4,3 $\bar{x}_1$

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1 trial

4,3

2 trials

Law of Large Numbers

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Law of Large Numbers

Law of Large Numbers

As the sample size increases, the average of the sample will tend to get closer to the average of the entire population.

Law of Large Numbers

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n : number of samples

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X_i : some estimate X for a sample size i

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as $n \rightarrow \infty$

Law of Large Numbers

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as $n \to \infty$ $$\frac{1}{n}\sum_{i=1}^{n} X_i \to \mathbb{E}[X] = \mu_X$$

Law of Large Numbers

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Law of Large Numbers

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n : number of samples

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as $n \to \infty$

$$\frac{X_1 + X_2 + X_3 + \ \ldots + \ X_n}{n} \longrightarrow \mathbb{E}[X] = \mu_X$$

UNDER CERTAIN CONDITIONS

as $n \to \infty$

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Law of Large Numbers

UNDER CERTAIN CONDITIONS

Law of Large Numbers

UNDER CERTAIN CONDITIONS

- Sample is randomly drawn.

Law of Large Numbers

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- Sample is randomly drawn.
- Sample size must be sufficiently large.

Law of Large Numbers

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Law of Large Numbers

UNDER CERTAIN CONDITIONS

- Sample is randomly drawn.
- Sample size must be sufficiently large.
- Independent observations.

DeepLearning.AI

Sample and Population

Central Limit Theorem

Central Limit Theorem (CLT) - Example 1

If $n = 1$

Central Limit Theorem (CLT) - Example 1

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Central Limit Theorem (CLT) - Example 1

$\mathbf{P}(H) = 0.5$

$\mathbf{P}(T) = 0.5$

If $n = 1$

Central Limit Theorem (CLT) - Example 1

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Random variable → X

If $n = 1$

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Random variable → X **number of heads when a coin is flipped n times**

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Discrete Random Variable

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What can we say about the probability distribution when the number of coin flips increases?

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Central Limit Theorem (CLT) - Example 1

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As n increases, the probability distribution becomes closer to a Gaussian distribution

Central Limit Theorem (CLT) - Example 1

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Random variable → X **number of heads when a coin is flipped n times**

Central Limit Theorem (CLT) - Example 1

$\mathbf{P}(H) = 0.5$

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Random variable → X **number of heads when a coin is flipped n times**

$$\mu = np$$

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$$\mu = np = n\mathbf{P}(H)$$

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Random variable → X **number of heads when a coin is flipped n times**

$$\mu = np \ = n\mathbf{P}(H)$$

$$\sigma^2 = np(1-p)$$

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$n = 1$

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$\mu = 1 \times 0.5$

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$\mu = 1 \times 0.5 \quad = 0.5$

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$\mu = np = n\mathbf{P}(H)$

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$n = 1$

$\mu = np$

$\mu = 1 \times 0.5 \quad = 0.5$

$\sigma^2 = np(1-p)$

$\sigma^2 = (1 \times 0.5)(0.5)$

Central Limit Theorem (CLT) - Example 1

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$\mu = np = n\mathbf{P}(H)$

$\sigma^2 = np(1-p) = n\mathbf{P}(H)\mathbf{P}(T)$

$n = 1$

$\mu = np$

$\mu = 1 \times 0.5 \quad = 0.5$

$\sigma^2 = np(1-p)$

$\sigma^2 = (1 \times 0.5)(0.5) = 0.25$

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As n increases, the probability distribution becomes closer to a gaussian distribution

Central Limit Theorem (CLT) - Example 1

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$\mu = 0.5$
$\sigma^2 = 0.25$

$\mu = 1$
$\sigma^2 = 0.5$

$\mu = 1.5$
$\sigma^2 = 0.75$

$\mu = 2$
$\sigma^2 = 1$

$\mu = 5$
$\sigma^2 = 2.5$

Central Limit Theorem (CLT) - Example 1

$\mu = 0.5$
$\sigma^2 = 0.25$

$\mu = 1$
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$\mu = 1.5$
$\sigma^2 = 0.75$

$\mu = 2$
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as n become sufficiently large we will get a normal distribution with parameters

Central Limit Theorem (CLT) - Example 1

$\mu = 0.5$ $\sigma^2 = 0.25$	$\mu = 1$ $\sigma^2 = 0.5$	$\mu = 1.5$ $\sigma^2 = 0.75$	$\mu = 2$ $\sigma^2 = 1$	$\mu = 5$ $\sigma^2 = 2.5$

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$$\sigma^2 = np(1-p)$$

Central Limit Theorem (CLT) - Example 1

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$\mu = 0.5$ $\sigma^2 = 0.25$	$\mu = 1$ $\sigma^2 = 0.5$	$\mu = 1.5$ $\sigma^2 = 0.75$	$\mu = 2$ $\sigma^2 = 1$	$\mu = 5$ $\sigma^2 = 2.5$

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Video 4b: Central Limit Theorem

- Example 2: Continuous Random Variable (Sample mean of a sampling distribution)

Uniform Distribution: Motivation

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You’re calling a tech support line.
They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

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They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

$X =$ ”Wait time for a called to be answered ”

$$X \sim \mathcal{U}(0,15)$$

Central Limit Theorem (CLT) - Example 2

$n = 1$ $\qquad$ $Y_1 = \frac{X_1}{1}$

Central Limit Theorem (CLT) - Example 2

$n = 1$ $\qquad Y_1 = \frac{X_1}{1}$

$n = 2$ $\qquad Y_2 = \frac{X_1 + X_2}{2}$

Central Limit Theorem (CLT) - Example 2

$$n = 1 \qquad Y_1 = \frac{X_1}{1}$$

$$n = 2 \qquad Y_2 = \frac{X_1 + X_2}{2}$$

$$n = 3 \qquad Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

$$\vdots \qquad \vdots$$

Central Limit Theorem (CLT) - Example 2

$$n = 1 \qquad Y_1 = \frac{X_1}{1}$$

$$n = 2 \qquad Y_2 = \frac{X_1 + X_2}{2}$$

$$n = 3 \qquad Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

$$\vdots \qquad \vdots$$

Record the average of all n experiments

Central Limit Theorem (CLT) - Example 2

$n = 1$ $\quad Y_1 = \frac{X_1}{1}$

$n = 2$ $\quad Y_2 = \frac{X_1 + X_2}{2}$

$n = 3$ $\quad Y_3 = \frac{X_1 + X_2 + X_3}{3}$

$\vdots \quad \vdots$

Record the average of all n experiments

$$Y_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Central Limit Theorem (CLT) - Example 2

$n = 1$ $\quad Y_1 = \frac{X_1}{1}$

$n = 2$ $\quad Y_2 = \frac{X_1 + X_2}{2}$

$n = 3$ $\quad Y_3 = \frac{X_1 + X_2 + X_3}{3}$

$\vdots \qquad \vdots$

Record the average of all n experiments

$$Y_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Can we say anything about the distribution of this average?

Central Limit Theorem (CLT) - Example 2

$$n = 1 \qquad Y_1 = \frac{X_1}{1}$$

Central Limit Theorem (CLT) - Example 2

$n = 1 \qquad Y_1 = \frac{X_1}{1}$

What happens to the distribution of these averages as n increases?

Central Limit Theorem (CLT) - Example 2

$n = 1 \qquad Y_1 = \frac{X_1}{1}$

Create many samples of Y_1 so you can get a pretty histogram

What happens to the distribution of these averages as n increases?

Central Limit Theorem (CLT) - Example 2

$$n = 1 \qquad Y_1 = \frac{X_1}{1}$$

Create many samples of Y_1 so you can get a pretty histogram

What happens to the distribution of these averages as n increases?

Central Limit Theorem (CLT) - Example 2

$$n = 2 \qquad Y_2 = \frac{X_1 + X_2}{2}$$

Create many samples of Y_2 so you can get a pretty histogram

What happens to the distribution of these averages as n increases

Central Limit Theorem (CLT) - Example 2

$$n = 2 \qquad Y_2 = \frac{X_1 + X_2}{2}$$

Create many samples of Y_2 so you can get a pretty histogram

What happens to the distribution of these averages as n increases

Central Limit Theorem (CLT) - Example 2

$$n = 3 \quad Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

Create many samples of Y_3 so you can get a pretty histogram

What happens to the distribution of these averages as n increases

Central Limit Theorem (CLT) - Example 2

$$n = 4 \quad Y_4 = \frac{X_1 + X_2 + X_3 + X_4}{4}$$

Create many samples of Y_4 so you can get a pretty histogram

What happens to the distribution of these averages as n increases

Central Limit Theorem (CLT) - Example 2

$$n = 5 \quad Y_5 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

Create many samples of Y_5 so you can get a pretty histogram

What happens to the distribution of these averages as n increases

Central Limit Theorem (CLT) - Example 2

Central Limit Theorem (CLT) - Example 2

$n = 1$

$n = 2$

Central Limit Theorem (CLT) - Example 2

$$= \frac{1}{n^2}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right]$$

$$= \frac{1}{n^2}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i]$$

$$= \frac{1}{n^2}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{1}{n}n\mathbb{E}[X] = \mathbb{E}[X]$$

$$= \frac{1}{n^2}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{1}{n}n\mathbb{E}[X] = \mathbb{E}[X] = 7.5$$

$$= \frac{1}{n^2}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{1}{n}n\mathbb{E}[X] = \mathbb{E}[X] = 7.5$$

$$Var[Y_n] = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{1}{n}n\mathbb{E}[X] = \mathbb{E}[X] = 7.5$$

$$Var[Y_n] = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{1}{n}n\mathbb{E}[X] = \mathbb{E}[X] = 7.5$$

$$Var[Y_n] = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)$$

$$= \frac{1}{n^2}nVar(X) = \frac{Var(X)}{n}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{1}{n}n\mathbb{E}[X] = \mathbb{E}[X] = 7.5$$

$$Var[Y_n] = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)$$

$$= \frac{1}{n^2}nVar(X) = \frac{Var(X)}{n} = \frac{18.75}{n}$$

Central Limit Theorem (CLT) - Example 2

Central Limit Theorem (CLT) - Example 2

Central Limit Theorem (CLT) - Example 2

$n = 1$

$n = 2$

When you average a large enough number of variables, the distribution will approximately follow a normal distribution

