Example Problem

Example Problem

Classification

Example Problem

Classification

Patient 1	
Age	29
Gender	Female
Height	169 cm
Weight	62 kg
Smoker	No
...	...
Heart rate	63
Blood pressure	120 90

- Is this patient healthy?

Example Problem

Classification

Patient 1	
Age	29
Gender	Female
Height	169 cm
Weight	62 kg
Smoker	No
...	...
Heart rate	63
Blood pressure	120 90

- Is this patient healthy?
- Calculate $\mathbf{P}(\text{healthy} \mid \text{symptoms and history})$

Example Problem

Example Problem

Sentiment analysis

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

Matsuo Bashō

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

Matsuo Bashō

- Is this a happy sentence?

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

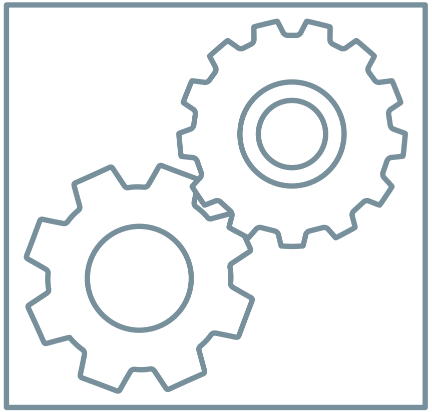
Matsuo Bashō

- Is this a happy sentence?
- Calculate P(happy I words in the sentence)

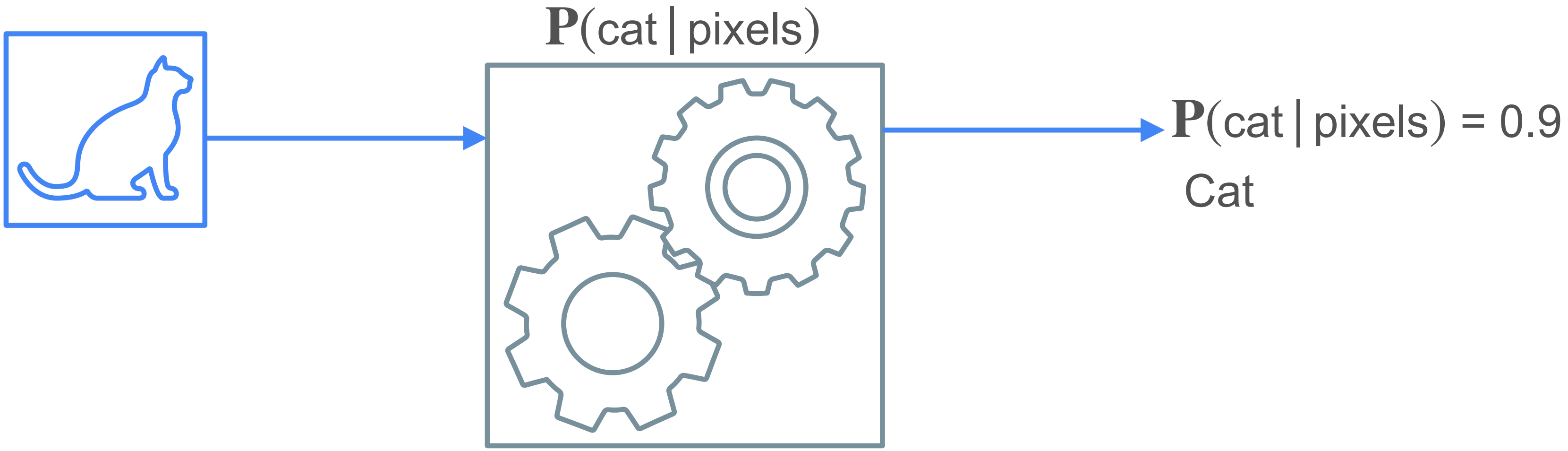
Example Solution

Example Solution

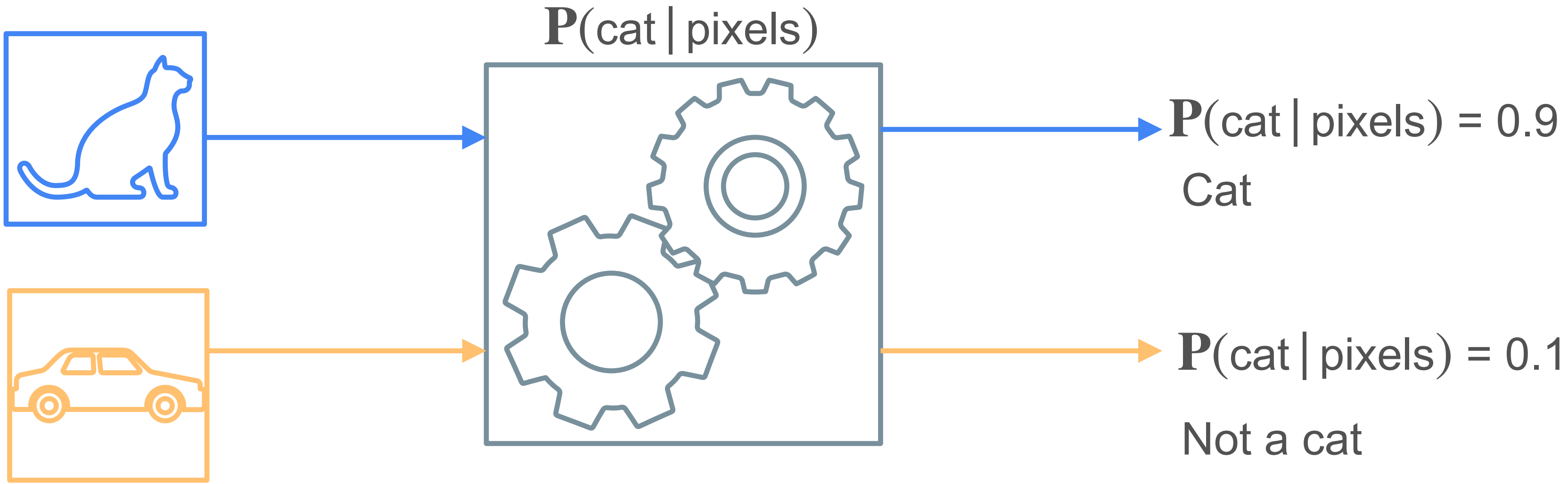
$\mathbf{P}(\text{cat} \mid \text{pixels})$



Example Solution



Example Solution



Example Problem: Generative Models

Example Problem: Generative Models

Face generation

Example Problem: Generative Models

Face generation

Image generated by a StyleGAN

Example Problem: Generative Models

Face generation

- Generate a group of pixels such that the resulting image looks like a human face.
- Goal: generate images such that $\mathbf{P}(\text{face} \mid \text{pixels})$ is high.

Image generated by a StyleGAN

W1 Lesson 2

Probability Distributions

DeepLearning.AI

Probability Distributions

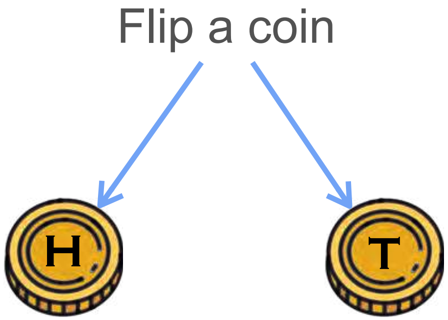
Random Variables

From Events to Random Variables

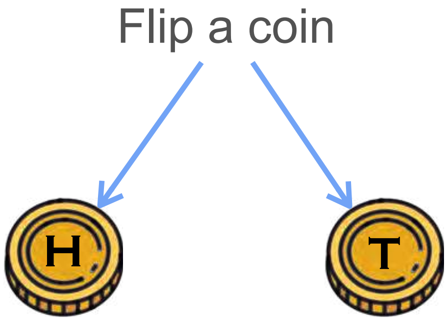
From Events to Random Variables

Flip a coin

From Events to Random Variables



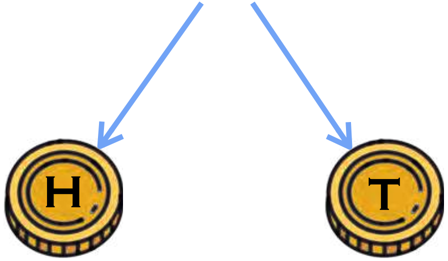
From Events to Random Variables



$$\mathbf{P}(H) = 0.5$$

From Events to Random Variables

Flip a coin

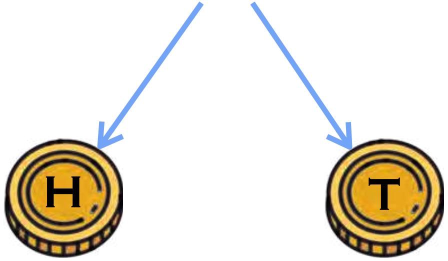


$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

From Events to Random Variables

$X =$ Number of heads

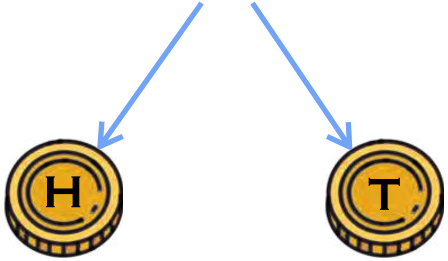
Flip a coin



$\mathbf{P}(H) = 0.5$ $\quad$ $\mathbf{P}(T) = 0.5$

From Events to Random Variables

Flip a coin



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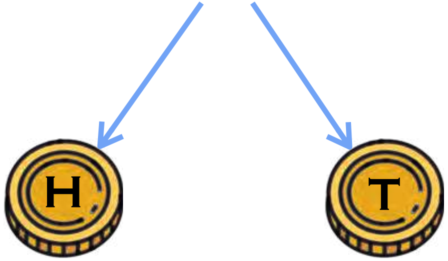
Flip a coin

From Events to Random Variables

Flip a coin



$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads 0 heads

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads

$X = 1$

0 heads

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads

$X = 1$

0 heads

$X = 0$

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads 0 heads

$X = 1$ $X = 0$

$\mathbf{P}(X = 1) = 0.5$

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads

$X = 1$

$\mathbf{P}(X = 1) = 0.5$

0 heads

$X = 0$

$\mathbf{P}(X = 0) = 0.5$

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\quad$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads $\quad$ 0 heads

$X = 1$ $\quad$ $X = 0$

$\mathbf{P}(X = 1) = 0.5$ $\quad$ $\mathbf{P}(X = 0) = 0.5$

X is a random variable

From Events to Random Variables

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 10$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 10$

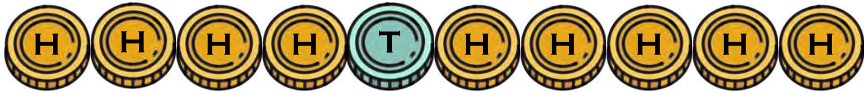
$X = 9$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 10$

$X = 9$



$X = 9$

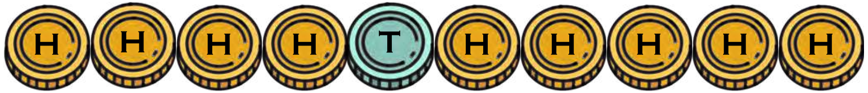
From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$

$X = 9$



$X = 9$

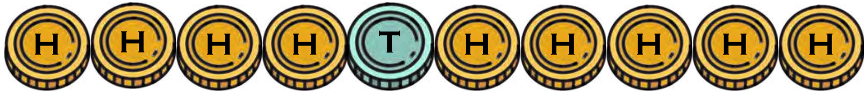
From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$X = 9$



$X = 9$

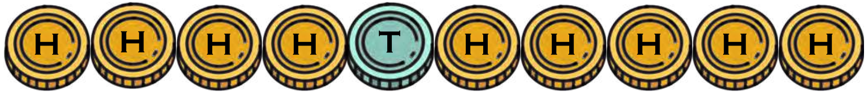
From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$X = 9$ $\quad 0.5^{9}0.5$



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From Events to Random Variables

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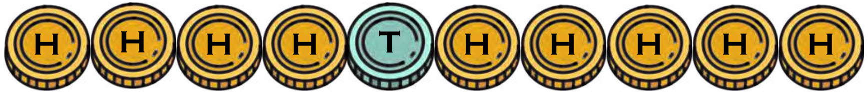


$X = 9$ $\quad 0.5^{9}0.5$

From Events to Random Variables

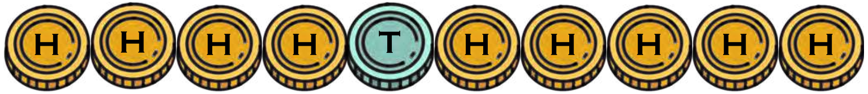
$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$\mathbf{P}(X = 0)?$

$X = 9$ $\quad 0.5^{9}0.5$



$X = 9$ $\quad 0.5^{9}0.5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

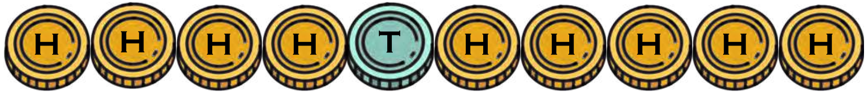
$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$\mathbf{P}(X = 0)?$

$\mathbf{P}(X = 1)?$

$X = 9$ $\quad 0.5^{9}0.5$



$X = 9$ $\quad 0.5^{9}0.5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

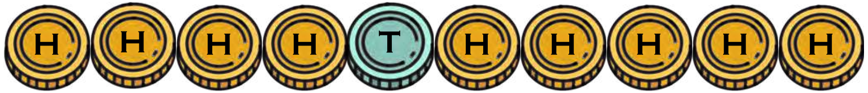
$X = 10$ $\quad 0.5^{10}$

$\mathbf{P}(X = 0)?$

$X = 9$ $\quad 0.5^{9}0.5$

$\mathbf{P}(X = 1)?$

$\dots$



$X = 9$ $\quad 0.5^{9}0.5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$X = 9$ $\quad 0.5^{9}0.5$

$X = 9$ $\quad 0.5^{9}0.5$

$\mathbf{P}(X = 0)?$

$\mathbf{P}(X = 1)?$

$\dots$

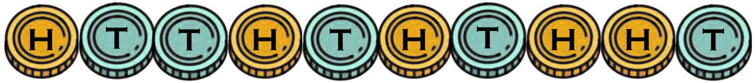
$\mathbf{P}(X = 10)?$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses



$X = 5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

Possible outcomes:

$X = 5$

$X = 6$

$X = 5$

⋮

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

⋮

Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	

$\mathbf{P}(H) = 0.5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	

$\mathbf{P}(H) = 0.5$

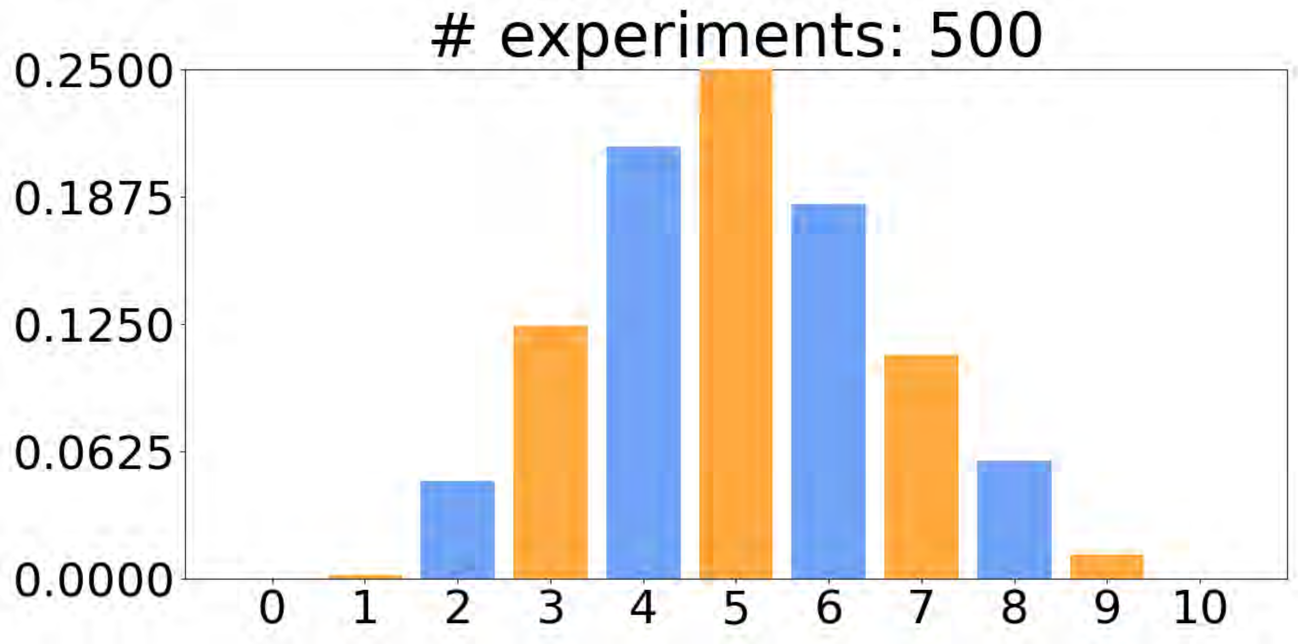
Repeat 500 times

Flipping a Fair Coin 500 Times

$$\mathbf{P}(H) = 0.5$$

Repeat 500 times

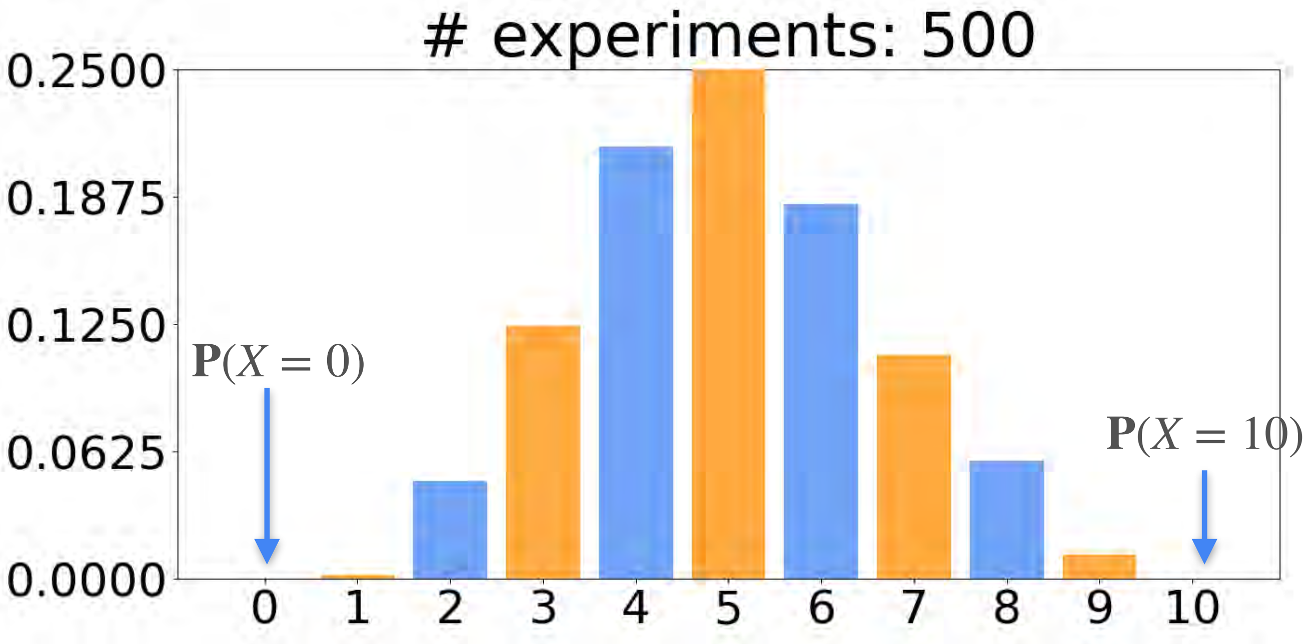
Flipping a Fair Coin 500 Times



$\mathbf{P}(H) = 0.5$

Repeat 500 times

Flipping a Fair Coin 500 Times



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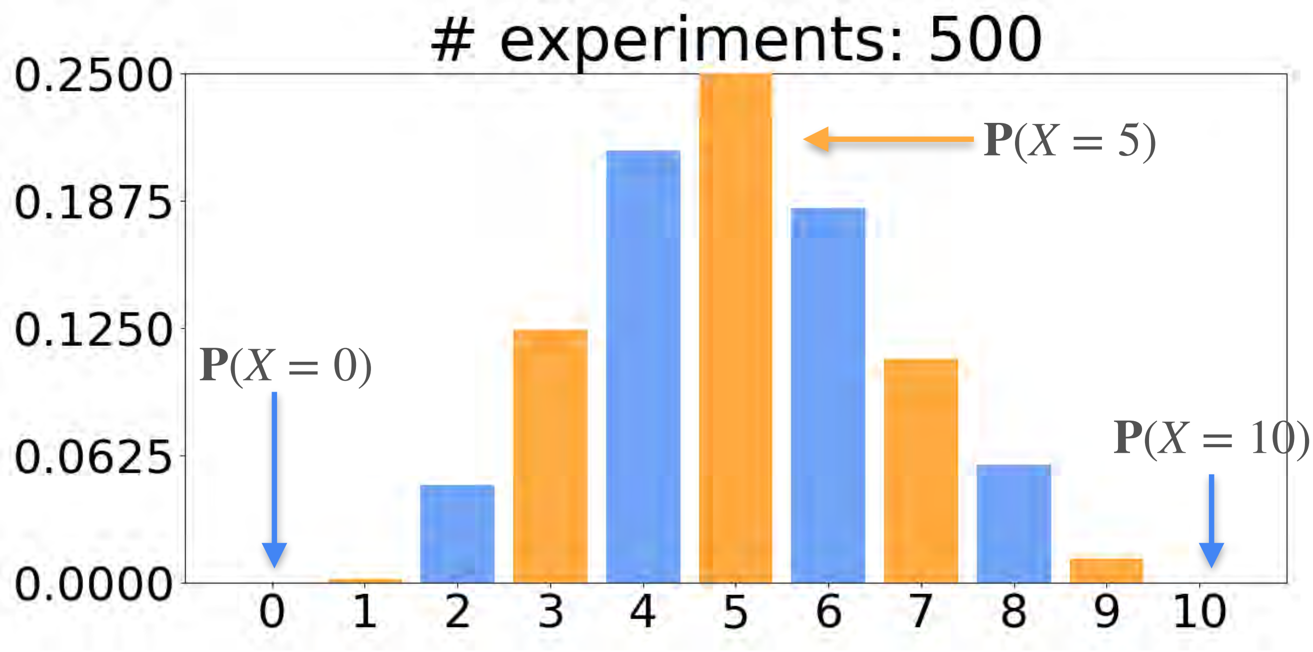
Repeat 500 times

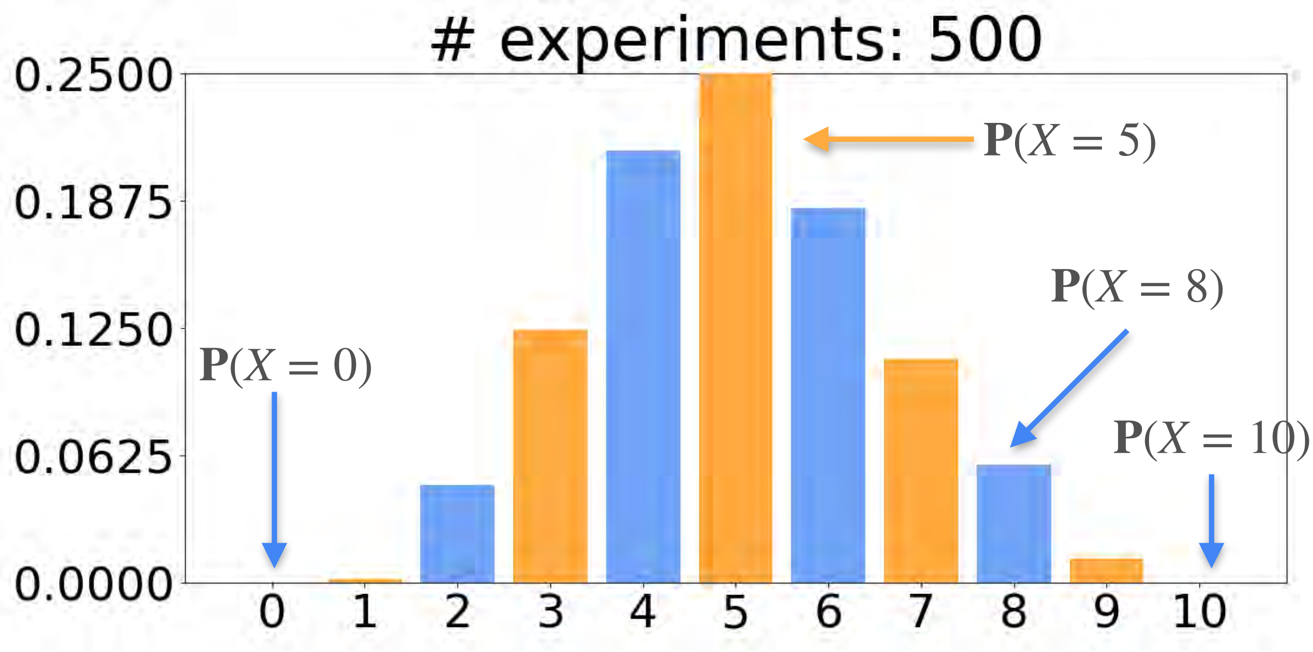
Flipping a Fair Coin 500 Times



$\mathbf{P}(H) = 0.5$

Repeat 500 times

Flipping a Fair Coin 500 Times



$\mathbf{P}(H) = 0.5$

Repeat 500 times

Why Random Variables?

- Random variables allow you to model the whole experiment at once

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$X =$ Number of heads

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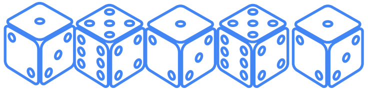
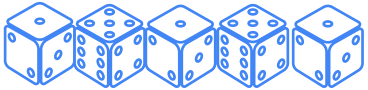


$X =$ Number of 1's

Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



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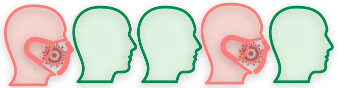
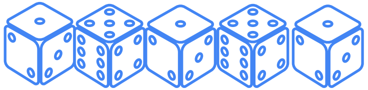


$X =$ Number of sick patients

Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



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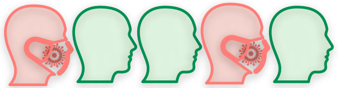

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?

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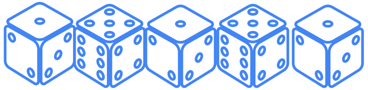
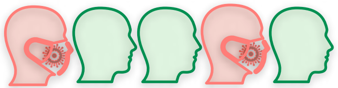


$X =$ Number of 1's



$X =$ Number of sick patients

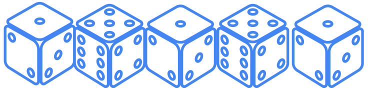
?

$\mathbf{P}(X = 1) = 0.5$

Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



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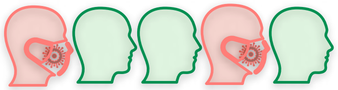


$X =$ Number of sick patients

?

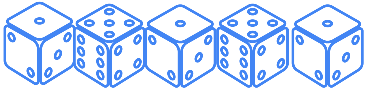
$\mathbf{P}(X = 1) = 0.5$

$\mathbf{P}(X = -7) = 0.2$

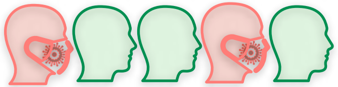
Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



$X =$ Number of 1's



$X =$ Number of sick patients

?

$$\mathbf{P}(X = 1) = 0.5$$
$$\mathbf{P}(X = -7) = 0.2$$
$$\mathbf{P}(X = 3.14159) = 0.3$$

Other Random Variables

Other Random Variables

Wait time until the next bus arrives

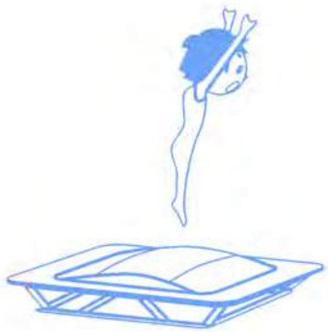
Other Random Variables

Wait time until the next bus arrives

Height of an gymnast's jump

Other Random Variables

Wait time until the next bus arrives

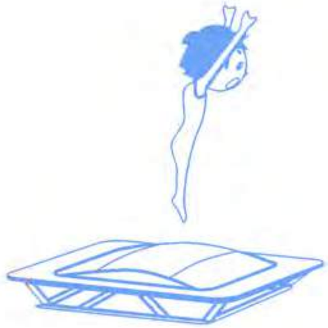


Height of an gymnast's jump

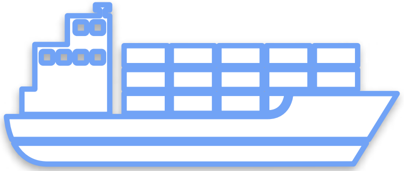
Number of defective products in a shipment

Other Random Variables

Wait time until the next bus arrives



Height of an gymnast's jump



Number of defective products in a shipment

mm. of rain in November

Discrete and Continuous Random Variables

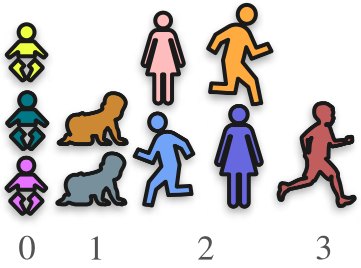
Discrete and Continuous Random Variables

Discrete random variables

Continuous random variables

Discrete and Continuous Random Variables

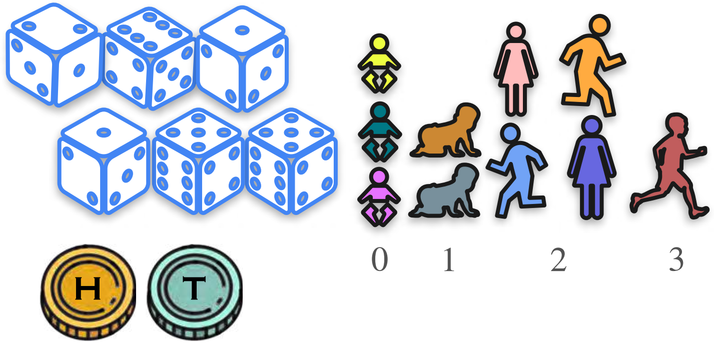
Discrete random variables



Continuous random variables

Discrete and Continuous Random Variables

Discrete random variables



Continuous random variables

Discrete and Continuous Random Variables

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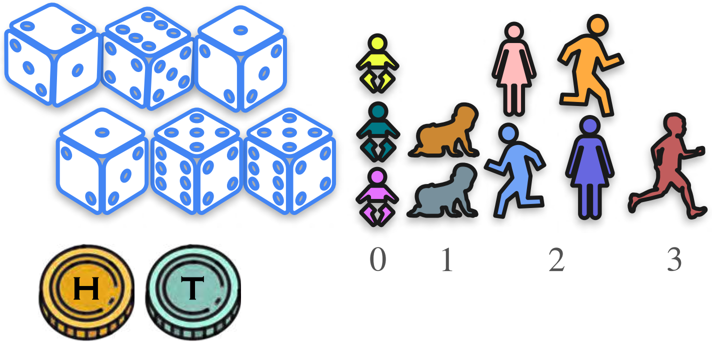
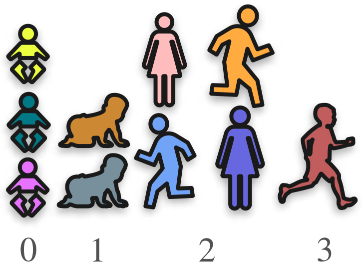


Finite number of values

Continuous random variables

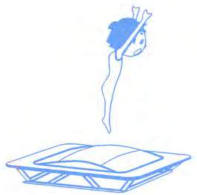
Discrete and Continuous Random Variables

Discrete random variables



Finite number of values

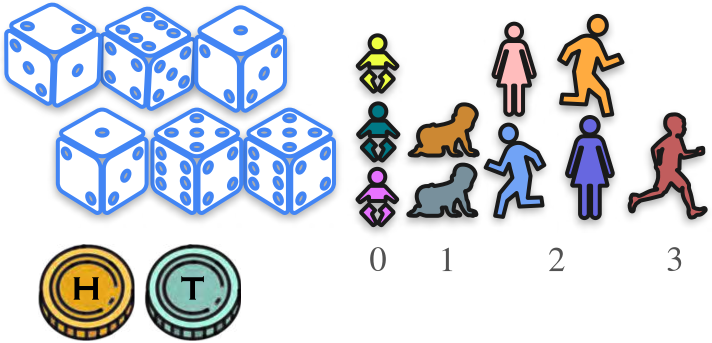
Continuous random variables



Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

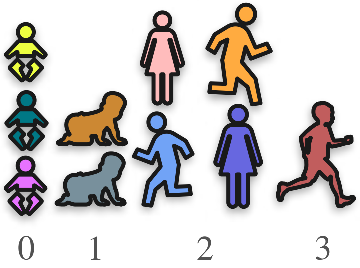
(Could be infinite too)

Continuous random variables

Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

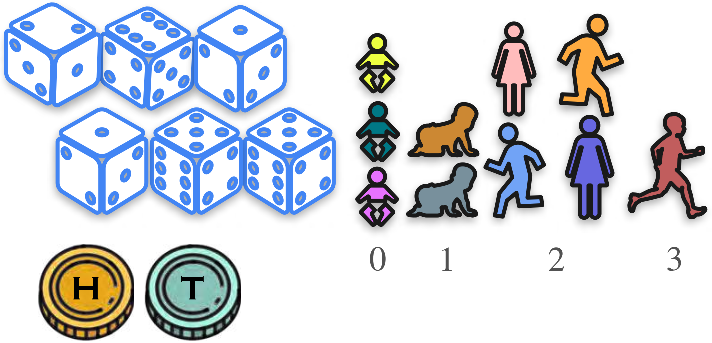
Can take only a **countable** number of values

Continuous random variables

Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

Can take only a **countable** number of values

Continuous random variables

Infinite number of values

Takes values on an interval

Random Variable Vs. Deterministic Variable

Random Variable Vs. Deterministic Variable

Deterministic

Random

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2,\ f(x) = x^2$$

Random

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2,\ f(x) = x^2$$

Random

X = number of defective items in a shipment

Random Variable Vs. Deterministic Variable

Deterministic	**Random**
$x = 2,\ f(x) = x^2$	X = number of defective items in a shipment
Fixed outcome	

Random Variable Vs. Deterministic Variable

Deterministic

$x = 2,\ f(x) = x^2$

Fixed outcome

Random

X = number of defective items in a shipment

Uncertain outcome

Probability Distributions

DeepLearning.AI

Probability Distributions (Discrete)

Discrete Distributions: Flip Three Coins

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses

Discrete Distributions: Flip Three Coins

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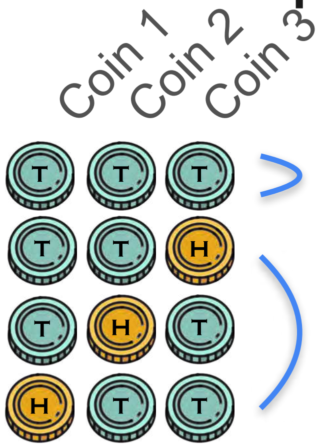
All three tails $(X = 0)$

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses



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Discrete Distributions: Flip Three Coins

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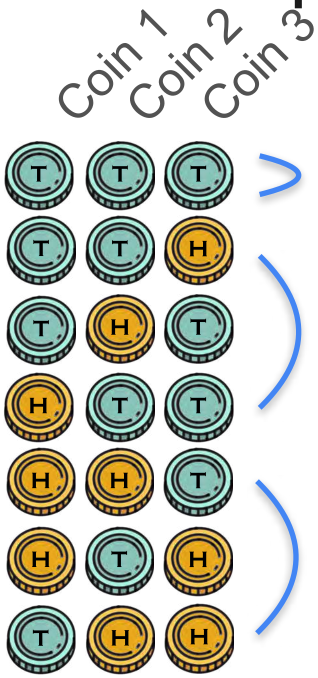

All three tails $(X = 0)$

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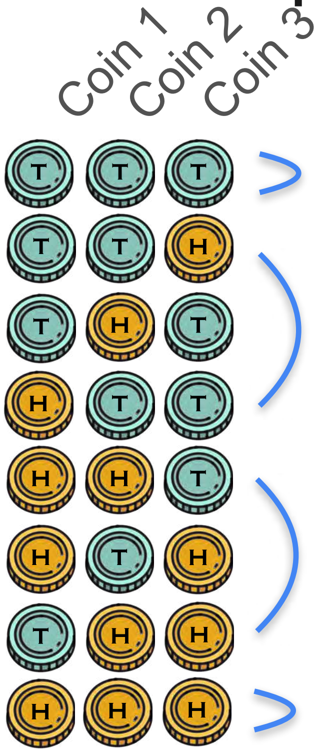
Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses



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One heads $(X = 1)$

Two heads $(X = 2)$

All three heads $(X = 3)$

Discrete Distributions: Flip Three Coins

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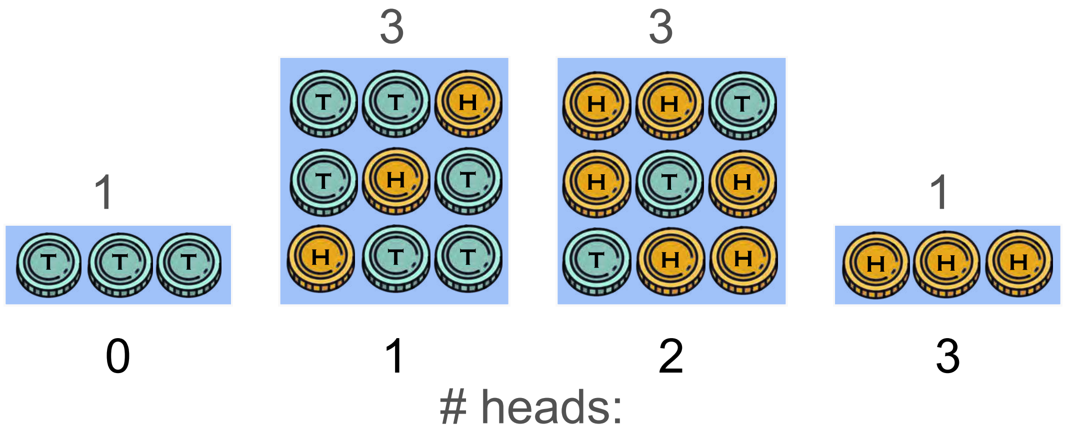

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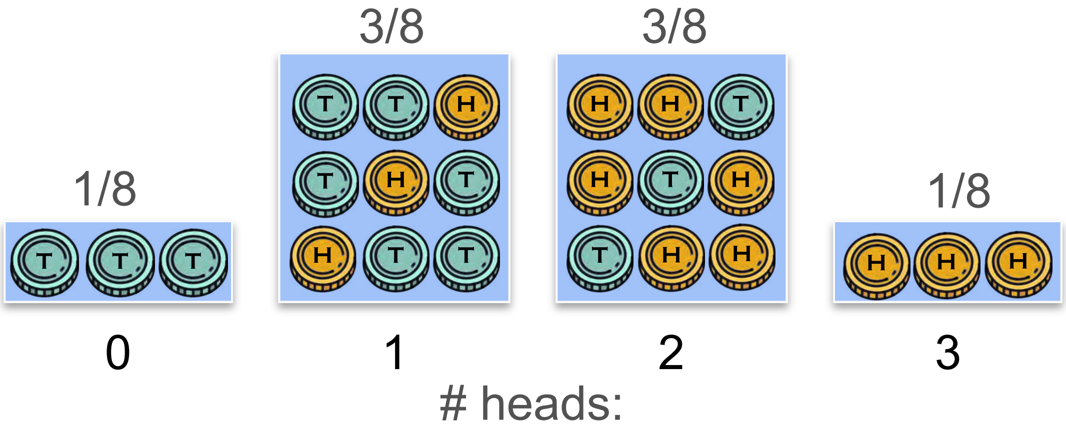

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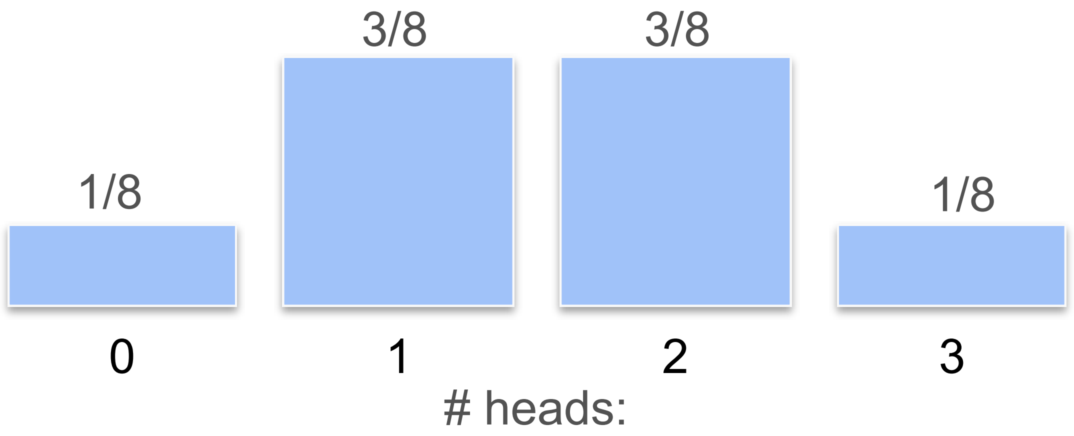

Discrete Distributions: Flip Four Coins

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X_2: number of heads in **4** coin tosses

Discrete Distributions: Flip Four Coins

X_2: number of heads in **4** coin tosses

0	1	2	3	4

heads:

Discrete Distributions: Flip Four Coins

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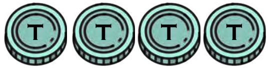

0 1 2 3 4

heads:

Discrete Distributions: Flip Four Coins

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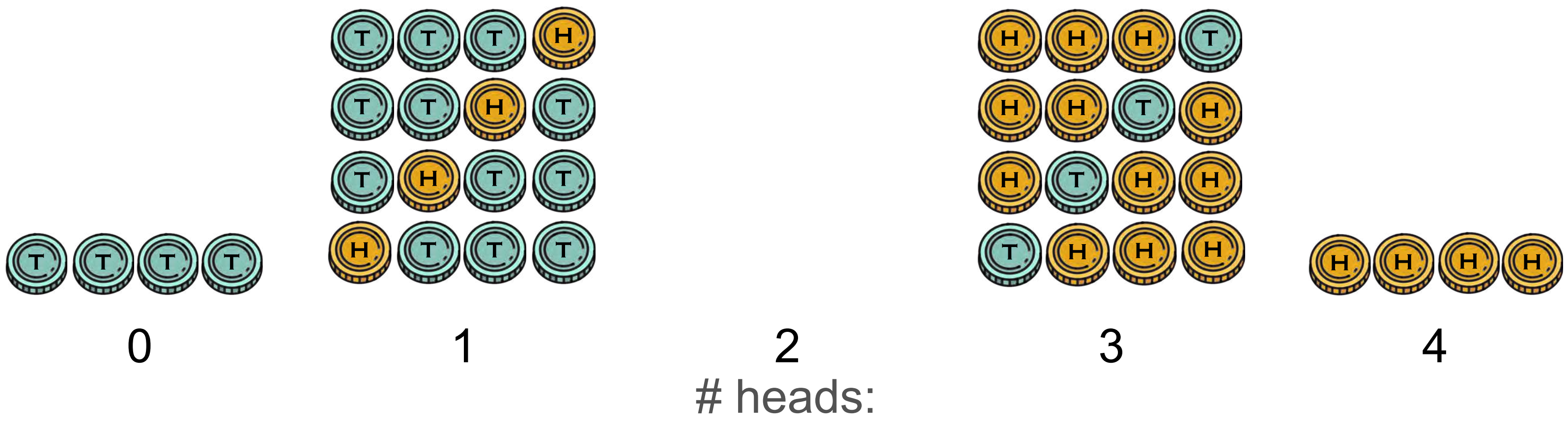

Discrete Distributions: Flip Four Coins

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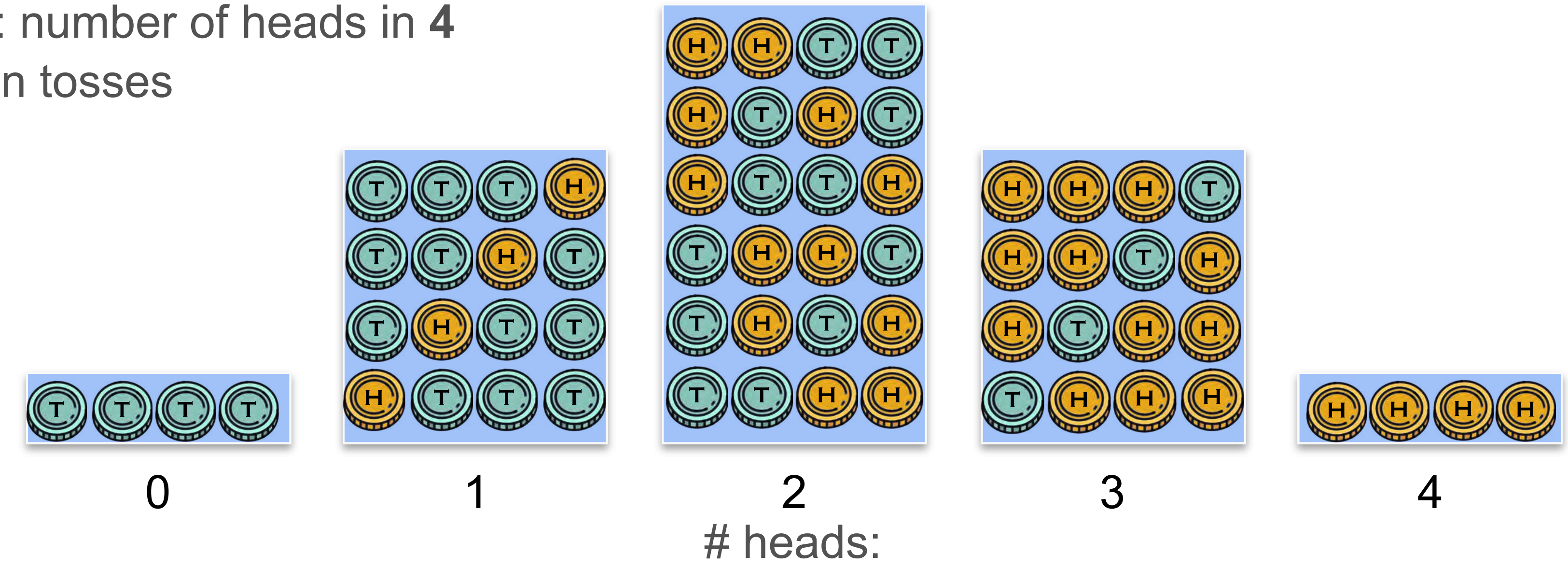

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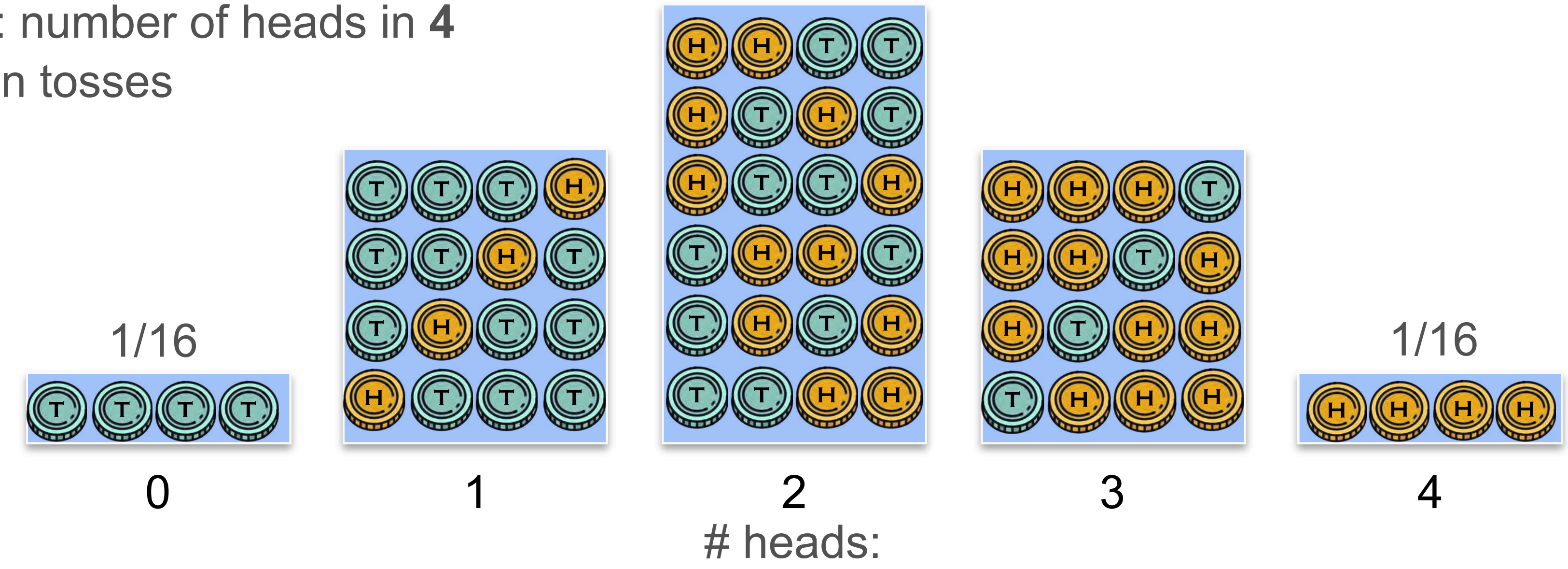

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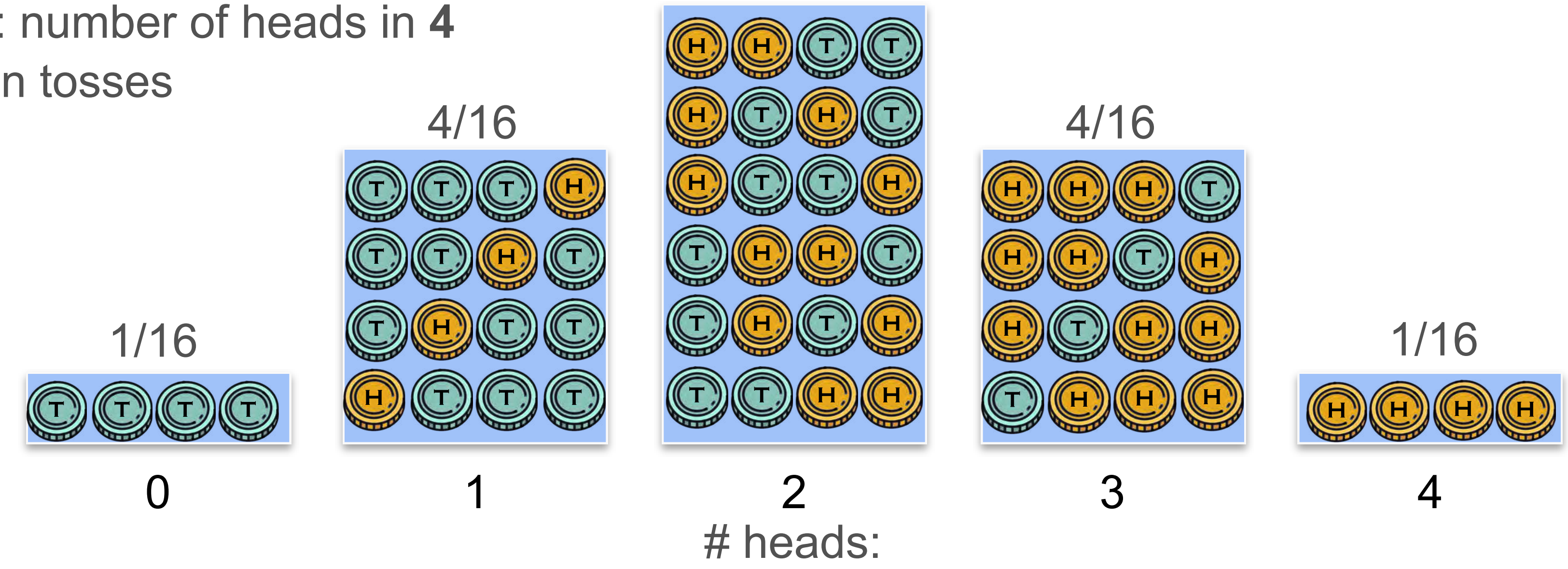

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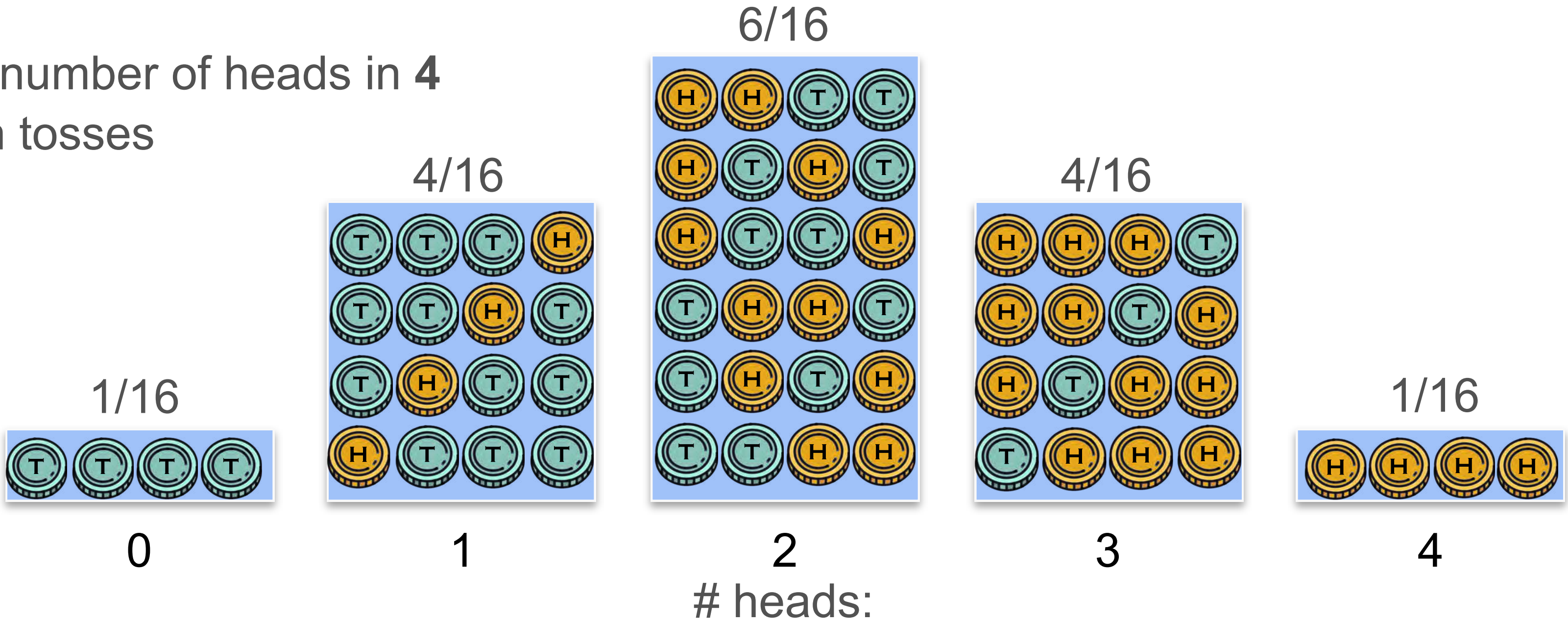
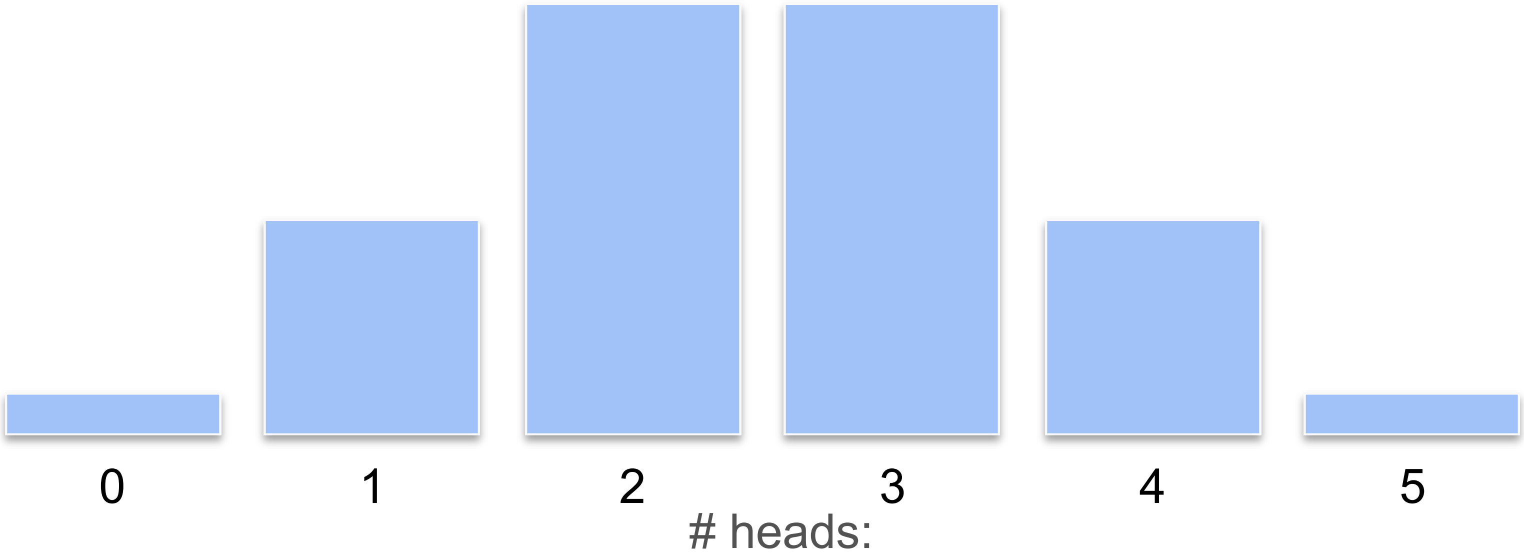
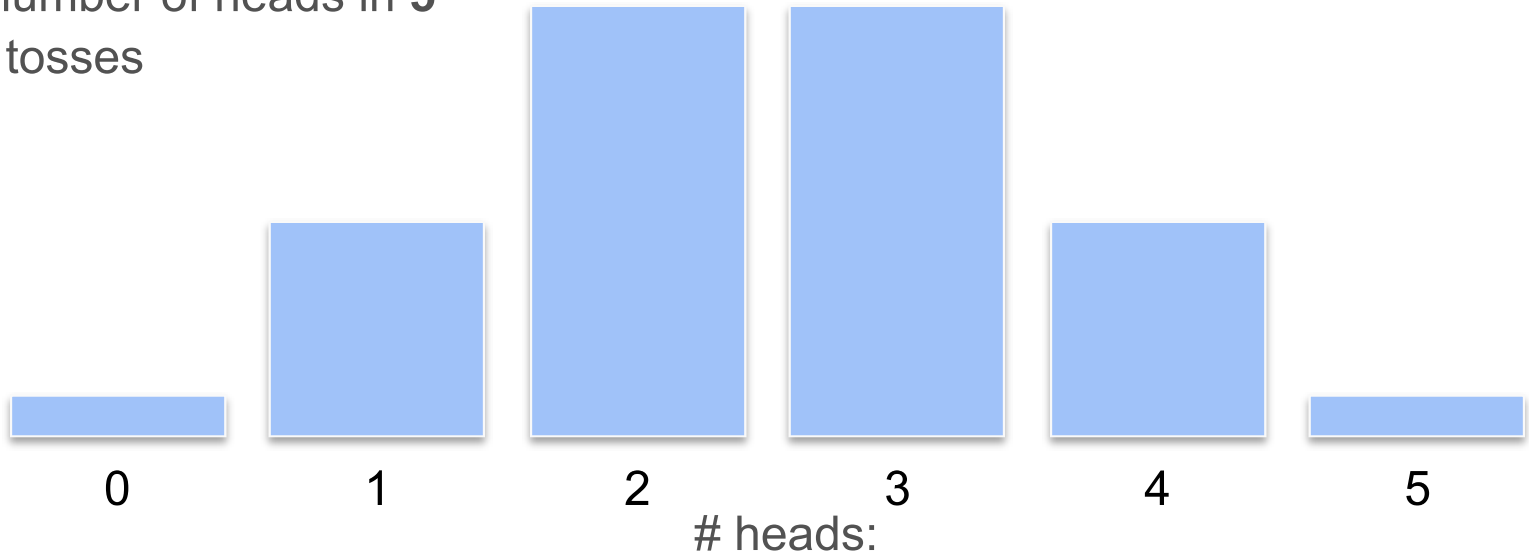


Discrete Distributions: Flip Five Coins



Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



Discrete Distributions: Flip Five Coins

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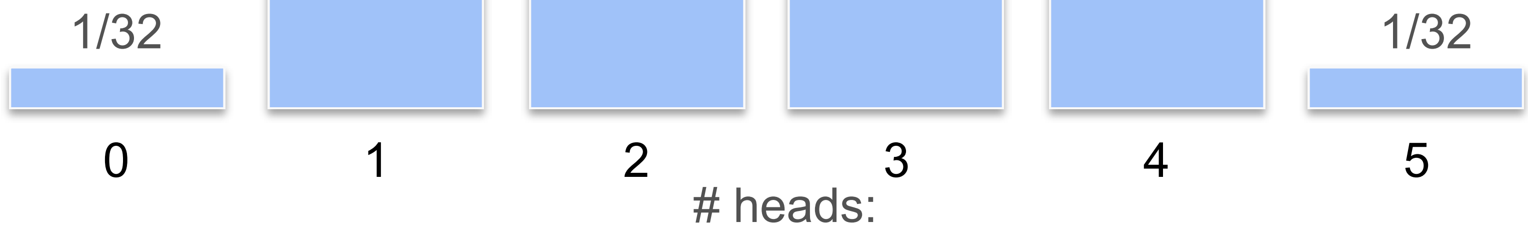

Discrete Distributions: Flip Five Coins

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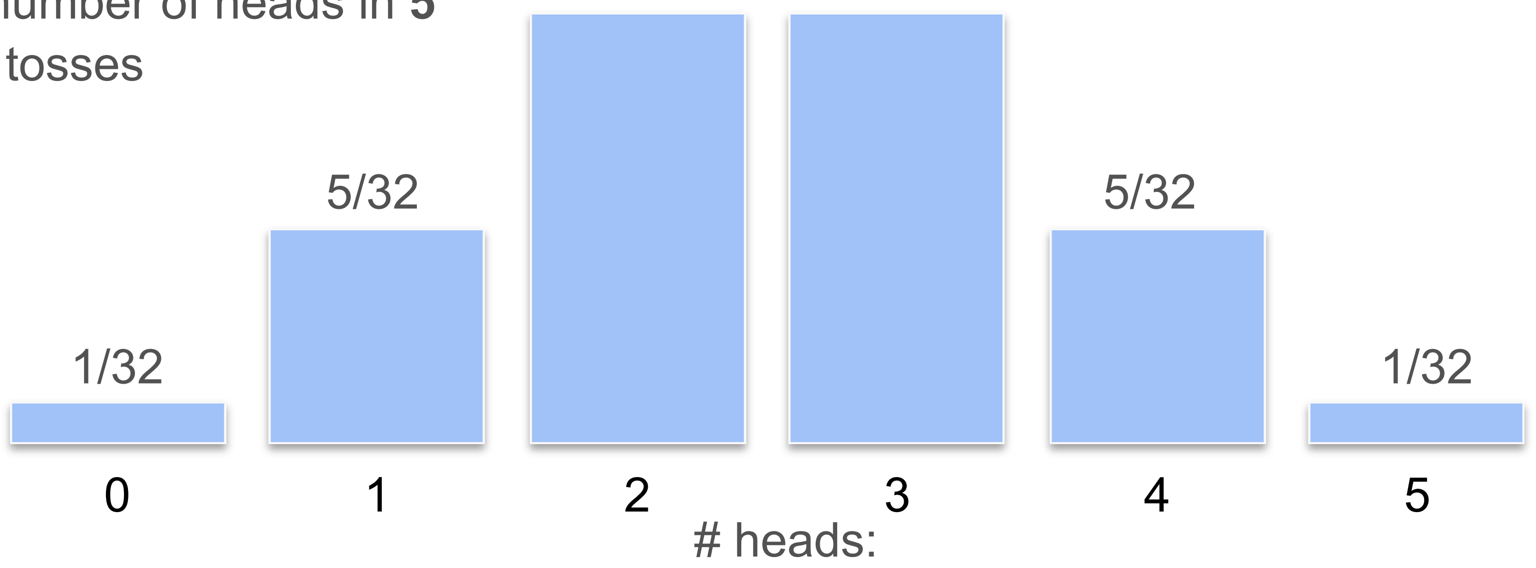

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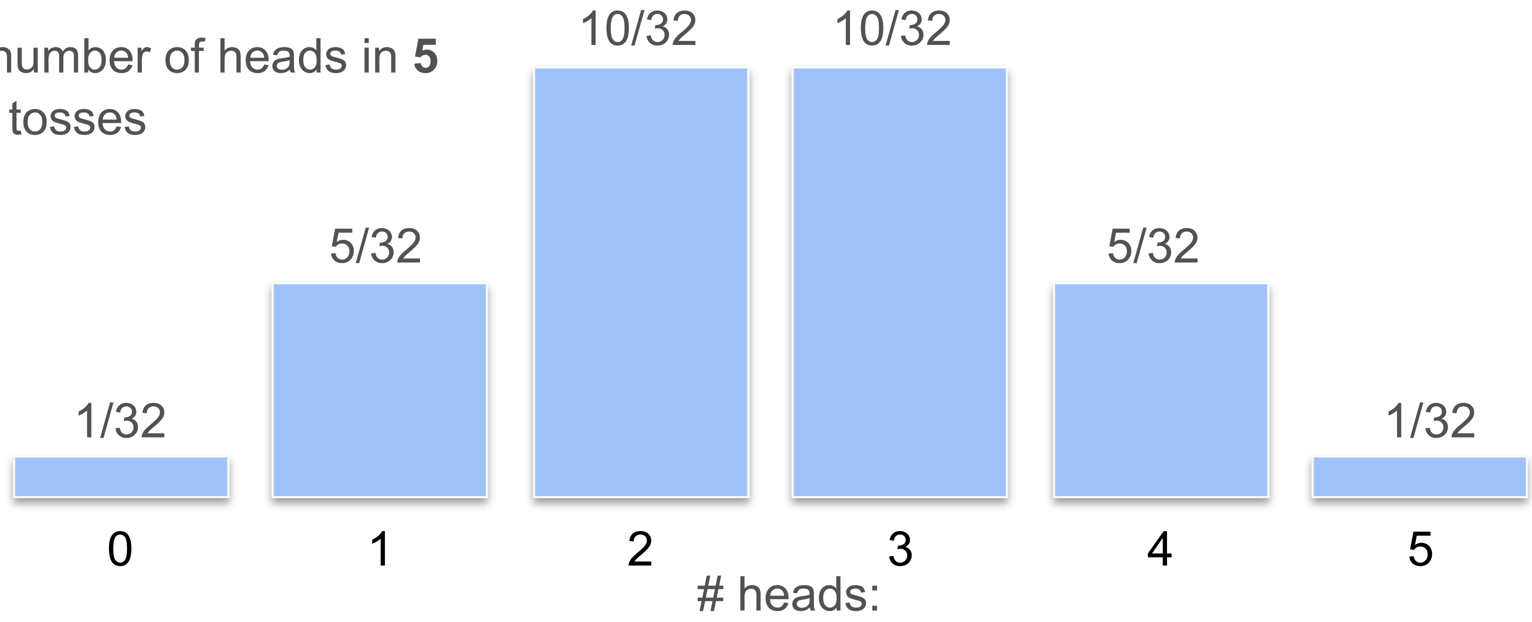

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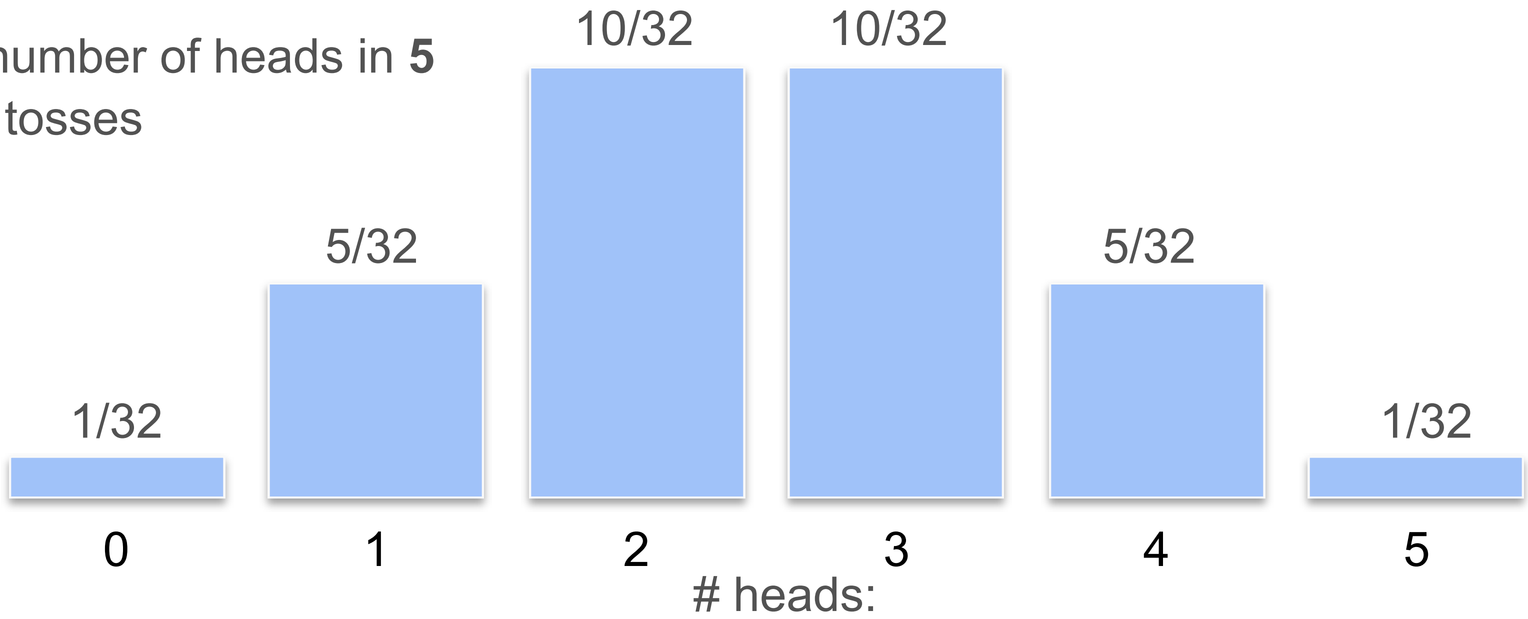

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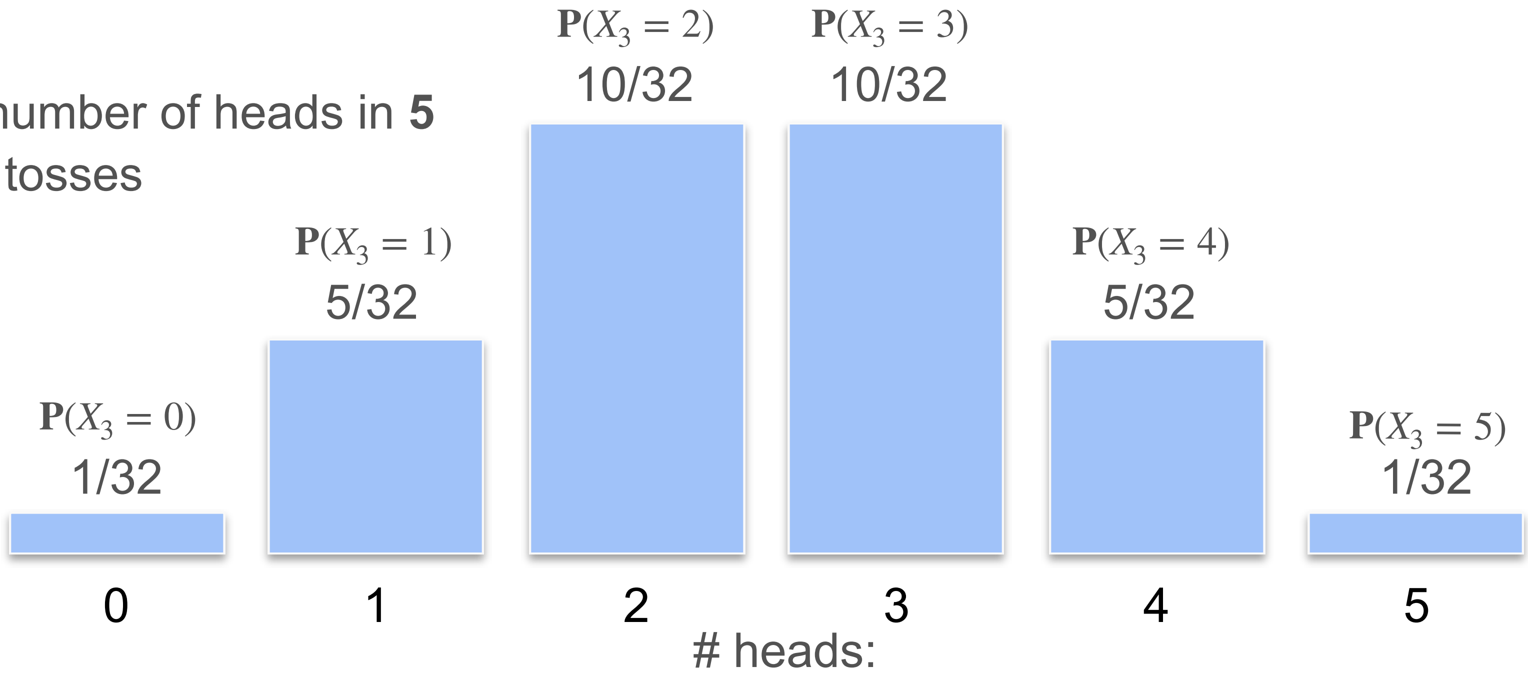

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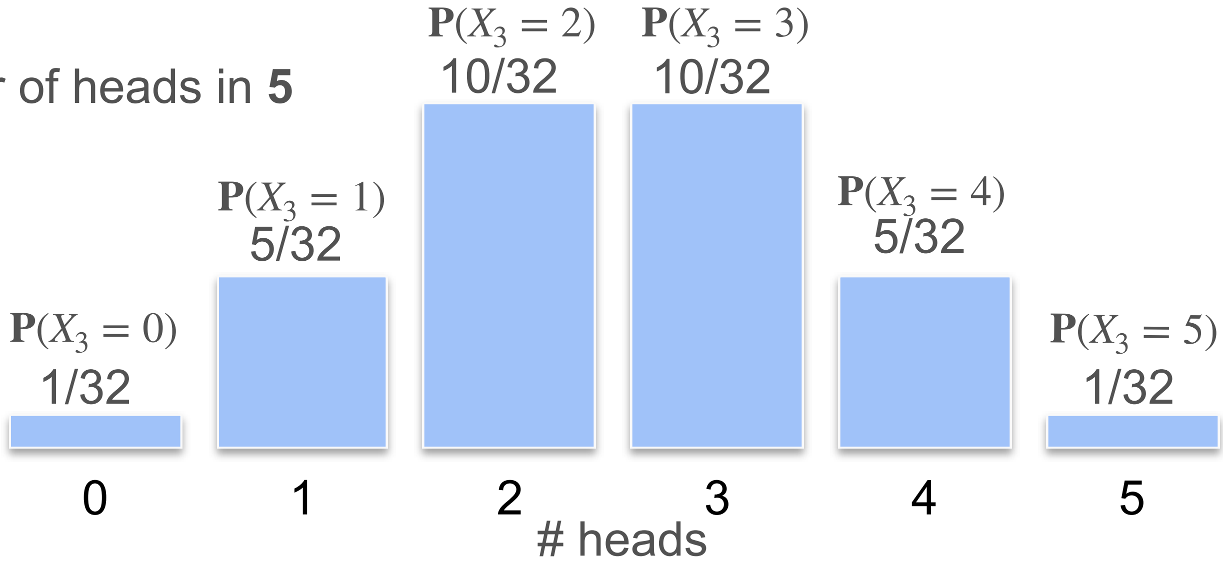

Discrete Distributions: Flip Five Coins

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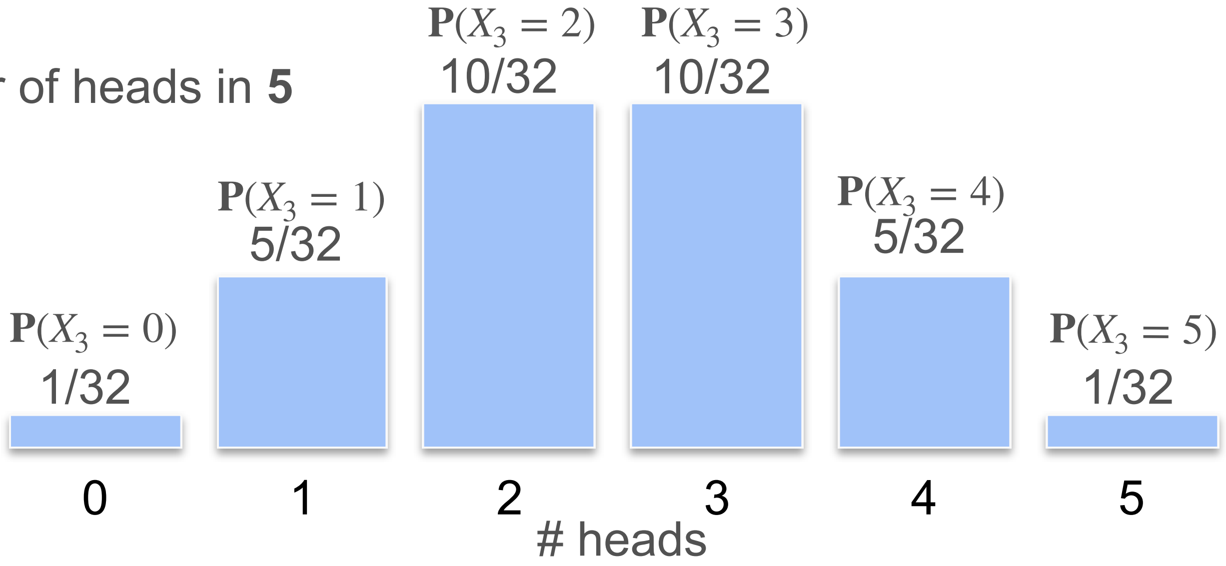

$$\mathbf{P}(X_3 = x), \quad x = 0,1,2,3,4,5$$

Discrete Distributions: Flip Five Coins

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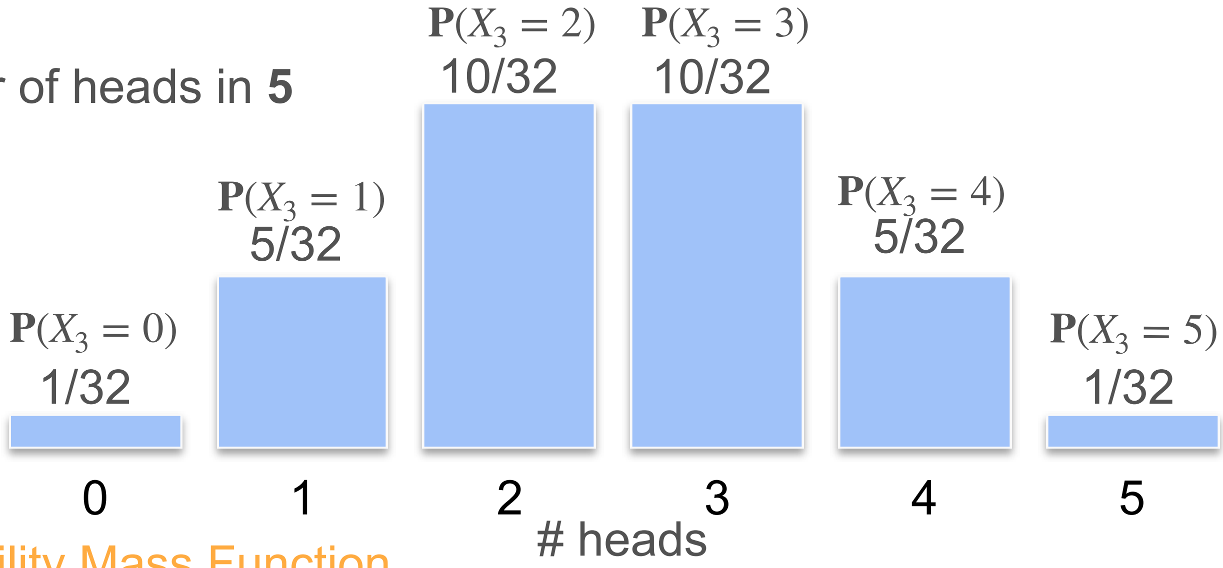

Probability Mass Function (PMF)

$$p_X(x) = \mathbf{P}(X_3 = x), \quad x = 0,1,2,3,4,5$$

Discrete Distributions: Flip Five Coins

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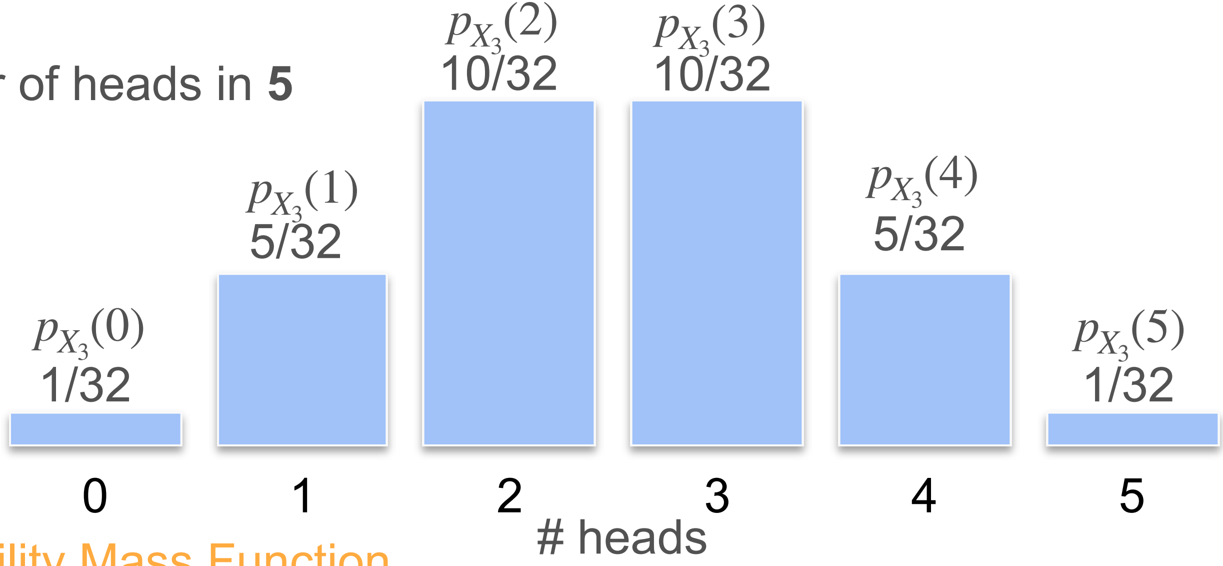

