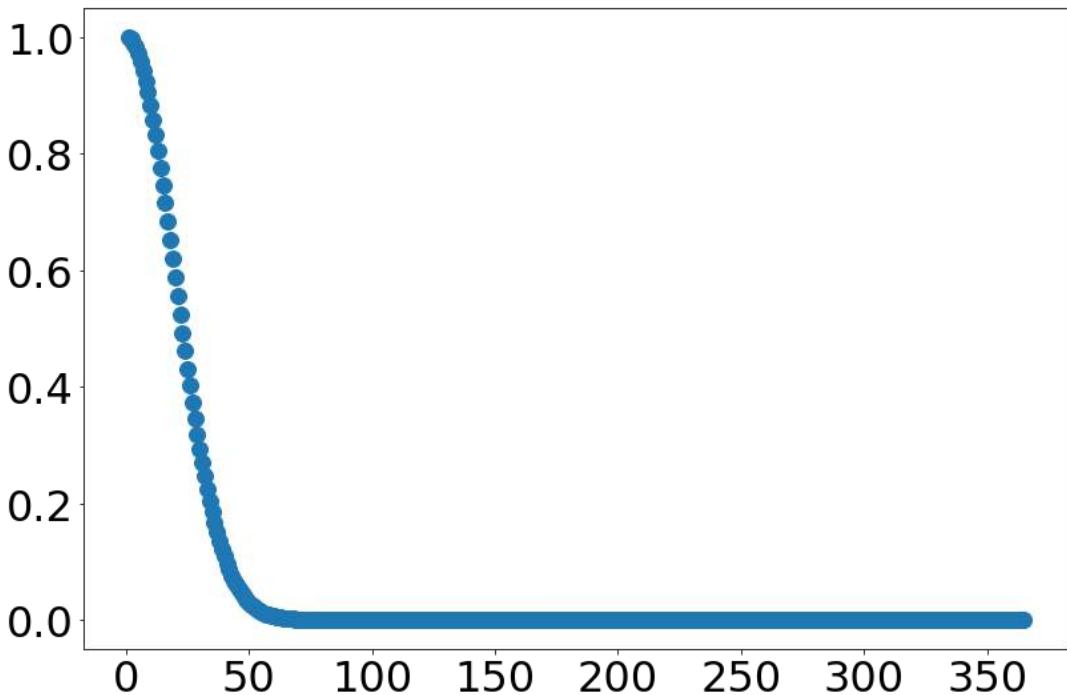
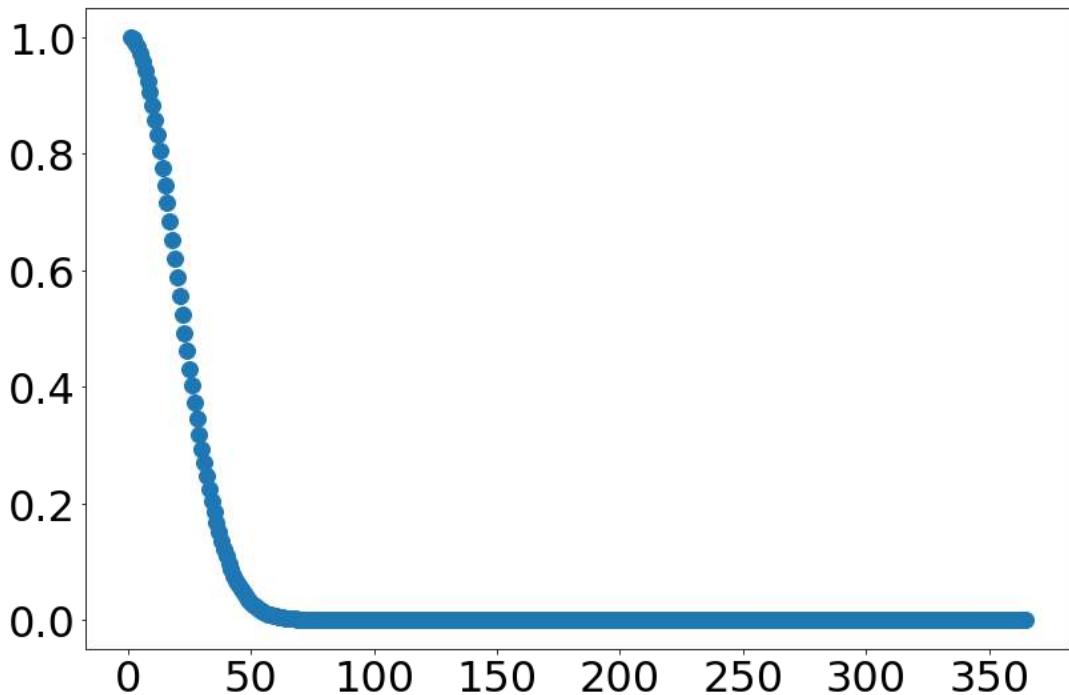


