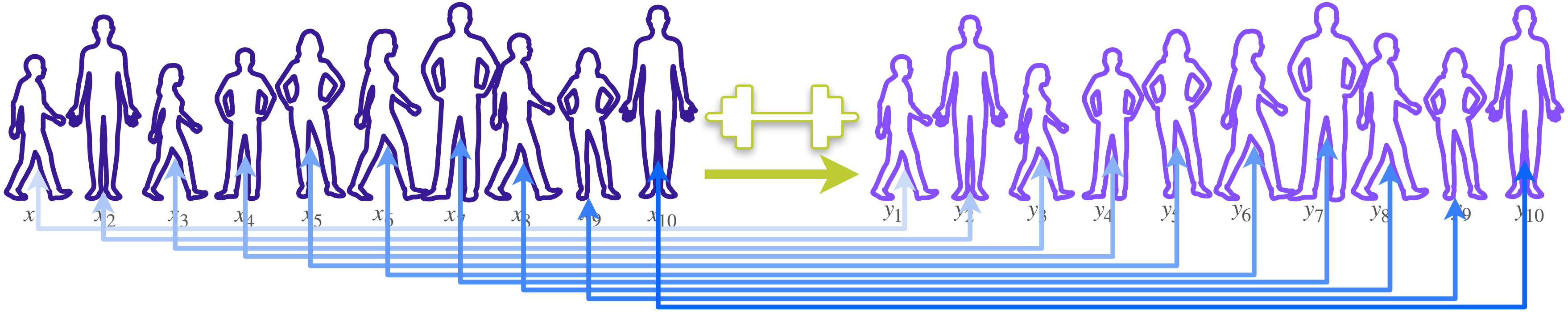
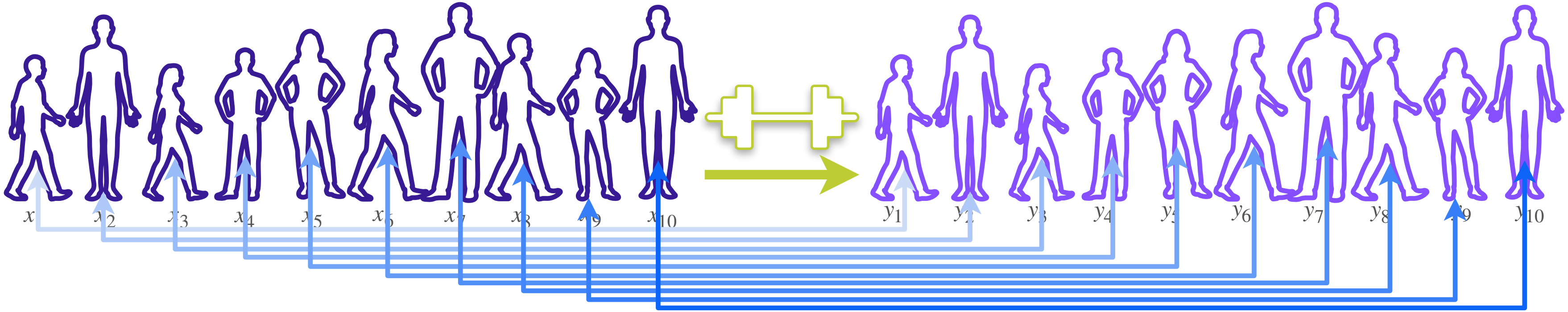
Paired t-Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

Paired *t*-Test: Statistic

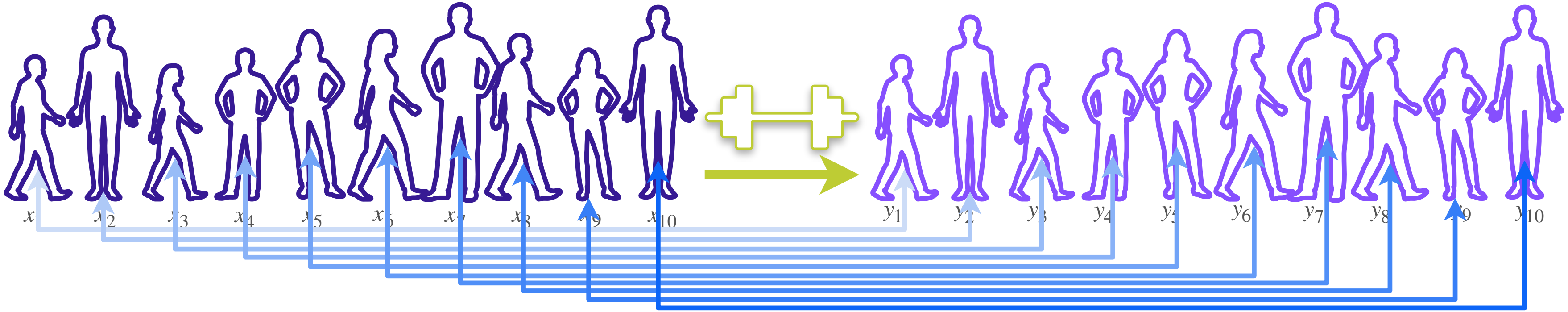


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

D_1

Paired t-Test: Statistic

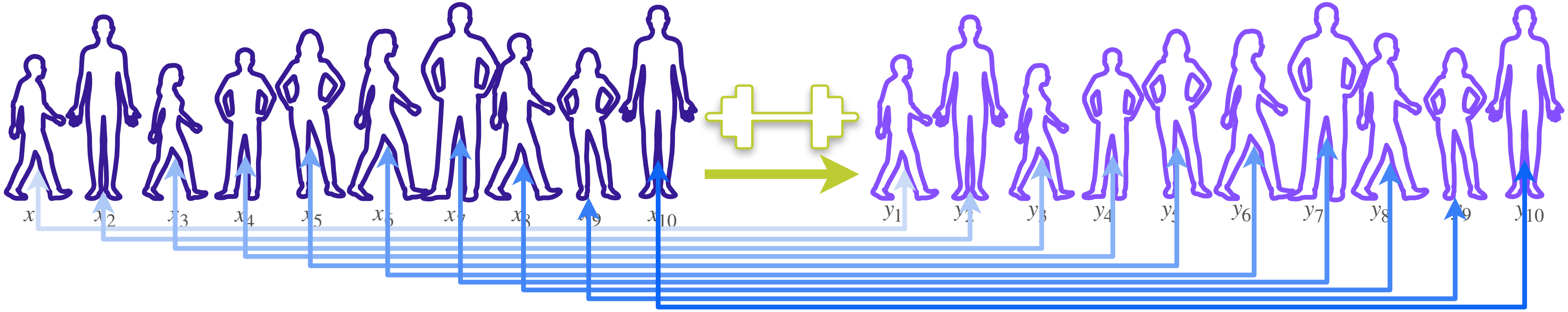


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

D_1 D_2

Paired *t*-Test: Statistic

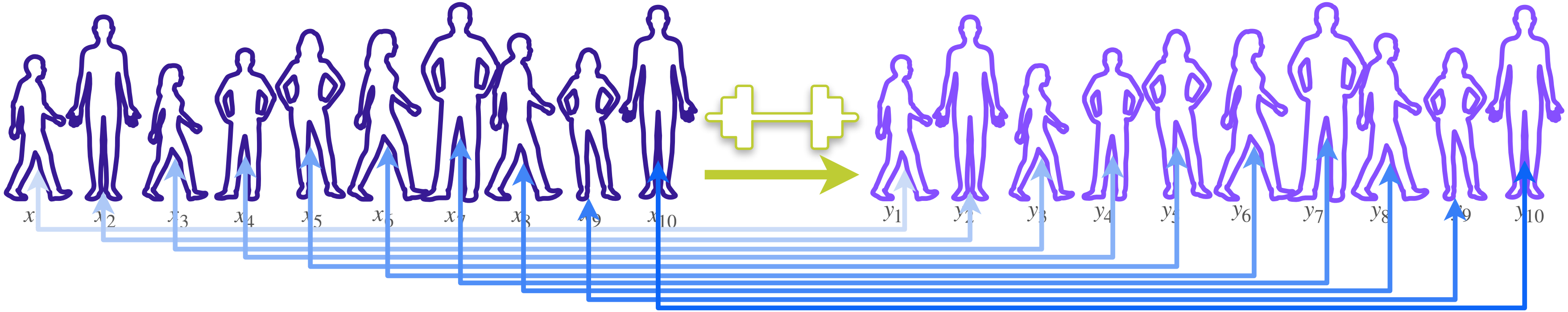


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired *t*-Test: Statistic



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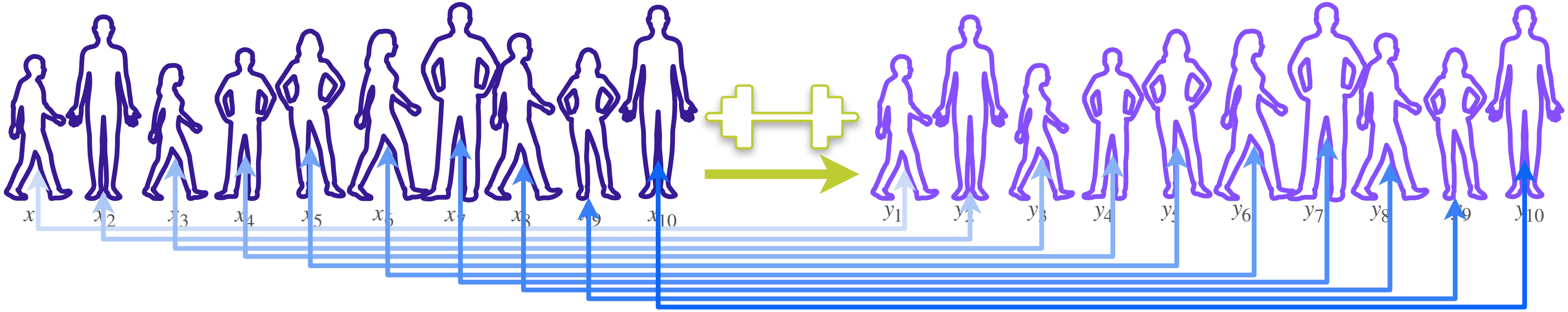
$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired t-Test: Statistic