Central Limit Theorem (CLT) - Example 2

$$\frac{Y_n - 7.5}{\sqrt{18.75/n}}$$

Central Limit Theorem (CLT) - Example 2

$$\frac{Y_n - 7.5}{\sqrt{18.75/n}} \xrightarrow{n\uparrow} \mathcal{N}(0,1)$$

Central Limit Theorem (CLT) - Example 2

Central Limit Theorem (CLT) - Example 2

$$Y_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Central Limit Theorem (CLT) - Example 2

$$Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

mean of Y_n

$$\mu_{Y_n} = \mu$$

population mean

Central Limit Theorem (CLT) - Example 2

$$Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

mean of Y_n

$$\mu_{Y_n} = \mu$$

population mean

population variance

variance of Y_n

$$\sigma^2_{Y_n} = \frac{\sigma^2}{n}$$

Central Limit Theorem (CLT) - Formal Definition

Central Limit Theorem (CLT) - Formal Definition

As $n \to \infty$ $$\frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mathbb{E}[X]}{\sigma_X}\sqrt{n} \sim \mathcal{N}\left(0,1^2\right)$$

Central Limit Theorem (CLT) - Formal Definition

As $n \to \infty$ $$\frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mathbb{E}[X]}{\sigma_X}\sqrt{n} \sim \mathcal{N}\left(0,1^2\right)$$

As $n \to \infty$ $$\frac{1}{n}\left(\frac{\sum_{i=1}^{n} X_i - \frac{1}{n}n\mathbb{E}[X]}{\sigma_X}\right)\sqrt{n} \sim \mathcal{N}\left(0,1^2\right)$$

Central Limit Theorem (CLT) - Formal Definition

As $n \to \infty$ $$\frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mathbb{E}[X]}{\sigma_X}\sqrt{n} \sim \mathcal{N}\left(0, 1^2\right)$$

As $n \to \infty$ $$\frac{1}{n}\left(\frac{\sum_{i=1}^{n} X_i - \frac{1}{\cancel{n}}\cancel{n}\mathbb{E}[X]}{\sigma_X}\right)\sqrt{n} \sim \mathcal{N}\left(0, 1^2\right)$$

Central Limit Theorem (CLT) - Formal Definition

As $n \to \infty$ $$\frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mathbb{E}[X]}{\sigma_X}\sqrt{n} \sim \mathcal{N}\left(0, 1^2\right)$$

As $n \to \infty$ $$\frac{1}{\sqrt{n}\cancel{n}}\left(\frac{\sum_{i=1}^{n} X_i - \frac{1}{\cancel{n}}\cancel{n}\mathbb{E}[X]}{\sigma_X}\right)\cancel{\sqrt{n}} \sim \mathcal{N}\left(0, 1^2\right)$$

Central Limit Theorem (CLT) - Formal Definition

As $n \to \infty$ $$\frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mathbb{E}[X]}{\sigma_X}\sqrt{n} \sim \mathcal{N}\left(0,1^2\right)$$

As $n \to \infty$

Central Limit Theorem (CLT) - Formal Definition

As $n \to \infty$ $$\frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mathbb{E}[X]}{\sigma_X}\sqrt{n} \sim \mathcal{N}\left(0,1^2\right)$$

As $n \to \infty$ $$\frac{\sum_{i=1}^{n} X_i - n\mathbb{E}[X]}{\sqrt{n}\sigma_X} \sim \mathcal{N}\left(0,1^2\right)$$

W3 Lesson 2

DeepLearning.AI

Point Estimation

What is Point Estimation?

Point Estimates

Point Estimates

Point Estimates

Mean of the population?

$$\mu \approx \frac{1}{10}\sum_{i=1}^{10} x_i = \bar{x}$$

Point Estimates

x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10}

Mean of the population?

$$\mu \approx \frac{1}{10}\sum_{i=1}^{10} x_i = \bar{x}$$

Variance of the population?

$$\sigma^2 \approx \frac{1}{10-1}\sum_{i=1}^{10} \left(x_i - \bar{x}\right)^2 = s^2$$

Point Estimates

x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10}

Mean of the population? $\mu \approx \frac{1}{10}\sum_{i=1}^{10} x_i = \bar{x}$

Variance of the population? $\sigma^2 \approx \frac{1}{10-1}\sum_{i=1}^{10} \left(x_i - \bar{x}\right)^2 = s^2$

$\bar{x}$ and s^2 are point estimates

Point Estimates

Point Estimates

A **point estimate** is a **single numerical value** based on **sample data** that is used to **approximate an unknown parameter** of a population or model parameter.

Point Estimates

Point Estimates

$\mathbf{P}(H)?$

Point Estimates

$\mathbf{P}(H)$?

Point Estimates

$\mathbf{P}(H)$?

10 throws
7 heads
3 tails

Point Estimates

$\mathbf{P}(H)$?

10 throws
7 heads
3 tails

$$\mathbf{P}(H) = \frac{7}{10}$$

Point Estimates

	x	y
0		
1		
2		
⋮		
50		

Point Estimates

	x	y
0		
1		
2		
⋮		
50		

Point Estimates

	x	y
0		
1		
2		
⋮		
50		

$$y = \beta_0 + \beta_1 x$$

$$\beta_0, \beta_1 = ??$$

Point Estimates

	x	y
0		
1		
2		
⋮		
50		

$$y = \beta_0 + \beta_1 x$$

$$\beta_0, \beta_1 = ??$$

Point Estimates

	x	y
0		
1		
2		
⋮		
50		

$$y = \beta_0 + \beta_1 x$$

$$\beta_0, \beta_1 = ??$$

DeepLearning.AI

Point Estimation

Maximum Likelihood Estimation: Motivation

There’s Popcorn on the Floor. What Happened?

There’s Popcorn on the Floor. What Happened?

There’s Popcorn on the Floor. What Happened?

Movies

There’s Popcorn on the Floor. What Happened?

Movies

Board Games

There’s Popcorn on the Floor. What Happened?

Movies

Board Games

Nap

Quiz

What do you think happened?

A. People were watching a movie

B. People were playing boardgames

C. People were taking a nap

There’s Popcorn on the Floor. What Happened?

Movies

Board Games

Nap

There's Popcorn on the Floor. What Happened?

Movies

Board Games

Nap

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

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There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

Maximum Likelihood

Maximum Likelihood

Data

Maximum Likelihood

Model 1

Data

Maximum Likelihood

Model 1

Model 2

Data

Maximum Likelihood

Model 1

Model 2

Model 3

Data

Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Find model that most likely produced the data

Maximum Likelihood

Find model that most likely produced the data

Maximum Likelihood

Find model that most likely produced the data

Maximize
$\mathbf{P}(\text{Data}|\text{Model})$

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Model 1

Data

Example: Linear Regression

Model 1

Model 2

Data

Example: Linear Regression

Model 1

Model 2

Model 3

Data

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Find model that most likely produced the data

DeepLearning.AI

Point Estimation

MLE: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Coin 1

$\mathbf{P}(H) = 0.7$

$\mathbf{P}(T) = 0.3$

Coin 2

$\mathbf{P}(H) = 0.5$

$\mathbf{P}(T) = 0.5$

Coin 3

$\mathbf{P}(H) = 0.3$

$\mathbf{P}(T) = 0.7$

Quiz

Which of the coins is more likely?

A. Coin 1 (P(H) = 0.7)

B. Coin 2 (P(H) = 0.5)

C. Coin 3 (P(H) = 0.3)

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Coin 1	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.3	0.3 = 0.0051

Maximum Likelihood: Bernoulli Example

Coin 1	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.3	0.3	= 0.0051
Coin 2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	= 0.0010

Maximum Likelihood: Bernoulli Example

Coin 1	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.3	0.3 = 0.0051
Coin 2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5 = 0.0010
Coin 3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.30	0.7	0.7 = 0.00003

Maximum Likelihood: Bernoulli Example

Coin 1	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.3	0.3 = 0.0051
Coin 2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5 = 0.0010
Coin 3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.30	0.7	0.7 = 0.00003

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Coin 1

$\mathbf{P}(H) = 0.7$

8H 2T

Maximum Likelihood: Bernoulli Example

Coin 1
$\mathbf{P}(H) = 0.7$

Coin 2
$\mathbf{P}(H) = 0.5$

8H 2T

Maximum Likelihood: Bernoulli Example

Coin 1
$\mathbf{P}(H) = 0.7$

Coin 2
$\mathbf{P}(H) = 0.5$

Coin 3
$\mathbf{P}(H) = 0.3$

8H 2T

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Find coin that most likely produced the 8 heads and 2 tails

Maximum Likelihood: Bernoulli Example

Find coin that most likely produced the 8 heads and 2 tails

Maximize

$\mathbf{P}(8H, 2T \mid \text{Coin})$

Maximum Likelihood: Bernoulli Example

Find coin that most likely produced the 8 heads and 2 tails

Maximize

$\mathbf{P}(8H, 2T \mid \text{Coin})$

Maximum Likelihood: Bernoulli Example

Find coin that most likely produced the 8 heads and 2 tails

Maximize

$\mathbf{P}$(8H, 2T | Coin)

Maximum Likelihood: Bernoulli Example

Find coin that most likely produced the 8 heads and 2 tails

Maximize

$\mathbf{P}(8H, 2T \mid \text{Coin})$

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Can you do any better?