Probability That no Two People Have the Same Birthday



Probability That no Two People Have the Same Birthday

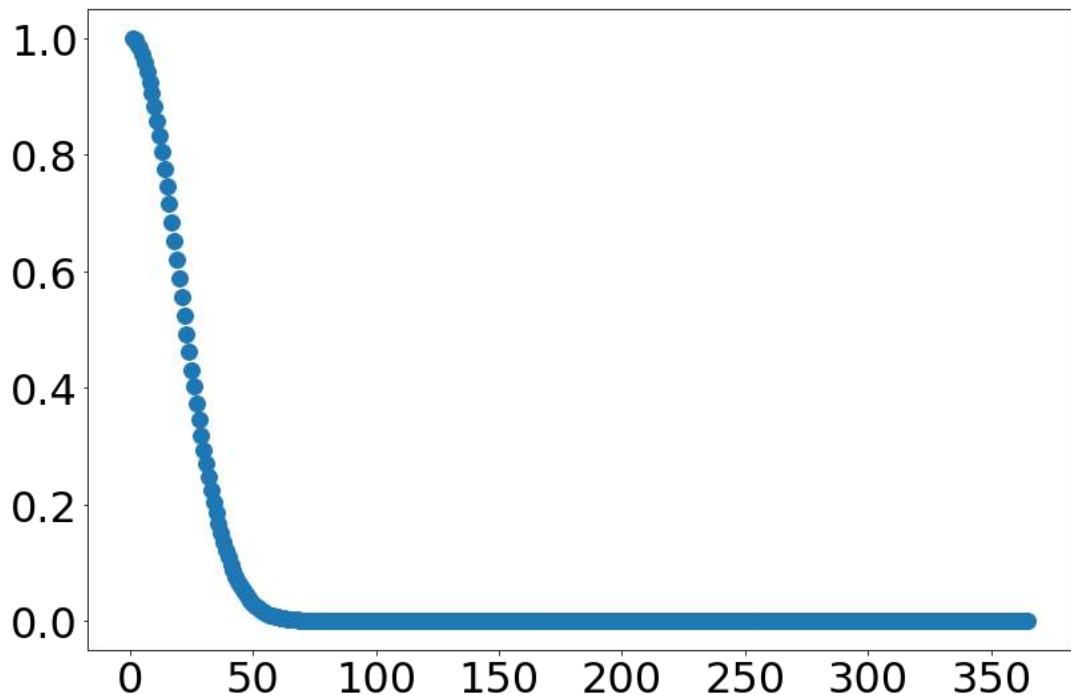
1 person: 1



Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

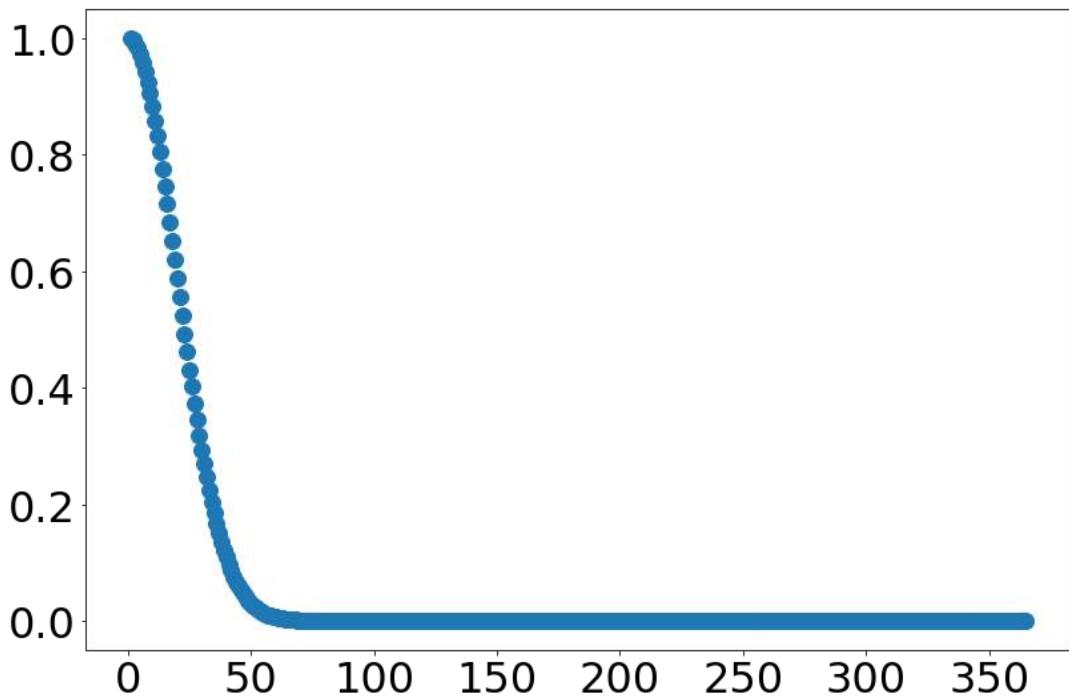


Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992



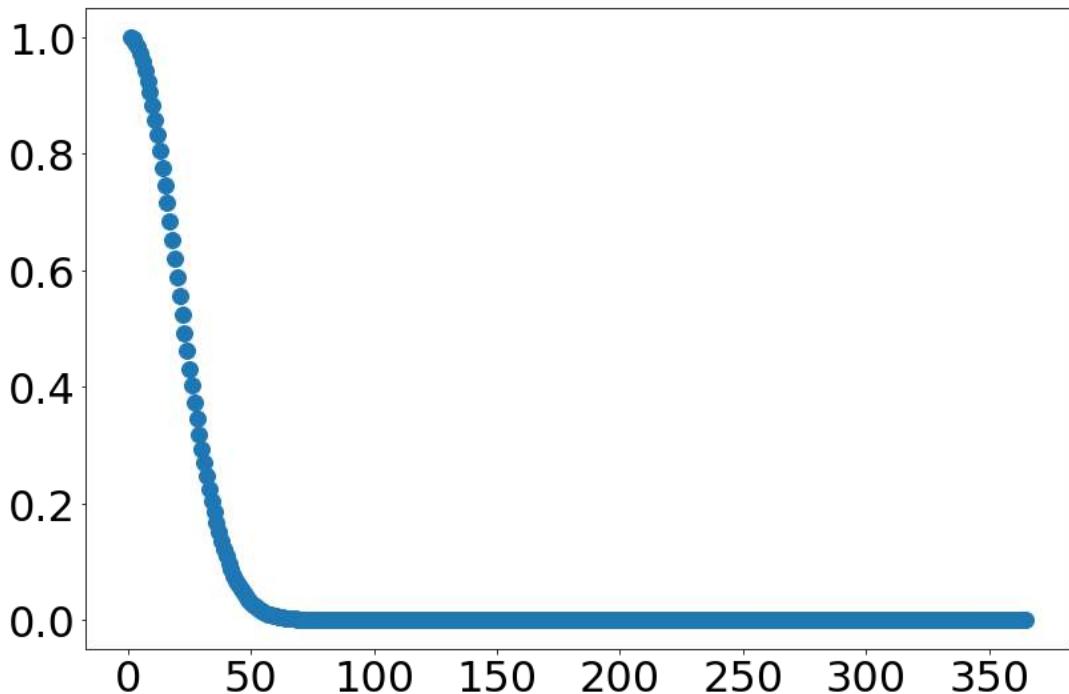
Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992