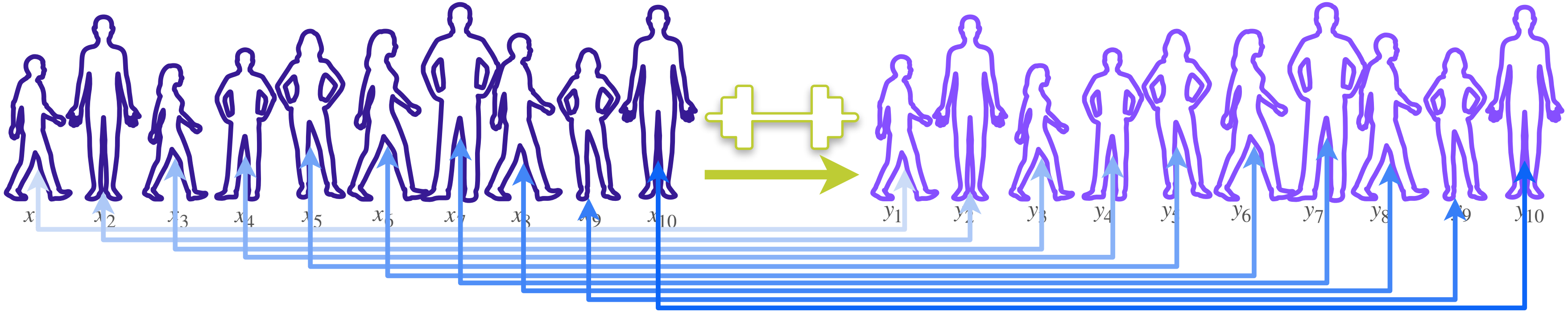


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

$$\overline{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired *t*-Test: Statistic



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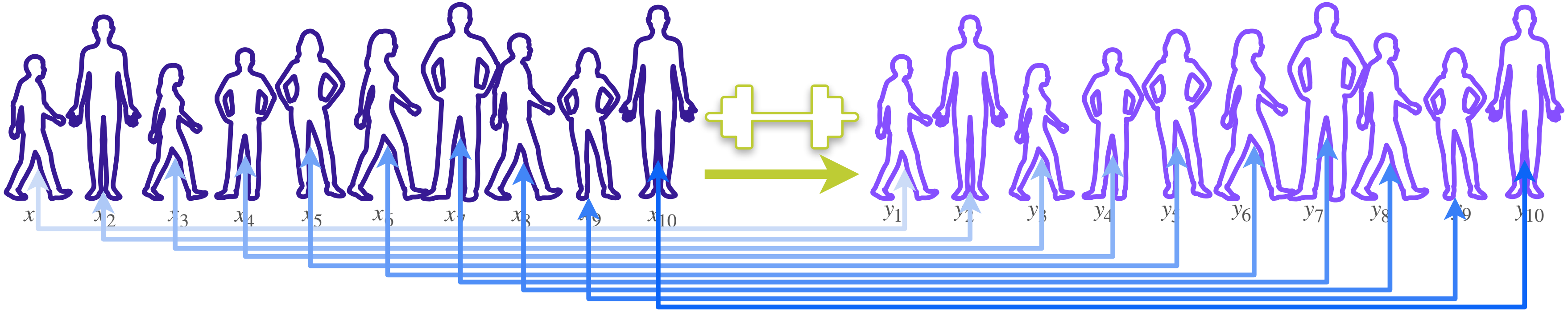
Paired t-Test: Statistic



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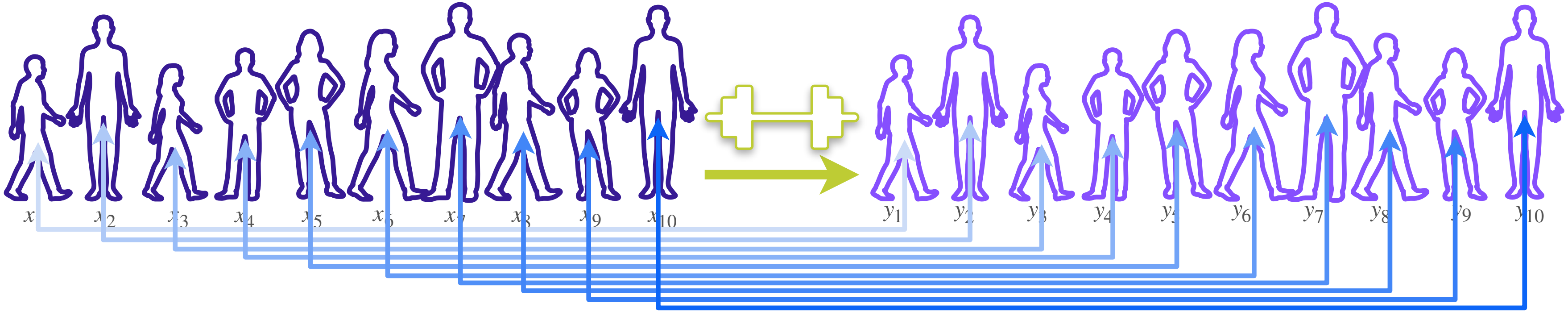
$$\overline{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

If X_i, Y_i are gaussian $\Rightarrow D_i = X_i - Y_i$ is gaussian.

$$D_i \overset{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

Paired t-Test: Statistic



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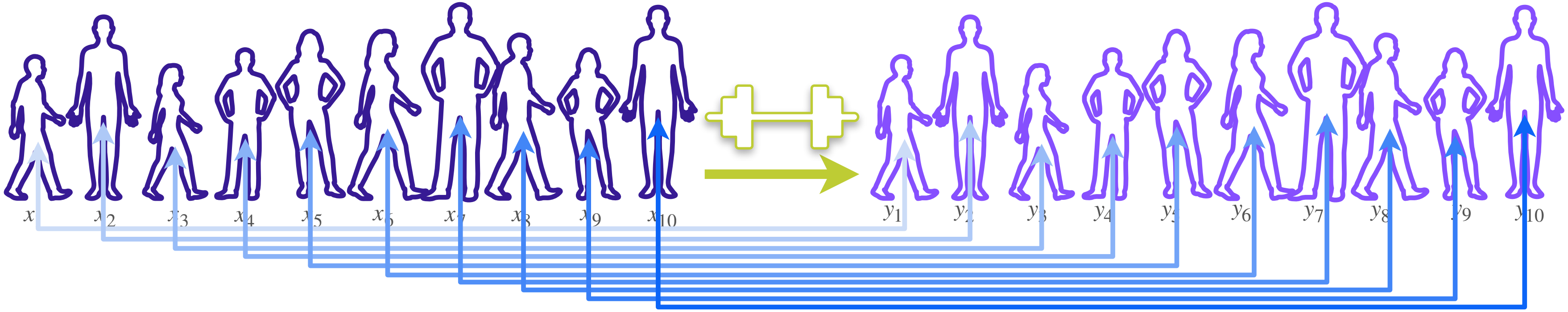

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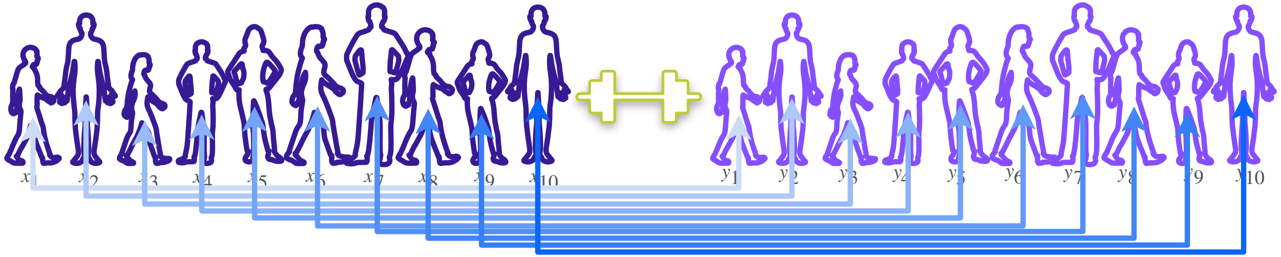

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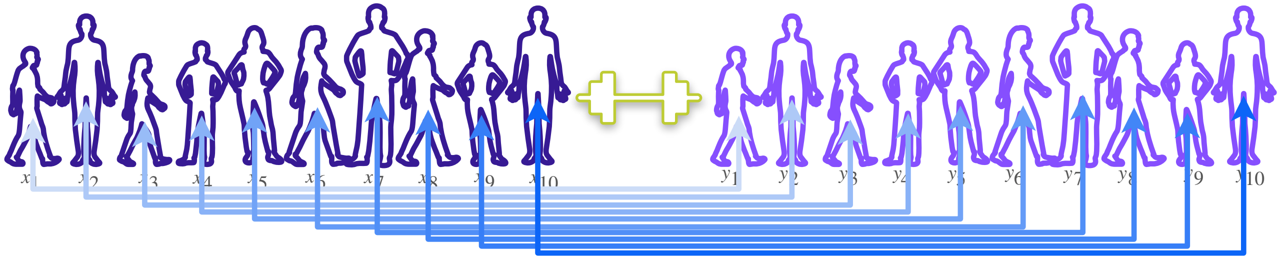
Paired t-Test: Statistic



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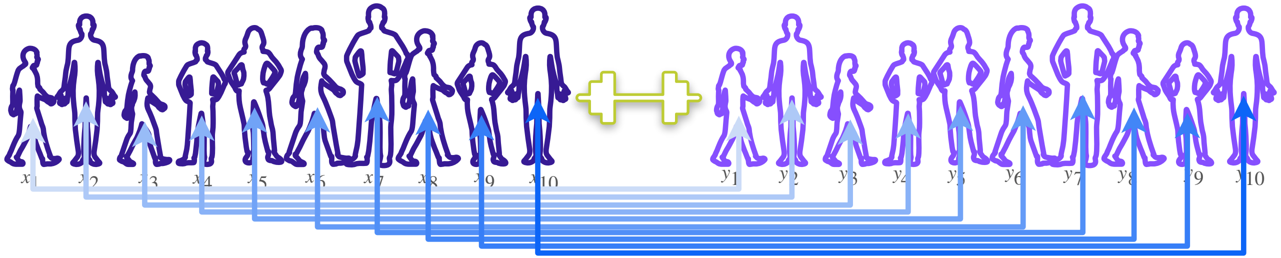
$$D_i = X_i - Y_i$$

$$D_i \overset{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

$$\frac{\overline{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

Paired t-Test: Statistic



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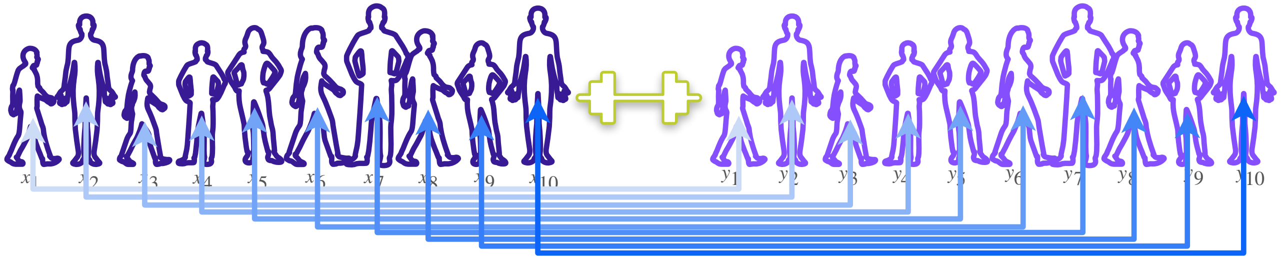
But σ_D is unknown