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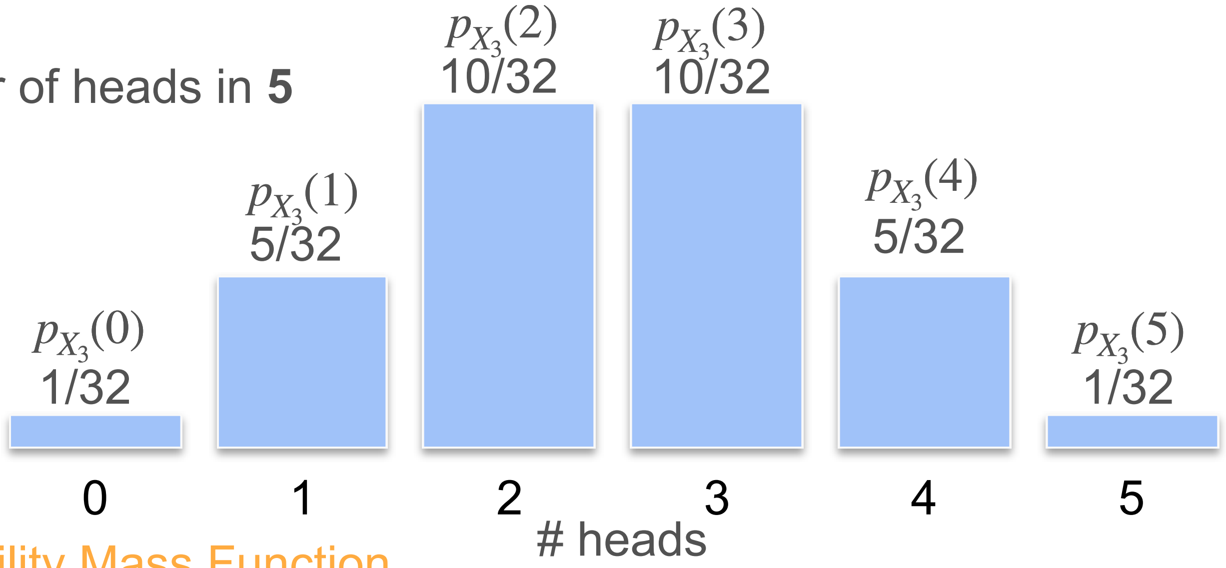

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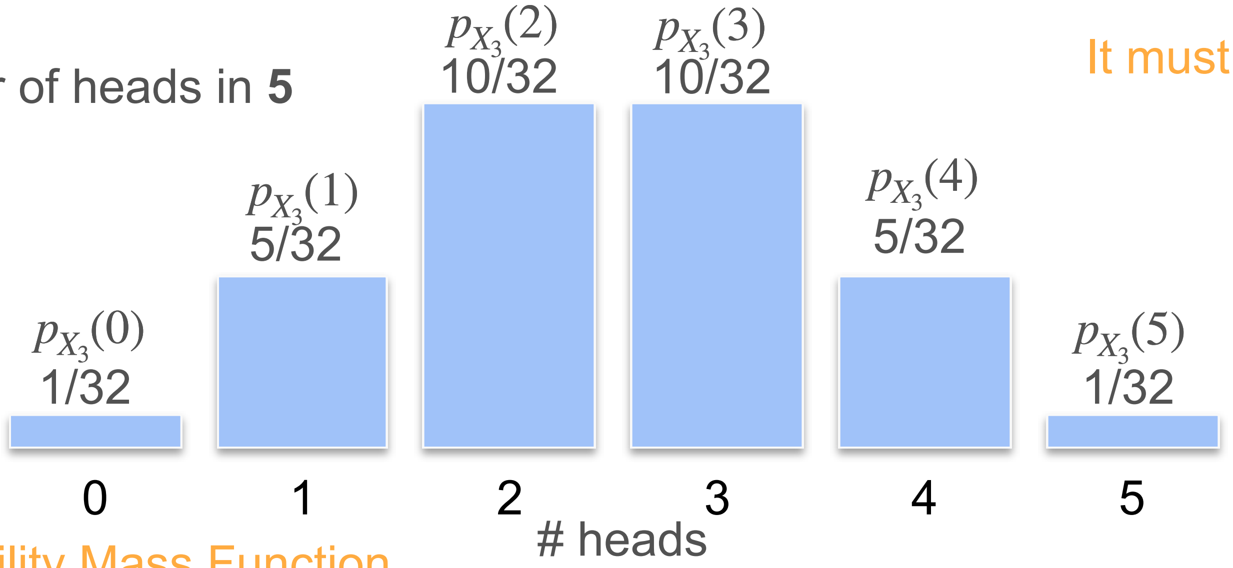
Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses

It must verify:



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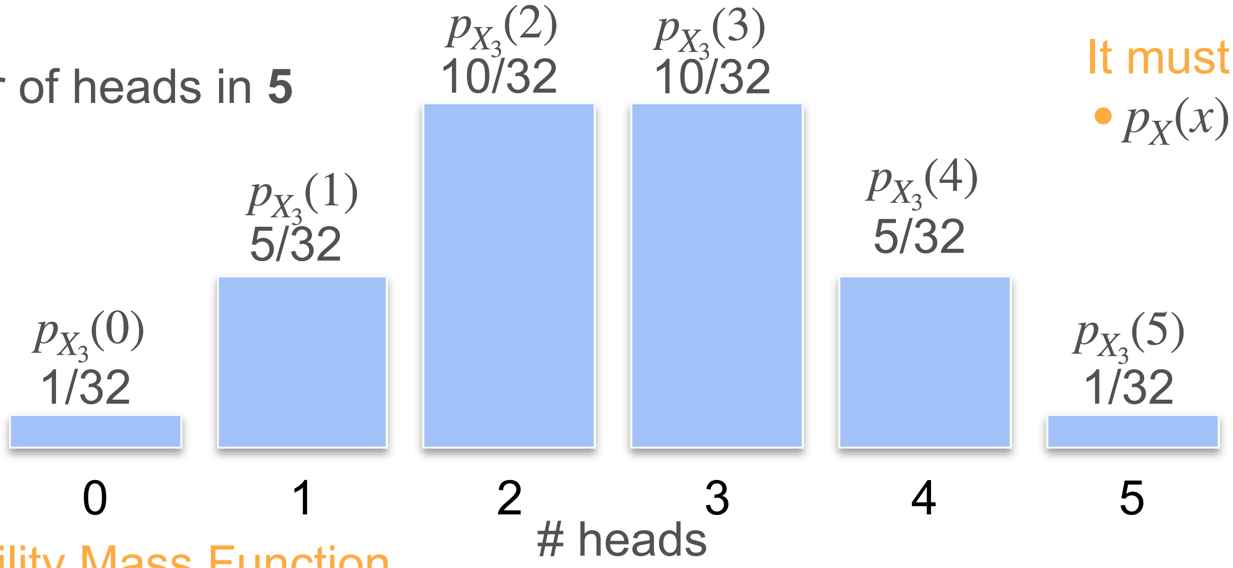

It must verify:

- $p_X(x) \geq 0$

Probability Mass Function (PMF)

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Discrete Distributions: Flip Five Coins

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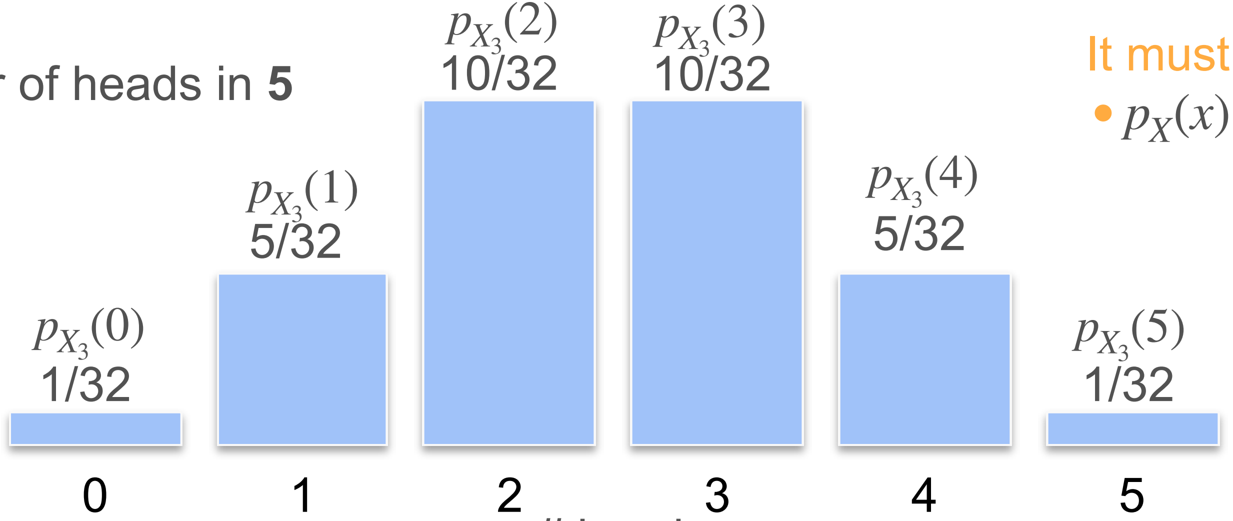


heads

It must verify:

- $p_X(x) \geq 0$

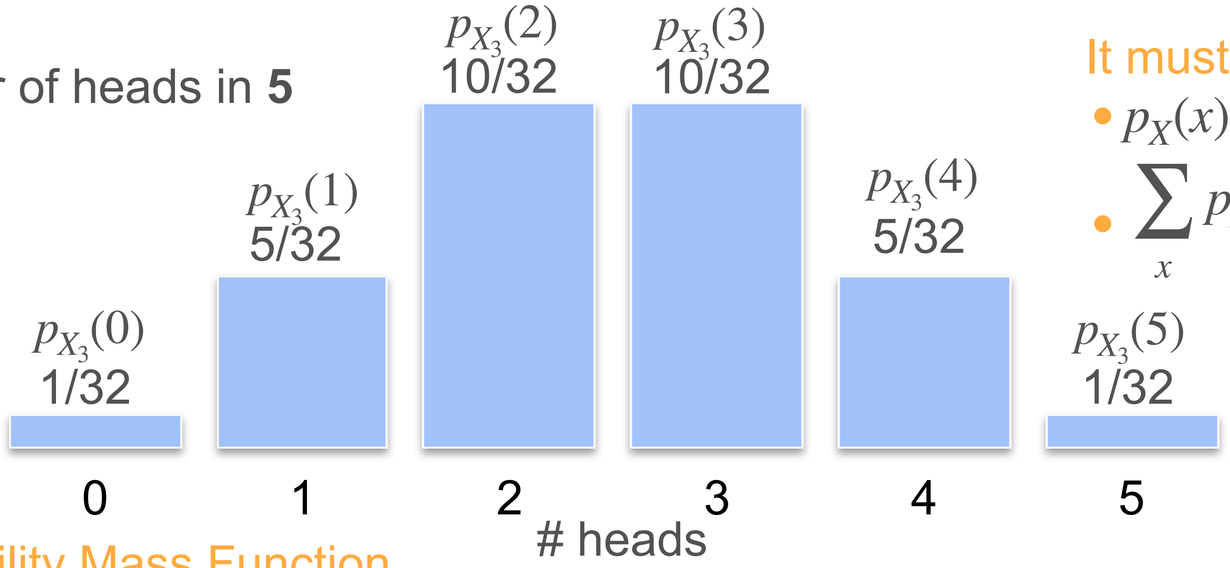
Probability Mass Function (PMF)

$$p_X(x) = \mathbf{P}(X_4 = x), \quad x = 0,1,2,3,4,5$$

$$p_{X_3}(0) + p_{X_3}(1) + p_{X_3}(2) + p_{X_3}(3) + p_{X_3}(4) + p_{X_3}(5) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32} \quad = 1$$

Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



It must verify:

- $p_X(x) \geq 0$
- $\sum_x p_X(x) = 1$

Probability Mass Function (PMF)

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Binomial Distribution

Binomial Distribution

X_1, X_2, X_3, X_4 are very similar

They all represent **number of heads in n experiments**

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Binomial Distribution

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The way the probability distributes along the possible outcomes seems to have a similar pattern

Could there be a **single model** to represent all this variables?

↓

Binomial distribution

DeepLearning.AI

Probability Distributions

Binomial Distribution

Binomial Distribution: Example

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

Binomial Distribution: Example

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0.5 0.5 0.5 0.5 0.5

Binomial Distribution: Example

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Binomial Distribution: Example

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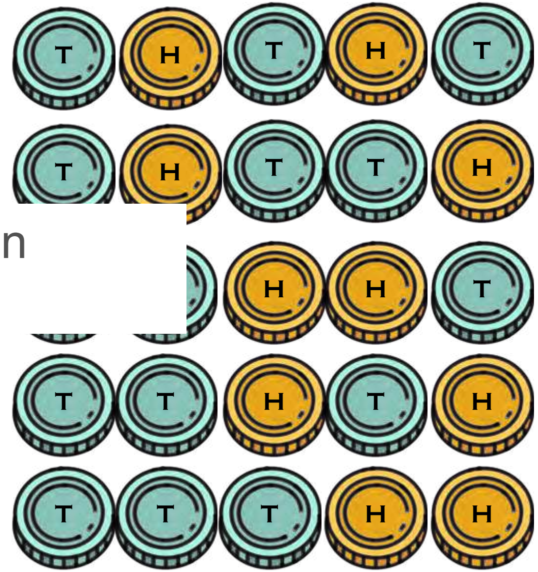
10 ways to have 2 heads in 5 coin tosses



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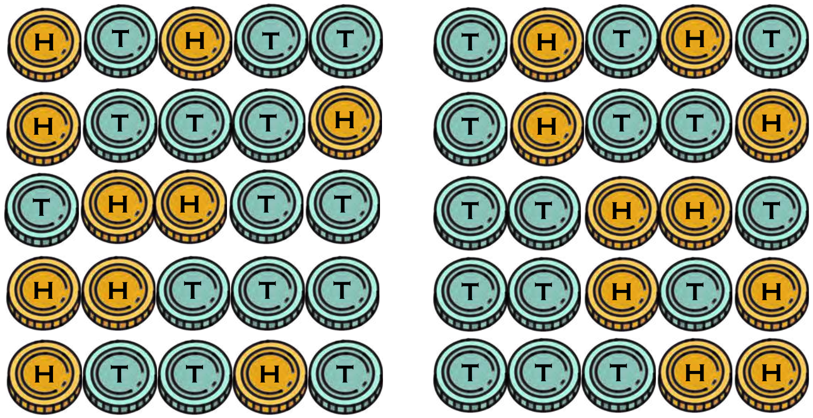

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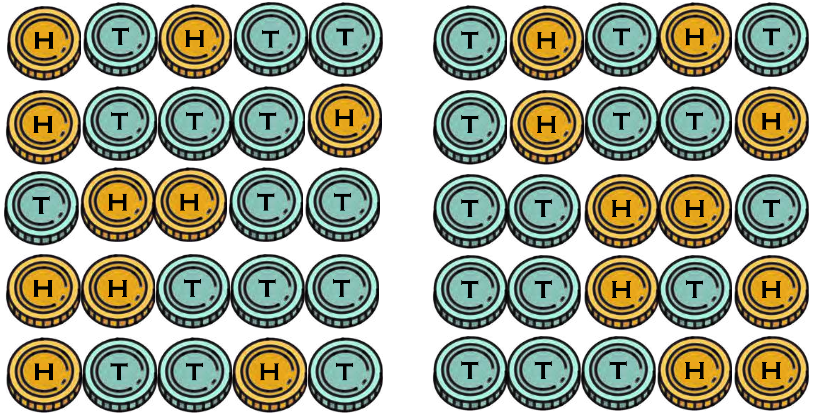
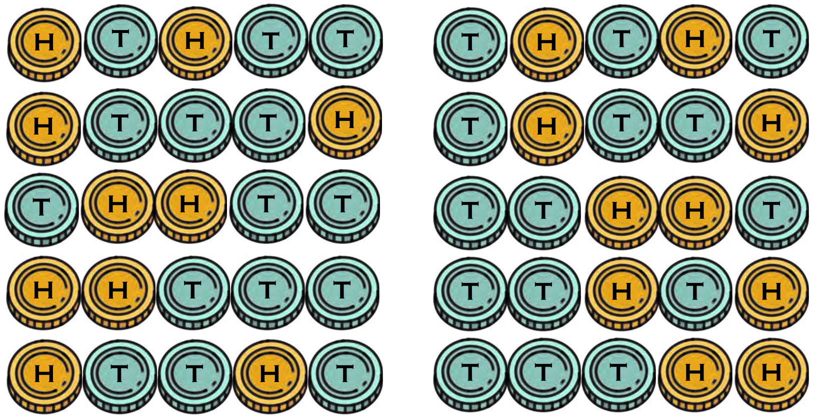


$$10 = \underline{\hspace{3cm}}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

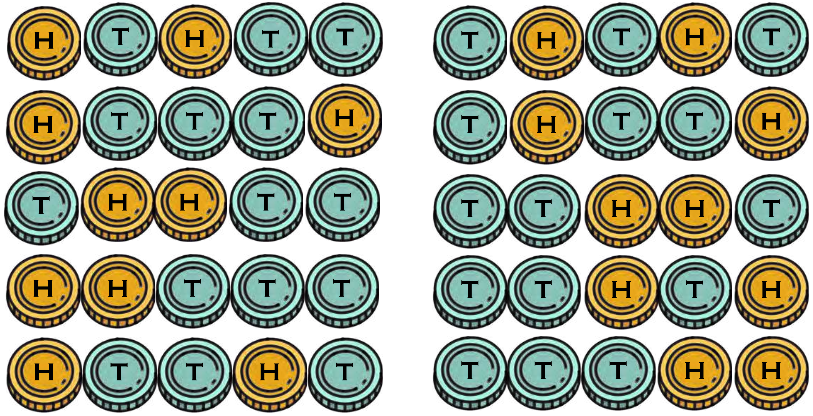


$$10 = \frac{5!}{\quad}$$

Number of ways you can order 5 coins

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



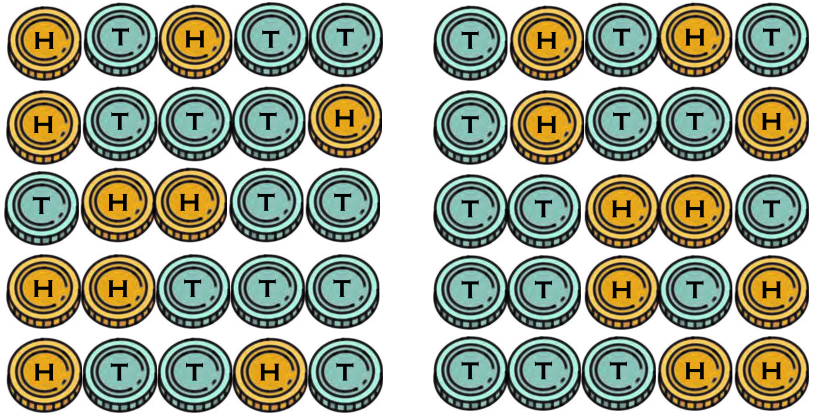
Number of ways you can order 5 coins

$$10 = \frac{5!}{2!}$$

Number of H

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



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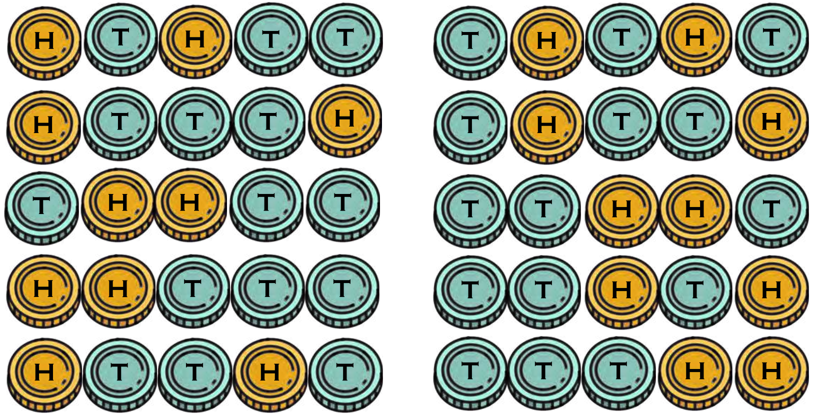

Number of ways you can order 5 coins

$$10 = \frac{5!}{2!(5-2)!}$$

Number of H

Number of T

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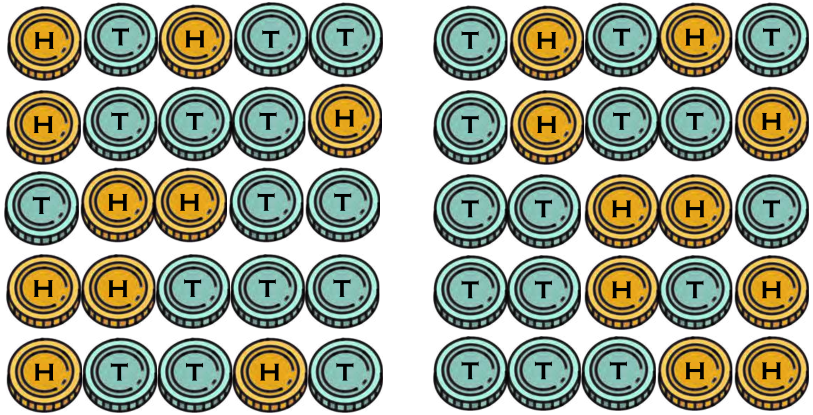

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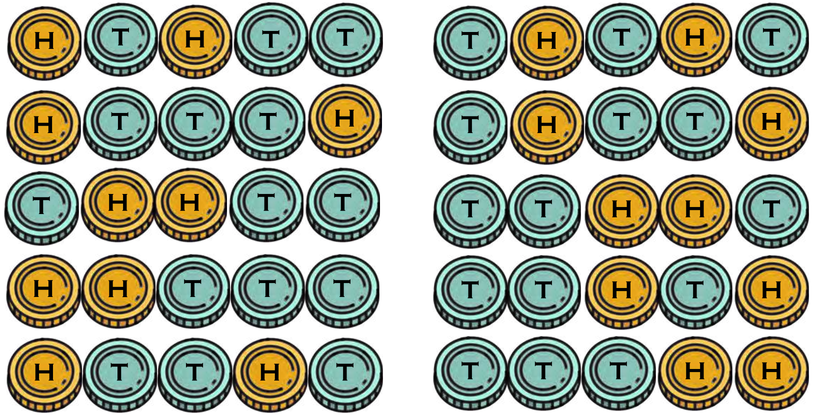

$$10 = \frac{5!}{2!(5-2)!} = \binom{5}{2}$$

Binomial coefficient

Number of ways you can get 2 heads in 5 coin tosses

Binomial Distribution: Binomial Coefficient

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In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

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Property**:**

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Property**:**

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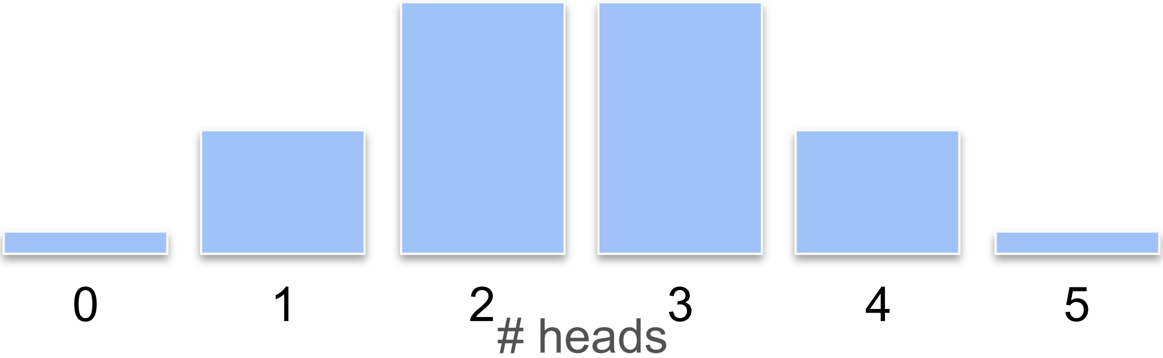
The PMF with a fair coin is symmetrical



Binomial Distribution: Binomial Coefficient

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General PMF for X : number of heads in 5 coin tosses?

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Your coin has $\mathbf{P}(H) = p$

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\boxed{p^x}$$

Probability of seeing x heads

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

Probability of seeing $5 - x$ tails

$$\boxed{p^x}\boxed{(1-p)^{5-x}}$$

Probability of seeing x heads

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

Probability of seeing $5 - x$ tails

All the possible orders

$$\boxed{\binom{5}{x}} \boxed{p^x} \boxed{(1-p)^{5-x}}$$

Probability of seeing x heads

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1-p)^{5-x}$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

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Binomial Distribution

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Binomial Distribution

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X **follows a binomial distribution**

$$X \sim Binomial(\boxed{5}, \boxed{p})$$

Number of flips $\quad \mathbf{P}(H)$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

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$$X \boxed{\sim} Binomial(\boxed{5},\boxed{p})$$

Number of flips

$\mathbf{P}(H)$

Binomial Distribution

Binomial Distribution

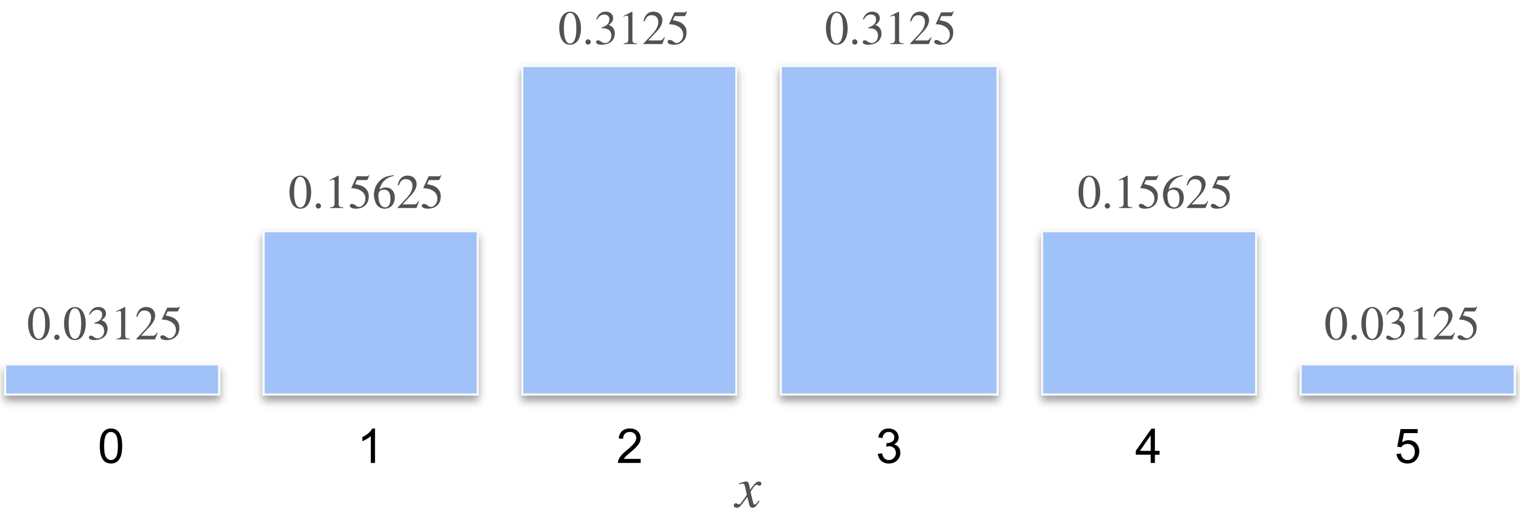
$$\boxed{\begin{array}{c} n = 5 \\ p = 0.5 \end{array}}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

Binomial Distribution

$$\boxed{\begin{array}{c} n = 5 \\ p = 0.5 \end{array}}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

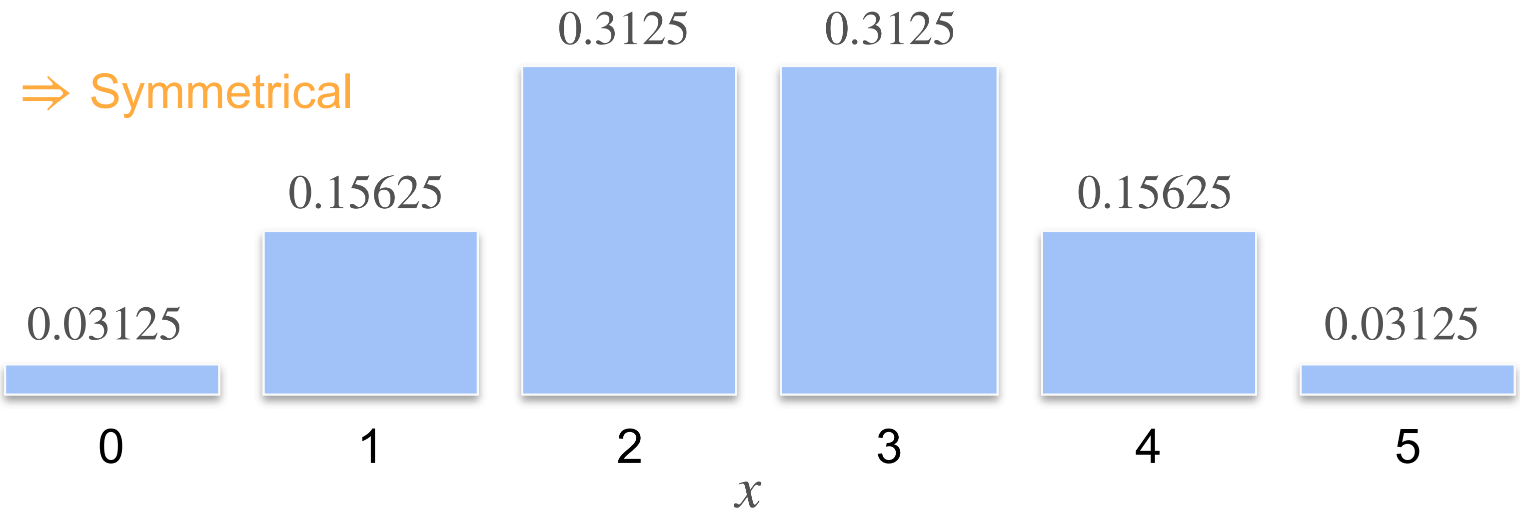


Binomial Distribution

$$\boxed{\begin{matrix} n = 5 \\ p = 0.5 \end{matrix}}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

$p = 0.5 \Rightarrow$ Symmetrical



Binomial Distribution

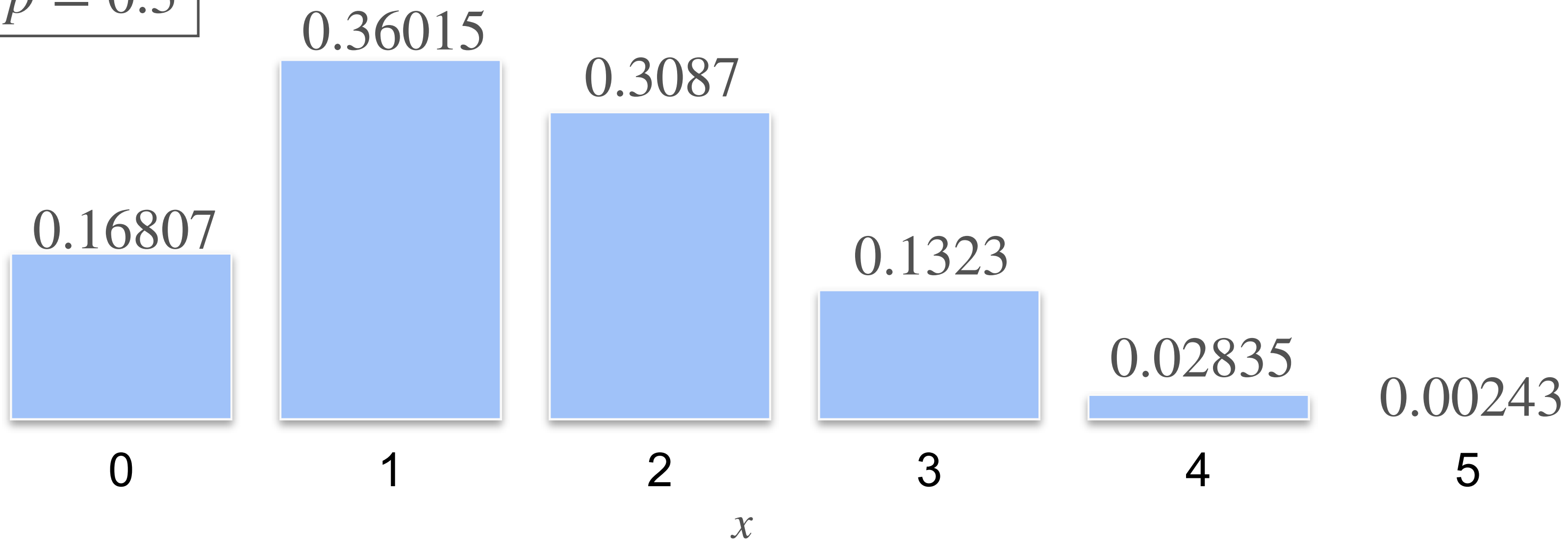
$$\boxed{\begin{array}{c} n = 5 \\ p = 0.3 \end{array}}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$

Binomial Distribution

$$n = 5$$
$$p = 0.3$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$



Binomial Distribution

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

$$X \sim Binomial(5,p)$$

Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,1,2,\ldots,n$$

$$X \sim Binomial(n, p)$$

Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,1,2,\ldots,n$$

$$X \sim Binomial(n,p)$$

n and p are called the **parameters** of the binomial distribution

Binomial Coefficient

Binomial Coefficient

Pick 1st number

n options

Binomial Coefficient

Pick 1st number   Pick 2nd number

n options $n-1$ options

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number
n options	$n-1$ options	$n-2$ options

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number

n options $\quad$ $n-1$ options $\quad$ $n-2$ options $\quad \cdots$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	$n-1$ options	$n-2$ options	$\dots$	$n-(k-1)$options

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options $\quad n-1$ options $\quad n-2$ options $\quad \ldots \quad n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options $\quad n-1$ options $\quad n-2$ options $\quad \dots \quad n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-(k-1))$$

Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k-th number

n options $n-1$ options $n-2$ options $\ldots$ $n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

Some of them repeat

$$n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-(k-1))$$

Binomial Coefficient

Binomial Coefficient

Pick 1st number

n options

Binomial Coefficient

Pick 1st number | Pick 2nd number

n options | n-1 options

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number

n options | n-1 options | n-2 options

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number	
n options	n-1 options	n-2 options	...

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → n-1 options → n-2 options → ... → n-(k-1) options

Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → n-1 options → n-2 options → ... → n-(k-1) options

Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

$$n * (n-1) * (n-2) * \ldots * (n-(k-1))$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

Some of them repeat

$$n * (n-1) * (n-2) * \ldots * (n-(k-1))$$

Binomial Coefficient

Binomial Coefficient

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st Pick 2nd

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st Pick 2nd Pick 3rd

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st　Pick 2nd　Pick 3rd　Pick 4th

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options			

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options		

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options	2 options	

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options	2 options	1 option

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options	2 options	1 option

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

$$4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

Binomial Coefficient

Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
4 options	3 options	2 options	1 option

$$1,2,3,4$$
$$1,2,4,3$$
$$1,3,2,4$$
$$1,3,4,2$$
$$\dots$$
$$4,3,2,1$$

$$4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

For five numbers:

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$$

Binomial Coefficient

Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
4 options	3 options	2 options	1 option

$1,2,3,4$

$1,2,4,3$

$1,3,2,4$

$1,3,4,2$

$\dots$

$4,3,2,1$

$$4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

For five numbers:

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$$

General solution: $k!$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options $\quad n-1$ options $\quad n-2$ options $\quad \ldots \quad n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2 \qquad n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))$

$n,1,3,4,\ldots,2$

$\ldots$

Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — Pick k-th number

n options $\quad n-1$ options $\quad n-2$ options $\quad \dots \quad n-(k-1)$options

Ordered sets of length k ⟶ Unordered sets of length k

$1,2,3,4,\dots,k$

$2,1,3,4,\dots,k$

$5,1,3,k,\dots,2$

$n,1,3,4,\dots,2$

$\dots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options $\quad n-1$ options $\quad n-2$ options $\quad \ldots \quad n-(k-1)$options

Ordered sets of length k $\longrightarrow$ Unordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!} = \underline{\hspace{3cm}}$$

Binomial Coefficient

Pick 1st number — n options

Pick 2nd number — $n-1$ options

Pick 3rd number — $n-2$ options

$\ldots$

Pick k-th number — $n-(k-1)$options

Ordered sets of length k ⟶ Unordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!}=\frac{n!}{(n-k)!}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options | $n-1$ options | $n-2$ options | $\dots$ | $n-(k-1)$ options

Ordered sets of length k → Unordered sets of length k

$1,2,3,4,\dots,k$

$2,1,3,4,\dots,k$

$5,1,3,k,\dots,2$

$n,1,3,4,\dots,2$

$\dots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!}=\frac{n!}{(n-k)!\,k!}$$

Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — Pick k-th number

n options $\quad$ $n-1$ options $\quad$ $n-2$ options $\quad \ldots \quad$ $n-(k-1)$options

Ordered sets of length k $\longrightarrow$ Unordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-(k-1))}{k!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$n * (n-1) * (n-2) * \ldots * (n-(k-1))$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!}$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!} = \frac{}{}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → n-1 options → n-2 options → ... → n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!} = \frac{n!}{(n-k)!}$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!} = \frac{n!}{(n-k)! * k!}$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

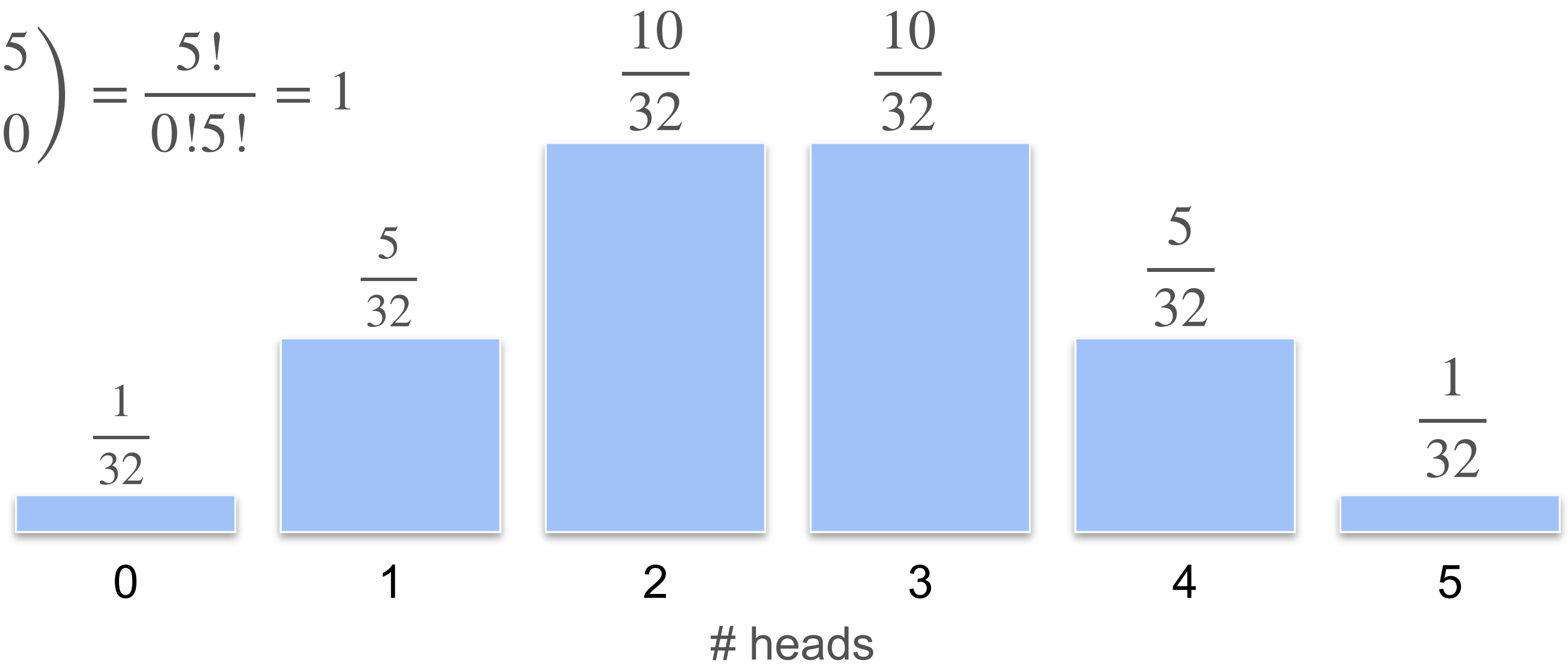
n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!} = \frac{n!}{(n-k)! * k!} = \binom{n}{k}$$

Binomial Distribution: Fair Coins

$$\binom{5}{0} = \frac{5!}{0!5!} = 1$$



Binomial Distribution: Fair Coins

$50\,\%$

$50\,\%$

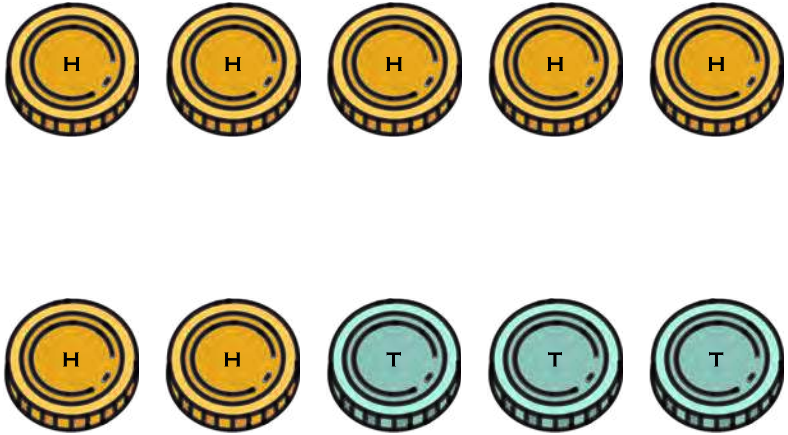
$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

Binomial Distribution: Biased Coins

30 %

70 %



Binomial Distribution: Biased Coins

30 %

70 %

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$

Binomial Distribution: Biased Coins

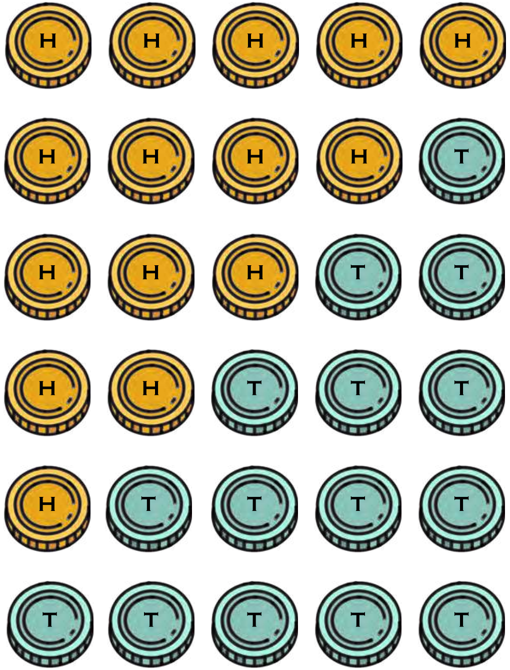
30 %

70 %

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.01323$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7$$

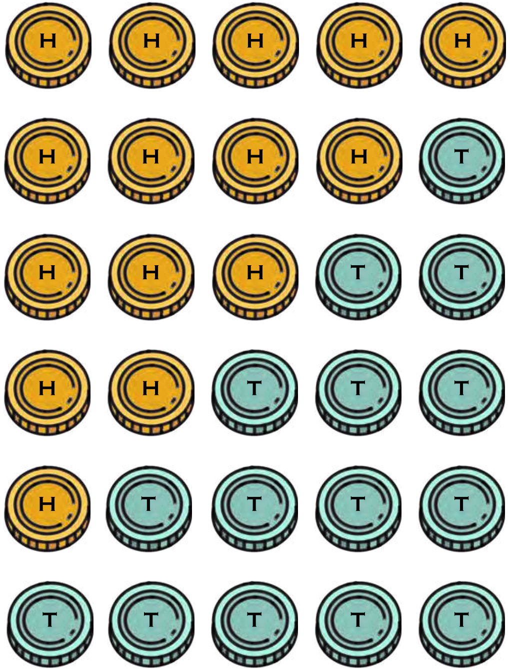
$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7$$

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

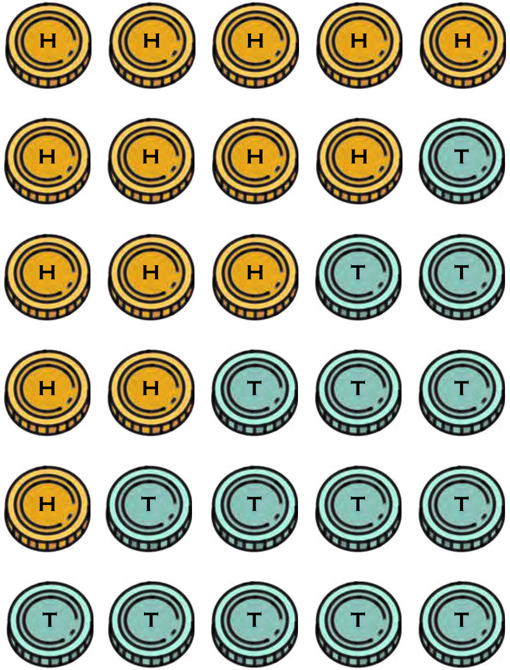
$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

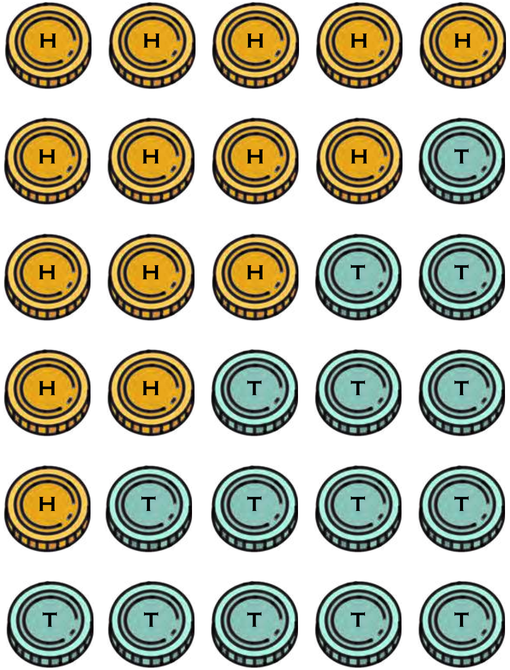
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

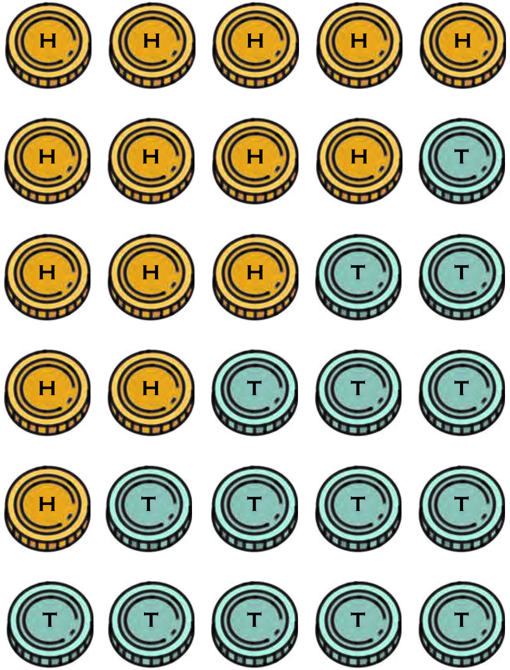
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

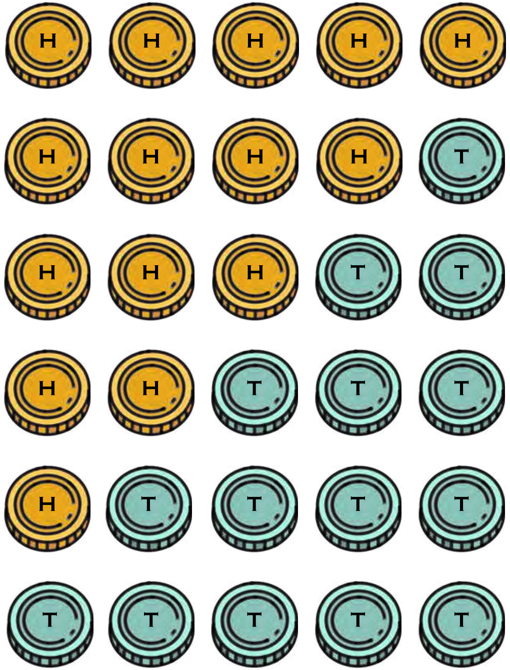
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 = 0.3^4 \cdot 0.7^1$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 = 0.3^3 \cdot 0.7^2$$

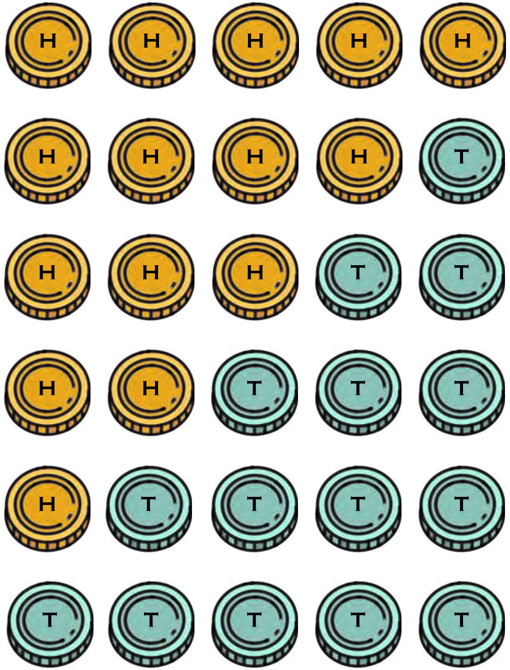
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^2 \cdot 0.7^3$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^1 \cdot 0.7^4$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3^5 \cdot 0.7^0$$

$$0.3^4 \cdot 0.7^1$$

$$0.3^3 \cdot 0.7^2$$

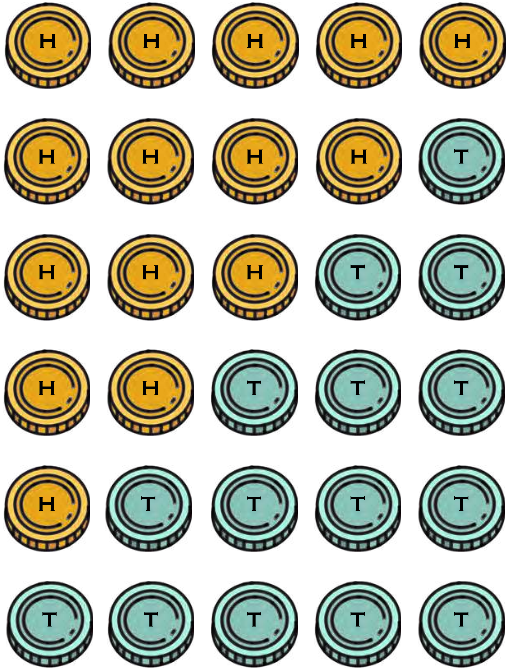
$$0.3^2 \cdot 0.7^3$$

$$0.3^1 \cdot 0.7^4$$

$$0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$0.3^5 \cdot 0.7^0$

$0.3^4 \cdot 0.7^1$

$0.3^3 \cdot 0.7^2$

$0.3^2 \cdot 0.7^3$

$0.3^1 \cdot 0.7^4$

$0.3^0 \cdot 0.7^5$

$$= 0.3^k \cdot 0.7^{n-k} \Rightarrow \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Account for all possible orders of heads and tails

Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins

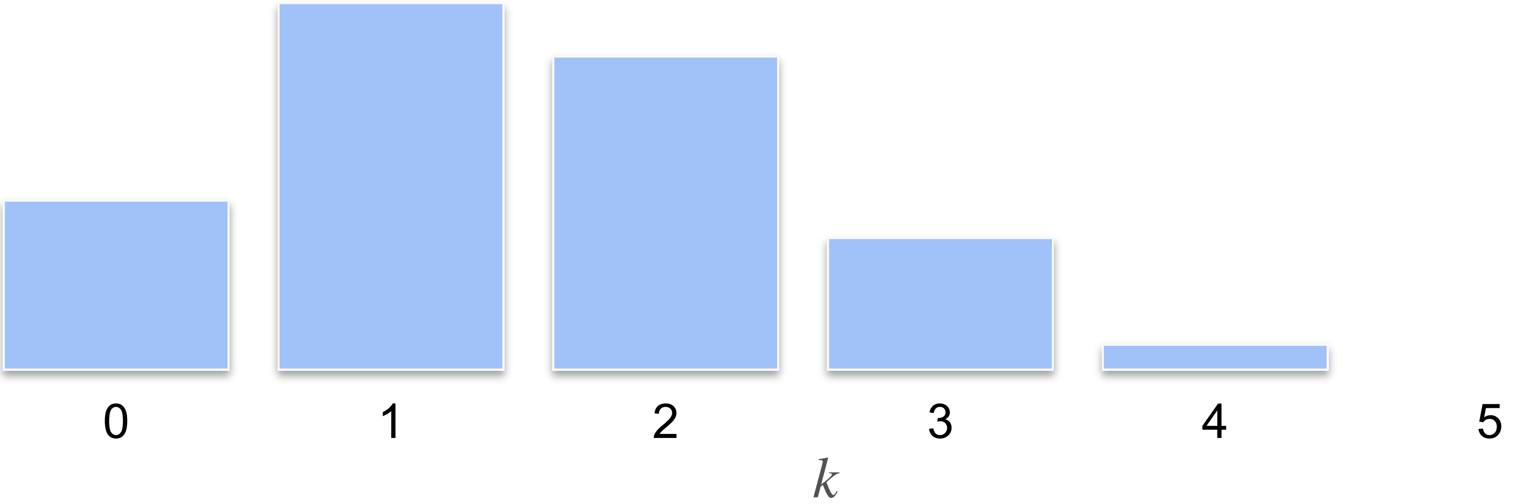
n = 5

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

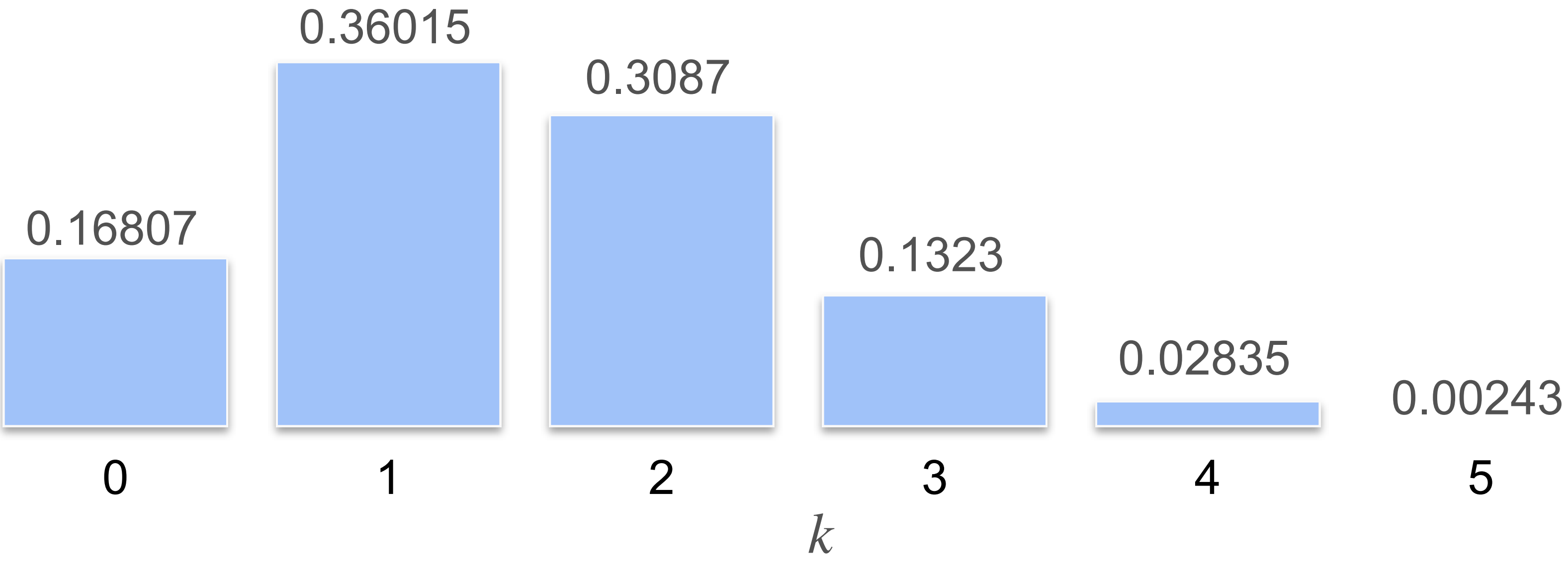
n = 5
p = 0.3



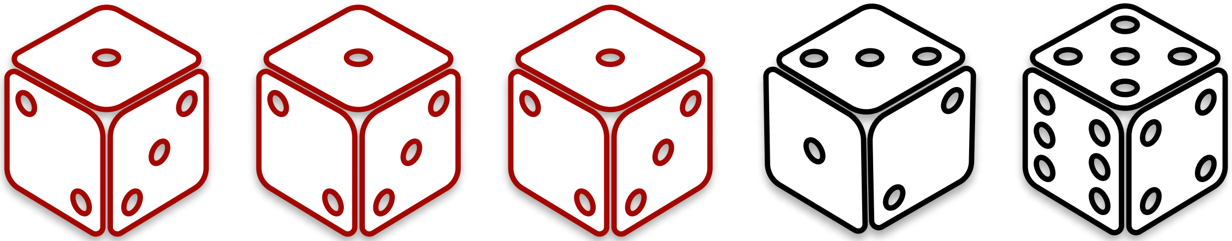
Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

n = 5
p = 0.3

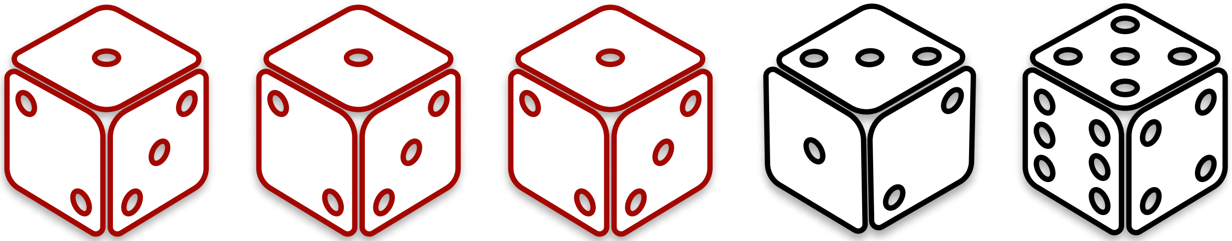


Binomial Distribution: Quiz

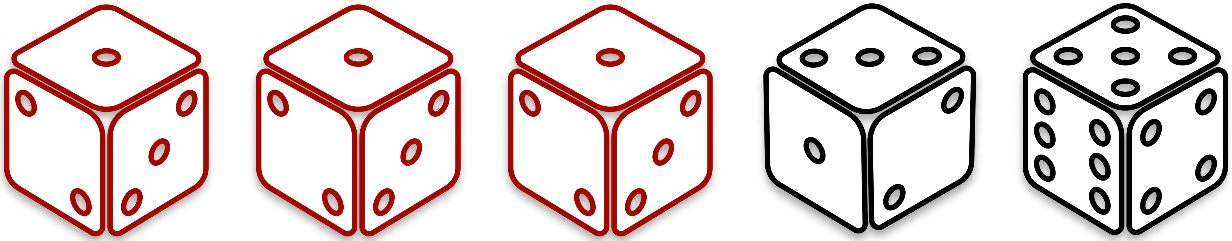


Binomial Distribution: Quiz

What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?

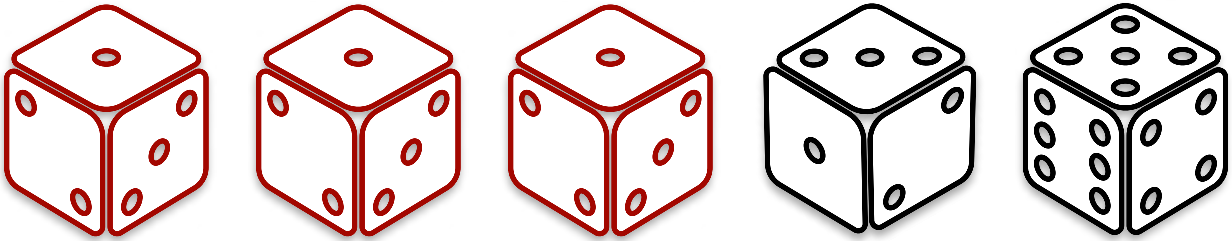


Binomial Distribution: Quiz

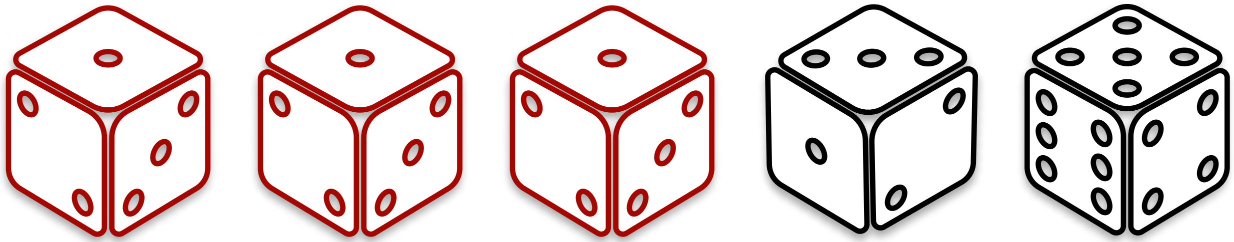


Binomial Distribution: Quiz

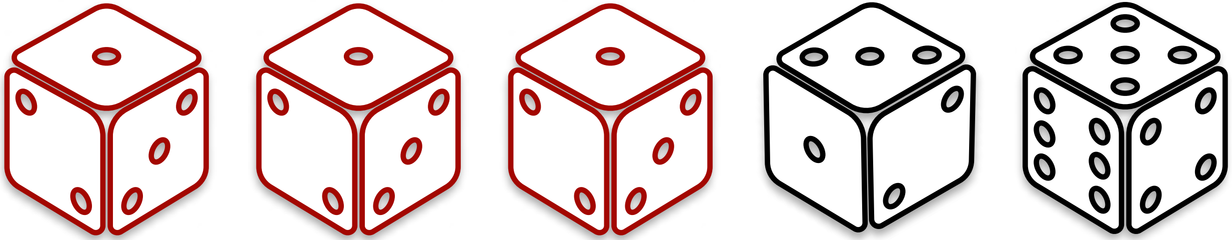
What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?



Binomial Distribution: Dice Is a Biased Coin!

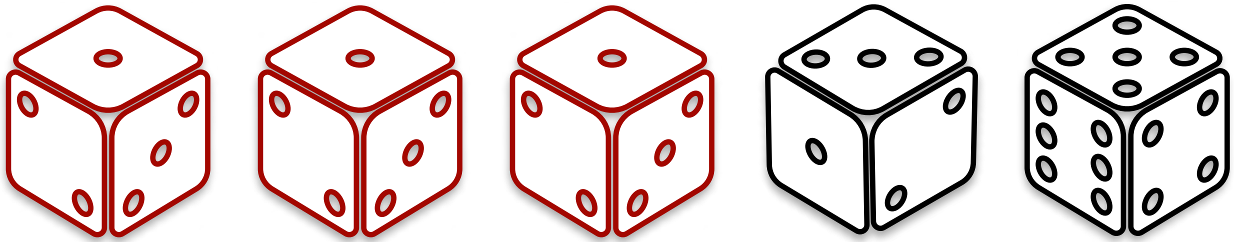


Binomial Distribution: Dice Is a Biased Coin!



one	one	one	not one	not one
$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{5}{6}$	$p = \frac{5}{6}$

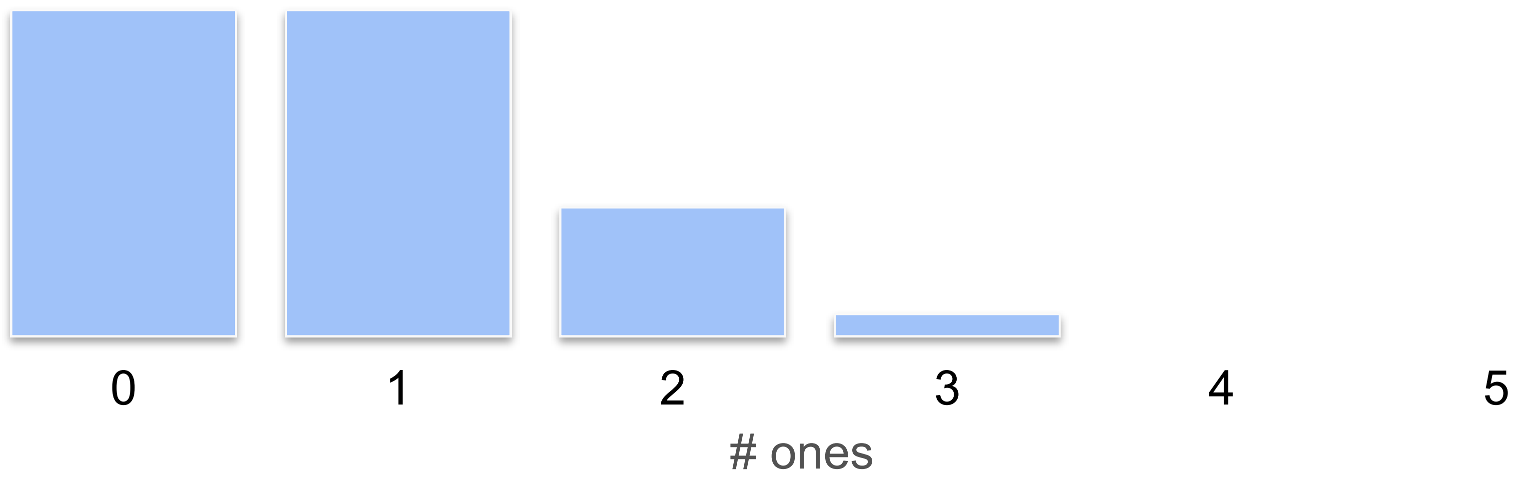
Binomial Distribution: Dice Is a Biased Coin!



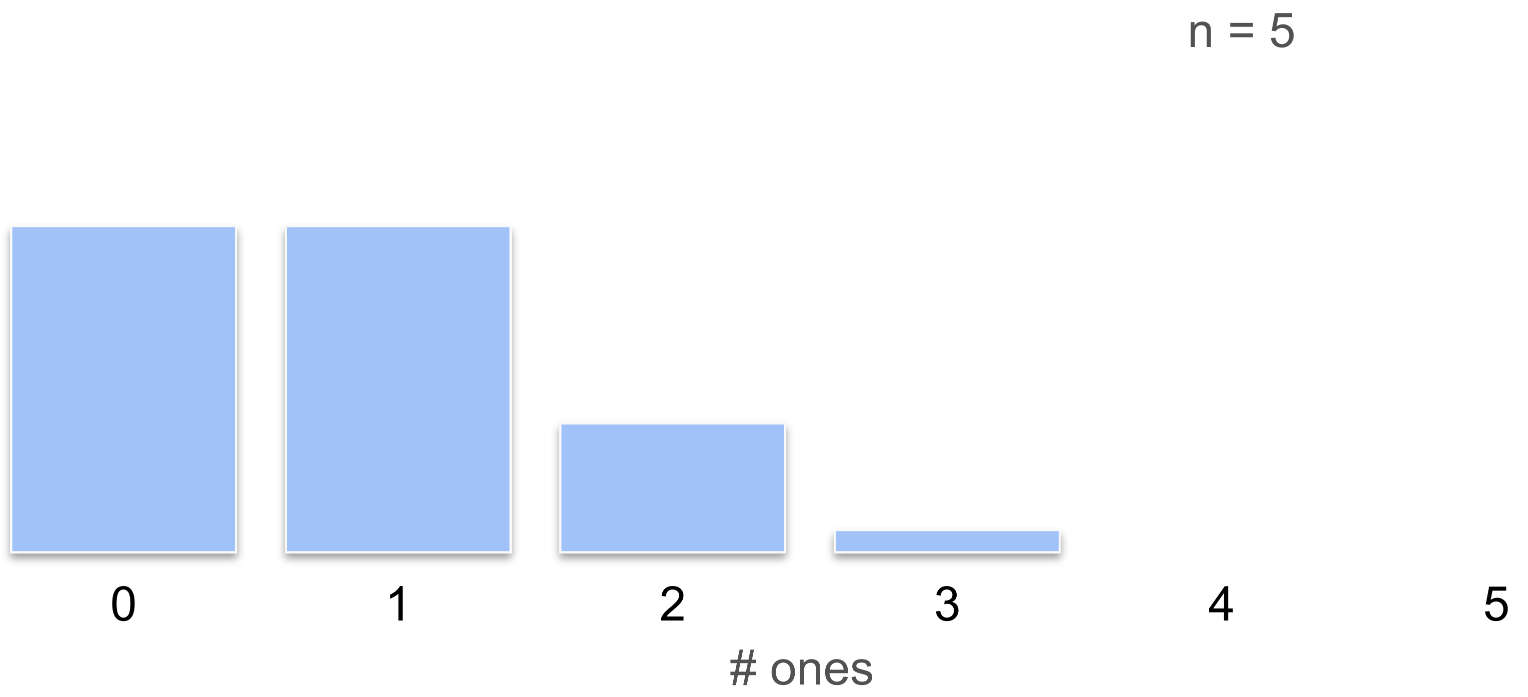
one	one	one	not one	not one
$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{5}{6}$	$p = \frac{5}{6}$

heads	heads	heads	not heads	not heads
$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{5}{6}$	$p = \frac{5}{6}$

Binomial Distribution: Dice Is a Biased Coin!

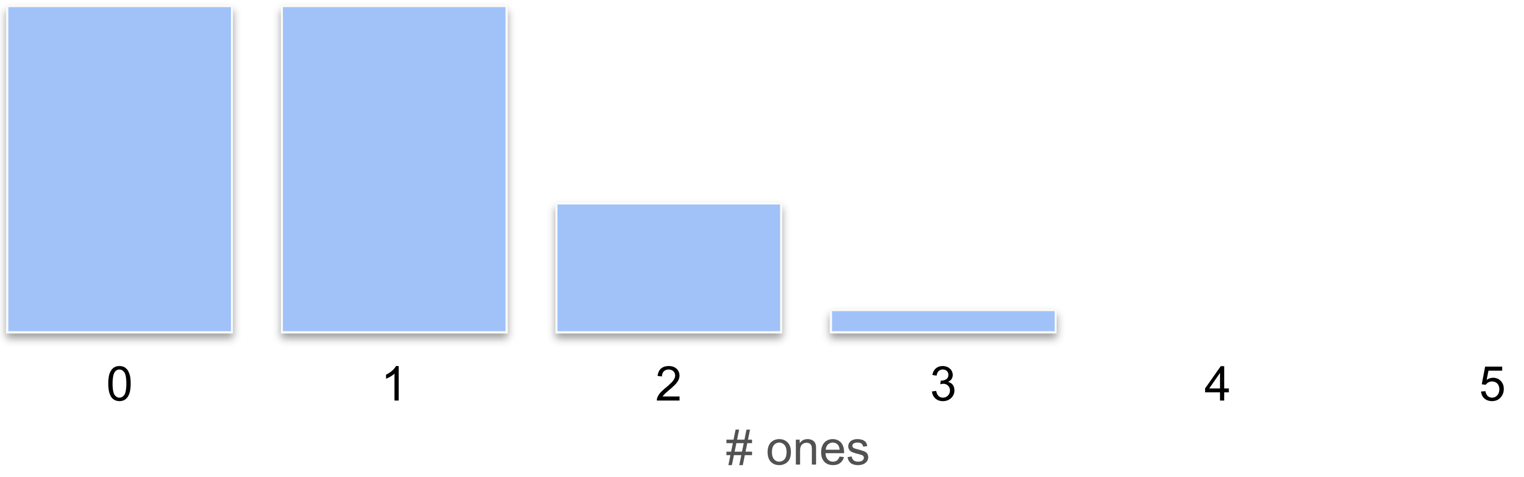


Binomial Distribution: Dice Is a Biased Coin!