4 people: 0.984



Probability That no Two People Have the Same Birthday

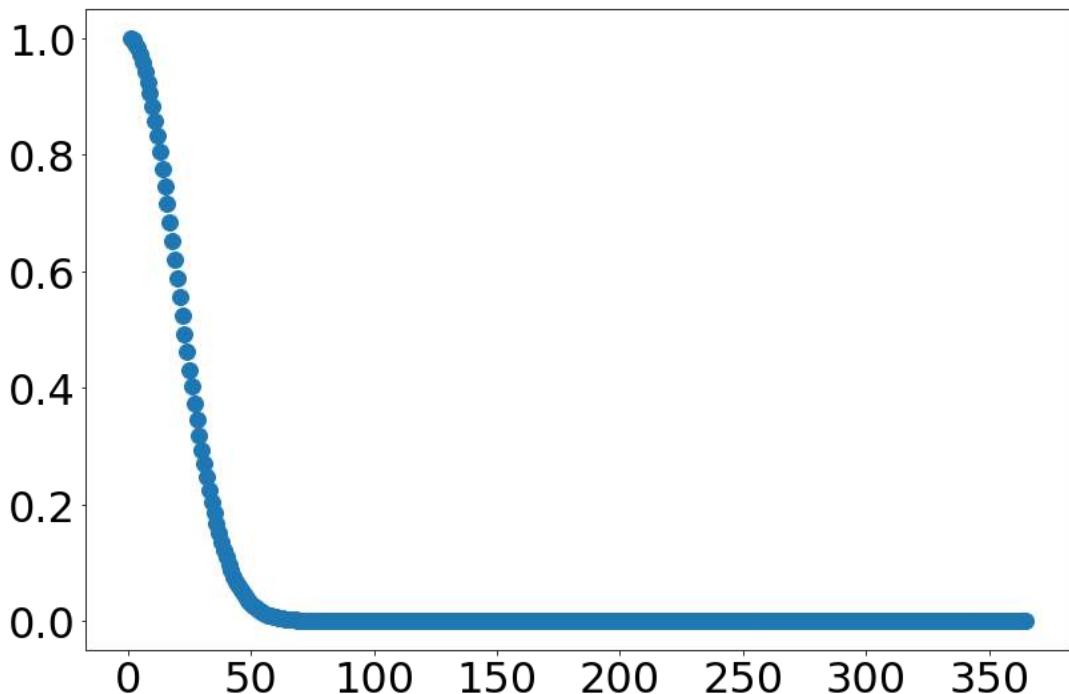
1 person: 1

2 people: 0.997

3 people: 0.992

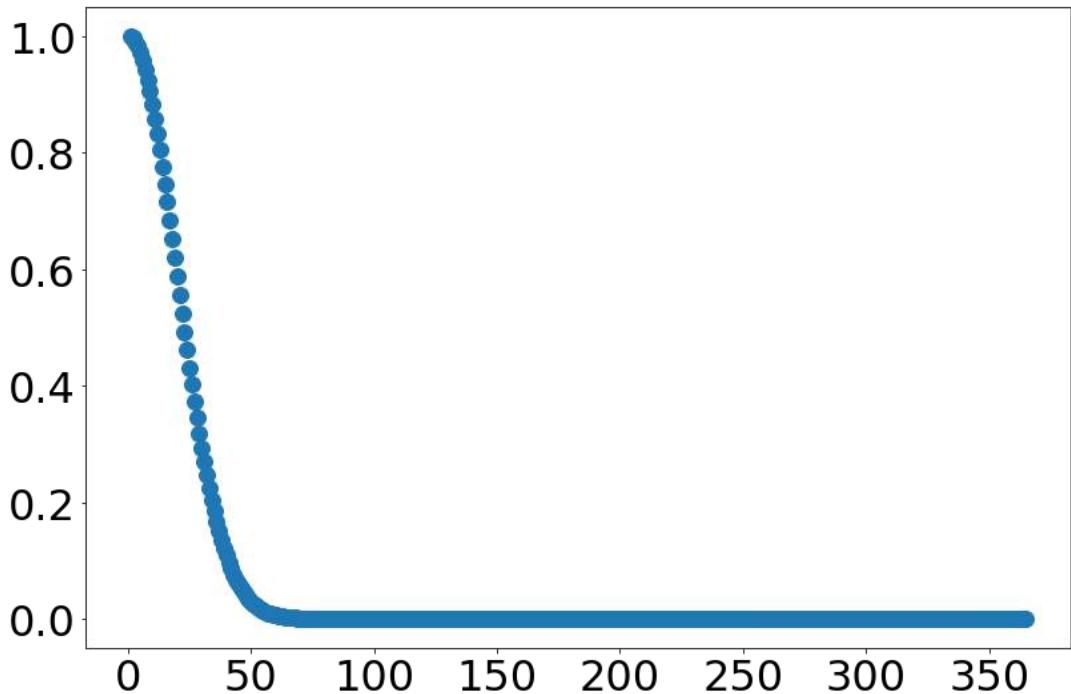
4 people: 0.984

5 people: 0.973



Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883



Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

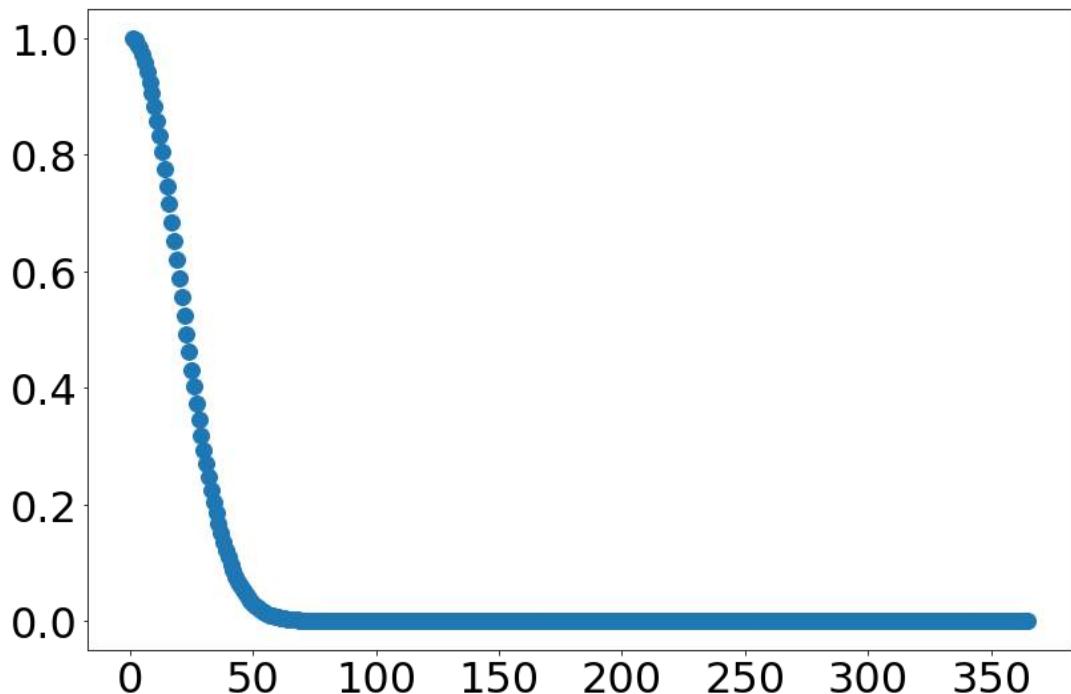
3 people: 0.992

4 people: 0.984

5 people: 0.973

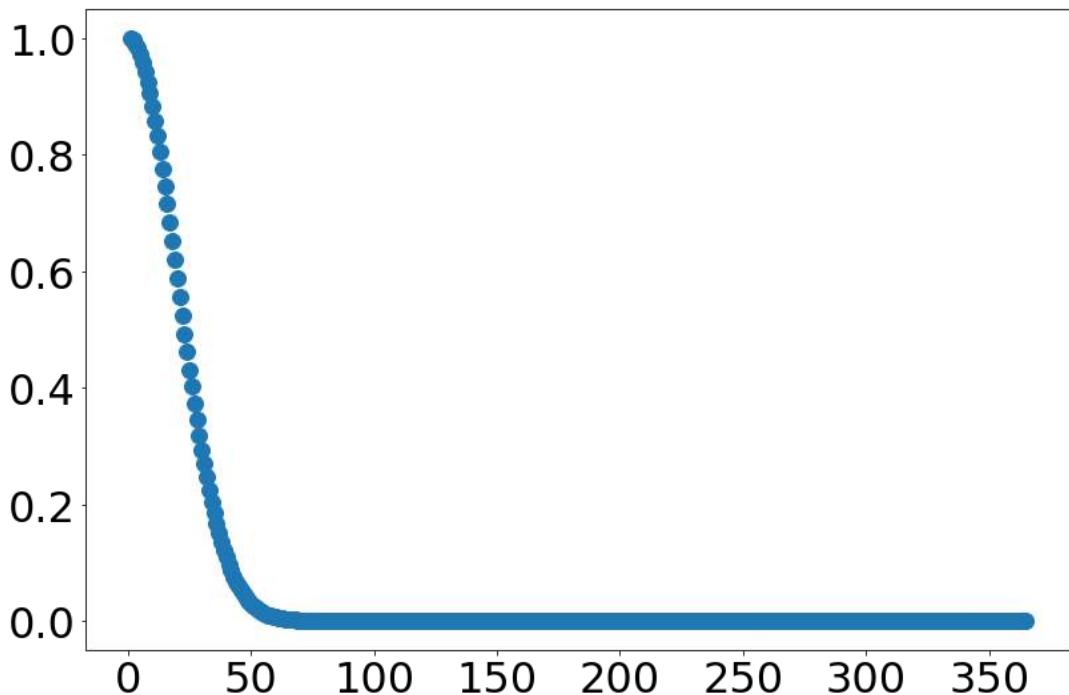
10 people: 0.883

20 people: 0.589



Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493



Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992

4 people: 0.984