Maximum Likelihood: Bernoulli Example

$p \quad p \quad p \quad p \quad p \quad p \quad p \quad p \quad 1-p \quad 1-p$

Can you do any better?

$$p = \mathbf{P}(H)$$

Maximum Likelihood: Bernoulli Example

$p \quad p \quad p \quad p \quad p \quad p \quad p \quad p \quad 1-p \quad 1-p$

Can you do any better?

$p = \mathbf{P}(H)$

$p^8(1-p)^2$

Maximum Likelihood: Bernoulli Example

$p \quad p \quad p \quad p \quad p \quad p \quad p \quad p \quad 1-p \quad 1-p$

Can you do any better?

$p = \mathbf{P}(H)$ $\qquad$ $p^8(1-p)^2$

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ $\qquad$ $p^8(1-p)^2$

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

Function of p

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

Function of p

You want p that maximizes the chances of seeing 8H

$$\log\left((p^8(1-p)^2\right)$$

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

Function of p

You want p that maximizes the chances of seeing 8H

$$\log\left((p^8(1-p)^2\right) = 8\log(p) + 2\log(1-p)$$

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

Function of p

You want p that maximizes the chances of seeing 8H

Log-likelihood $\ell(p; 8H) = \log\left((p^8(1-p)^2\right) = 8\log(p) + 2\log(1-p)$

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

Function of p

You want p that maximizes the chances of seeing 8H

Log-likelihood $\ell(p; 8H) = \log\left((p^8(1-p)^2\right) = 8\log(p) + 2\log(1-p)$

$$\frac{d}{dp}\left(8\log(p) + 2\log(1-p)\right)$$

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

Function of p

You want p that maximizes the chances of seeing 8H

Log-likelihood $\ell(p; 8H) = \log\left((p^8(1-p)^2\right) = 8\log(p) + 2\log(1-p)$

$$\frac{d}{dp}\left(8\log(p) + 2\log(1-p)\right) = \frac{8}{p} + \frac{2}{1-p}(-1)$$

Maximum Likelihood: Bernoulli Example

$p = \mathbf{P}(H)$ Likelihood $L(p; 8H) = p^8(1-p)^2$

Function of p

You want p that maximizes the chances of seeing 8H

Log-likelihood $\ell(p; 8H) = \log\left((p^8(1-p)^2\right) = 8\log(p) + 2\log(1-p)$

$$\frac{d}{dp}\left(8\log(p) + 2\log(1-p)\right) = \frac{8}{p} + \frac{2}{1-p}(-1) = 0 \rightarrow \hat{p} = \frac{8}{10}$$

The General Case

Maximum Likelihood: Bernoulli Example

n coins

k heads

Maximum Likelihood: Bernoulli Example

n coins
k heads

Maximum Likelihood: Bernoulli Example

n coins
k heads

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Maximum Likelihood: Bernoulli Example

n coins

k heads

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} p_{X_i}(x_i)$$

Maximum Likelihood: Bernoulli Example

n coins

k heads

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} p_{X_i}(x_i) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i}$$

Maximum Likelihood: Bernoulli Example

n coins

k heads

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} p_{X_i}(x_i) = \prod_{i=1}^{n} \underbrace{p^{x_i}(1-p)^{1-x_i}}$$

If $x_i = 1, p^{[x_i]}(1-p)^{[1-x_i]} = p$

If $x_i = 0, p^{[x_i]}(1-p)^{[1-x_i]} = (1-p)$

Maximum Likelihood: Bernoulli Example

n coins

k heads

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} p_{X_i}(x_i) = \prod_{i=1}^{n} \underbrace{p^{x_i}(1-p)^{1-x_i}}$$

If $x_i = 1, p^{[x_i]}(1-p)^{[1-x_i]} = p$

If $x_i = 0, p^{[x_i]}(1-p)^{[1-x_i]} = (1-p)$

$\sum_{i=1}^{n} x_i =$ # heads

$n - \sum_{i=1}^{n} x_i =$ # tails

Maximum Likelihood: Bernoulli Example

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} p_{X_i}(x_i) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i}$$

Maximum Likelihood: Bernoulli Example

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} p_{X_i}(x_i) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} = p^{\left(\sum_{i=1}^{n} x_i\right)}(1-p)^{\left(n-\sum_{i=1}^{n} x_i\right)}$$

Maximum Likelihood: Bernoulli Example

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} p_{X_i}(x_i) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} = p^{\left(\sum_{i=1}^{n} x_i\right)}(1-p)^{\left(n-\sum_{i=1}^{n} x_i\right)}$$

Log-likelihood

$$\ell(p; \mathbf{x}) = \log\left((p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}\right)$$

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Likelihood

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Log-likelihood

$$\ell(p; \mathbf{x}) = \log\left((p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}\right) = \left(\sum_{i=1}^{n} x_i\right)\log(p) + \left(n - \sum_{i=1}^{n} x_i\right)\log(1-p)$$

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$$\mathbf{X} = (X_1, \ldots, X_n)$$

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Find the maximum!

$$\frac{d}{dp}\ell(p; \mathbf{x}) = \frac{d}{dp}\left(\left(\sum_{i=1}^n x_i\right)\log(p) + \left(n - \sum_{i=1}^n x_i\right)\log(1-p)\right)$$

$$= \frac{\sum_{i=1}^n x_i}{p} + \frac{n - \sum_{i=1}^n x_i}{1-p}(-1) = 0$$

Maximum Likelihood: Bernoulli Example

$$\mathbf{X} = (X_1, \ldots, X_n)$$

$$X_i \overset{i.i.d}{\sim} Bernoulli(p)$$

$$\ell(p; \mathbf{x}) = \log\left((p^{\sum_{i=1}^n x_i}(1-p)^{n-\sum_{i=1}^n x_i}\right) = \left(\sum_{i=1}^n x_i\right)\log(p) + \left(n - \sum_{i=1}^n x_i\right)\log(1-p)$$

Find the maximum!

$$\frac{d}{dp}\ell(p; \mathbf{x}) = \frac{d}{dp}\left(\left(\sum_{i=1}^n x_i\right)\log(p) + \left(n - \sum_{i=1}^n x_i\right)\log(1-p)\right)$$

$$= \frac{\sum_{i=1}^n x_i}{p} + \frac{n - \sum_{i=1}^n x_i}{1-p}(-1) = 0 \quad \Rightarrow \quad \hat{p} = \frac{\sum_{i=1}^n x_i}{n} = \bar{x}$$

DeepLearning.AI

Point Estimation

MLE: Gaussian Example

Maximum Likelihood: Gaussian Example

Maximum Likelihood: Gaussian Example

Observations

Maximum Likelihood: Gaussian Example

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

$\mathcal{N}(10,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(10,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
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0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(10,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(10,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(10,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(10,1^2)$
Heights almost 0
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(10,1^2)$
Heights almost 0
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
The $\mathcal{N}(2,1^2)$ is more likely!
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(10,1^2)$
Heights almost 0
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

$\mathcal{N}(2,1^2)$
The $\mathcal{N}(2,1^2)$ is more likely!
Density
0.4
0.3
0.2
0.1
0.0
−2
0
2
4
6
8
10
12
x

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Maximum Likelihood: Gaussian Example

Candidates

Observations

Gaussians With Three Different Means

Candidates

Observations

Gaussians With Three Different Means

Candidates

Observations

Gaussians With Three Different Means

Candidates

$\mathcal{N}(-1, 1^2)$

Observations

Gaussians With Three Different Means

Candidates

$\mathcal{N}(-1,1^2)$

$\mathcal{N}(0,1^2)$

Observations

Gaussians With Three Different Means

Candidates

$\mathcal{N}(-1,1^2)$

$\mathcal{N}(0,1^2)$

$\mathcal{N}(1,1^2)$

Observations

Gaussians With Three Different Means

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

$0.399 \cdot 0.054 = 0.022$

Candidates

Observations

$0.399 \cdot 0.054 = 0.022$

$0.242 \cdot 0.242 = 0.059$

Candidates

Observations

$0.399 \cdot 0.054 = 0.022$

$0.242 \cdot 0.242 = 0.059$

$0.054 \cdot 0.399 = 0.022$

Candidates

Observations

$0.399 \cdot 0.054 = 0.022$

The $\mathcal{N}(0,1)$ is more likely!