Paired t-Test: Statistic



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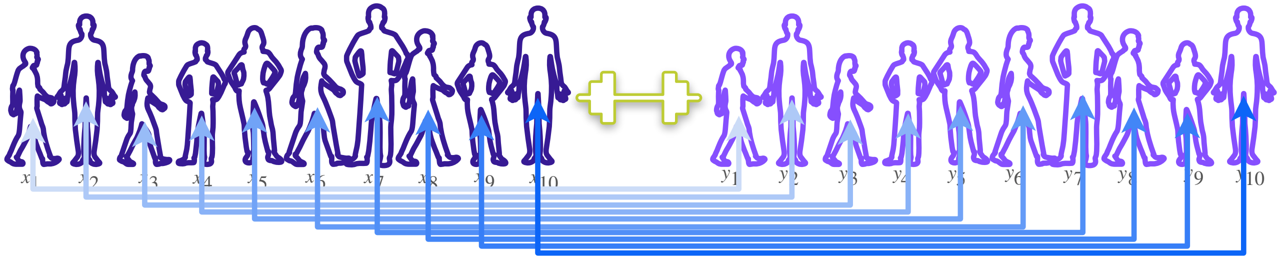
$$\frac{\overline{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

But σ_D is unknown $\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\dfrac{\sum_{i=1}^{10} (D_i - \overline{D})^2}{10 - 1}}$

Paired t-Test: Statistic



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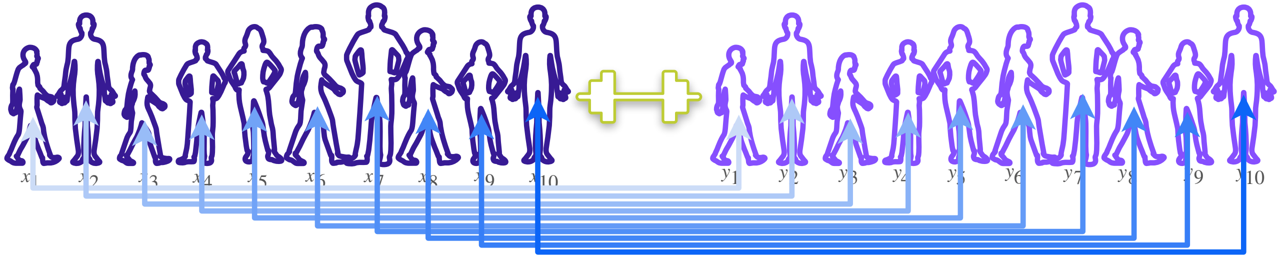
Paired t-Test: Statistic



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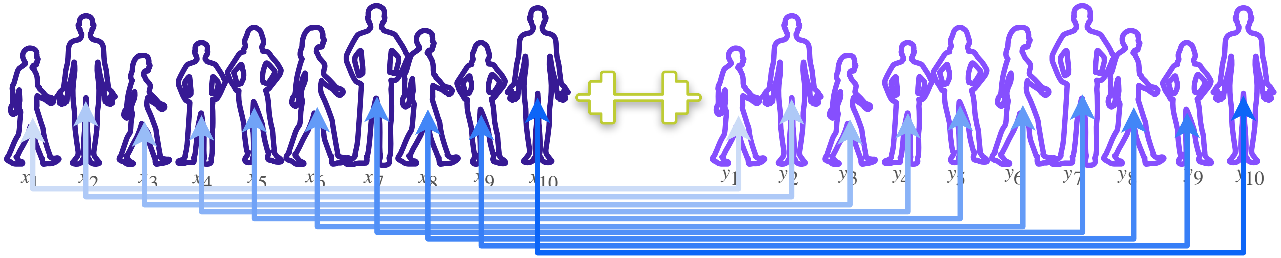
Paired t-Test: Statistic



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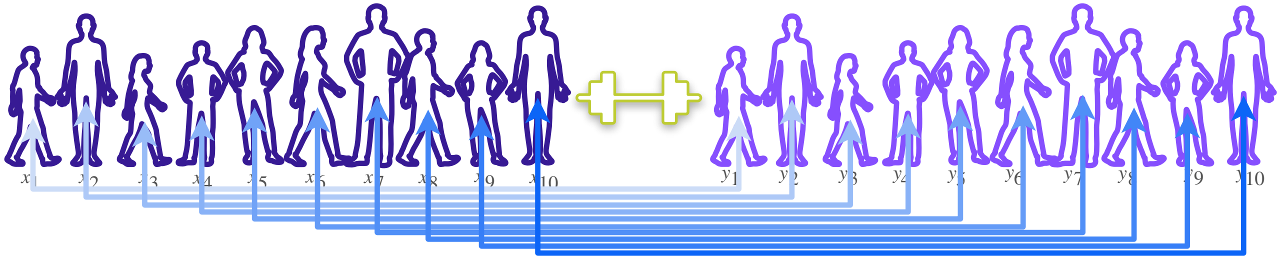
$H_0 : \mu_D = 0$

Paired t-Test: Statistic



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$D_i \overset{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$

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$H_0 : \mu_D = 0$

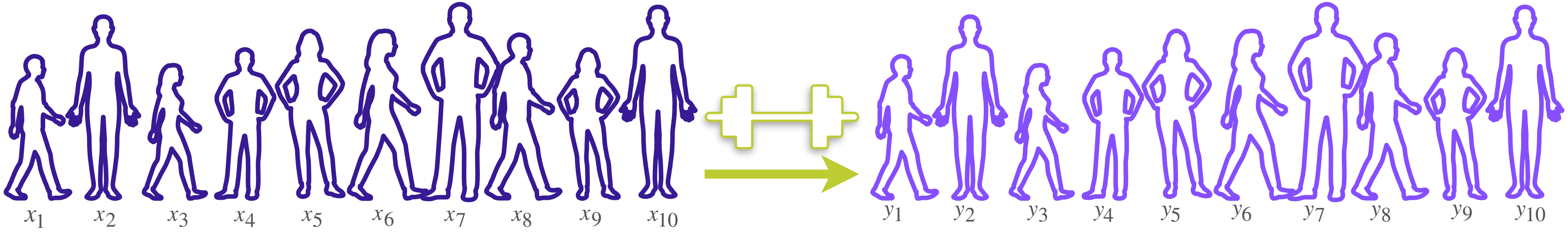
Test statistic

Paired t-Test: Observations

$=$

d_i

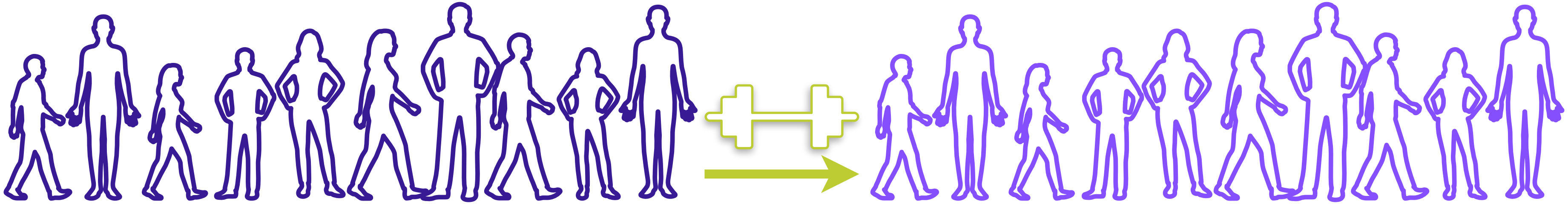
Paired t-Test: Observations



$=$

d_i

Paired t-Test: Observations



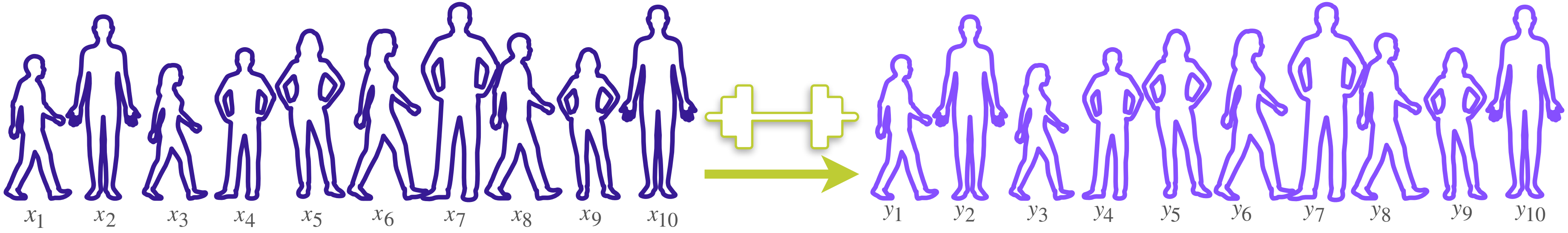
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

$=$

d_i

Paired t-Test: Observations



251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

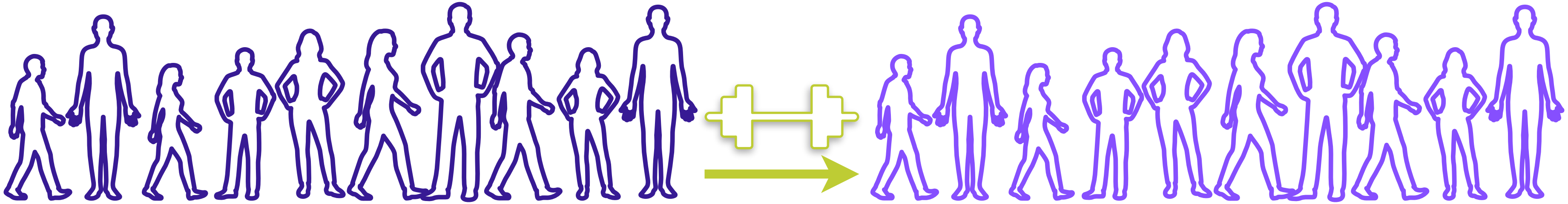
251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