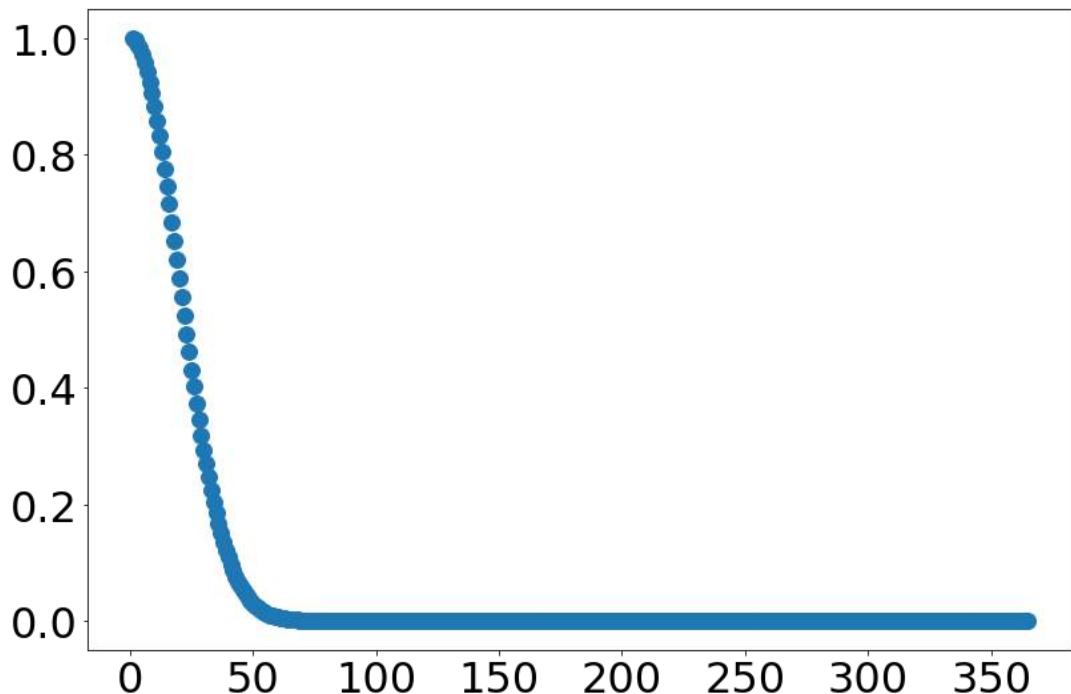
5 people: 0.973

10 people: 0.883

20 people: 0.589

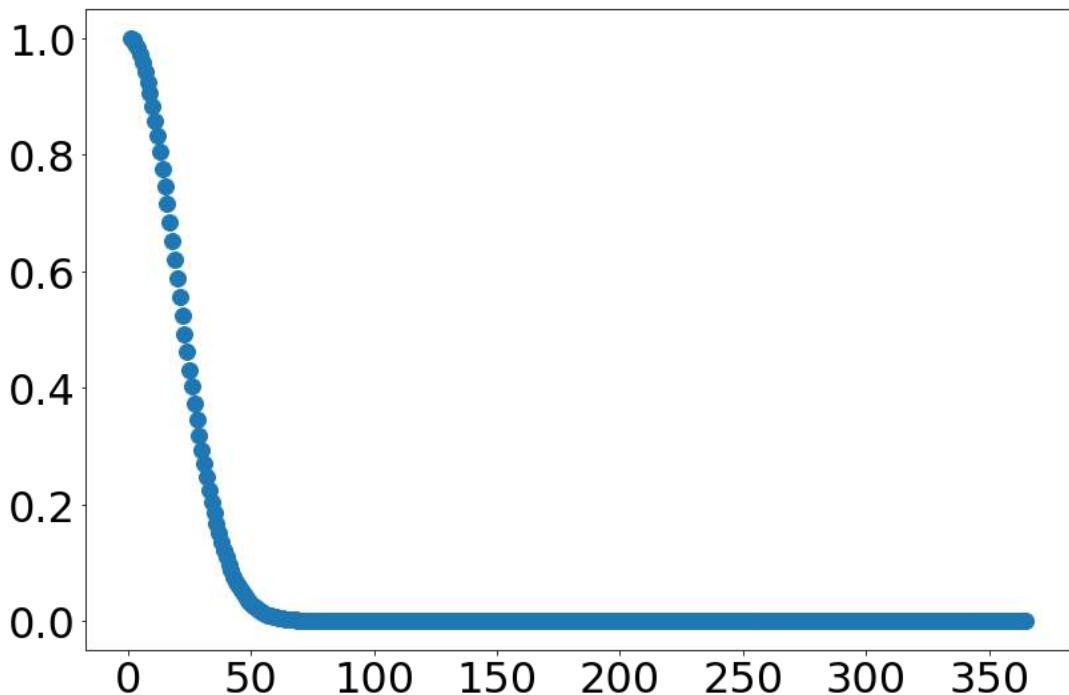
23 people: 0.493

30 people: 0.294



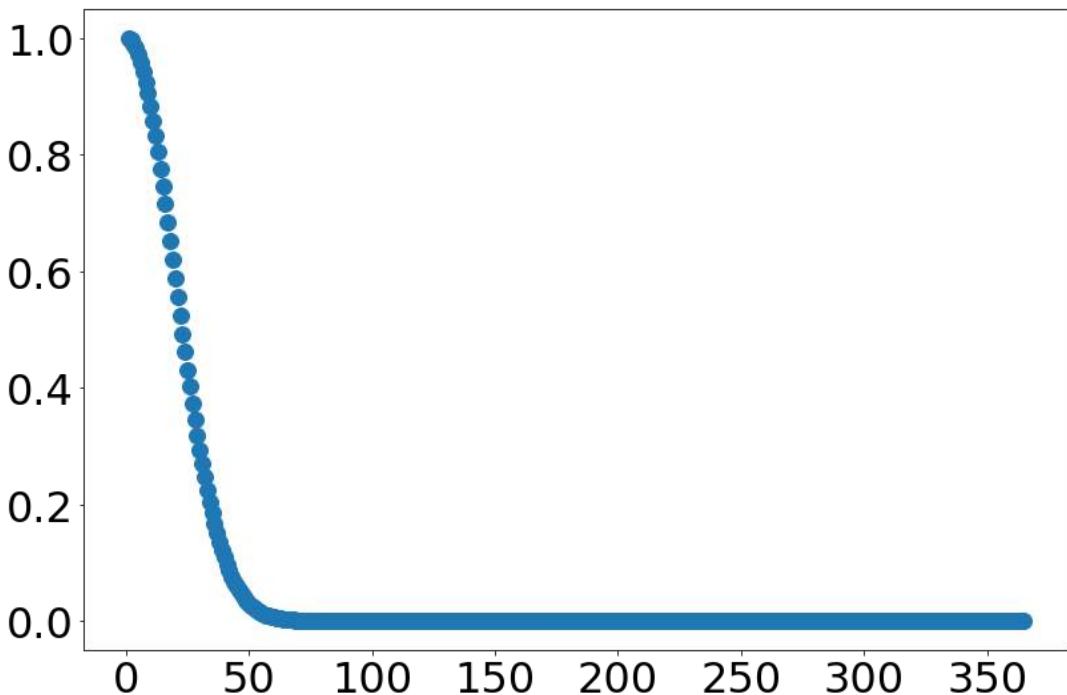
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030



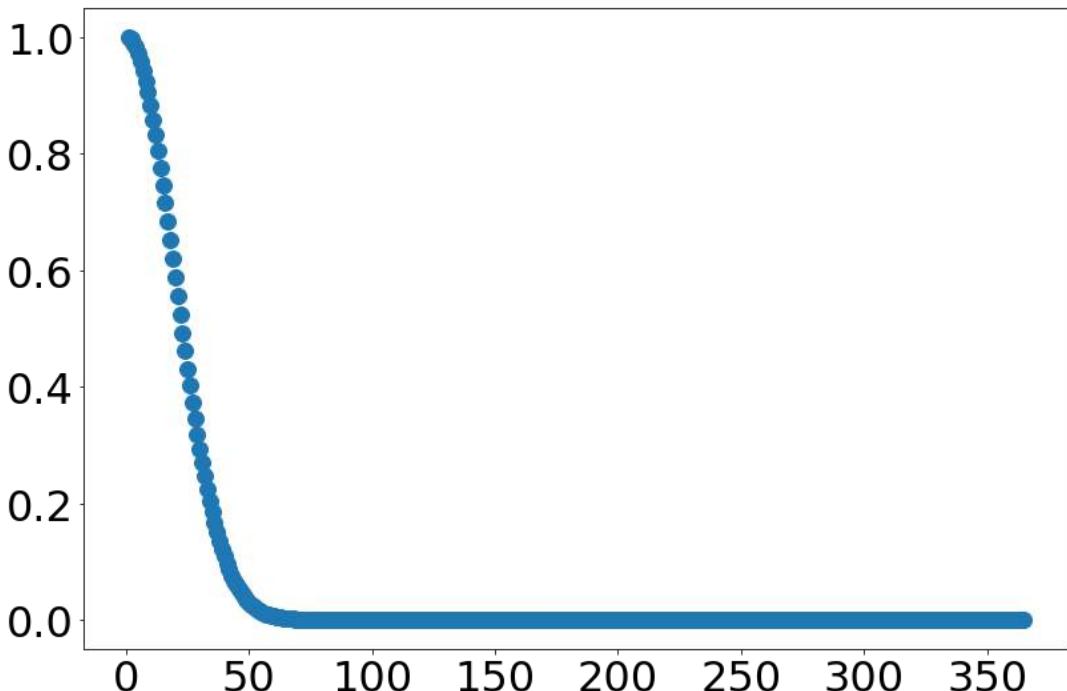
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003



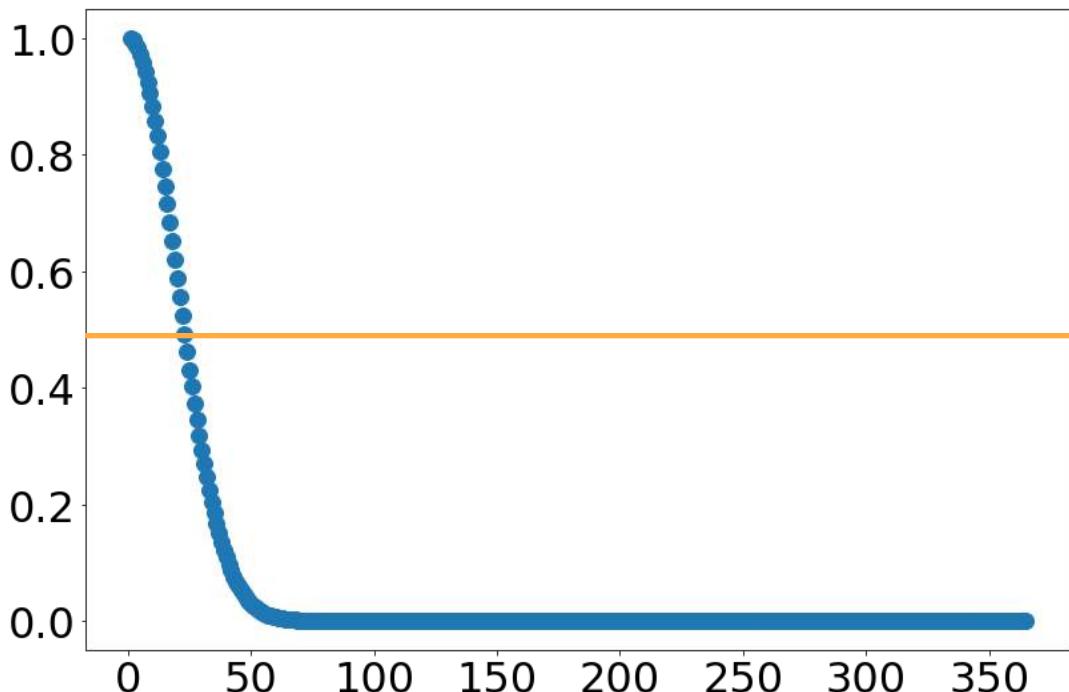
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



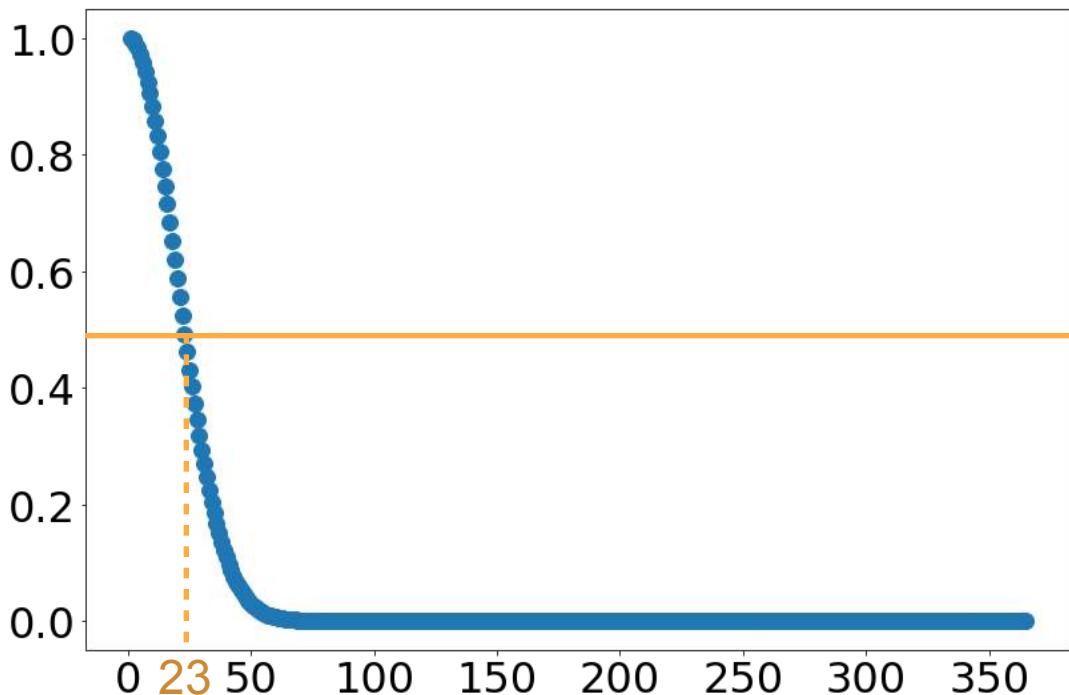
Probability That no Two People Have the Same Birthday

1 person: 1
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3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