Binomial Distribution: Dice Is a Biased Coin!

n = 5
p = 0.1666



Binomial Distribution: Dice Is a Biased Coin!

n = 5
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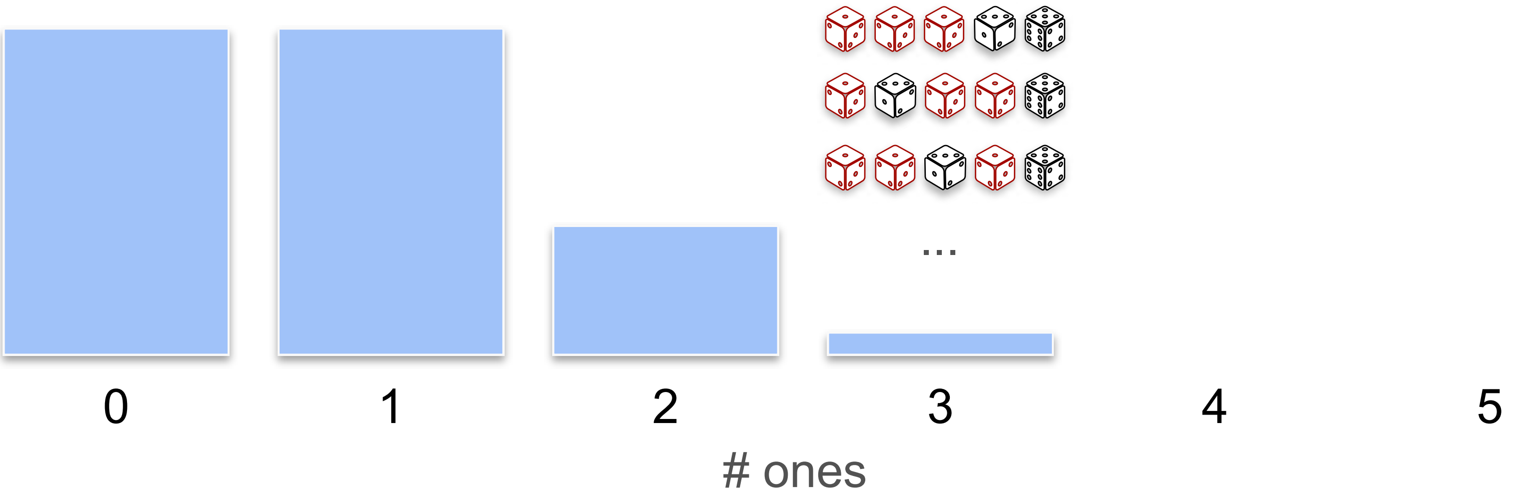
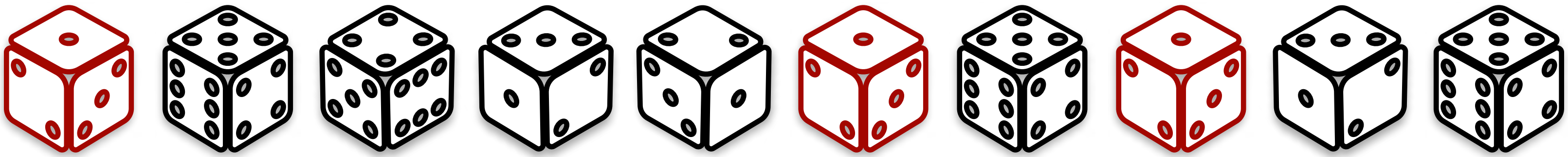
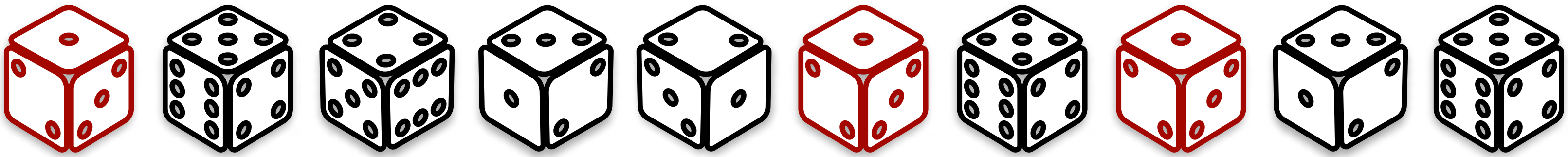


Binomial Distribution: Quiz

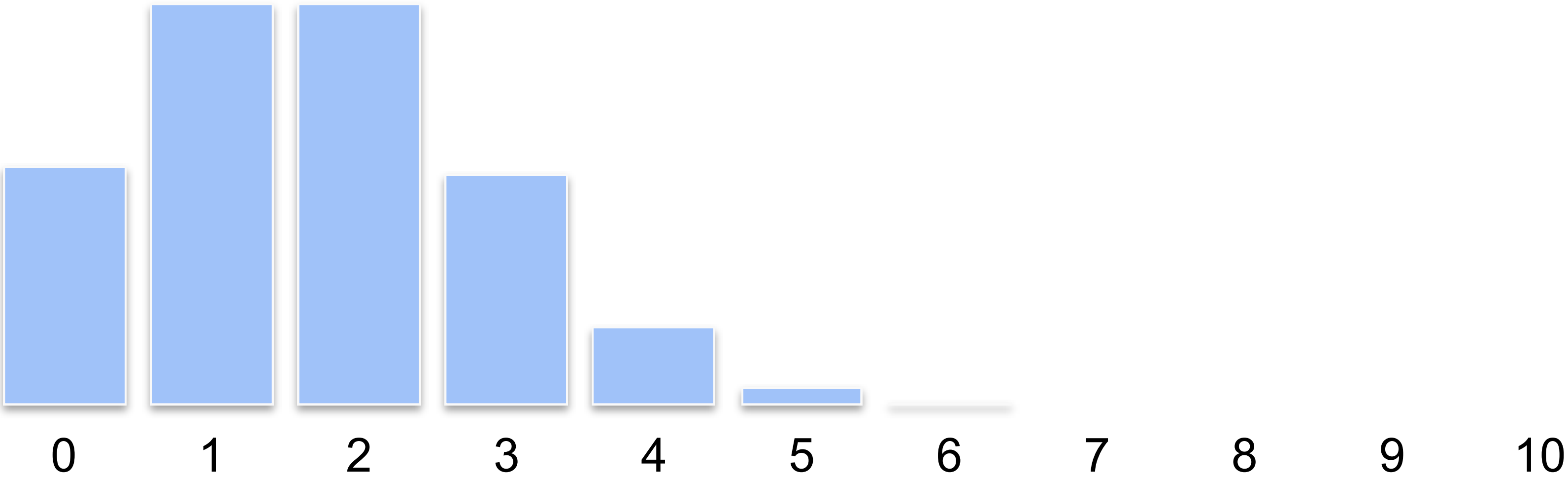


Binomial Distribution: Quiz

- Quiz: What are the parameters for the following binomial distribution:
  - I roll 10 dice
  - I want to record the number of times I obtain the number 1

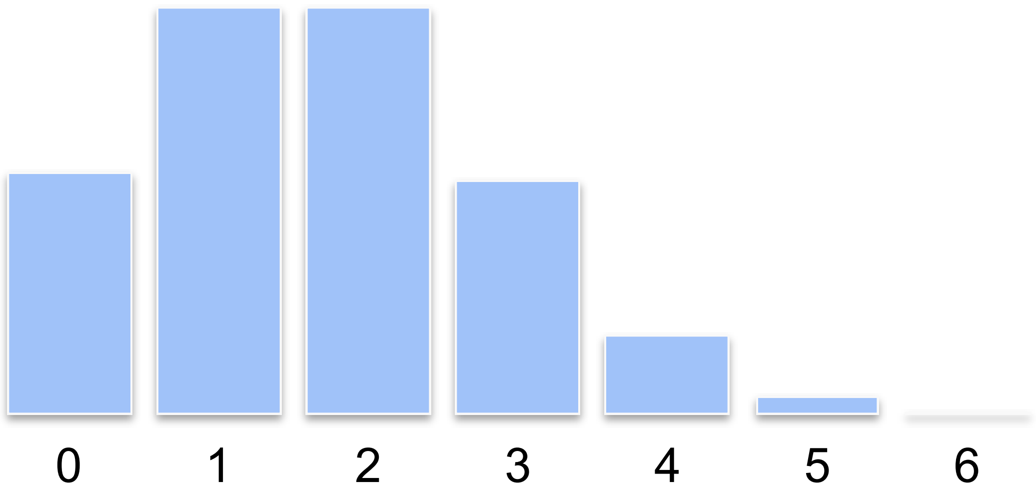


Binomial Distribution: Quiz



Binomial Distribution: Quiz

$n = 10$



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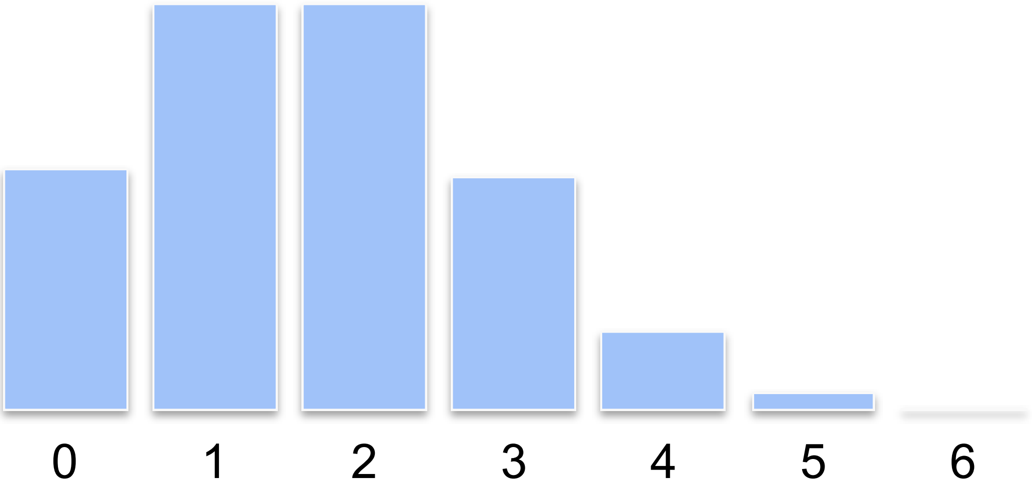
Binomial Distribution: Quiz

$n = 10$
$p = 0.1666$



Bernoulli Distribution

Bernoulli Distribution

$X =$ Number of heads

Bernoulli Distribution

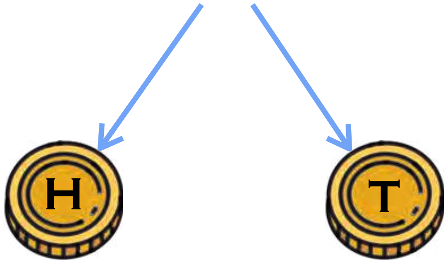
$X =$ Number of heads

Flip a coin

Bernoulli Distribution

$X =$ Number of heads

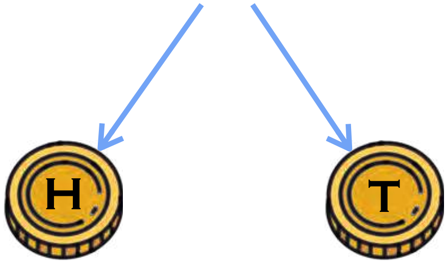
Flip a coin



Bernoulli Distribution

$X =$ Number of heads

Flip a coin

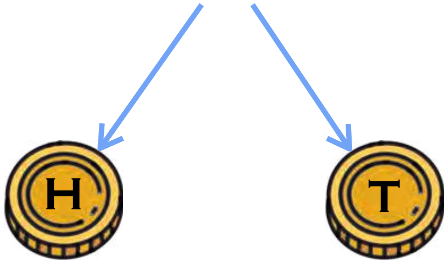


$\mathrm{P}(X = 1) = 0.5$

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

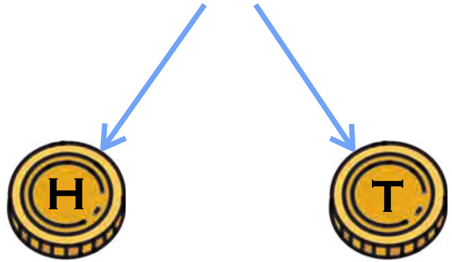


$\mathbf{P}(X = 1) = 0.5$ $\mathbf{P}(X = 0) = 0.5$

Bernoulli Distribution

$X =$ Number of heads

Flip a coin



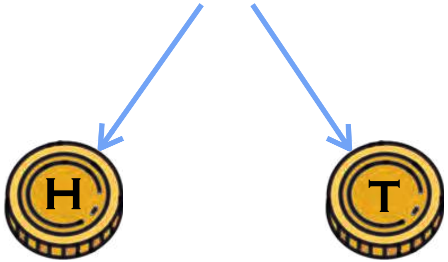
$\mathrm{P}(X = 1) = 0.5$ $\mathrm{P}(X = 0) = 0.5$

Success

Bernoulli Distribution

$X =$ Number of heads

Flip a coin



$\mathrm{P}(X = 1) = 0.5$ $\mathrm{P}(X = 0) = 0.5$

Success Failure

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

$\mathrm{P}(X=1) = 0.5$

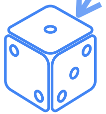
Success

$\mathrm{P}(X=0) = 0.5$

Failure

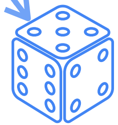
$X =$ Number of 1's

Throw a dice



$\mathrm{P}(X=1) = \frac{1}{6}$

Success



$\mathrm{P}(X=0) = \frac{5}{6}$

Failure

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

$\mathrm{P}(X = 1) = 0.5$

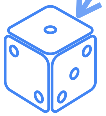
Success

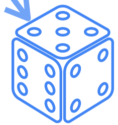
$\mathrm{P}(X = 0) = 0.5$

Failure

$X =$ Number of 1's

Throw a dice





$\mathrm{P}(X = 1) = \frac{1}{6}$

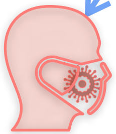
Success

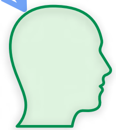
$\mathrm{P}(X = 0) = \frac{5}{6}$

Failure

$X =$ Number of sick patients

Flip a coin





$\mathrm{P}(X = 1) = p$

Success

$\mathrm{P}(X = 0) = 1 - p$

Failure

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

$\mathrm{P}(X = 1) = 0.5$ $\mathrm{P}(X = 0) = 0.5$

Success Failure

$X =$ Number of 1's

Throw a dice

$\mathrm{P}(X = 1) = \frac{1}{6}$ $\mathrm{P}(X = 0) = \frac{5}{6}$

Success Failure

$X =$ Number of sick patients

Flip a coin

$\mathrm{P}(X = 1) = p$ $\mathrm{P}(X = 0) = 1 - p$

Success Failure

$X \sim Bernoulli(p)$

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

$\mathrm{P}(X = 1) = 0.5$

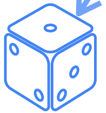
Success

$\mathrm{P}(X = 0) = 0.5$

Failure

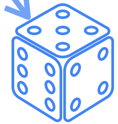
$X =$ Number of 1's

Throw a dice



$\mathrm{P}(X = 1) = \frac{1}{6}$

Success

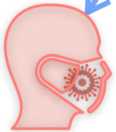


$\mathrm{P}(X = 0) = \frac{5}{6}$

Failure

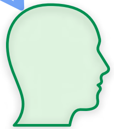
$X =$ Number of sick patients

Flip a coin



$\mathrm{P}(X = 1) = p$

Success



$\mathrm{P}(X = 0) = 1 - p$

Failure

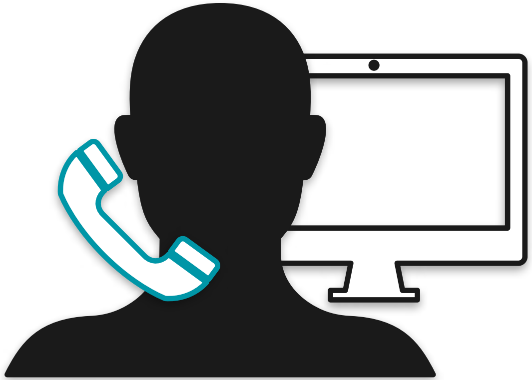
$X \sim Bernoulli(p)$ ← p is the parameter of the Bernoulli distribution

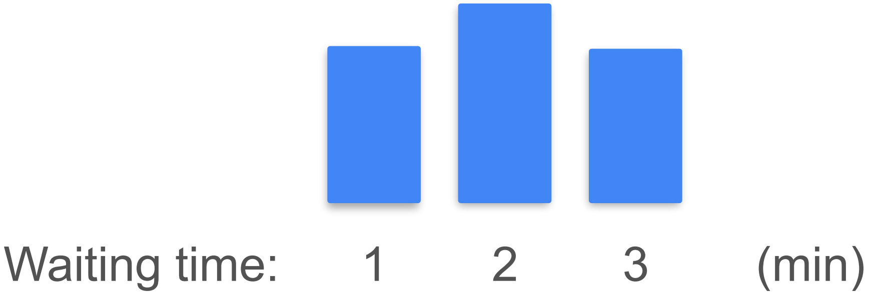
DeepLearning.AI

Probability Distributions

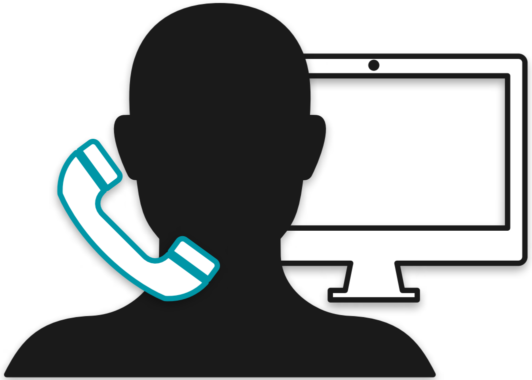
Probability Distributions (Continuous)

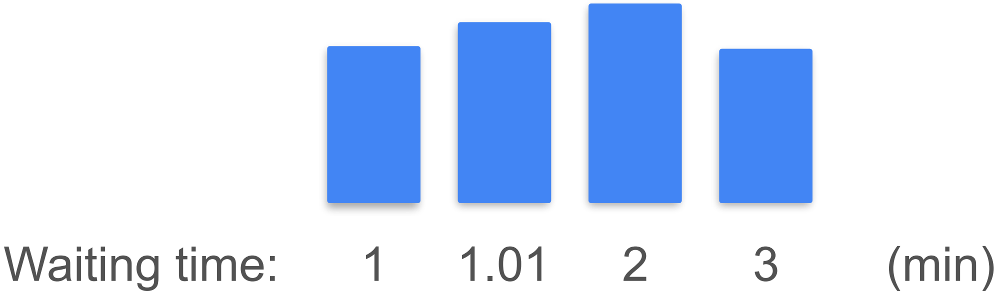
From Discrete to Continuous Distributions



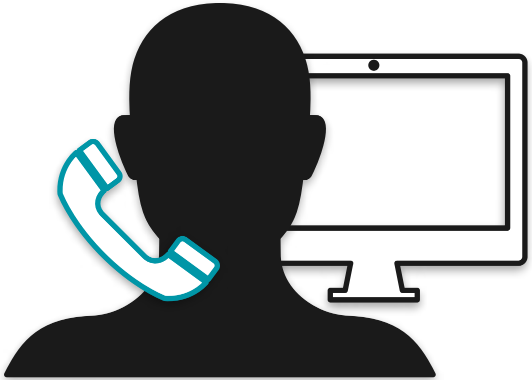


From Discrete to Continuous Distributions

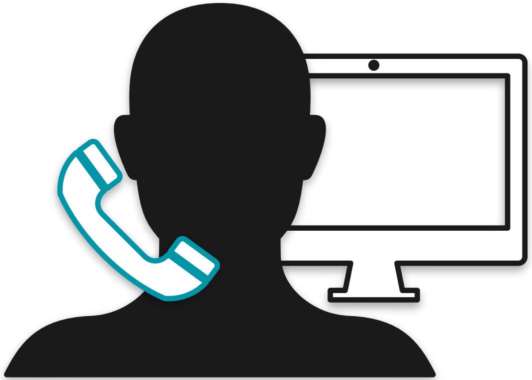


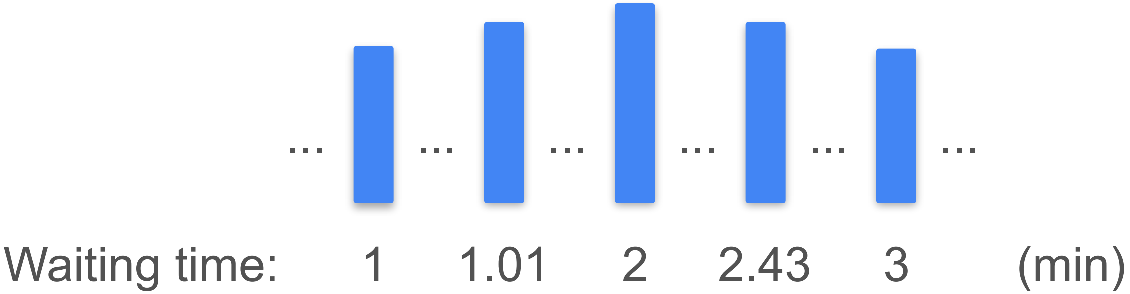


From Discrete to Continuous Distributions

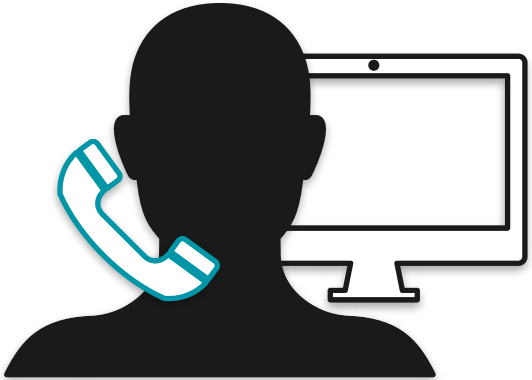


From Discrete to Continuous Distributions

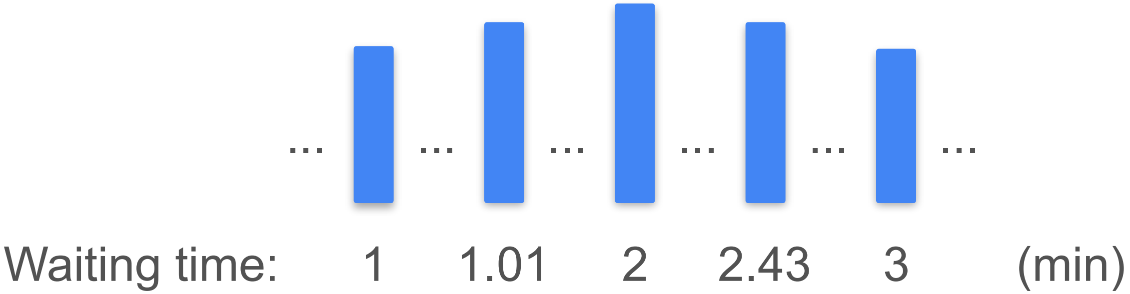




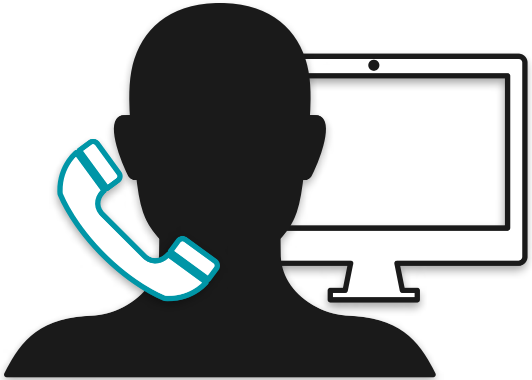
From Discrete to Continuous Distributions



This is a continuous distribution!



From Discrete to Continuous Distributions



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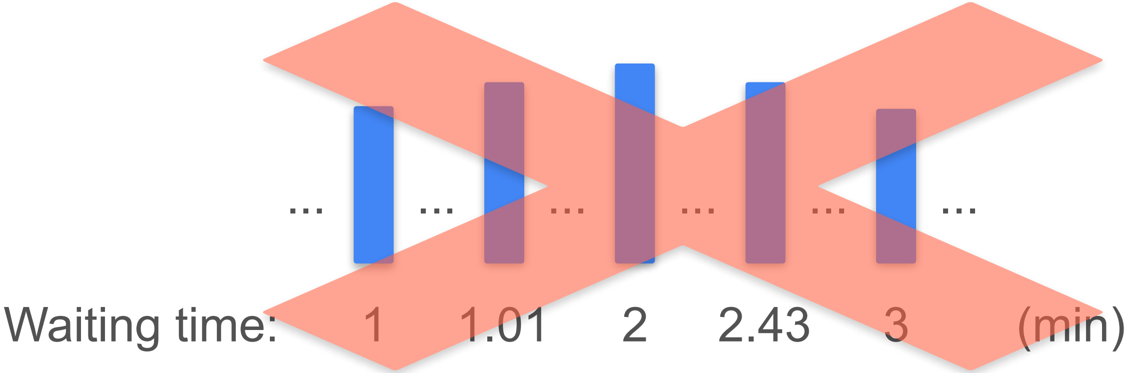
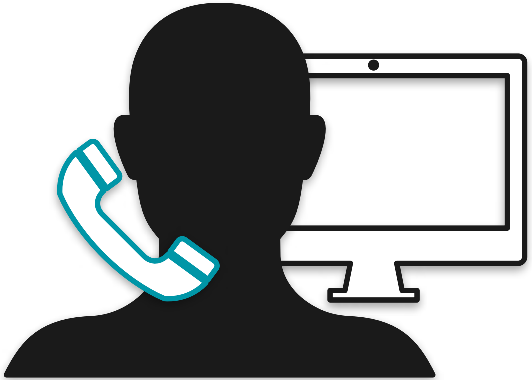


From Discrete to Continuous Distributions



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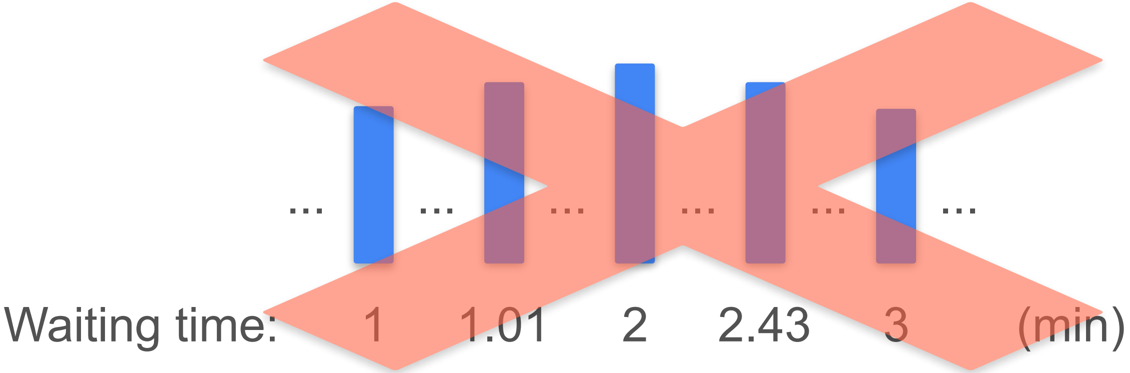
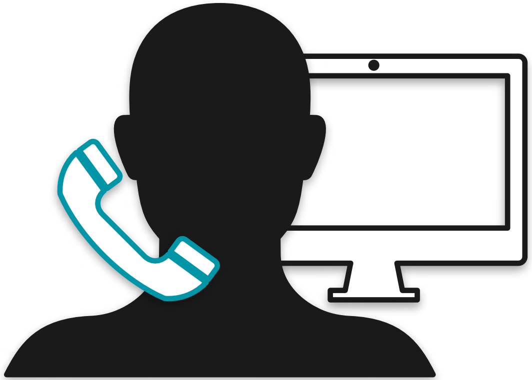
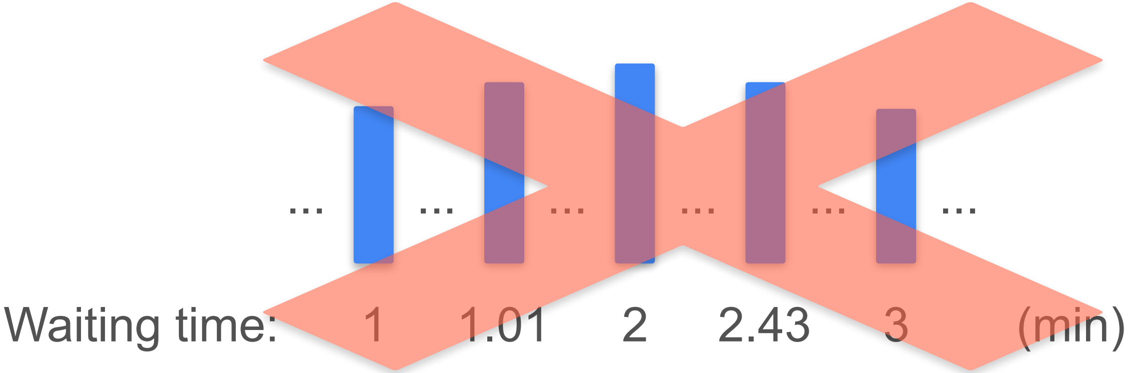


What is the probability that you will wait EXACTLY one minute for the call?

From Discrete to Continuous Distributions



This is a continuous distribution!



What is the probability that you will wait EXACTLY one minute for the call?

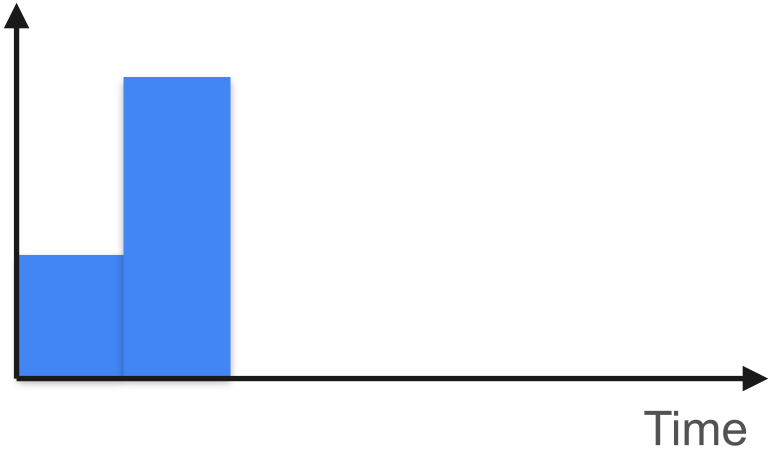
Answer: ZERO

From Discrete to Continuous Distributions

From Discrete to Continuous Distributions

$\mathbf{P}$(between 0 and 1 mins)

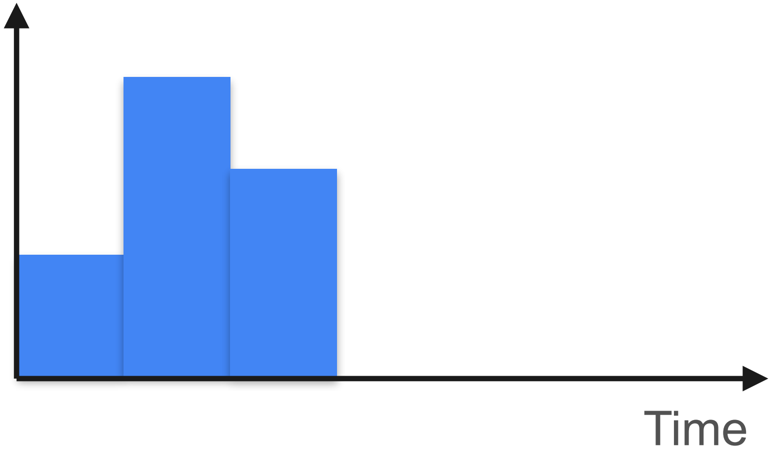
From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 1 mins)

$\mathbf{P}$(between 1 and 2 mins)

From Discrete to Continuous Distributions

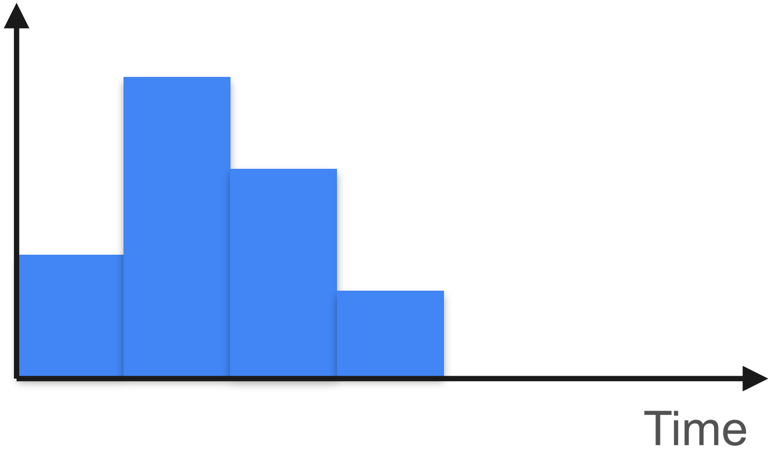


$\mathbf{P}$(between 0 and 1 mins)

$\mathbf{P}$(between 1 and 2 mins)

$\mathbf{P}$(between 2 and 3 mins)

From Discrete to Continuous Distributions



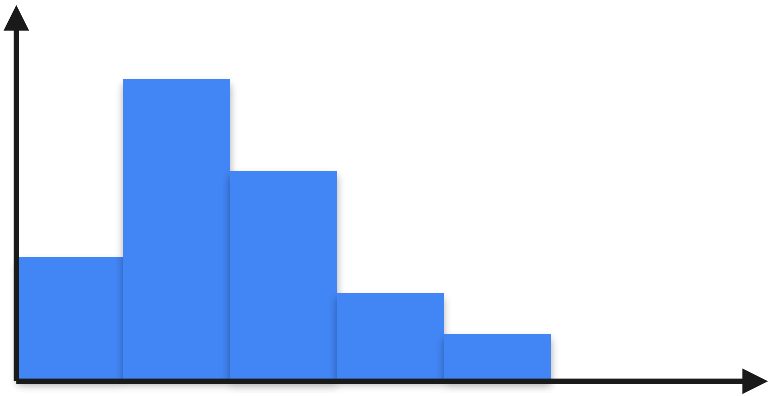
$\mathbf{P}$(between 0 and 1 mins)

$\mathbf{P}$(between 1 and 2 mins)

$\mathbf{P}$(between 2 and 3 mins)

$\mathbf{P}$(between 3 and 4 mins)

From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 1 mins)

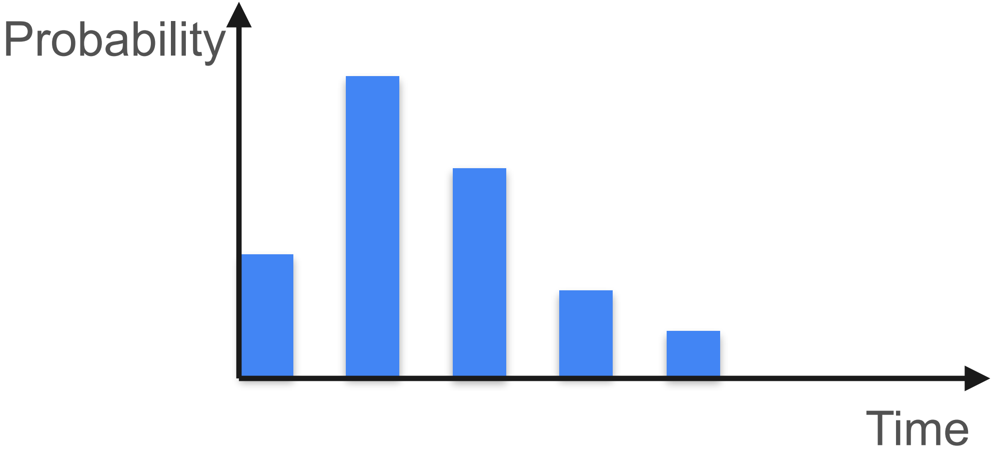
$\mathbf{P}$(between 1 and 2 mins)

$\mathbf{P}$(between 2 and 3 mins)

$\mathbf{P}$(between 3 and 4 mins)

$\mathbf{P}$(between 4 and 5 mins)

From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 1 mins)

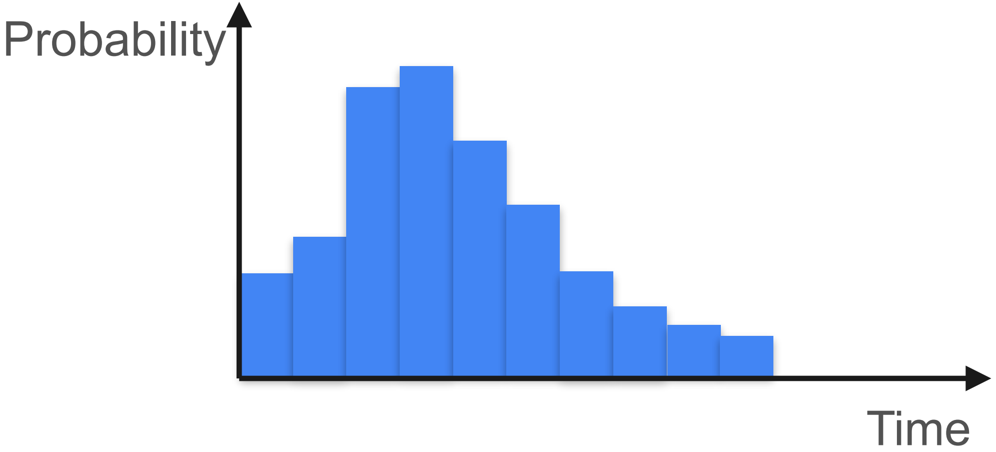
$\mathbf{P}$(between 1 and 2 mins)

$\mathbf{P}$(between 2 and 3 mins)

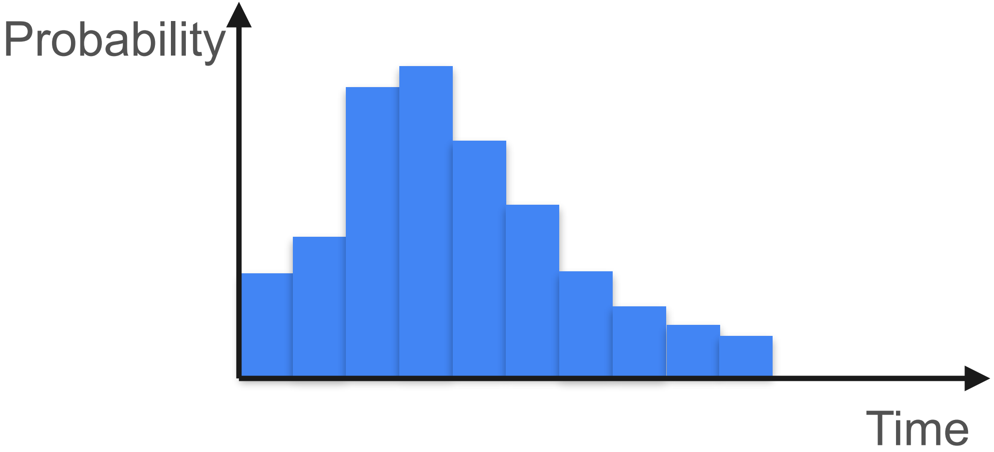
$\mathbf{P}$(between 3 and 4 mins)

$\mathbf{P}$(between 4 and 5 mins)

From Discrete to Continuous Distributions

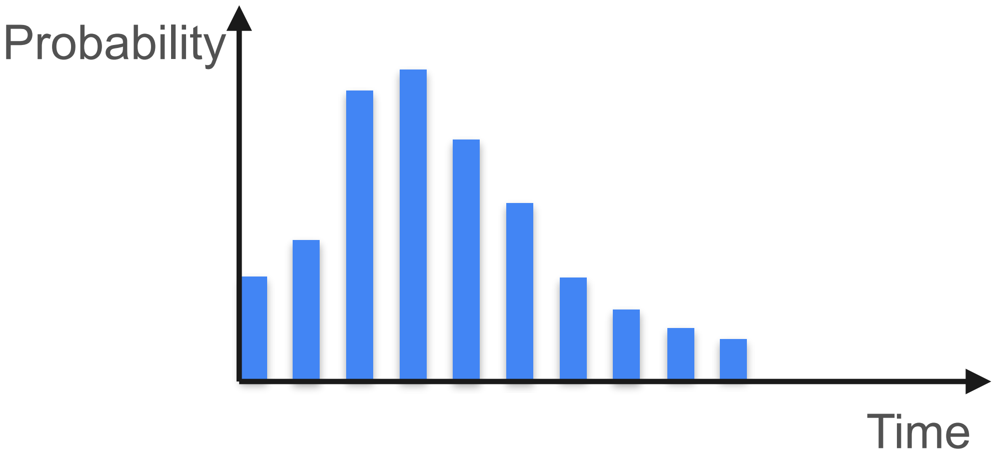


From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 0.5 mins)
$\mathbf{P}$(between 0.5 and 1 mins)
$\mathbf{P}$(between 1 and 1.5 mins)
⋮
$\mathbf{P}$(between 3.5 and 4 mins)
$\mathbf{P}$(between 4 and 4.5 mins)
$\mathbf{P}$(between 4.5 and 5 mins)

From Discrete to Continuous Distributions

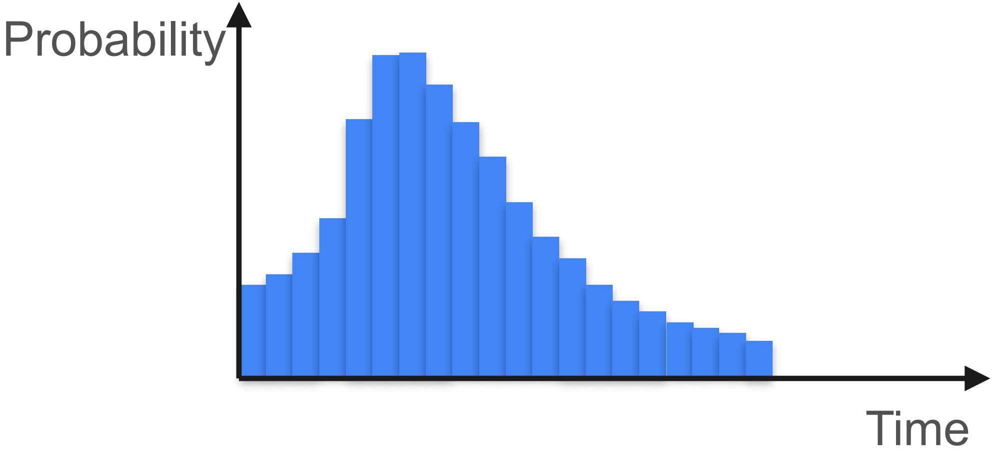


P(between 0 and 0.5 mins)
P(between 0.5 and 1 mins)
P(between 1 and 1.5 mins)

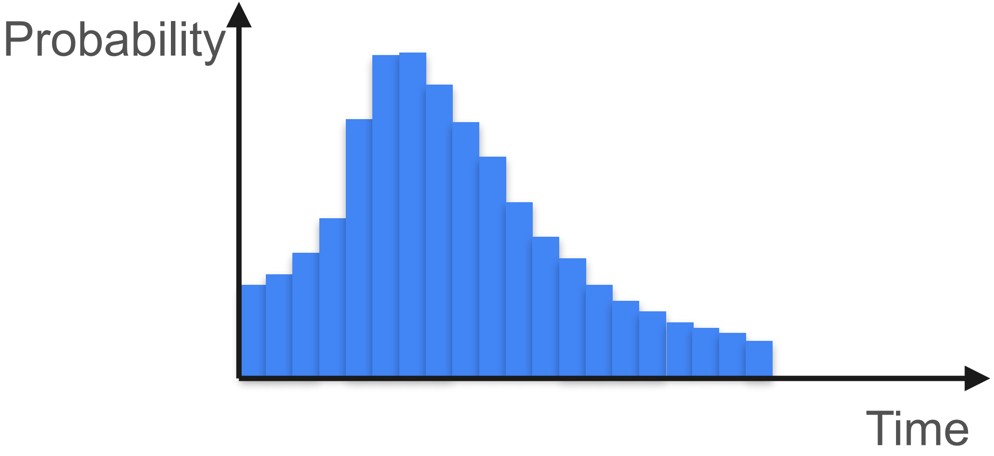
⋮

P(between 3.5 and 4 mins)
P(between 4 and 4.5 mins)
P(between 4.5 and 5 mins)

From Discrete to Continuous Distributions



From Discrete to Continuous Distributions

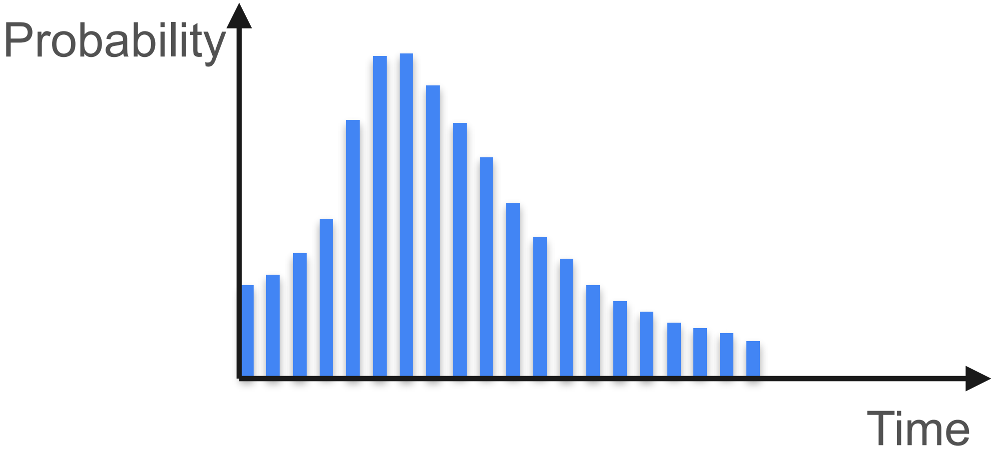


P(between 0 and 0.25 mins)
P(between 0.25 and 0.5 mins)
P(between 0.5 and 0.75 mins)

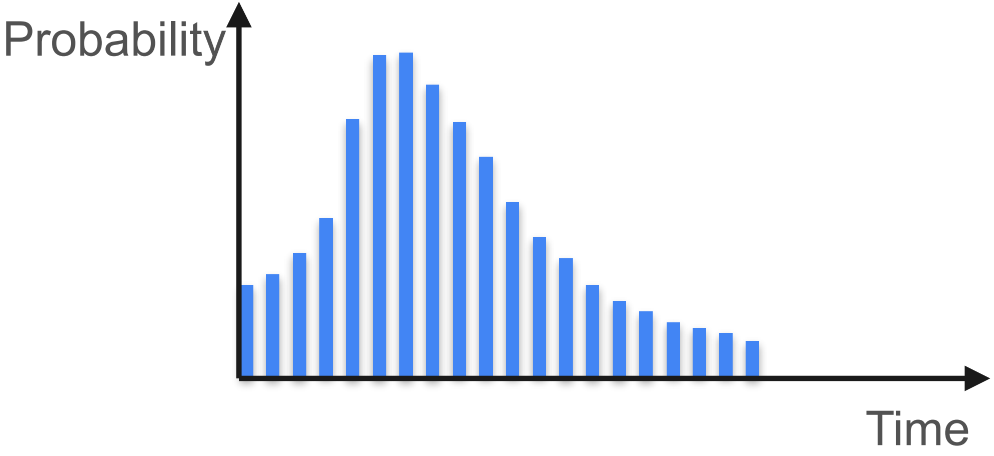
⋮

P(between 4.25 and 4.5 mins)
P(between 4.5 and 4.75 mins)
P(between 4.75 and 5 mins)

From Discrete to Continuous Distributions

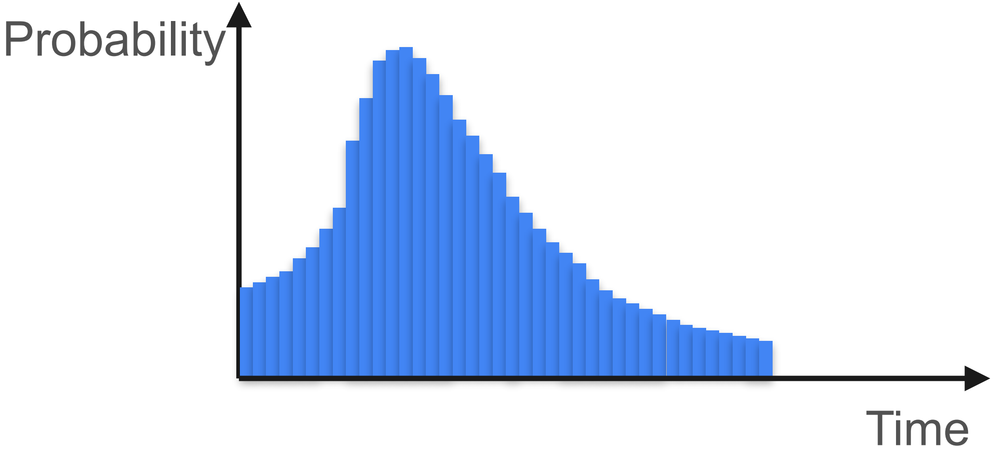


From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 0.25 mins)
$\mathbf{P}$(between 0.25 and 0.5 mins)
$\mathbf{P}$(between 0.5 and 0.75 mins)
⋮
$\mathbf{P}$(between 4.25 and 4.5 mins)
$\mathbf{P}$(between 4.5 and 4.75 mins)
$\mathbf{P}$(between 4.75 and 5 mins)

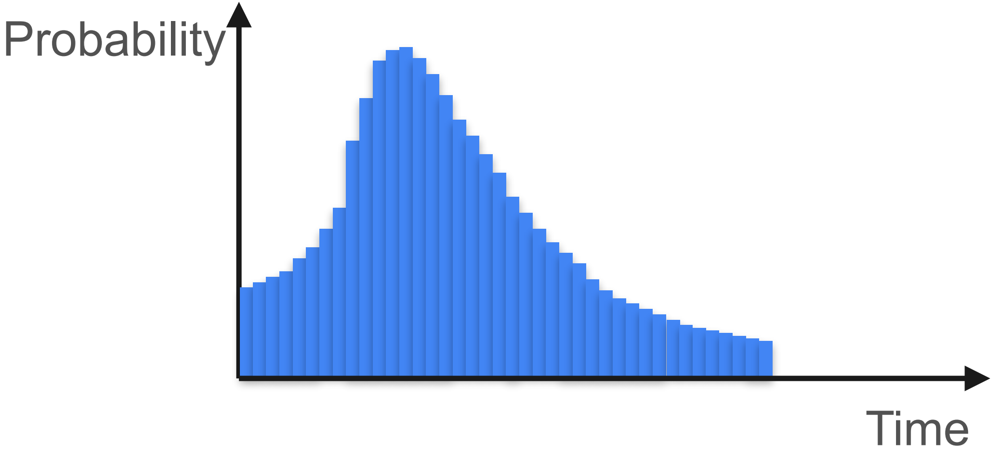
From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 0.125 mins)

$\mathbf{P}$(between 4.875 and 5 mins)

From Discrete to Continuous Distributions

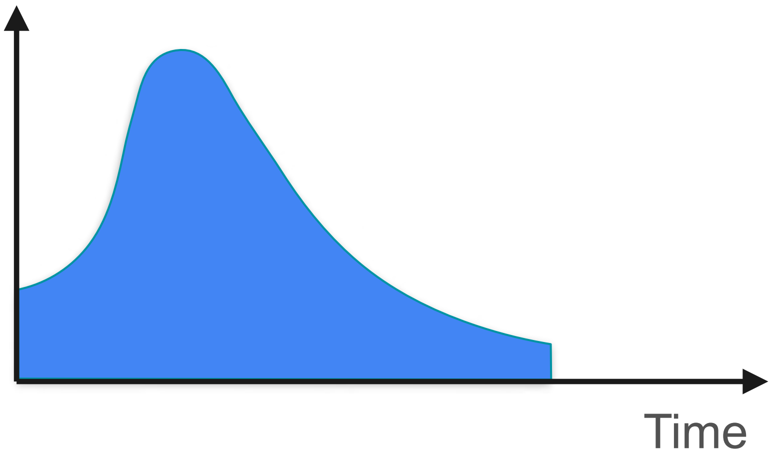


$\mathbf{P}$(between 0 and 0.125 mins)

⋮

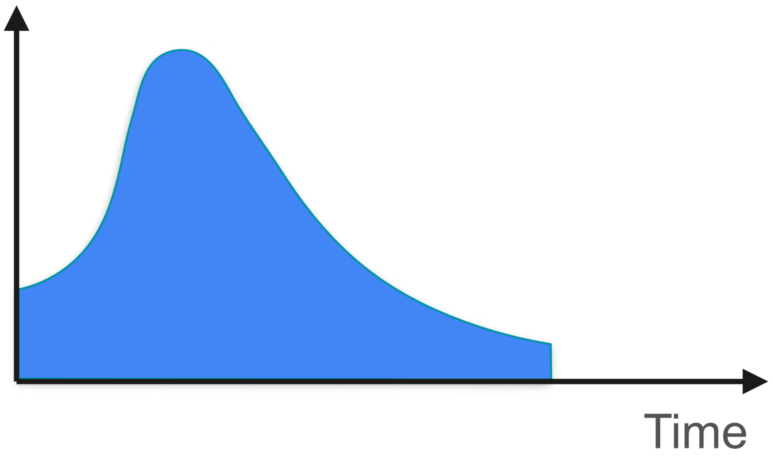
$\mathbf{P}$(between 4.875 and 5 mins)

From Discrete to Continuous Distributions

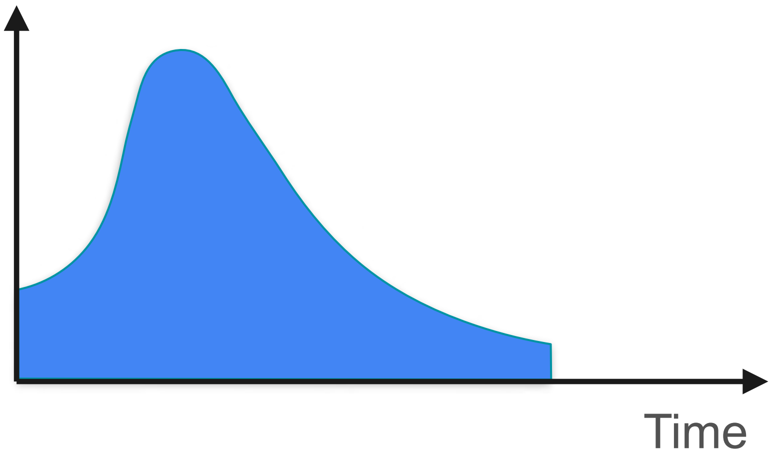


From Discrete to Continuous Distributions

- Discrete:
  - Sum of heights equals 1

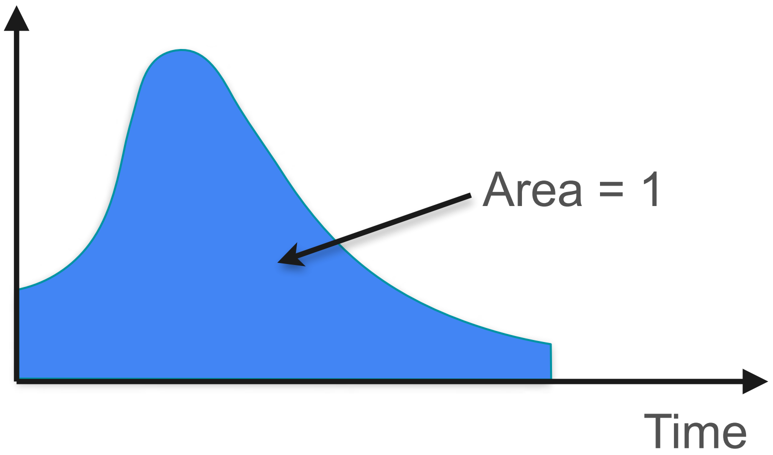


From Discrete to Continuous Distributions



- Discrete:
  - Sum of heights equals 1
- Continuous:
  - Area under the curve equals 1

From Discrete to Continuous Distributions



- Discrete:
  - Sum of heights equals 1
- Continuous:
  - Area under the curve equals 1

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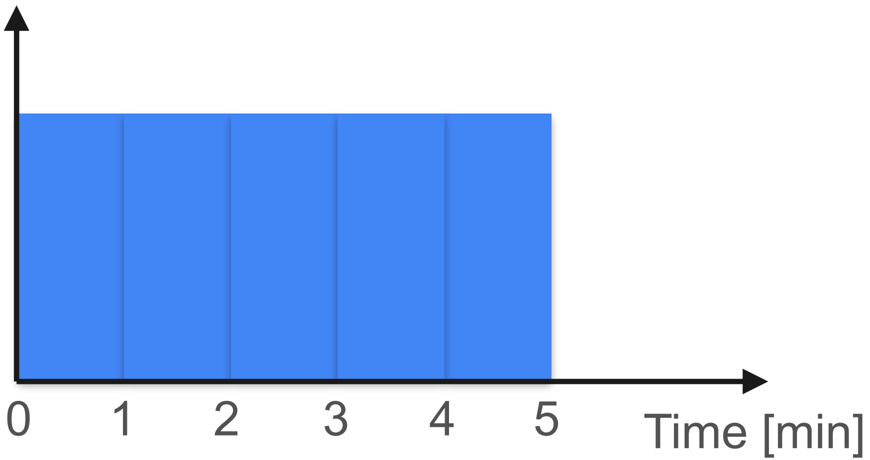
Probability Distributions

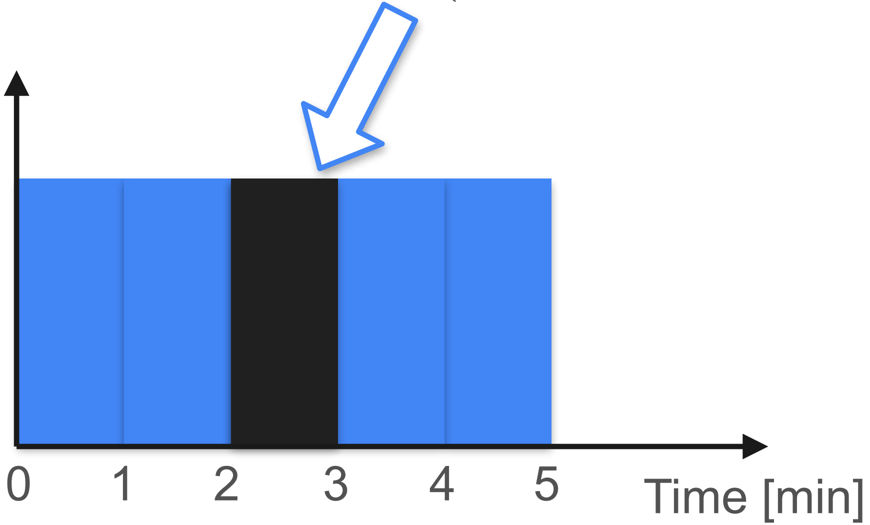
Probability density function

Probability Density Function

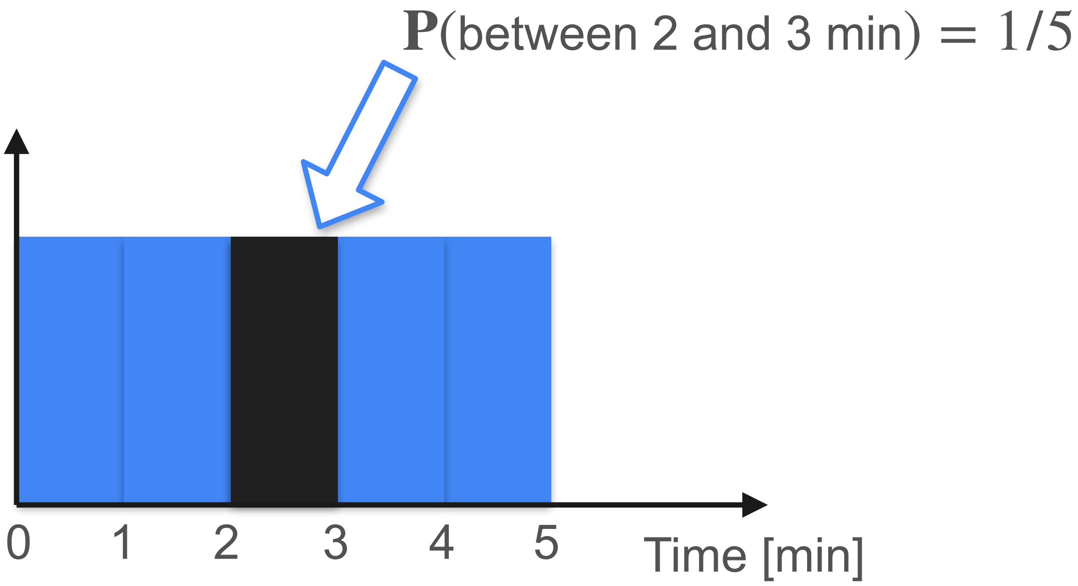


Probability Density Function

$\mathbf{P}(\text{between 2 and 3 min}) = ?$

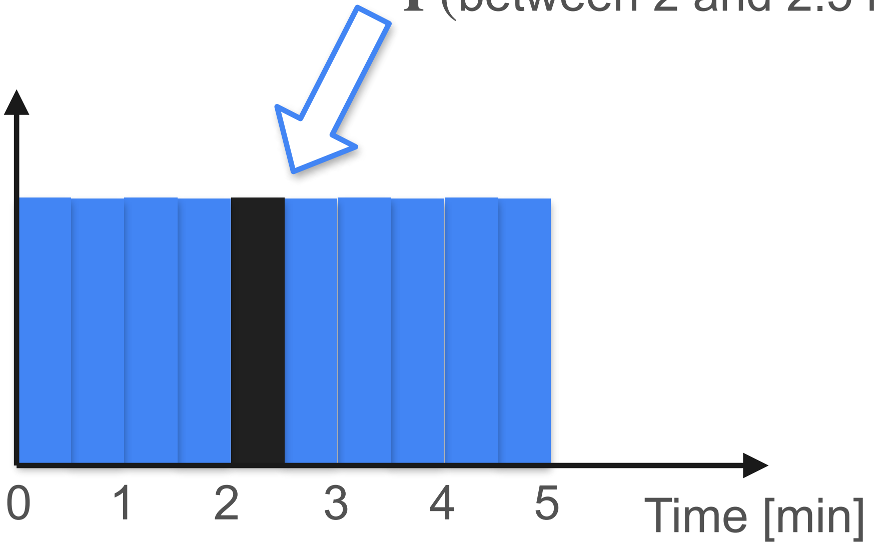


Probability Density Function



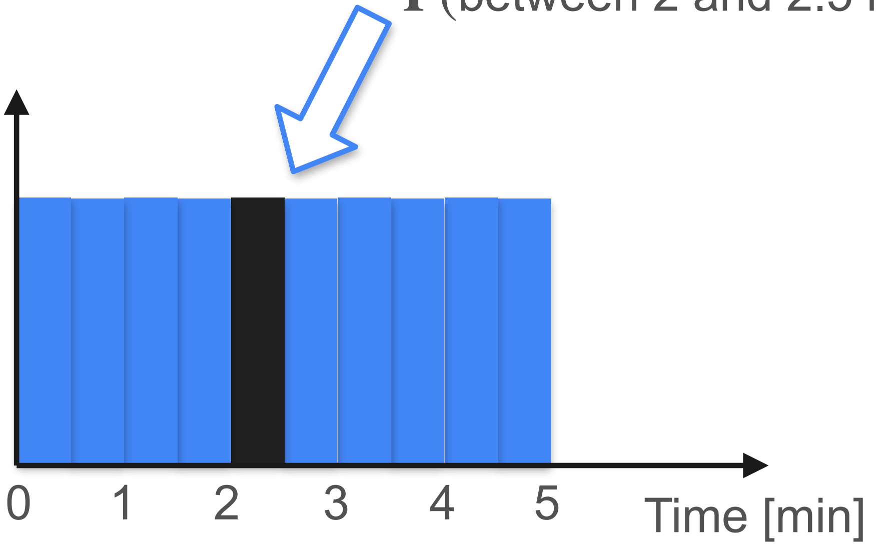
Probability Density Function

$\mathbf{P}(\text{between 2 and 2.5 min}) = ?$

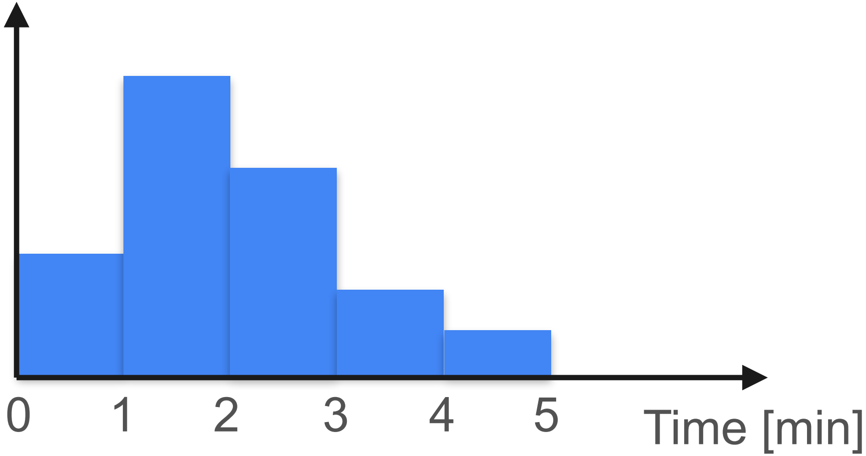


Probability Density Function

$\mathbf{P}(\text{between 2 and 2.5 min}) = 0.1$

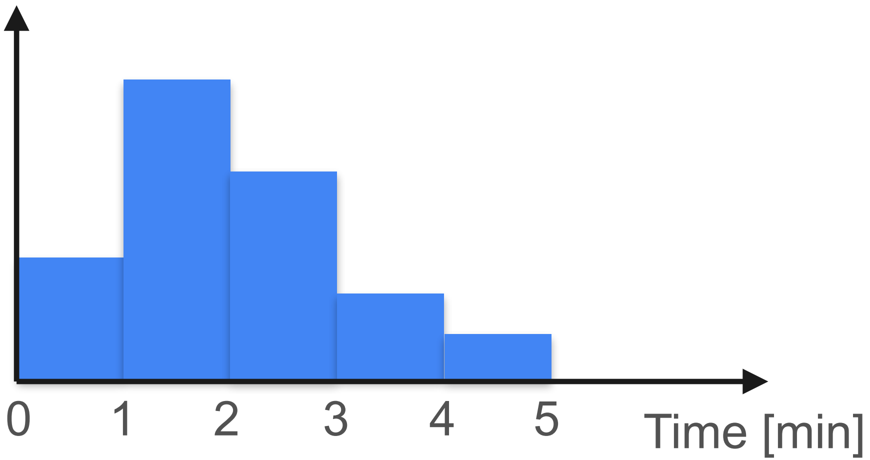


Probability Density Function

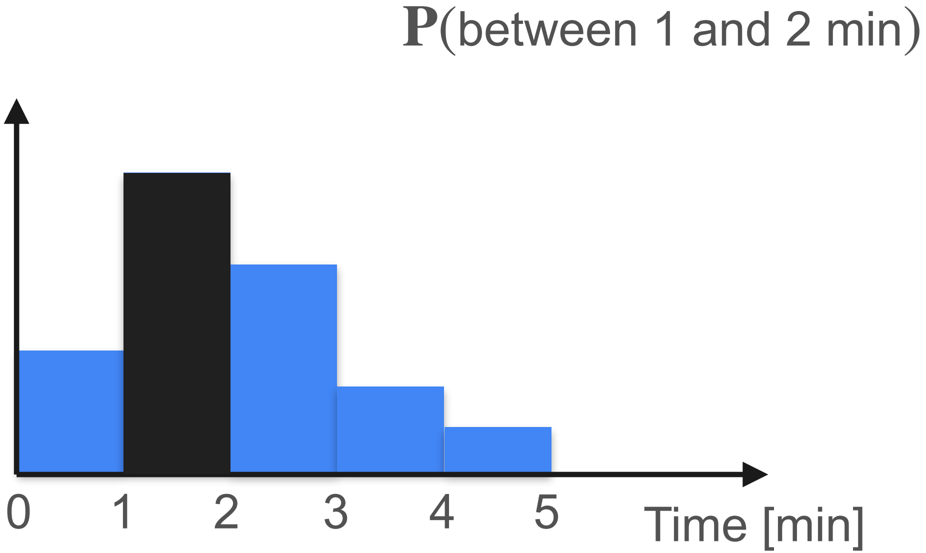


Probability Density Function

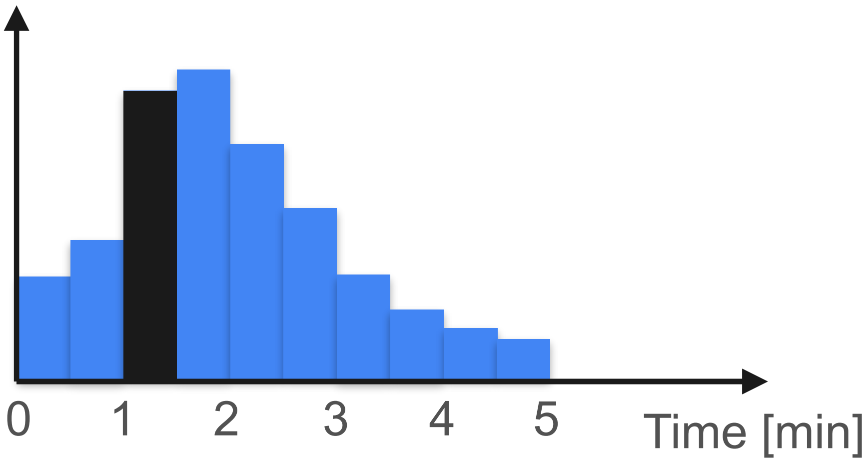
$\mathbf{P}$(between 1 and 2 min)



Probability Density Function

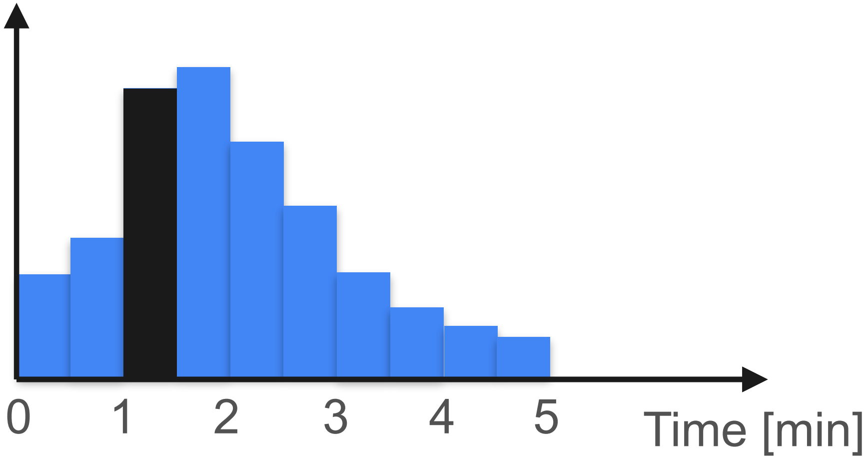


Probability Density Function

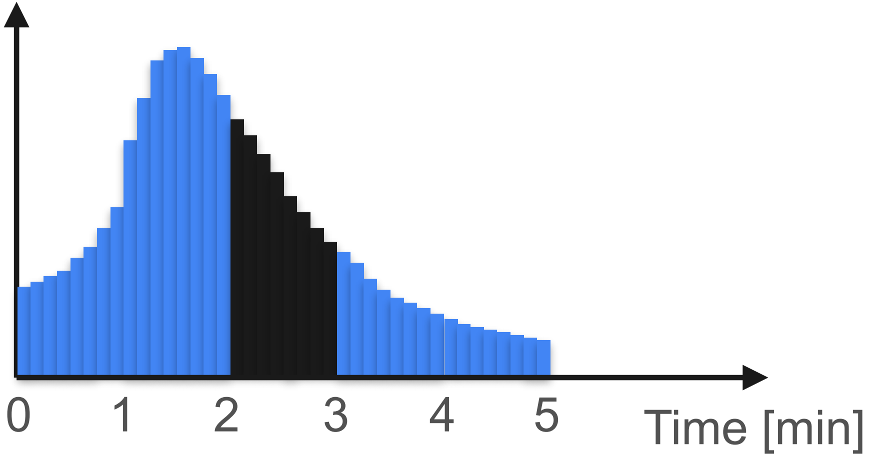


Probability Density Function

$\mathbf{P}$(between 1 and 1:30)

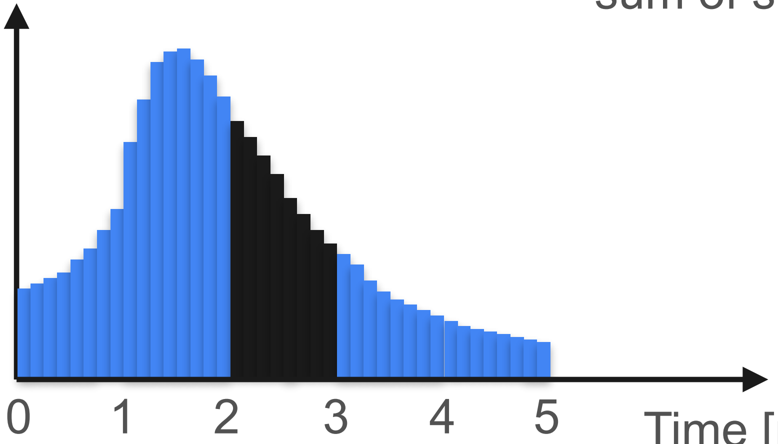


Probability Density Function

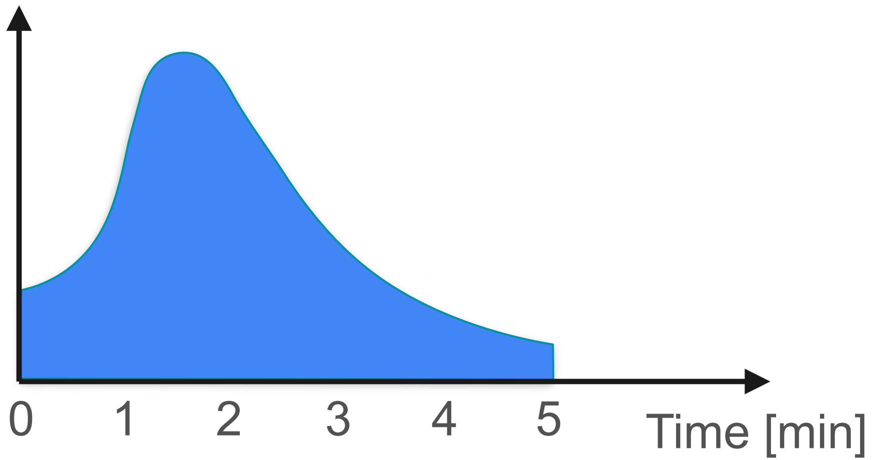


Probability Density Function

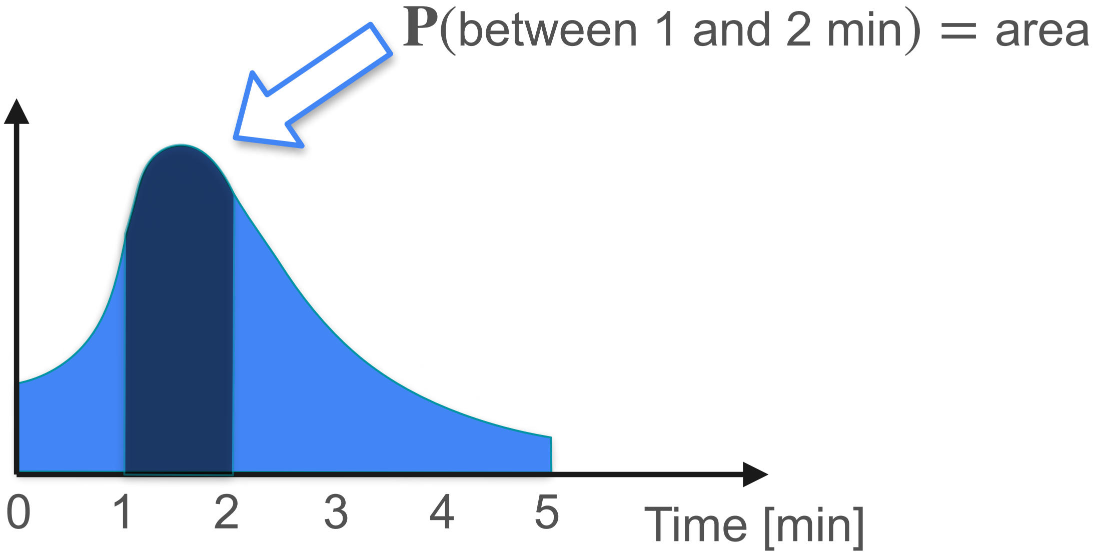
$\mathbf{P}$(between 2 and 3 min) $=$
sum of shaded areas



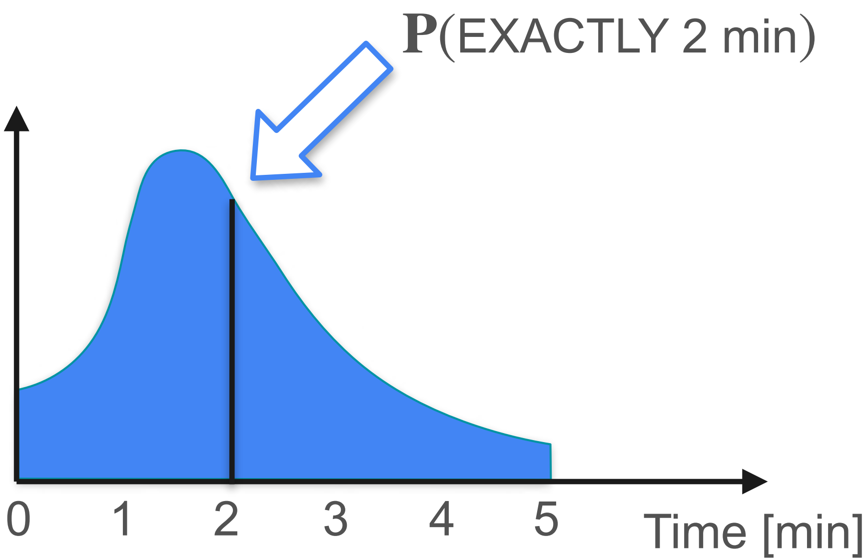
Probability Density Function



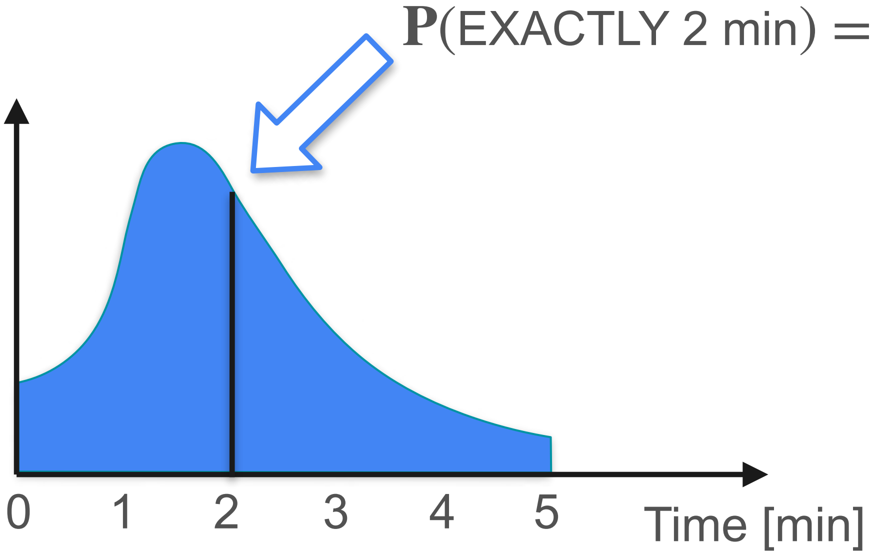
Probability Density Function



Probability Density Function

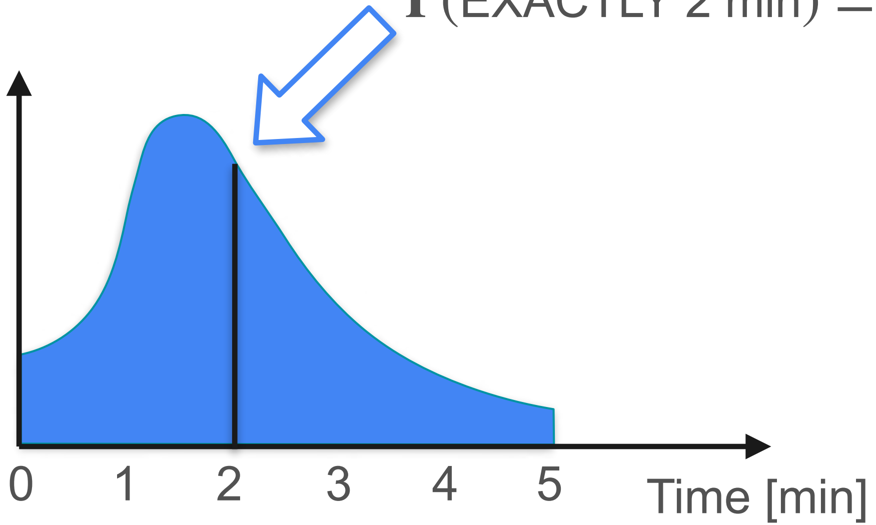


Probability Density Function



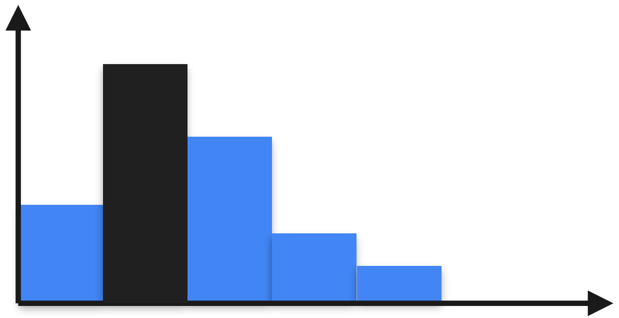
Probability Density Function

$\mathbf{P}(\text{EXACTLY 2 min}) = \text{area} \ = 0$

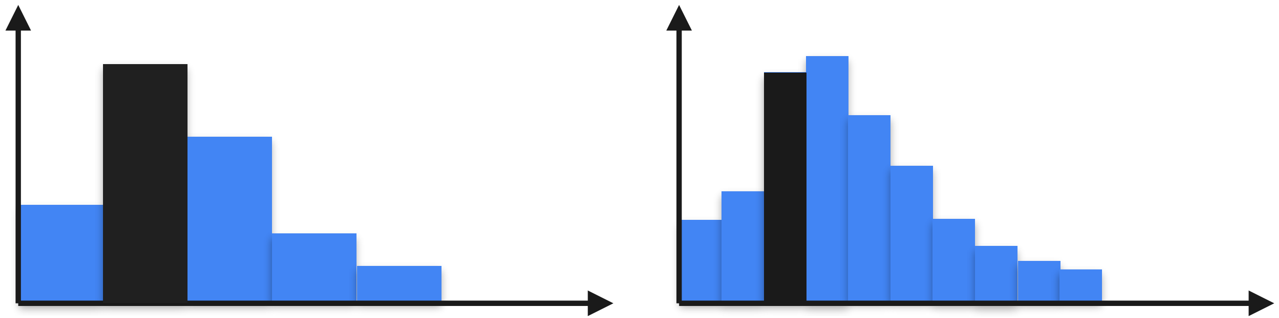


Probability Density Function

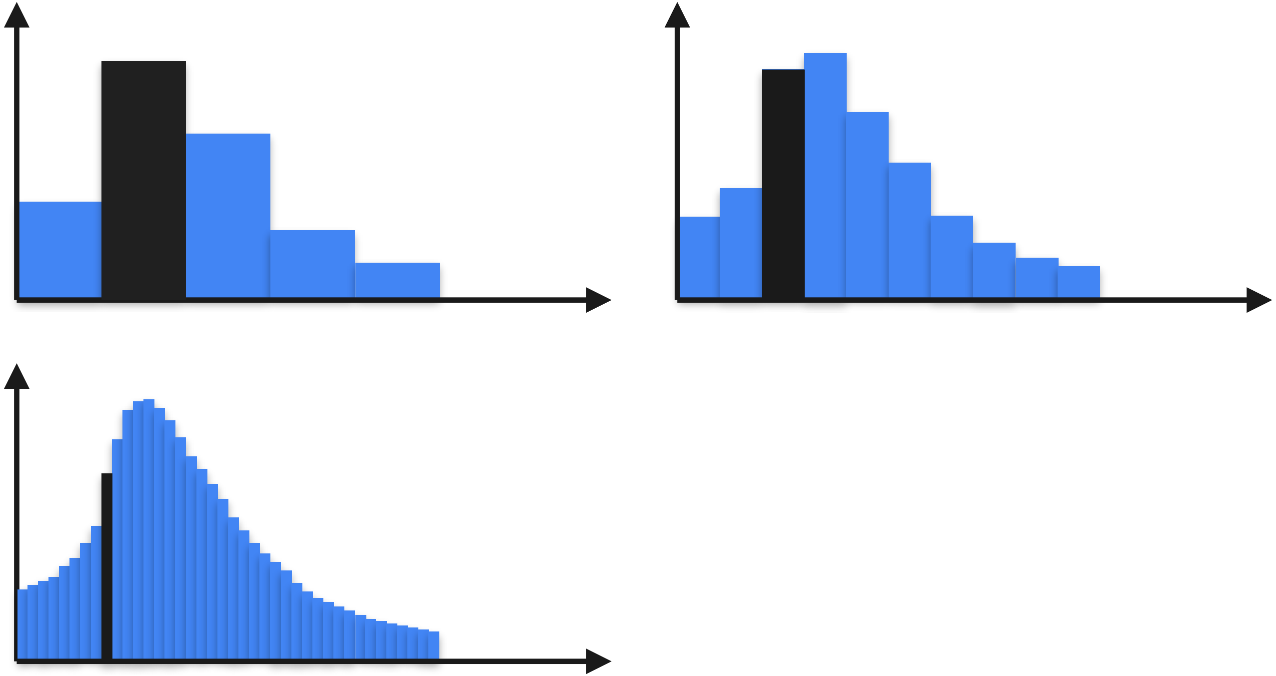
Probability Density Function



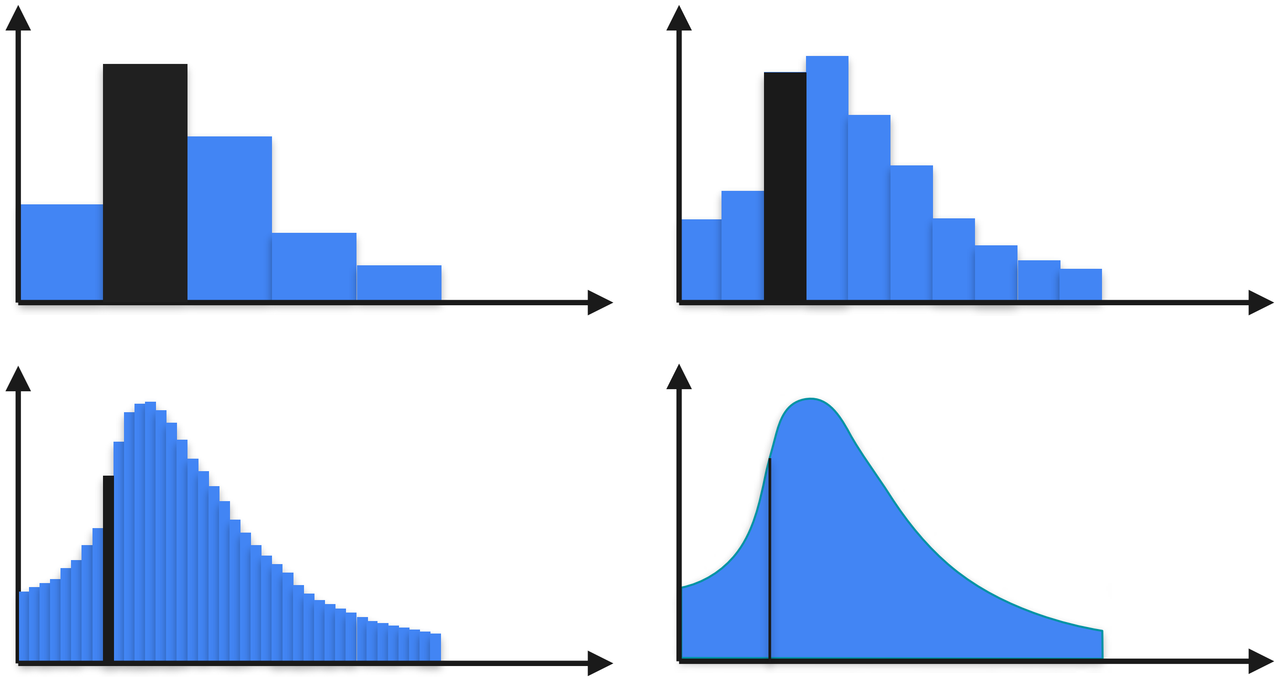
Probability Density Function



Probability Density Function



Probability Density Function



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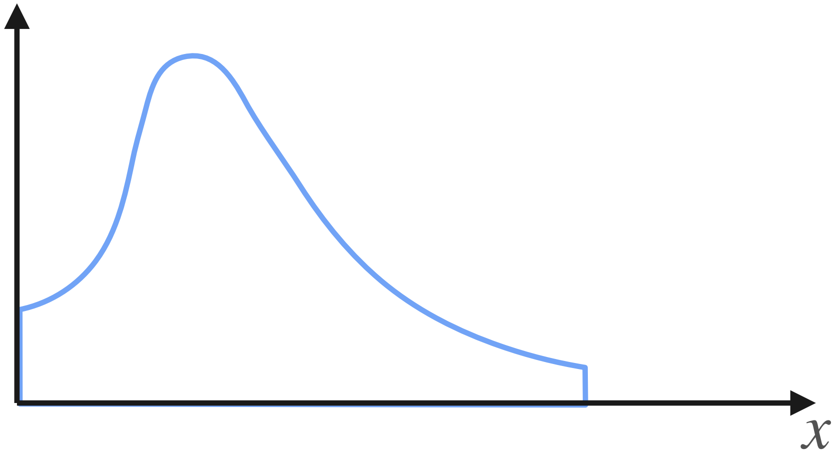
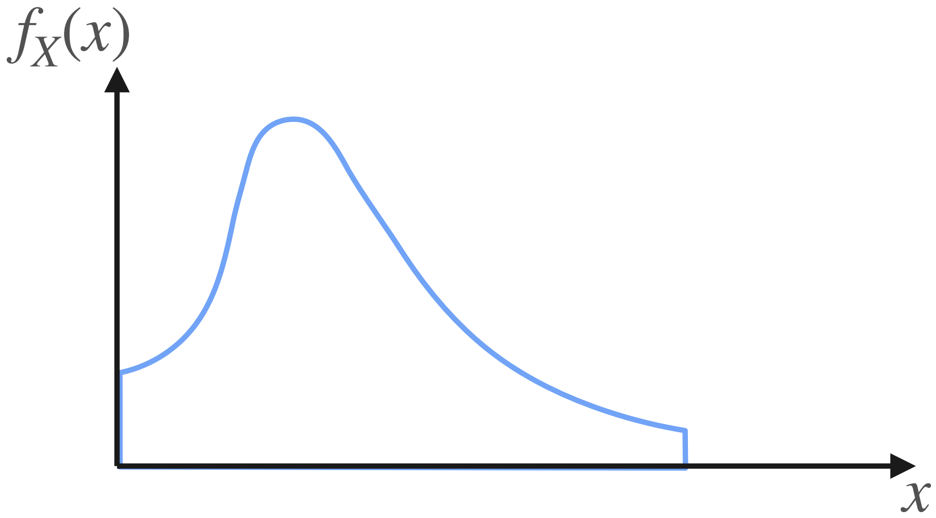
Probability Density Function: Formal Definition

Probability Density Function: Formal Definition



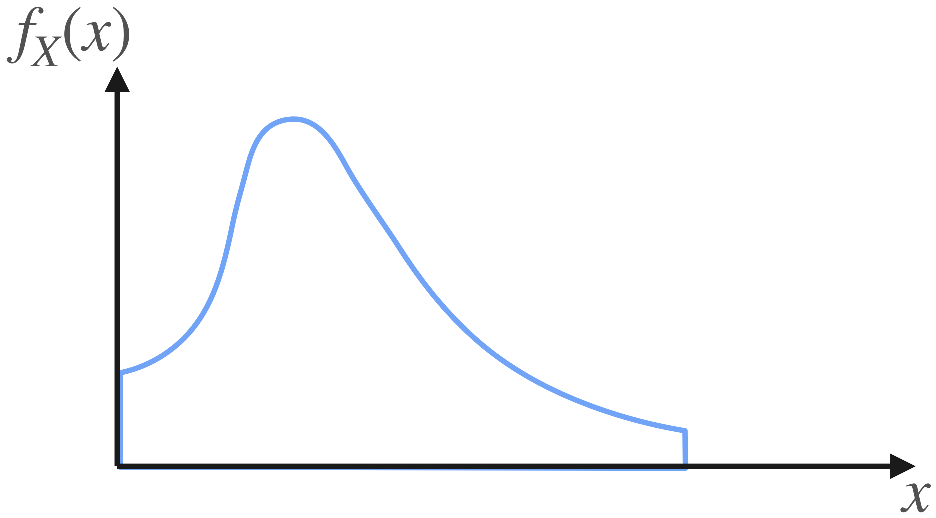
Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Probability Density Function: Formal Definition

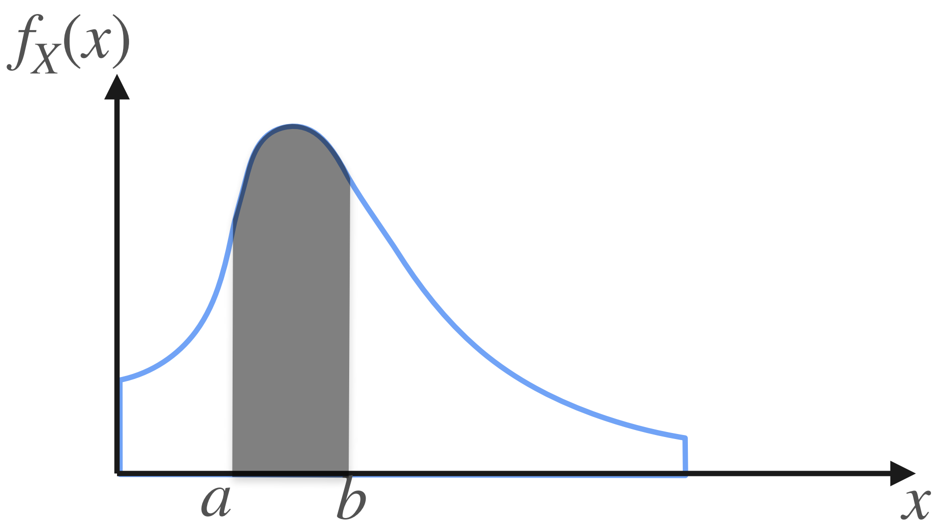


Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

Probability Density Function: Formal Definition



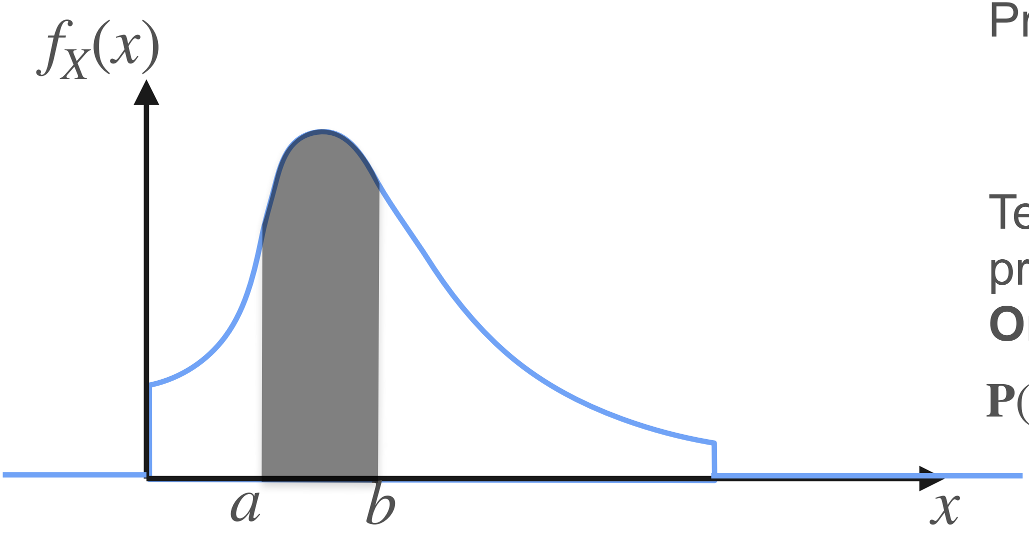
Probability Density Function (PDF)

$f_X(x)$

Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

$\mathbf{P}(a < X < b) =$ area under $f_X(x)$

Probability Density Function: Formal Definition



Probability Density Function (PDF)

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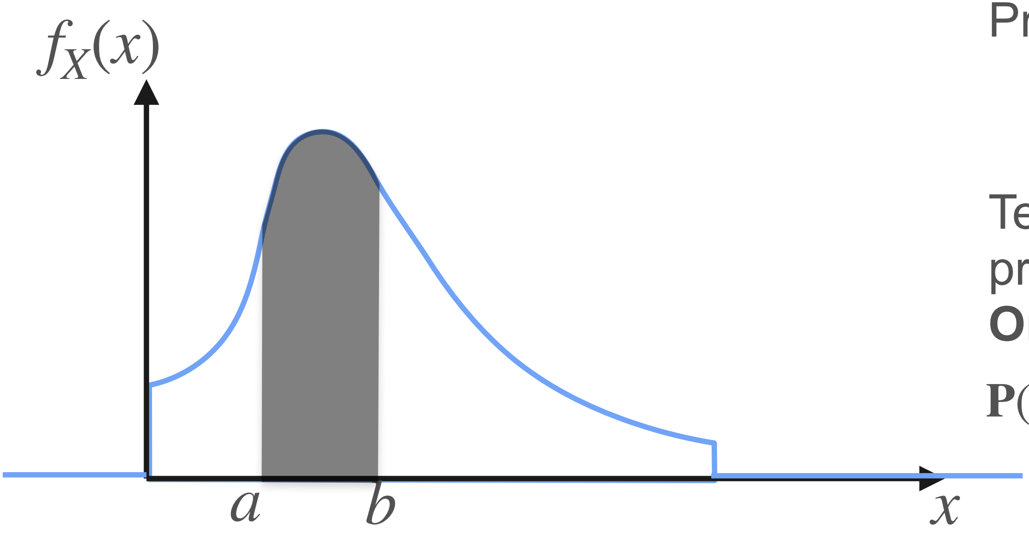
$f_X(x)$ needs to satisfy:
- It is defined for all numbers

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.