$=$

d_i

Paired t-Test: Observations



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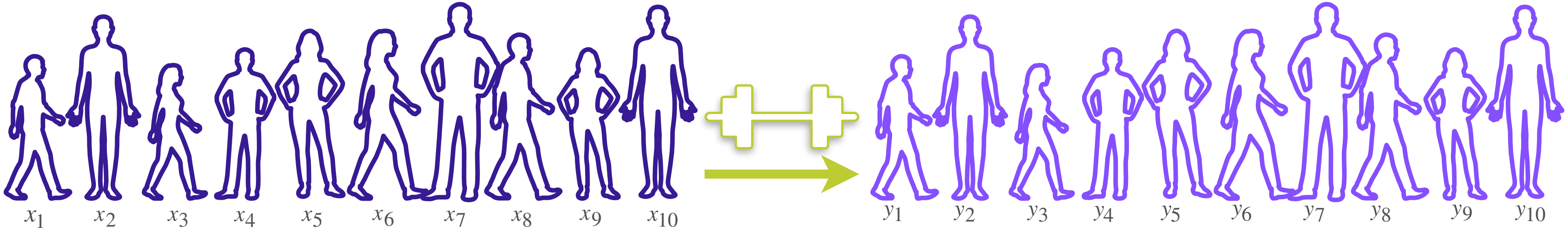
$=$

d_i

Paired t-Test: Observations



251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

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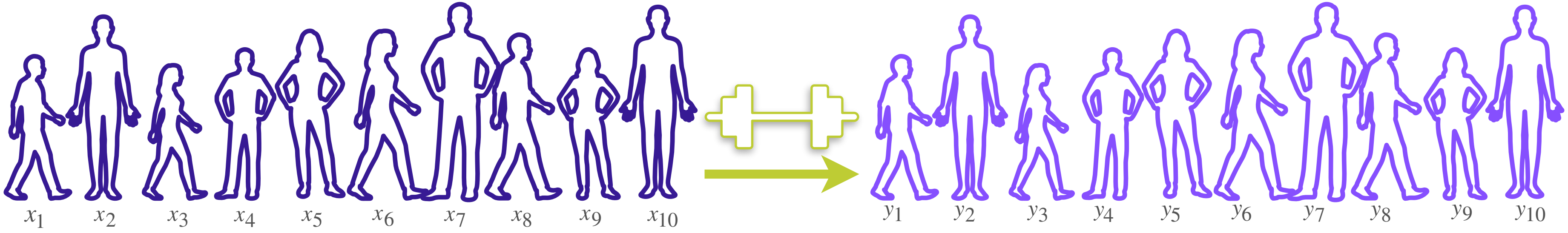
x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0
					$-$					
y_i	251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4
					$=$					
d_i										

Paired t-Test: Observations



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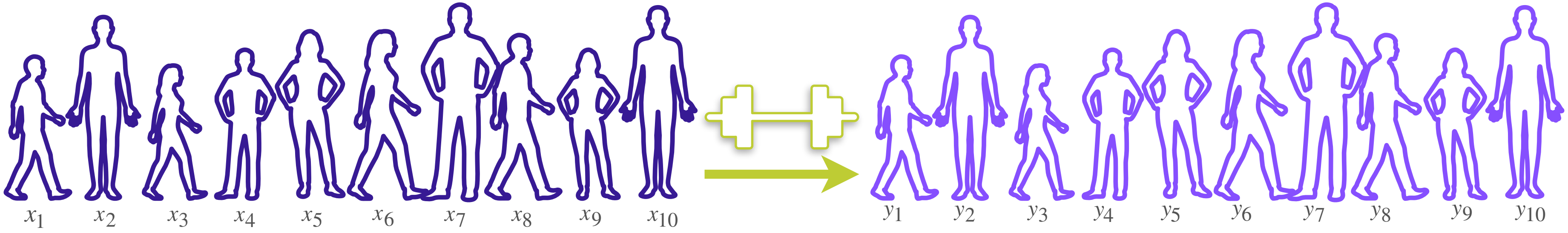
x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0
					$-$					
y_i	251.9	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4
					$=$					
d_i	0.4									

Paired t-Test: Observations



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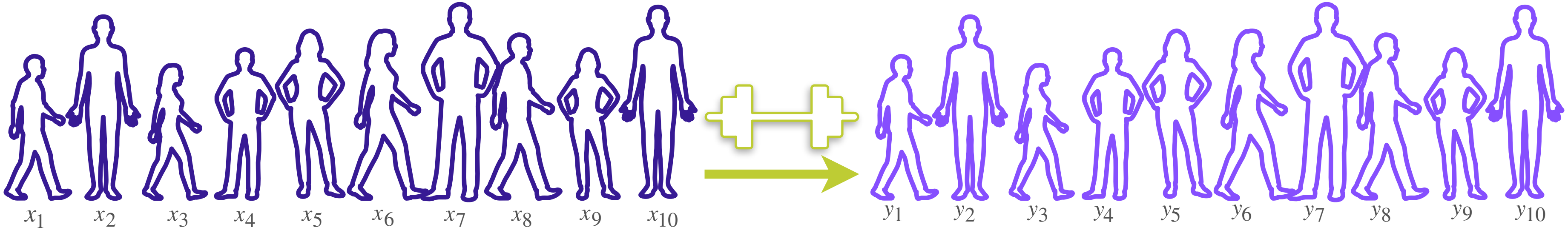
251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0
					$-$					
y_i	251.9	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4
					$=$					
d_i	0.4	1.3	2.7	1.1	1.2	-1.3	2.8	1.9	2.2	-1.4

Paired t-Test: Observations



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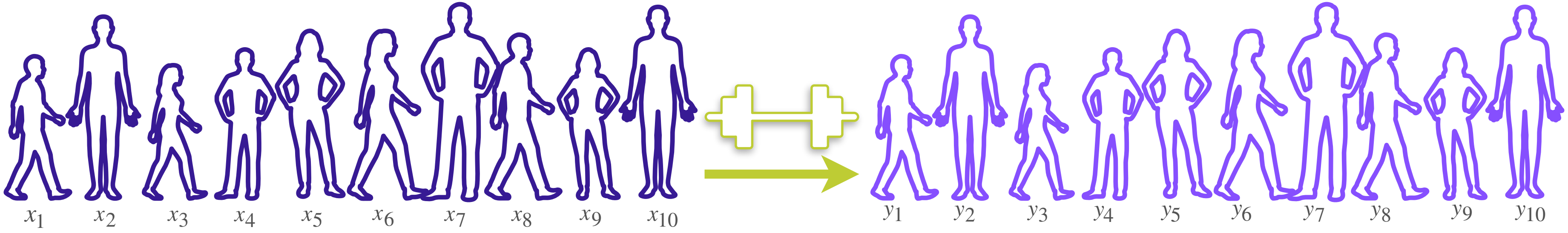
$$\bar{d} = 1.09$$

$$s_D = 1.485$$

Paired t-Test: Observations



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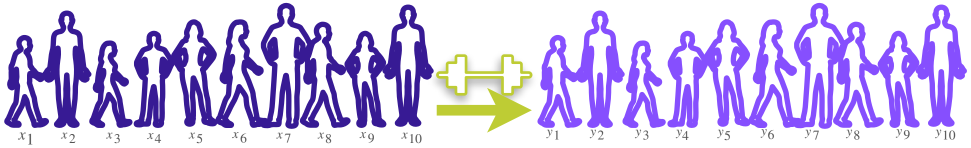
d_i	0.4	1.3	2.7	1.1	1.2	-1.3	2.8	1.9	2.2	-1.4

$$\bar{d} = 1.09$$

$$s_D = 1.485$$

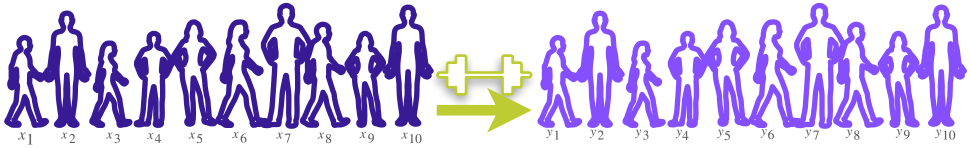
$$t = \frac{1.09 - 0}{1.485/\sqrt{10}} = 2.321$$

Independent Two-Sample t-Test: Right Tailed Test



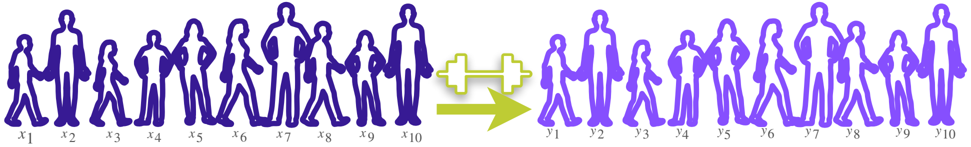
$\bar{d} = 1.09$ $\quad s_D = 1.485$ $\quad n = 10$

Independent Two-Sample t-Test: Right Tailed Test



$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\ =\ 2.321}$$

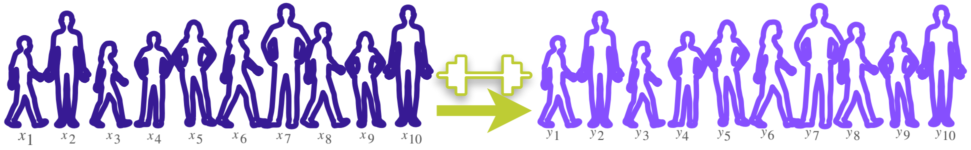
Independent Two-Sample t-Test: Right Tailed Test



$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

Independent Two-Sample t-Test: Right Tailed Test

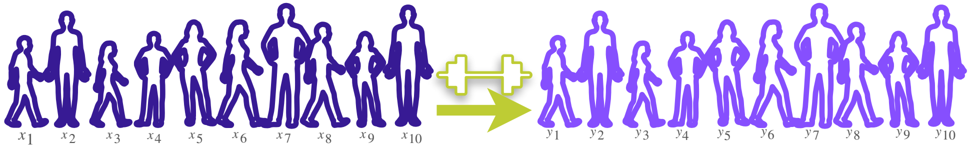


$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\,=\,2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$\alpha = 0.05$