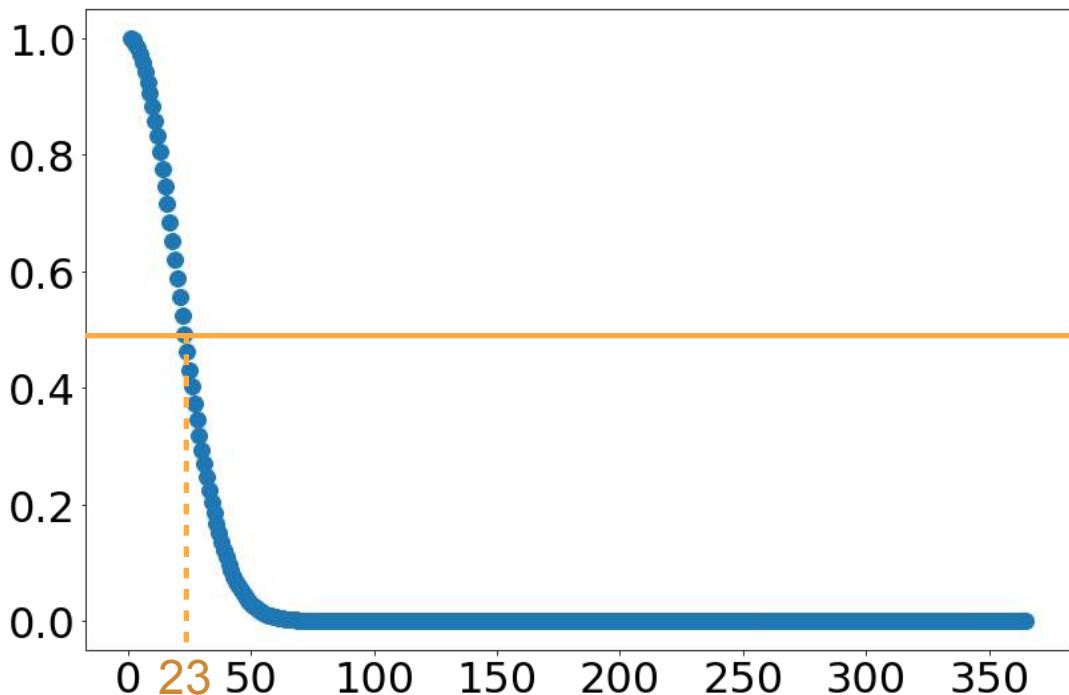
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0





DeepLearning.AI

Introduction to probability

Conditional probability

Conditional Probability: Coin Example 1



50% 50%

Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

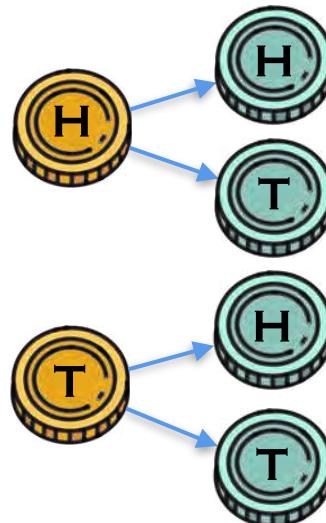
Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st 2nd



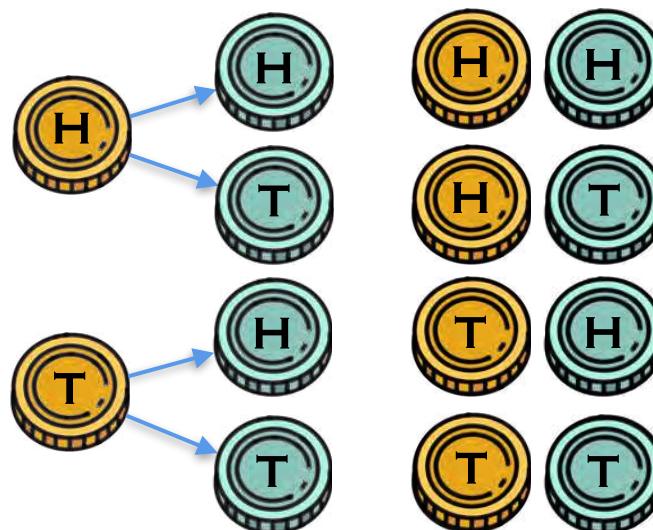
Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st 2nd



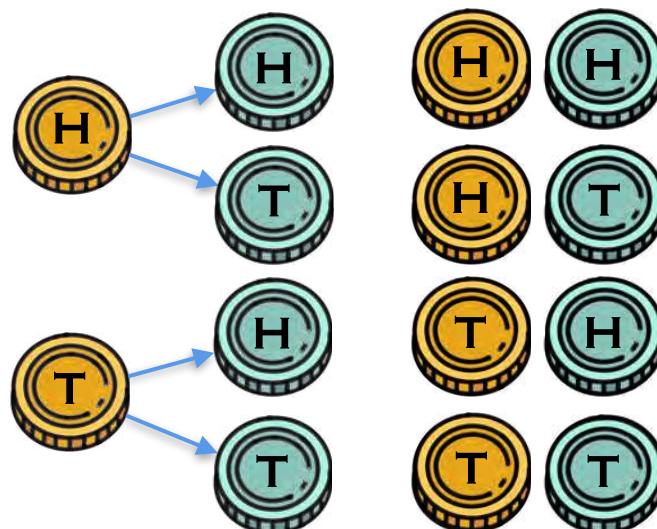
Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st 2nd



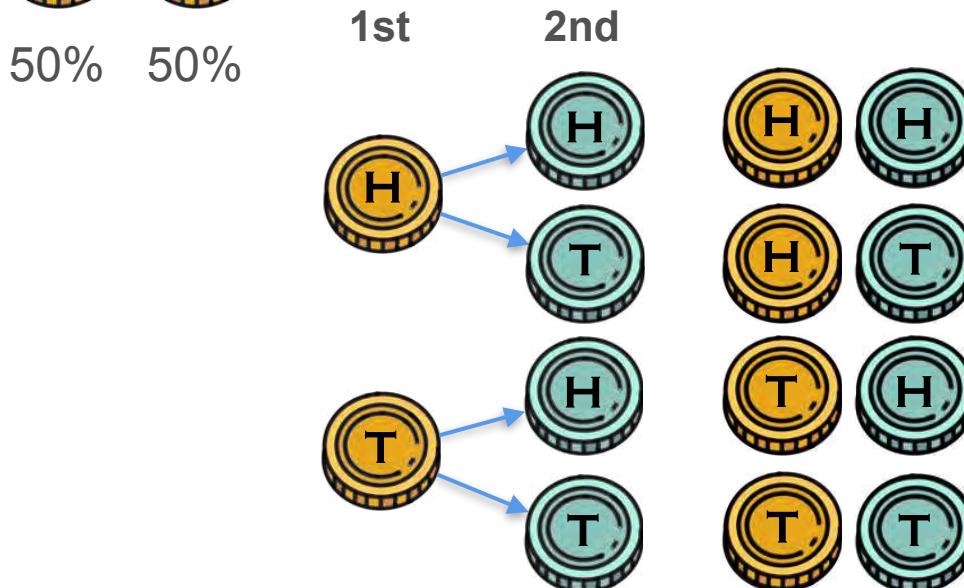
$$P(HH) = \underline{\hspace{2cm}}$$



Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?



$$P(HH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The numerator shows two coins both showing heads (H). The denominator shows all four possible outcomes: HH, HT, TH, TT.

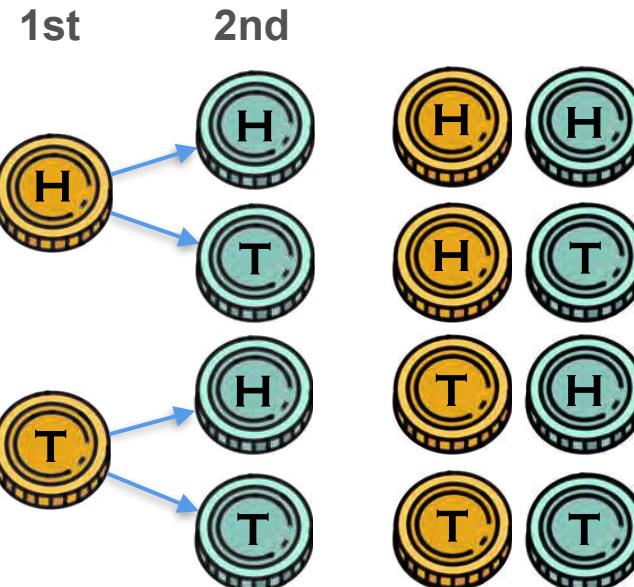
Two yellow coins, both with 'H' in blue, are shown side-by-side. Below them is a horizontal line. To the right of the line is a fraction bar. Below the fraction bar is a 2x2 grid of four coins: top-left yellow with 'H', top-right teal with 'H', bottom-left yellow with 'T', and bottom-right teal with 'T'.
Below the fraction bar is a 2x2 grid of four coins: top-left yellow with 'T', top-right teal with 'T', bottom-left yellow with 'T', and bottom-right teal with 'T'.

Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$

A vertical stack of four pairs of coins, each pair showing heads (H) and tails (T). The pairs are: (H, H), (H, T), (T, H), and (T, T). This represents the sample space of all possible outcomes of two coin flips.