$0.242 \cdot 0.242 = 0.059$

$0.054 \cdot 0.399 = 0.022$

Candidates
Density
0.4
0.3
0.2
0.1
0.0
−4
−2
0
2
4
x
Likelihood = 0.022
Likelihood = 0.059
Likelihood = 0.022
Observations

Candidates

$\mathcal{N}(-1,1^2)$
$\mathcal{N}(0,1^2)$
$\mathcal{N}(1,1^2)$
Density
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4
x

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mathcal{N}(1,1^2)$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mu = 0$
$\mathcal{N}(1,1^2)$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mu = 0$
$\mathcal{N}(1,1^2)$
$\mu = 1$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mu = 0$
$\hat{\mu} = 0$
$\mathcal{N}(1,1^2)$
$\mu = 1$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mu = 0$
$\hat{\mu} = 0$
$\mathcal{N}(1,1^2)$
$\mu = 1$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

Gaussians With Three Different Variance

Candidates

Observations

Gaussians With Three Different Variance

Candidates

Observations

Gaussians With Three Different Variance

Candidates

$\mathcal{N}(0,0.5^2)$

Observations

Gaussians With Three Different Variance

Candidates

Observations

$\mathcal{N}(0,0.5^2)$

$\mathcal{N}(0,1^2)$

Gaussians With Three Different Variance

Candidates

$\mathcal{N}(0,0.5^2)$

$\mathcal{N}(0,1^2)$

$\mathcal{N}(0,2^2)$

Observations

Gaussians With Three Different Variance

Candidates

Observations

Gaussians With Three Different Variance

Candidates

Observations

Gaussians With Three Different Variance

Candidates

Observations

Gaussians With Three Different Variance

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

$0.044 \cdot 0.044 = 0.002$

Candidates

Observations

$0.044 \cdot 0.044 = 0.002$

$0.242 \cdot 0.242 = 0.059$

Candidates

Observations

$0.044 \cdot 0.044 = 0.002$

$0.242 \cdot 0.242 = 0.059$

$0.176 \cdot 0.176 = 0.031$

Candidates

Observations

0.044 · 0.044 = 0.002

The $\mathcal{N}(0,1^2)$ is more likely!

0.242 · 0.242 = 0.059

0.176 · 0.176 = 0.031

Candidates
Observations
$\mathcal{N}(0,0.5^2)$
0.044
0.044
Likelihood = 0.002
Observations
$\mathcal{N}(0,1^2)$
0.242
0.242
Likelihood = 0.059
$\mathcal{N}(0,2^2)$
0.176
0.176
Likelihood = 0.031
Density
x

Candidates

Observations

Candidates

Observations

Variance of the observations

Candidates

Observations

Variance of the observations

$$\widehat{\sigma^2} = \frac{1}{2}\left((0-1)^2 + (0+1)^2\right)$$

Candidates

Observations

Variance of the observations

$$\widehat{\sigma^2} = \frac{1}{2}\left((0-1)^2 + (0+1)^2\right) = 1$$

Candidates

Observations

Variance of the observations

$$\widehat{\sigma^2} = \frac{1}{2}\left((0-1)^2 + (0+1)^2\right) = 1$$

Candidates

Observations

Variance of the observations

$$\widehat{\sigma^2} = \frac{1}{2}\left((0-1)^2 + (0+1)^2\right) = 1$$

Candidates

Observations

Variance of the observations

$$\widehat{\sigma^2} = \frac{1}{2}\left((0-1)^2 + (0+1)^2\right) = 1$$

Candidates

Observations

Variance of the observations

$$\widehat{\sigma^2} = \frac{1}{2}\left((0-1)^2 + (0+1)^2\right) = 1$$

The best distribution is the one where the **variance** of the distribution is the **variance** of the sample

Candidates

$\mathcal{N}(-1,1^2)$
$\mathcal{N}(0,1^2)$
$\mathcal{N}(1,1^2)$
Density
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4
x

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mu = 0$
$\mathcal{N}(1,1^2)$
$\mu = 1$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mu = 0$
$\hat{\mu} = 0$
$\mathcal{N}(1,1^2)$
$\mu = 1$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

$\mathcal{N}(-1,1^2)$
$\mu = -1$
$\mathcal{N}(0,1^2)$
$\mu = 0$
$\hat{\mu} = 0$
$\mathcal{N}(1,1^2)$
$\mu = 1$
Density
x
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4

Candidates

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

Candidates

Observations

The best distribution is the one where the **variance** of the distribution is the **variance** of the sample

Best Gaussian Fit

Best Gaussian Fit

$$\hat{\mu} = 0$$

Best Gaussian Fit

$$\hat{\mu} = 0$$

$$\widehat{\sigma^2} = 1$$

Best Gaussian Fit

$$\hat{\mu} = 0$$

$$\widehat{\sigma^2} = 1$$

Best Gaussian Fit

$$\hat{\mu} = 0$$

$$\widehat{\sigma^2} = 1$$

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

Best Gaussian Fit

$\mu = 0$

$\hat{\mu} = 0$

$\widehat{\sigma^2} = 1$

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

Best Gaussian Fit

$\mu = 0$

$\hat{\mu} = 0$

$\widehat{\sigma^2} = 1$

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

The best distribution is the one where the **variance** of the distribution is the **variance** of the sample

Best Gaussian Fit

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

The best distribution is the one where the **variance** of the distribution is the **variance** of the sample

Best Gaussian Fit

Best Gaussian Fit

Best Gaussian Fit

Best Gaussian Fit

Best Gaussian Fit

Maximum Likelihood: Gaussian Example

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

You measure 10 people
(i.e. sample size =10)

$$\mathbf{X} = (X_1, X_2, \ldots, X_{10})$$

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

You measure 10 people
(i.e. sample size =10)

$$\mathbf{X} = (X_1, X_2, \ldots, X_{10})$$

$$X_1, X_2, \ldots, X_{10} \overset{i.i.d}{\sim} \mathcal{N}(\mu, \sigma)$$

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

You measure 10 people
(i.e. sample size =10)

$$\mathbf{X} = (X_1, X_2, \ldots, X_{10})$$

$$X_1, X_2, \ldots, X_{10} \overset{i.i.d}{\sim} \mathcal{N}(\mu, \sigma)$$

μ?

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

You measure 10 people
(i.e. sample size =10)

$$\mathbf{X} = (X_1, X_2, \ldots, X_{10})$$

$$X_1, X_2, \ldots, X_{10} \overset{i.i.d}{\sim} \mathcal{N}(\mu, \sigma)$$

μ?

$\mu = 64$

$\mu = 68$

$\mu = 66$

$\mu = 70$

$\mu = 64$
$\sigma = 3.11$

$\mu = 68$
$\sigma = 3.11$

$\mu = 66$
$\sigma = 3.11$

$\mu = 70$
$\sigma = 3.11$

$\mu = 64$
$\sigma = 3.11$
$\mu = 68$
$\sigma = 3.11$
$\mu = 66$
$\sigma = 3.11$
$\mu = 70$
$\sigma = 3.11$
Density
Height
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
60.0
62.5
65.0
67.5
70.0
72.5
75.0
77.5

$\mu = 64$
$\sigma = 3.11$
$\mu = 68$
$\sigma = 3.11$
$\mu = 66$
$\sigma = 3.11$
$\mu = 70$
$\sigma = 3.11$
Density
Height
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
60.0
62.5
65.0
67.5
70.0
72.5
75.0
77.5

$\mu = 64$
$\sigma = 3.11$
$\mu = 68$
$\sigma = 3.11$
$\mu = 66$
$\sigma = 3.11$
$\mu = 70$
$\sigma = 3.11$
Density
Height
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
60.0
62.5
65.0
67.5
70.0
72.5
75.0
77.5

$\mu = 64$
$\sigma = 3.11$
$\mu = 68$
$\sigma = 3.11$
$\mu = 66$
$\sigma = 3.11$
$\mu = 70$
$\sigma = 3.11$
Density
Height
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
60.0
62.5
65.0
67.5
70.0
72.5
75.0
77.5

$\mu = 64$
$\sigma = 3.11$
$\mu = 68$
$\sigma = 3.11$
$\mu = 66$
$\sigma = 3.11$
$\mu = 70$
$\sigma = 3.11$
Density
Height
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
60.0
62.5
65.0
67.5
70.0
72.5
75.0
77.5

For which of these is the product of lines (likelihoods) the highest?

For which of these is the product of lines (likelihoods) the highest?

For which of these is the product of lines (likelihoods) the highest?

$\mu = 64$
$\sigma = 3.11$
$4.92 \cdot 10^{-16}$
$\mu = 68$
$\sigma = 3.11$
$1.20 \cdot 10^{-11}$
$\mu = 66$
$\sigma = 3.11$
$6.08 \cdot 10^{-13}$
$3.79 \cdot 10^{-12}$
Density
Height
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
60.0
62.5
65.0
67.5
70.0
72.5
75.0
77.5

$$\frac{d}{d\mu}\ell(\mu; \mathbf{x}) = \frac{d}{d\mu}\left[\log\left(\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{10}\right) - \frac{1}{2}\frac{\sum_{i=1}^{10}(x_i - \mu)^2}{\sigma^2}\right]$$

Likelihood $L(\mu; \underline{x}) = \prod_{i=1}^{10} f_\mu(x_i)$

$$\frac{d}{d\mu}\ell(\mu; \mathbf{x}) = \frac{d}{d\mu}\left[\log\left(\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{10}\right) - \frac{1}{2}\frac{\sum_{i=1}^{10}(x_i - \mu)^2}{\sigma^2}\right]$$

Likelihood $L(\mu; \underline{x})$

$$= \prod_{i=1}^{10} f_\mu(x_i)$$

$$= \prod_{i=1}^{10} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right)$$

$$\frac{d}{d\mu}\ell(\mu; \mathbf{x}) = \frac{d}{d\mu}\left[\log\left(\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{10}\right) - \frac{1}{2}\frac{\sum_{i=1}^{10}(x_i-\mu)^2}{\sigma^2}\right]$$

Likelihood $L(\mu; \underline{x})$

$$= \prod_{i=1}^{10} f_\mu(x_i)$$

$$= \prod_{i=1}^{10} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right)$$

Observations

x_i are fixed!

$$\frac{d}{d\mu}\ell(\mu; \mathbf{x}) = \frac{d}{d\mu}\left[\log\left(\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{10}\right) - \frac{1}{2}\frac{\sum_{i=1}^{10}(x_i-\mu)^2}{\sigma^2}\right]$$

Likelihood $L(\mu; \underline{x})$ $= \prod_{i=1}^{10} f_\mu(x_i)$

$$= \prod_{i=1}^{10} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right)$$

Observations x_i are fixed!