Independent Two-Sample t-Test: Right Tailed Test

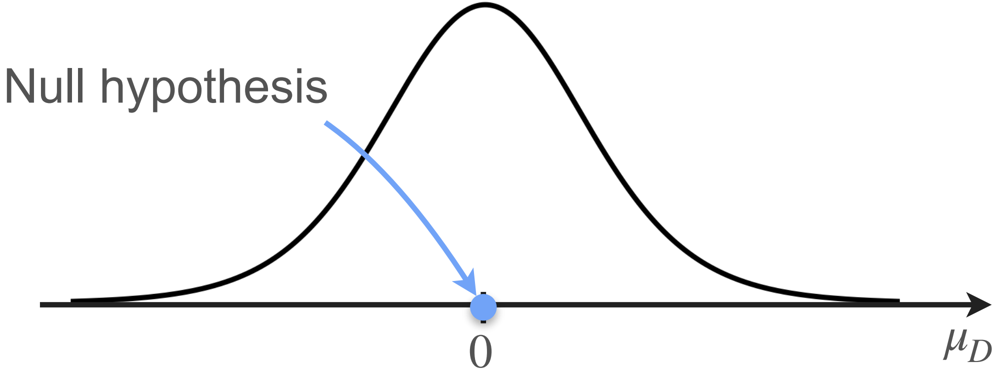


$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

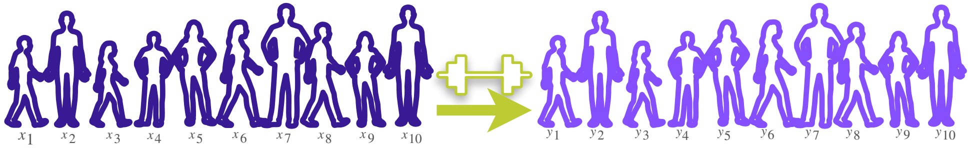
$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$



Independent Two-Sample t-Test: Right Tailed Test



$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

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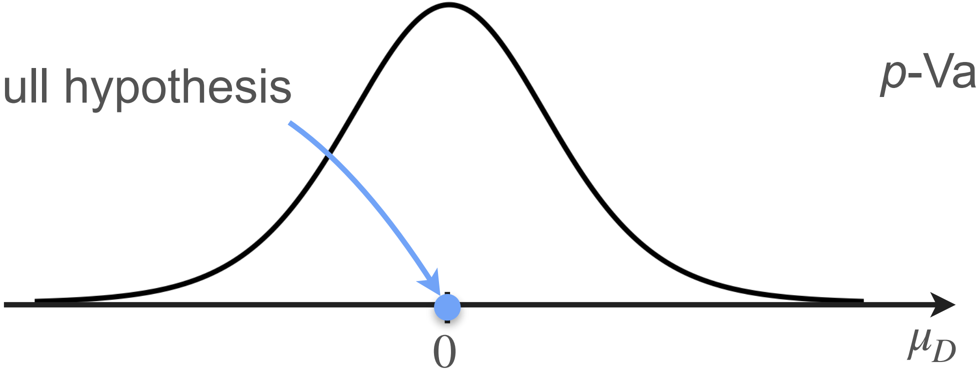
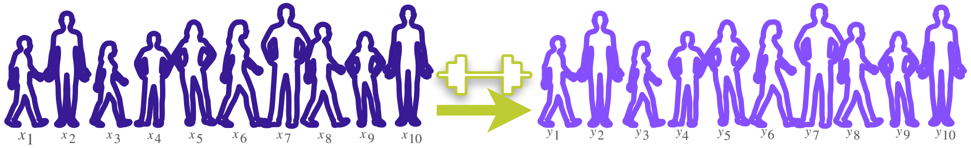


p-Value:

Independent Two-Sample t-Test: Right Tailed Test



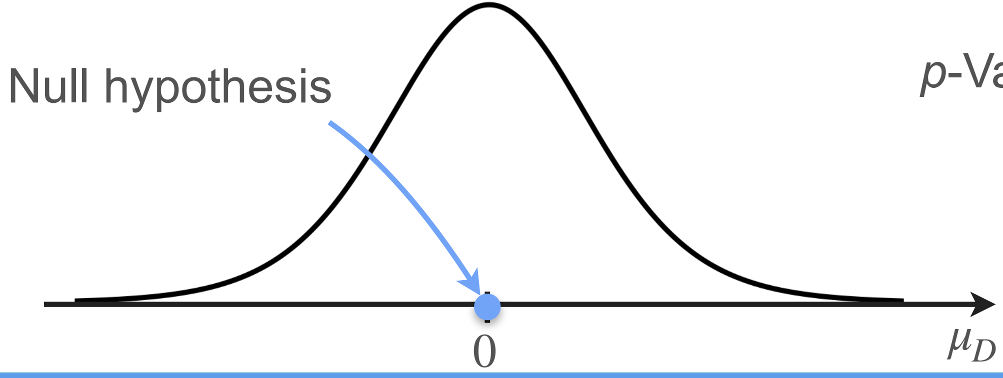
$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

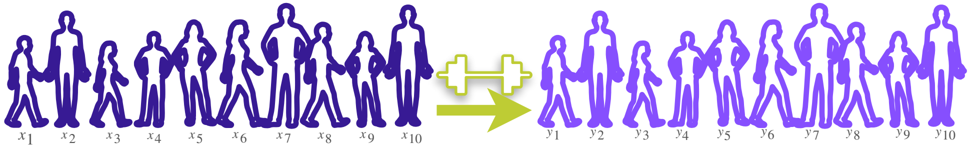
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\overline{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$

p-Value: $\mathbf{P}\left(T > 2.321 \;|\, \mu_D = 0\right)$



Independent Two-Sample t-Test: Right Tailed Test



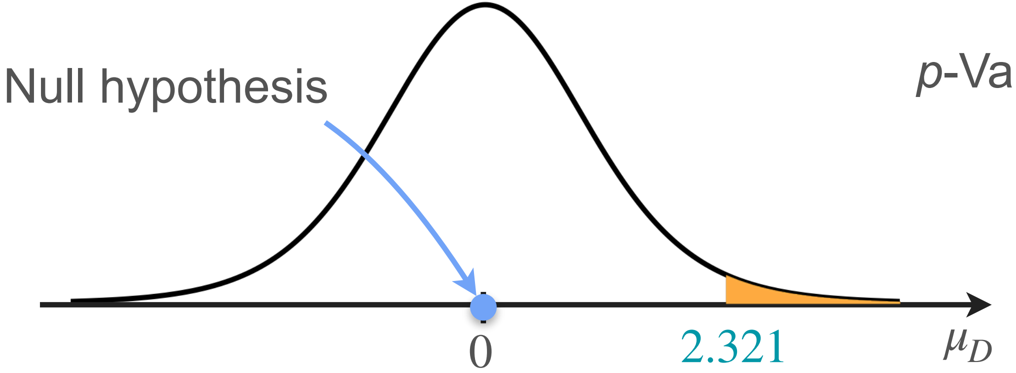
$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\ =\ 2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

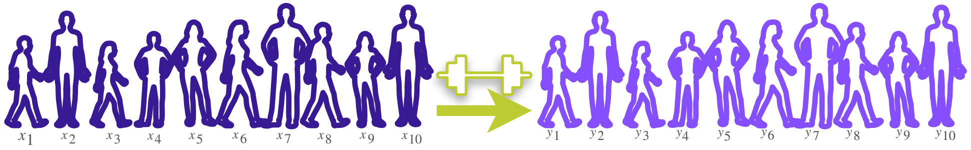
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\overline{D} - 0}{S_D/\sqrt{10}} \sim t_{10-1}$

p-Value: $\mathbf{P}\left(T > 2.321 \mid \mu_D = 0\right)$



Independent Two-Sample t-Test: Right Tailed Test

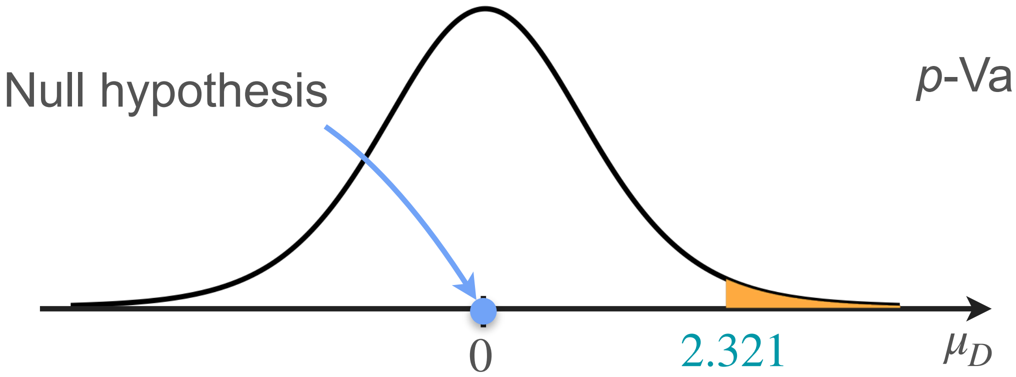


$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\, =\, 2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$\alpha = 0.05$

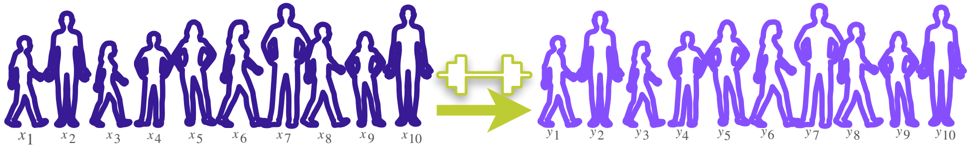
If H_0 is true: $T = \dfrac{\overline{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$



p-Value: $\mathbf{P}\left(T > 2.321 \mid \mu_D = 0\right)$

$= 0.0227$

Independent Two-Sample t-Test: Right Tailed Test

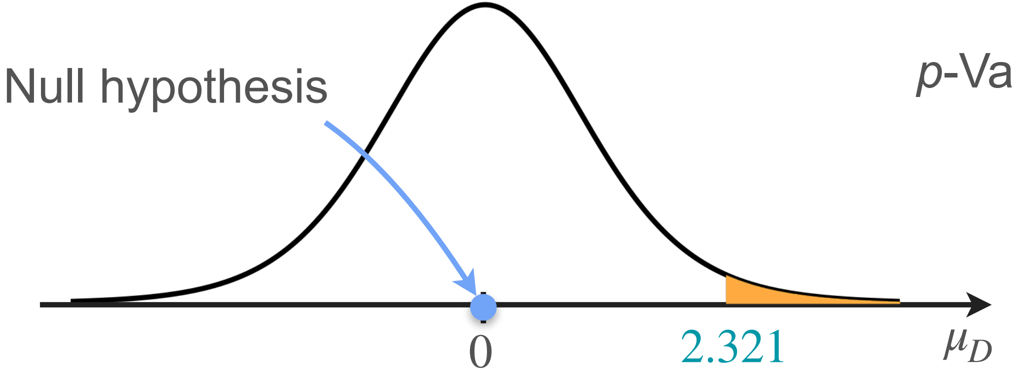


$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\,=\,2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$



p-Value: $\mathbf{P}\left(T > 2.321 \mid \mu_D = 0\right)$

$= 0.0227 < 0.05$

$\Rightarrow$ Reject H_0 (and accept H_1)