Conditional Probability: Coin Example 1

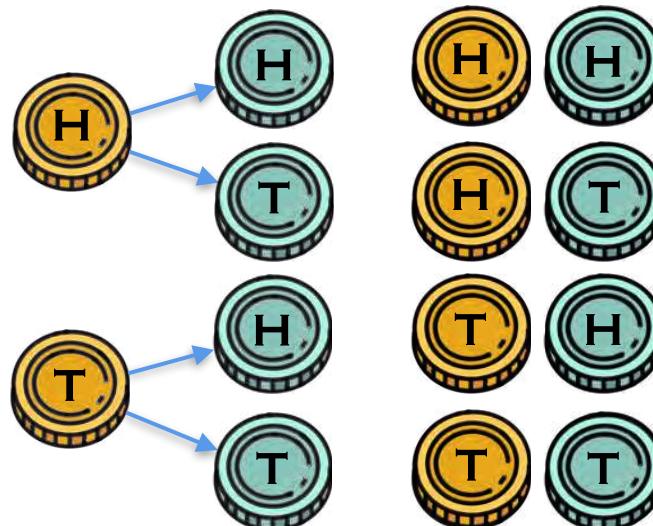


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 1

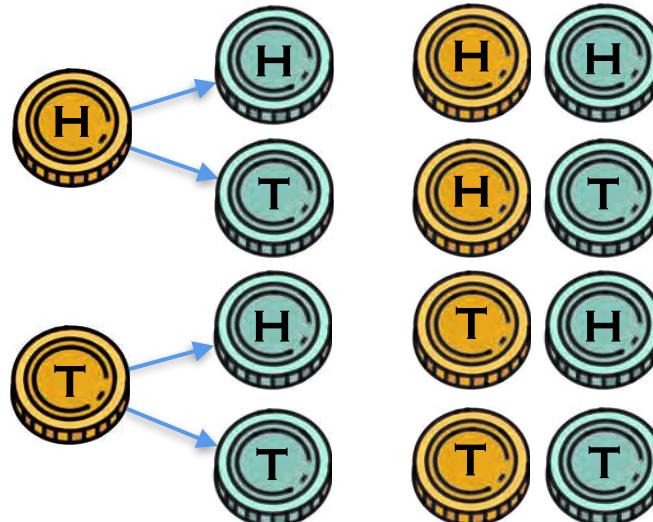


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 1

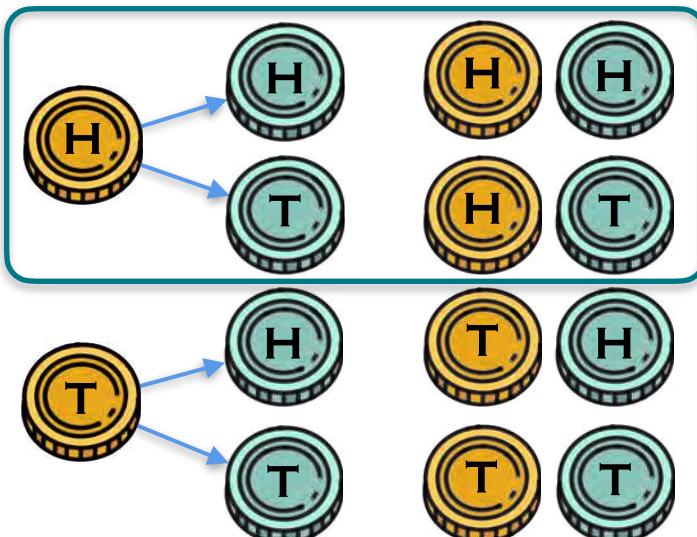


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



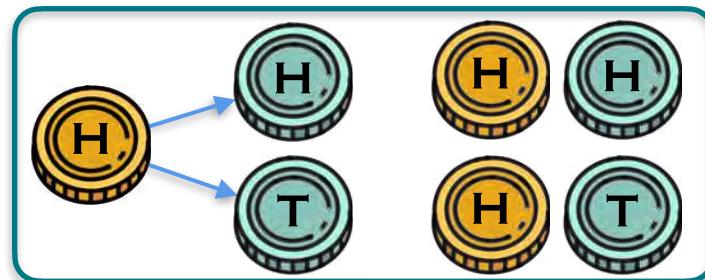
Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4}$$



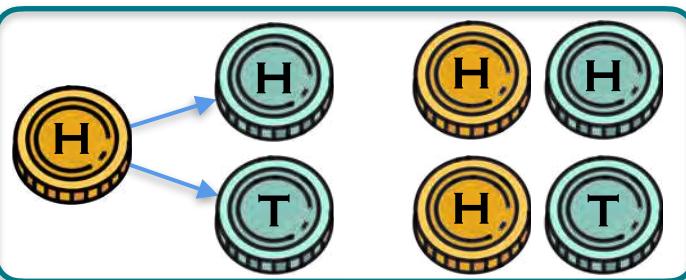
Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$
A diagram illustrating conditional probability. It shows two rows of four coins each. The top row represents the actual outcomes of two coin flips: the first coin is heads (H) and the second is heads (H). The bottom row represents the possible outcomes for the second flip, given that the first flip was heads (H). The second coin in the bottom row is tails (T), while the other three coins are heads (H). This illustrates that there is only one favorable outcome (HH) out of four possible outcomes (HH, HT, TH, TT) when the first coin is heads.

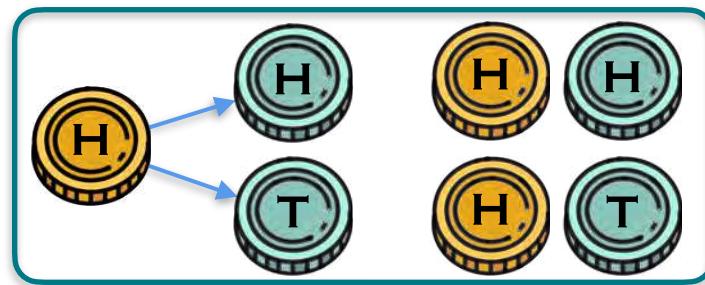
Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH \mid \text{1st is } H) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}} = \frac{1}{2}$$

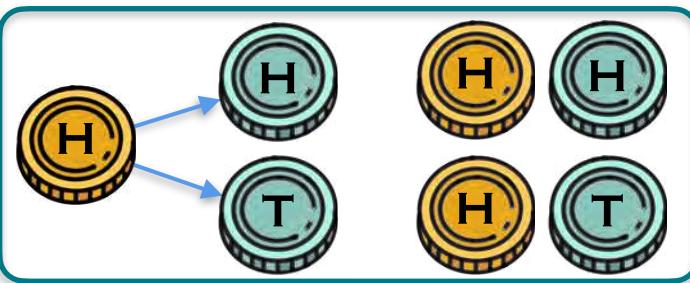


Conditional Probability: Coin Example 1

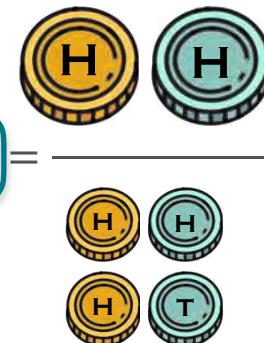


What is the probability of landing on heads twice?

1st 2nd **GIVEN that the first one is heads**



$$P(HH \mid \text{1st is } H) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}} = \frac{1}{2}$$



Conditional Probability: Coin Example 1

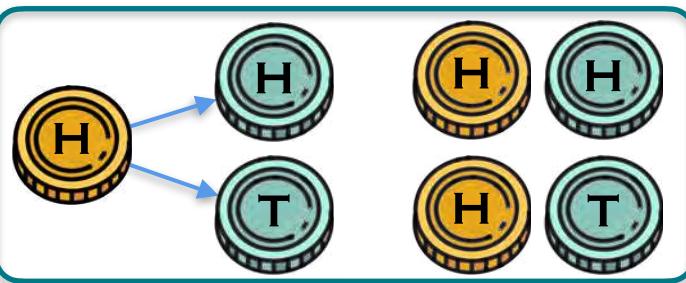


50% 50%

What is the probability of landing on heads twice?

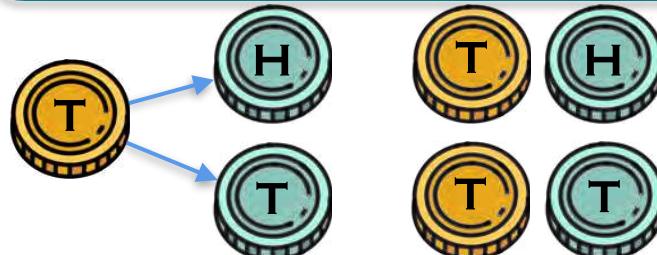
1st 2nd

GIVEN that the first one is heads



$$P(HH | \text{1st is } H) =$$

$$= \frac{1}{2}$$



Conditional Probability: Coin Example 1

What is the probability of landing on heads twice?