It's a function of μ

$$\frac{d}{d\mu}\ell(\mu; \mathbf{x}) = \frac{d}{d\mu}\left[\log\left(\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{10}\right) - \frac{1}{2}\frac{\sum_{i=1}^{10}(x_i-\mu)^2}{\sigma^2}\right]$$

Likelihood $L(\mu; \underline{x}) = \prod_{i=1}^{10} f_\mu(x_i)$

$$= \boxed{\prod_{i=1}^{10} \frac{1}{\sqrt{2\pi}\,\sigma}} \exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right)$$

$$\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{10}$$

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Derivative $\frac{d}{d\mu}\ell(\mu; \mathbf{x}) = \frac{d}{d\mu}\left[\log\left(\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{10}\right) - \frac{1}{2}\frac{\sum_{i=1}^{10}(x_i-\mu)^2}{\sigma^2}\right]$

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$$= \quad 0 \quad -\frac{1}{2}\frac{1}{\sigma^2}\sum_{i=1}^{10} 2(x_i-\mu)(-1)$$

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This needs to be zero

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$$\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\left(\sum_{i=1}^{n}x_i\right) - n\,\mu = 0 \quad \Rightarrow \quad \mu = \frac{1}{n}\sum_{i=1}^{n}x_i$$

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

Maximum Likelihood: Gaussian Example

Maximum Likelihood: Gaussian Example

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

Maximum Likelihood: Gaussian Example

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

$$\hat{\mu} = \frac{66.75 + 70.24 + 67.19 + 67.09 + 63.65 + 64.64 + 69.81 + 69.79 + 73.52 + 71.74}{10}$$

Maximum Likelihood: Gaussian Example

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$$\hat{\mu} = \frac{66.75 + 70.24 + 67.19 + 67.09 + 63.65 + 64.64 + 69.81 + 69.79 + 73.52 + 71.74}{10}$$

$$= 68.442$$

Maximum Likelihood: Gaussian Example

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

Maximum Likelihood: Gaussian Example

What about the variance?

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

Maximum Likelihood: Gaussian Example

What about the variance?

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

If you repeat the process, assuming unknown variance, you'll get that

Maximum Likelihood: Gaussian Example

What about the variance?

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

If you repeat the process, assuming unknown variance, you'll get that

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{10} (x_i - \bar{x})^2}{10}$$

Maximum Likelihood: Gaussian Example

What about the variance?

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

If you repeat the process, assuming unknown variance, you'll get that

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{10}(x_i - \bar{x})^2}{10} = \frac{1}{10}\big((66.75 - 68.442)^2 + (70.24 - 68.442)^2 + (67.19 - 68.442)^2 + (67.09 - 68.442)^2 + (63.65 - 68.442)^2 + (64.64 - 68.442)^2 + (69.81 - 68.442)^2 + (69.79 - 68.442)^2 + (73.52 - 68.442)^2 + (71.74 - 68.442)^2\big)$$

$$= 8.72$$

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

$$\mathcal{N}(\mu, \sigma)$$

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

$$\mathcal{N}(\mu, \sigma)$$

$$\hat{\mu} = 68.442$$

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

$$\mathcal{N}(\mu, \sigma)$$

$$\hat{\mu} = 68.442$$

$$\hat{\sigma} = 8.72$$

DeepLearning.AI

Point Estimation

MLE: Linear Regression

Maximum Likelihood

Maximum Likelihood

Data

Maximum Likelihood

Model 1

Data

Maximum Likelihood

Model 1

Model 2

Data

Maximum Likelihood

Model 1

Model 2

Model 3

Data

Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Find model that most likely produced the data

Maximum Likelihood

Find model that most likely produced the data

Maximum Likelihood

Find model that most likely produced the data

Maximize
$\mathbf{P}(\text{Data}|\text{Model})$

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Model 1

Data

Example: Linear Regression

Model 1

Model 2

Data

Example: Linear Regression

Model 1

Model 2

Model 3

Data

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Example: Linear Regression

Find model that most likely produced the data

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

Model

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

How Exactly Does a Line Produce Points?

Linear Regression

Linear Regression

Line that best produced the points

Linear Regression

Line that best produced the points

Line that best fits the data (linear regression)

Linear Regression

Line that best produced the points

Line that best fits the data (linear regression)

Linear Regression

Line that best produced the points

How?

Line that best fits the data (linear regression)

Linear Regression and Likelihood

Linear Regression and Likelihood

Linear Regression and Likelihood

Linear Regression and Likelihood

Linear Regression and Likelihood

Linear Regression and Likelihood

Linear Regression and Likelihood

Likelihood:

Linear Regression and Likelihood

Likelihood:

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Likelihood:

Linear Regression and Likelihood

Likelihood:

Linear Regression and Likelihood

Likelihood:

Linear Regression and Likelihood

Likelihood:

Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_1^2}$$

Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_1^2}$$

Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_5^2}$$

Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_5^2}$$

Maximize

Linear Regression and Likelihood

Likelihood:

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}$$

Maximize

Linear Regression and Likelihood

Likelihood:

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

Linear Regression and Likelihood

Likelihood:

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$e^{-\frac{1}{2}\left(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2\right)}$$

Linear Regression and Likelihood

Likelihood:

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$\boxed{\cancel{e^{-\frac{1}{2}}}}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

Linear Regression and Likelihood

Likelihood:

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$\boxed{\cancel{e^{-\frac{1}{2}}}}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

Minimize

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Linear Regression and Likelihood

Likelihood:

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$\boxed{\cancel{e^{-\frac{1}{2}}}}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

Minimize

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Least squares error!

Linear Regression and Likelihood

Likelihood:

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$\boxed{\cancel{e^{-\frac{1}{2}}}}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

Minimize

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Least squares error!

Linear regression!

Picking the Right Model

Picking the Right Model

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Model 1:

Likelihood $= 4.91 \cdot 10^{-260}$

Picking the Right Model

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Model 2:
Likelihood $= 8.16 \cdot 10^{-28}$

Picking the Right Model

Model 1:
Likelihood $= 4.91 \cdot 10^{-260}$

Model 2:
Likelihood $= 8.16 \cdot 10^{-28}$

Model 3:
Likelihood $= 3.48 \cdot 10^{-49}$

Picking the Right Model

Model 1:
Likelihood $= 4.91 \cdot 10^{-260}$

Model 2:
Likelihood $= 8.16 \cdot 10^{-28}$

Model 3:
Likelihood $= 3.48 \cdot 10^{-49}$

DeepLearning.AI

Point Estimation

Frequentist vs Bayesian Statistics

Frequentist Vs. Bayesian Statistics

Frequentist Vs. Bayesian Statistics

Frequentists	Bayesians

Frequentist Vs. Bayesian Statistics

Frequentists	Bayesians
• Probabilities represent long term frequency of events	

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events

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- Probabilities represent the degree of belief (or certainty)

Frequentist Vs. Bayesian Statistics

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- Concept of Likelihood

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Frequentist Vs. Bayesian Statistics

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- Probabilities represent the degree of belief (or certainty)
- Concept of Prior
- Goal: update prior belief based on observations

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events
- Concept of Likelihood
- Goal: Find the model that most likely generated the observed data

Bayesians

- Probabilities represent the degree of belief (or certainty)
- Concept of Prior
- Goal: update prior belief based on observations

Frequentist Approach

Frequentist Approach

Frequentist Approach

p = probability of heads

Frequentist Approach

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$$p = 0.8$$

Frequentist Approach

p = probability of heads

$$p = 0.8$$

Maximum likelihood

Bayesian Approach

p = probability of heads

Bayesian Approach

p = probability of heads

p could be anything between 0 and 1

Bayesian Approach

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p could still be anything between 0 and 1, but it may be closer to 1

Bayesian Approach

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p could still be anything between 0 and 1, but it's probably around 0.5

Bayesian Approach

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Bayesian Approach

p = probability of heads

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…

Bayesian Approach

p = probability of heads

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p could still be anything between 0 and 1, but it may be closer to 1

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p could still be anything between 0 and 1, but it's more likely closer to 0.66

…

p could still be anything between 0 and 1, but it's very probably close to 0.8

Bayesian Approach

p = probability of heads

Bayesian Approach

p = probability of heads

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Bayesian Statistics

Bayesian Statistics

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

Bayesian Statistics

Remember Bayes' theorem? $\boxed{\mathbf{P}(B \mid A)} = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

Posterior

Bayesian Statistics

Remember Bayes' theorem? $\boxed{\mathbf{P}(B \mid A)} = \dfrac{\mathbf{P}(A \mid B)\boxed{\mathbf{P}(B)}}{\mathbf{P}(A)}$

Prior

Posterior

Bayesian Statistics

Remember Bayes' theorem? $\boxed{\mathbf{P}(B \mid A)} = \dfrac{\mathbf{P}(A \mid B)\boxed{\mathbf{P}(B)}}{\boxed{\mathbf{P}(A)}}$

Prior

Posterior

Normalizing constant

Bayesian Statistics

The parameters (or model) you want to estimate

Remember Bayes' theorem?

$$\boxed{\mathbf{P}(B \mid A)} = \frac{\mathbf{P}(A \mid B)\boxed{\mathbf{P}(B)}}{\boxed{\mathbf{P}(A)}}$$