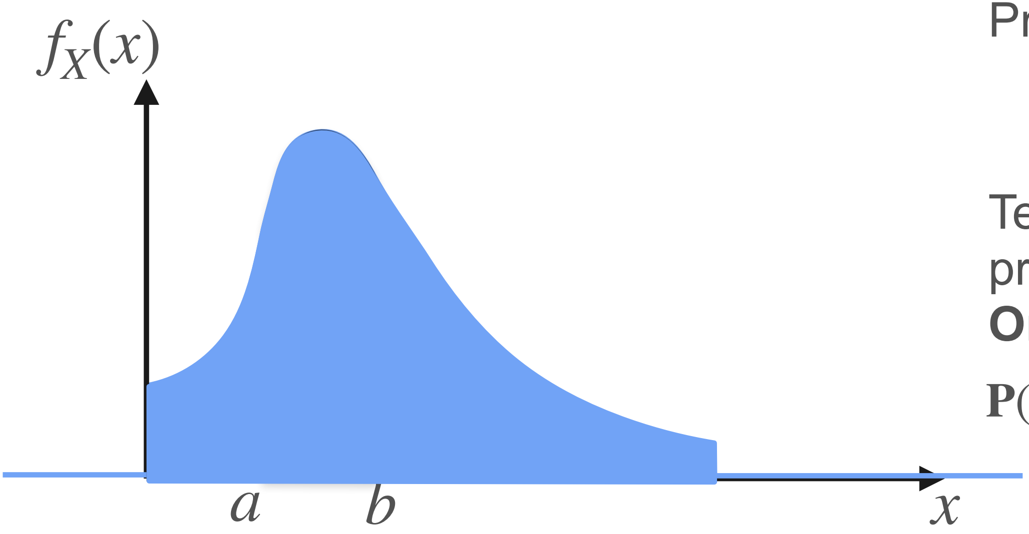
Only defined for continuous variables!

$\mathbf{P}(a < X < b) =$ area under $f_X(x)$

$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$f_X(x)$

Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

$\mathbf{P}(a < X < b) =$ area under $f_X(x)$

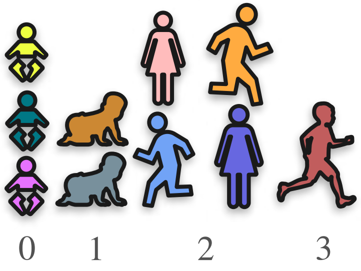
$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$
- Area under $f_X(x) = 1$

Discrete and Continuous Random Variables

Discrete and Continuous Random Variables

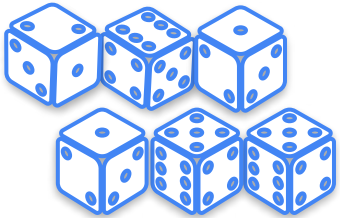
Discrete random variables

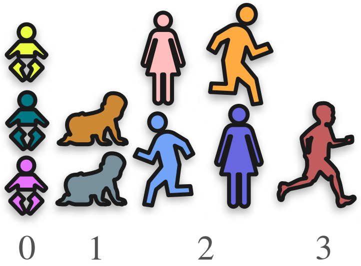


Can take only a **finite** or at most countable number of values

Discrete and Continuous Random Variables

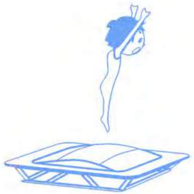
Discrete random variables





Can take only a **finite** or at most countable number of values

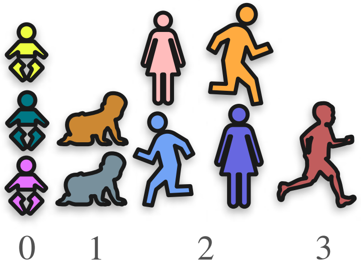
Continuous random variables



Takes values on an interval (infinite possibilities!)

Discrete and Continuous Random Variables

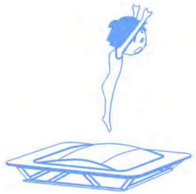
Discrete random variables



Can take only a **finite** or at most countable number of values

PMF: $p_X(x) = \mathbf{P}(X = x)$

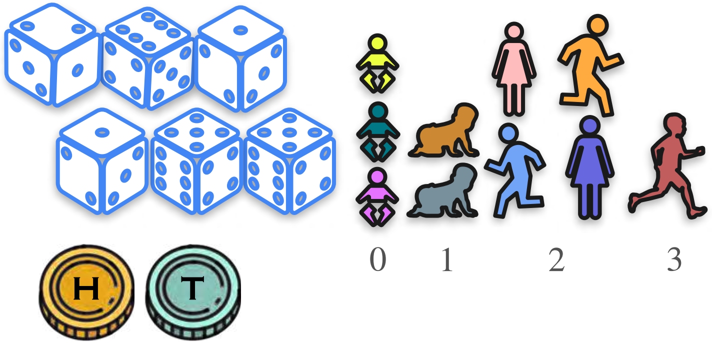
Continuous random variables



Takes values on an interval (infinite possibilities!)

Discrete and Continuous Random Variables

Discrete random variables



Can take only a **finite** or at most countable number of values

PMF: $p_X(x) = \mathbf{P}(X = x)$

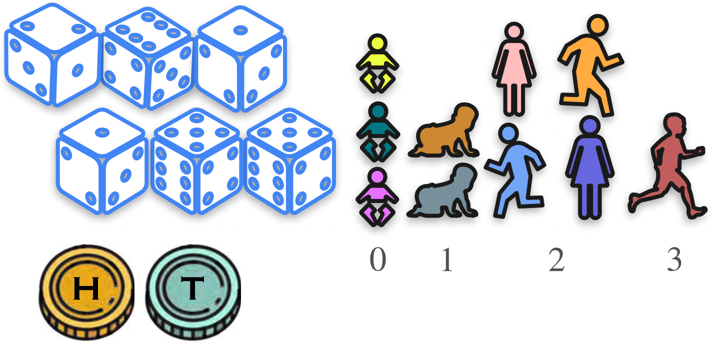
Continuous random variables

Takes values on an interval (infinite possibilities!)

PDF: $f_X(x)$

Discrete and Continuous Random Variables

Discrete random variables



Can take only a **finite** or at most countable number of values

PMF: $p_X(x) = \mathbf{P}(X = x)$

Continuous random variables

Takes values on an interval (infinite possibilities!)

PDF: $f_X(x)$

$\mathbf{P}(X = x) = 0 \quad \forall x$

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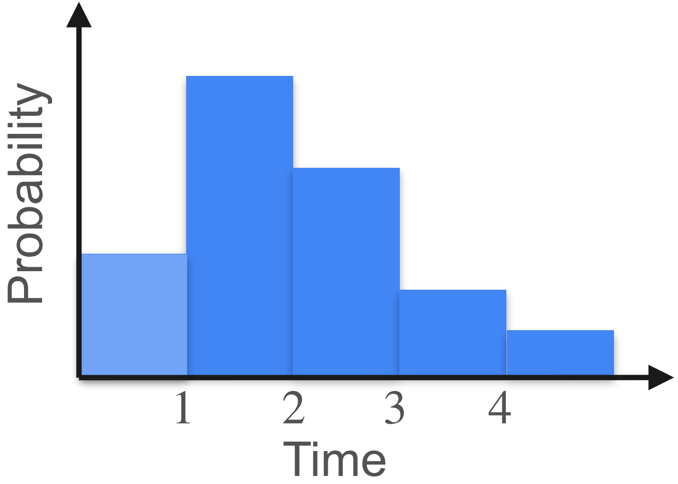
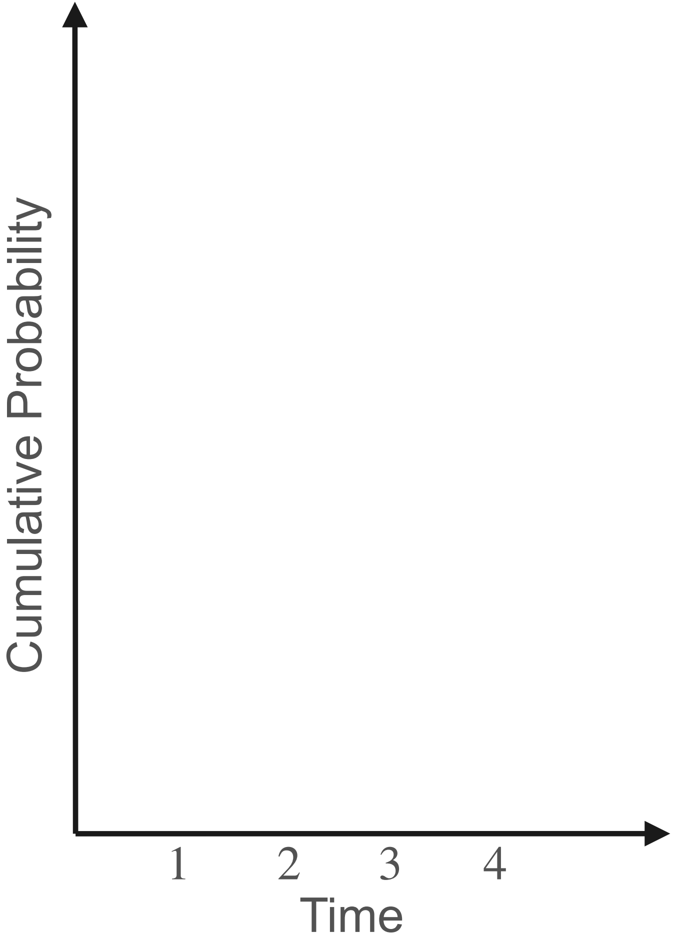
Probability Distributions

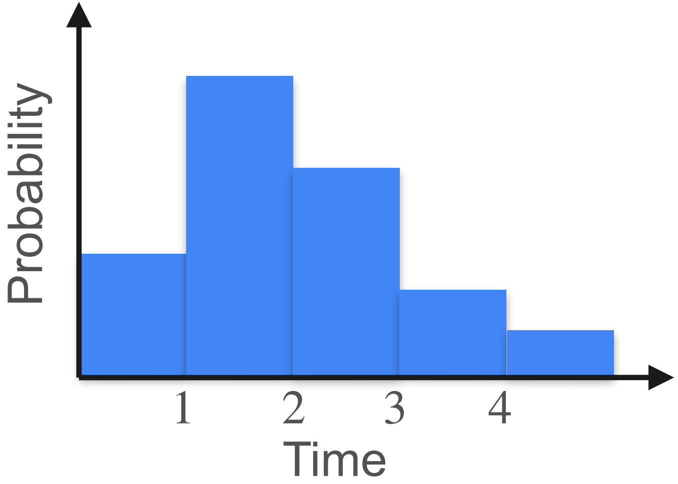
Cumulative Distribution Function

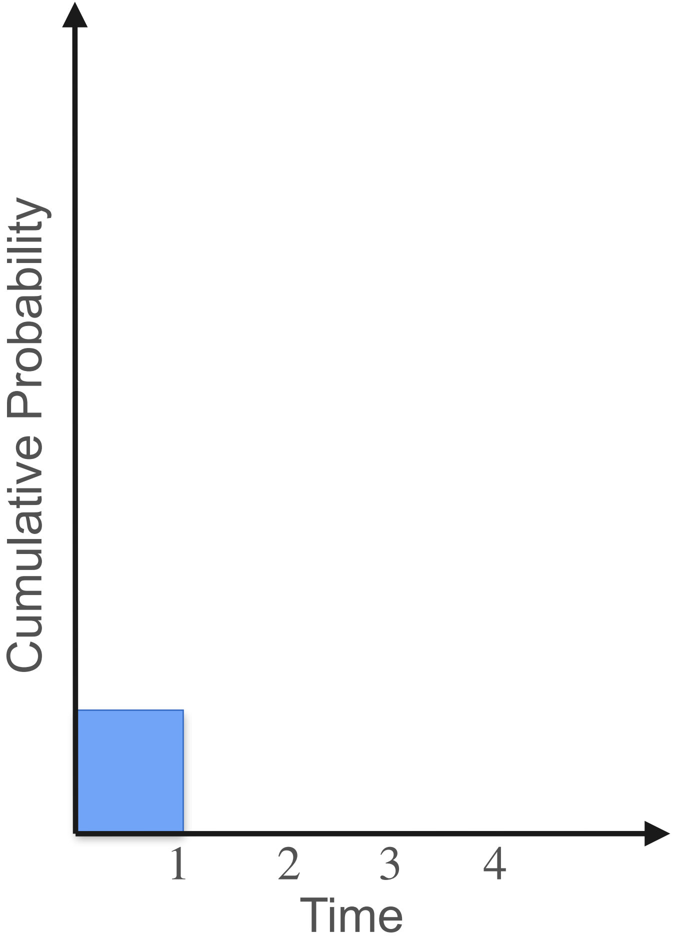
Cumulative Distribution



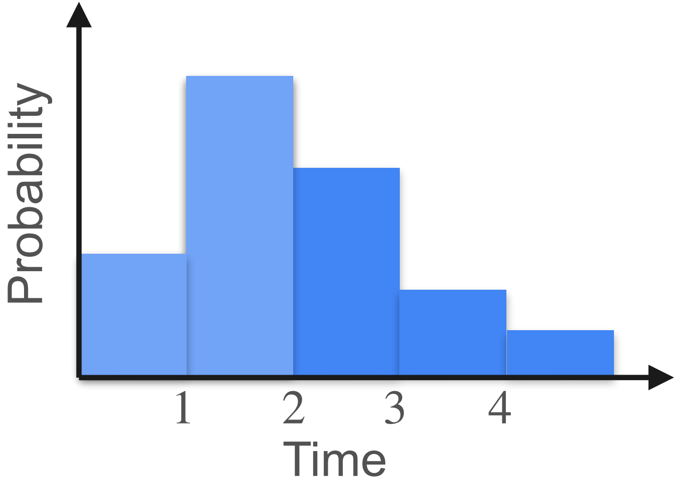


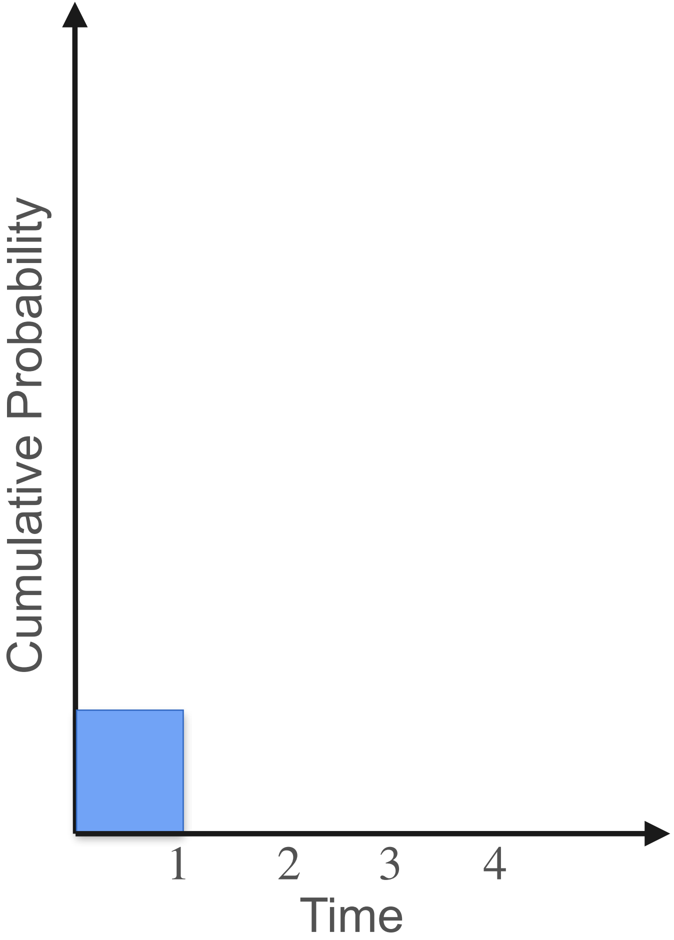
Cumulative Distribution



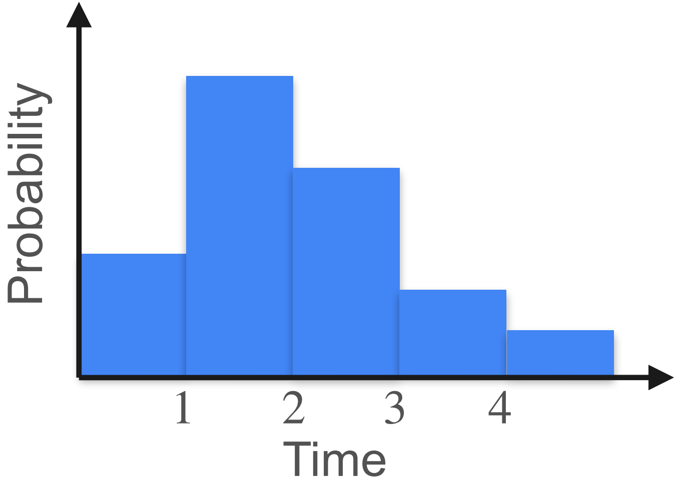


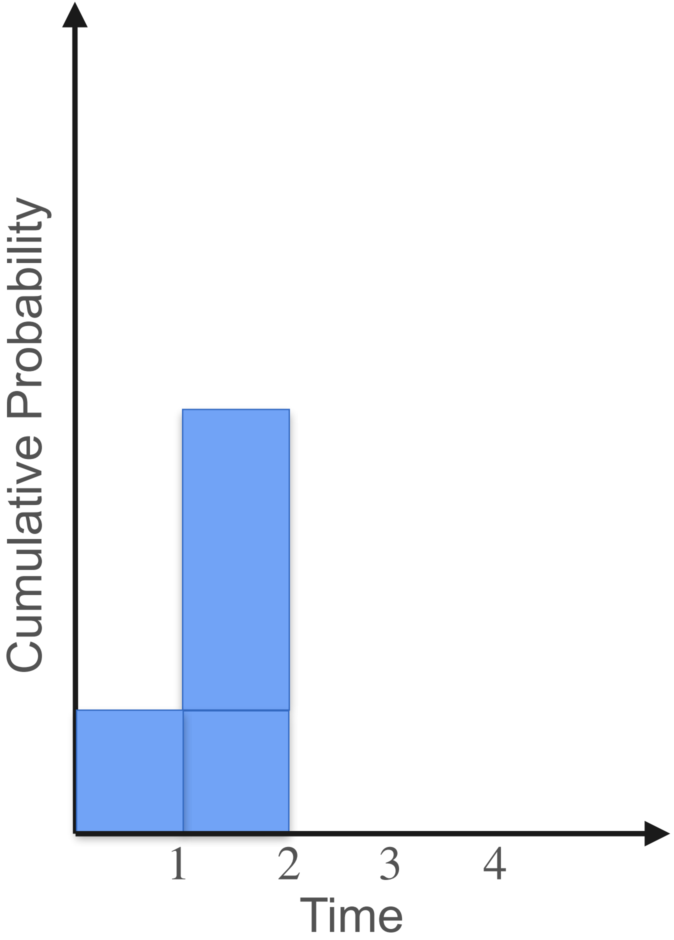
Cumulative Distribution

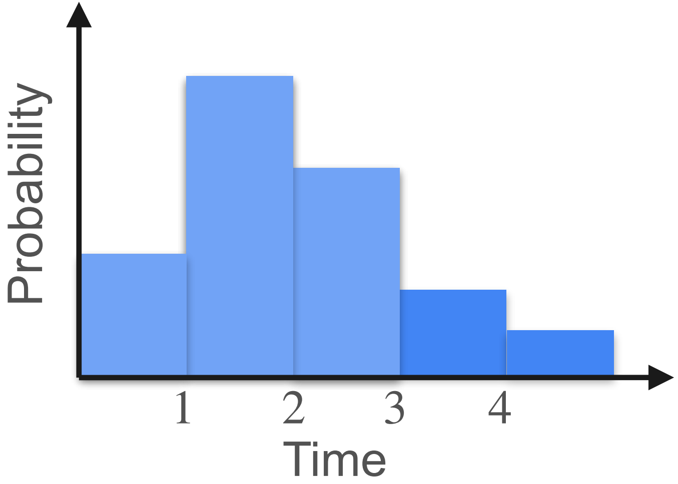


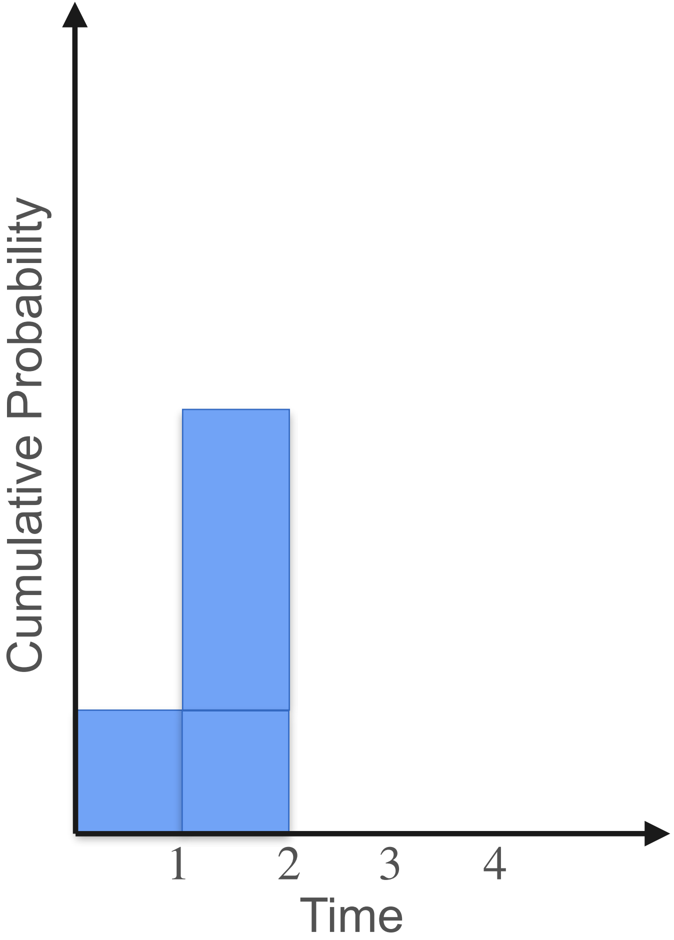
Cumulative Distribution



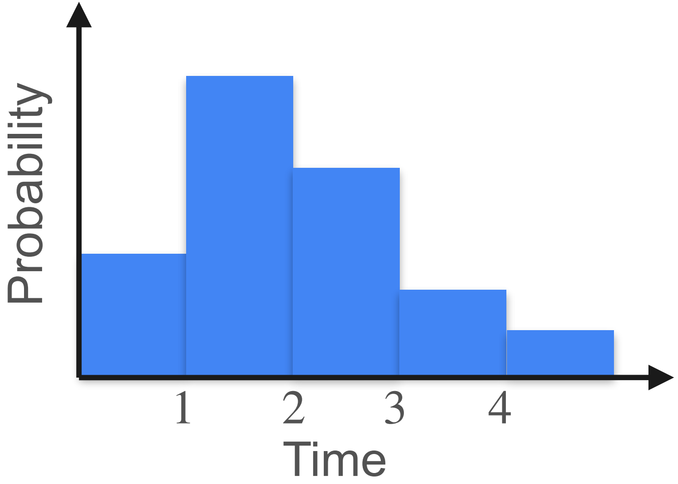


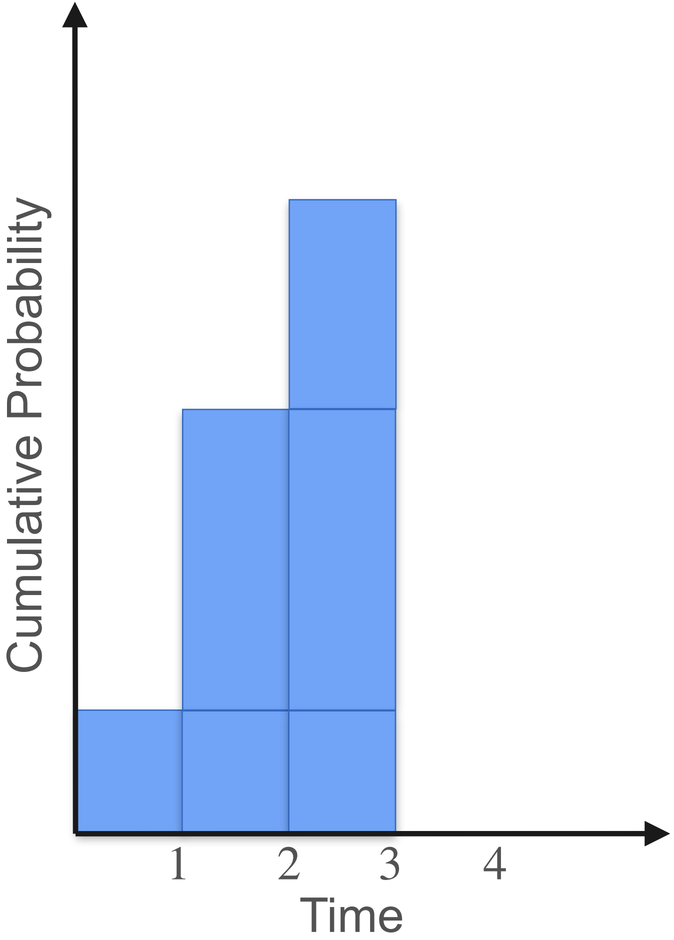
Cumulative Distribution



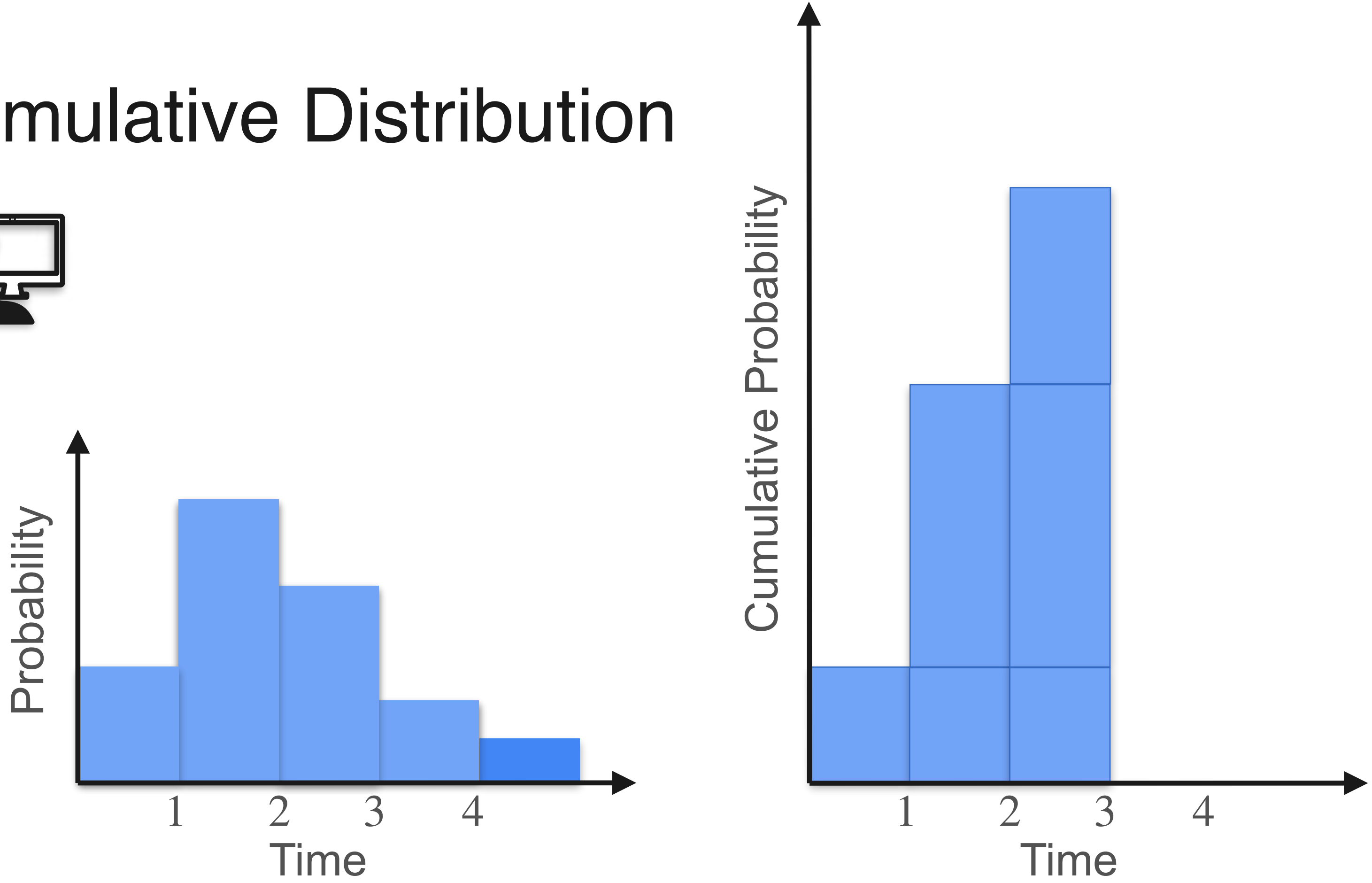


Cumulative Distribution

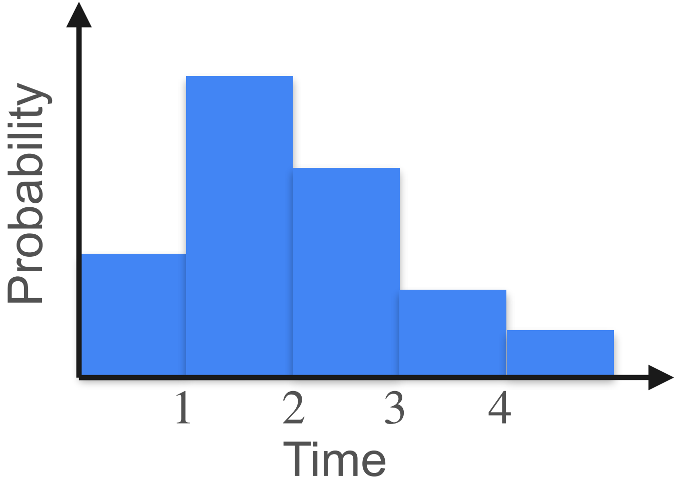


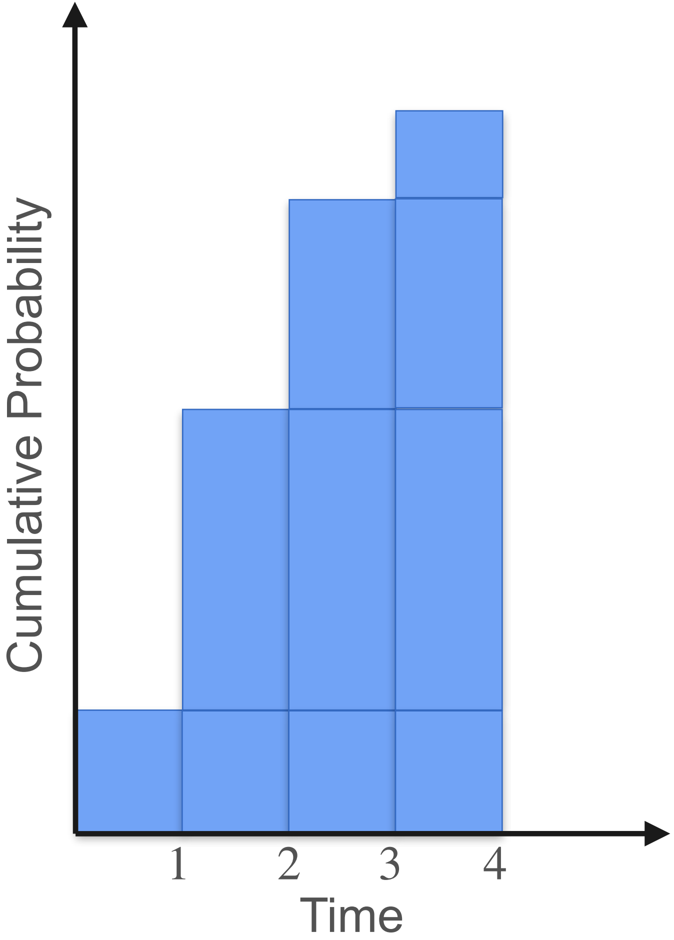


Cumulative Distribution

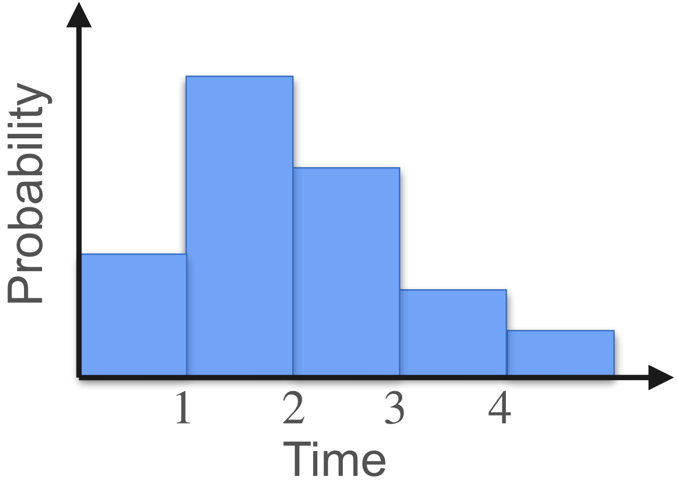


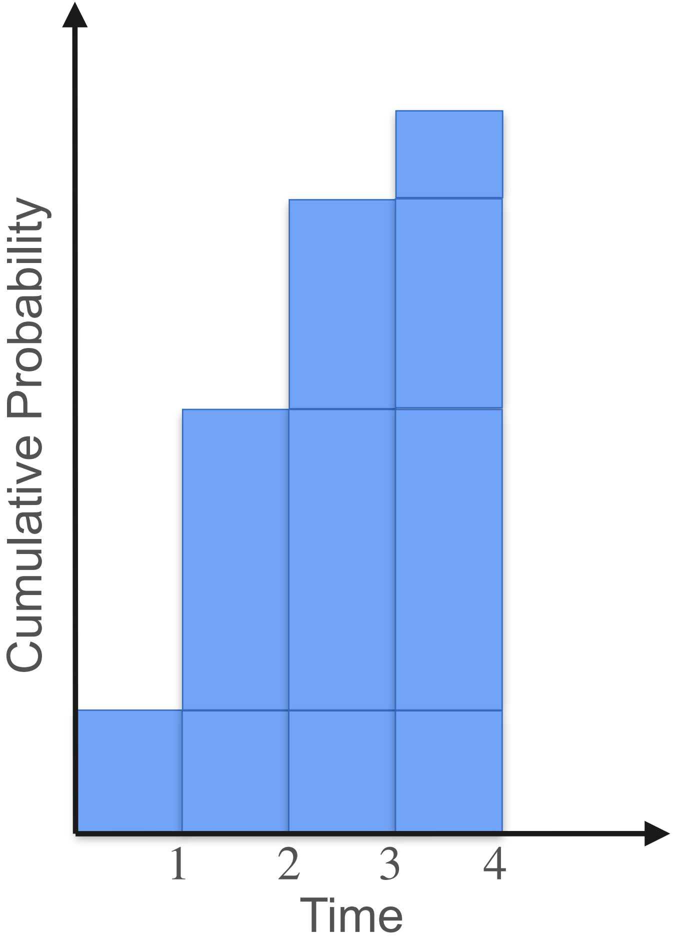
Cumulative Distribution



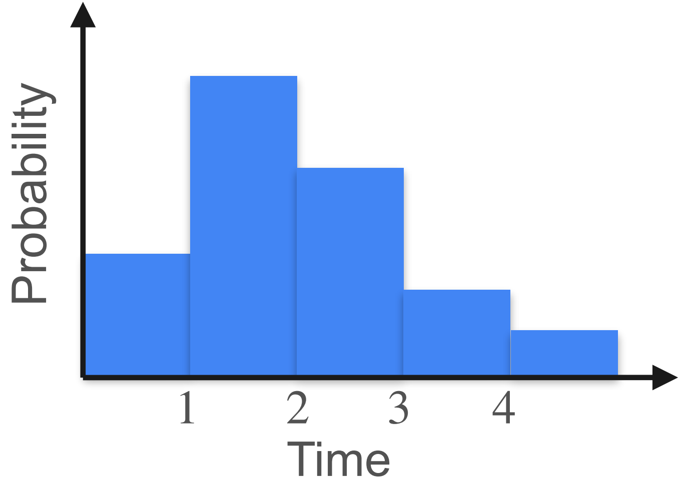


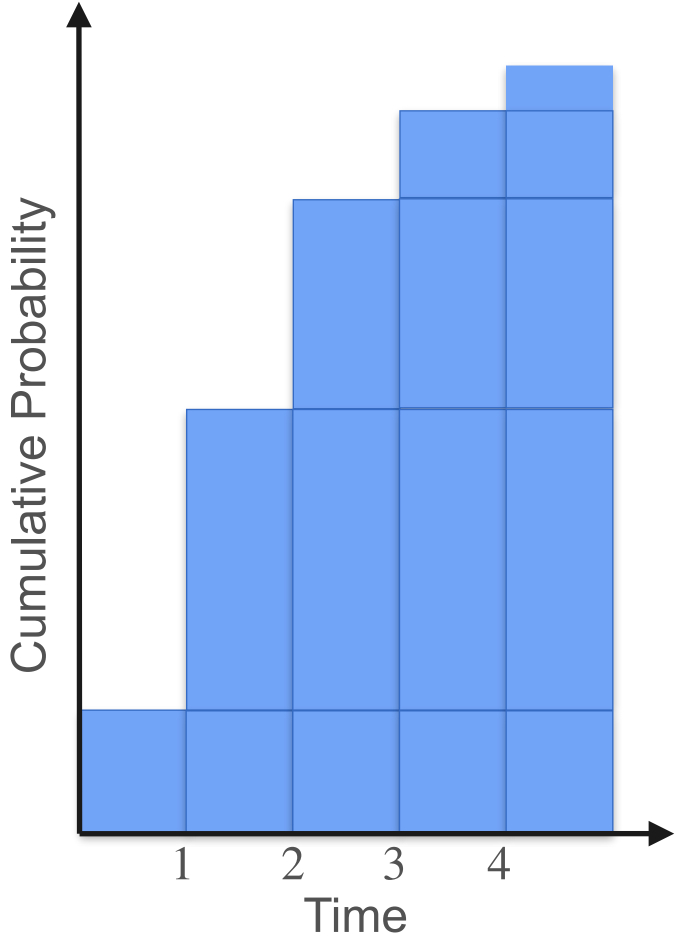
Cumulative Distribution



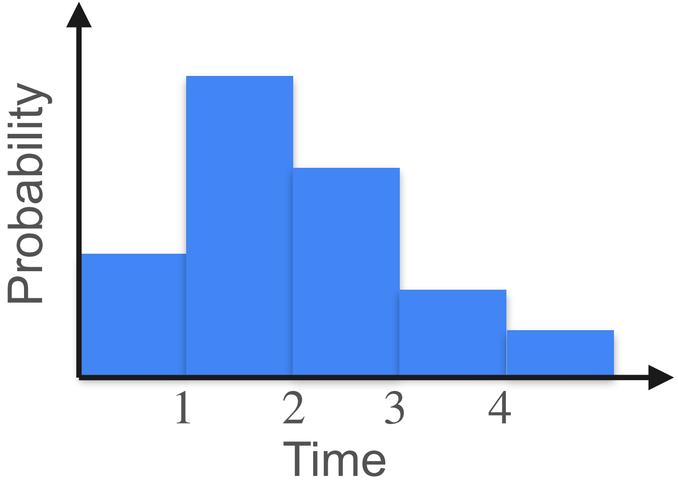


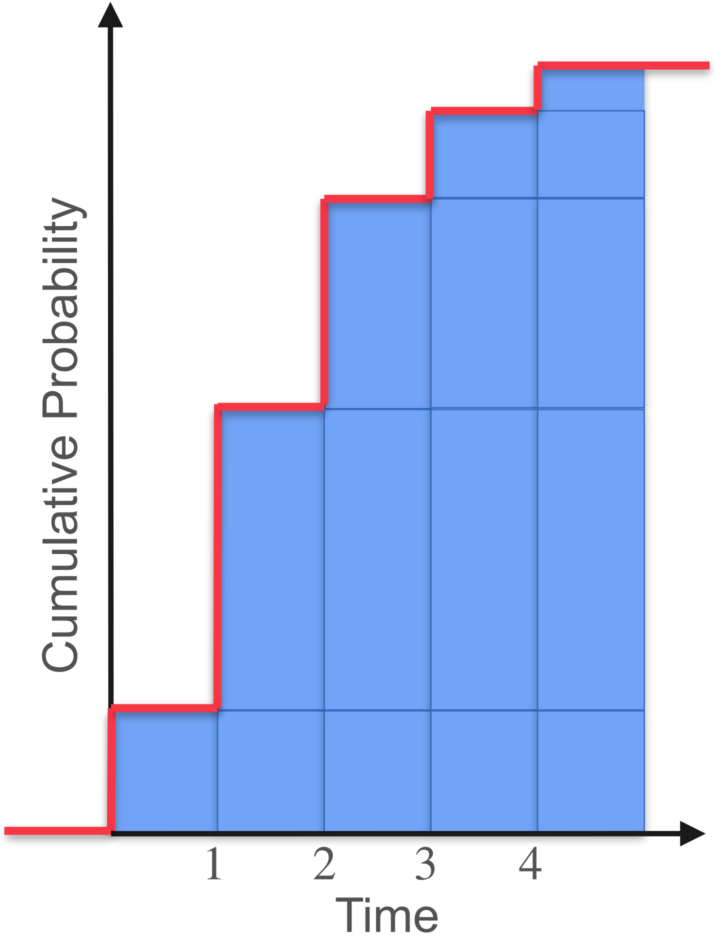
Cumulative Distribution





Cumulative Distribution





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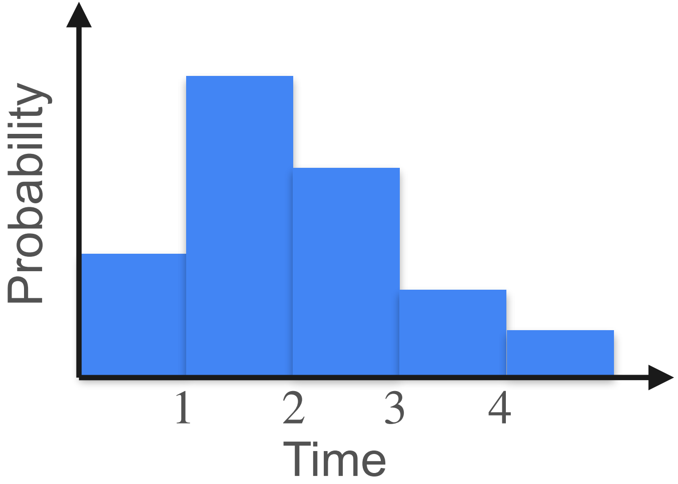

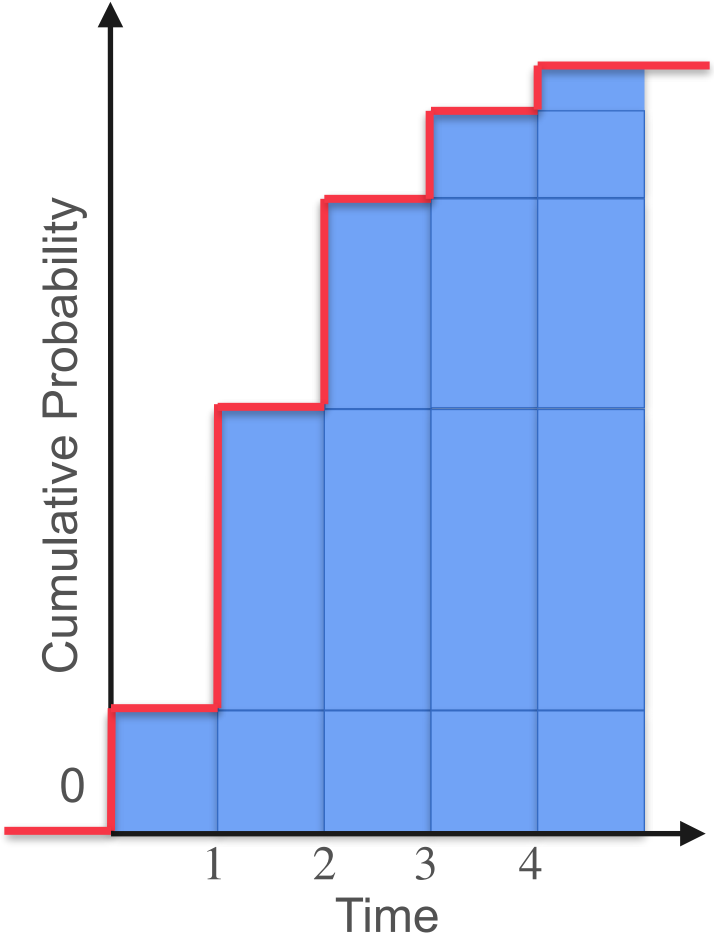

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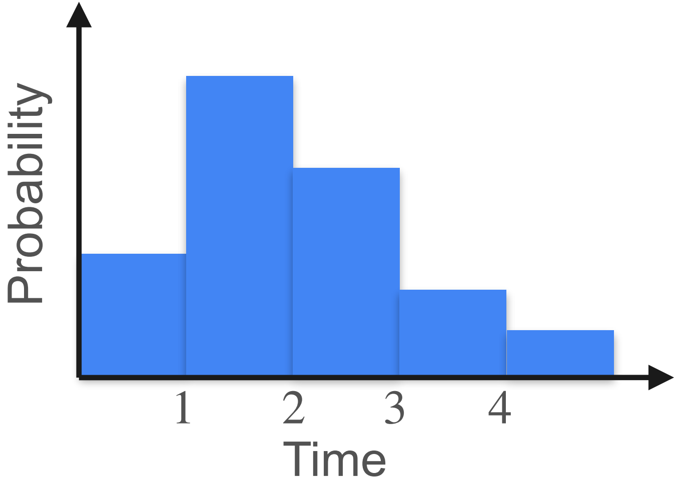
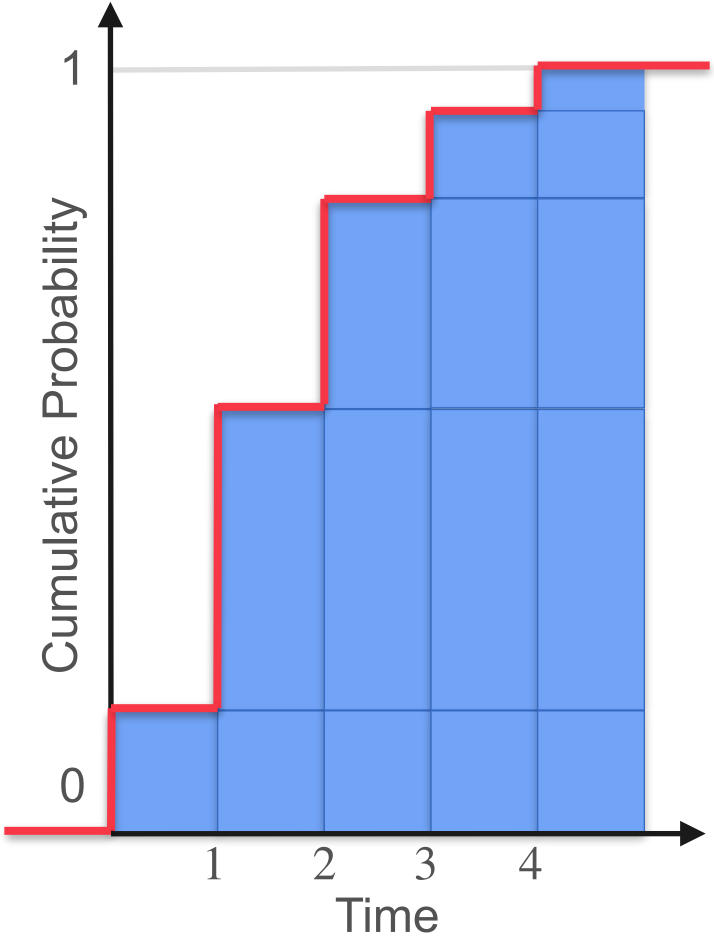


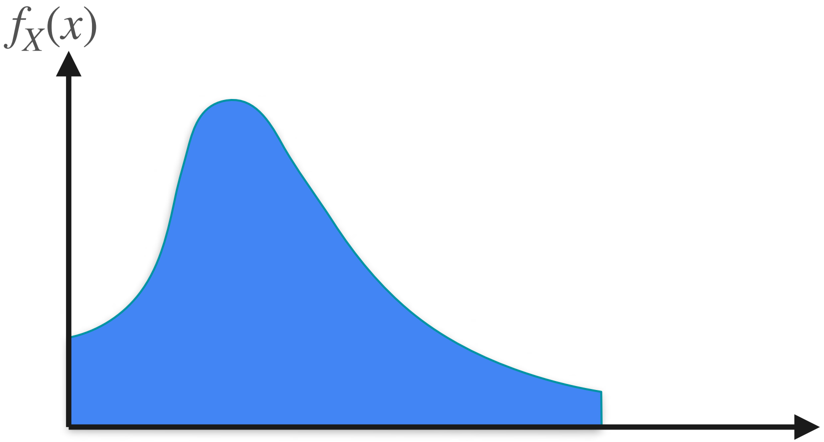


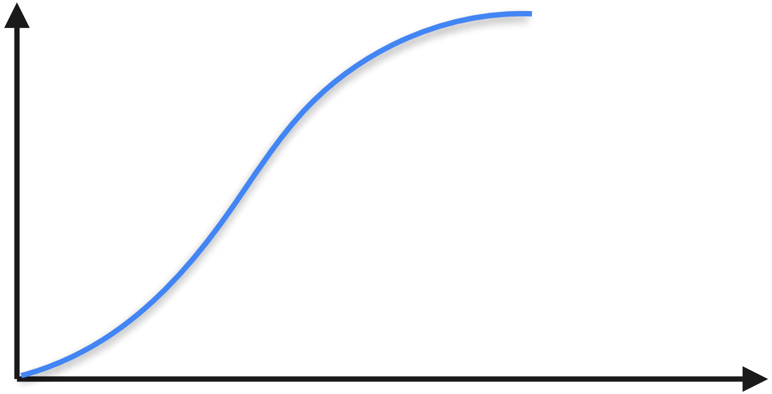
Cumulative Distribution

CDF: Cumulative distribution function

Cumulative Distribution

CDF: Cumulative distribution function





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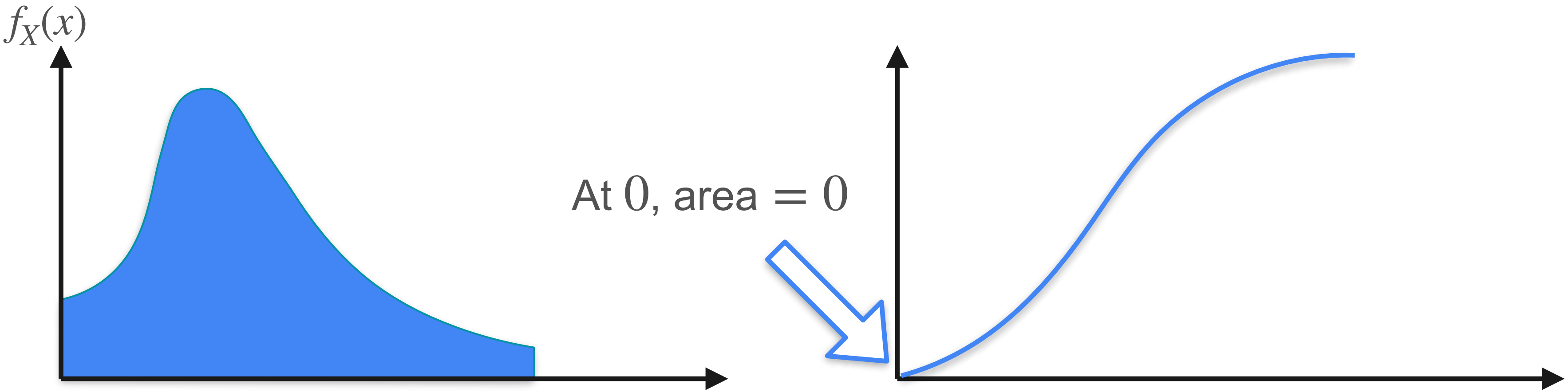

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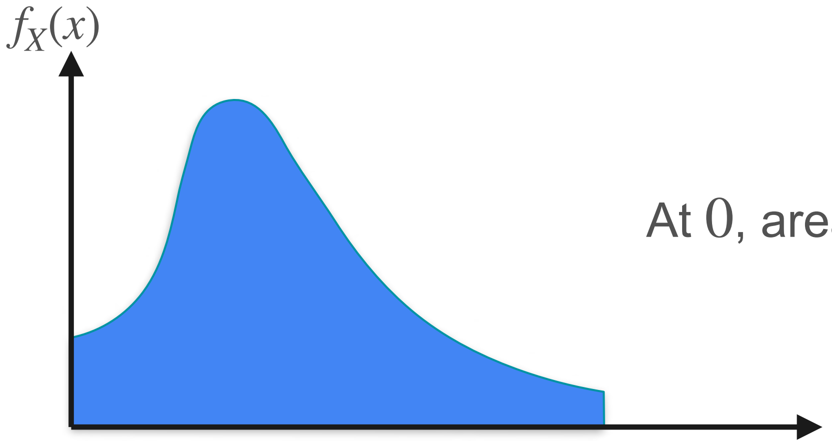

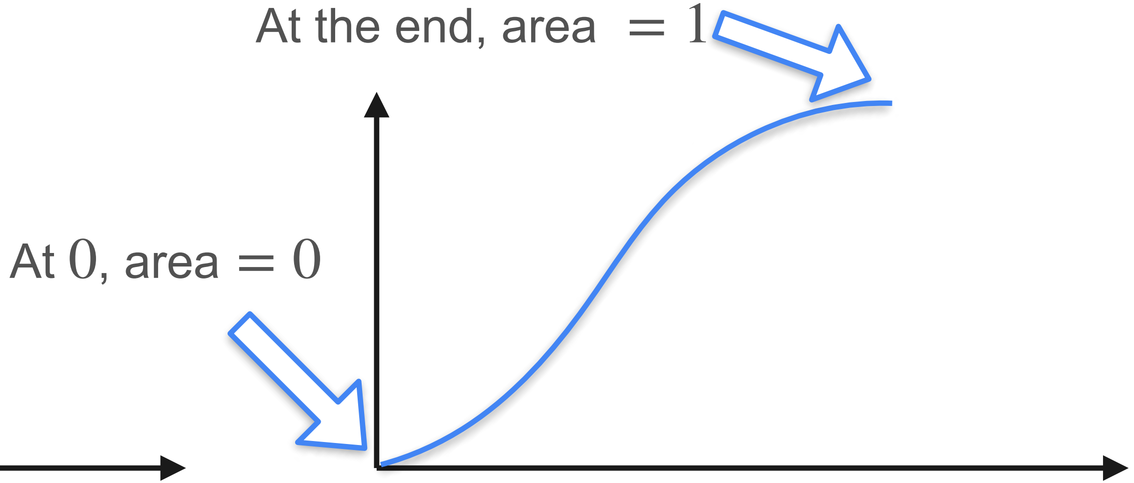

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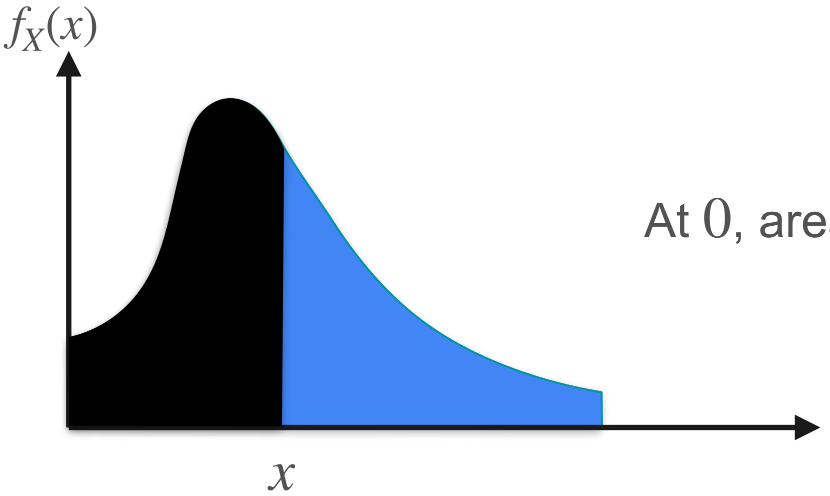

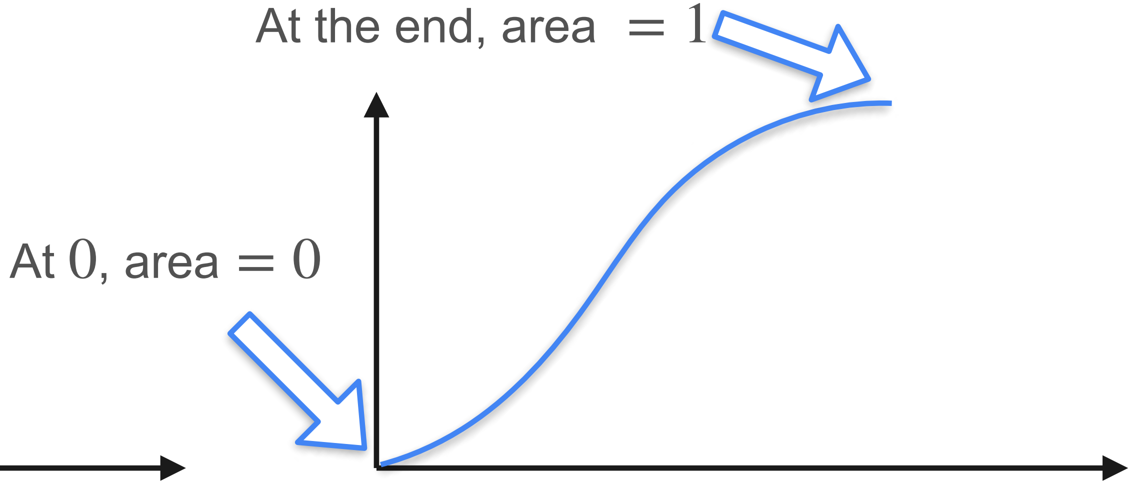

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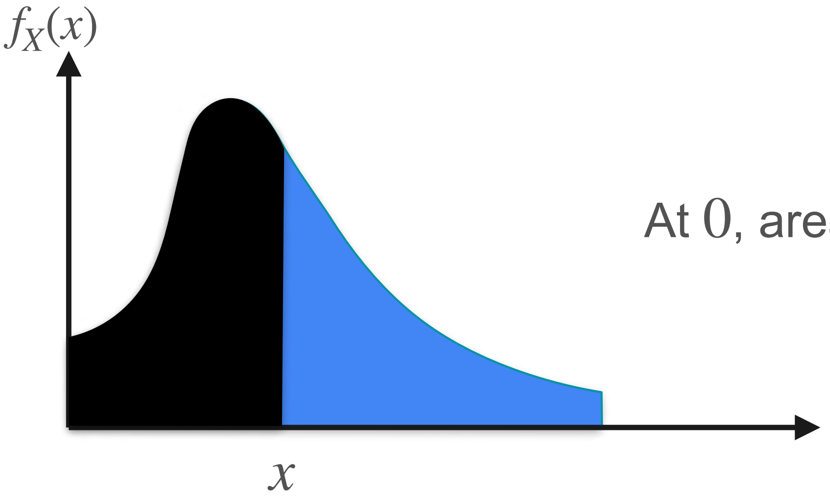

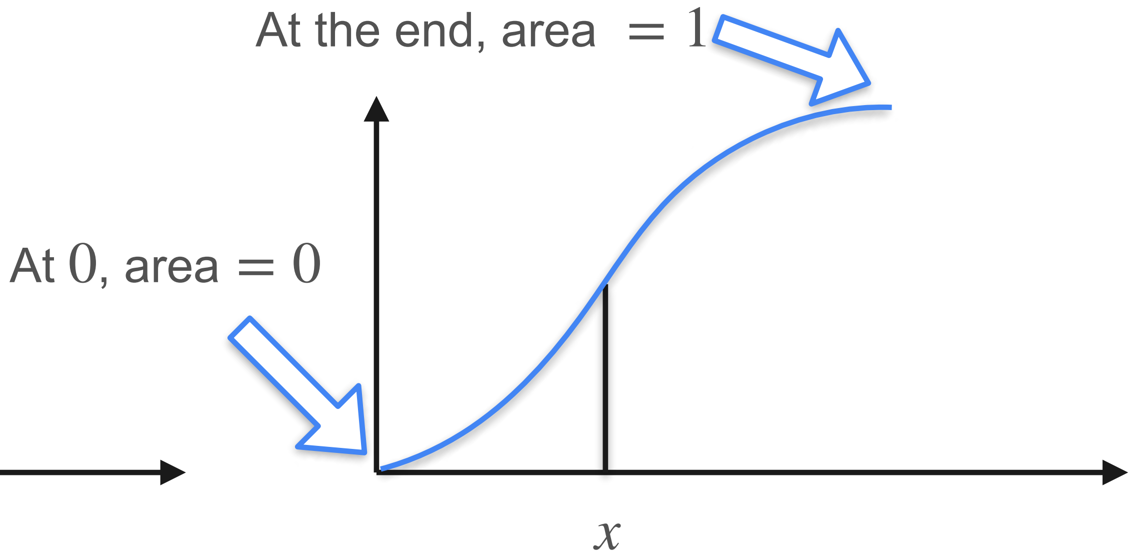

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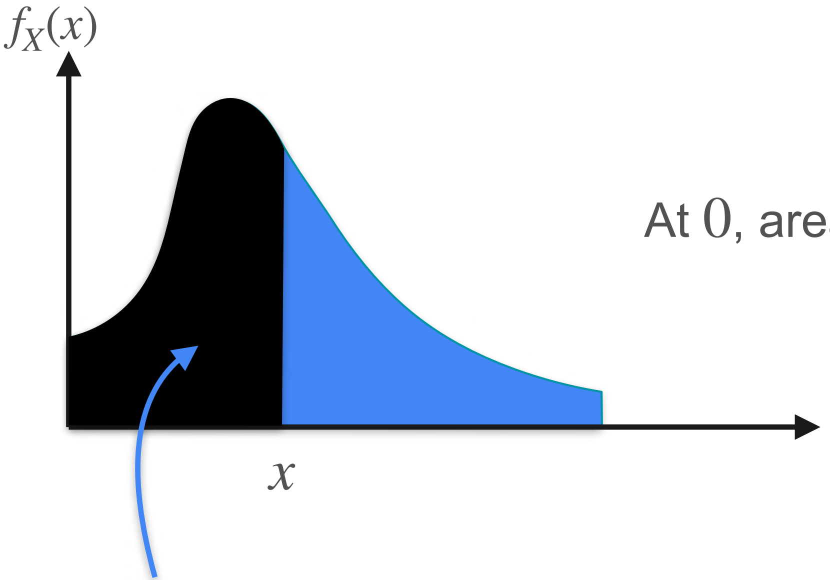

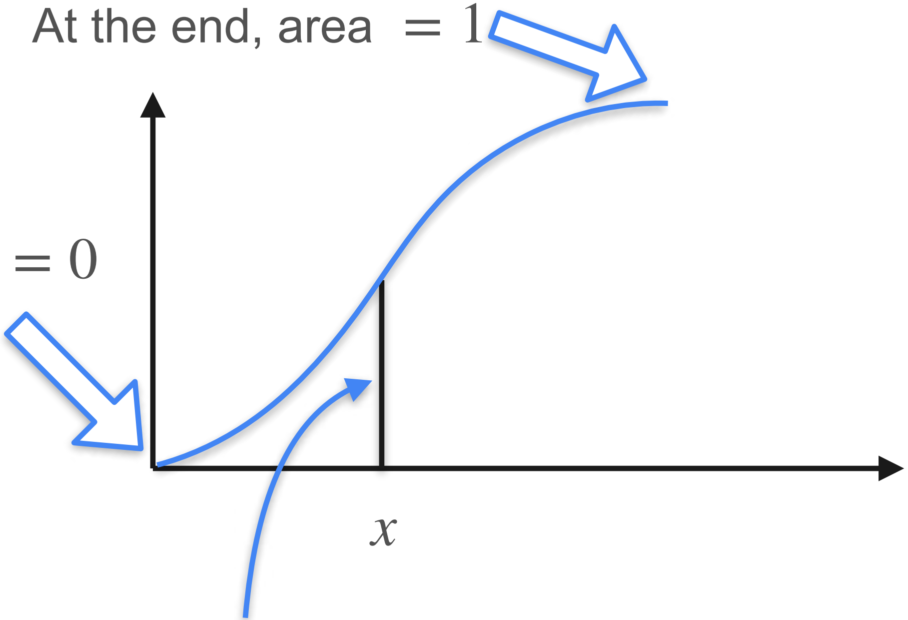

$\mathbf{P}$(less than or equal to 2 minutes) $= 0.5$

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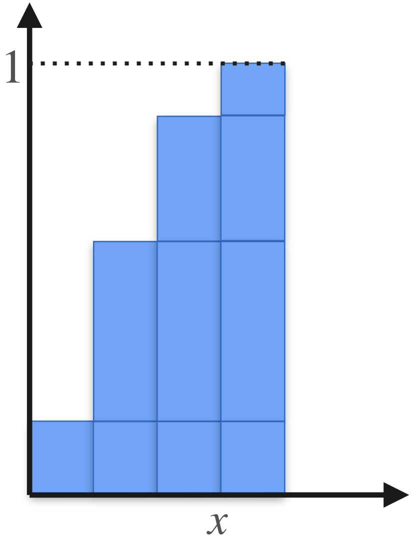
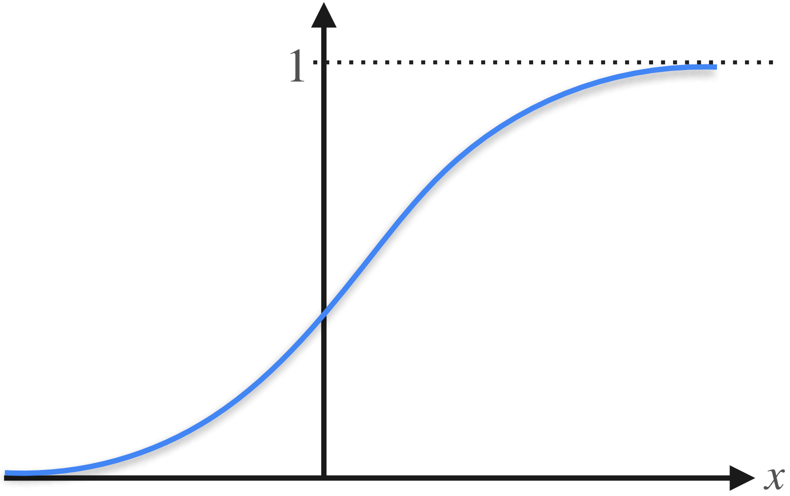
Cumulative Distribution Function: Formal Definition

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

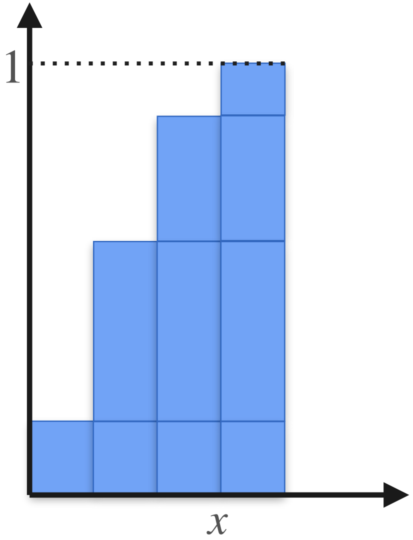


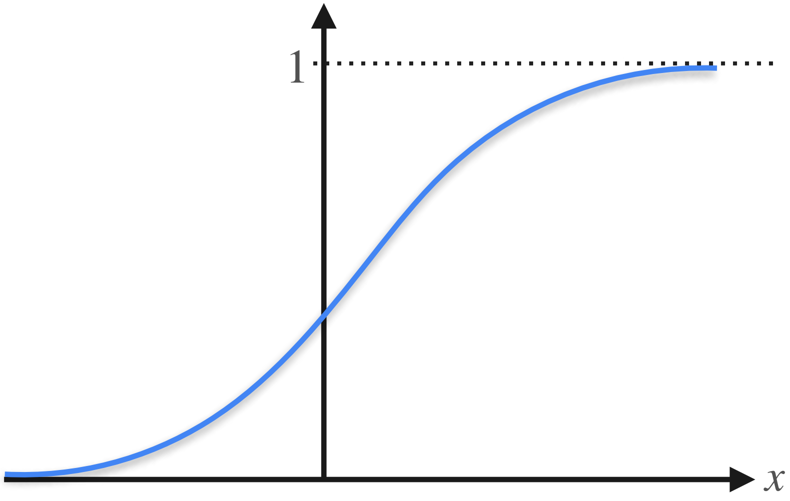


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $\text{CDF}(x) = \mathbf{P}(X \leq x)$

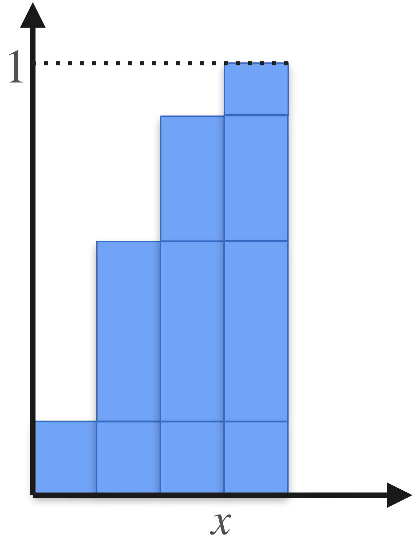


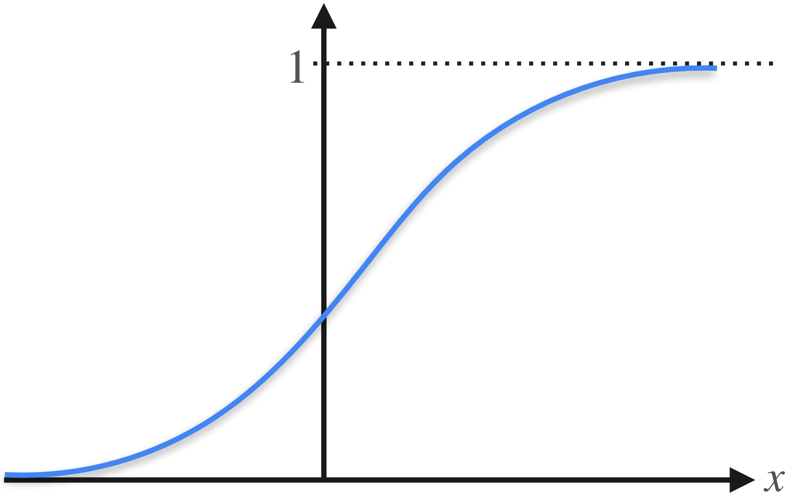


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $\text{CDF}(x) = \mathbf{P}(X \leq x)$ It is defined for every real number

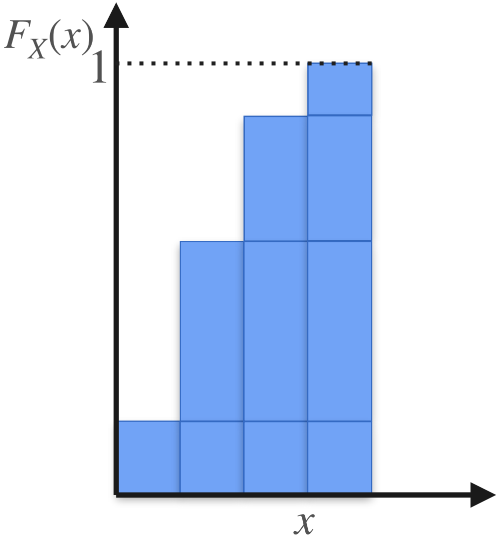


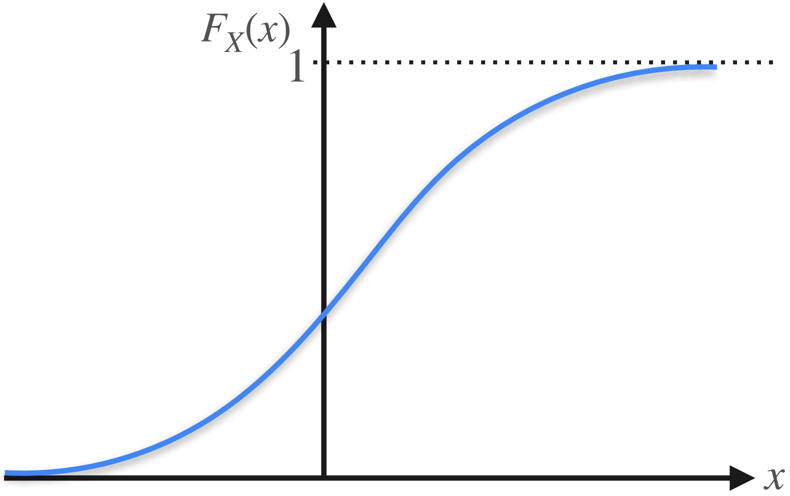


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $F_X(x) = \mathbf{P}(X \leq x)$ ← It is defined for every real number





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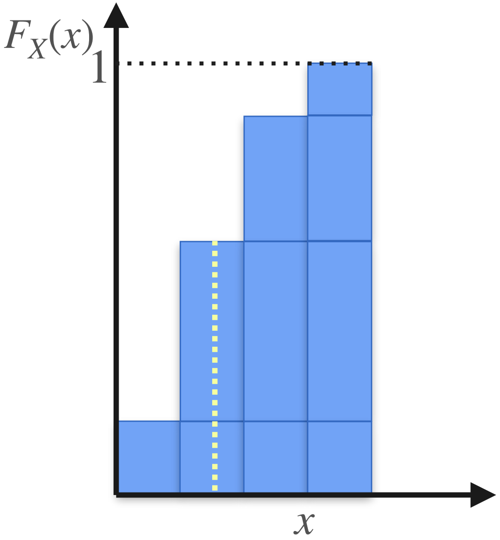

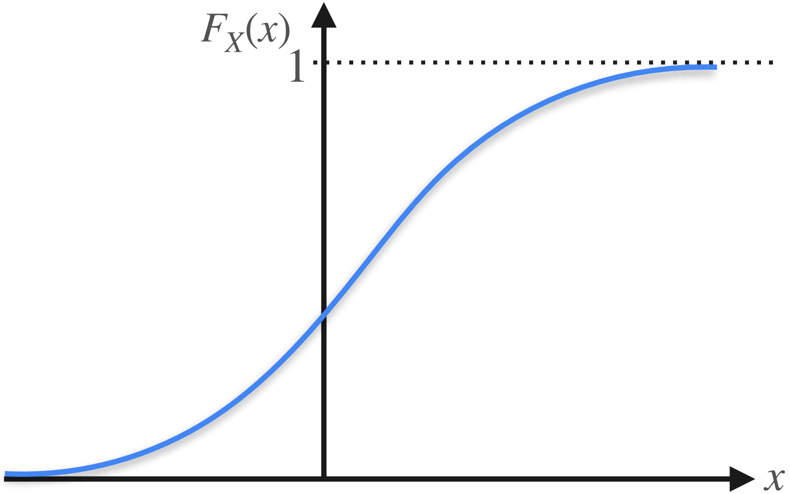

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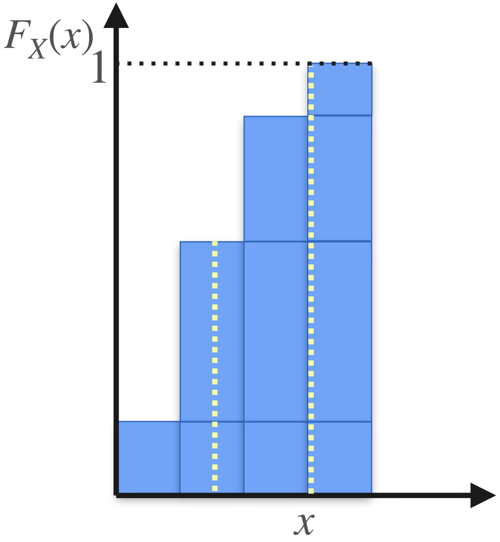