GIVEN that the first one is heads

$$P(HH | \text{1st is } H)$$

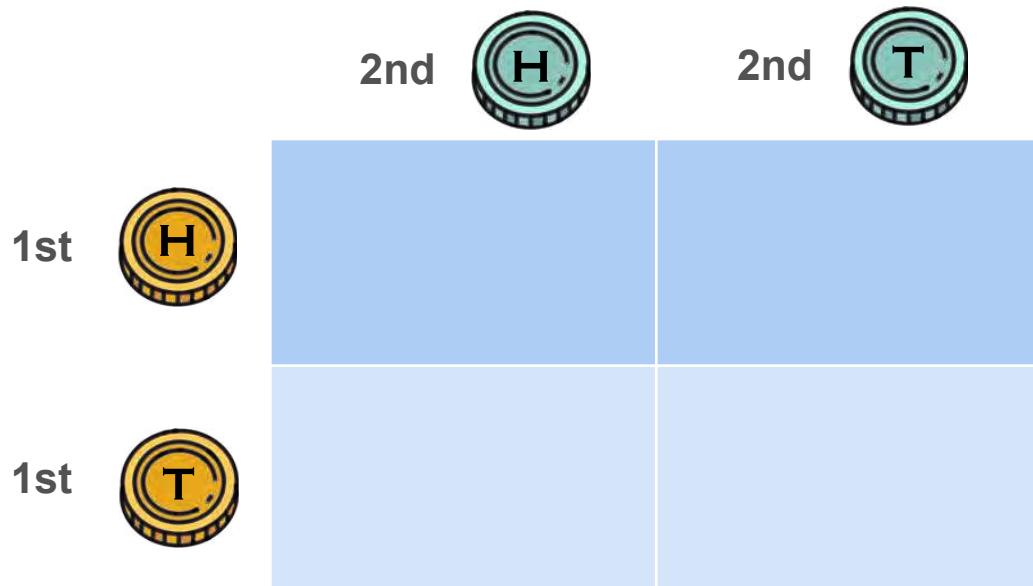


Conditional Probability: Coin Example 1

Conditional Probability: Coin Example 1



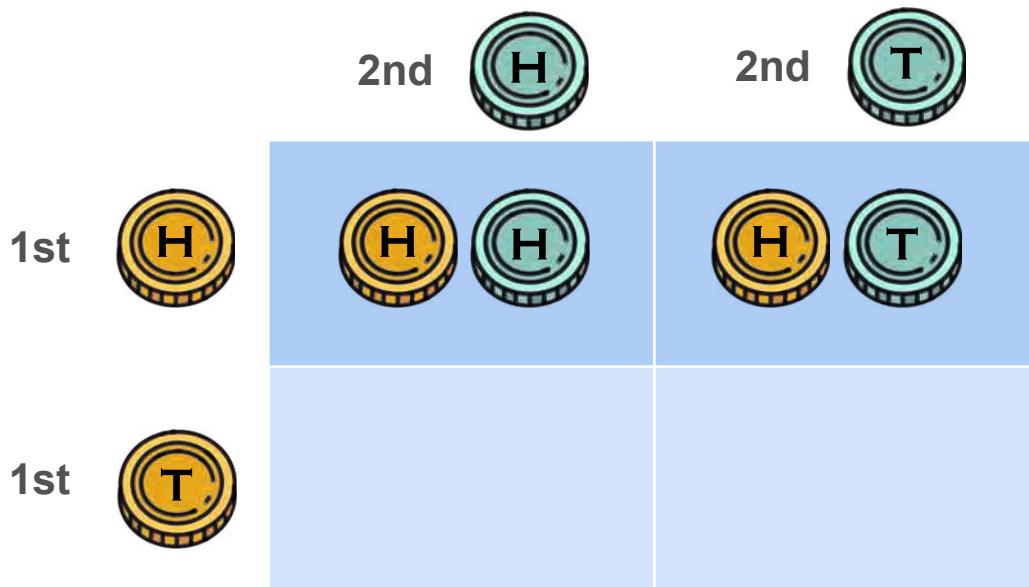
Conditional Probability: Coin Example 1



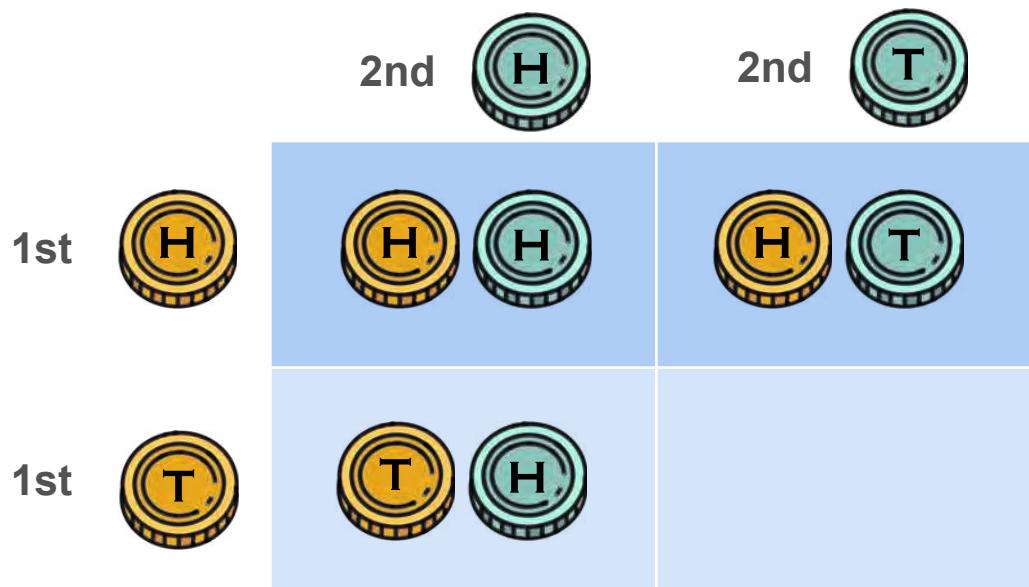
Conditional Probability: Coin Example 1



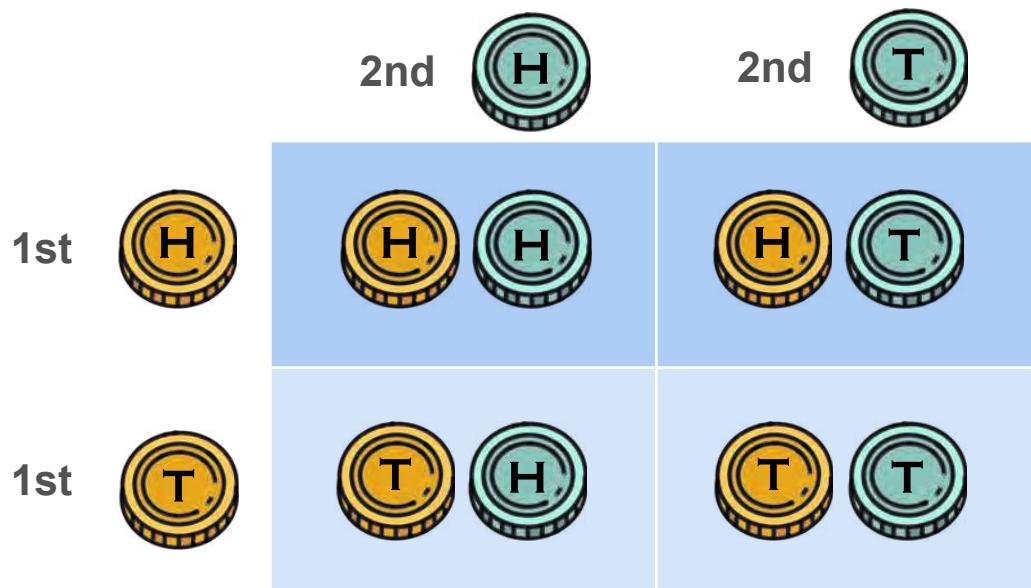