Prior

Posterior

Normalizing constant

Bayesian Statistics

The parameters (or model) you want to estimate

Your sample vector

Remember Bayes' theorem?

$$\boxed{\mathbf{P}(B \mid A)} = \frac{\mathbf{P}(A \mid B)\boxed{\mathbf{P}(B)}}{\boxed{\mathbf{P}(A)}}$$

Prior

Posterior

Normalizing constant

Bayesian Statistics

The parameters (or model) you want to estimate

Your sample vector

Probability of samples given model B

Remember Bayes' theorem?

$$\mathbf{P}(B \mid A) = \frac{\mathbf{P}(A \mid B)\,\mathbf{P}(B)}{\mathbf{P}(A)}$$

Prior

Posterior

Normalizing constant

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

Bayesian Statistics: Bernoulli Example

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Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

$$\Theta = \mathbf{P}(H)$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

Θ is a random variable $\longrightarrow$ $\Theta = \mathbf{P}(H)$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

Θ is a random variable $\longrightarrow$ $\Theta = \mathbf{P}(H)$

$X_i = 1$ if H,0 if T

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Observations: $\sum_{i=1}^{10} X_i = 8$

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Suppose that Θ takes on the particular value θ

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$\mathbf{P}(A \mid B)$

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Observations: $\sum_{i=1}^{10} X_i = 8 \longleftarrow A$

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Θ is a random variable $\rightarrow$ $\Theta = \mathbf{P}(H)$ $\qquad X_i = 1$ if H,0 if T

Observations: $\sum_{i=1}^{10} X_i = 8 \leftarrow A$

Suppose that $\boxed{\Theta \text{ takes on the particular value } \theta} \leftarrow B$

$\mathbf{P}(A \mid B)$

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$\mathbf{P}(A \mid B) \longrightarrow \mathbf{P}\left(\sum_{i=1}^{10} X_i = 8 \mid \Theta = \theta\right) =$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

$\theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad 1-\theta \quad 1-\theta$

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Observations: $\sum_{i=1}^{10} X_i = 8 \longleftarrow A$

Suppose that $\boxed{\Theta \text{ takes on the particular value } \theta} \longleftarrow B$

$$\mathbf{P}(A \mid B) \longrightarrow \mathbf{P}\left(\sum_{i=1}^{10} X_i = 8 \mid \Theta = \theta\right) = \theta^8(1-\theta)^2$$

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Observations: $\sum_{i=1}^{10} X_i = 8 \longleftarrow A$

Suppose that $\boxed{\Theta \text{ takes on the particular value } \theta} \longleftarrow B$

$\mathbf{P}(A \mid B) \longrightarrow \mathbf{P}\left(\sum_{i=1}^{10} X_i = 8 \mid \Theta = \theta\right) = \theta^8(1-\theta)^2 \qquad X_i \mid \Theta = \theta \sim Bernoulli(\theta)$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

$\theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad 1-\theta \quad 1-\theta$

$X_i \mid \Theta = \theta \sim Bernoulli(\theta)$

How do you choose the prior?

Bayesian Statistics: Bernoulli Example

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$X_i \mid \Theta = \theta \sim Bernoulli(\theta)$

How do you choose the prior?

If before tossing the coins you have no idea whether the coin is fair or not, then your prior belief about Θ should not favor any value.
You can model this with a uniform distribution

$\Theta \sim Uniform(0,1)$ $\qquad f_\Theta(\theta) = 1, \quad 0 \le \theta \le 1$

Bayesian Statistics: Bernoulli Example

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$$\boxed{f_\Theta(\theta) = 1, \quad 0 \le \theta \le 1} \longleftarrow \text{Prior } (\mathbf{P}(B))$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

$\theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad 1-\theta \quad 1-\theta$

$X_i \mid \Theta = \theta \sim Bernoulli(\theta) \longleftarrow (\mathbf{P}(A \mid B))$

$\Theta \sim Uniform(0,1) \longleftarrow$ Prior $(\mathbf{P}(B))$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

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How do you get the posterior? Bayes!

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

$\theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad 1-\theta \quad 1-\theta$

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How do you get the posterior? Bayes!

$$f_{\Theta \mid \mathbf{X}=\mathbf{x}}(\theta) = \frac{p_{\mathbf{X} \mid \Theta=\theta}(\mathbf{x}) f_{\Theta}(\theta)}{p_{\mathbf{X}}(\mathbf{x})}$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $\mathbf{P}(B \mid A) = \dfrac{\mathbf{P}(A \mid B)\mathbf{P}(B)}{\mathbf{P}(A)}$

$\theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad \theta \quad 1-\theta \quad 1-\theta$

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Normalizing constant

Posterior $(\mathbf{P}(B \mid A))$

Bayesian Statistics: MAP Estimator

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) \propto p_{\mathbf{X}|\Theta=\theta}(\mathbf{x}) f_{\Theta}(\theta)$$

Bayesian Statistics: MAP Estimator

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) \propto \boxed{p_{\mathbf{X}|\Theta=\theta}(\mathbf{x})}\boxed{f_{\Theta}(\theta)}$$

$\mathbf{P}$(data | model) $\mathbf{P}$(model)

Bayesian Statistics: MAP Estimator

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) \propto \boxed{p_{\mathbf{X}|\Theta=\theta}(\mathbf{x})}\boxed{f_{\Theta}(\theta)}$$

Does this look familiar? $\mathbf{P}(\text{data} \mid \text{model})$ $\mathbf{P}(\text{model})$

Bayesian Statistics: MAP Estimator

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) \propto \boxed{p_{\mathbf{X}|\Theta=\theta}(\mathbf{x})}\boxed{f_{\Theta}(\theta)}$$

Does this look familiar? $\mathbf{P}(\text{data} \mid \text{model})$ $\mathbf{P}(\text{model})$

Point estimators?

Bayesian Statistics: MAP Estimator

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) \propto \boxed{p_{\mathbf{X}|\Theta=\theta}(\mathbf{x})}\boxed{f_{\Theta}(\theta)}$$

Does this look familiar? $\mathbf{P}(\text{data} \mid \text{model})$ $\mathbf{P}(\text{model})$

Point estimators?

$$\hat{\theta} = \arg\max_{\theta} f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) = \arg\max_{\theta} p_{\mathbf{X}|\Theta=\theta}(\mathbf{x}) f_{\Theta}(\theta)$$

Bayesian Statistics: MAP Estimator

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) \propto \boxed{p_{\mathbf{X}|\Theta=\theta}(\mathbf{x})}\boxed{f_{\Theta}(\theta)}$$

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Point estimators?

$$\hat{\theta} = \arg\max_{\theta} f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) = \arg\max_{\theta} p_{\mathbf{X}|\Theta=\theta}(\mathbf{x}) f_{\Theta}(\theta)$$

This is called the Maximum a Posteriori (MAP) estimator

Bayesian Statistics: MAP Estimator

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) \propto \boxed{p_{\mathbf{X}|\Theta=\theta}(\mathbf{x})}\boxed{f_{\Theta}(\theta)}$$

Does this look familiar? $\mathbf{P}(\text{data} \mid \text{model})$ $\mathbf{P}(\text{model})$

Point estimators?

$$\hat{\theta} = \arg\max_{\theta} f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) = \arg\max_{\theta} p_{\mathbf{X}|\Theta=\theta}(\mathbf{x}) f_{\Theta}(\theta)$$

This is called the Maximum a Posteriori (MAP) estimator

Notice that if the prior is non informative, then MAP = MLE

DeepLearning.AI

Point Estimation

Regularization

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Loss

10

Example: Polynomial Regression

Loss

10

2

Example: Polynomial Regression

Loss

10

2

0.1

Example: Polynomial Regression

Loss

10

2

0.1

Example: Polynomial Regression

Loss	Equation
10	
2	
0.1	

Example: Polynomial Regression

Loss	Equation
10	$y = 4x + 3$
2	
0.1	

Example: Polynomial Regression

Loss	Equation
10	$y = 4x + 3$
2	$y = 2x^2 - 4x + 5$
0.1	

Example: Polynomial Regression

Loss	Equation
10	$y = 4x + 3$
2	$y = 2x^2 - 4x + 5$
0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$

Example: Polynomial Regression

Loss	Equation	Penalty
10	$y = 4x + 3$	
2	$y = 2x^2 - 4x + 5$	
0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$	

Example: Polynomial Regression

Loss	Equation	Penalty
10	$y = 4x + 3$	$L_2 = 4^2$
2	$y = 2x^2 - 4x + 5$	
0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$	

Example: Polynomial Regression

Loss	Equation	Penalty
10	$y = \boxed{4}x + 3$	$L_2 = 4^2$ $= 16$
2	$y = 2x^2 - 4x + 5$	
0.1	$y = 4x^{10} - 9x^8 - 2x^6$ $+3x^5 - 6x^4 - 10x + 4$	

Example: Polynomial Regression

Loss	Equation	Penalty
10	$y = \boxed{4}x + 3$	$L_2 = 4^2$ $= 16$
2	$y = \boxed{2}x^2 - \boxed{4}x + 5$	$L_2 = 2^2 + (-4)^2$
0.1	$y = 4x^{10} - 9x^8 - 2x^6$ $+3x^5 - 6x^4 - 10x + 4$	

Example: Polynomial Regression

Loss	Equation	Penalty
10	$y = \boxed{4}x + 3$	$L_2 = 4^2$ $= 16$
2	$y = \boxed{2}x^2 - \boxed{4}x + 5$	$L_2 = 2^2 + (-4)^2$ $= 20$
0.1	$y = 4x^{10} - 9x^8 - 2x^6$ $+3x^5 - 6x^4 - 10x + 4$	