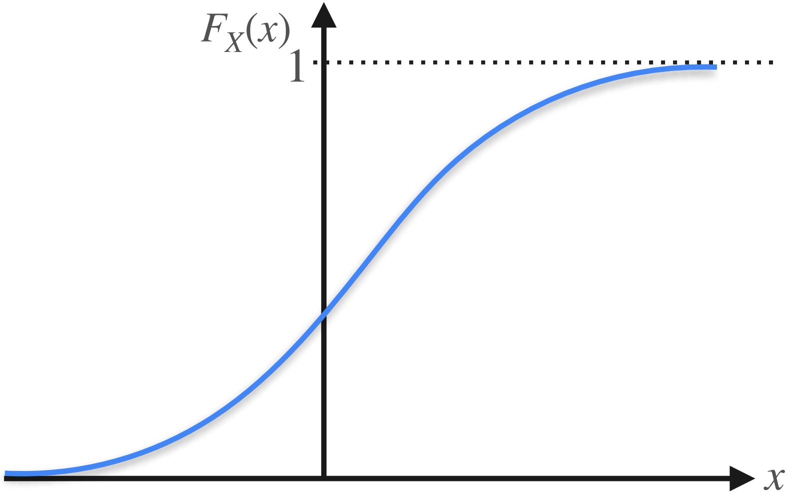

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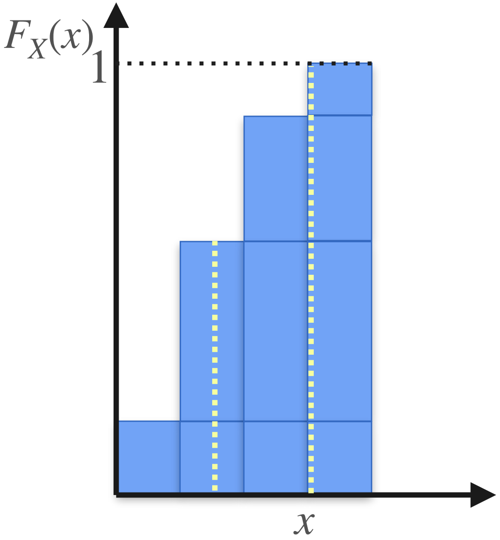

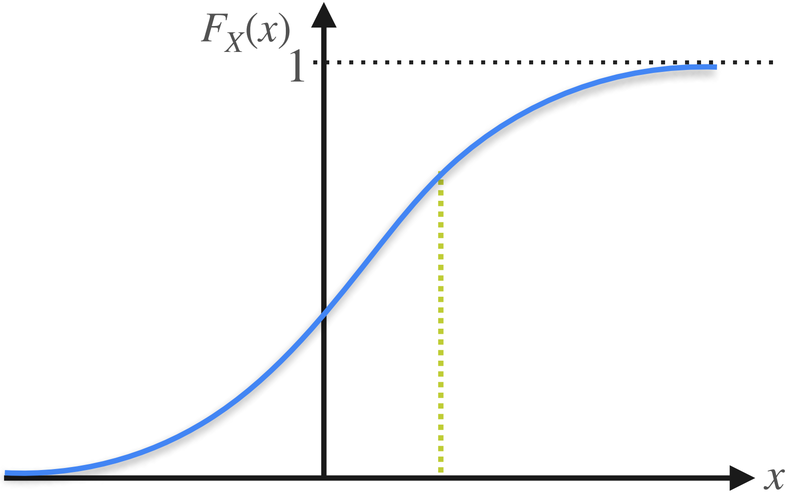

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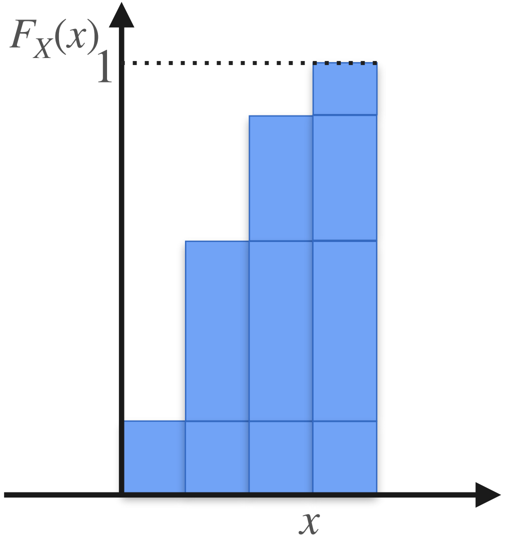

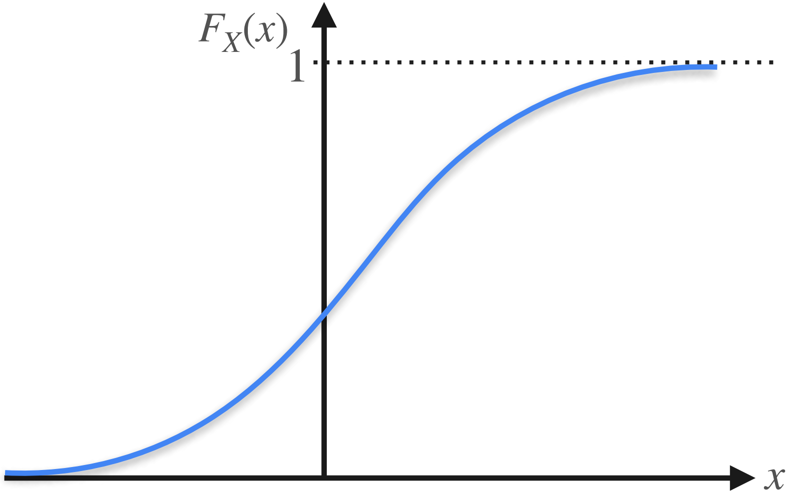

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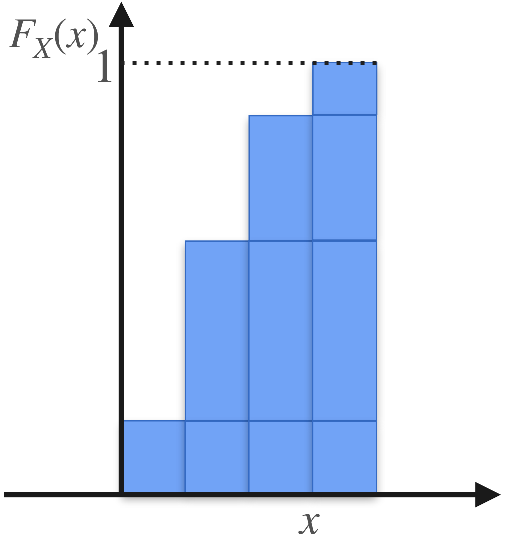

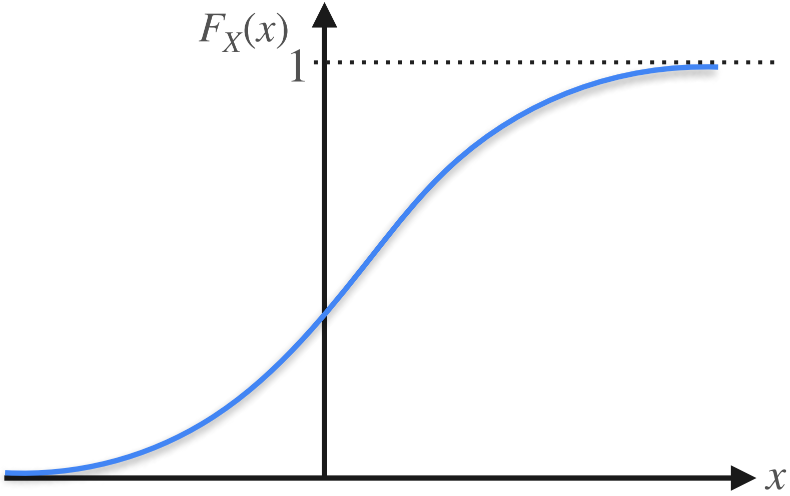

Properties

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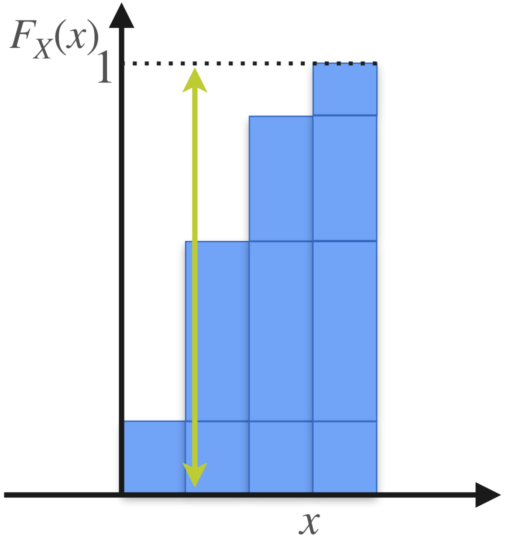

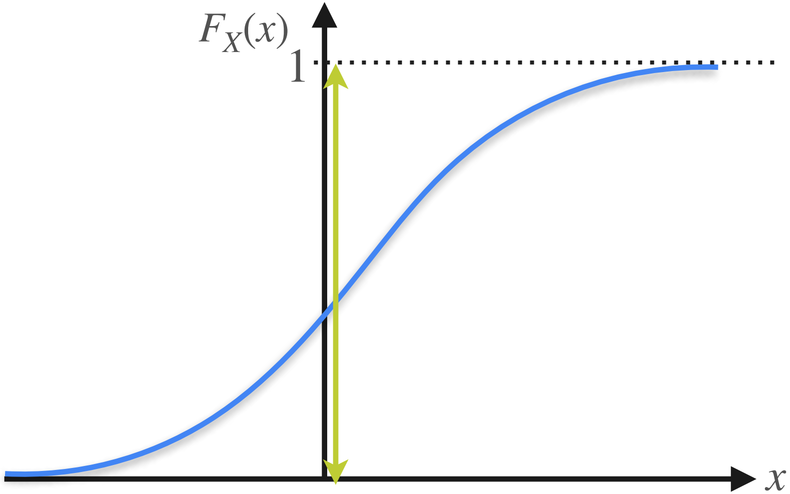

Properties

- $0 \leq F_X(x) \leq 1$

Cumulative Distribution Function: Formal Definition

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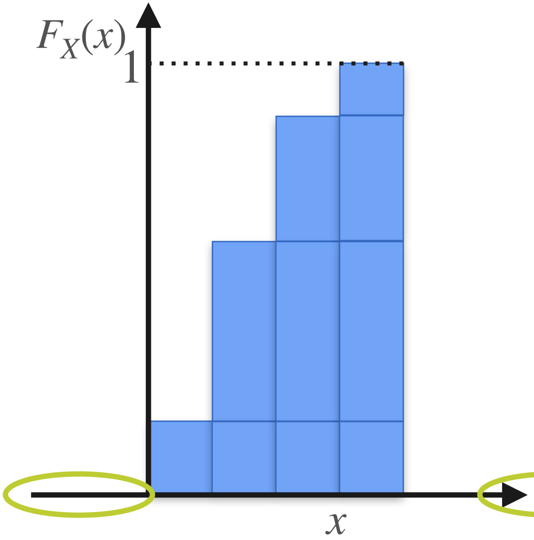

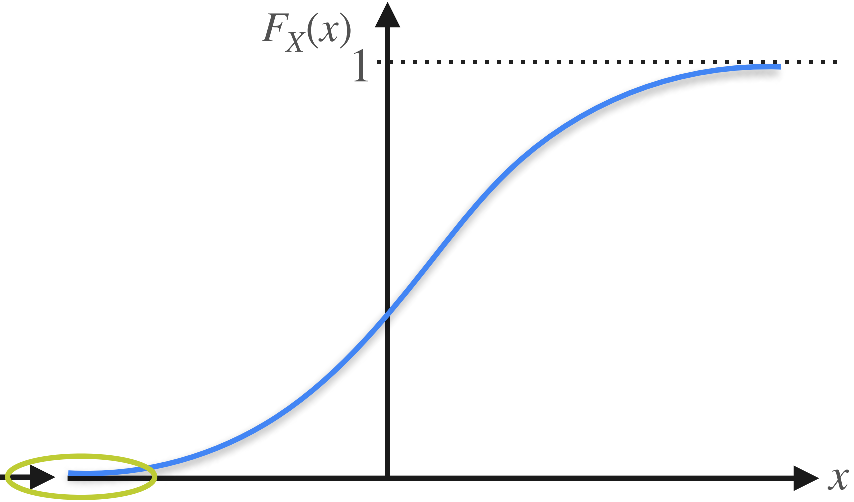

Properties

- $0 \leq F_X(x) \leq 1$
- Left “endpoint” is 0

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

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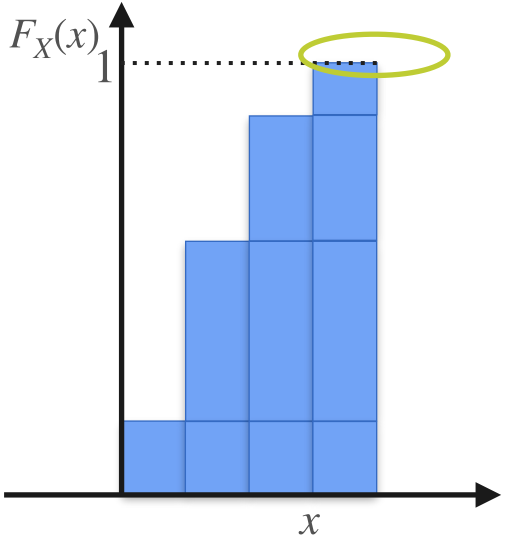

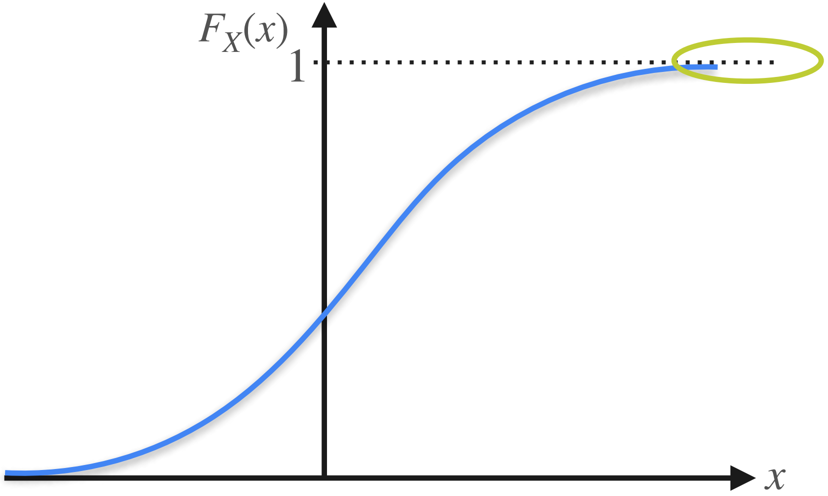

Properties

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- Right "endpoint" is 1

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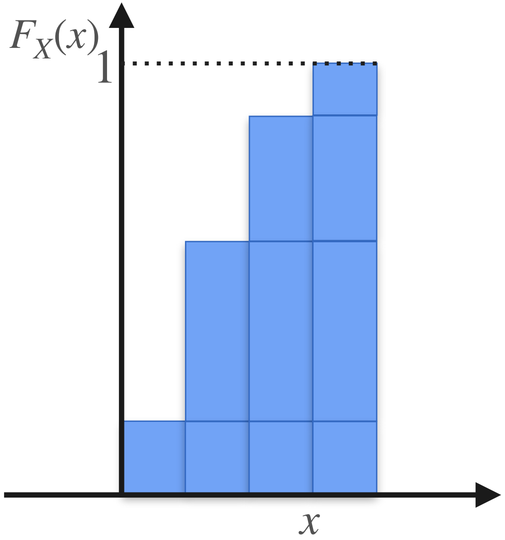
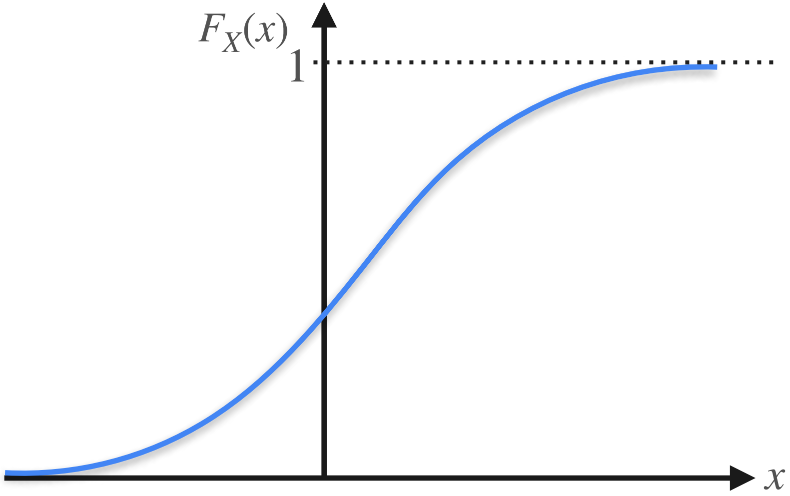


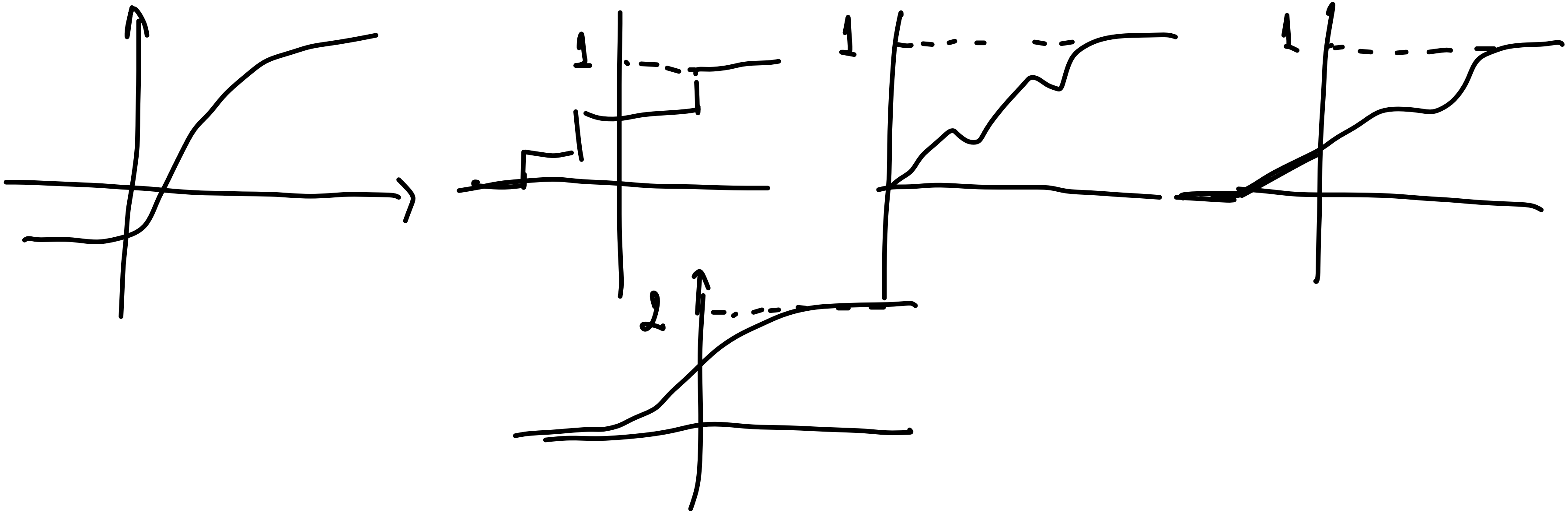


Properties

- $0 \leq F_X(x) \leq 1$
- Left "endpoint" is 0
- Right "endpoint" is 1
- **Never decreases**

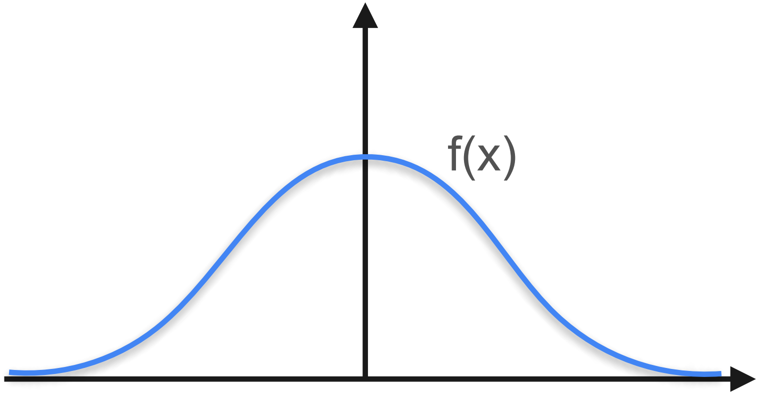
Quiz

- Which of the following functions could be a CDF?



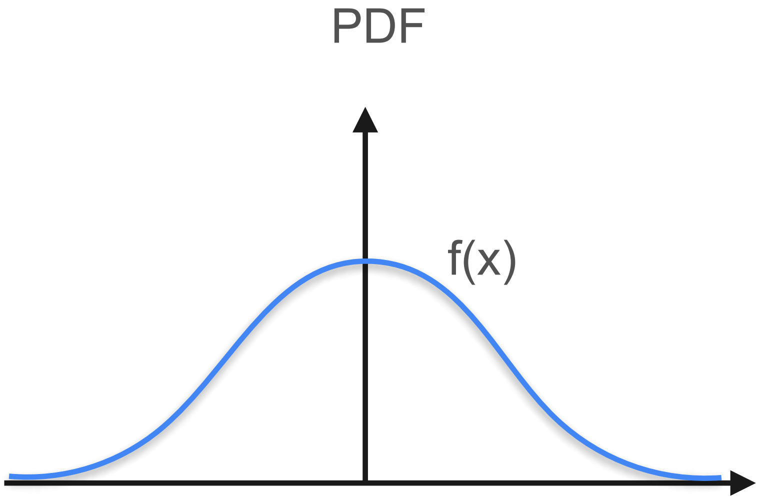
PDF and CDF Summary

PDF



- area = 1
- Always positive

PDF and CDF Summary



- area = 1
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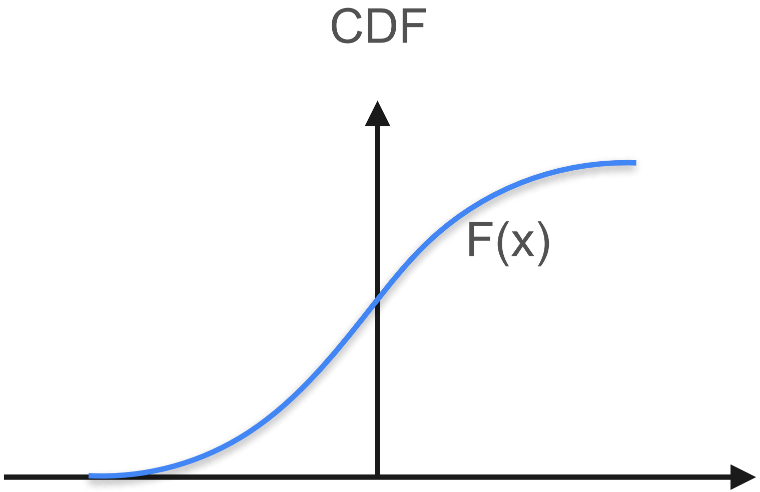

- left “endpoint” is 0
- right “endpoint" is 1
- (endpoints can be at infinity)
- Always positive and increasing

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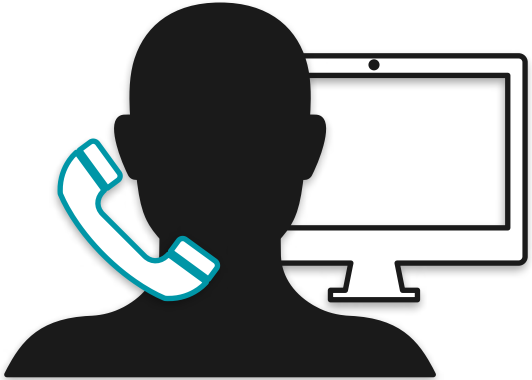
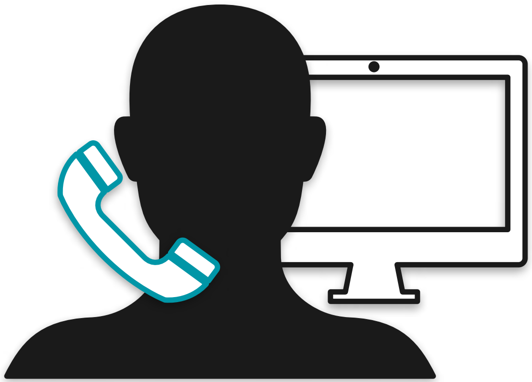
Probability Distributions

Uniform Distribution

Uniform Distribution: Motivation

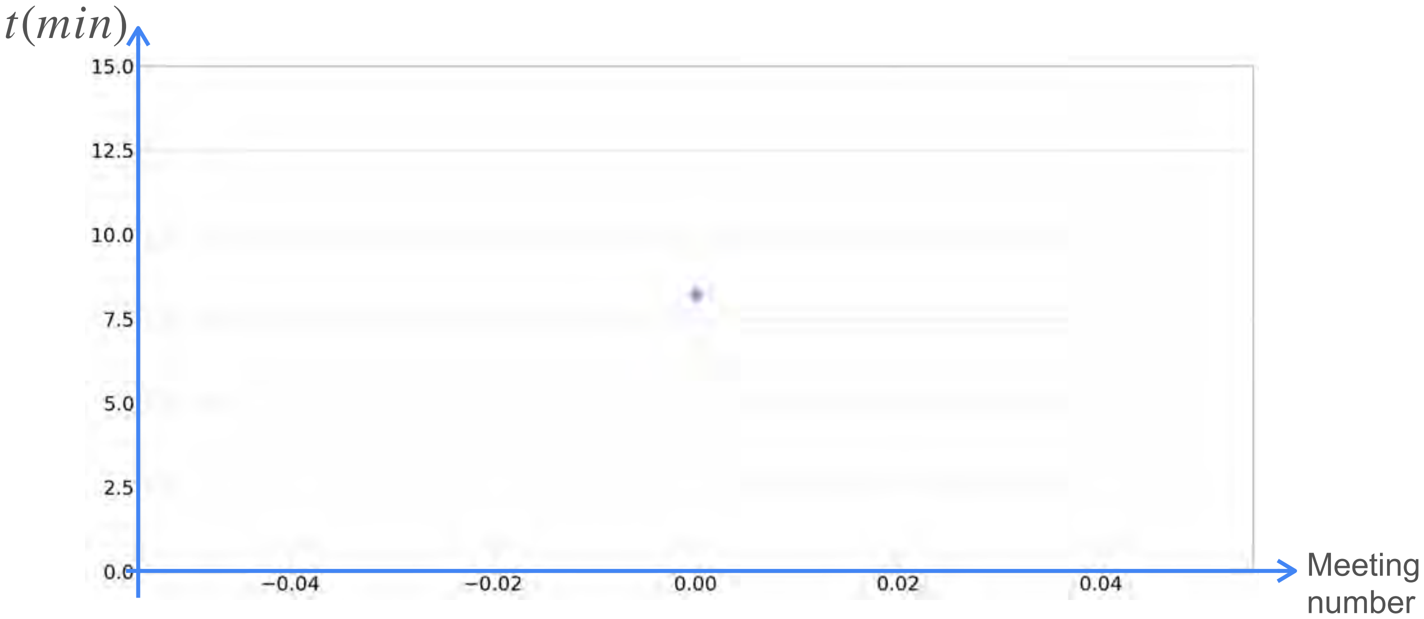


Uniform Distribution: Motivation



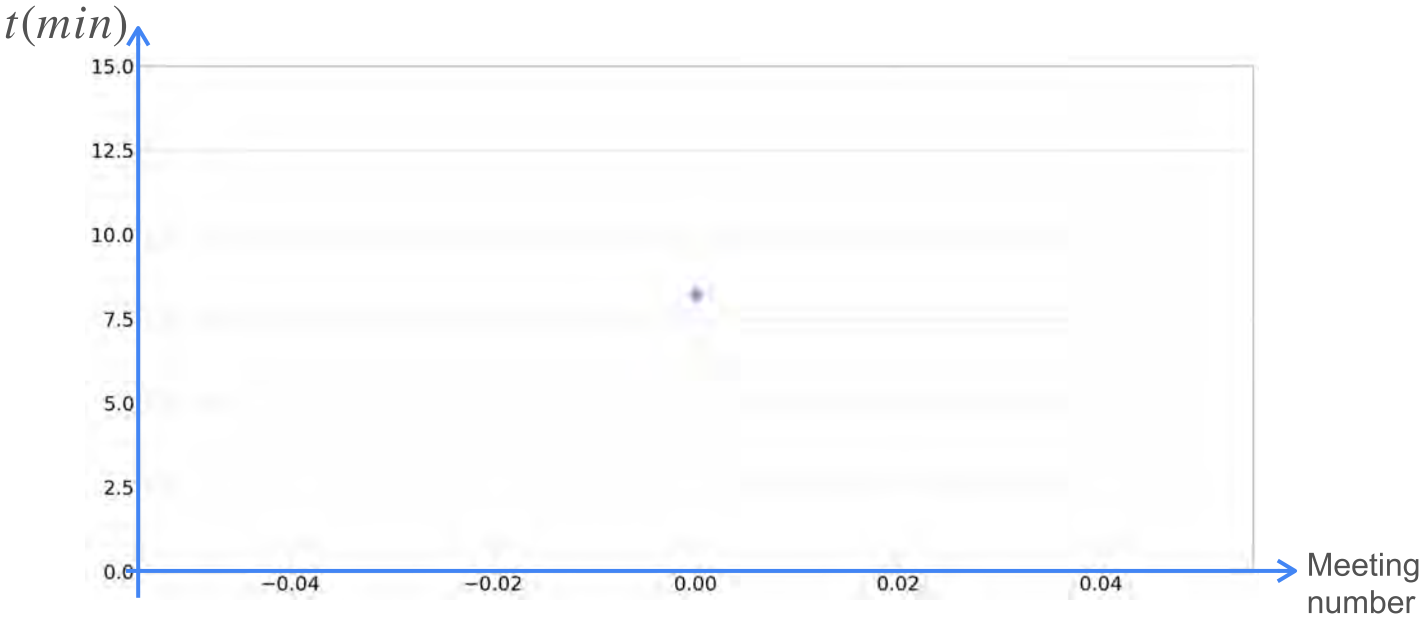
You're calling a tech support line.
They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

Uniform Distribution: Motivation



Uniform Distribution: Motivation

Last 200 times you called them, you took down notes of how long they took to respond



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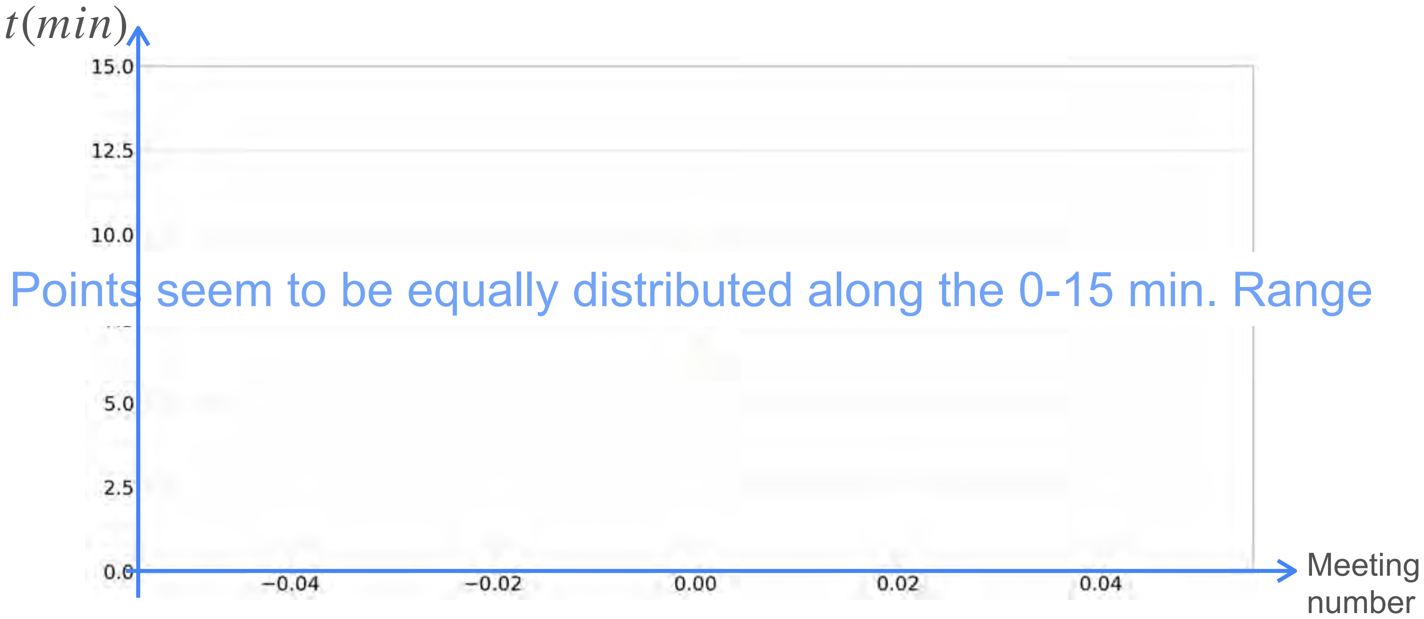
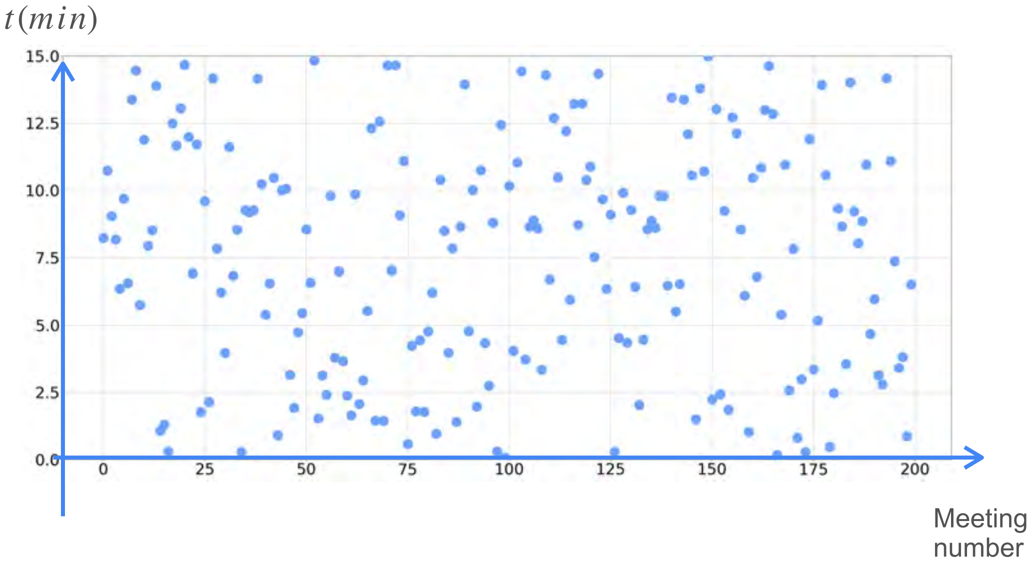


Uniform Distribution: Motivation

$t(min)$

Uniform Distribution: Motivation



$t(min)$

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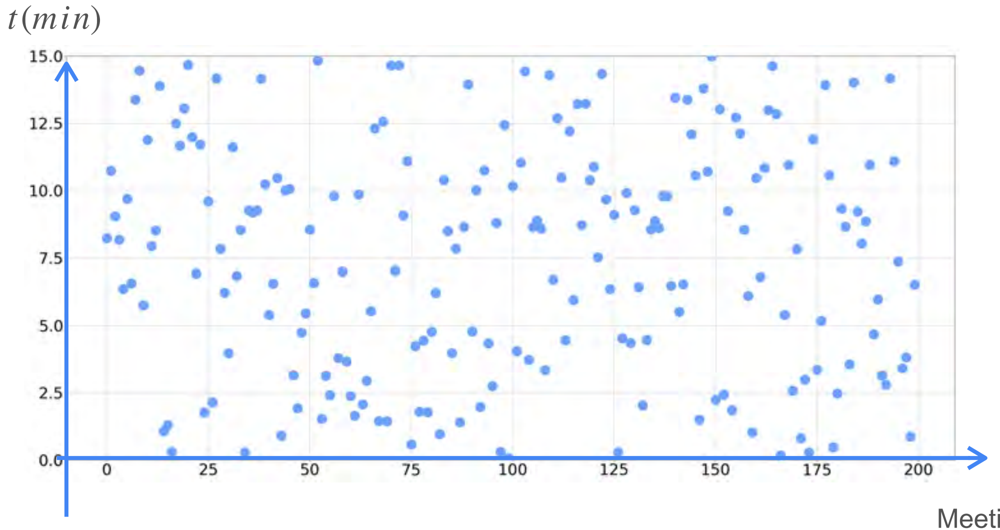

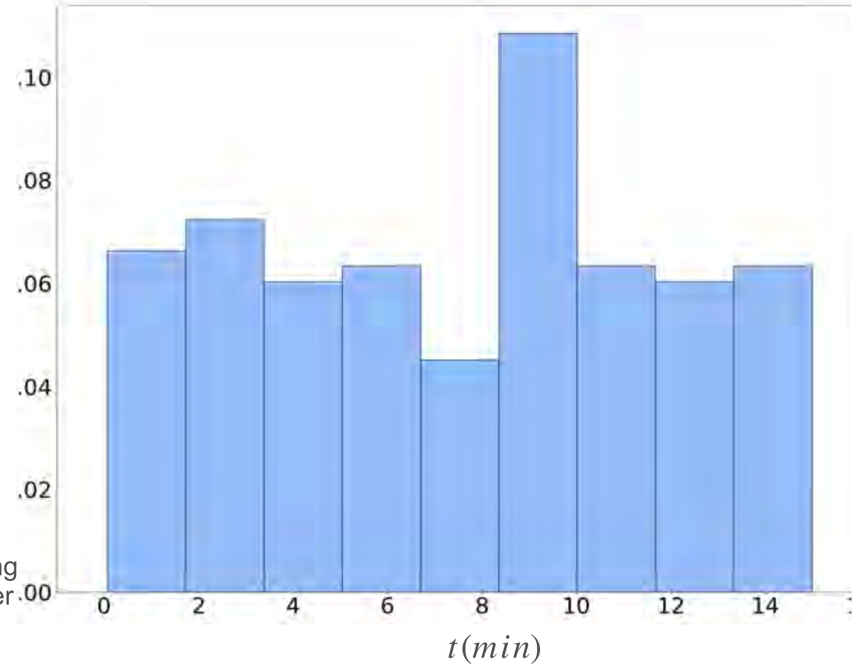

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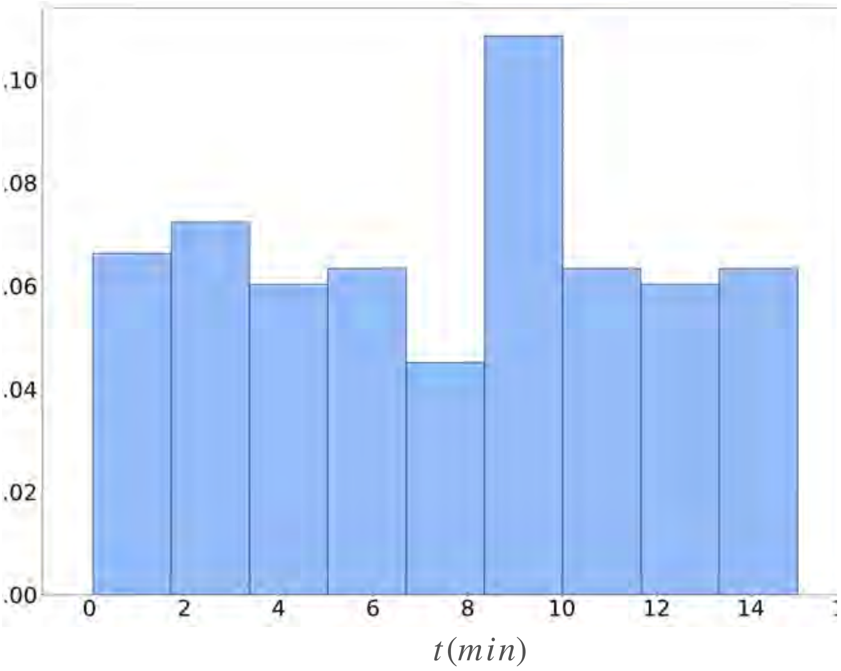
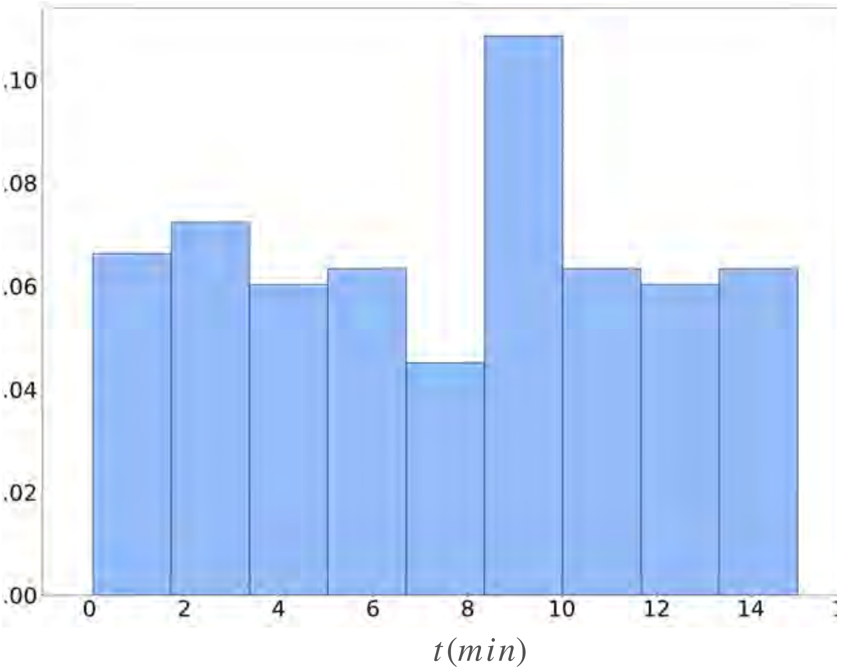


Uniform Distribution: Motivation

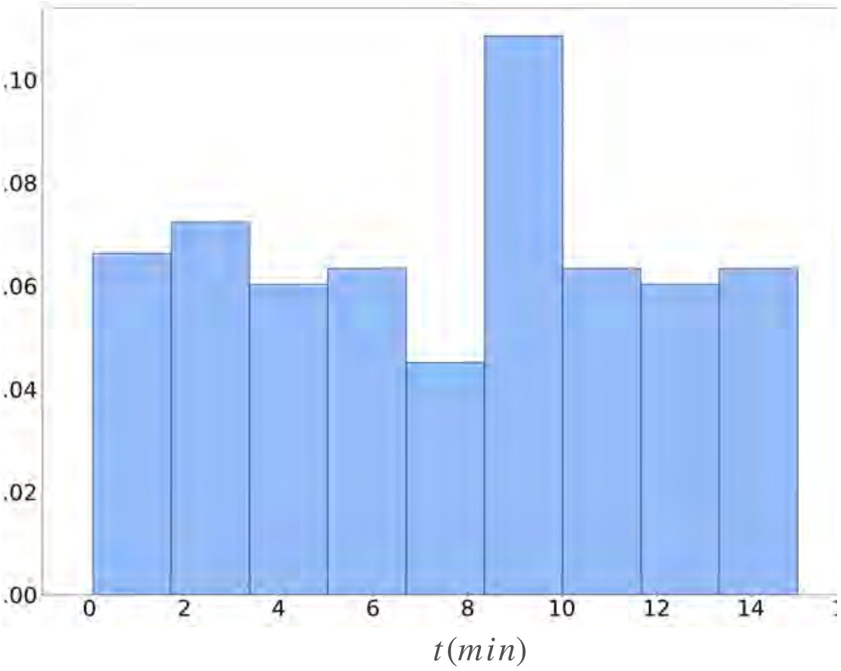
T: time (in minutes) you have to wait



Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.



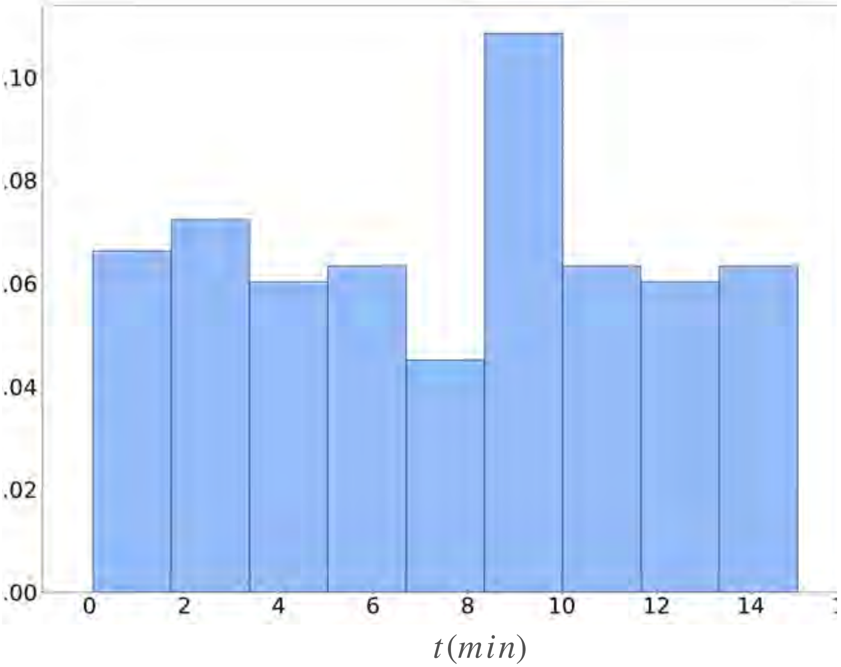
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

↓

The pdf must be constant for all values in the interval (0,15)



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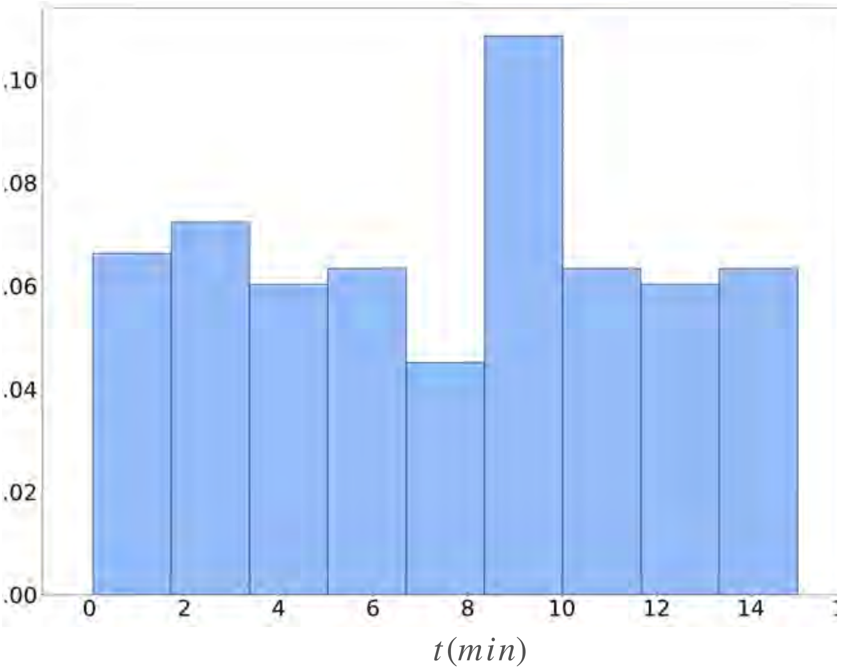
↓

Which constant?



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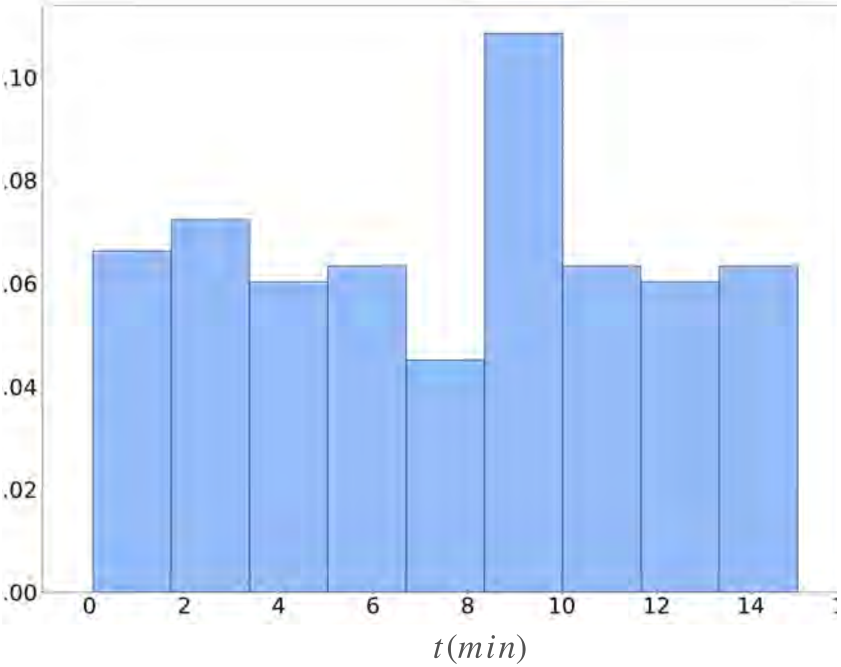
↓

Which constant? → $15 \times h = 1$



Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

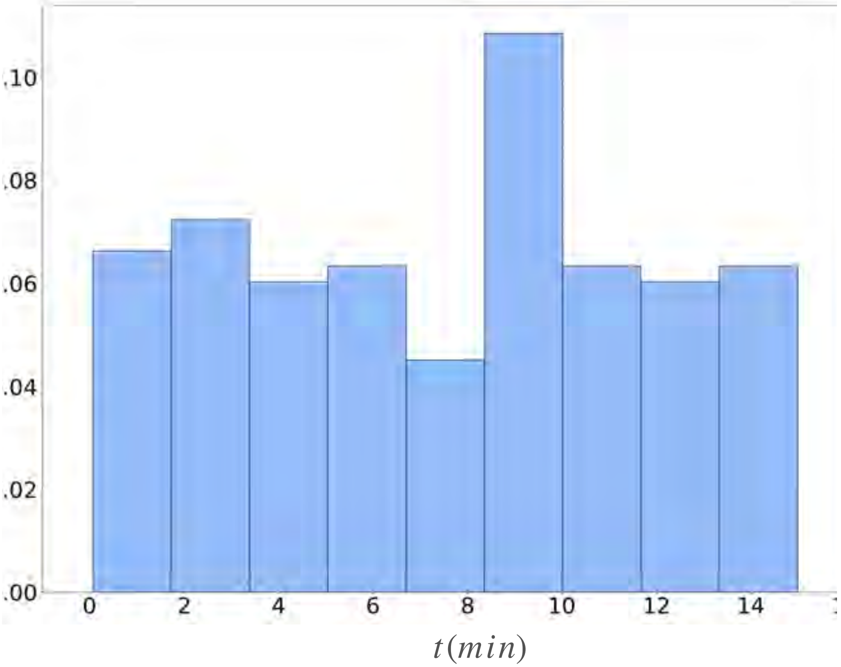
↓

The pdf must be constant for all values in the interval (0,15)

↓

Which constant? → $15 \times h = 1$ → $h = \frac{1}{15}$

$= 0.06$



Uniform Distribution: Motivation

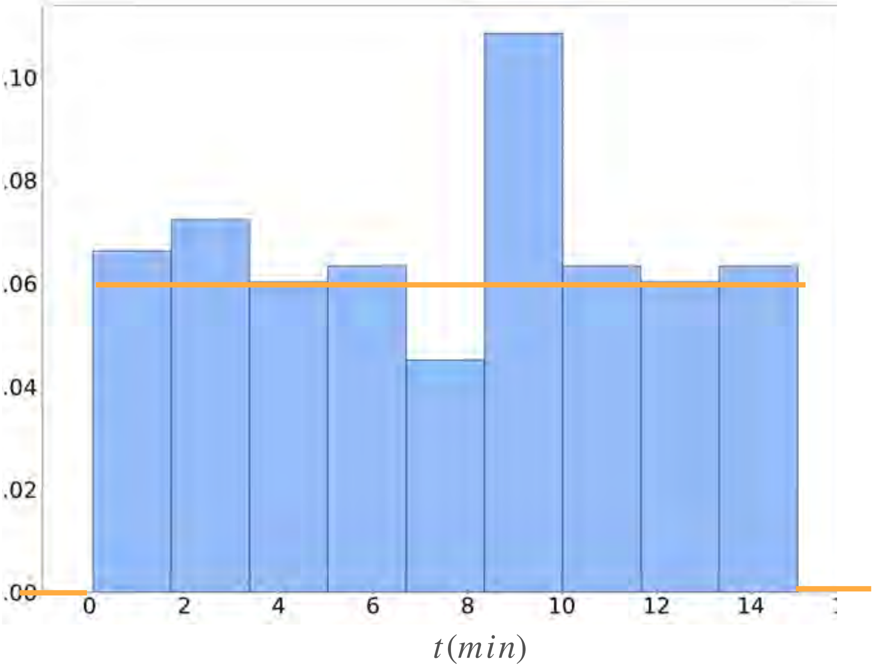
T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant? $\rightarrow 15 \times h = 1 \rightarrow h = \frac{1}{15}$

$= 0.06$



Uniform Distribution: Model

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

- a: beginning of the interval

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

- a: beginning of the interval
- b: end of the interval

x

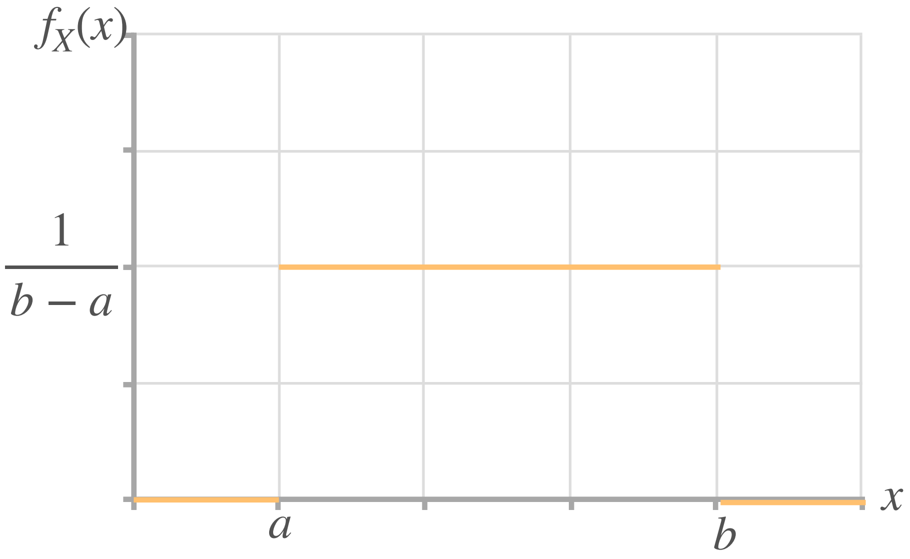
Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

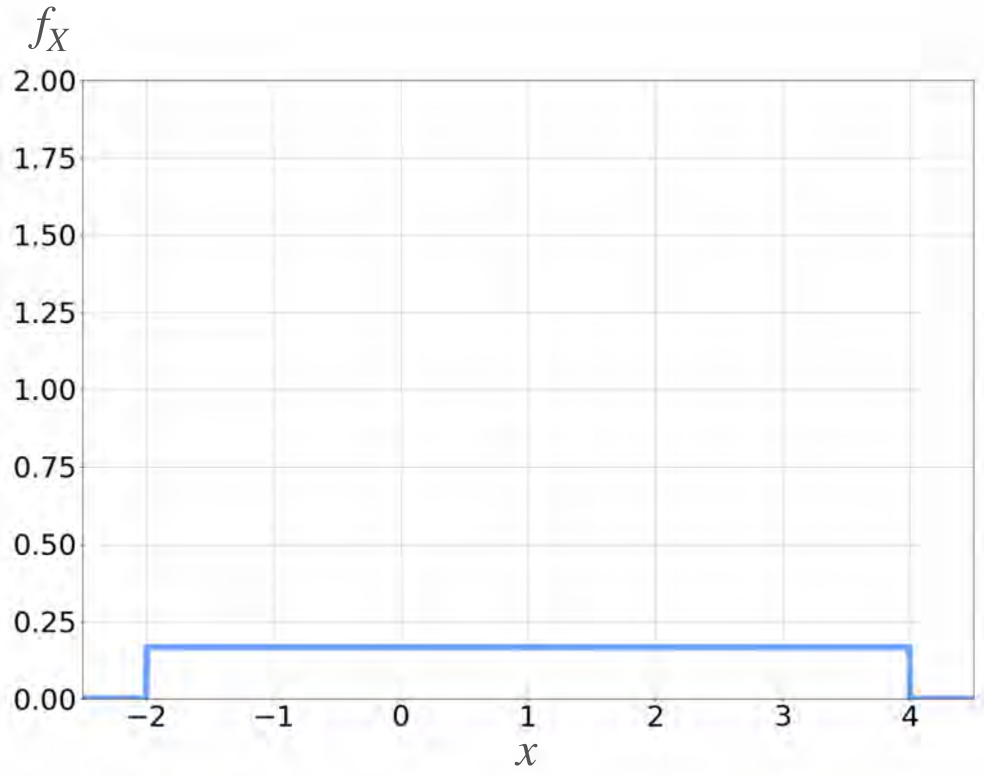
Parameters:

- a: beginning of the interval
- b: end of the interval

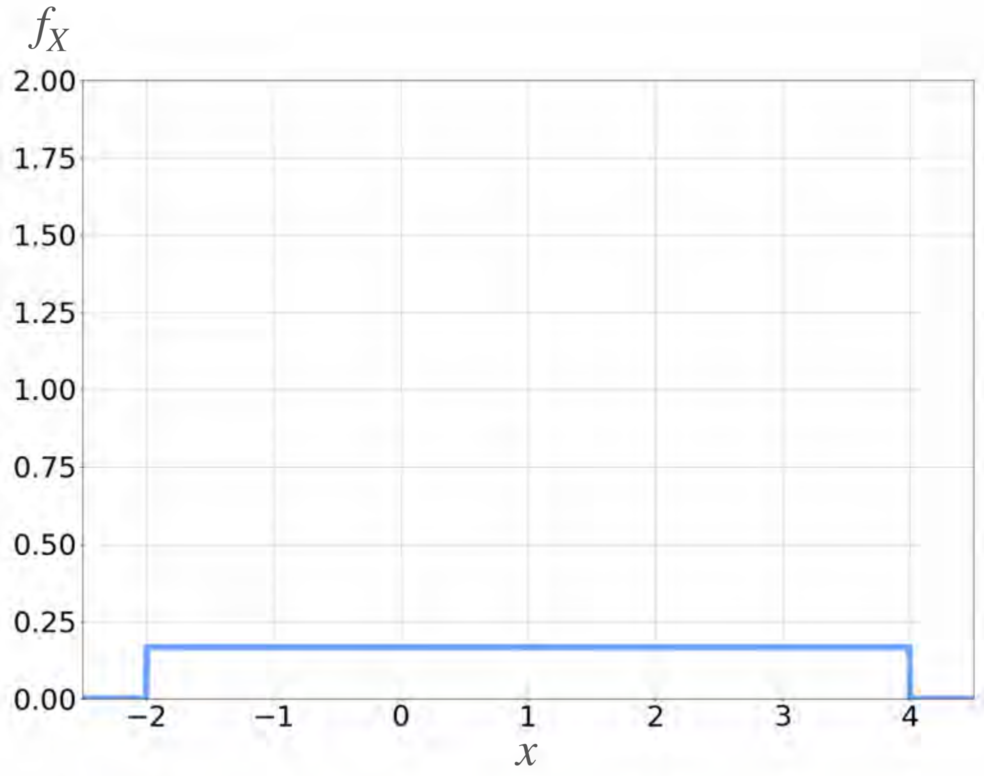
$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x \notin (a,b) \end{cases}$$



Uniform Distribution: PDF



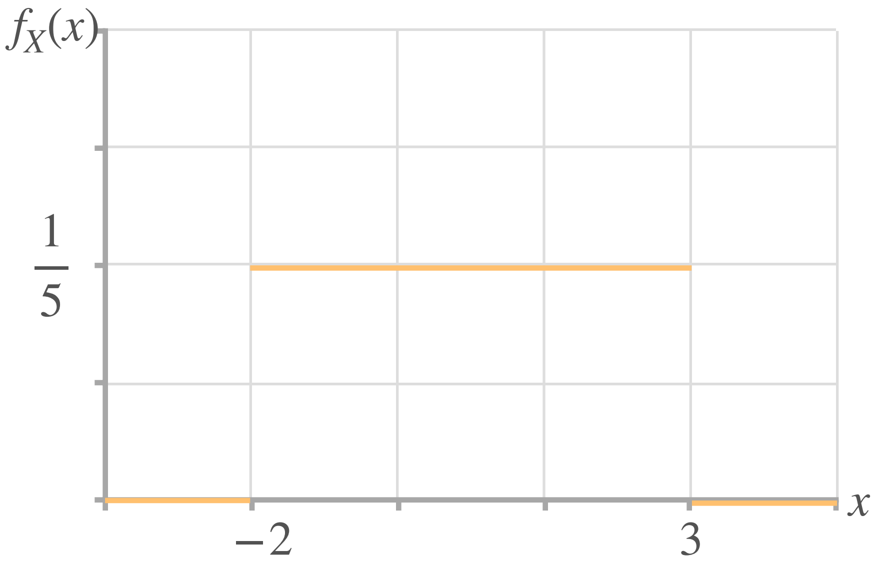
Uniform Distribution: PDF



Quiz

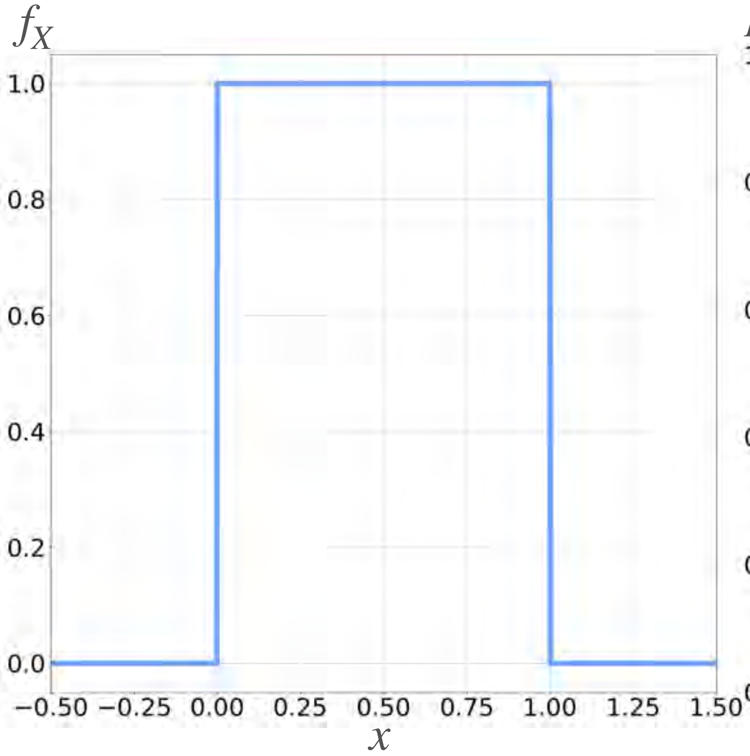
What is the probability of X being between -1 and 3?

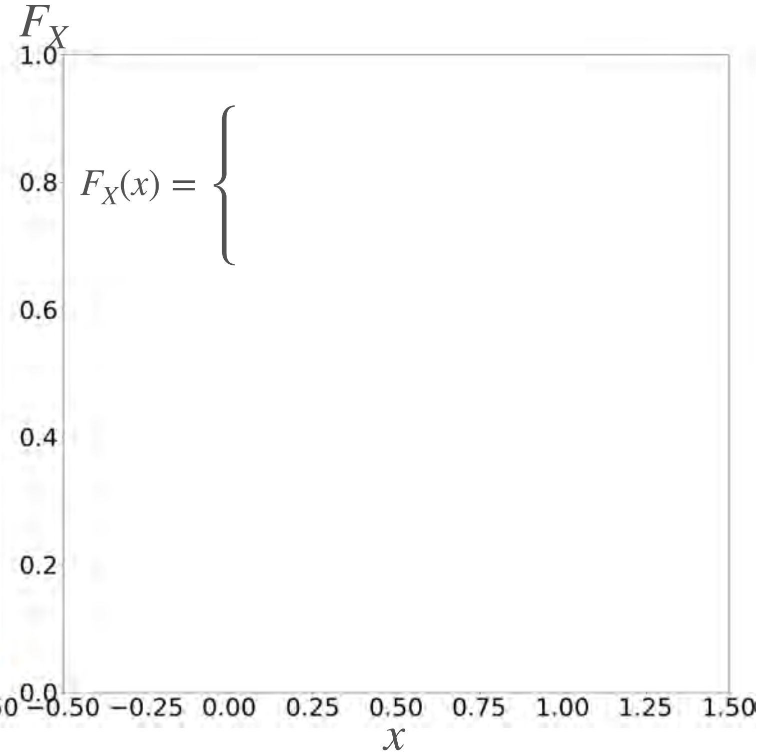
What is the probability of X being between -1 and 4?



Uniform Distribution: CDF

Uniform Distribution: CDF





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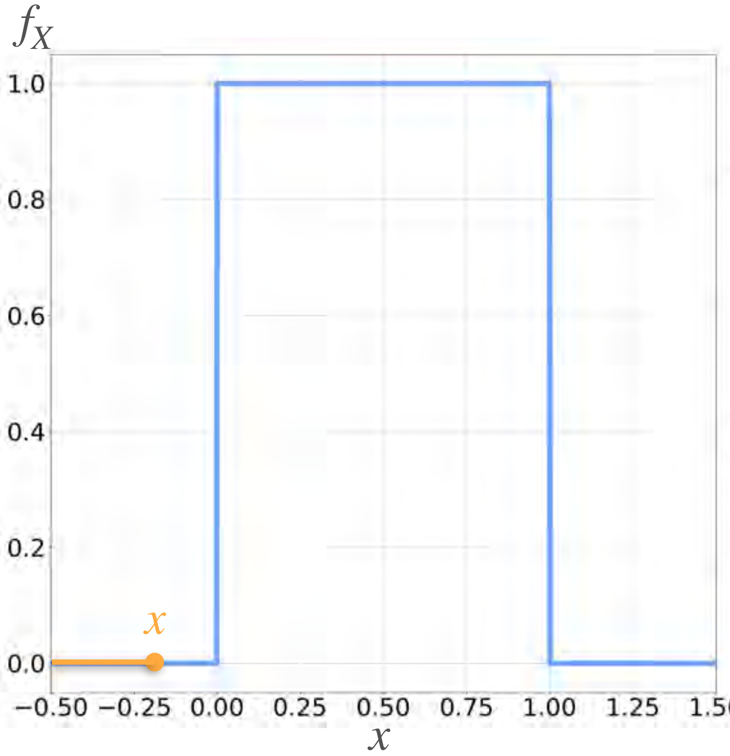
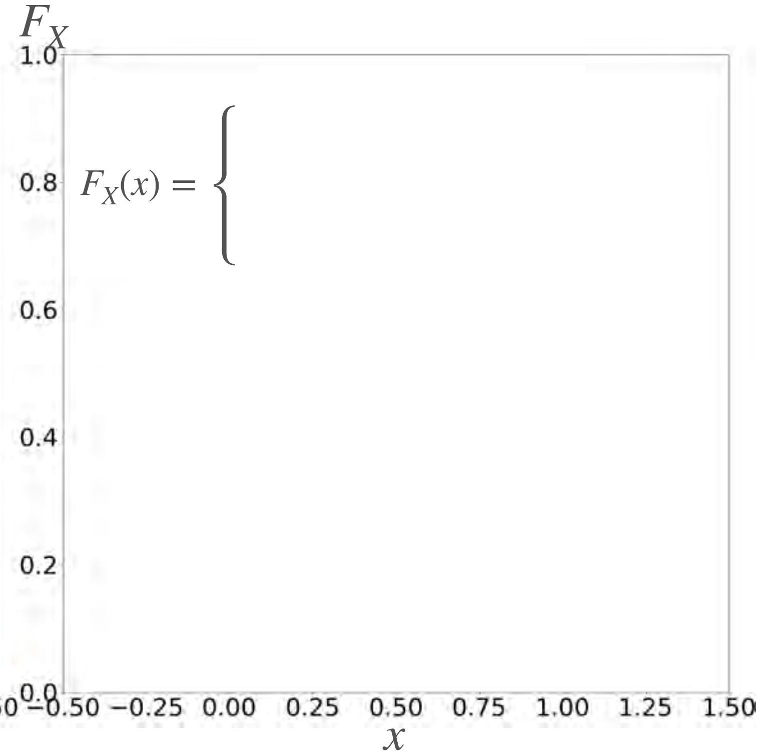
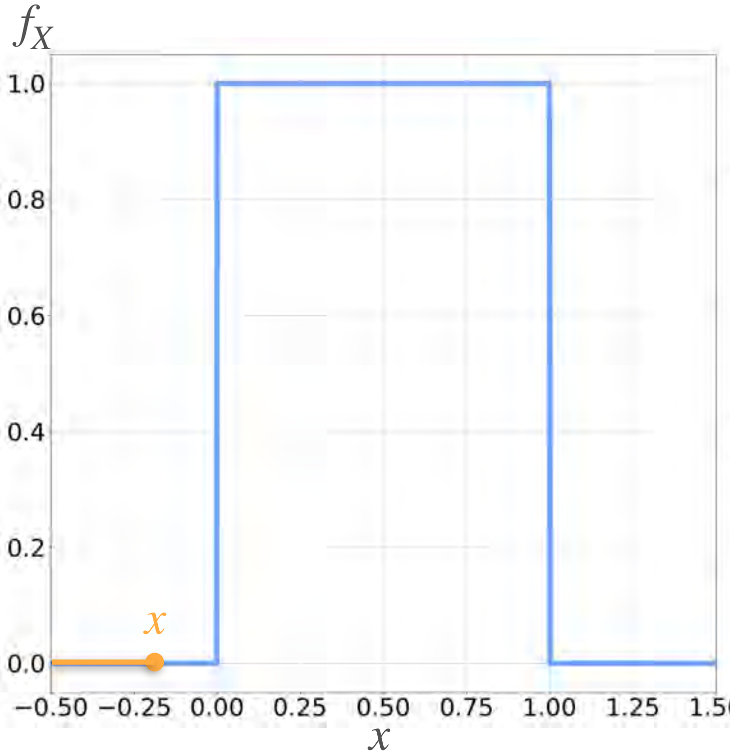
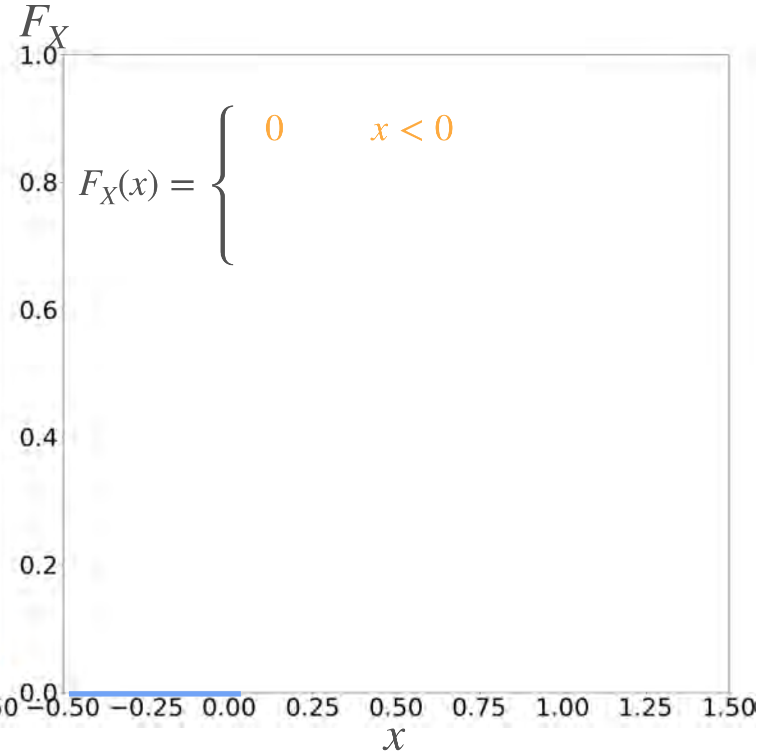
Uniform Distribution: CDF