(with a 5% significance level)

Hypothesis Testing

ML Application: A/B testing

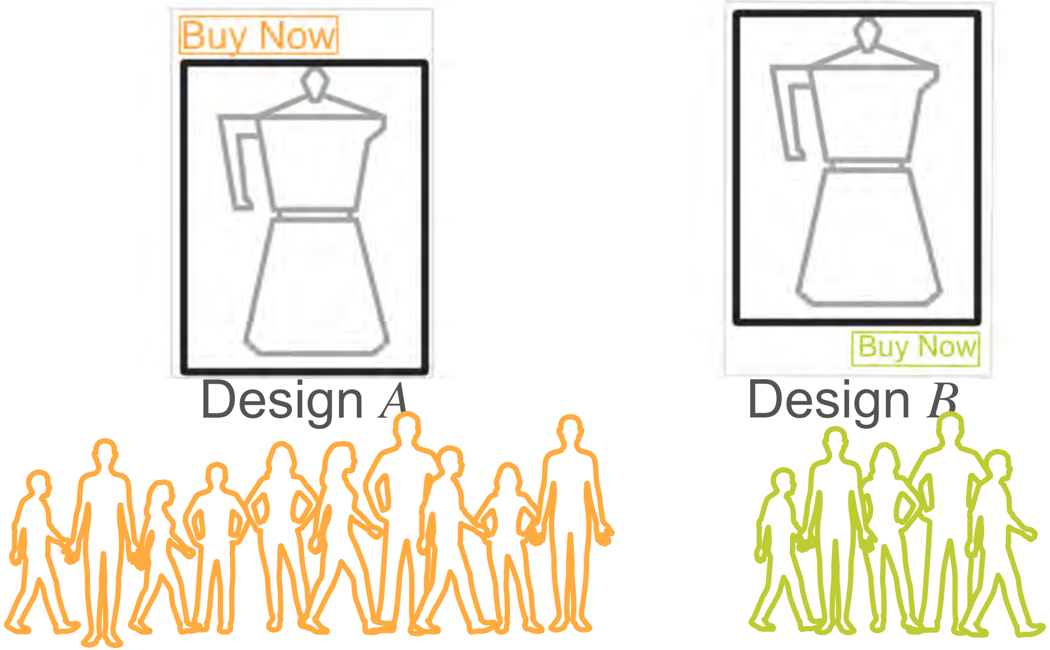
A/B Testing: Purchase Amount

A/B Testing: Purchase Amount

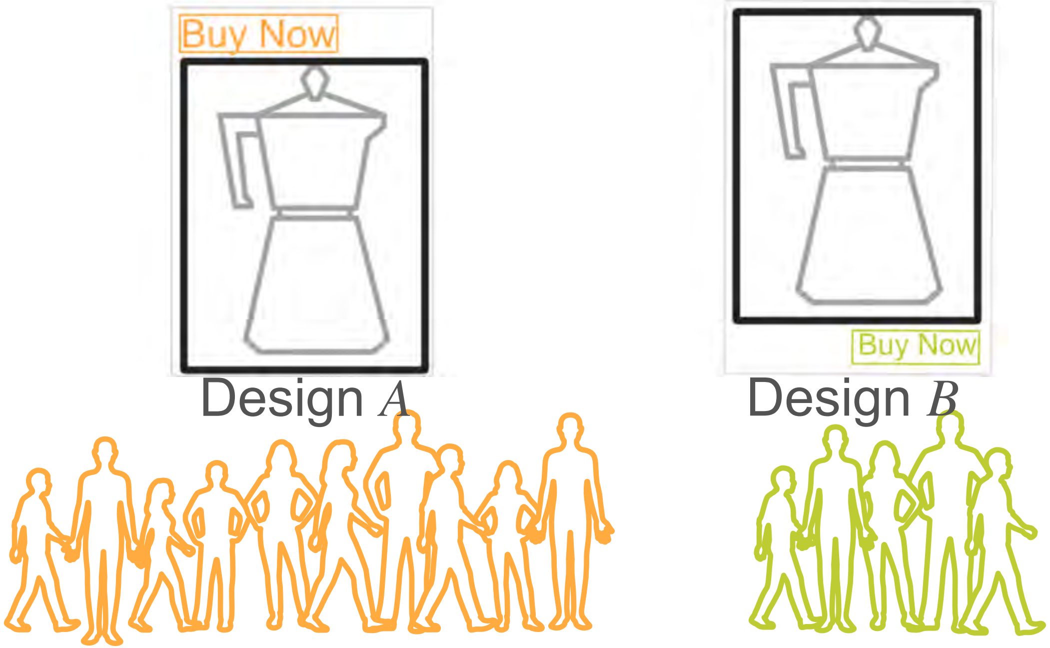
Design A

Design B

A/B Testing: Purchase Amount



A/B Testing: Purchase Amount



A/B Testing: Purchase Amount

Design A

$n_A = 80$

Design B

$n_B = 20$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad n_B = 20$

$s_A = 10$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$

$s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$

$s_B = 15$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad n_B = 20$ $\quad \bar{y} = 55$

$s_A = 10$ $\quad s_B = 15$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\ ^2}{\quad} + \frac{\ ^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\bar{x} = 50$ $s_A = 10$

$n_B = 20$ $\bar{y} = 55$ $s_B = 15$

$H_0 : \mu_A = \mu_B$ vs. $H_1 : \mu_A < \mu_B$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\ ^2}{\quad} + \frac{\ ^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\ ^2}{\quad} + \frac{\ ^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\quad^2}{\quad} + \frac{\quad^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\qquad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\quad^2}{\quad} + \frac{\quad^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

$n_A = 80$ $\quad \bar{x} = 50$

$s_A = 10$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$

Design B

$n_B = 20$ $\quad \bar{y} = 55$

$s_B = 15$

$Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

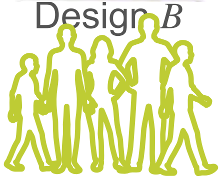
If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\ ^2}{\ } + \frac{\ ^2}{\ }}}$$

A/B Testing: Purchase Amount

Design A

Design B



$n_A = 80$ $\bar{x} = 50$ $n_B = 20$ $\bar{y} = 55$

$s_A = 10$ $s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\dfrac{10^2}{80} + \dfrac{15^2}{20}}}$$

-1.482

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50 \quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55 \quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\qquad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

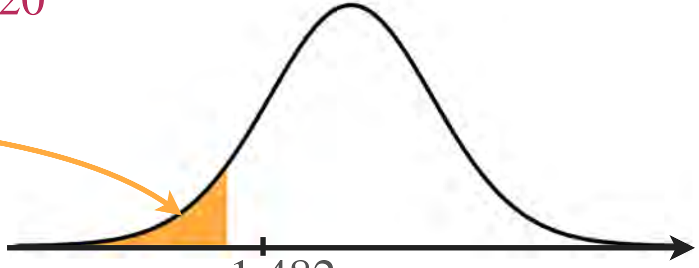
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\bar{X} - \bar{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}}$$

-1.482

p-Value:



A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\bar{x} = 50$

$s_A = 10$

$n_B = 20$ $\bar{y} = 55$

$s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$

$Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

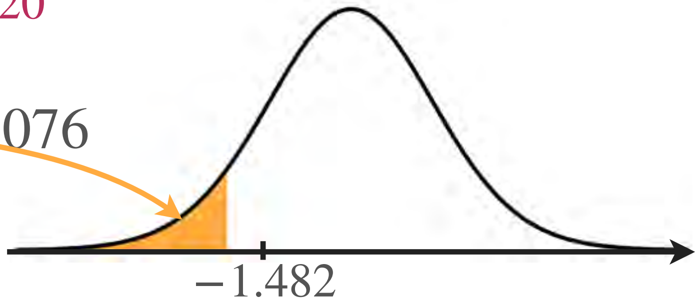
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\dfrac{10^2}{80} + \dfrac{15^2}{20}}}$$

-1.482

p-Value: 0.076



A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\qquad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0: \mu_A - \mu_B = 0$ vs. $H_1: \mu_A - \mu_B < 0$

$\alpha = 0.05$

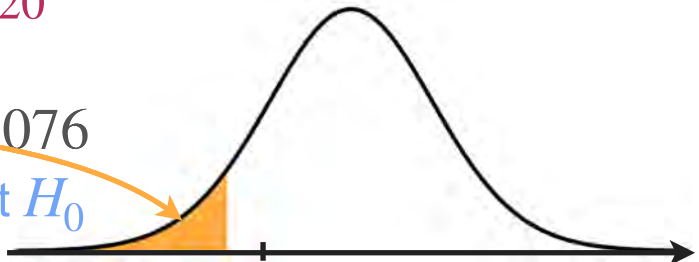
If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\dfrac{10^2}{80} + \dfrac{15^2}{20}}}$$

-1.482

p-Value: 0.076

Don't reject H_0



A/B Testing and t-Tests

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

Propose variations (A/B)