Example: Polynomial Regression

Loss	Equation	Penalty
10	$y = \boxed{4}x + 3$	$L_2 = 4^2$ $= 16$
2	$y = \boxed{2}x^2 - \boxed{4}x + 5$	$L_2 = 2^2 + (-4)^2$ $= 20$
0.1	$y = \boxed{4}x^{10} - \boxed{9}x^8 - \boxed{2}x^6$ $+ \boxed{3}x^5 - \boxed{6}x^4 - \boxed{10}x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2$ $+ 3^2 + (-6)^2 + (-10)^2$

Example: Polynomial Regression

Loss	Equation	Penalty
10	$y = \boxed{4}x + 3$	$L_2 = 4^2$ $= 16$
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Example: Polynomial Regression

Loss	Equation	Penalty	New loss
10	$y = \boxed{4}x + 3$	$L_2 = 4^2$ $= 16$	
2	$y = \boxed{2}x^2 - \boxed{4}x + 5$	$L_2 = 2^2 + (-4)^2$ $= 20$	
0.1	$y = \boxed{4}x^{10} - \boxed{9}x^8 - \boxed{2}x^6$ $+ \boxed{3}x^5 - \boxed{6}x^4 - \boxed{10}x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2$ $+3^2 + (-6)^2 + (-10)^2$ $= 246$	

Example: Polynomial Regression

Loss	Equation	Penalty	New loss
$\boxed{10}$	$y = \boxed{4}x + 3$	$L_2 = 4^2 = \boxed{16}$	26
2	$y = \boxed{2}x^2 - \boxed{4}x + 5$	$L_2 = 2^2 + (-4)^2 = 20$	
0.1	$y = \boxed{4}x^{10} - \boxed{9}x^8 - \boxed{2}x^6 + \boxed{3}x^5 - \boxed{6}x^4 - \boxed{10}x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2 + 3^2 + (-6)^2 + (-10)^2 = 246$	

Example: Polynomial Regression

Loss	Equation	Penalty	New loss
$\boxed{10}$	$y = \boxed{4}x + 3$	$L_2 = 4^2 = \boxed{16}$	26
$\boxed{2}$	$y = \boxed{2}x^2 - \boxed{4}x + 5$	$L_2 = 2^2 + (-4)^2 = \boxed{20}$	22
0.1	$y = \boxed{4}x^{10} - \boxed{9}x^8 - \boxed{2}x^6 + \boxed{3}x^5 - \boxed{6}x^4 - \boxed{10}x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2 + 3^2 + (-6)^2 + (-10)^2 = 246$	

Example: Polynomial Regression

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Example: Polynomial Regression

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Regularization Term

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

Regularization Term

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Log-loss: $\ell\ell$

Regularization Term

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Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \cdots + a_1^2$

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \cdots + a_1^2$

Regularization parameter: λ

Regularization Term

Model: $y = \boxed{a_n} x^n + \boxed{a_{n-1}} x^{n-1} + \cdots + \boxed{a_1} x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \cdots + a_1^2$

Regularization parameter: λ

Regularization Term

Model: $y = \boxed{a_n} x^n + \boxed{a_{n-1}} x^{n-1} + \cdots + \boxed{a_1} x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \cdots + a_1^2$

Regularization parameter: λ

Regularized error:

Regularization Term

Model: $y = \boxed{a_n} x^n + \boxed{a_{n-1}} x^{n-1} + \cdots + \boxed{a_1} x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \cdots + a_1^2$

Regularization parameter: λ

Regularized error: $\ell\ell + \lambda\,(a_n^2 + a_{n-1}^2 + \cdots + a_1^2)$

DeepLearning.AI

Point Estimation

Back to Bayesics

There’s Popcorn on the Floor. What Happened?

Movies

Board Games

Nap

There's Popcorn on the Floor. What Happened?

Movies

Board Games

Nap

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

There’s Popcorn on the Floor. What Happened?

Movies

Popcorn throwing contest

There's Popcorn on the Floor. What Happened?

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$$\mathbf{P}(\text{Popcorn}|\text{Movies}) < \mathbf{P}(\text{Popcorn}|\text{Contest})$$

There's Popcorn on the Floor. What Happened?

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There's Popcorn on the Floor. What Happened?

$$P(\text{Popcorn}|\text{Movies}) < P(\text{Popcorn}|\text{Contest})$$

There's Popcorn on the Floor. What Happened?

$\mathbf{P}(\text{Movies})$ Movies

High

$\mathbf{P}(\text{Popcorn}|\text{Movies})$

$\mathbf{P}(\text{Movies}) \gg \mathbf{P}(\text{Contest})$

Very high

$\mathbf{P}(\text{Contest})$ Popcorn throwing contest

$\mathbf{P}(\text{Popcorn}|\text{Contest})$

$\mathbf{P}(\text{Popcorn}|\text{Movies}) < \mathbf{P}(\text{Popcorn}|\text{Contest})$

There's Popcorn on the Floor. What Happened?

$$P(\text{Popcorn}|\text{Movies})\ P(\text{Movies}) < P(\text{Popcorn}|\text{Contest})\ P(\text{Contest})$$

There's Popcorn on the Floor. What Happened?

$$P(\text{Popcorn}|\text{Movies})\ P(\text{Movies}) > P(\text{Popcorn}|\text{Contest})\ P(\text{Contest})$$

There's Popcorn on the Floor. What Happened?

$$\mathbf{P}(\text{Popcorn}|\text{Movies})\ \mathbf{P}(\text{Movies}) > \mathbf{P}(\text{Popcorn}|\text{Contest})\ \mathbf{P}(\text{Contest})$$

$$\boxed{\mathbf{P}(A \mid B)\mathbf{P}(B) = \mathbf{P}(A \cap B)}$$

There's Popcorn on the Floor. What Happened?

$$\mathbf{P}(\text{Popcorn}|\text{Movies})\ \mathbf{P}(\text{Movies}) > \mathbf{P}(\text{Popcorn}|\text{Contest})\ \mathbf{P}(\text{Contest})$$

$$\mathbf{P}(\text{Popcorn} \cap \text{Movie})$$

$$\boxed{\mathbf{P}(A \mid B)\mathbf{P}(B) = \mathbf{P}(A \cap B)}$$

There's Popcorn on the Floor. What Happened?

$$\mathbf{P}(\text{Popcorn}|\text{Movies})\ \mathbf{P}(\text{Movies}) > \mathbf{P}(\text{Popcorn}|\text{Contest})\ \mathbf{P}(\text{Contest})$$

$$\mathbf{P}(\text{Popcorn} \cap \text{Movie}) \qquad \mathbf{P}(\text{Popcorn} \cap \text{Contest})$$

$$\boxed{\mathbf{P}(A \mid B)\mathbf{P}(B) = \mathbf{P}(A \cap B)}$$

There's Popcorn on the Floor. What Happened?

$$\mathbf{P}(\text{Popcorn}|\text{Movies})\ \mathbf{P}(\text{Movies}) > \mathbf{P}(\text{Popcorn}|\text{Contest})\ \mathbf{P}(\text{Contest})$$

$$\boxed{\mathbf{P}(\text{Popcorn} \cap \text{Movie}) > \mathbf{P}(\text{Popcorn} \cap \text{Contest})}$$

$$\boxed{\mathbf{P}(A \mid B)\mathbf{P}(B) = \mathbf{P}(A \cap B)}$$

What Does This Have To Do With Regularization?

DeepLearning.AI

Point Estimation

Bayes Theorem and Regularization

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Model 1

Data

Example: Polynomial Regression

Model 1

Model 2

Data

Example: Polynomial Regression

Model 1

Model 2

Model 3

Data

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Example: Polynomial Regression

Maximum likelihood

Polynomial regression

Maximum likelihood

Polynomial regression

Log-loss

Maximum likelihood

Polynomial regression

$\mathbf{P}(\text{Data}|\text{Model})$

Log-loss

Maximum likelihood with Bayes

Polynomial regression

$\mathbf{P}(\text{Data}|\text{Model})$

Log-loss

Maximum likelihood with Bayes

Polynomial regression

$\mathbf{P}(\text{Data}|\text{Model})$

$\cdot$

$\mathbf{P}(\text{Model})$

Log-loss

Maximum likelihood with Bayes

Polynomial regression with regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$\cdot$

$\mathbf{P}(\text{Model})$

Log-loss

Maximum likelihood with Bayes

Polynomial regression with regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$\cdot$

$\mathbf{P}(\text{Model})$

Log-loss

$+$

Regularization term

Maximum likelihood with Bayes

Polynomial regression with regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$\cdot$

$\mathbf{P}(\text{Model})$

Take logarithms!

Log-loss

$+$

Regularization term

Maximum likelihood with Bayes

Polynomial regression with regularization

$\mathbf{P}$(Data|Model)

$\cdot$

$\mathbf{P}$(Model)

Log-loss

$+$

Regularization term

Maximum likelihood with Bayes

Polynomial regression with regularization

$\mathbf{P}$(Data|Model)

$\cdot$

$\boxed{\mathbf{P}(\text{Model})}$

?

Log-loss

$+$

Regularization term

What Is the Probability of a Model?

What Is the Probability of a Model?

What Is the Probability of a Model?

What Is the Probability of a Model?

What Is the Probability of a Model?

$\mathbf{P}(\text{Model 1})$

$\mathbf{P}(\text{Model 2})$

$\mathbf{P}(\text{Model 3})$

What Is the Probability of a Model?