Uniform Distribution: CDF





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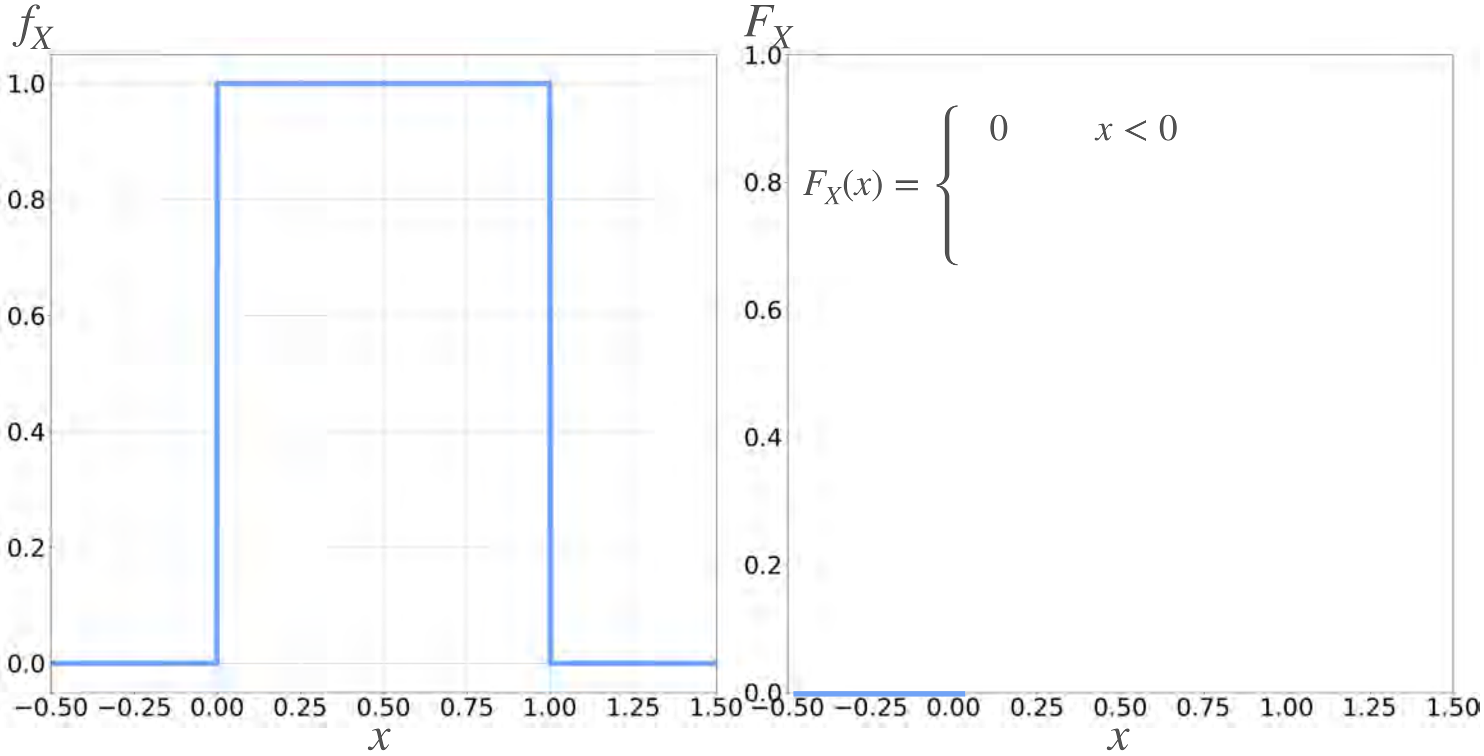
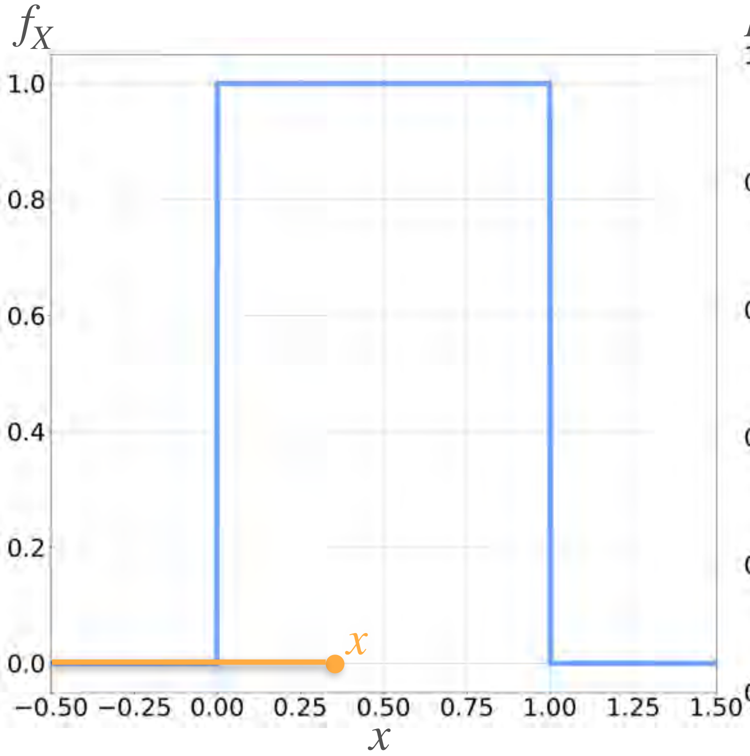
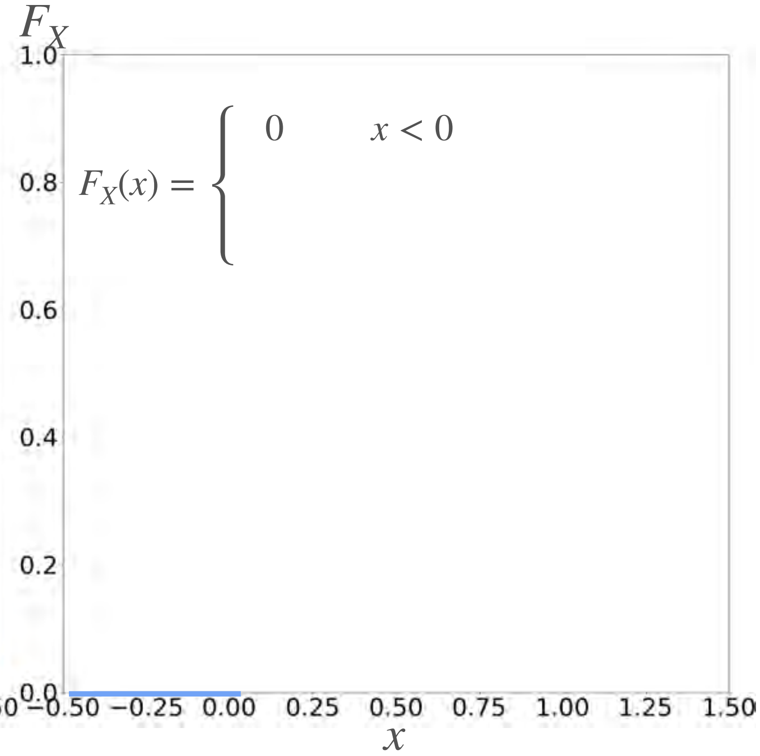
Uniform Distribution: CDF



$$F_X(x) = \begin{cases} 0 & x < 0 \\ & \\ & \end{cases}$$

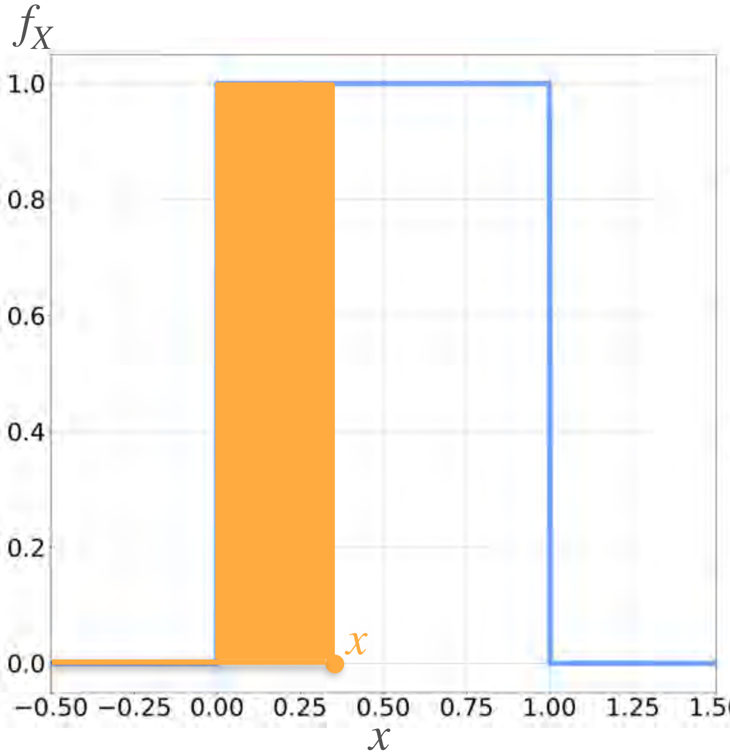
Uniform Distribution: CDF

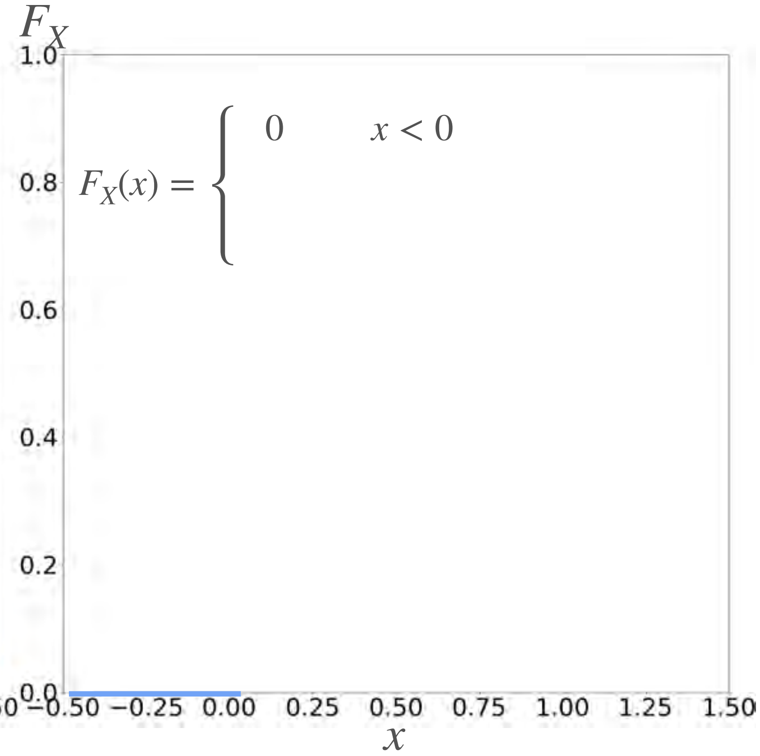




$$F_X(x) = \begin{cases} 0 & x < 0 \\ & \\ & \end{cases}$$

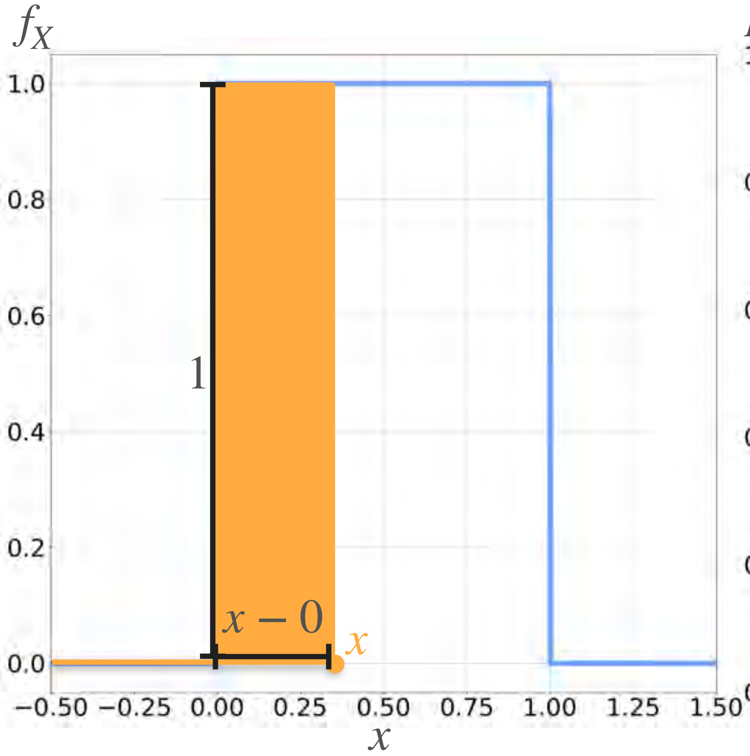
Uniform Distribution: CDF

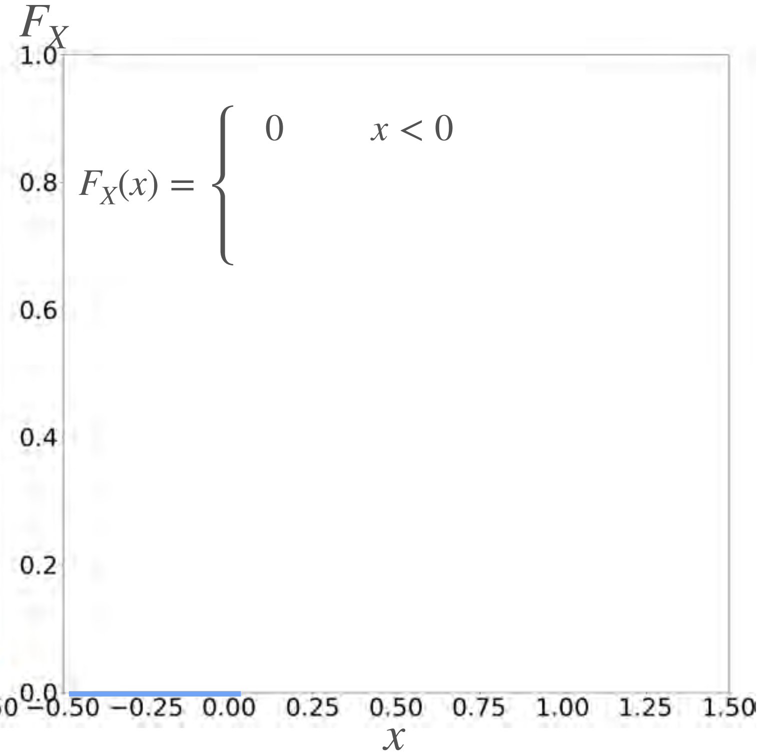




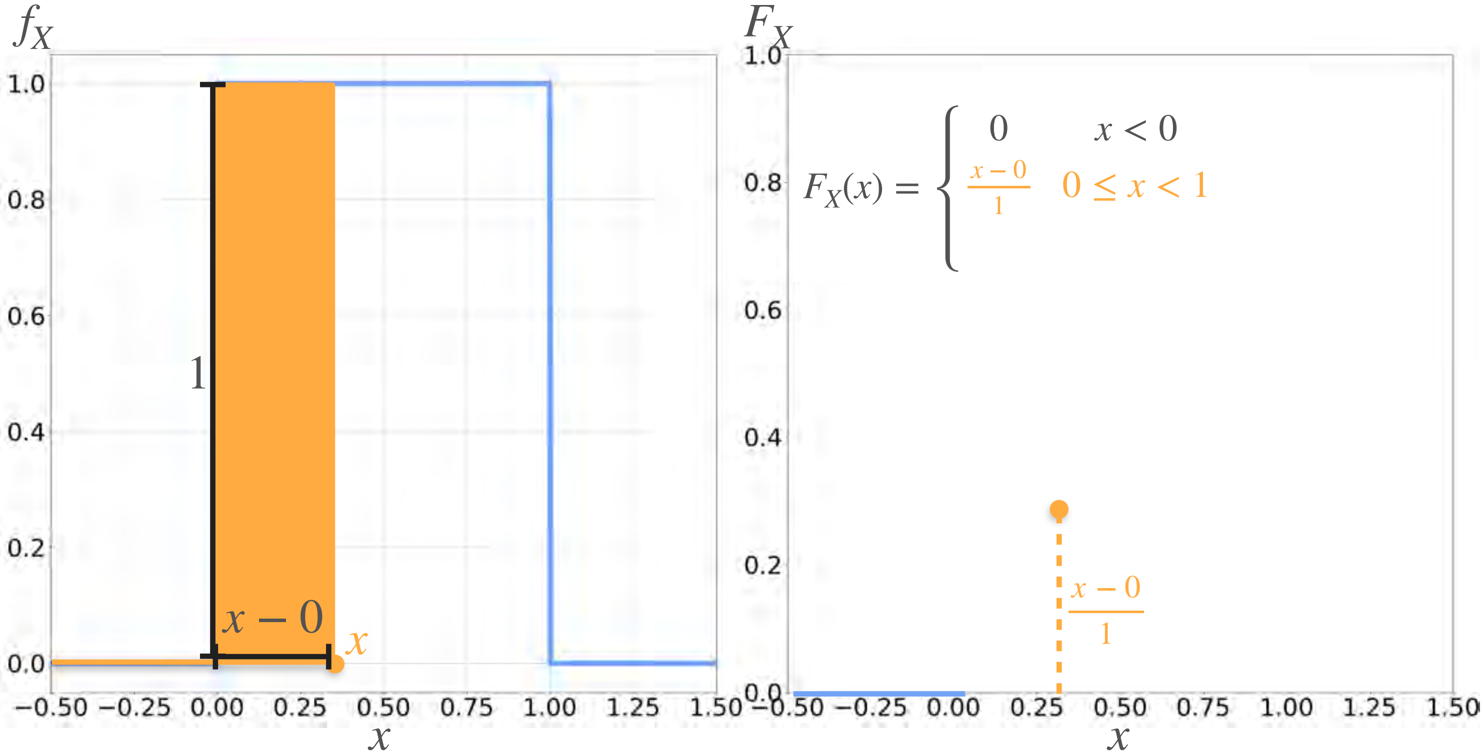
$$F_X(x) = \begin{cases} 0 & x < 0 \\ & \\ & \end{cases}$$

Uniform Distribution: CDF

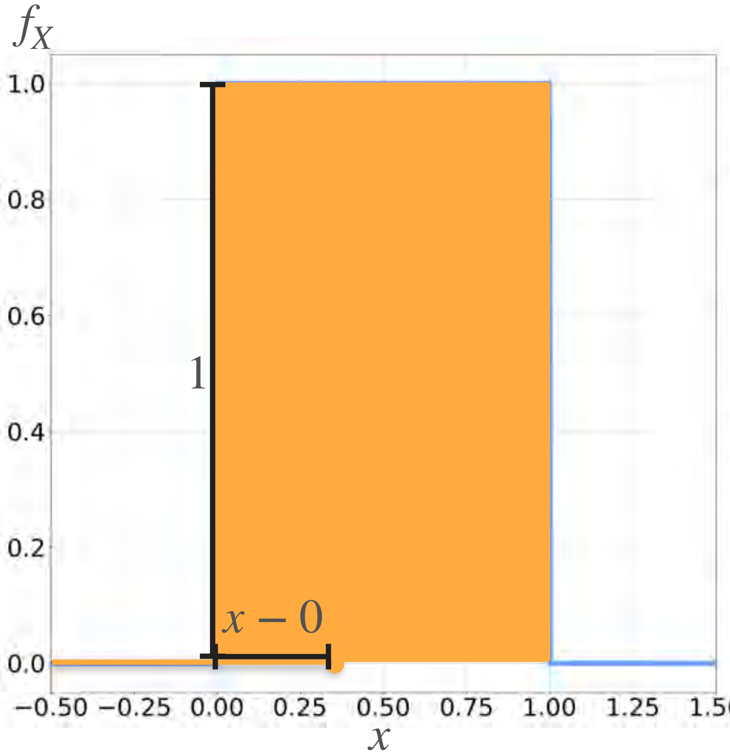


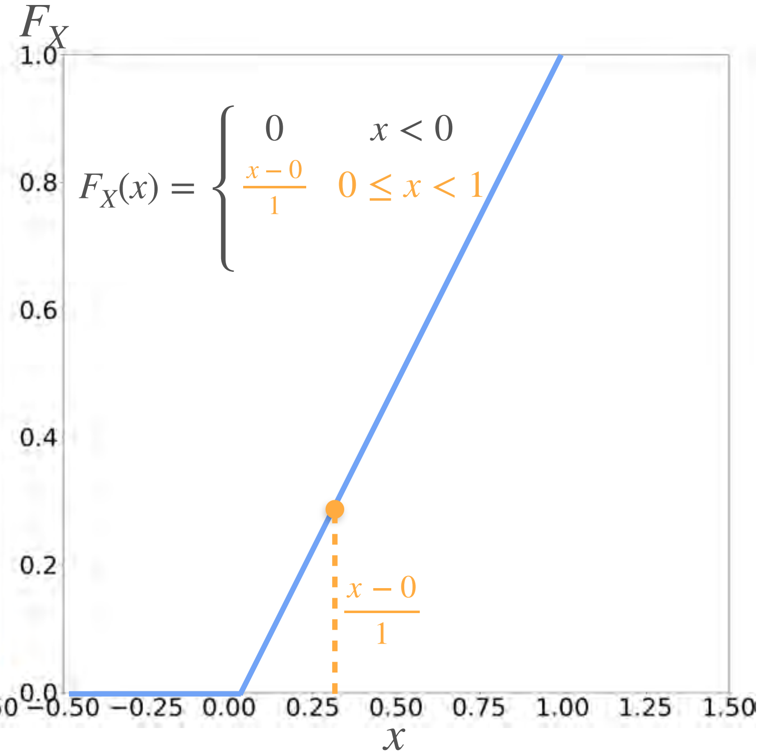


Uniform Distribution: CDF

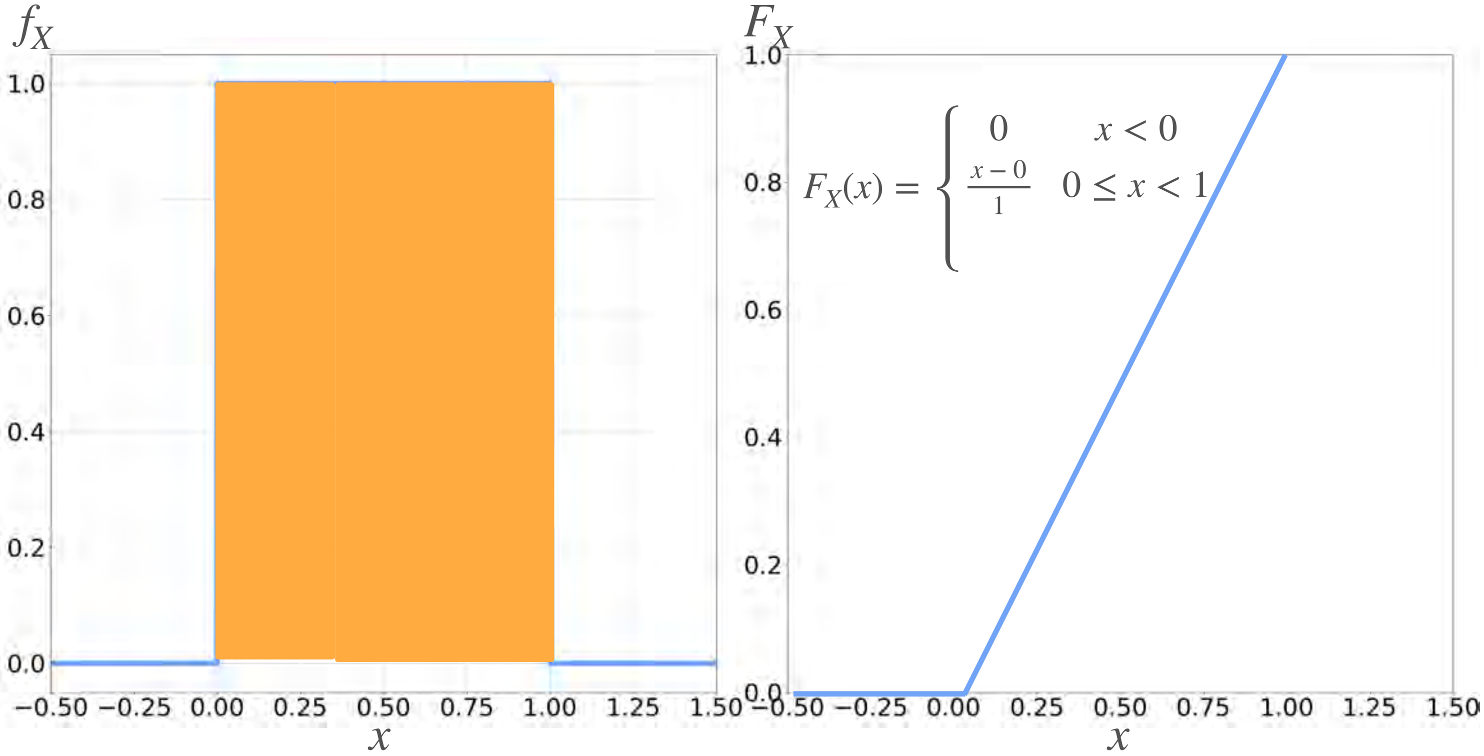


Uniform Distribution: CDF



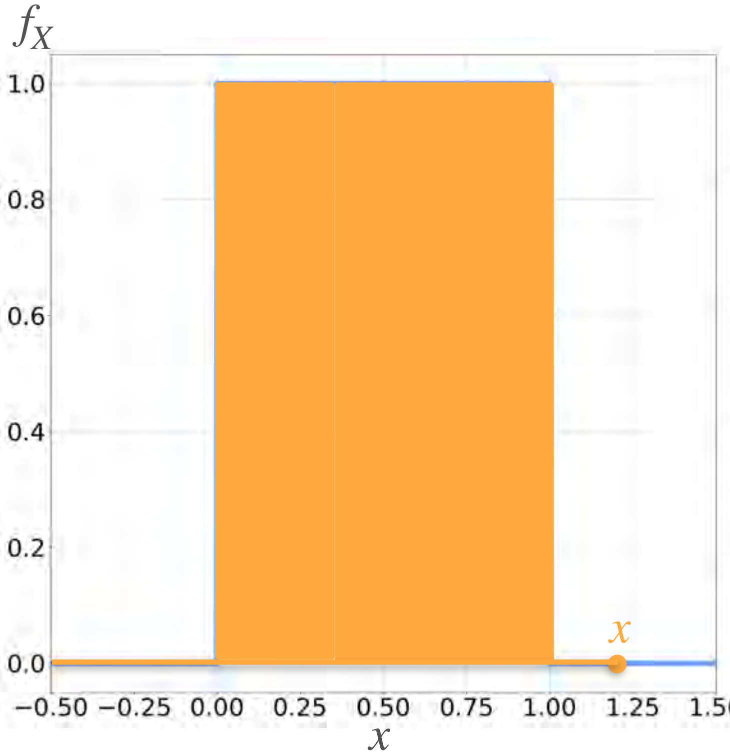


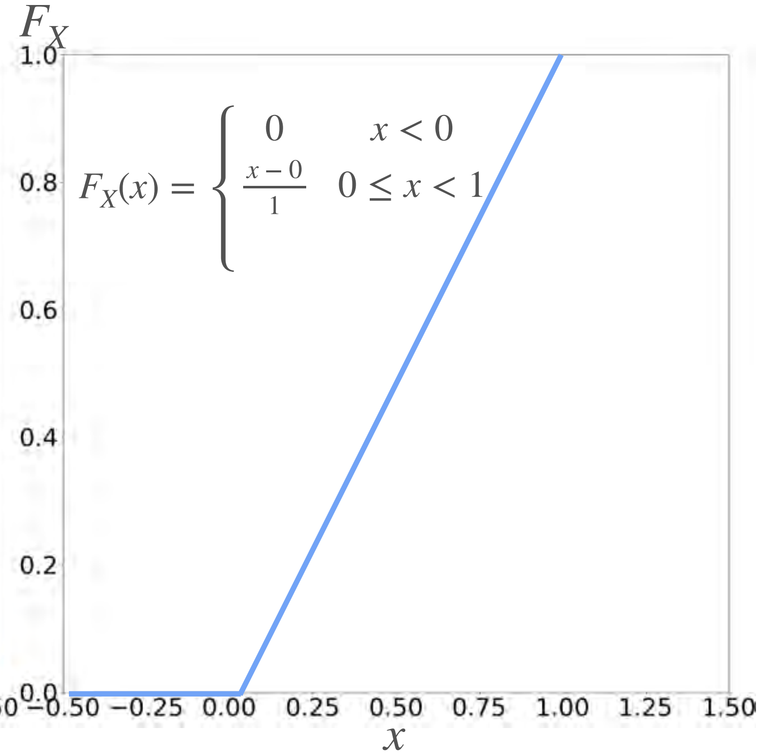
Uniform Distribution: CDF



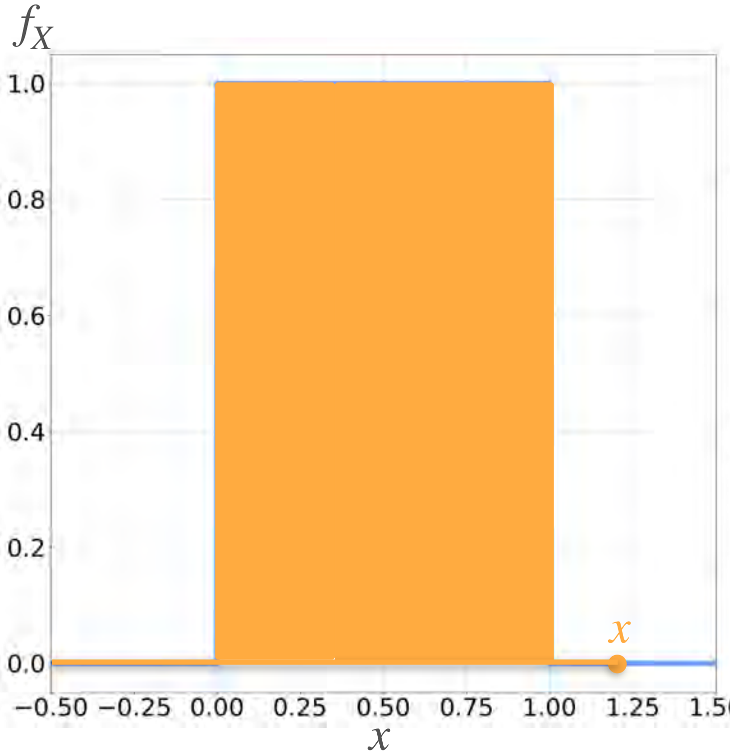
$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x-0}{1} & 0 \le x < 1 \end{cases}$$

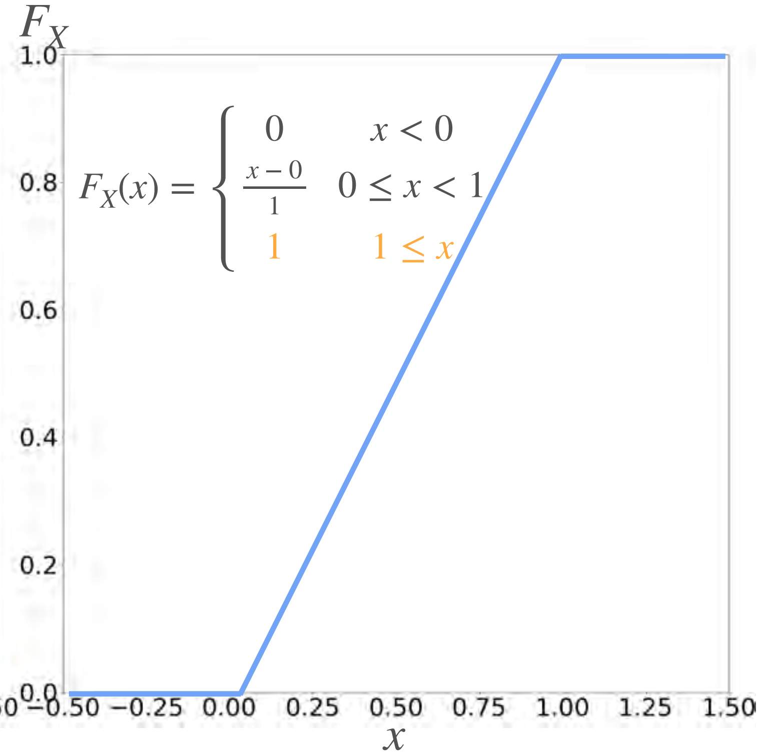
Uniform Distribution: CDF





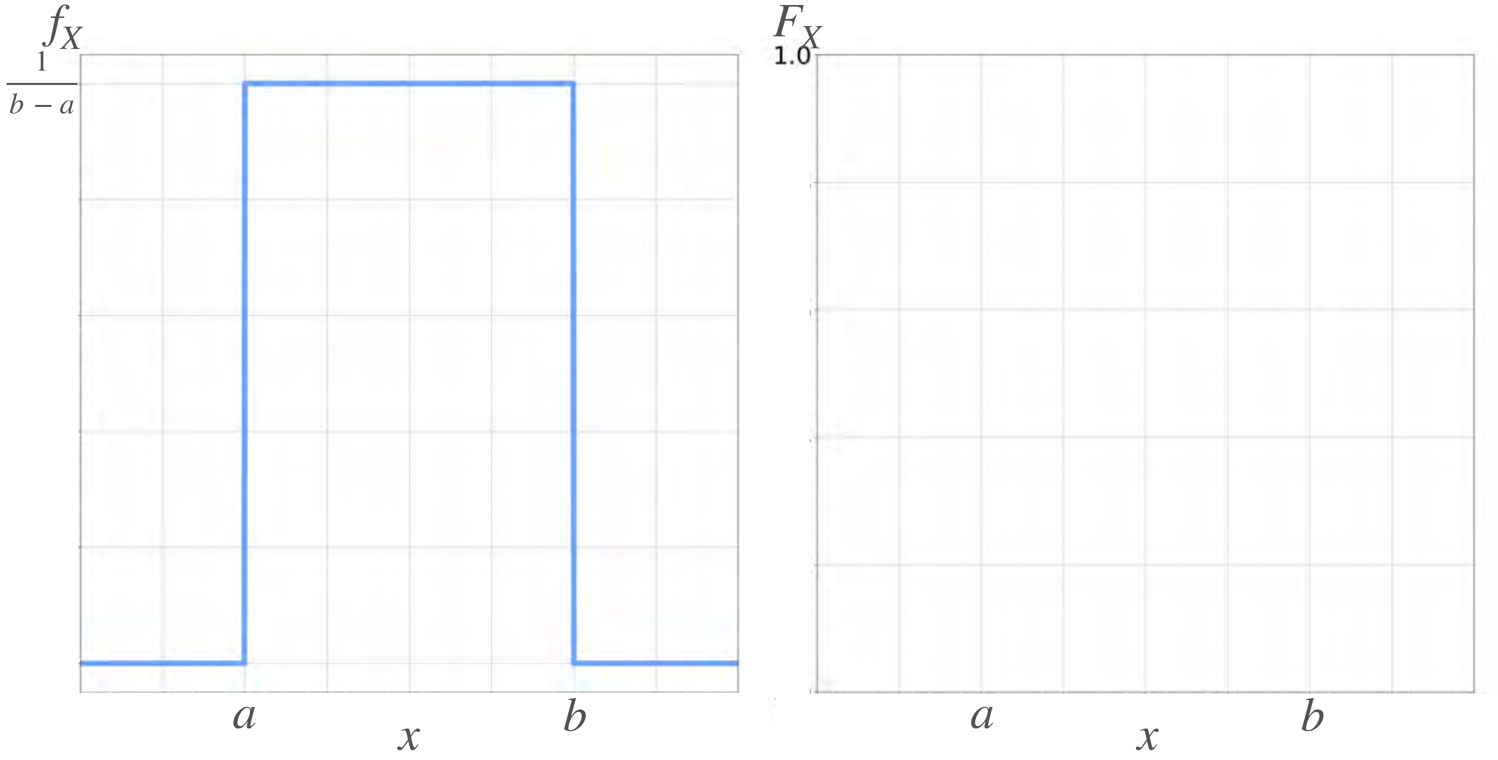
Uniform Distribution: CDF



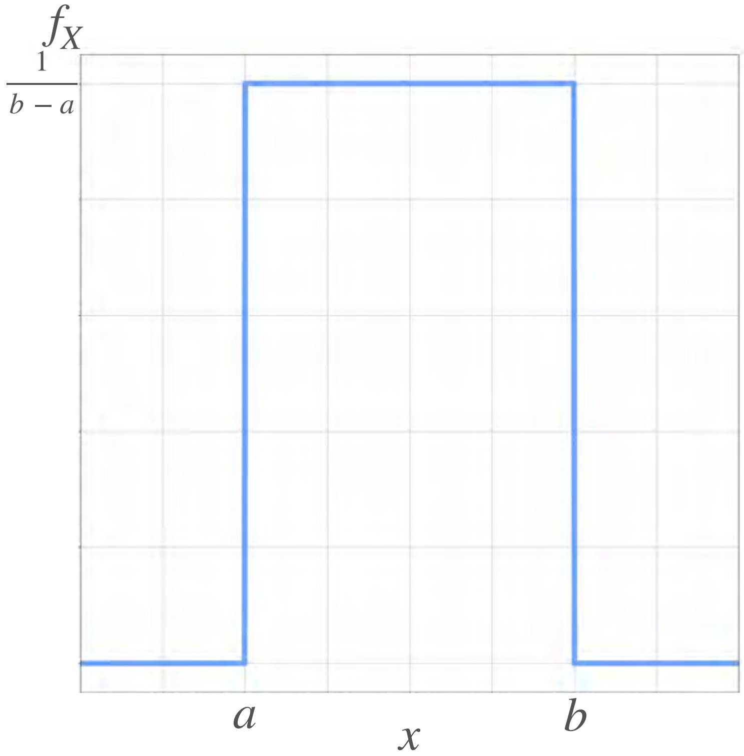


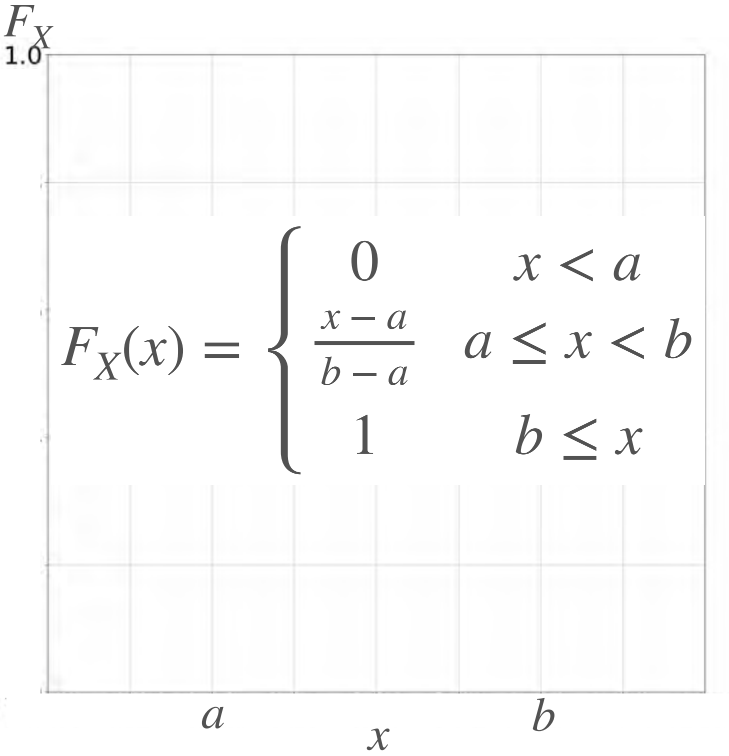
$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x-0}{1} & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

Uniform Distribution: CDF



Uniform Distribution: CDF





$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & b \le x \end{cases}$$

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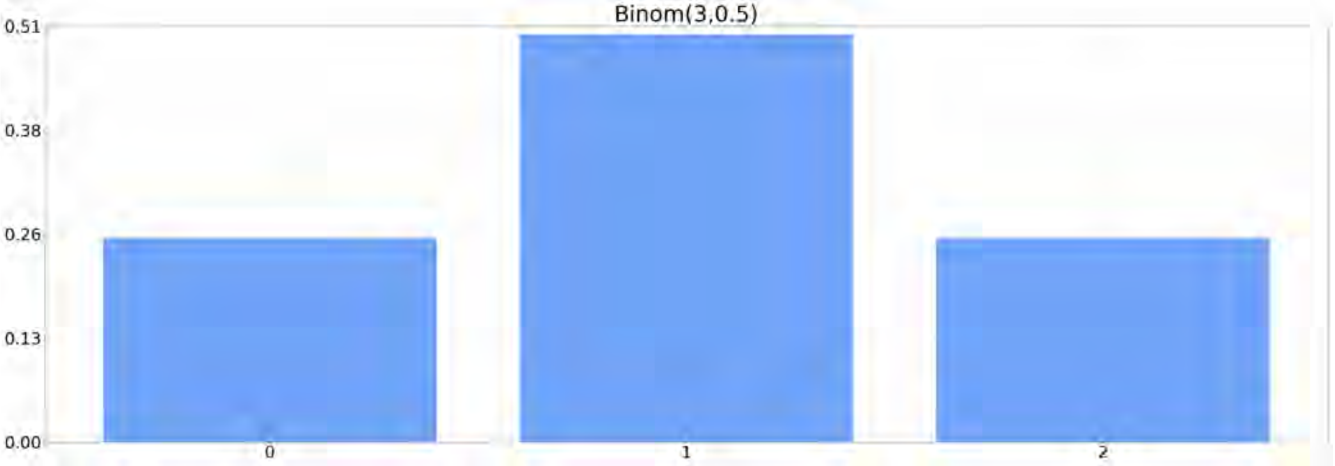
Probability Distributions

Normal distribution

Binomial Distribution With Very Large n

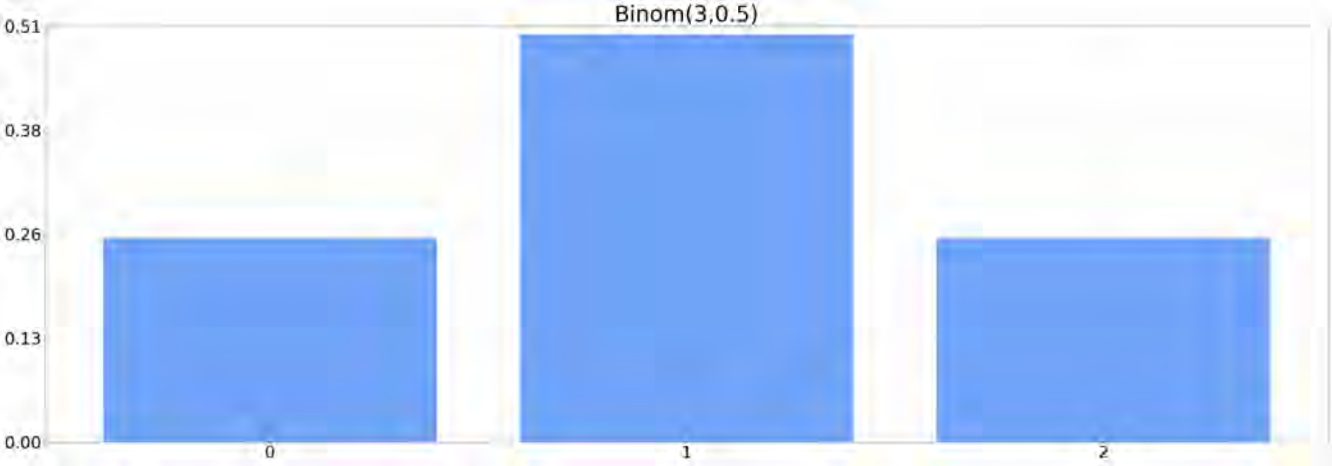
Binomial Distribution With Very Large n

Binomial Distribution With Very Large n



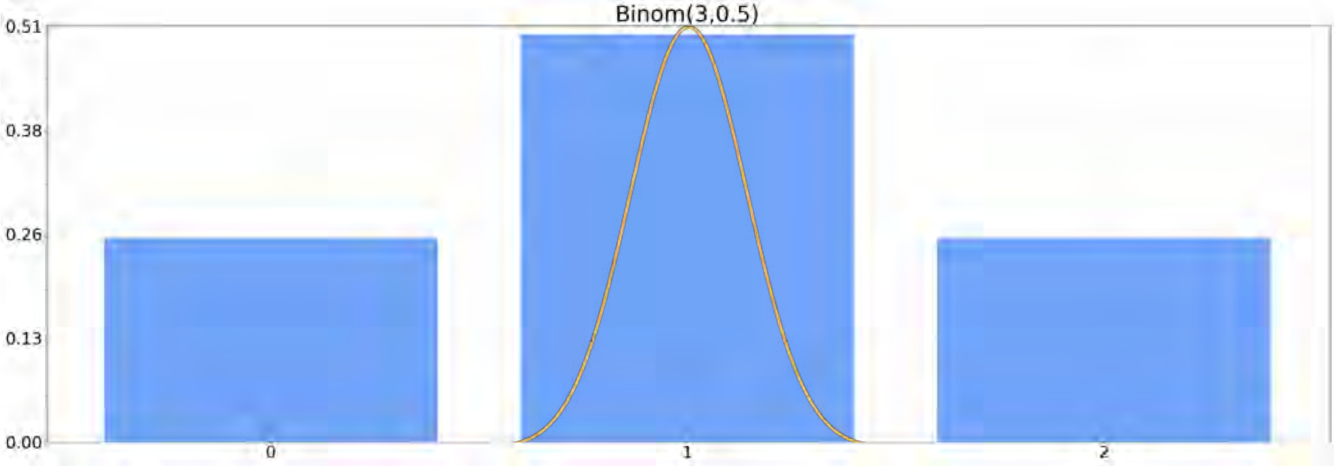
Number of throws = 2

Binomial Distribution With Very Large n



Number of throws $= n$

Binomial Distribution With Very Large n

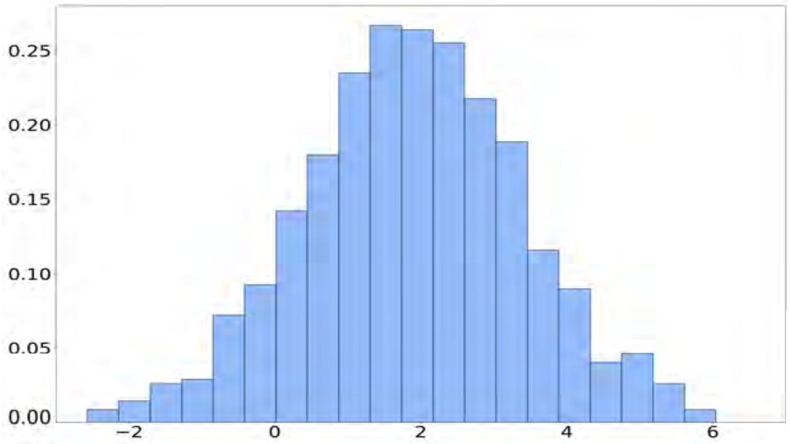


Number of throws $= n$

Bell Shaped Data

Bell Shaped Data

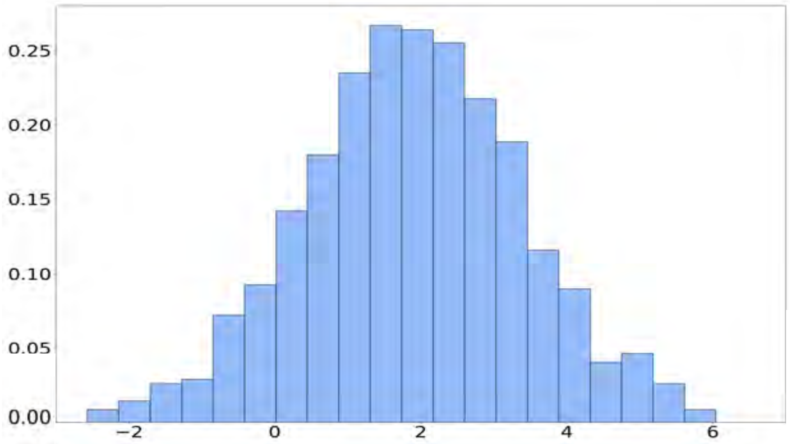
Data



Bell Shaped Data

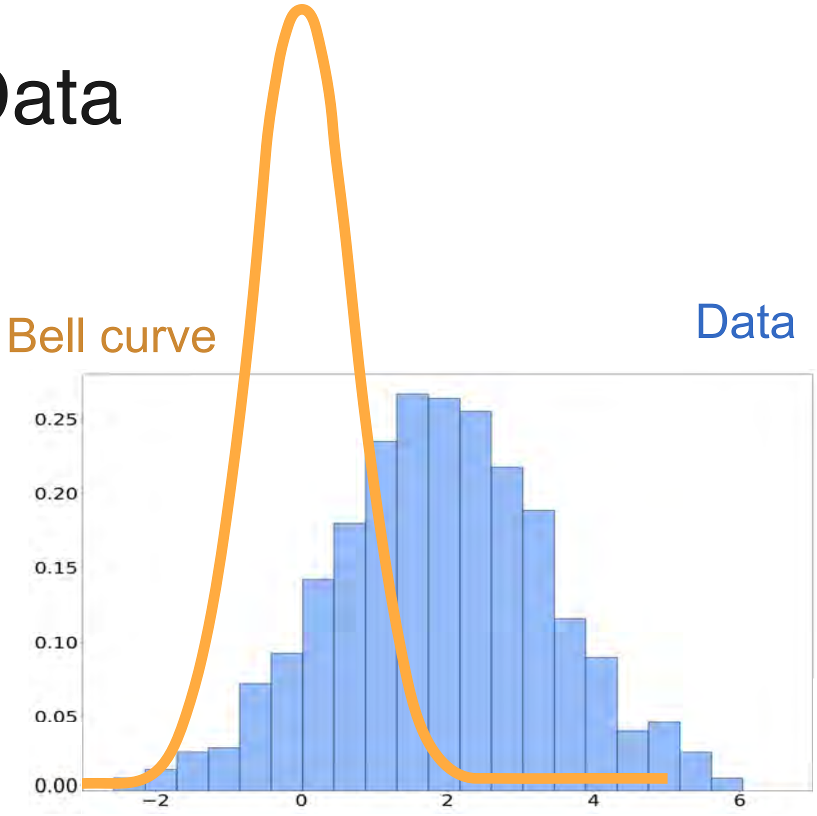
$$e^{-x^2}$$

Data



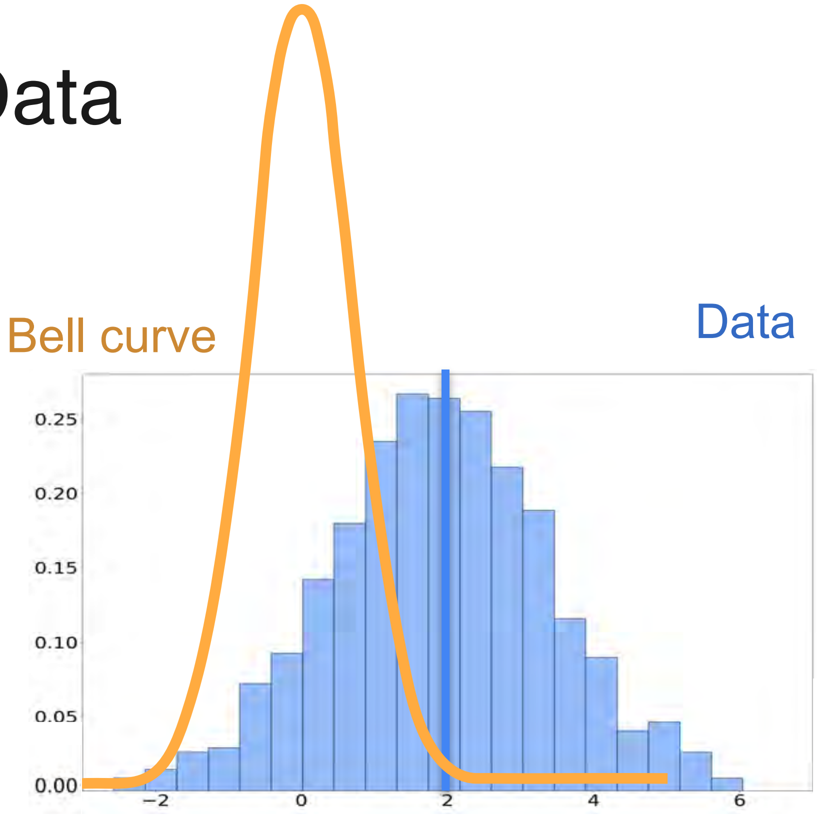
Bell Shaped Data

$$e^{-x^2}$$



Bell Shaped Data

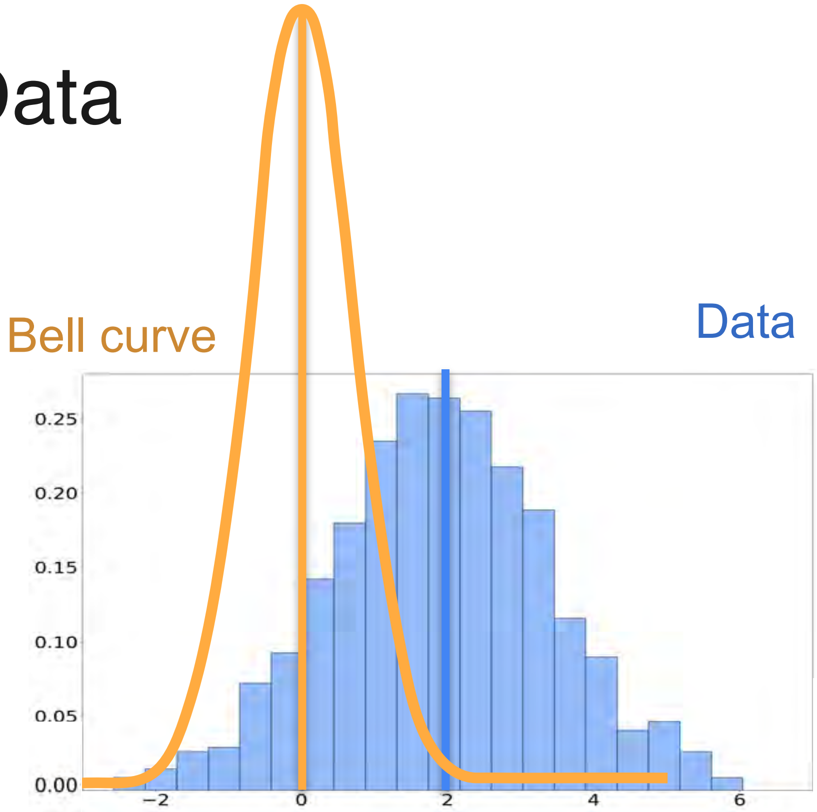
$$e^{-x^2}$$



$$\mu = 2$$

Bell Shaped Data

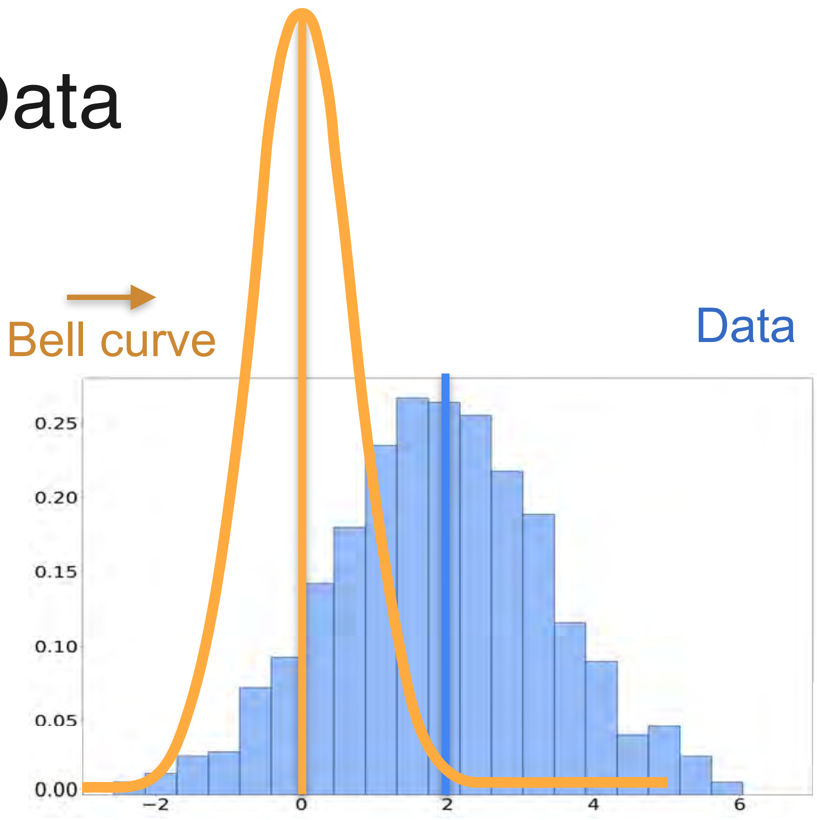
$$e^{-x^2}$$



$\mu = 0$ $\mu = 2$

Bell Shaped Data

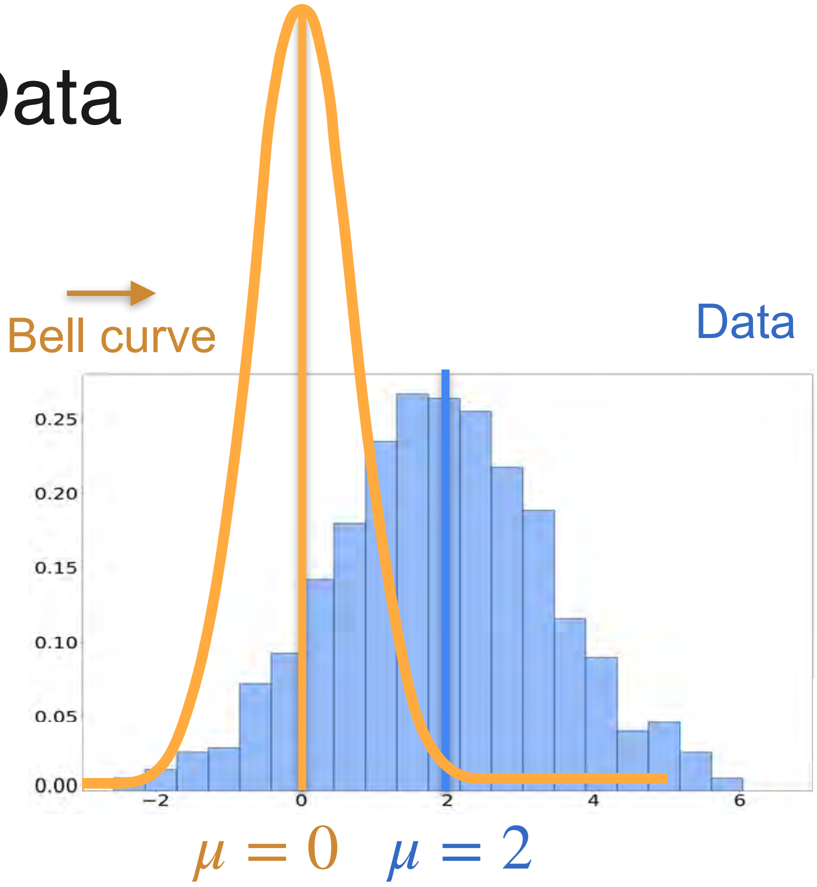
$$e^{-x^2}$$



$\mu = 0$ $\mu = 2$

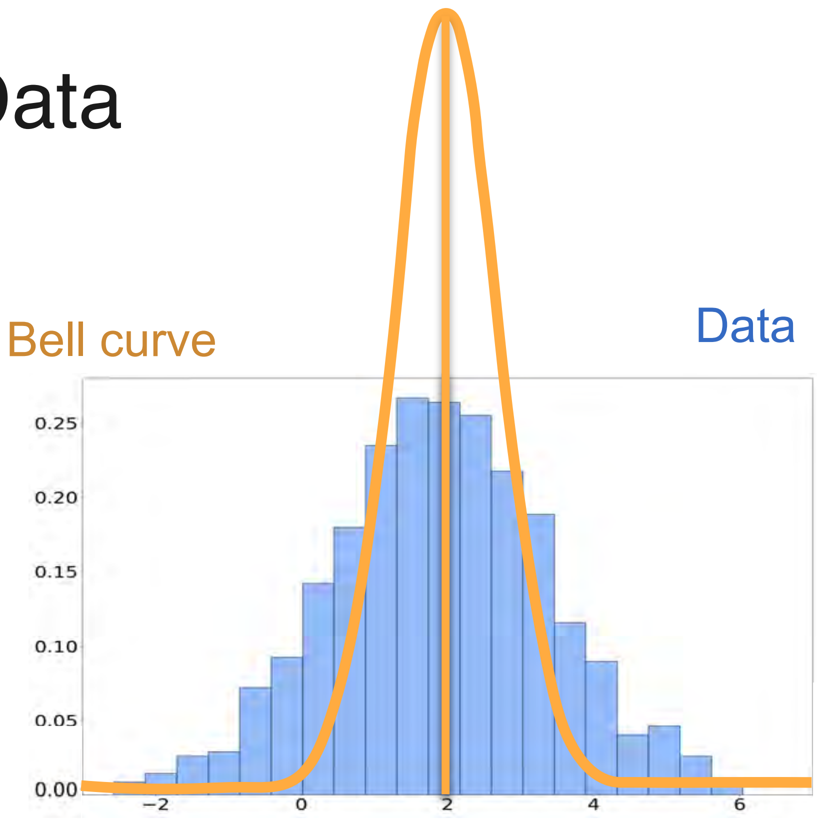
Bell Shaped Data

$$e^{-(x-2)^2}$$

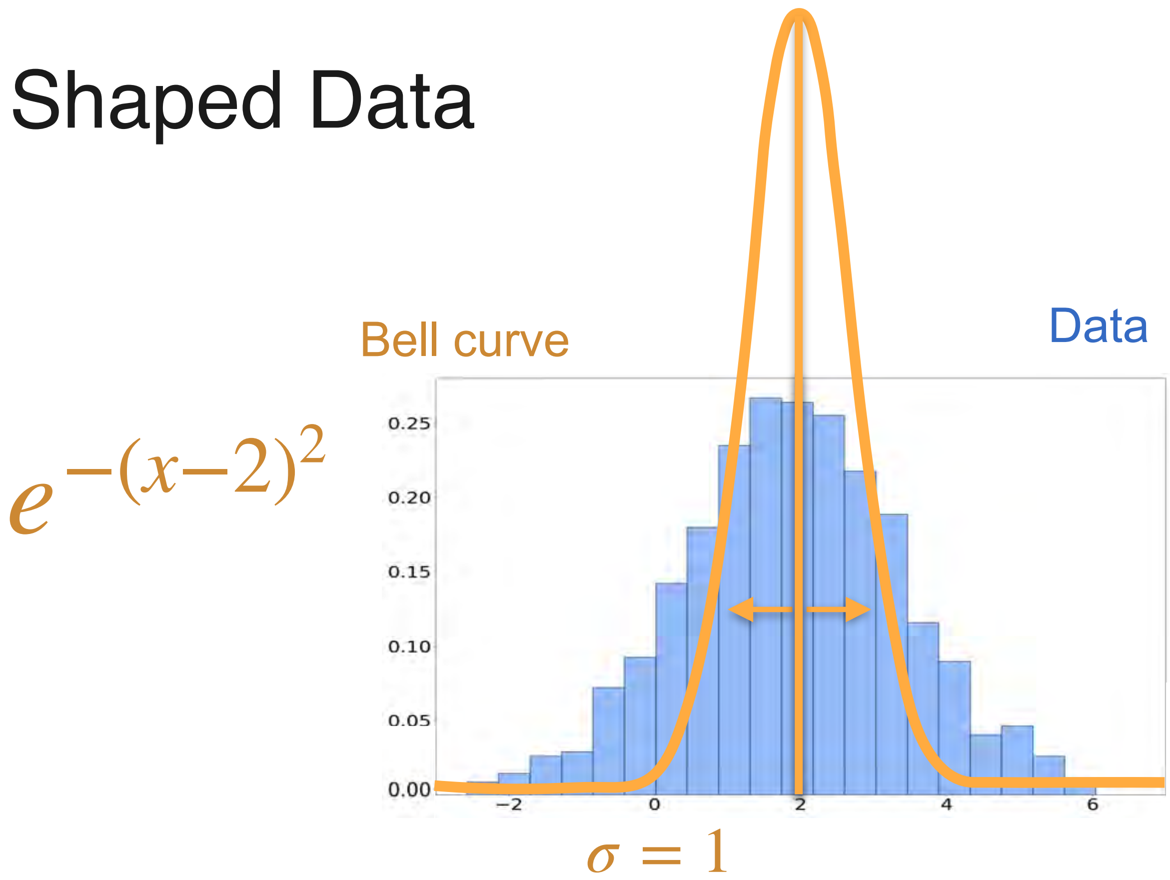


Bell Shaped Data

$$e^{-(x-2)^2}$$



Bell Shaped Data

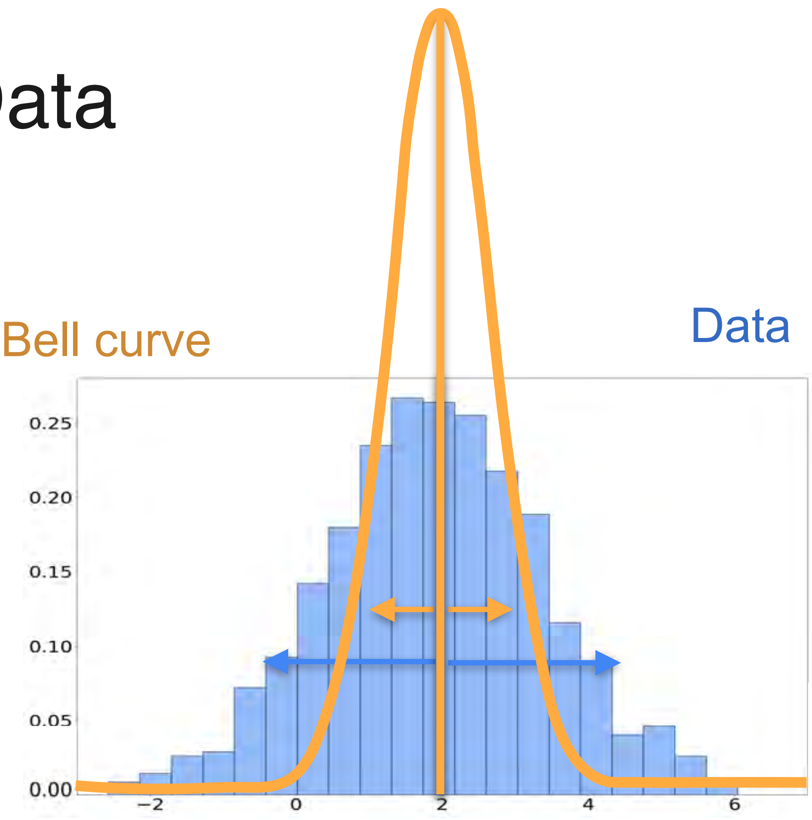


Bell Shaped Data

$$e^{-(x-2)^2}$$

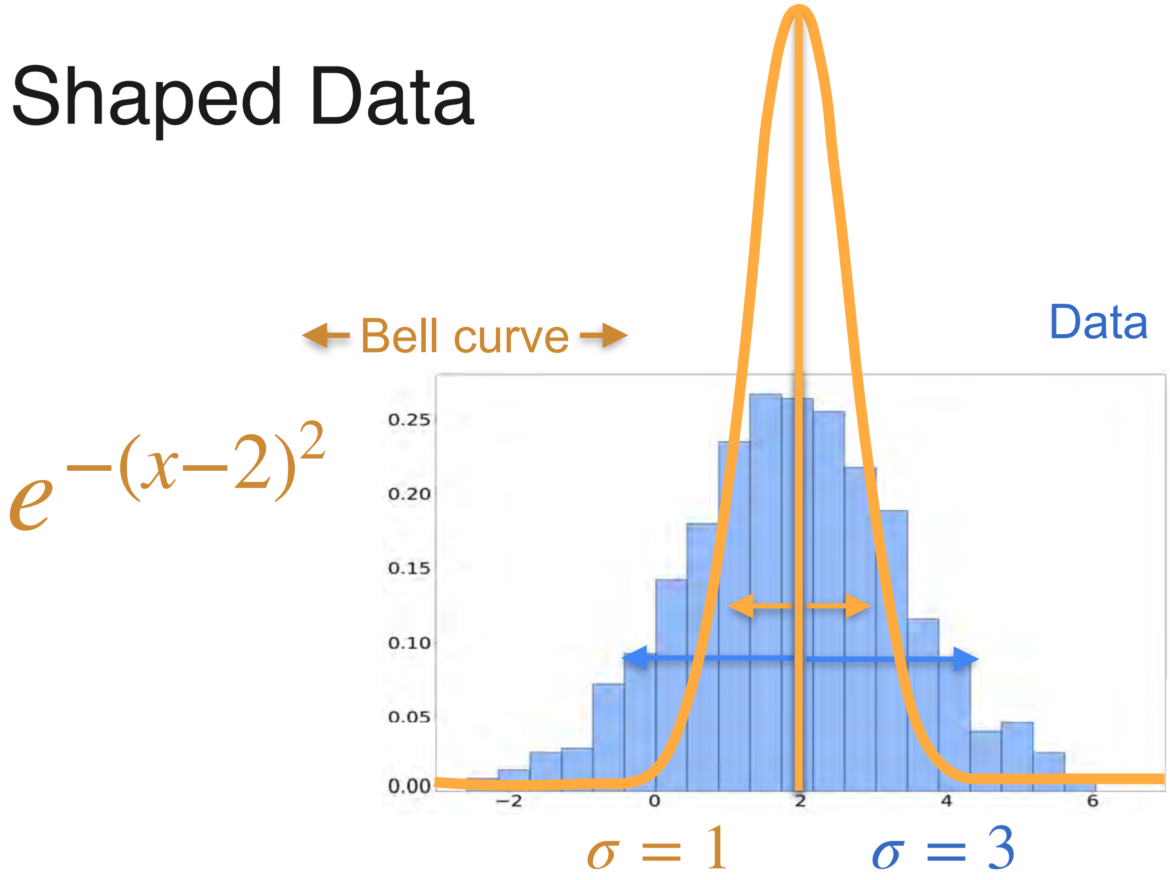
Bell curve

Data

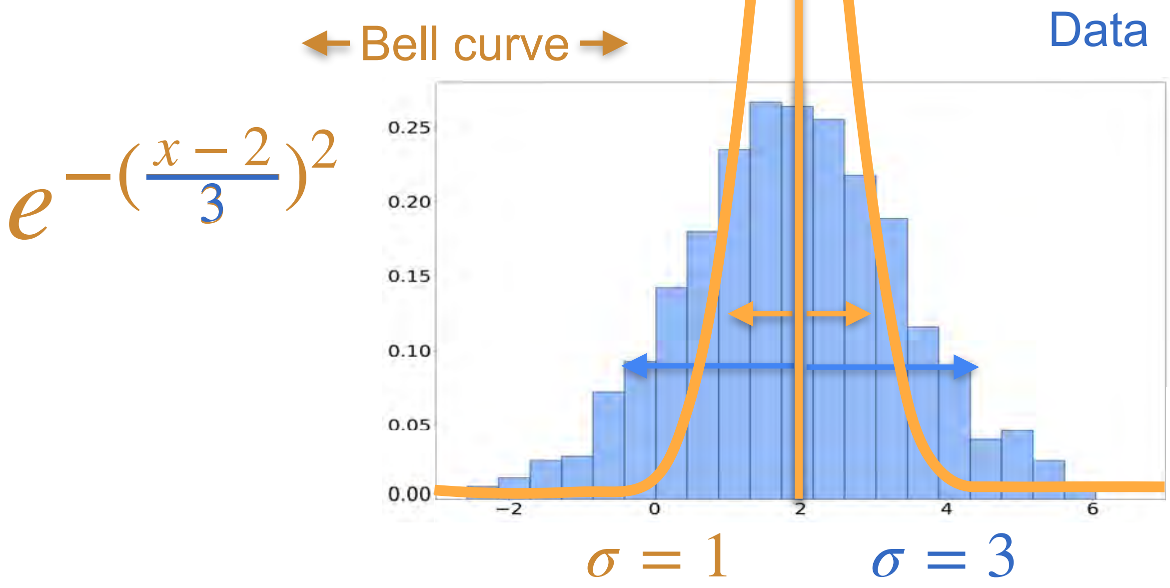


$\sigma = 1$ $\sigma = 3$

Bell Shaped Data

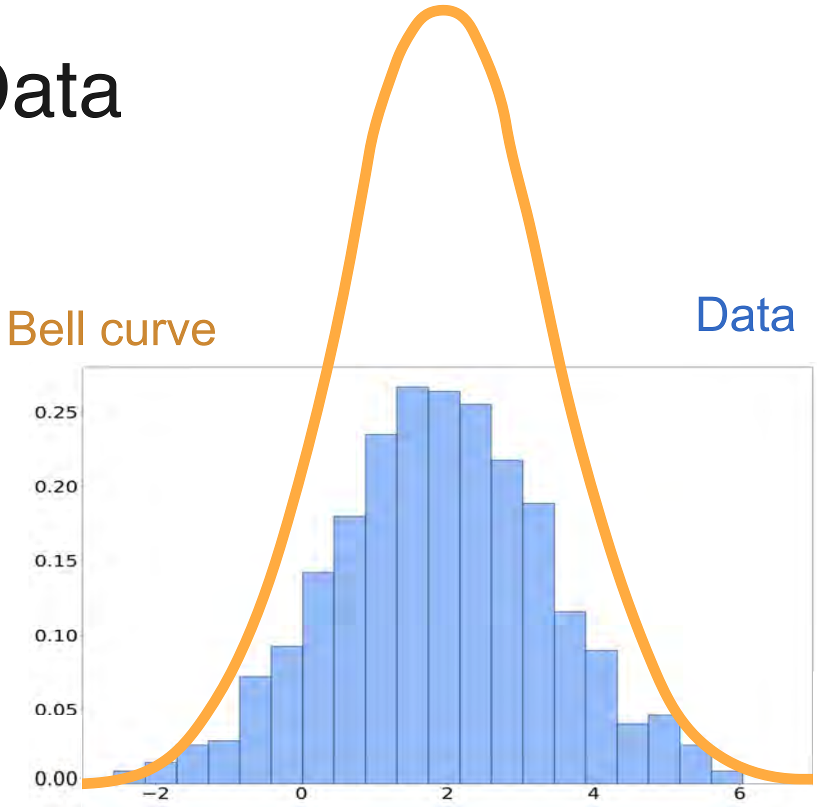


Bell Shaped Data



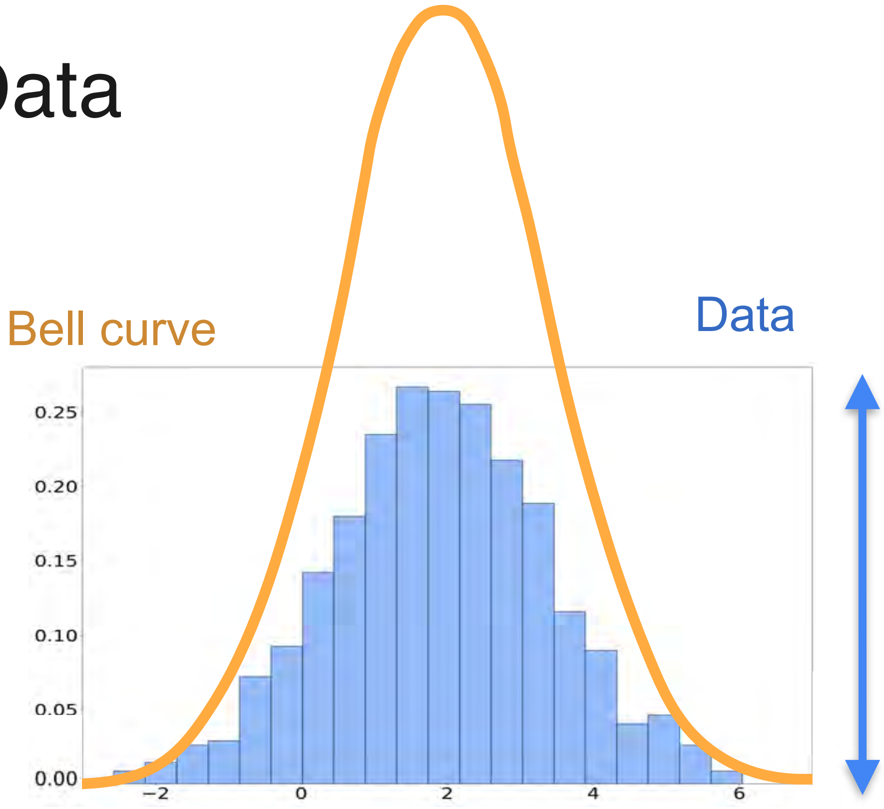
Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



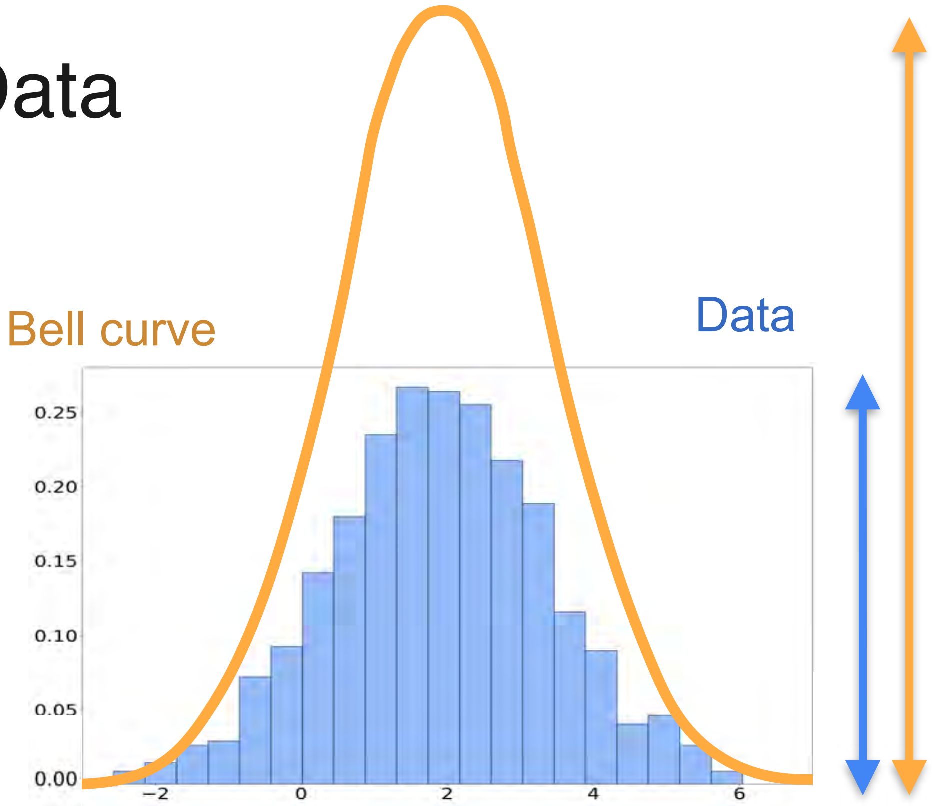
Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



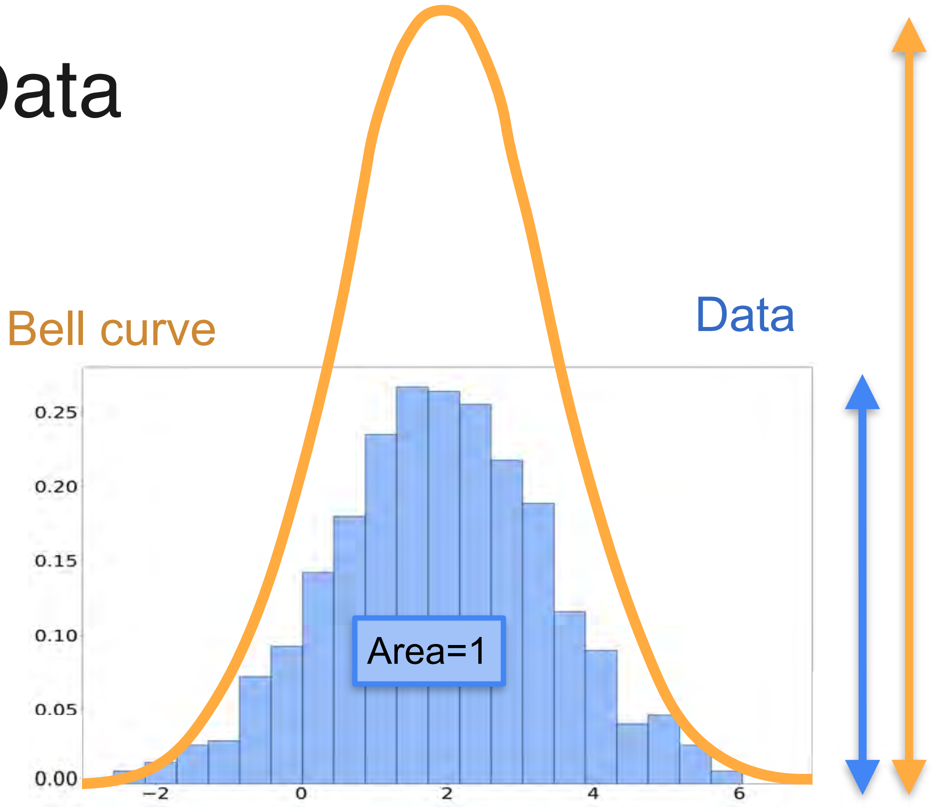
Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



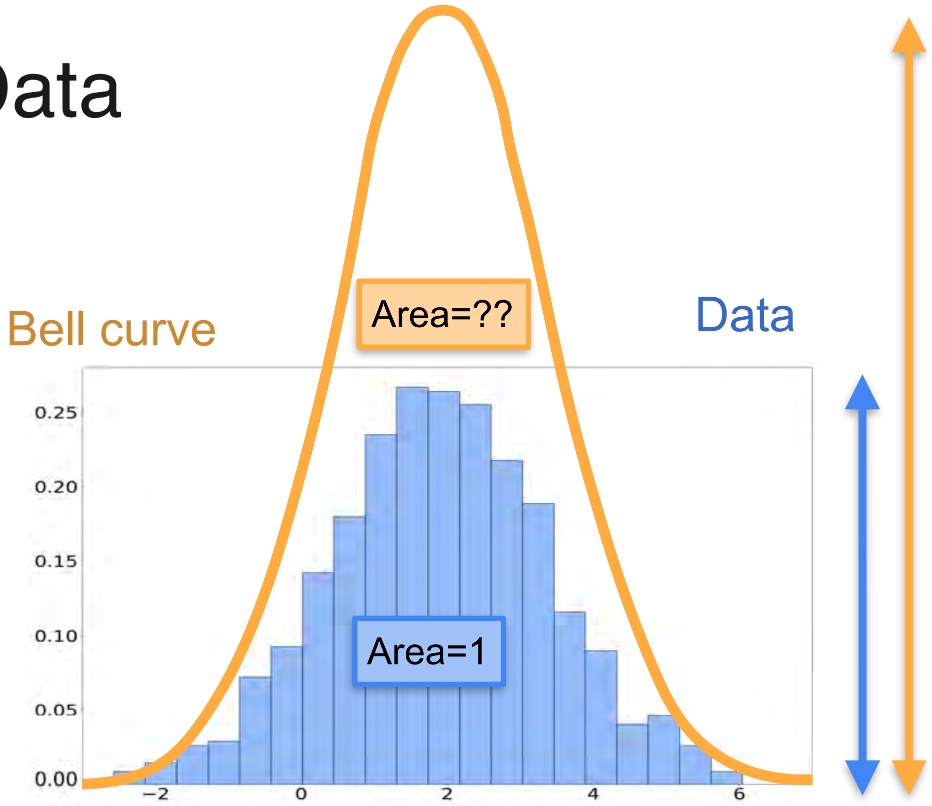
Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



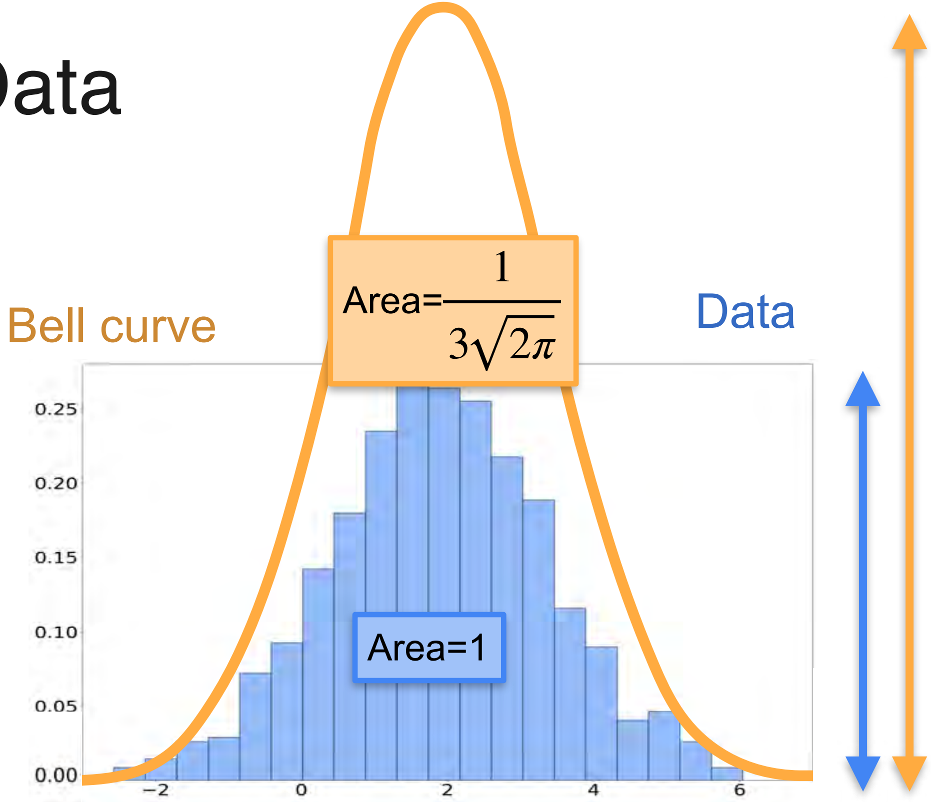
Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



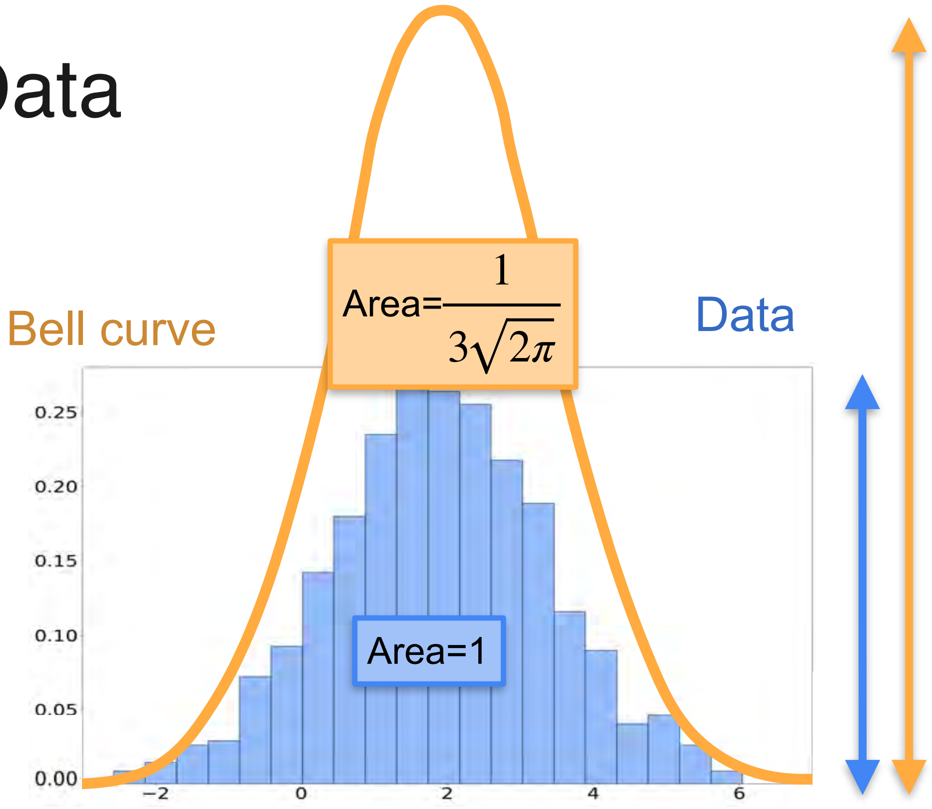
Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



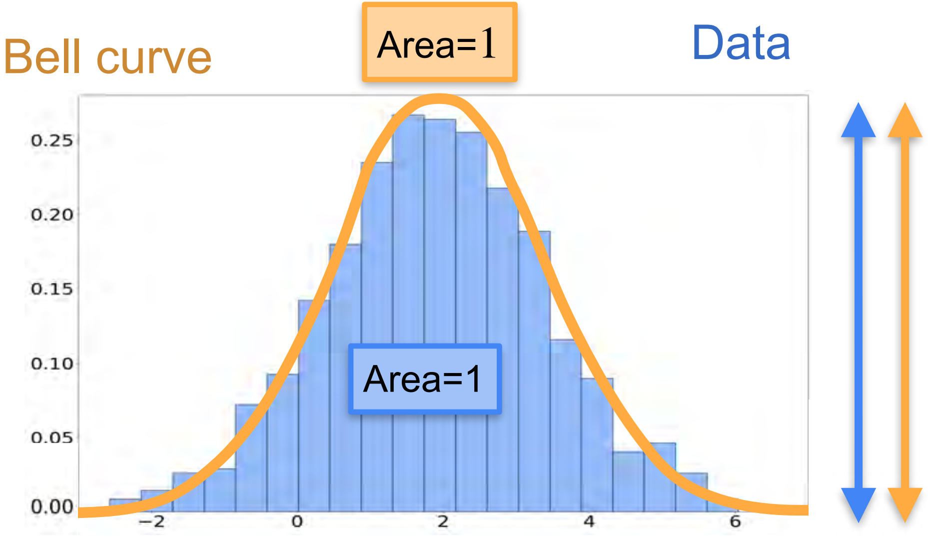
Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}}\, e^{-\left(\frac{x-2}{3}\right)^2}$$

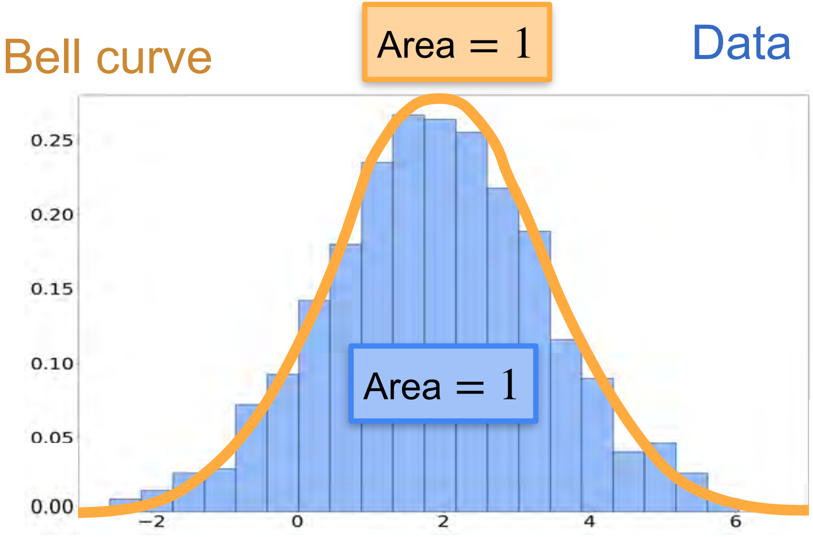


Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}}\, e^{-\left(\frac{x-2}{3}\right)^2}$$