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

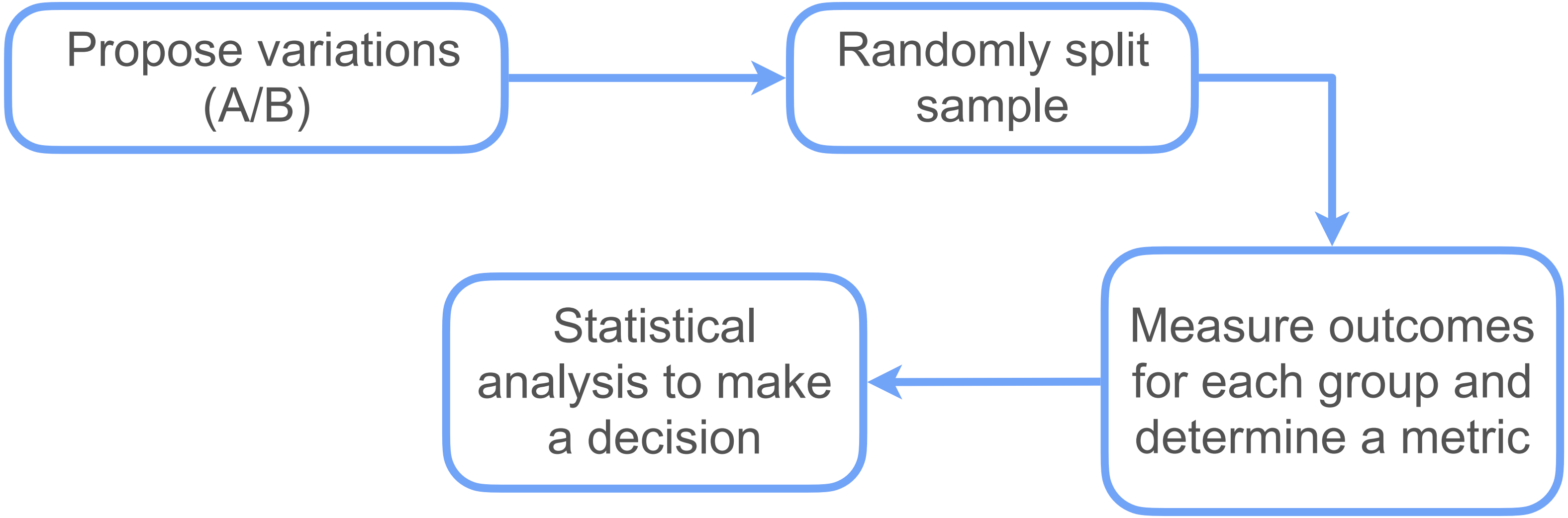
Propose variations (A/B) → Randomly split sample

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

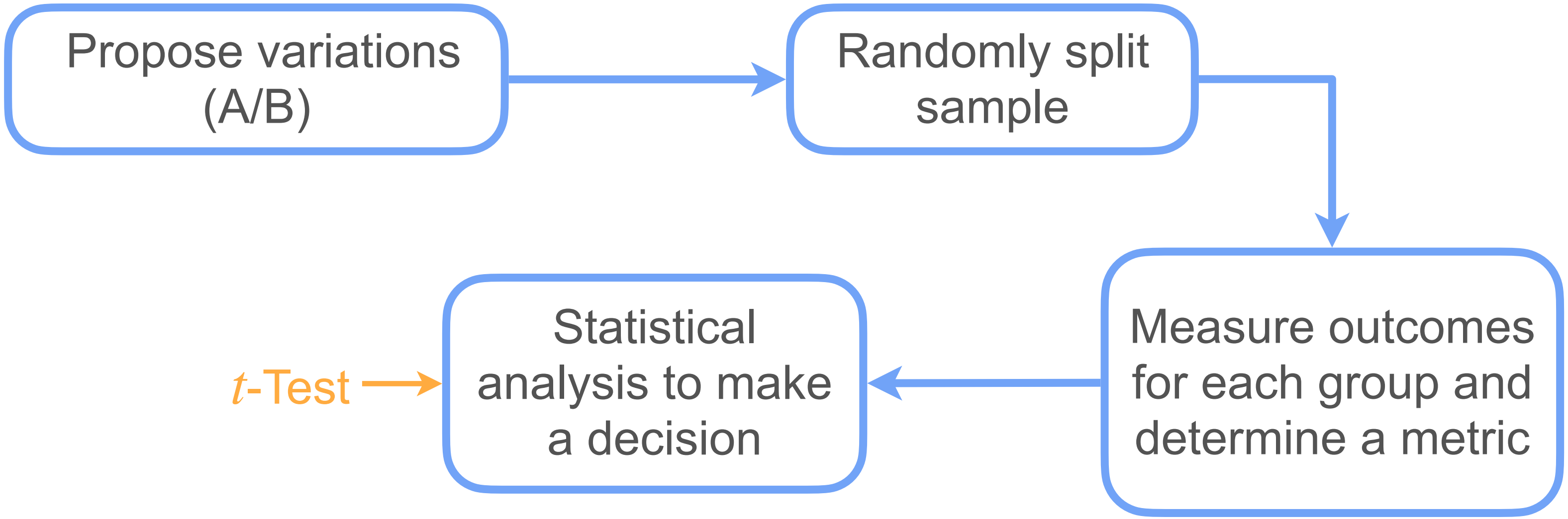
A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)



A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)



A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Design A

Design B

A/B Testing: Conversion Rates

Design A Design B

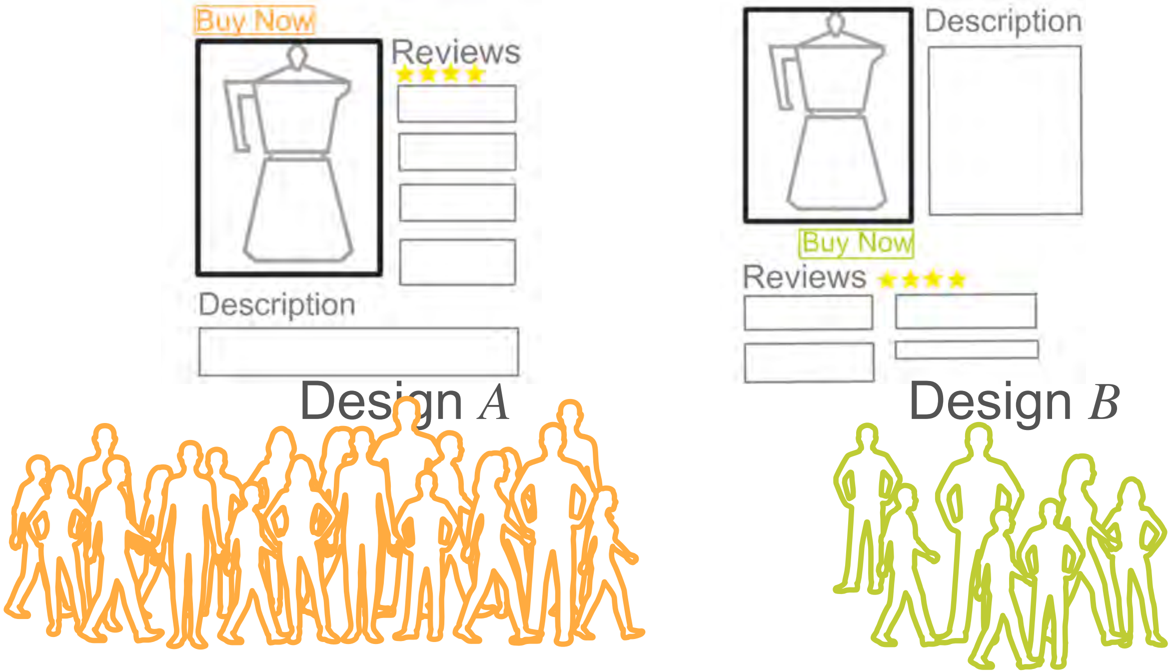
Higher conversion rates?

A/B Testing: Conversion Rates

Design A Design B

Higher conversion rates?