$\mathbf{P}$(Model 1)

$$a_1 x + b$$

$\mathbf{P}$(Model 2)

$$a_1 x^2 + a_2 x + b$$

$\mathbf{P}$(Model 3)

$$a_1 x^{10} + a_2 x^9 + \cdots + a_{10} x + b$$

What Is the Probability of a Model?

$\mathbf{P}$(Model 1)

$\boxed{a_1}x + b$

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$\mathbf{P}$(Model 3)

$a_1x^{10} + a_2x^9 + \cdots + a_{10}x + b$

$a_i \sim \mathcal{N}(0,1)$

What Is the Probability of a Model?

$\mathbf{P}$(Model 1)

$\boxed{a_1}x + b$

$\mathbf{P}$(Model 2)

$a_1x^2 + a_2x + b$

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$\mathbf{P}$(Model 3)

$a_1x^{10} + a_2x^9 + \cdots + a_{10}x + b$

What Is the Probability of a Model?

$$\mathbf{P}(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

$$\boxed{a_1}x + b$$

$\mathbf{P}$(Model 2)

$$a_1x^2 + a_2x + b$$

$\mathbf{P}$(Model 3)

$$a_1x^{10} + a_2x^9 + \cdots + a_{10}x + b$$

What Is the Probability of a Model?

$\mathbf{P}$(Model 1) $= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$

$\boxed{a_1}x + b$

$\mathbf{P}$(Model 2)

$\boxed{a_1}x^2 + a_2x + b$

$\mathbf{P}$(Model 3)

$a_1x^{10} + a_2x^9 + \cdots + a_{10}x + b$

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$\mathbf{P}$(Model 1) $= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$

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$\boxed{a_1}x^2 + \boxed{a_2}x + b$

$\mathbf{P}$(Model 3)

$a_1x^{10} + a_2x^9 + \cdots + a_{10}x + b$

What Is the Probability of a Model?

$$\mathbf{P}(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

$$\boxed{a_1} x + b$$

$\mathbf{P}$(Model 2)

$$\boxed{a_1} x^2 + \boxed{a_2} x + b = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

$\mathbf{P}$(Model 3)

$$a_1 x^{10} + a_2 x^9 + \cdots + a_{10} x + b$$

What Is the Probability of a Model?

$$\mathbf{P}(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

$$\boxed{a_1} x + b$$

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$$\boxed{a_1} x^2 + \boxed{a_2} x + b = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

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$$\boxed{a_1} x^{10} + \boxed{a_2} x^9 + \cdots + \boxed{a_{10}} x + b$$

What Is the Probability of a Model?

$$\mathbf{P}(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

$$\boxed{a_1} x + b$$

$\mathbf{P}$(Model 2)

$$\boxed{a_1} x^2 + \boxed{a_2} x + b = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

$\mathbf{P}$(Model 3)

$$\boxed{a_1} x^{10} + \boxed{a_2} x^9 + \cdots + \boxed{a_{10}} x + b = \prod_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_i^2}$$

Bayes and Regularization

Bayes and Regularization

Bayes and Regularization

Bayes and Regularization

$\mathbf{P}(\text{Data}|\text{Model})$

Bayes and Regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$\mathbf{P}(\text{Model})$

Bayes and Regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$\mathbf{P}(\text{Model})$

Bayes and Regularization

P(Data|Model)

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_5^2}$$

P(Model)

Bayes and Regularization

P(Data|Model)

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_5^2}$$

P(Model)

$$a_1x^2 + a_2x + b$$

Bayes and Regularization

P(Data|Model)

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_5^2}$$

P(Model)

$$\boxed{a_1}x^2 + \boxed{a_2}x + b$$

Bayes and Regularization

P(Data|Model) **P**(Model)

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_5^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}a_1^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}a_2^2}$$

$$\boxed{a_1}x^2 + \boxed{a_2}x + b$$

Bayes and Regularization

P(Data|Model) **P**(Model)

$$\cancel{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2}} \cdot \cancel{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2}} \cdot \cancel{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_3^2}} \cdot \cancel{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_4^2}} \cdot \cancel{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_5^2}} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}a_2^2}$$

$$\boxed{a_1}x^2 + \boxed{a_2}x + b$$

Bayes and Regularization

P(Data|Model) **P**(Model)

$$\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}a_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}a_2^2}$$

$$\boxed{a_1}x^2 + \boxed{a_2}x + b$$

Bayes and Regularization

P(Data|Model)

$$\underbrace{\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}}_{\boxed{\text{log}}} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}a_1^2} \cdot \cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}a_2^2}$$

P(Model)

$$-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

$$\boxed{a_1}x^2 + \boxed{a_2}x + b$$

Bayes and Regularization

P(Data|Model)

$$\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}}_{\boxed{\text{log}}}$$

$$-\frac{1}{2}(d_1^2+d_2^2+d_3^2+d_4^2+d_5^2)$$

P(Model)

$$\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_2^2}}_{\boxed{\text{log}}}$$

$$-\frac{1}{2}(a_1^2+a_2^2)$$

$$\boxed{a_1}x^2+\boxed{a_2}x+b$$

Bayes and Regularization

P(Data|Model) **P**(Model)

$$\underbrace{\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2}\cdot\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2}\cdot\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2}\cdot\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2}\cdot\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}}_{\boxed{\text{log}}}\cdot\underbrace{\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}a_1^2}\cdot\cancel{\frac{1}{\sqrt{2\pi}}}e^{-\frac{1}{2}a_2^2}}_{\boxed{\text{log}}}$$

$$-\frac{1}{2}(d_1^2+d_2^2+d_3^2+d_4^2+d_5^2) \quad + \quad -\frac{1}{2}(a_1^2+a_2^2)$$

$$\boxed{a_1}x^2+\boxed{a_2}x+b$$

Bayes and Regularization

P(Data|Model) **P**(Model)

$$\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}}_{\boxed{\text{log}}}\cdot\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_2^2}}_{\boxed{\text{log}}}$$

Maximize

$$-\frac{1}{2}(d_1^2+d_2^2+d_3^2+d_4^2+d_5^2) \quad \overset{\boxed{\text{log}}}{+} \quad -\frac{1}{2}(a_1^2+a_2^2)$$

$$\boxed{a_1}x^2+\boxed{a_2}x+b$$

Bayes and Regularization

P(Data|Model) **P**(Model)

$$\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}}_{\boxed{\text{log}}}\cdot\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_2^2}}_{\boxed{\text{log}}}$$

Maximize

$$-\frac{1}{2}(d_1^2+d_2^2+d_3^2+d_4^2+d_5^2) \quad + \quad -\frac{1}{2}(a_1^2+a_2^2)$$

Minimize

$$\boxed{a_1}x^2+\boxed{a_2}x+b$$

Bayes and Regularization

P(Data|Model) **P**(Model)

$$\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_2^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_3^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_4^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}d_5^2}}_{\boxed{\log}}\cdot\underbrace{\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_1^2}\cdot\frac{1}{\cancel{\sqrt{2\pi}}}e^{-\frac{1}{2}a_2^2}}_{\boxed{\log}}$$

$$\boxed{a_1}x^2 + \boxed{a_2}x + b$$

Maximize

$$-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2) \quad + \quad -\frac{1}{2}(a_1^2 + a_2^2)$$

Minimize $d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 \quad + \quad a_1^2 + a_2^2$

Regularization

Regularization

Regularization

$\mathbf{P}$(Model 1)

$\mathbf{P}$(Model 2)

$\mathbf{P}$(Model 3)

Regularization

$\mathbf{P}$(Model 1)

Minimize x_1^2

$\mathbf{P}$(Model 2)

Minimize $x_1^2 + x_2^2$

$\mathbf{P}$(Model 3)

Minimize $x_1^2 + \cdots + x_{10}^2$

Regularization

$\mathbf{P}$(Model 1)

Minimize x_1^2

$\mathbf{P}$(Data|Model 1)

Model 1

$\mathbf{P}$(Model 2)

Minimize $x_1^2 + x_2^2$

$\mathbf{P}$(Data|Model 2)

Model 2

$\mathbf{P}$(Model 3)

Minimize $x_1^2 + \cdots + x_{10}^2$

$\mathbf{P}$(Data|Model 3)

Model 3

Regularization

$\mathbf{P}$(Model 1)

Minimize x_1^2

$\mathbf{P}$(Data|Model 1)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Model 1

$\mathbf{P}$(Model 2)

Minimize $x_1^2 + x_2^2$

$\mathbf{P}$(Data|Model 2)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Model 2

$\mathbf{P}$(Model 3)

Minimize $x_1^2 + \cdots + x_{10}^2$

$\mathbf{P}$(Data|Model 3)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Model 3

Regularization

New Loss

P(Model 1)

Minimize x_1^2

P(Data|Model 1)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Model 1

P(Model 2)

Minimize $x_1^2 + x_2^2$

P(Data|Model 2)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Model 2

P(Model 3)

Minimize $x_1^2 + \cdots + x_{10}^2$

P(Data|Model 3)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Model 3

Regularization

New Loss

$\mathbf{P}$(Model 1) $\mathbf{P}$(Data|Model 1)

Minimize $x_1^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$

$\mathbf{P}$(Model 2) $\mathbf{P}$(Data|Model 2)

Minimize $x_1^2 + x_2^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$

$\mathbf{P}$(Model 3) $\mathbf{P}$(Data|Model 3)

Minimize $x_1^2 + \cdots + x_{10}^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$

Model 1

Model 2

Model 3

DeepLearning.AI

Point Estimation

Conclusion