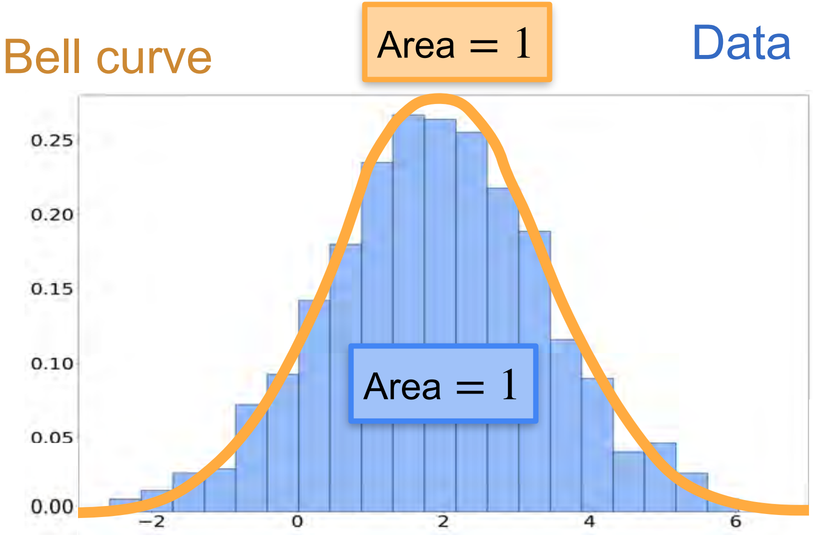


Bell Shaped Data



Bell Shaped Data

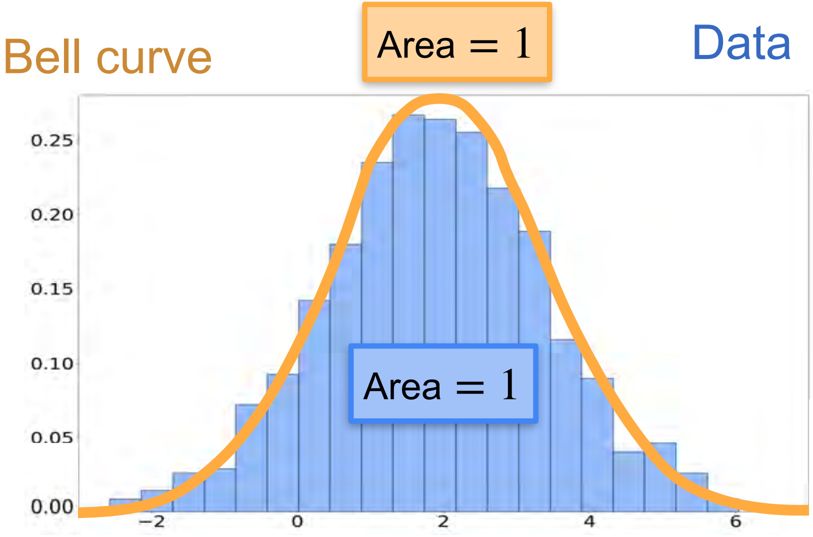
Mean = μ



Bell Shaped Data

Mean = μ

Standard deviation = σ

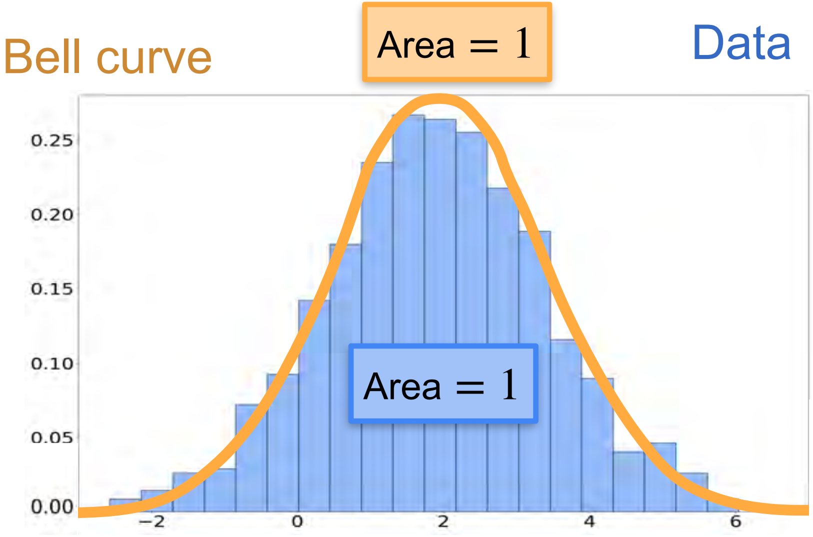


Bell Shaped Data

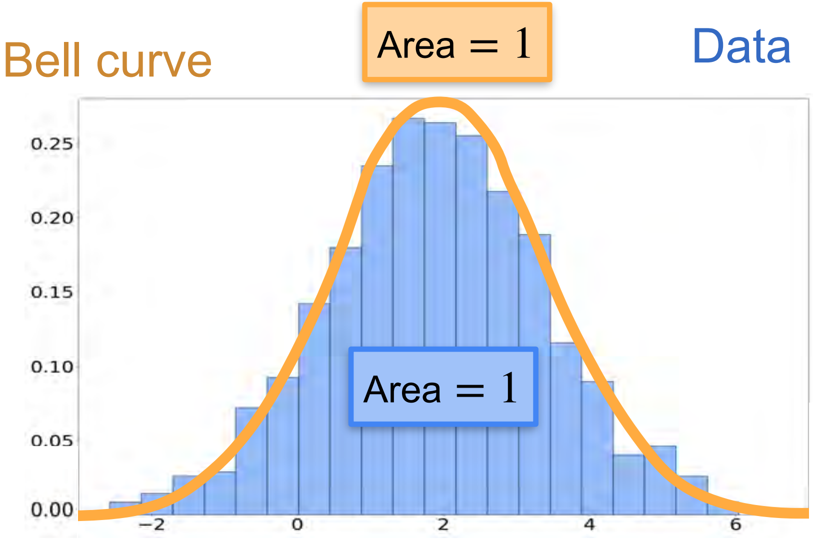
Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

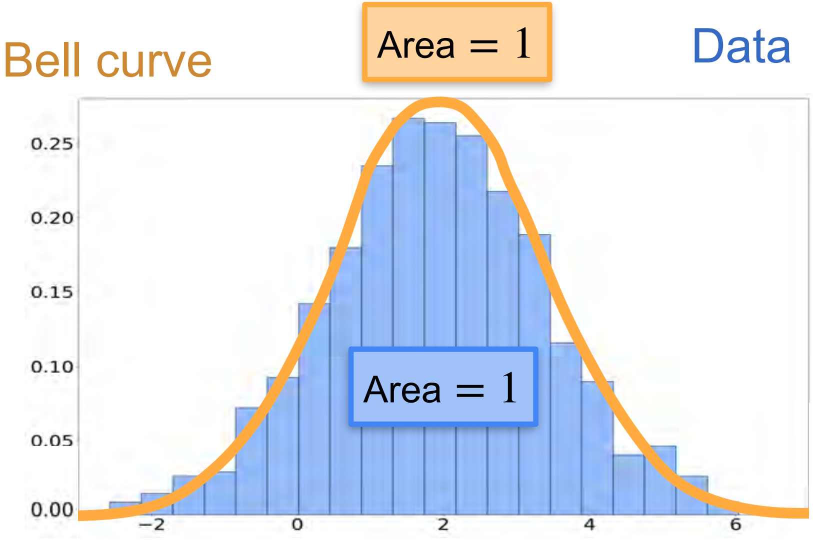


Bell Shaped Data



Bell Shaped Data

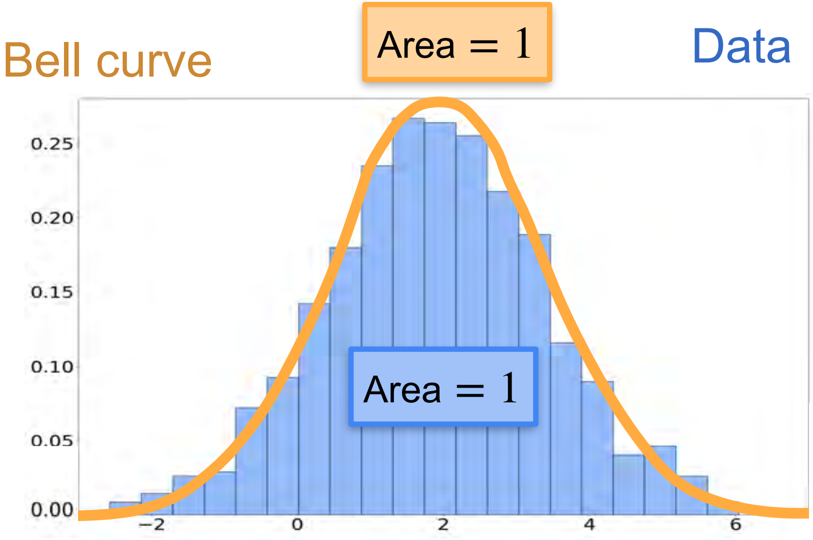
Mean = μ



Bell Shaped Data

Mean = μ

Standard deviation = σ

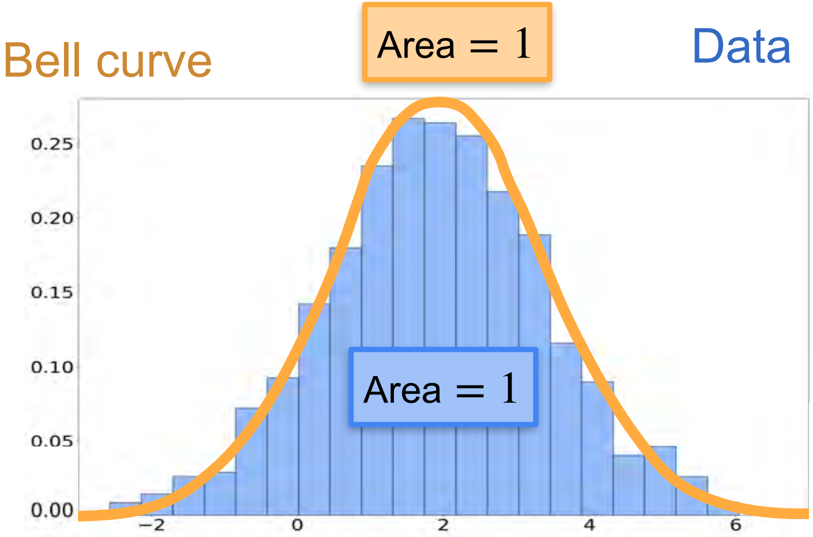


Bell Shaped Data

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

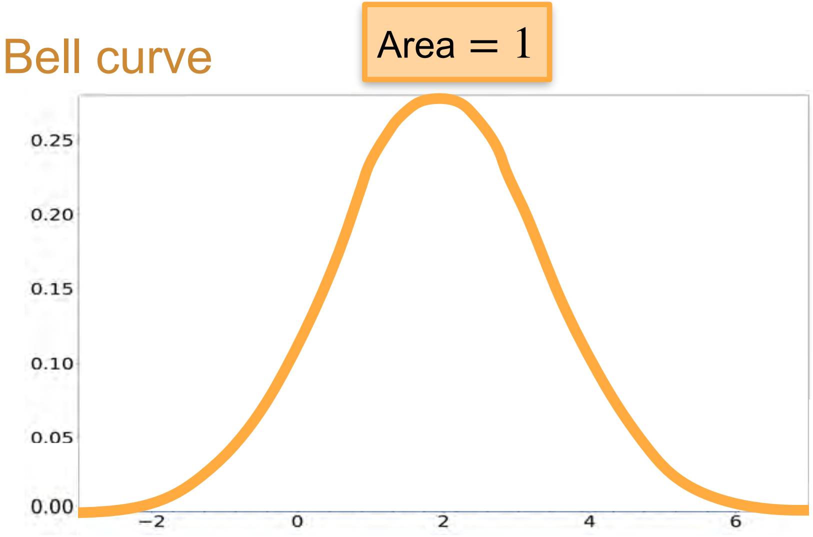


Normal Distribution

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

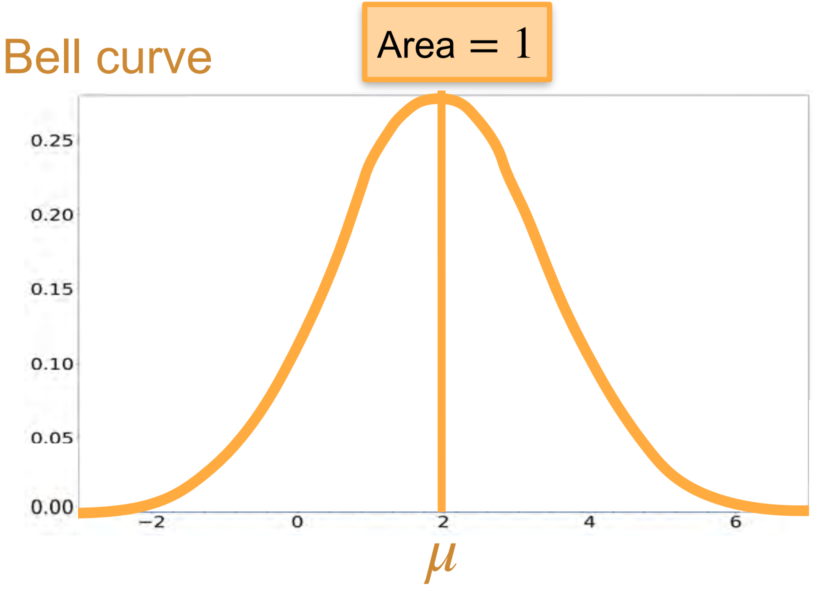


Normal Distribution

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$



Normal Distribution

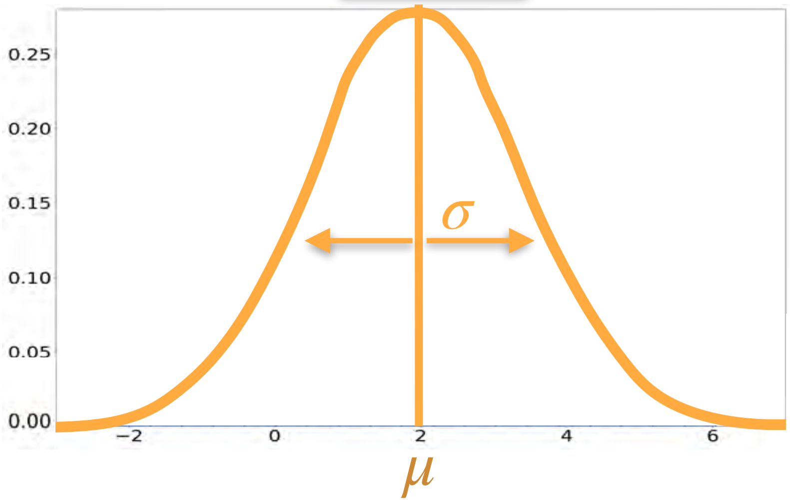
Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

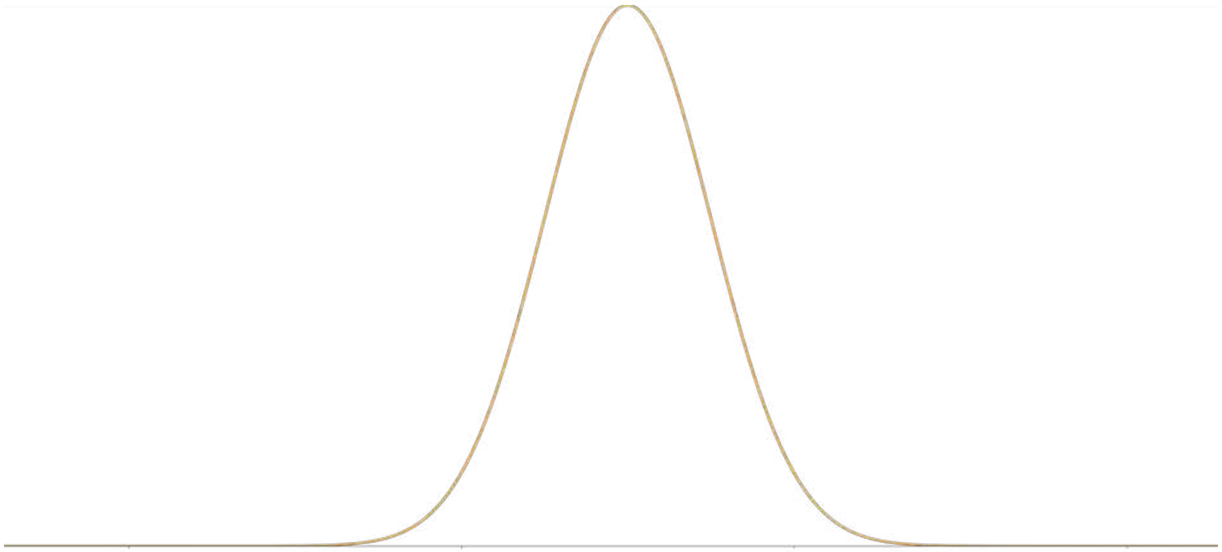
Bell curve

Area $= 1$



Normal Distribution

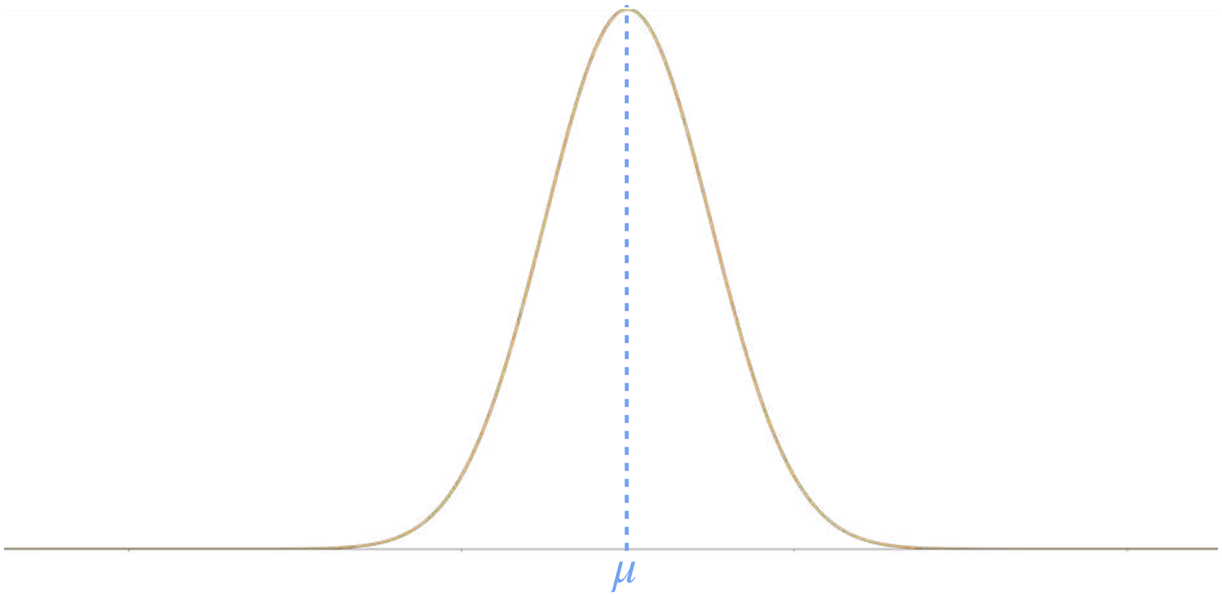
Normal Distribution



Normal Distribution

Parameters:

- μ: center of the bell



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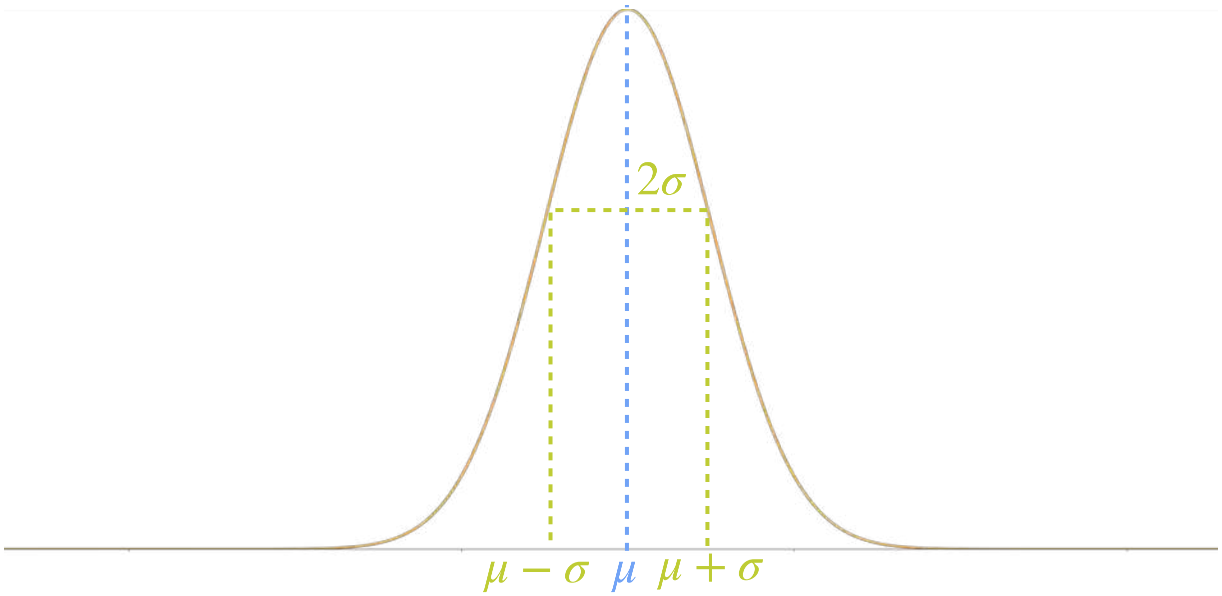

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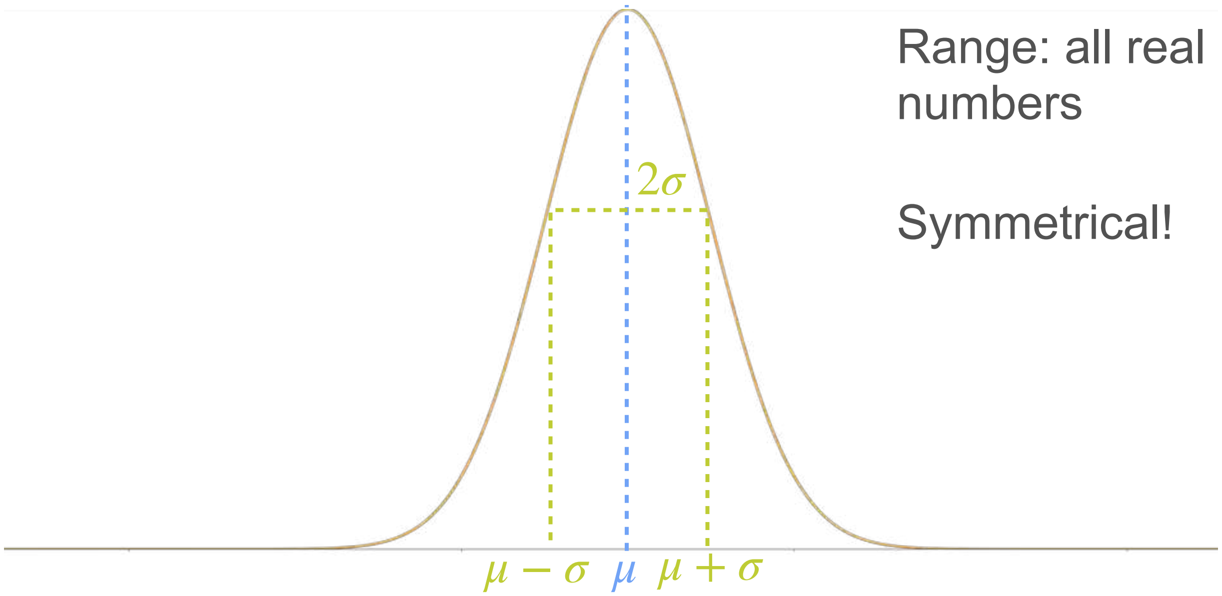


Range: all real numbers

Symmetrical!

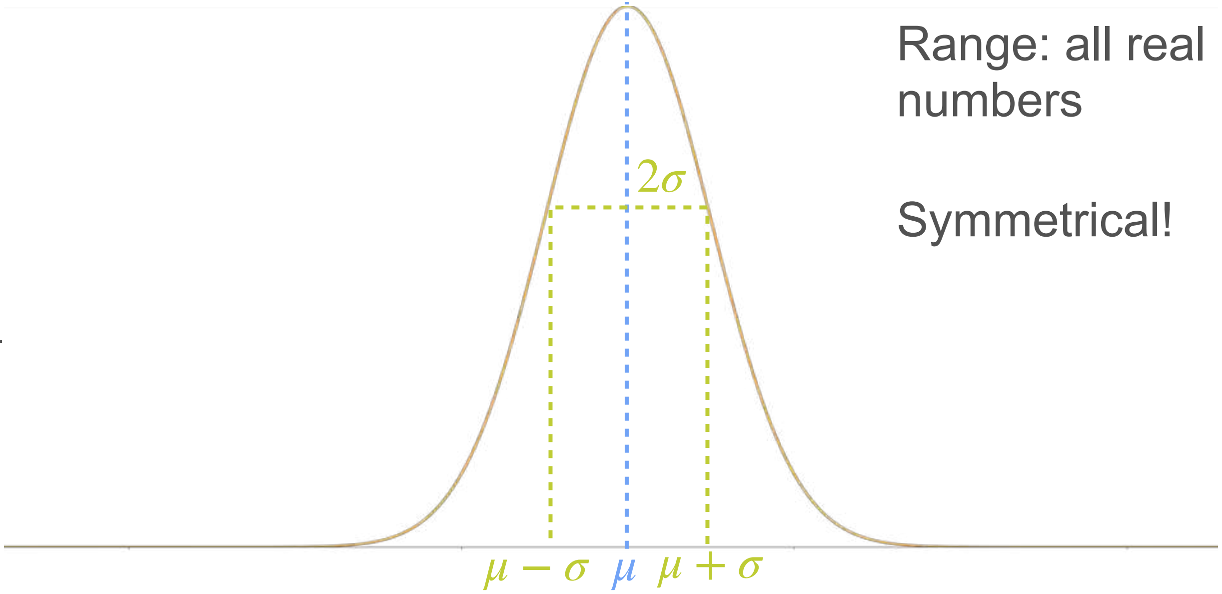
Normal Distribution

Parameters:

- μ: center of the bell
- σ: spread of the bell

$$f_X(x) = \boxed{\frac{1}{\sqrt{2\pi}\sigma}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Scaling constant



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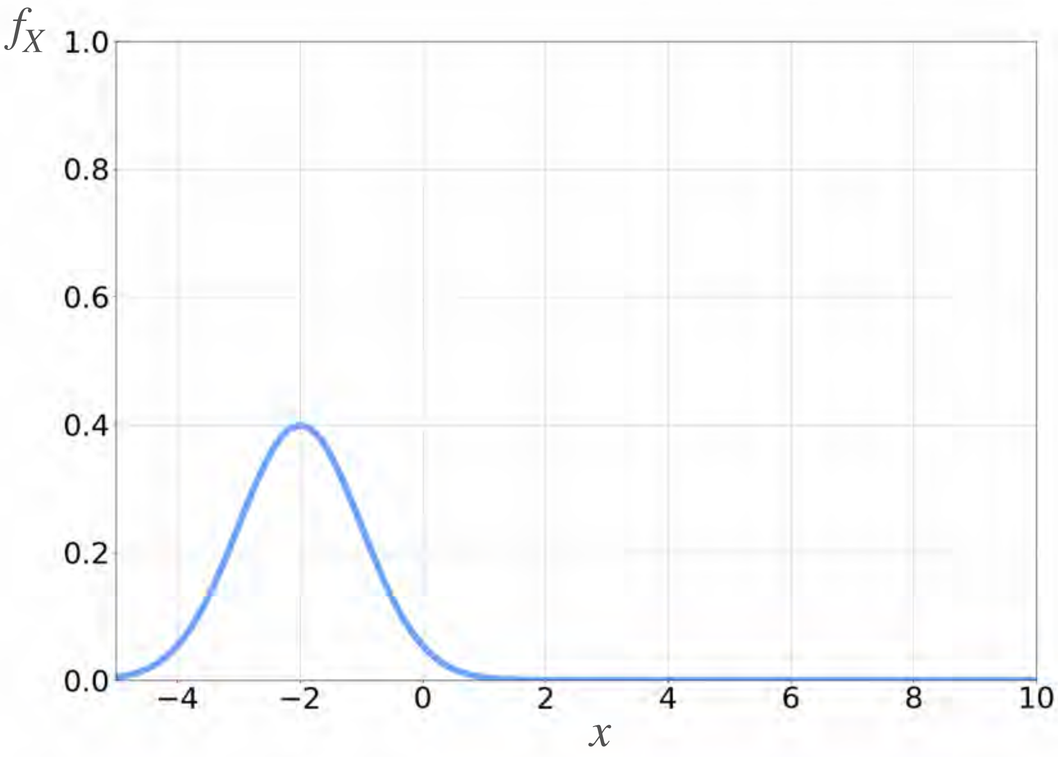

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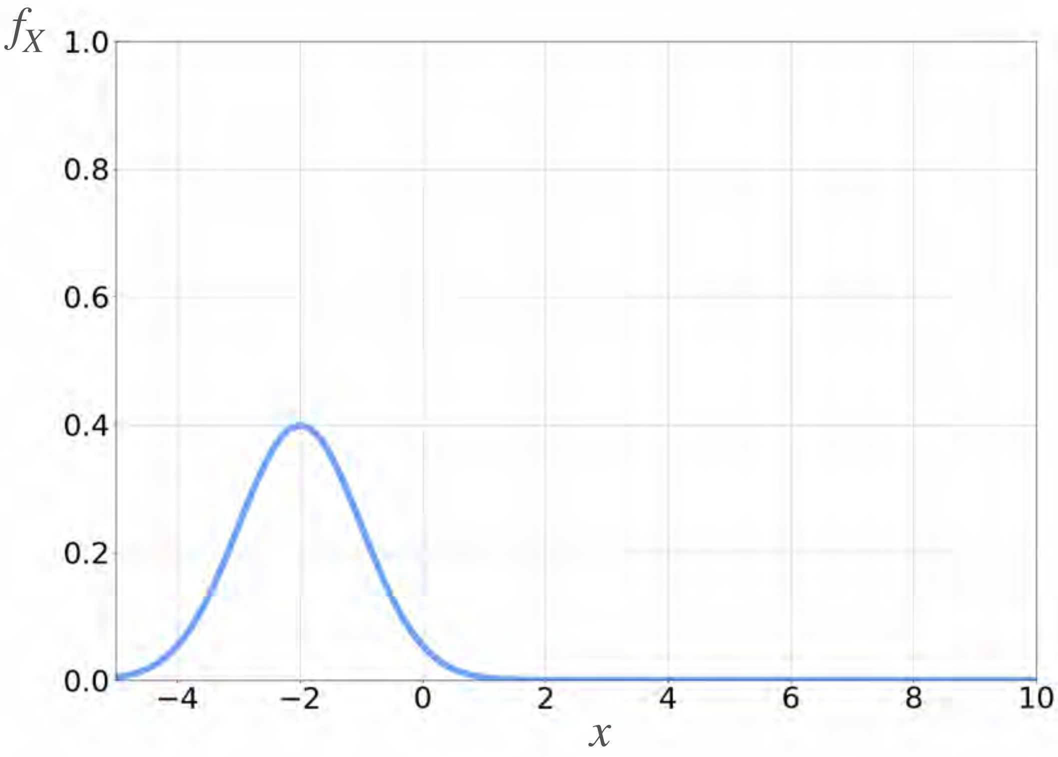

Normal Distribution - Notation

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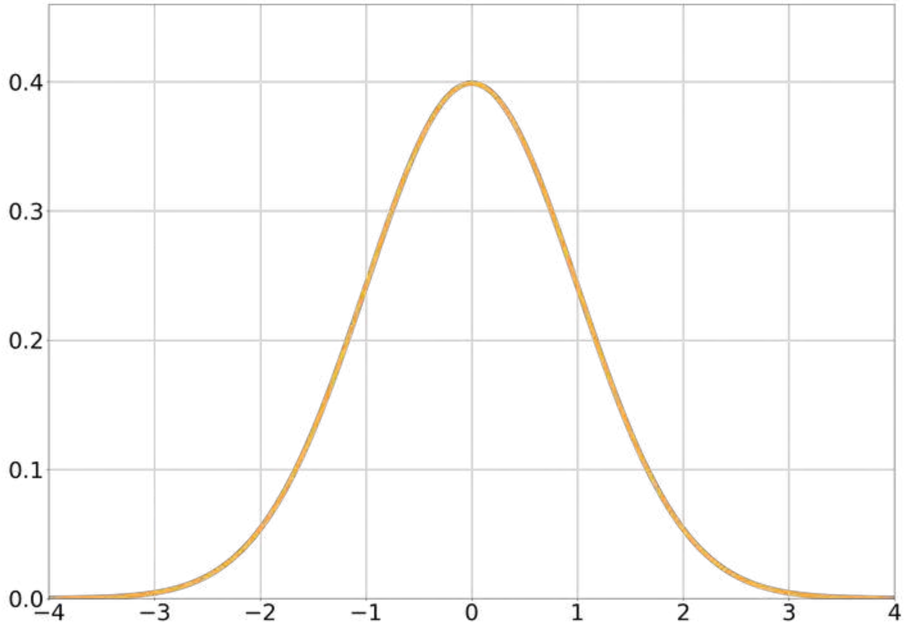
Standard Normal Distribution

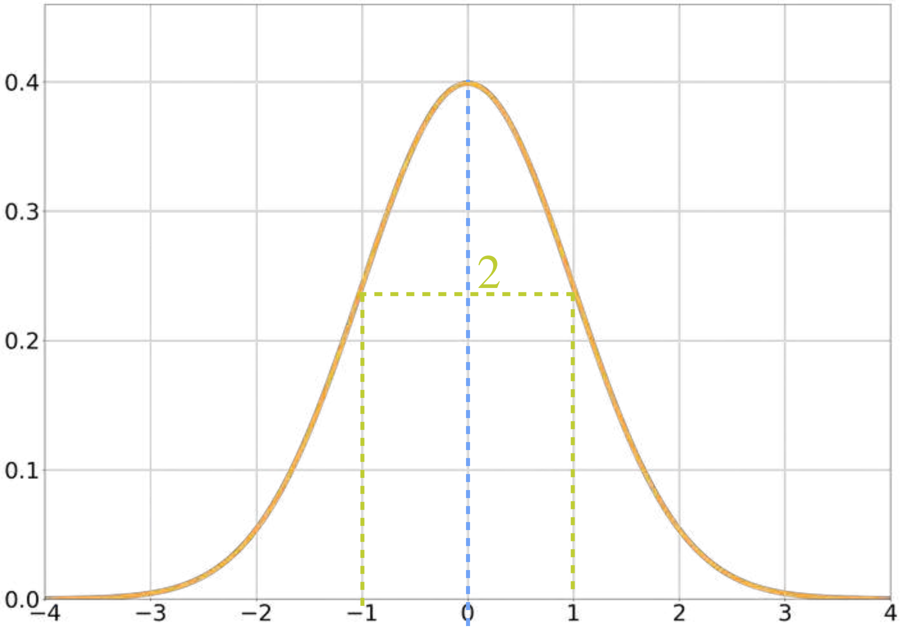


Standard Normal Distribution

Parameters:

- μ: 0
- σ: 1

$$X \sim \mathcal{N}(0, 1^2)$$



Standard Normal Distribution

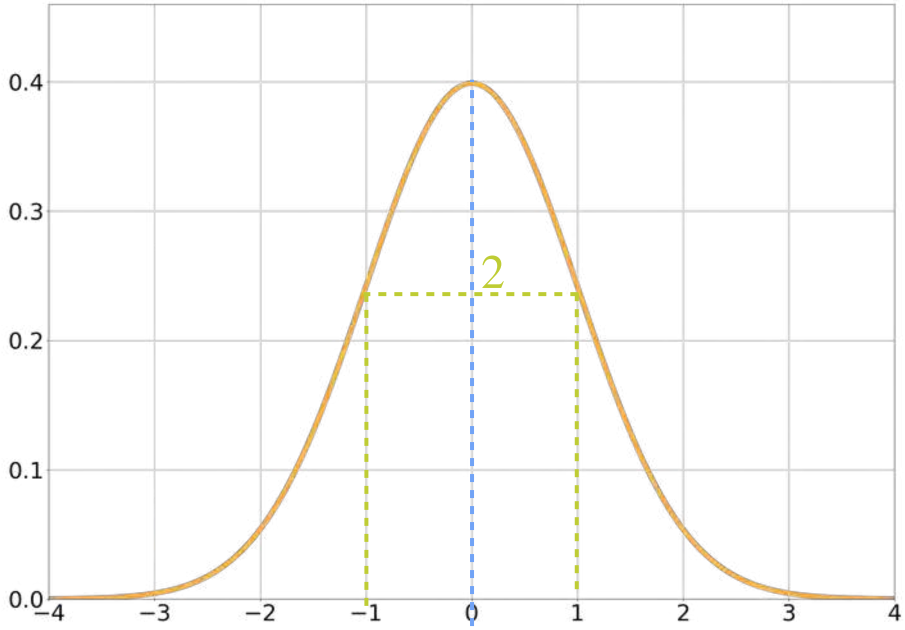
Parameters:

- μ: 0
- σ: 1

$$X \sim \mathcal{N}(0, 1^2)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}1} e^{-\frac{1}{2}\frac{(x-0)^2}{1^2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



Standardization

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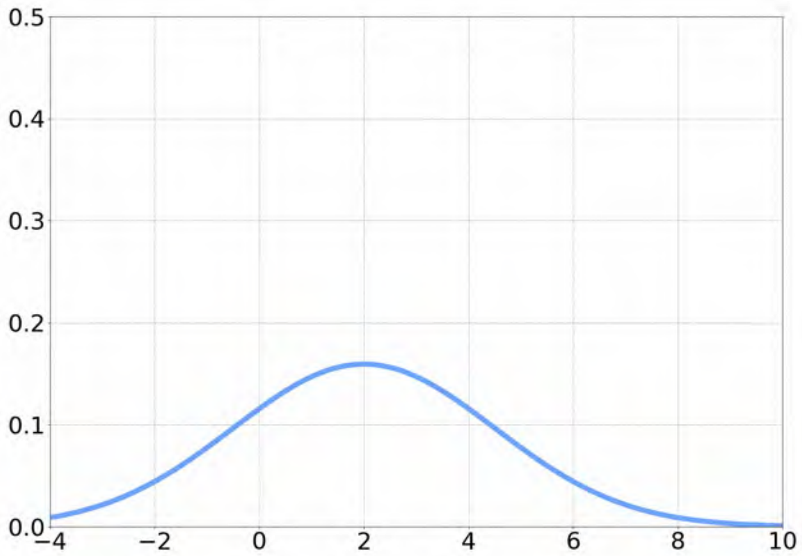
$$X - 2$$



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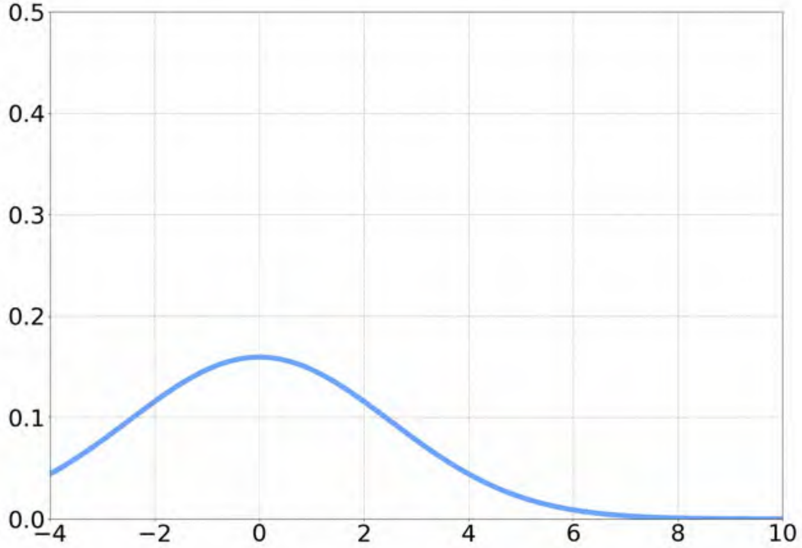
$$\frac{X-2}{2.5}$$



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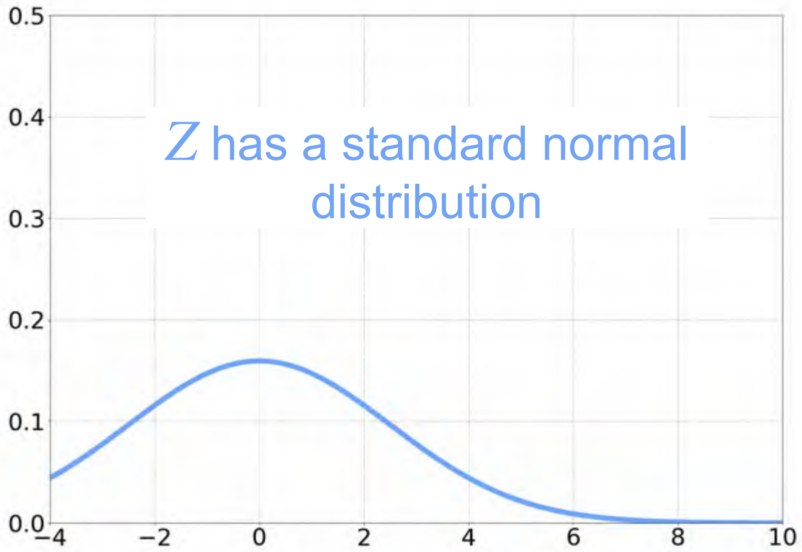
$$Z = \frac{X - 2}{2.5}$$



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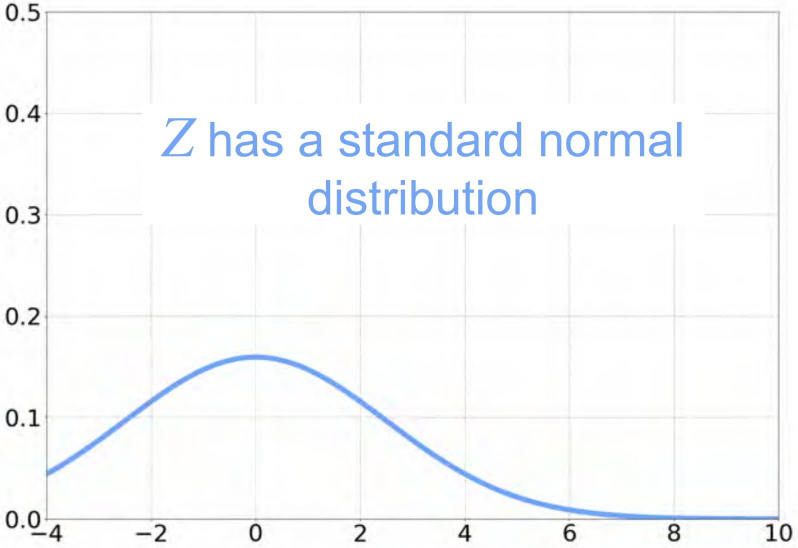

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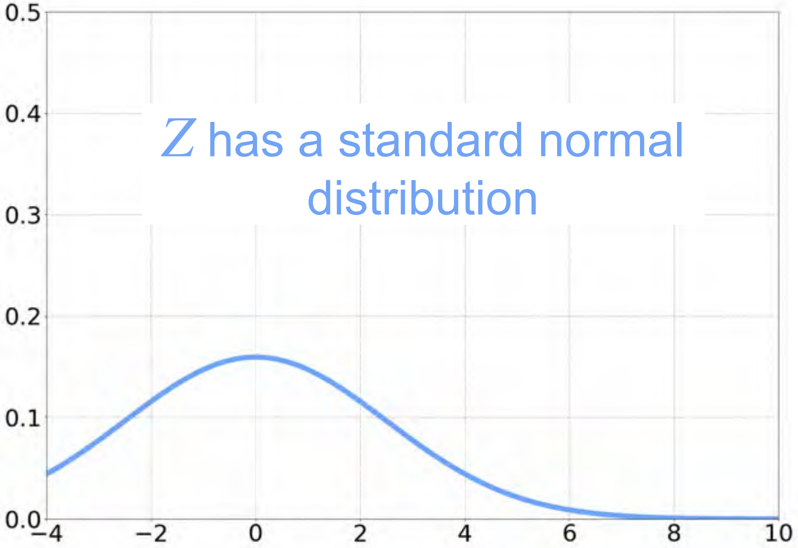
There's a really easy way to convert any normal distribution to the standard one!

X distributes normally with

$$\mu = 2, \ \sigma = 2.5$$

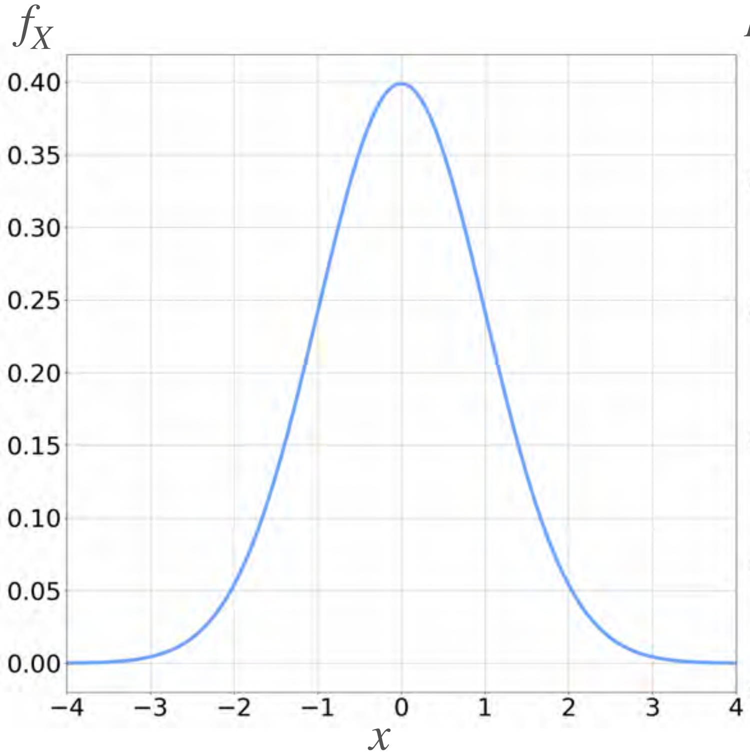
$$Z = \frac{X - \mu}{\sigma}$$

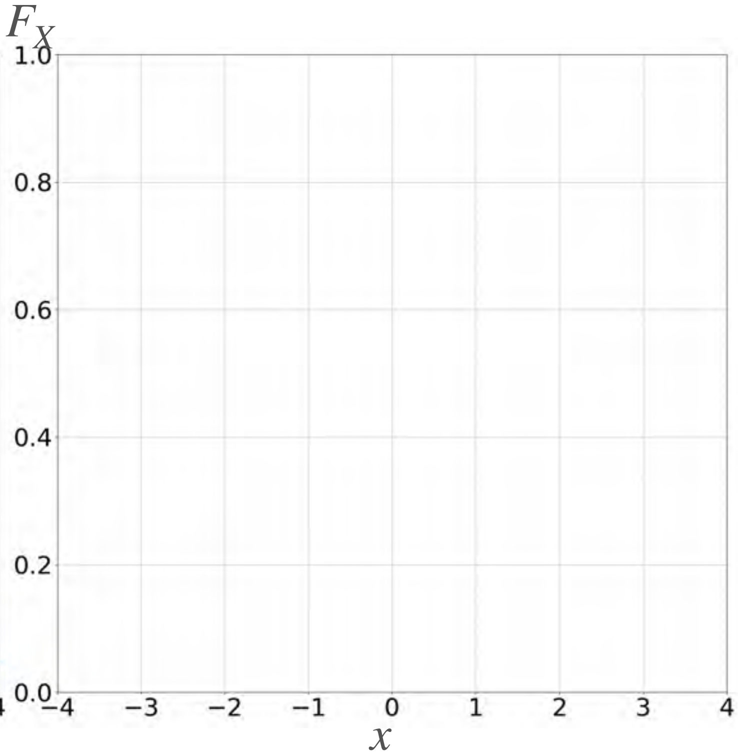
Standardization is crucial to compare variables of different magnitudes!



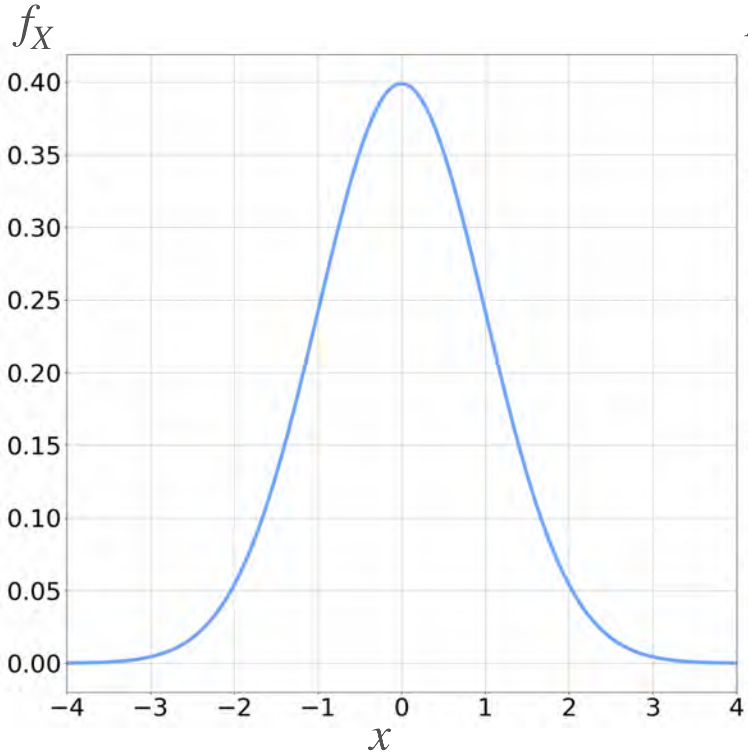
Z has a standard normal distribution

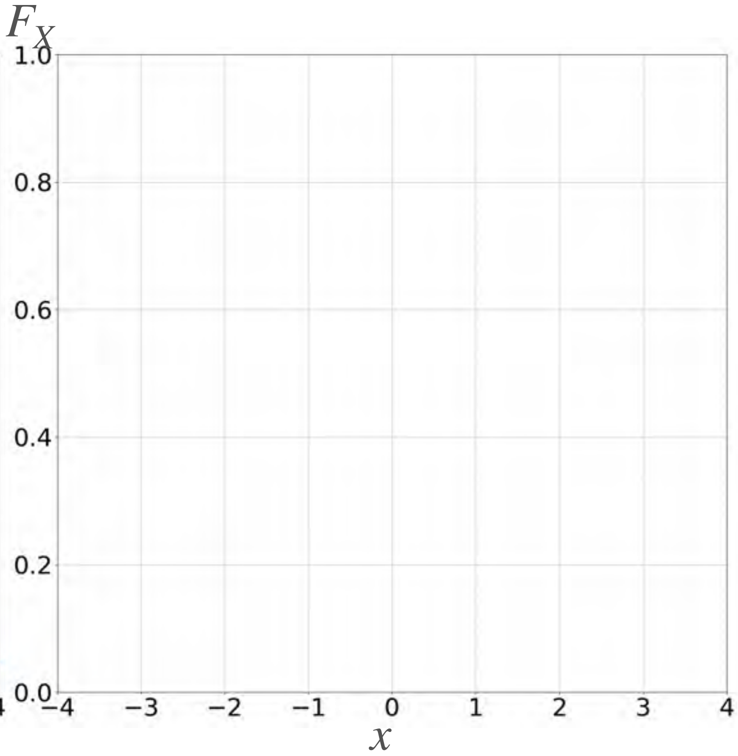
Normal Distribution: CDF





Normal Distribution: CDF





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Normal Distribution: Computing Probabilities

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This math can't be done by hand

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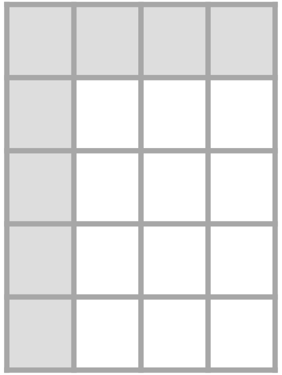

In the old days, people used tables of data

Now, you can use the help of some software to do the approximate area under the curve for you!

Normal Distribution: Applications

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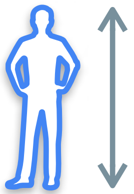
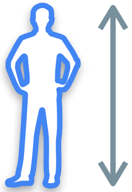


Height

Normal Distribution: Applications



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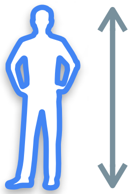

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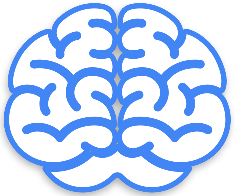
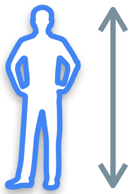


IQ

Normal Distribution: Applications



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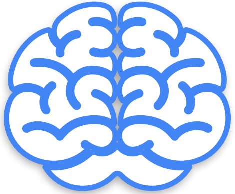
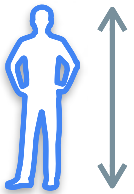


IQ

Noise in a
communication channel

Normal Distribution: Applications



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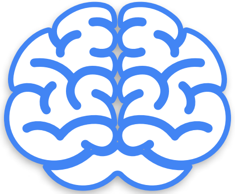
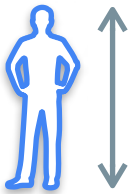


IQ

Noise in a communication channel

In general, characteristics that are the sum of many independent processes

Normal Distribution: Applications



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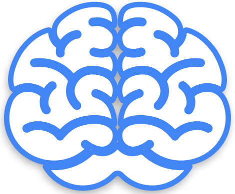


IQ

Noise in a communication channel

In general, characteristics that are the sum of many independent processes

Many models in ML are designed under the assumption that the variables follow a normal distribution

DeepLearning.AI

Probability Distributions

Chi-squared distribution

Chi-Square Distribution: Motivation

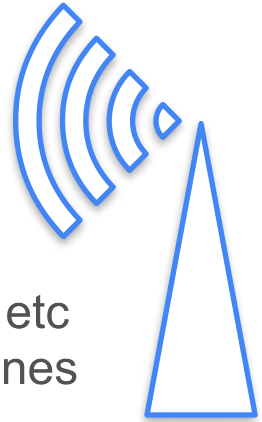
Chi-Square Distribution: Motivation

Message sent: 10010

Chi-Square Distribution: Motivation

Communication channel

Noise
Interference from other devices
Obstructions like walls, trees, etc.
Atmospheric conditions: rain, humidity, etc
Electrical interference, i.e. from power lines
Others



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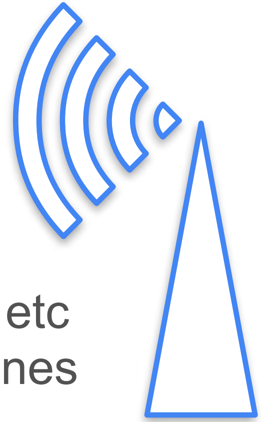
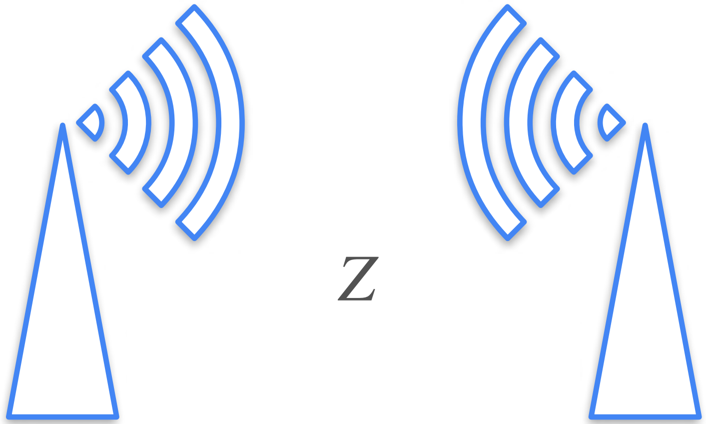


Message sent: 10010

Message received: 10010 $+Z$

Chi-Square Distribution: Motivation



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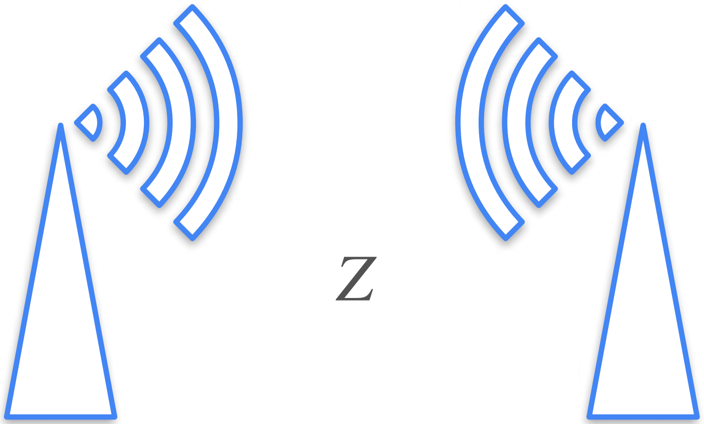
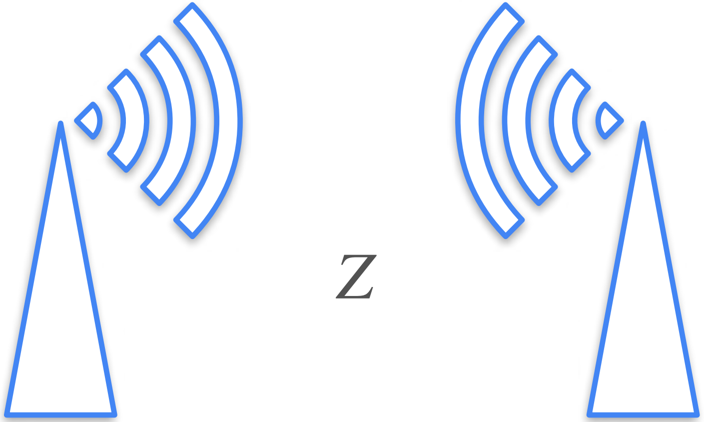


The communication channel has noise with a standard normal distribution

Chi-Square Distribution: Motivation



Z

What is the **power** of the noise in the channel?

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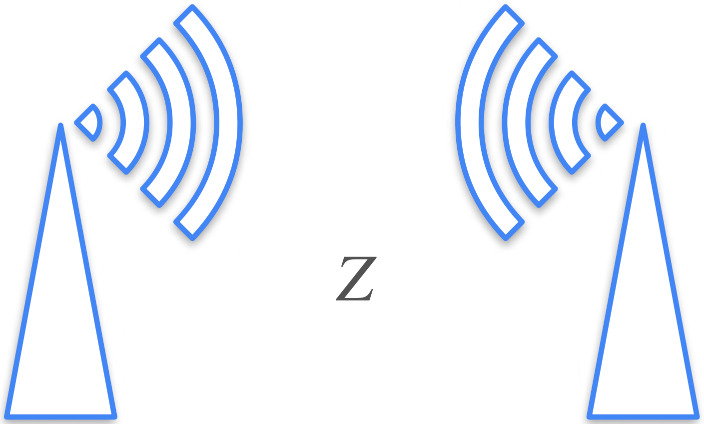

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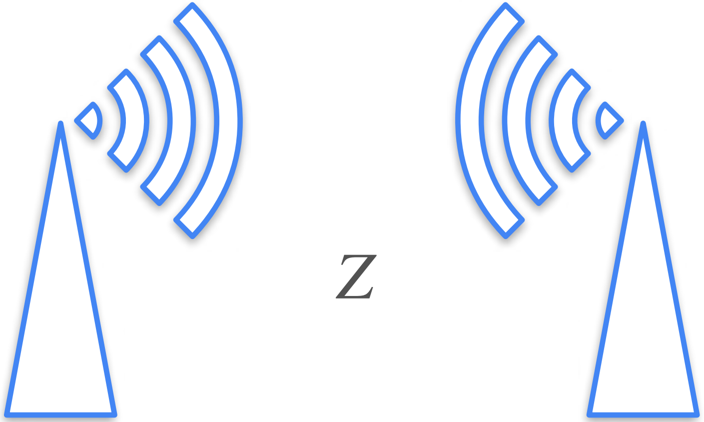

The communication channel has noise with a standard normal distribution

What is the **power** of the noise in the channel?

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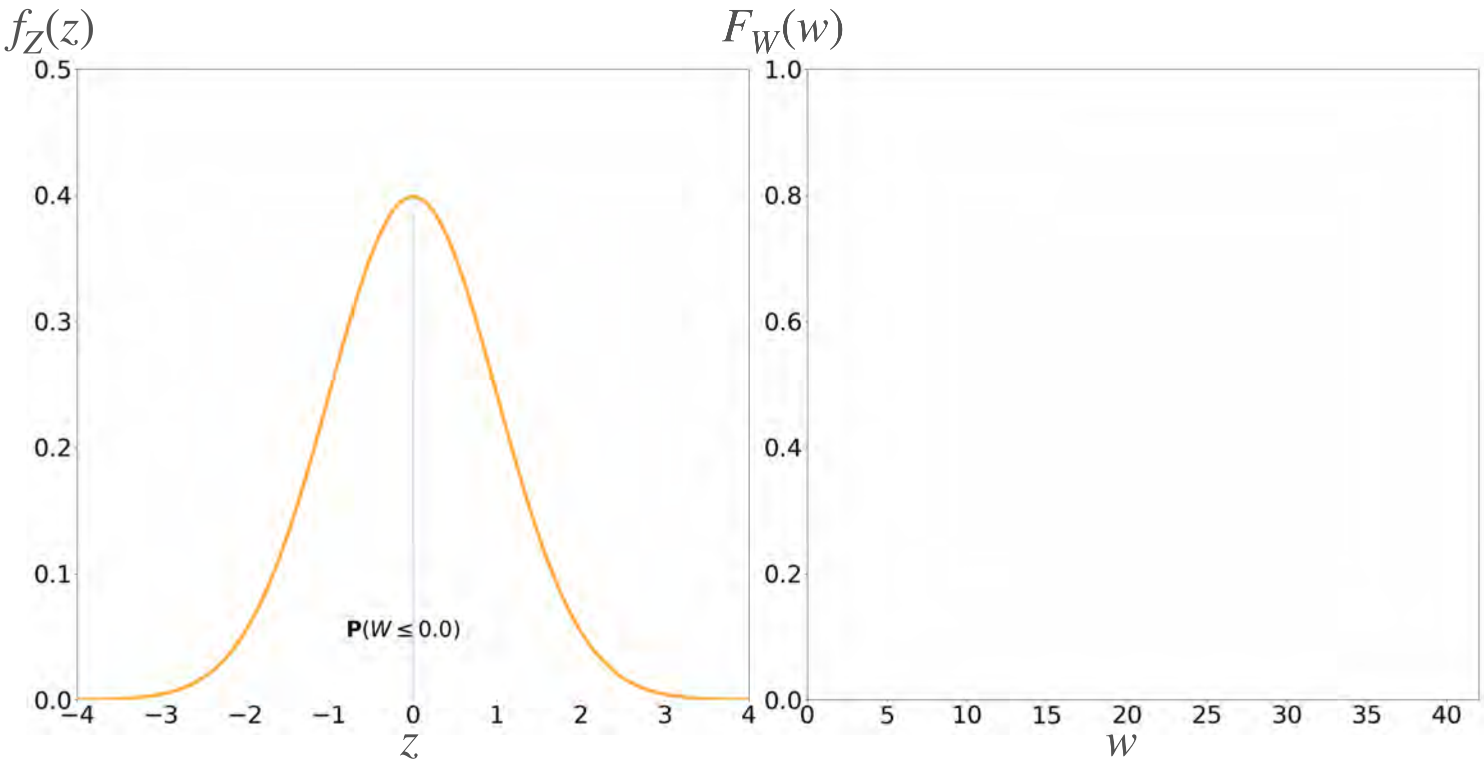
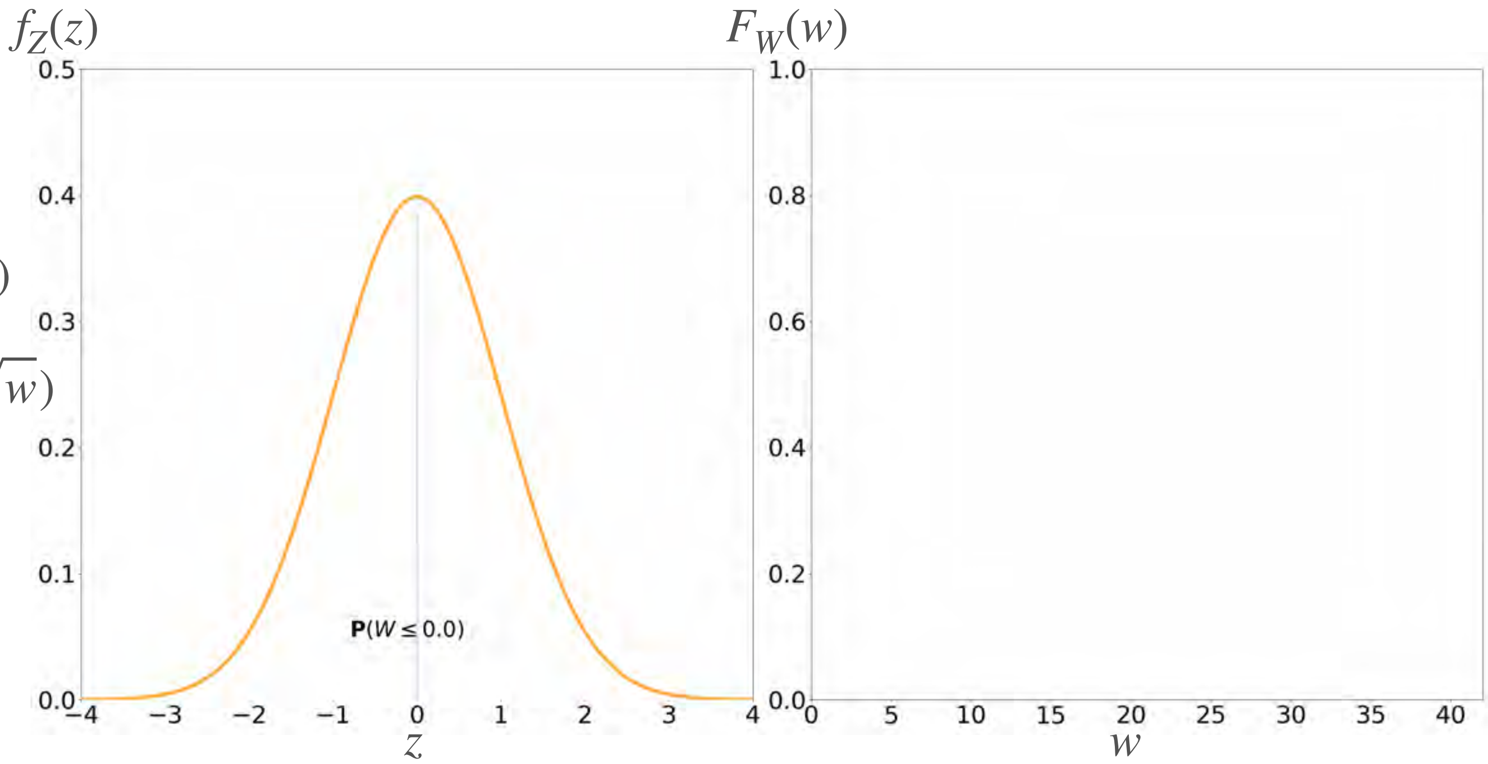
What is the distribution of W?

Chi Square Distribution



Chi Square Distribution

$$F_W(w) = \mathbf{P}(W \le w)$$
$$= \mathbf{P}(Z^2 \le w)$$
$$= \mathbf{P}(|Z| \le \sqrt{w})$$
$$= \mathbf{P}(-\sqrt{w} \le Z \le \sqrt{w})$$



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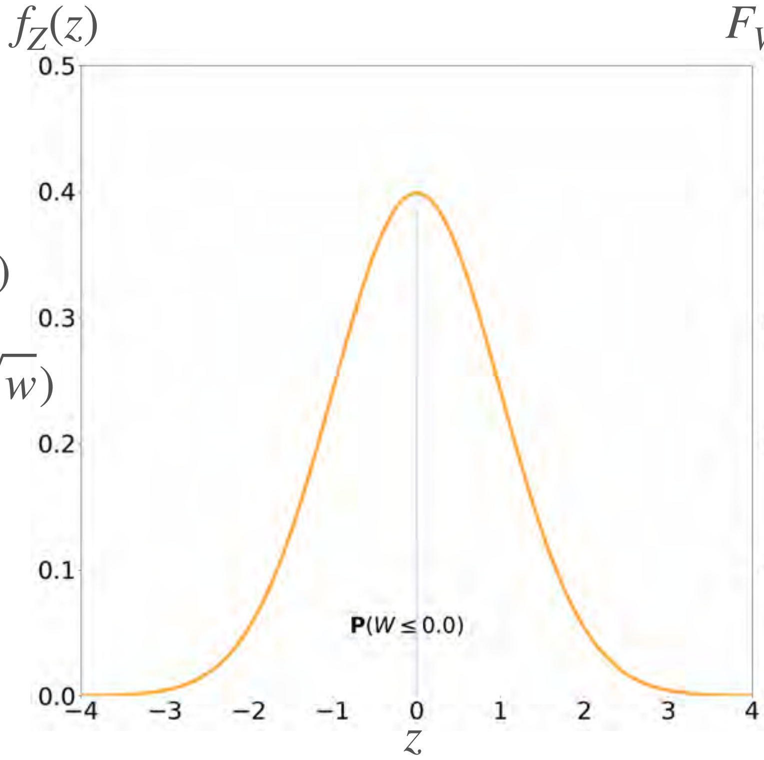

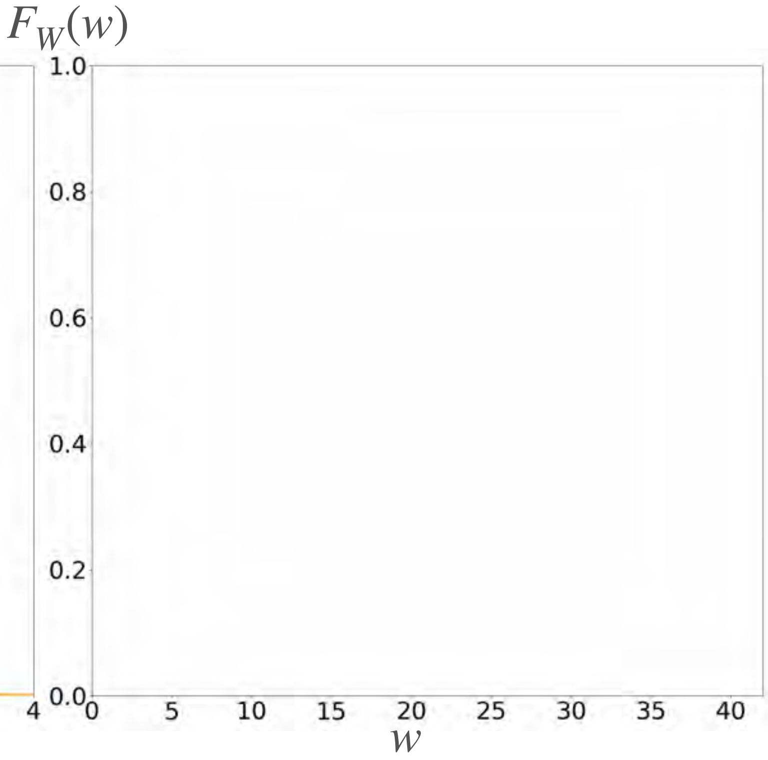

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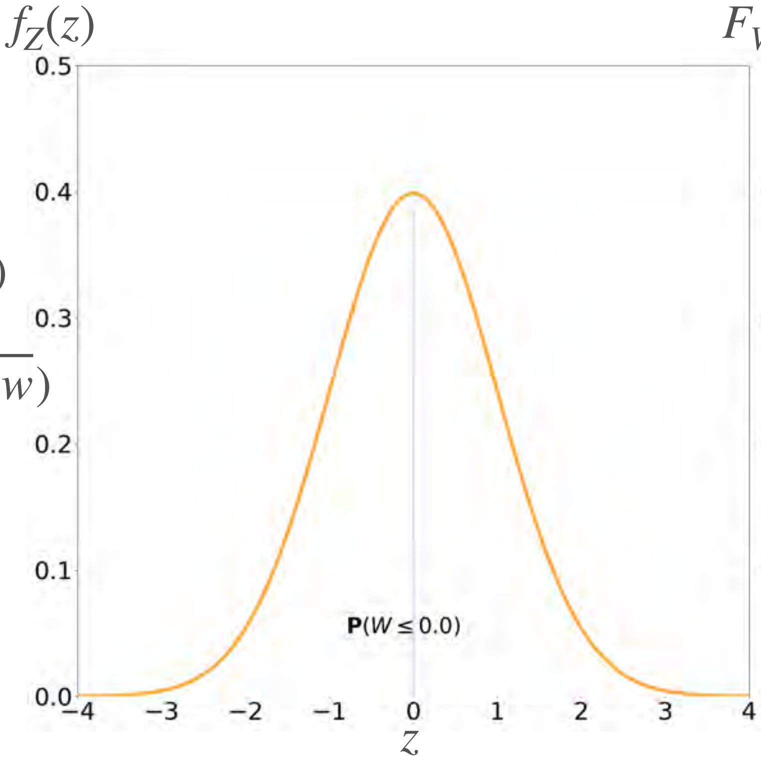

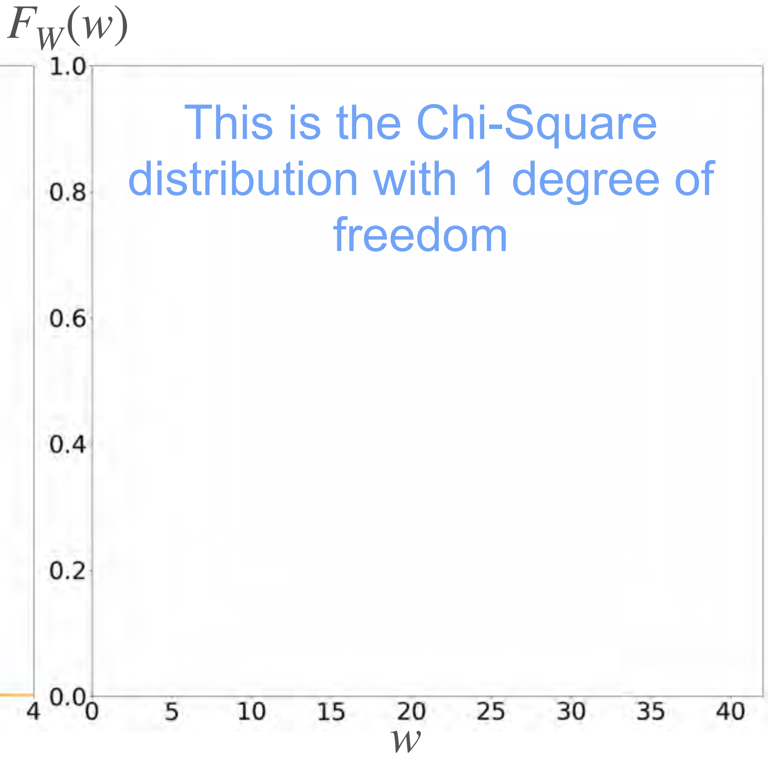

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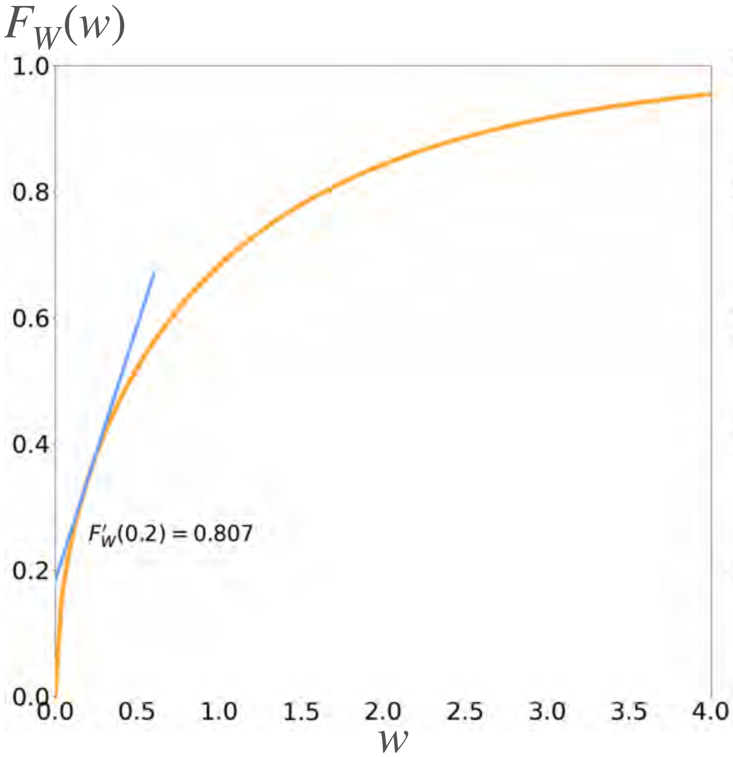
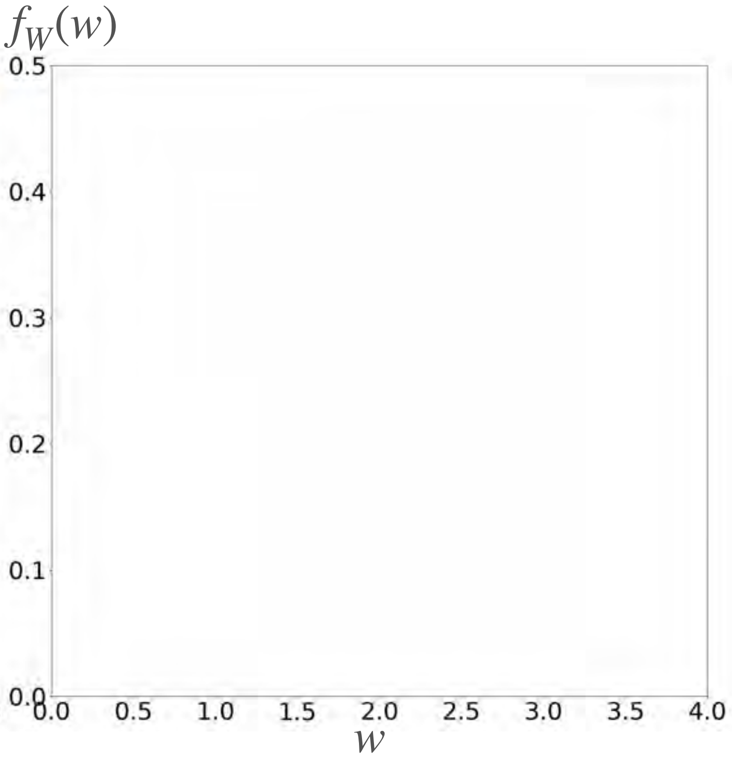
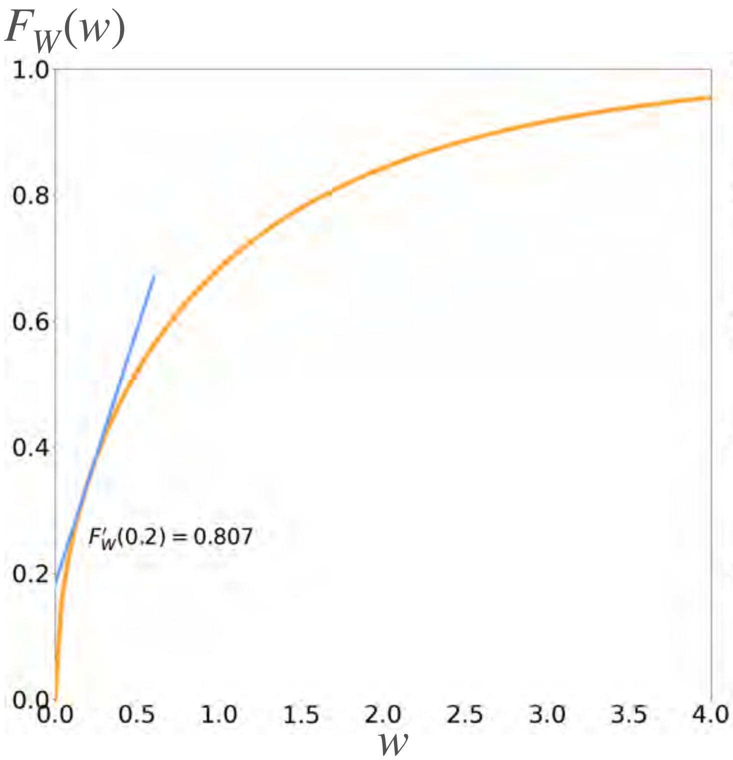
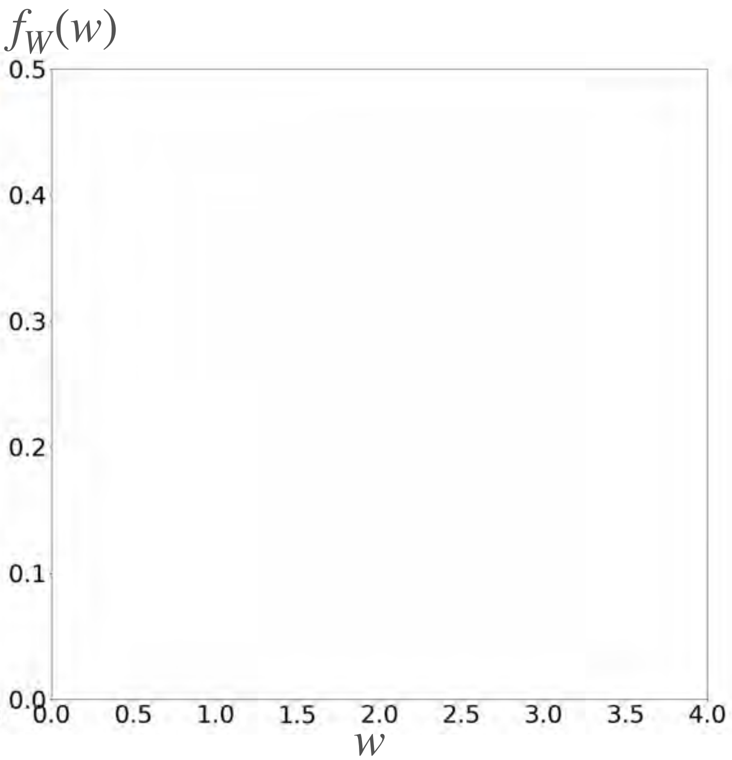