A/B Testing: Conversion Rates



A/B Testing: Conversion Rates

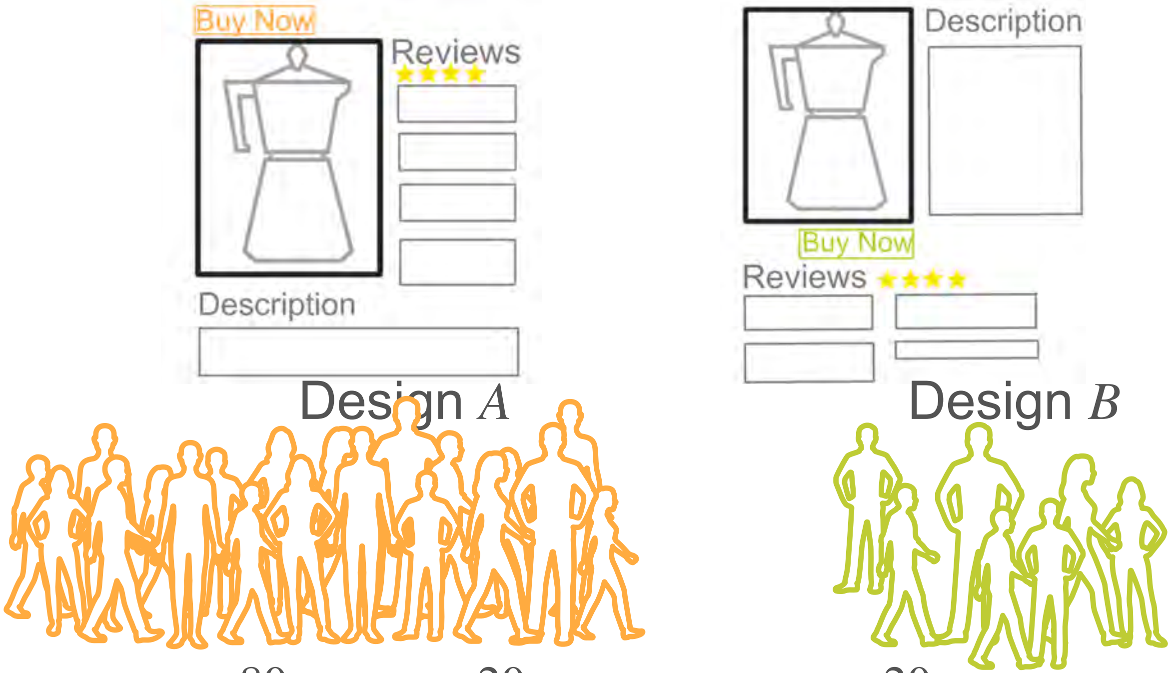
Design A

$n_A = 80$

Design B

$n_B = 20$

A/B Testing: Conversion Rates



$n_A = 80$ $x = 20$ $n_B = 20$

A/B Testing: Conversion Rates

$n_A = 80$ $x = 20$

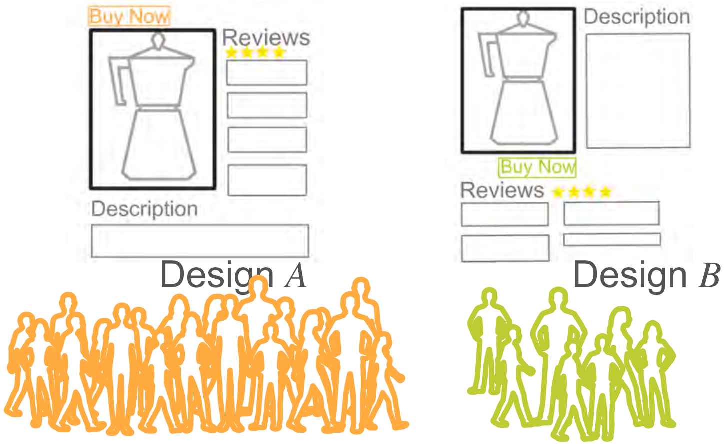
$n_B = 20$ $y = 8$

A/B Testing: Conversion Rates

$$n_A = 80$$
$$x = 20$$

$$n_B = 20$$
$$y = 8$$

A/B Testing: Conversion Rates



$n_A = 80$

$x = 20$

$n_B = 20$

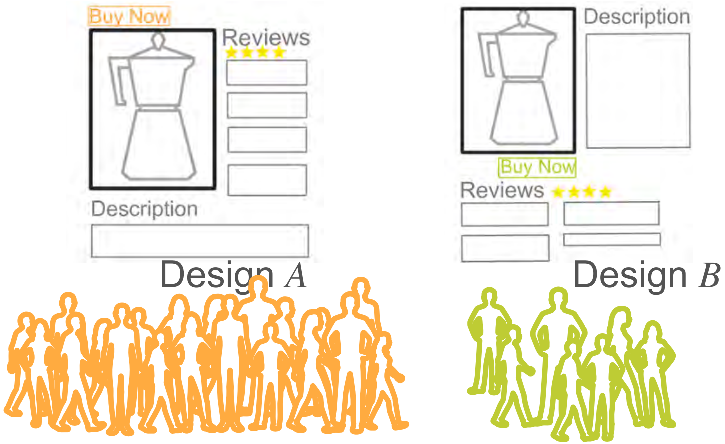
$y = 8$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$p_A =$ Conversion rate from Design A

$p_B =$ Conversion rate from Design B

A/B Testing: Conversion Rates



$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$p_A =$ Conversion rate from Design A

$p_B =$ Conversion rate from Design B

$\alpha = 0.05$

A/B Testing: Conversion Rates

Design A

$n_A = 80$

$x = 20$

Design B

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$p_A =$ Conversion rate from Design A

$p_B =$ Conversion rate from Design B

$\alpha = 0.05$

A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Statistic?

A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \to p_A$$

$$\frac{Y}{n_B} \to p_B$$

A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \to p_A \quad \xrightarrow{\text{C.L.T.}} \quad \frac{X}{n_A} \sim \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

$$\frac{Y}{n_B} \to p_B \quad \longrightarrow \quad \frac{Y}{n_B} \sim \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

A/B Testing: Conversion Rates

Statistic?

$$\frac{X}{n_A} \overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

$$\frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

A/B Testing: Conversion Rates

Statistic?

$$\left.\begin{aligned} \frac{X}{n_A} &\overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right) \\ \\ \frac{Y}{n_B} &\overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right) \end{aligned}\right\}$$

A/B Testing: Conversion Rates

Statistic?

$$\left.\begin{aligned} \frac{X}{n_A} &\overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right) \\[2em] \frac{Y}{n_B} &\overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right) \end{aligned}\right\} \frac{X}{n_A} - \frac{Y}{n_B} \rightarrow p_A - p_B$$

A/B Testing: Conversion Rates

Statistic?

$$\frac{X}{n_A} \overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

$$\frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

$$\frac{X}{n_A} - \frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}\right)$$

$$\frac{X}{n_A} - \frac{Y}{n_B} \to p_A - p_B$$

A/B Testing: Conversion Rates

Statistic?

$$\frac{X}{n_A} \overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

$$\frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

$$\frac{X}{n_A} - \frac{Y}{n_B} \to p_A - p_B$$

$$\frac{X}{n_A} - \frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}\right)$$

$$\Updownarrow$$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0,1)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0,1)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \;(1 - p \;)}{n_A} + \frac{p \;(1 - p \;)}{n_B}}} \overset{a}{\sim} \mathcal{N}\left(0, 1^2\right)$$

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$$= p(1-p)\left(\frac{1}{n_A} + \frac{1}{n_B}\right)$$

A/B Testing: Conversion Rates

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$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\boxed{\frac{p \ (1-p \ )}{n_A} + \frac{p \ (1-p \ )}{n_B}}}} \overset{a}{\sim} \mathcal{N}\left(0,1^2\right)$$

$$= p(1-p)\left(\frac{1}{n_A} + \frac{1}{n_B}\right) = p(1-p)(n_A + n_B)\frac{1}{n_A n_B}$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \quad (1 - p \quad)}{n_A} + \frac{p \quad (1 - p \quad)}{n_B}}} \overset{a}{\sim} \mathcal{N}\left(0, 1^2\right)$$

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A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \ (1-p \ )}{n_A} + \frac{p \ (1-p \ )}{n_B}}} \overset{a}{\sim} \mathcal{N}\left(0,1^2\right) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \overset{a}{\sim} \mathcal{N}\left(0,1^2\right)$$

But you don't know p

$\downarrow$

Replace it by estimation! $\hat{p} = \dfrac{X+Y}{n_A + n_B}$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \ (1 - p \ )}{n_A} + \frac{p \ (1 - p \ )}{n_B}}} \overset{a}{\sim} \mathcal{N}(0, 1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1 - p)}} \sqrt{n_A n_B} \overset{a}{\sim} \mathcal{N}(0, 1^2)$$

But you don't know p

$\downarrow$

Replace it by estimation! $\hat{p} = \dfrac{X + Y}{n_A + n_B}$ $\longrightarrow$

Test statistic

$$\boxed{\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X + Y)\left(1 - \frac{X + Y}{n_A + n_B}\right)}} \sqrt{n_A n_B} \overset{a}{\sim} \mathcal{N}(0, 1^2)}$$

A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Design A

Design B

A/B Testing: Conversion Rates

Design A

Design B

$$n_A = 80$$

$$x = 20$$

A/B Testing: Conversion Rates

$$n_A = 80$$

$$x = 20$$

$$n_B = 20$$

$$y = 8$$

A/B Testing: Conversion Rates

Design A

Design B

$$n_A = 80$$
$$x = 20$$

$$n_B = 20$$
$$y = 8$$

$$X \sim Binomial(n_A, p_A) \quad Y \sim Binomial(n_B, p_B)$$

A/B Testing: Conversion Rates

Design A

$n_A = 80$

$x = 20$

Design B

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

A/B Testing: Conversion Rates

Design A

Design B

$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$

A/B Testing: Conversion Rates

Design A

$n_A = 80$

$x = 20$

Design B

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$

If H_0 is true $\Rightarrow p_A = p_B = p$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A + n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

Design A Design B

$n_A = 80$ $n_B = 20$

$x = 20$ $y = 8$

$X \sim Binomial(n_A, p_A)$ $Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$ If H_0 is true $\Rightarrow p_A = p_B = p$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = \frac{\left(\frac{20}{80} - \frac{8}{20}\right) - 0}{\sqrt{(20 + 8)\left(1 - \frac{20+8}{80+20}\right)}} \sqrt{80\,20}$$

$z = -1.336$

A/B Testing: Conversion Rates

Design A

$n_A = 80$

$x = 20$

Design B

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

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-2.07

A/B Testing: Conversion Rates

Design A Design B

$n_A = 80$ $n_B = 20$

$x = 20$ $y = 8$

$X \sim Binomial(n_A, p_A)$ $Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$ If H_0 is true $\Rightarrow p_A = p_B = p$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A + n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$z = -1.336$

-2.07

A/B Testing: Conversion Rates

Design A

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$z = -1.336$

p-value

-2.07

A/B Testing: Conversion Rates

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$X \sim Binomial(n_A, p_A)$

Design B

$n_B = 20$

$y = 8$

$Y \sim Binomial(n_B, p_B)$

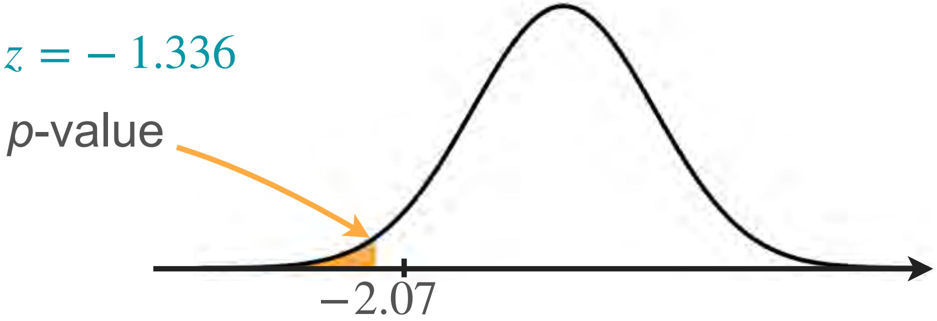
$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$

If H_0 is true $\Rightarrow p_A = p_B = p$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A + n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$z = -1.336$



A/B Testing: Conversion Rates

Design A

Design B

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$n_B = 20$

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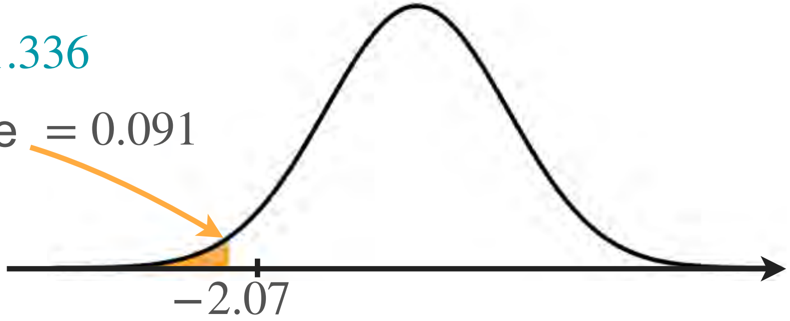
$z = -1.336$

p-value $= 0.091$



A/B Testing: Conversion Rates

Design A Design B

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$z = -1.336$

p-value $= 0.091$

Do not reject H_0