Chi Square Distribution

$$f_W(w) = F'_W(w)$$





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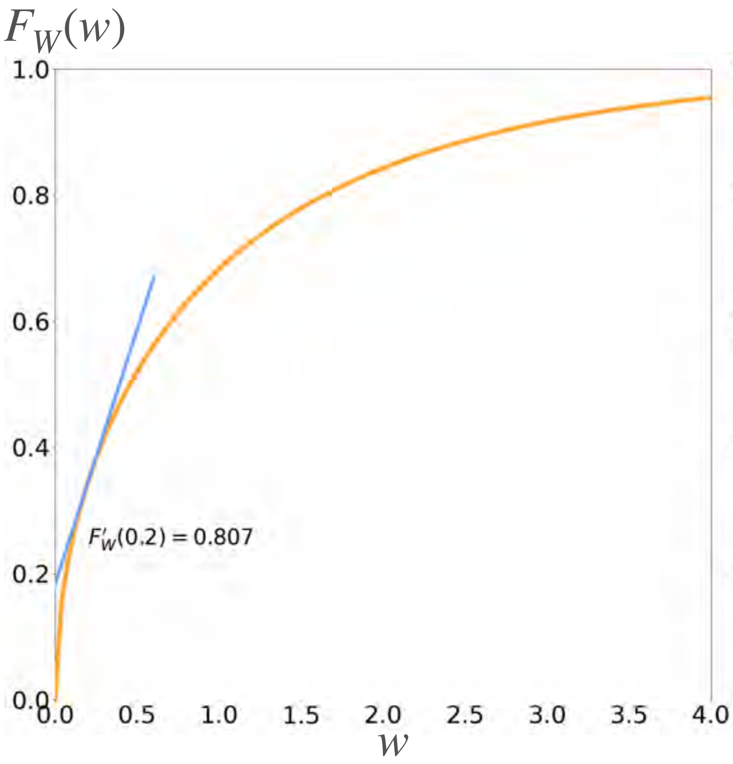

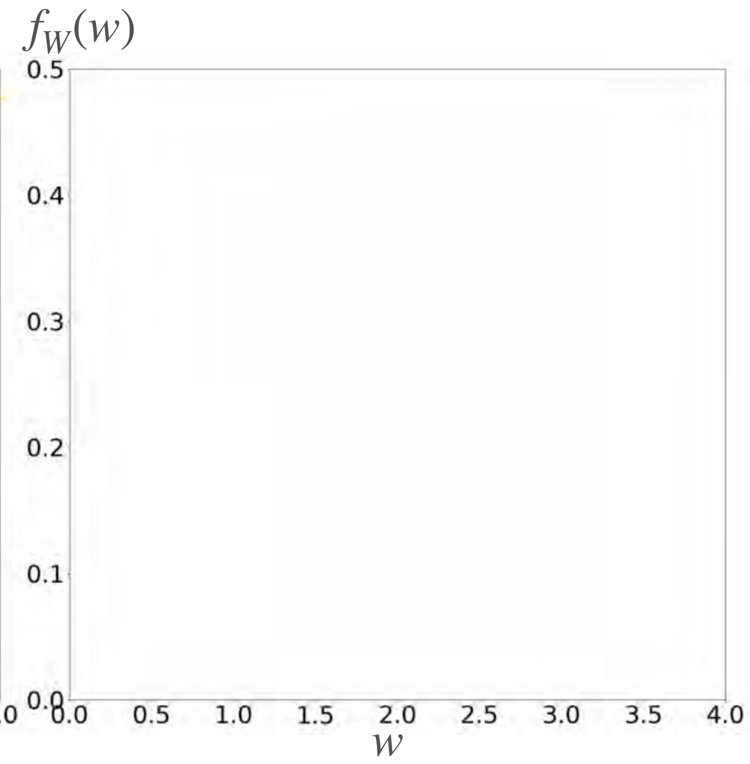

Chi-Square Distribution

Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square Distribution

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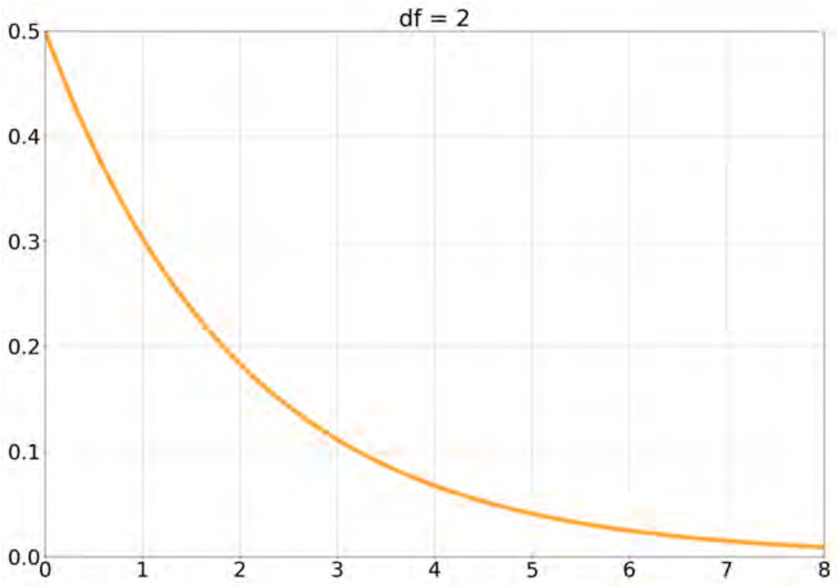
Chi-Square with 2 df



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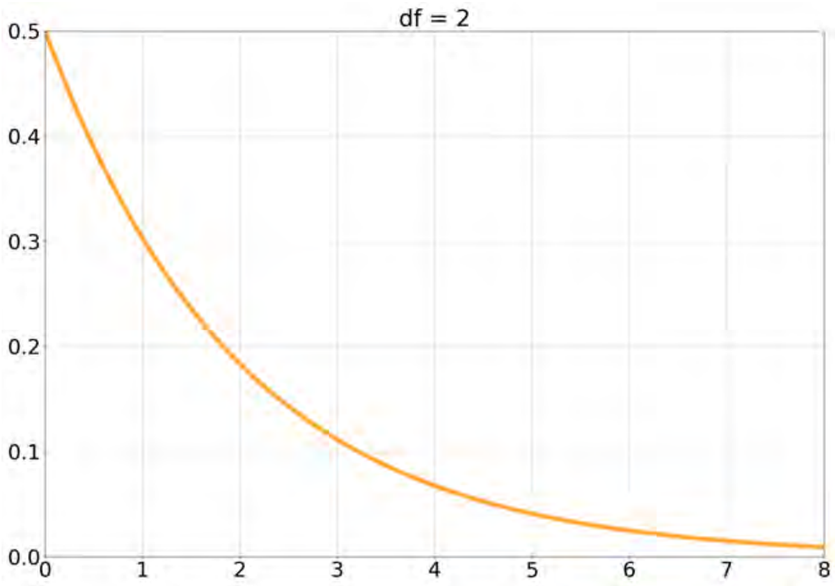
Chi-Square with 2 df



Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

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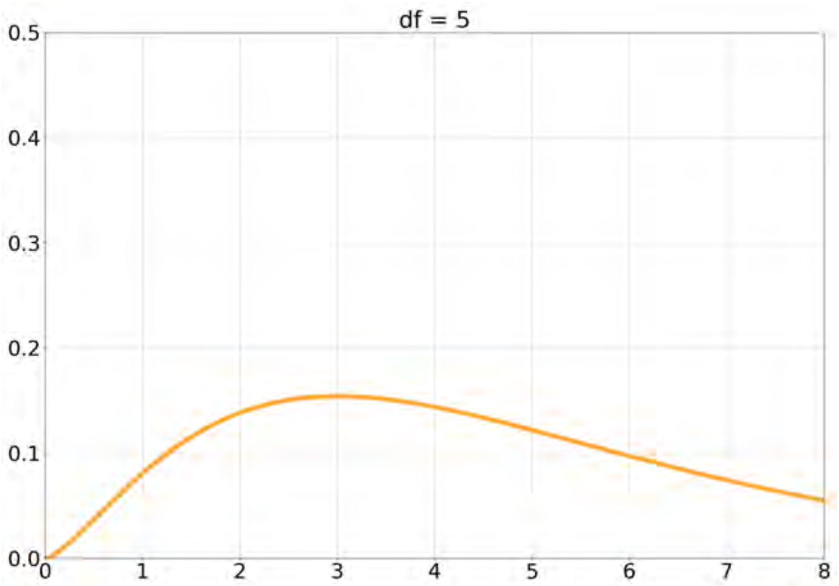
Chi-Square with 5 df



Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

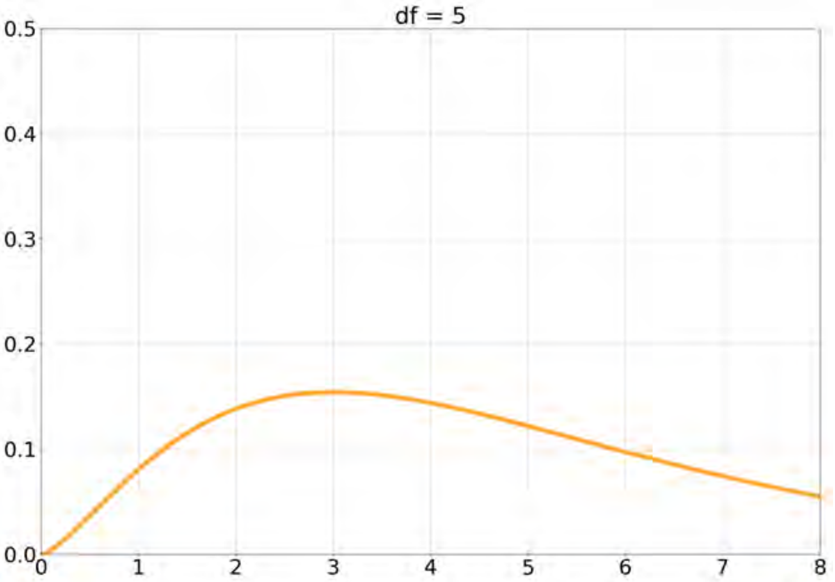
Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square with 5 df

Accumulated power over k transmissions?

$$W_k = \sum_{i=1}^{k} Z_i^2$$



Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

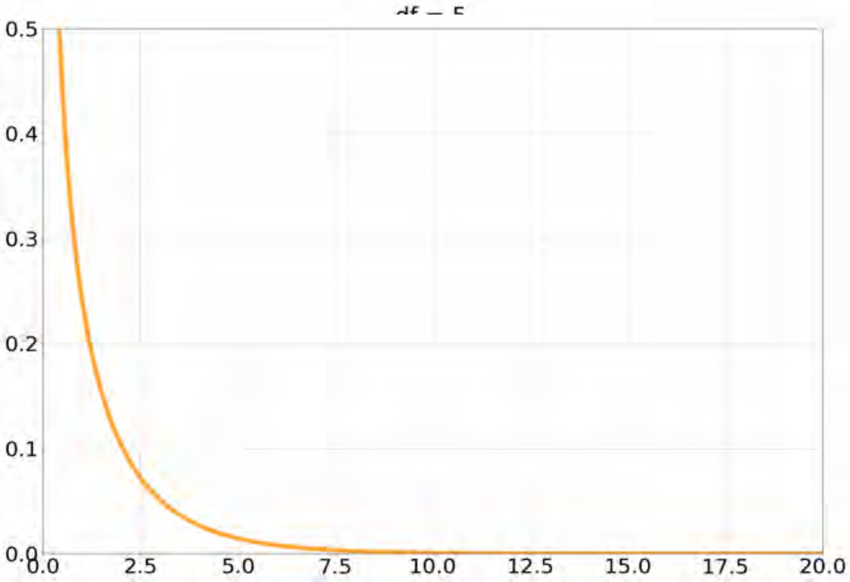
$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square with 5 df

Accumulated power over k transmissions?

$$W_k = \sum_{i=1}^{k} Z_i^2$$

Chi-Square with k df



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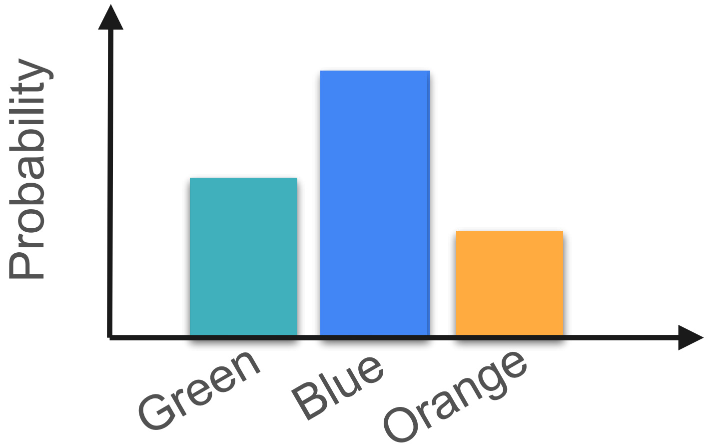
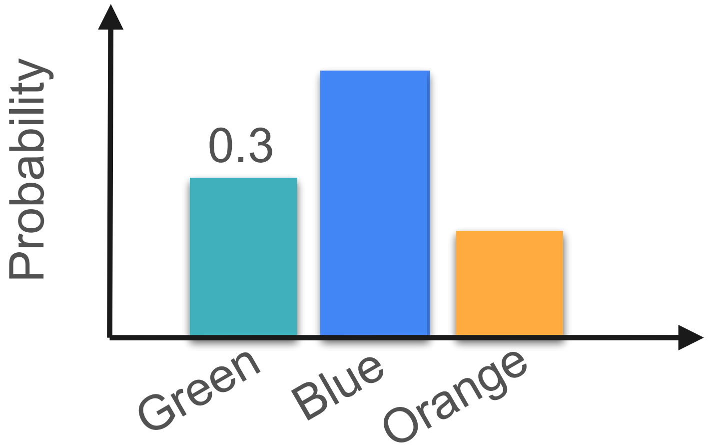
Probability Distributions

Sampling from a Distribution

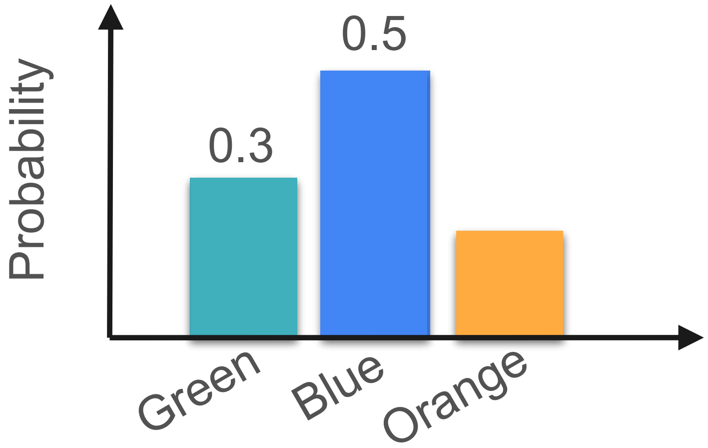
Sampling From a Distribution



Sampling From a Distribution



Sampling From a Distribution



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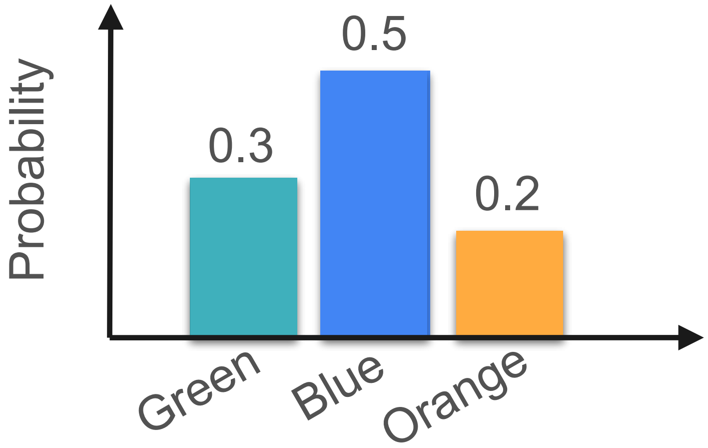

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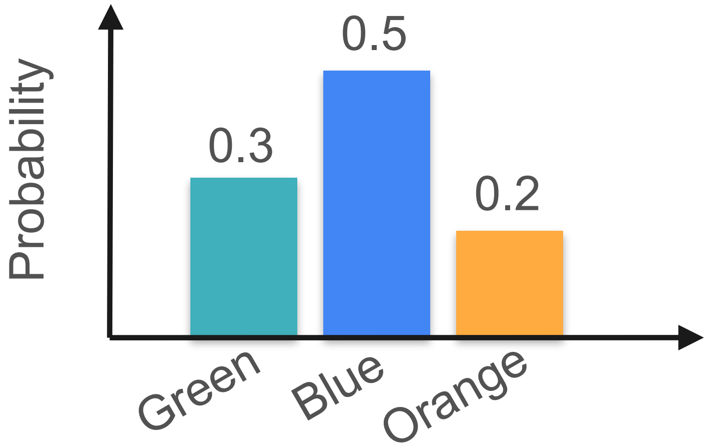

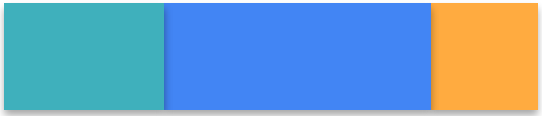

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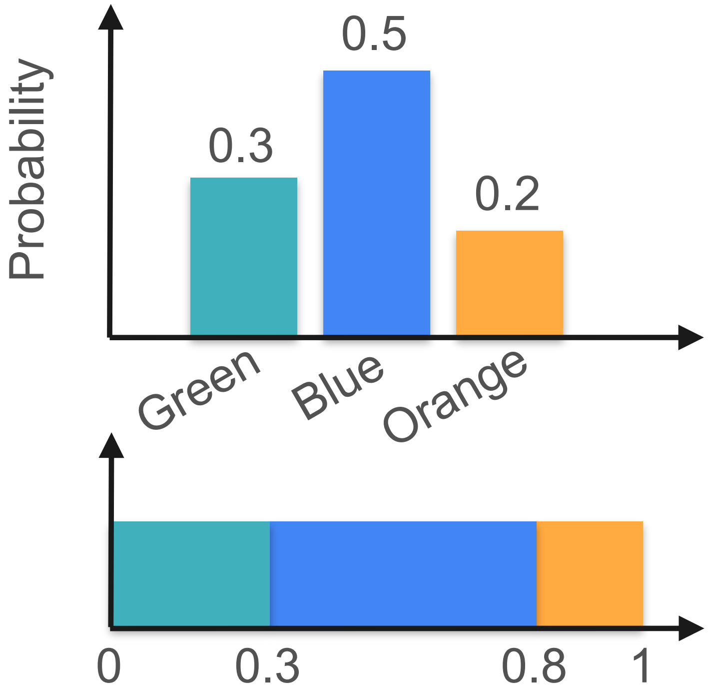
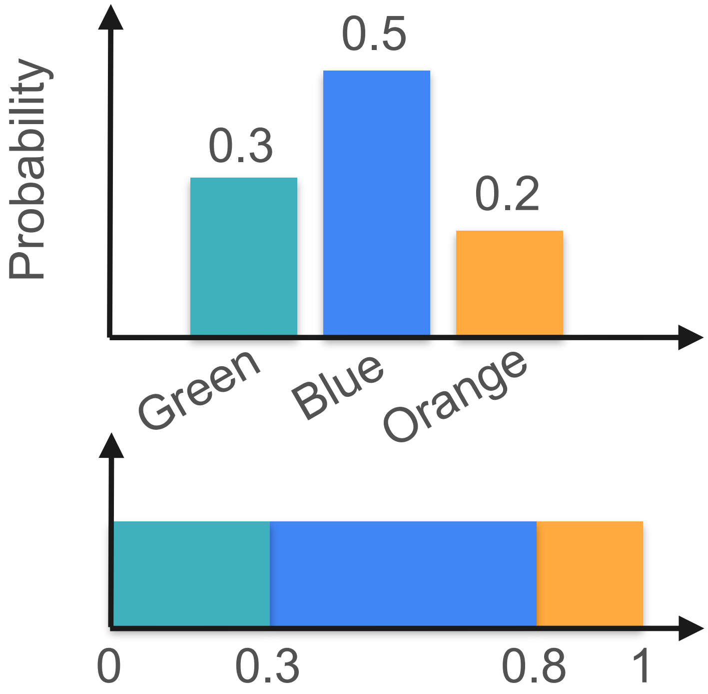
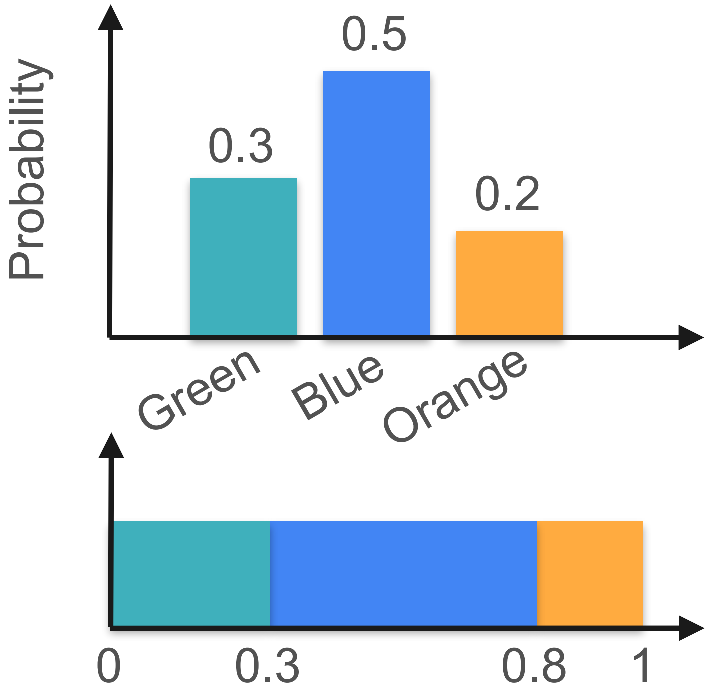


Sampling From a Distribution



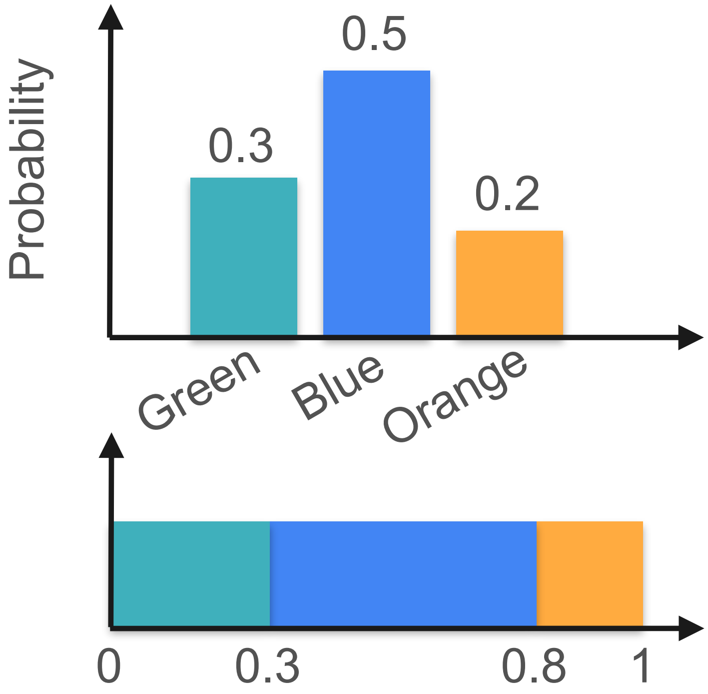
- **Step 1**: generate a random number between 0 and 1

Sampling From a Distribution



- **Step 1**: generate a random number between 0 and 1
- **Step 2**: find out which interval the number belongs to
    - [0, 0.3)
    - [0.3, 0.8)
    - [0.8, 1]

Sampling From a Distribution



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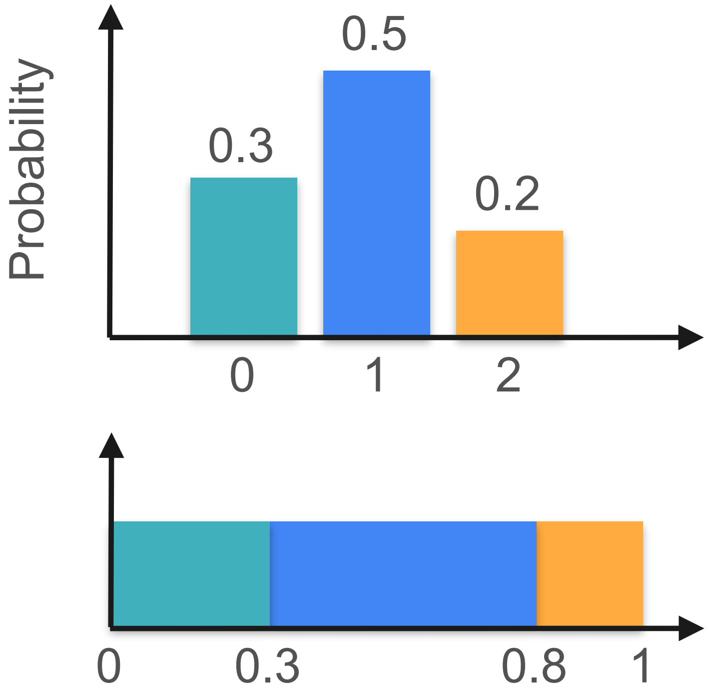
- **Step 3**: Assign an outcome based on the interval

Sampling From a Distribution



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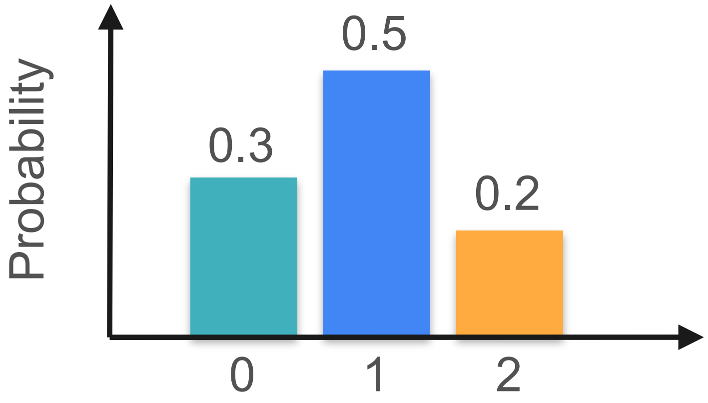
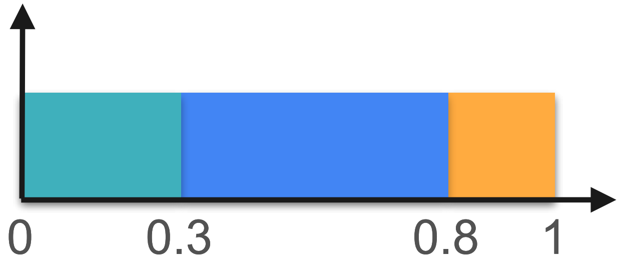
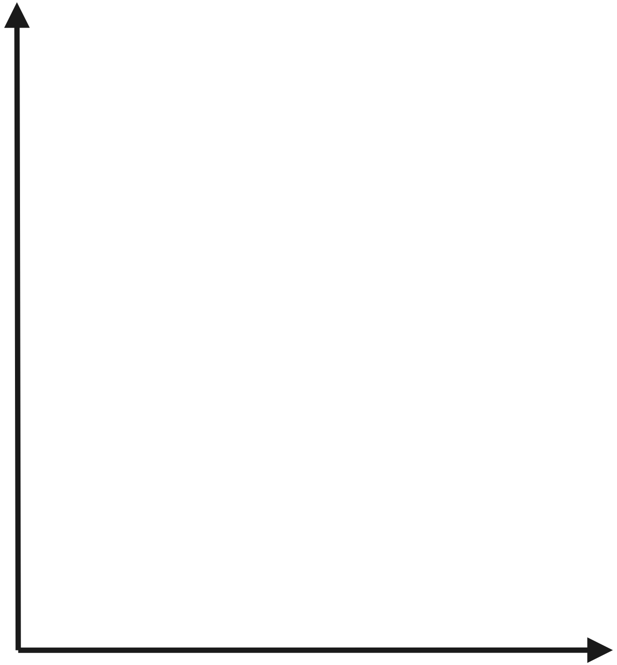
Sampling From a Distribution







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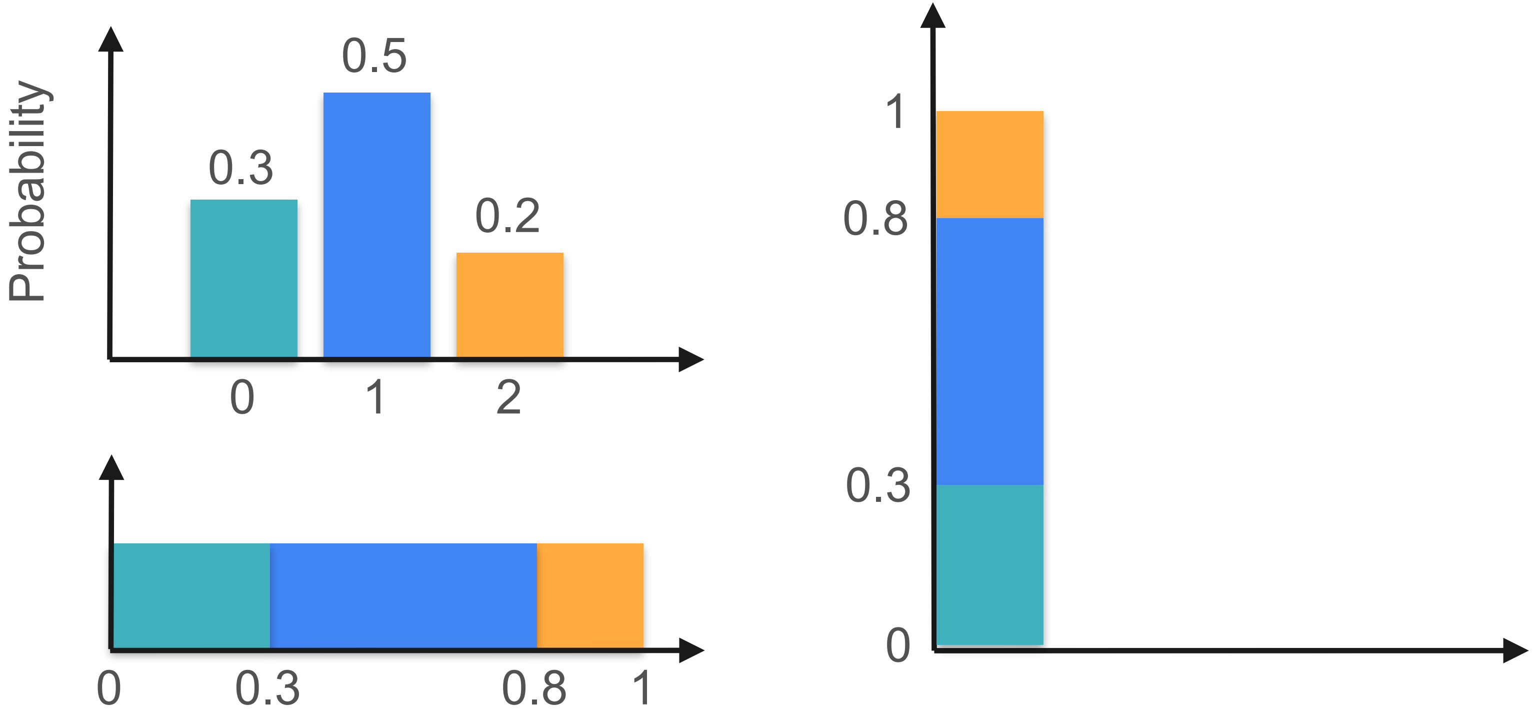
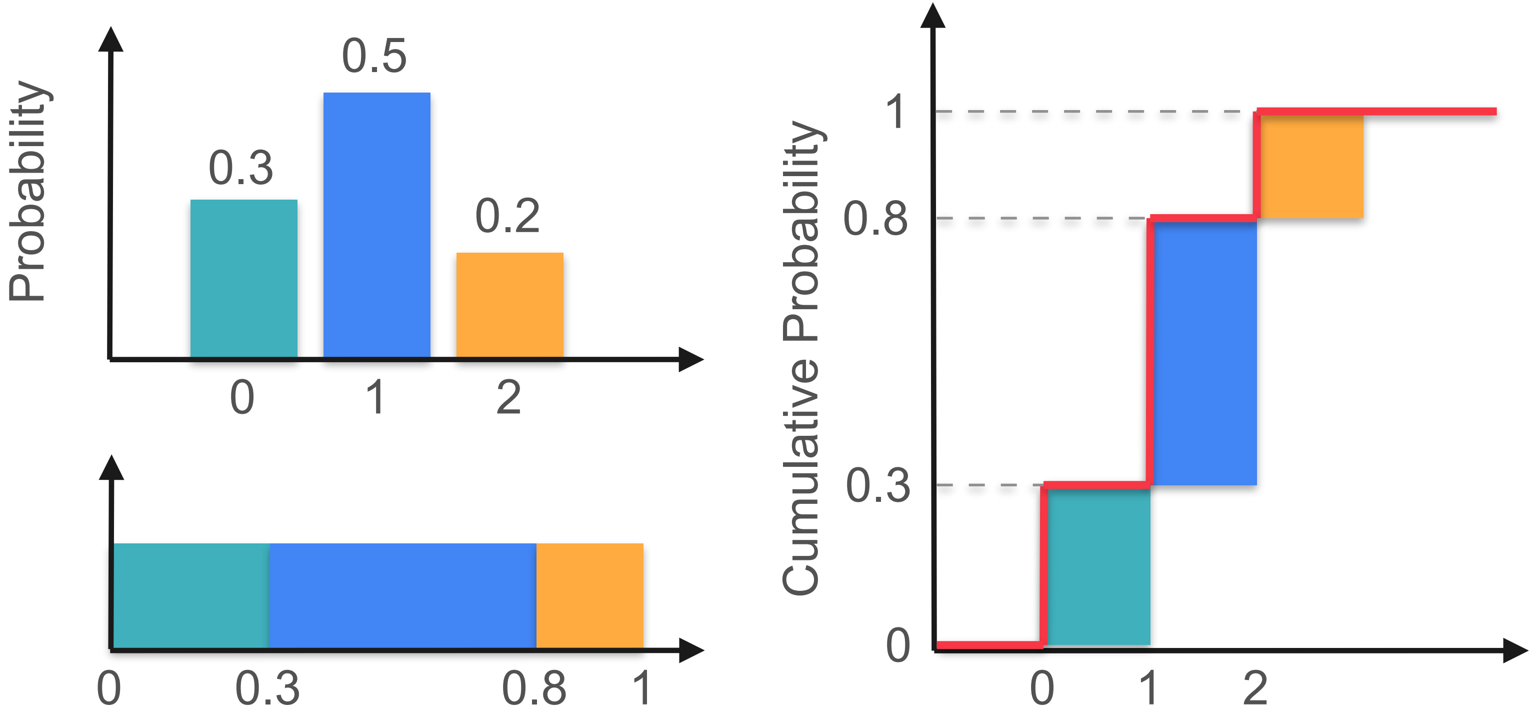
Sampling From a Distribution



Sampling From a Distribution



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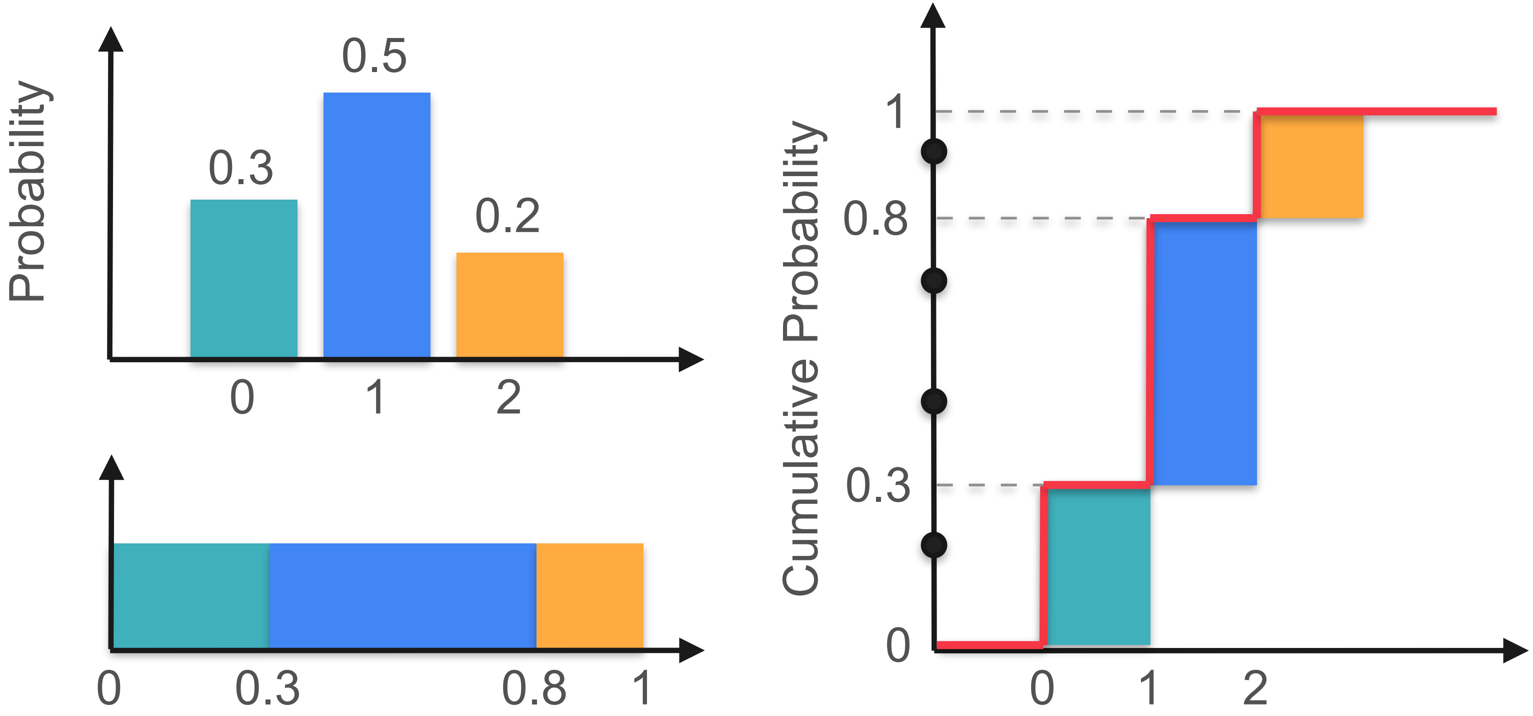

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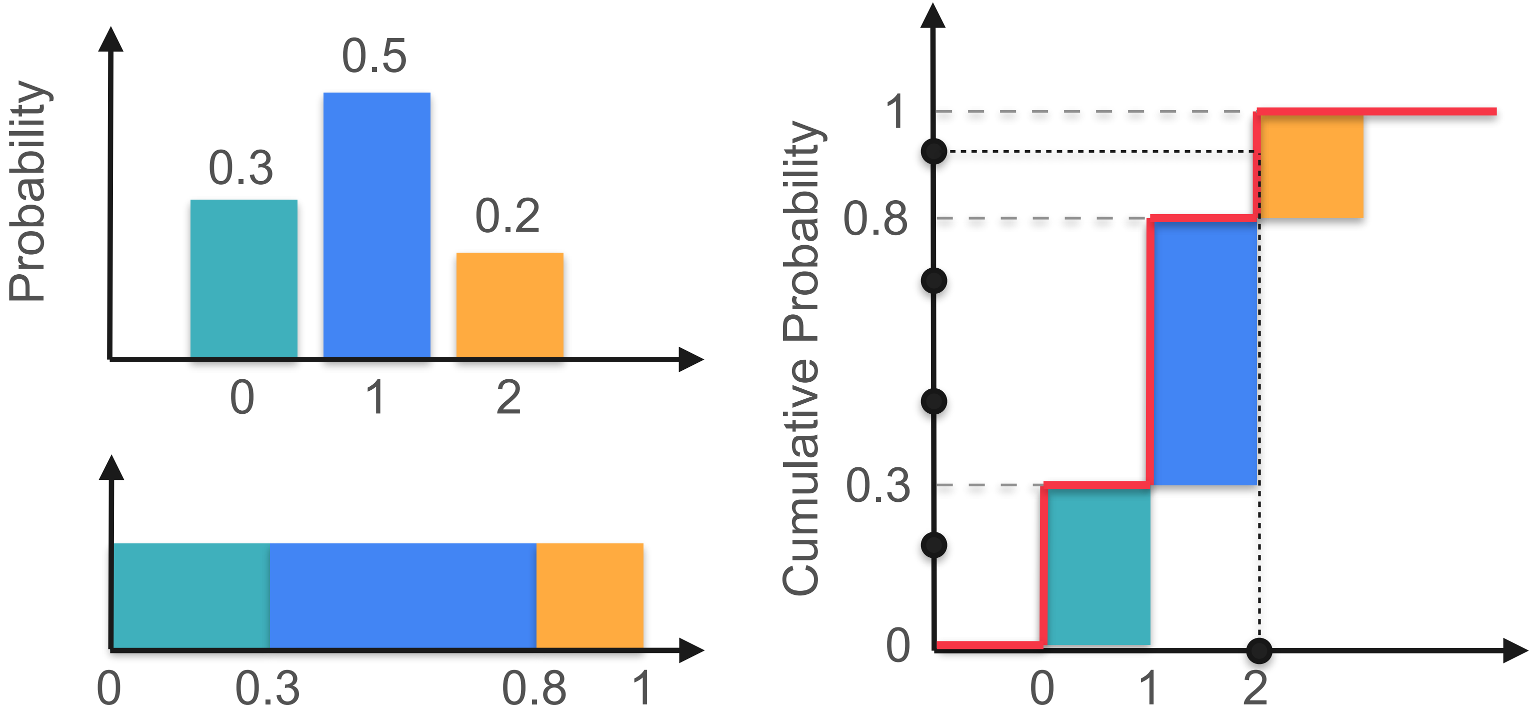

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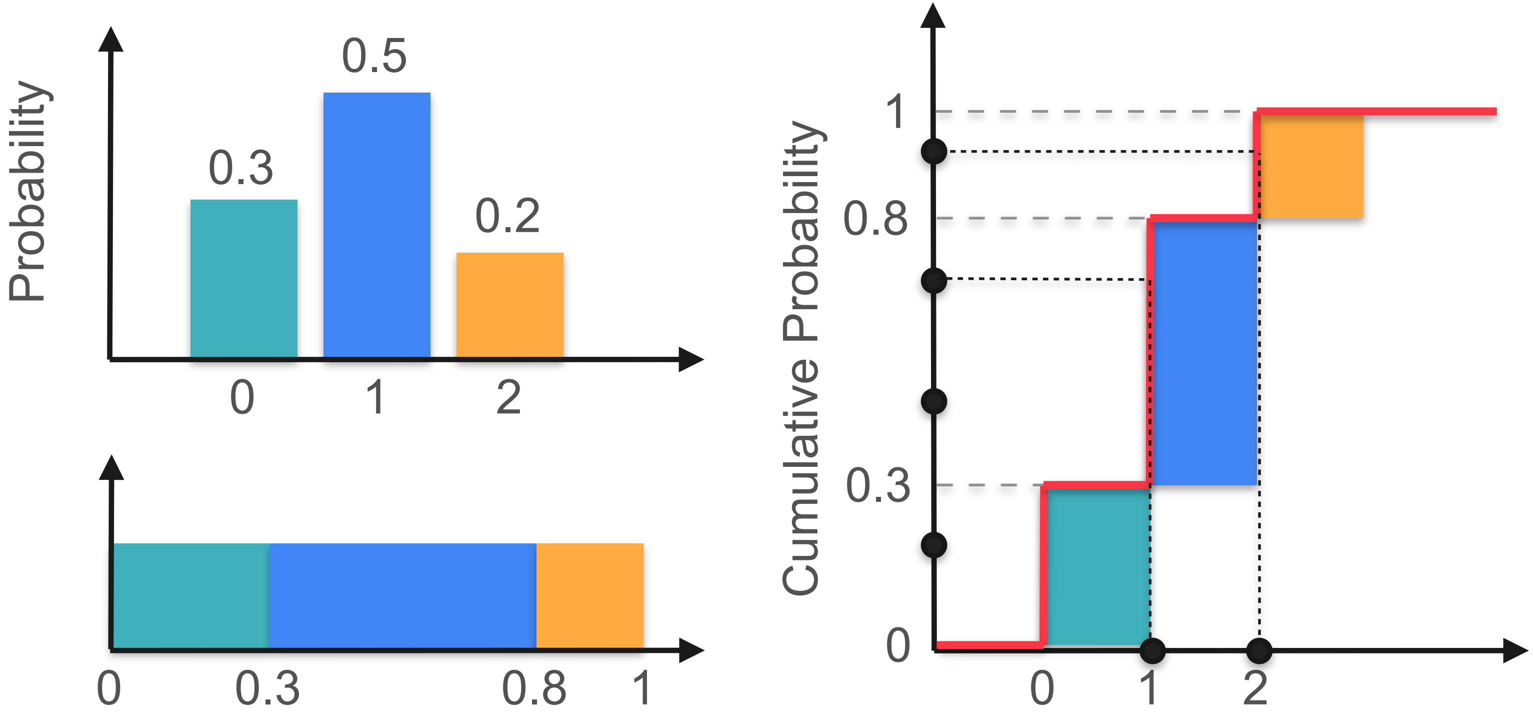

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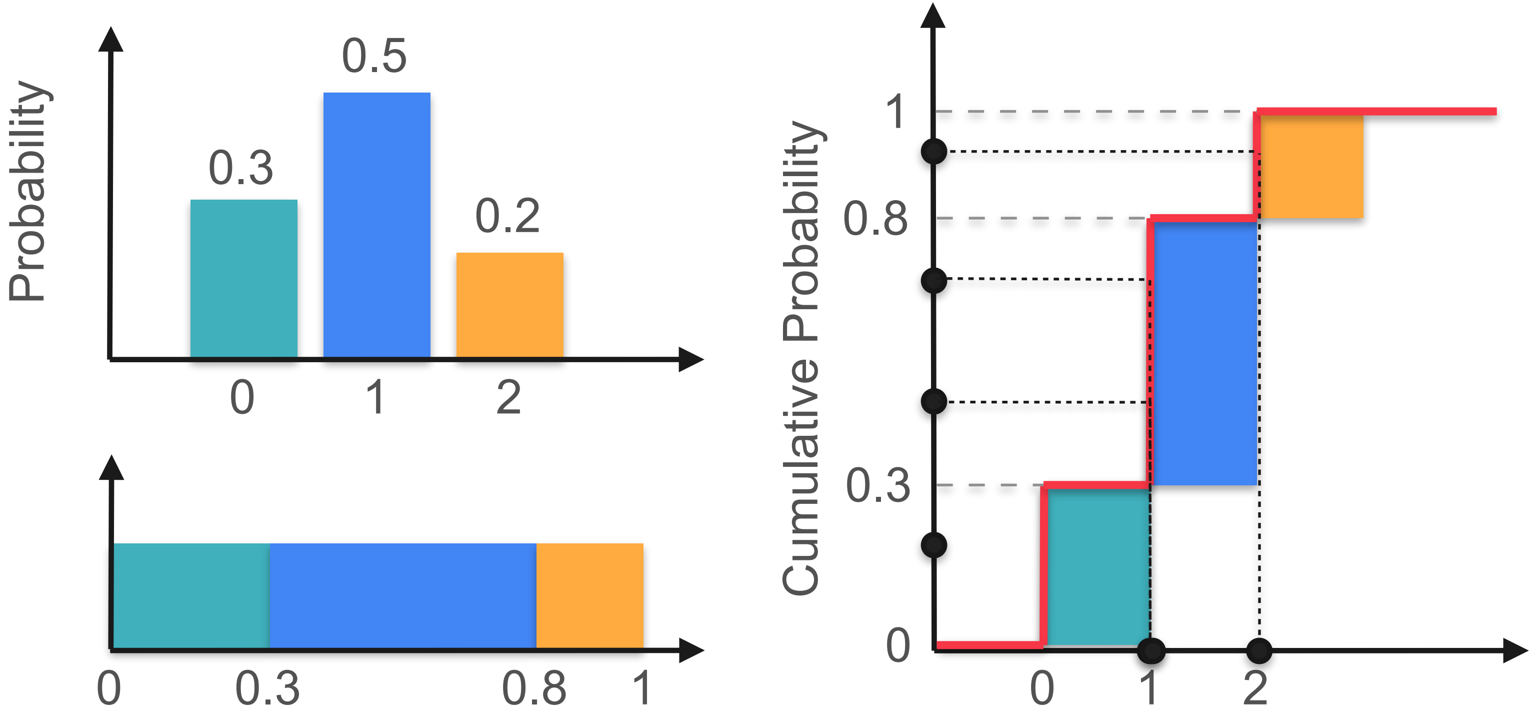

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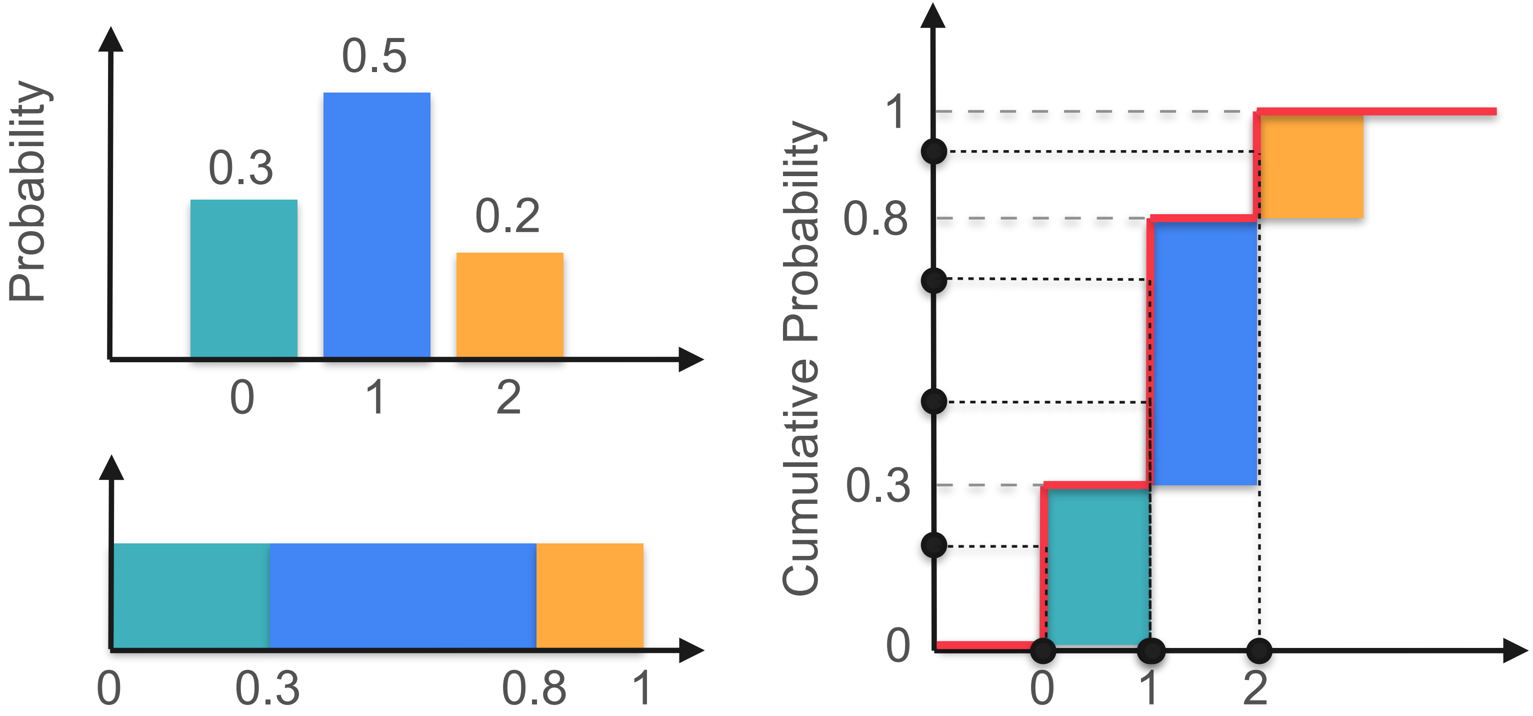
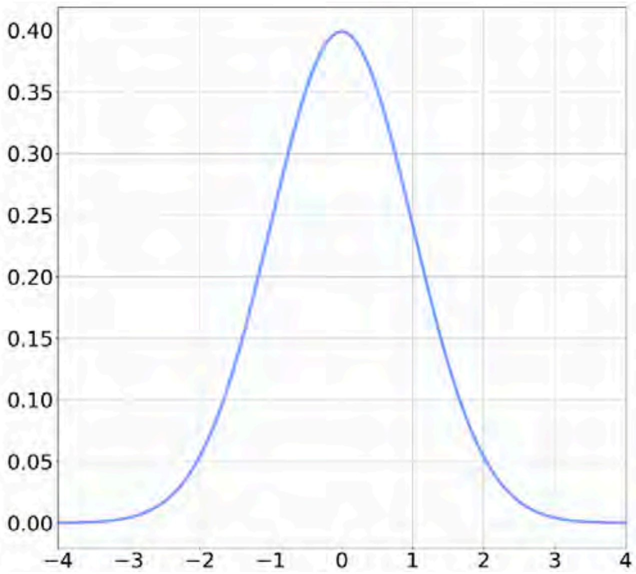
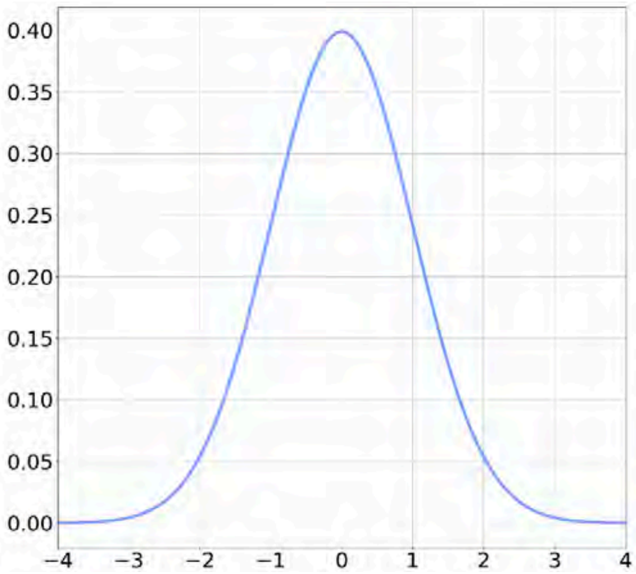
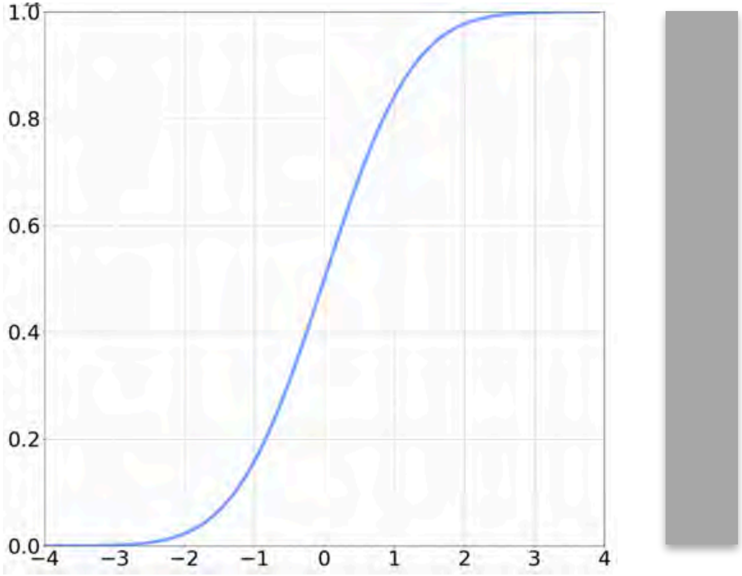


Sampling From a Normal Distribution



Sampling From a Normal Distribution





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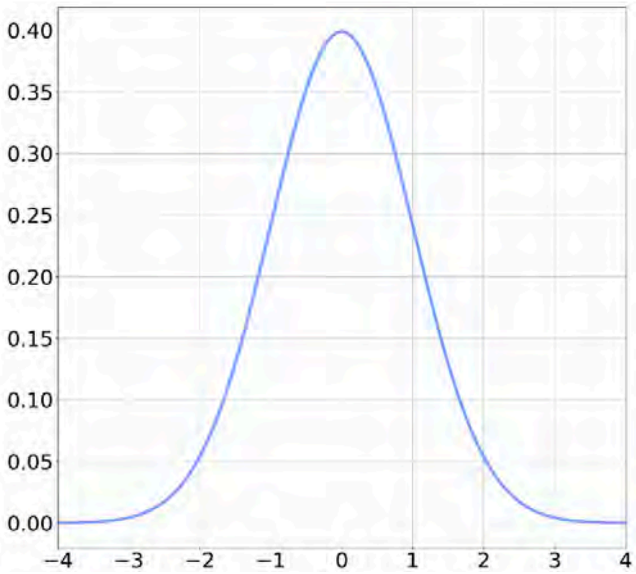

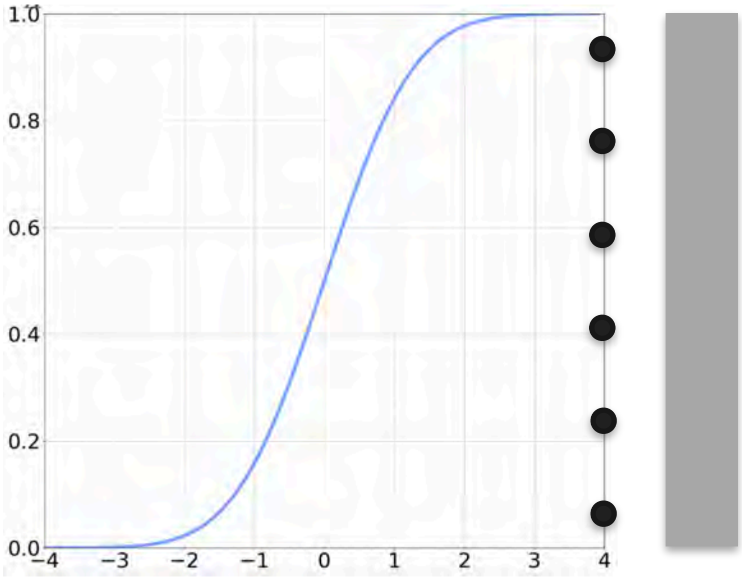

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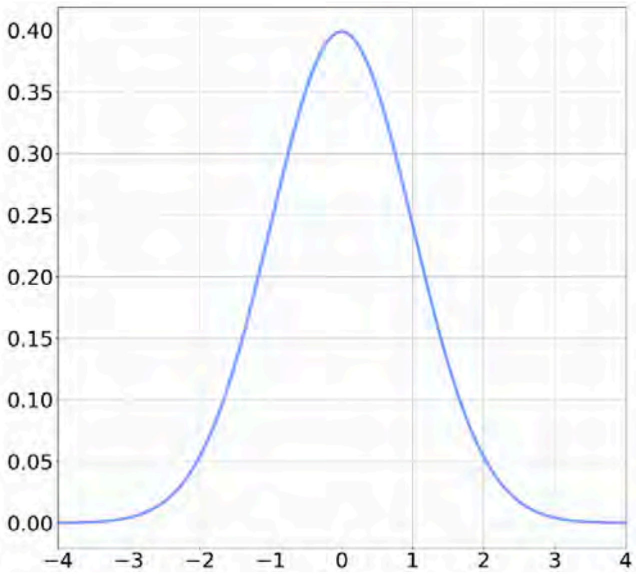

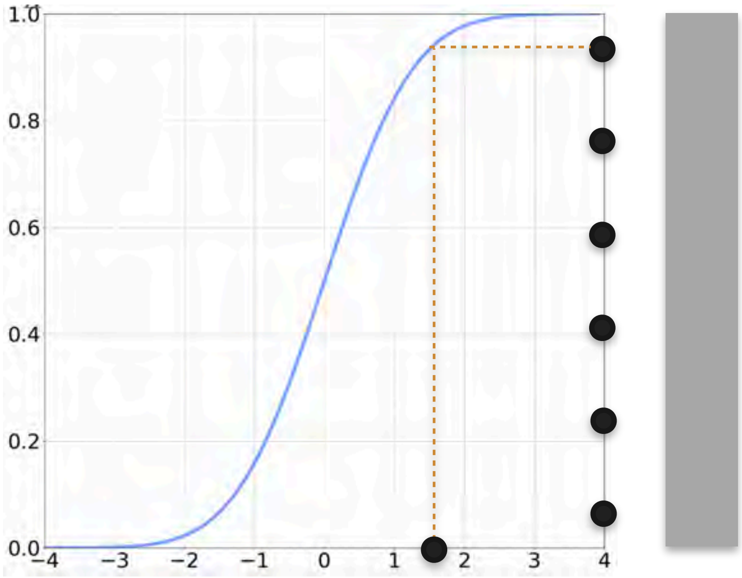

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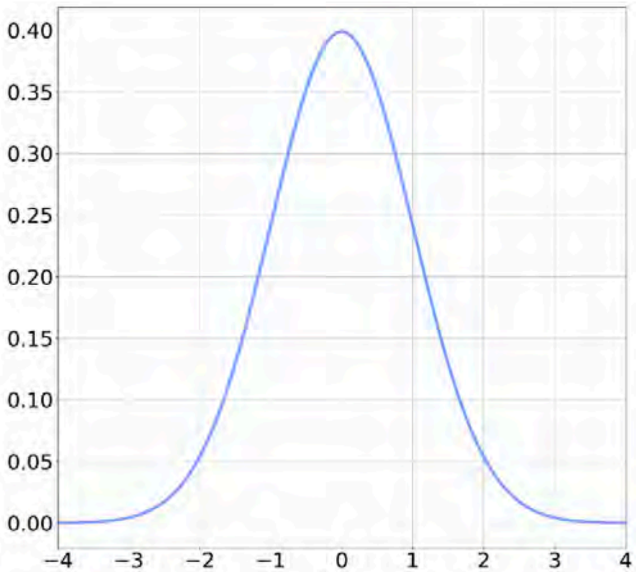

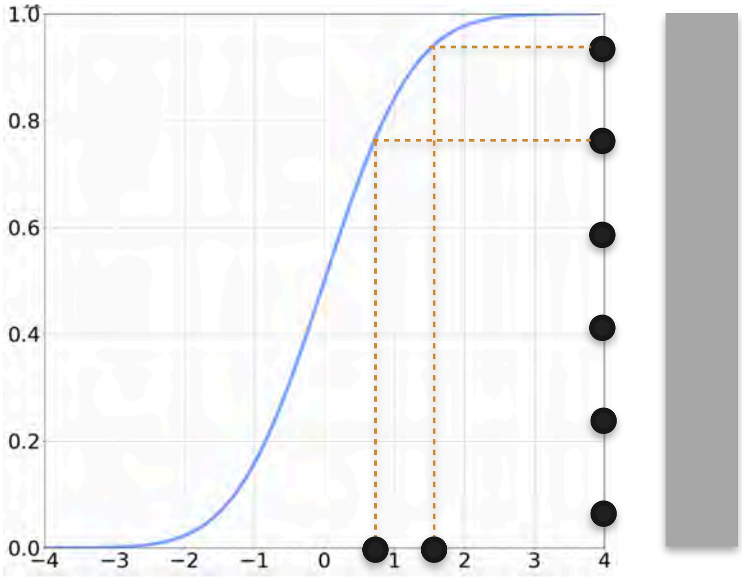

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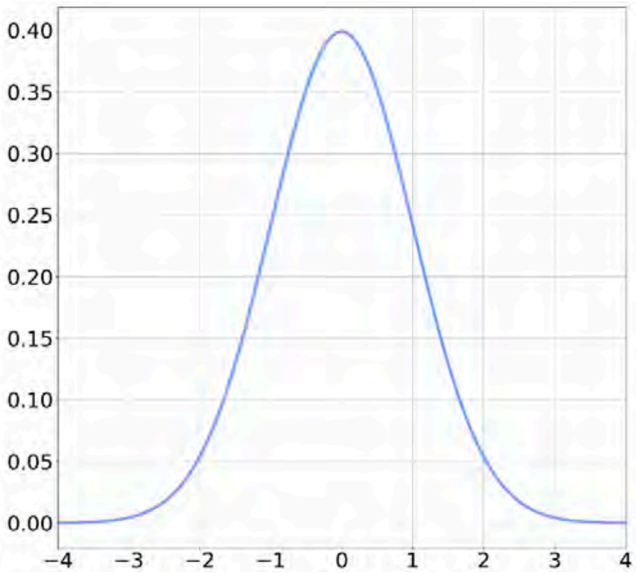

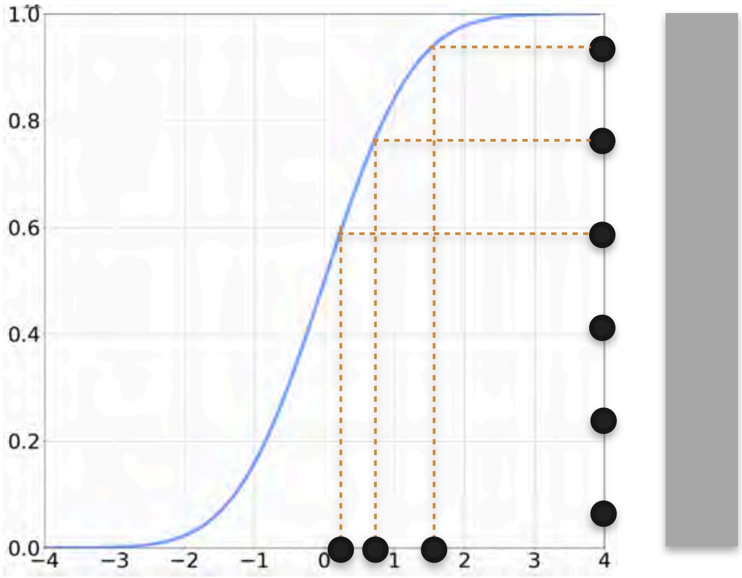

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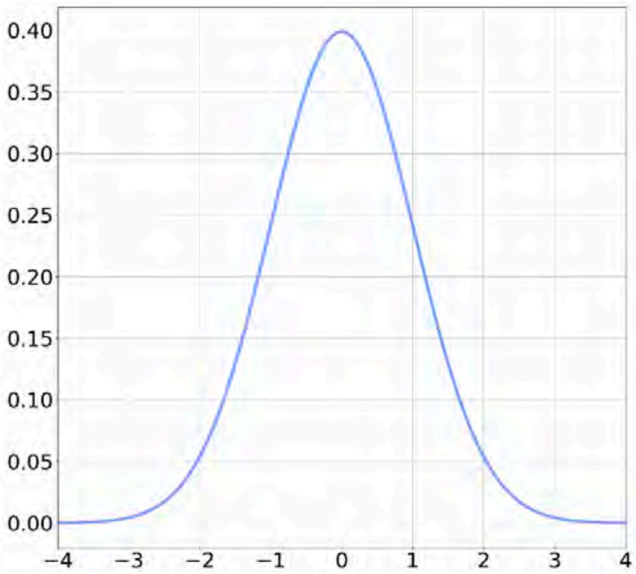

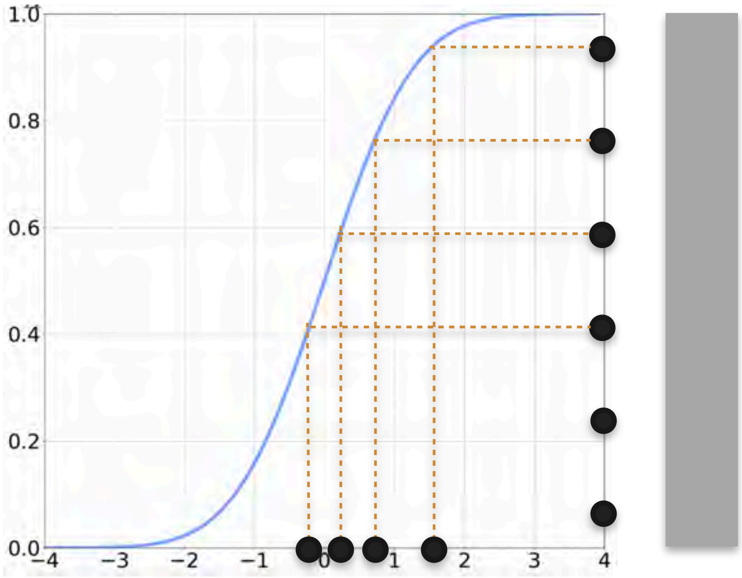

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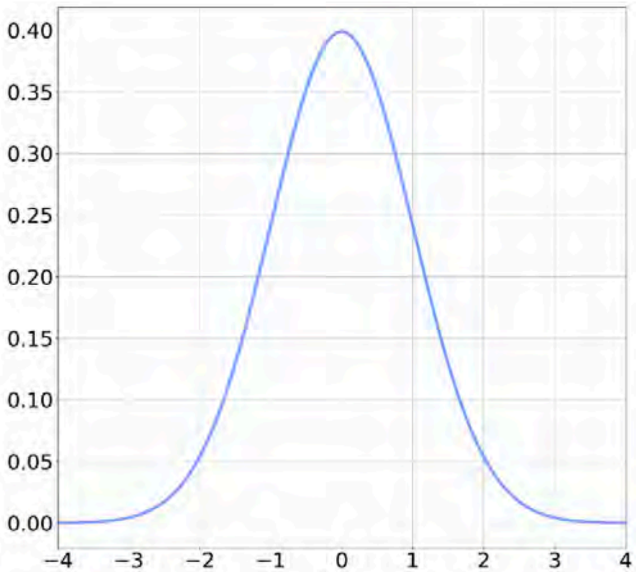

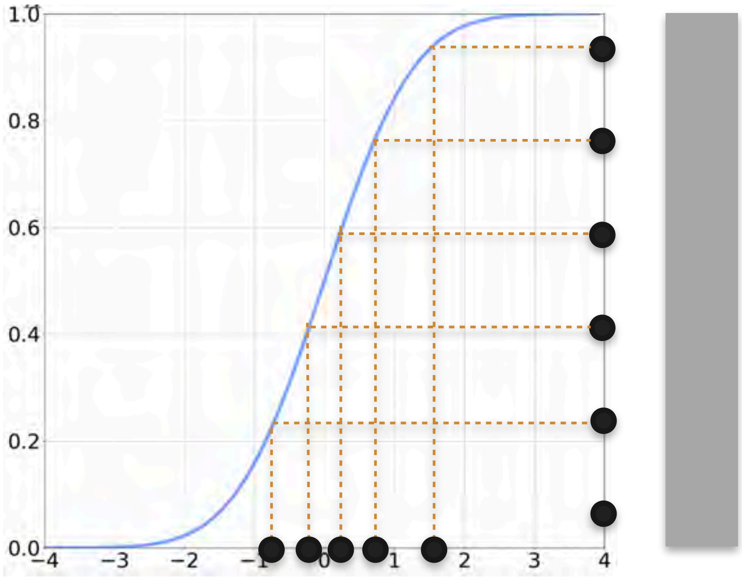

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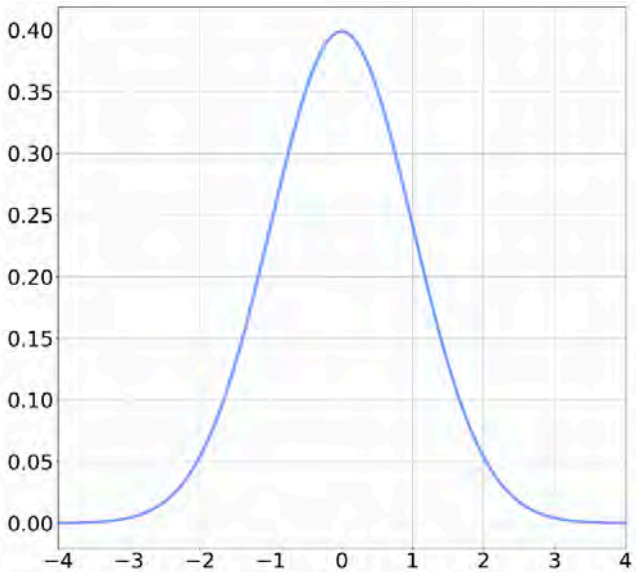

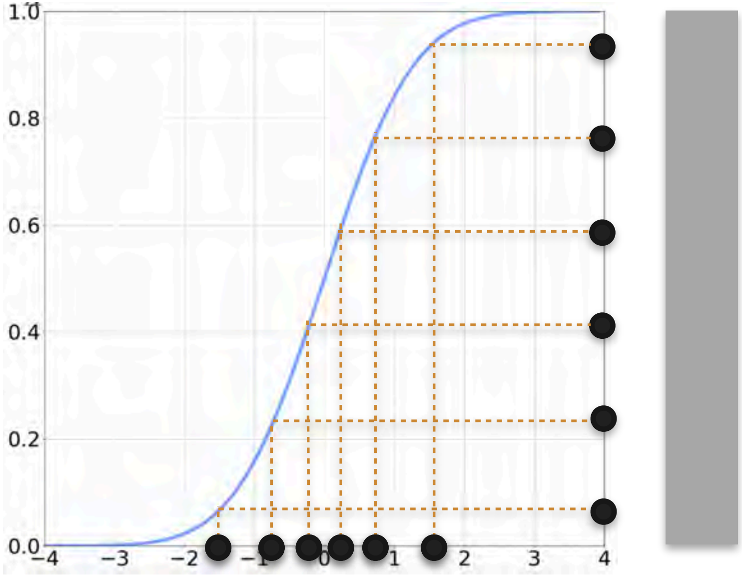

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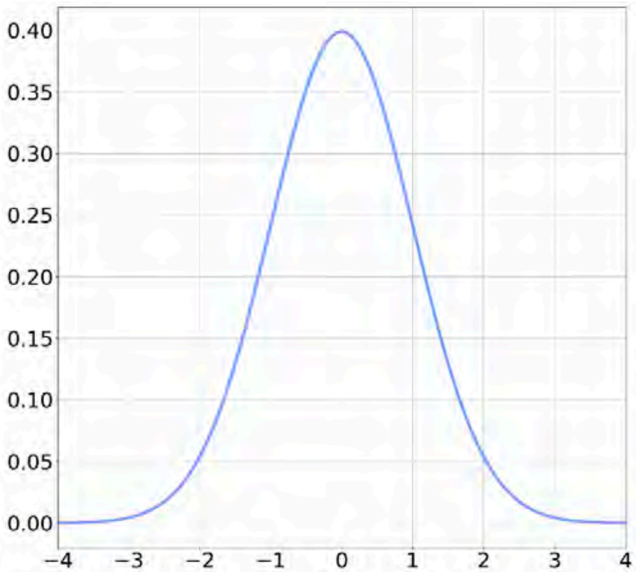

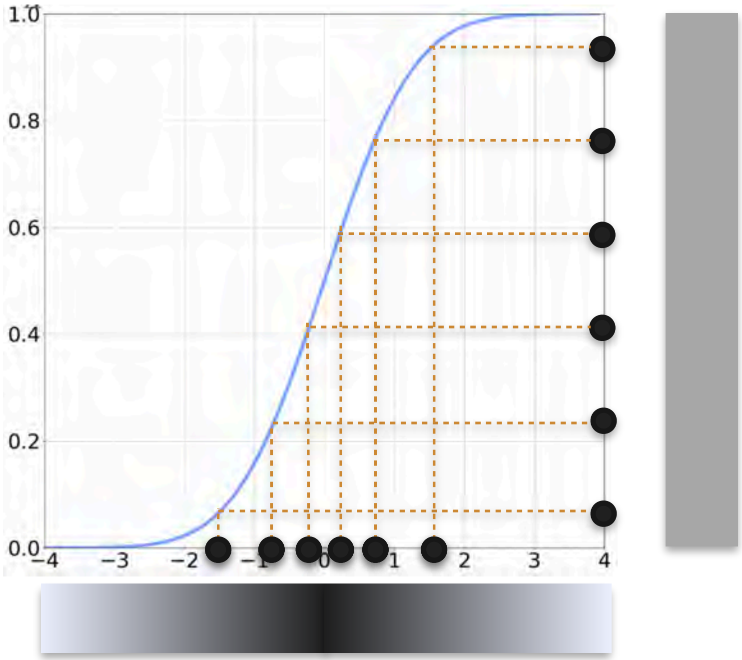

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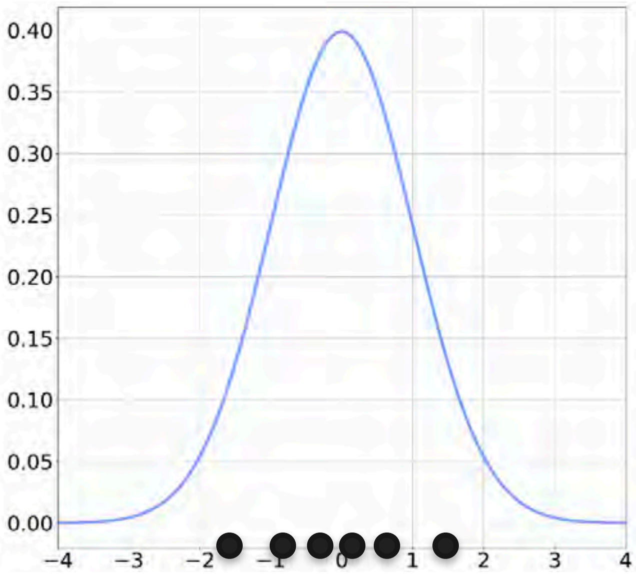

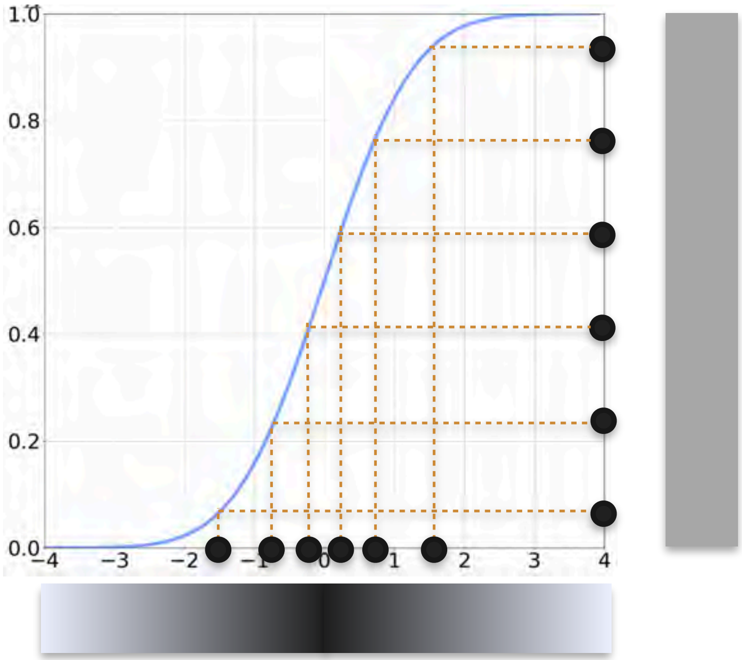

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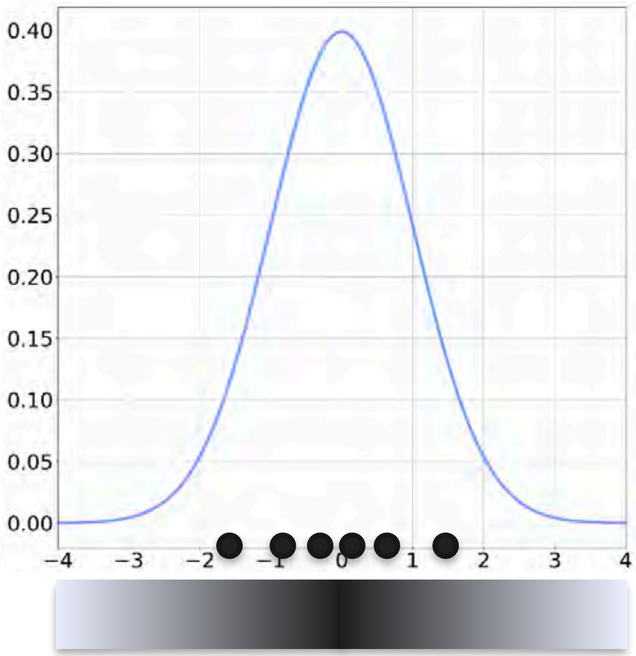

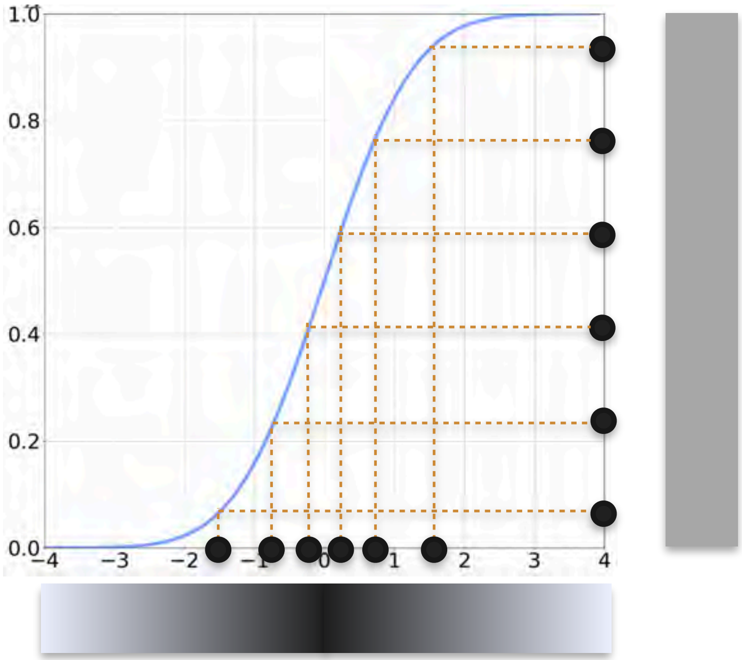

Probability Distributions

Conclusion