Conditional Probability: Coin Example 1



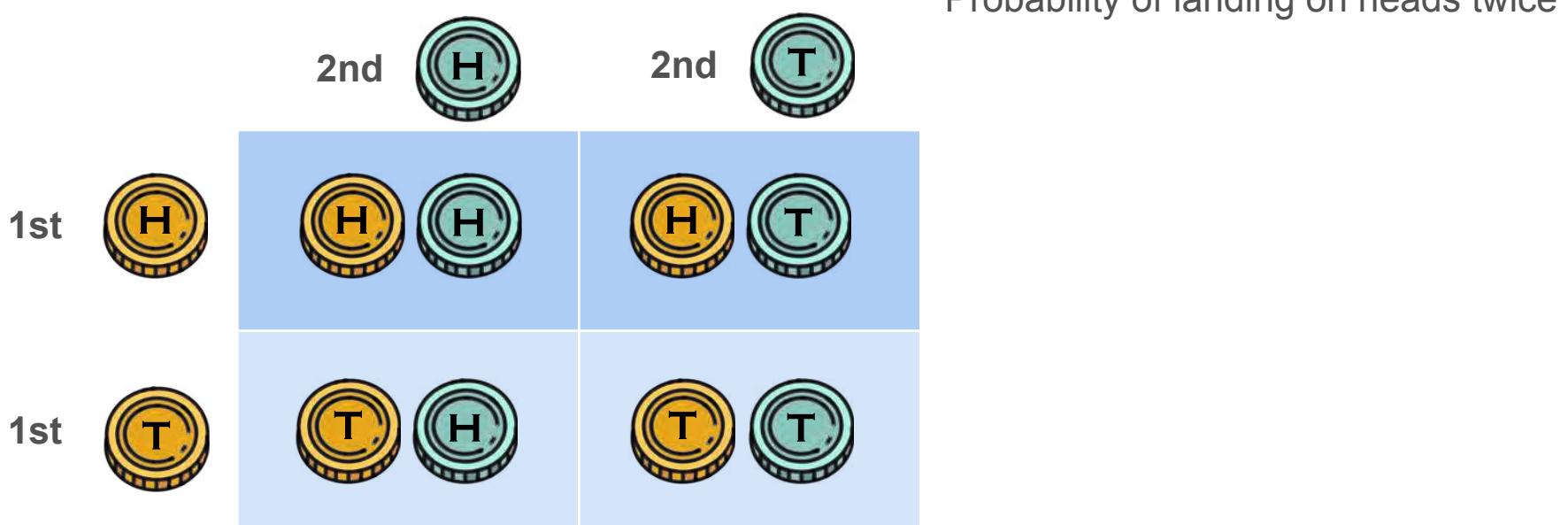
Conditional Probability: Coin Example 1



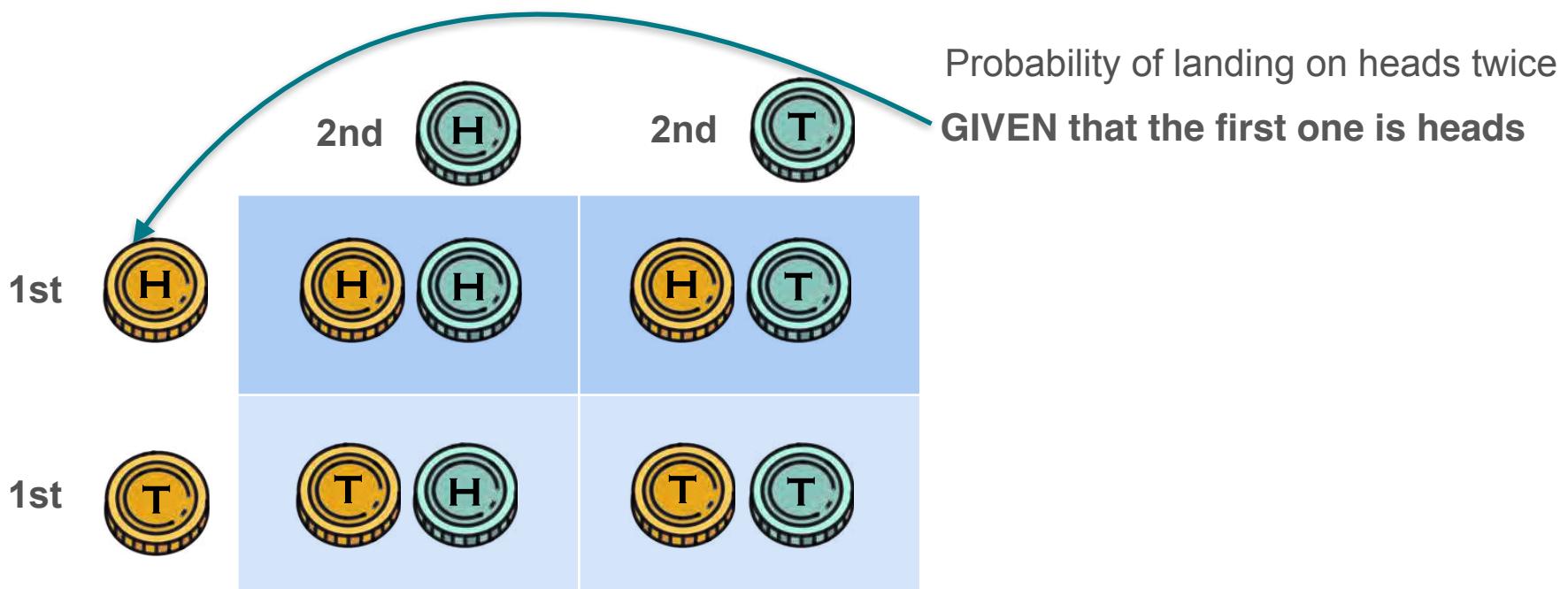
Conditional Probability: Coin Example 1



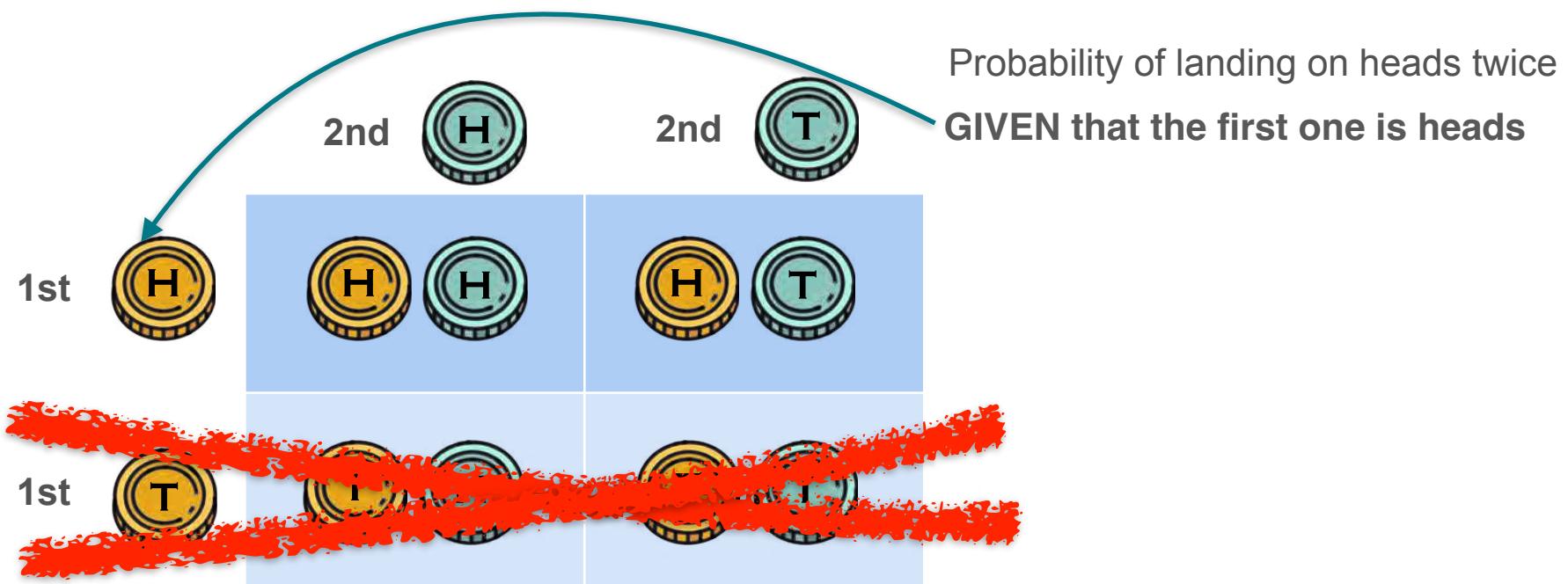
Conditional Probability: Coin Example 1



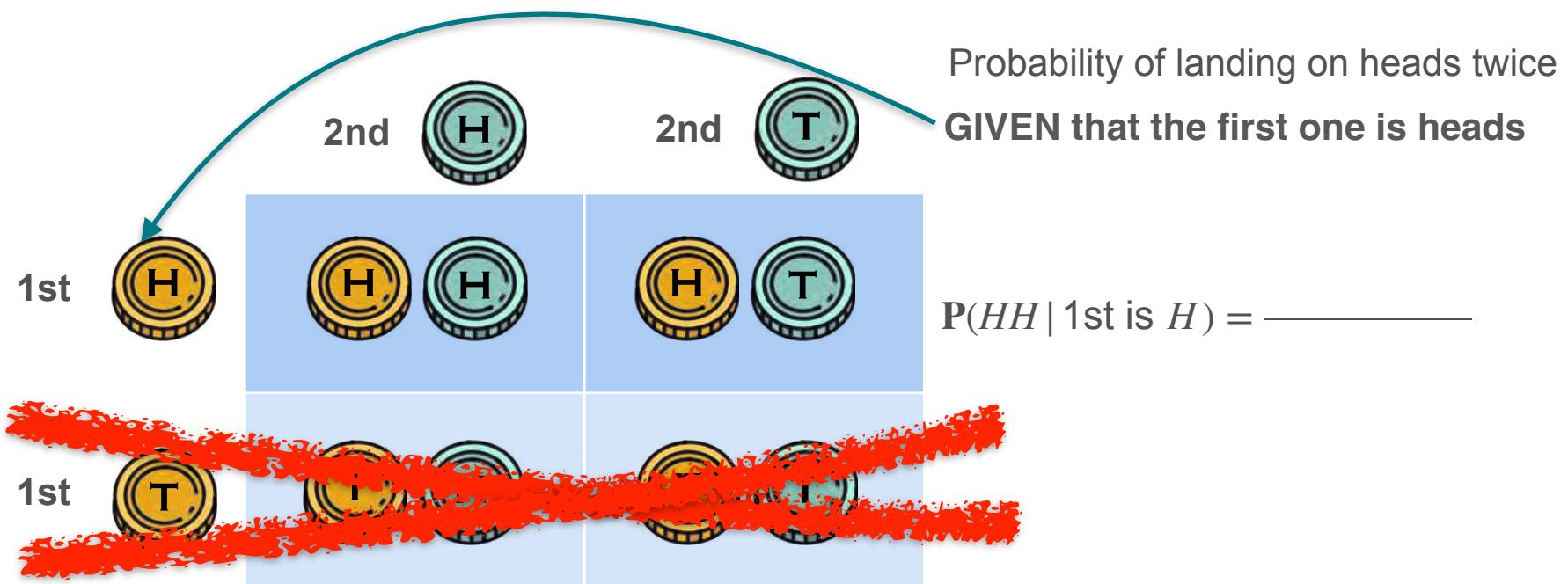
Conditional Probability: Coin Example 1



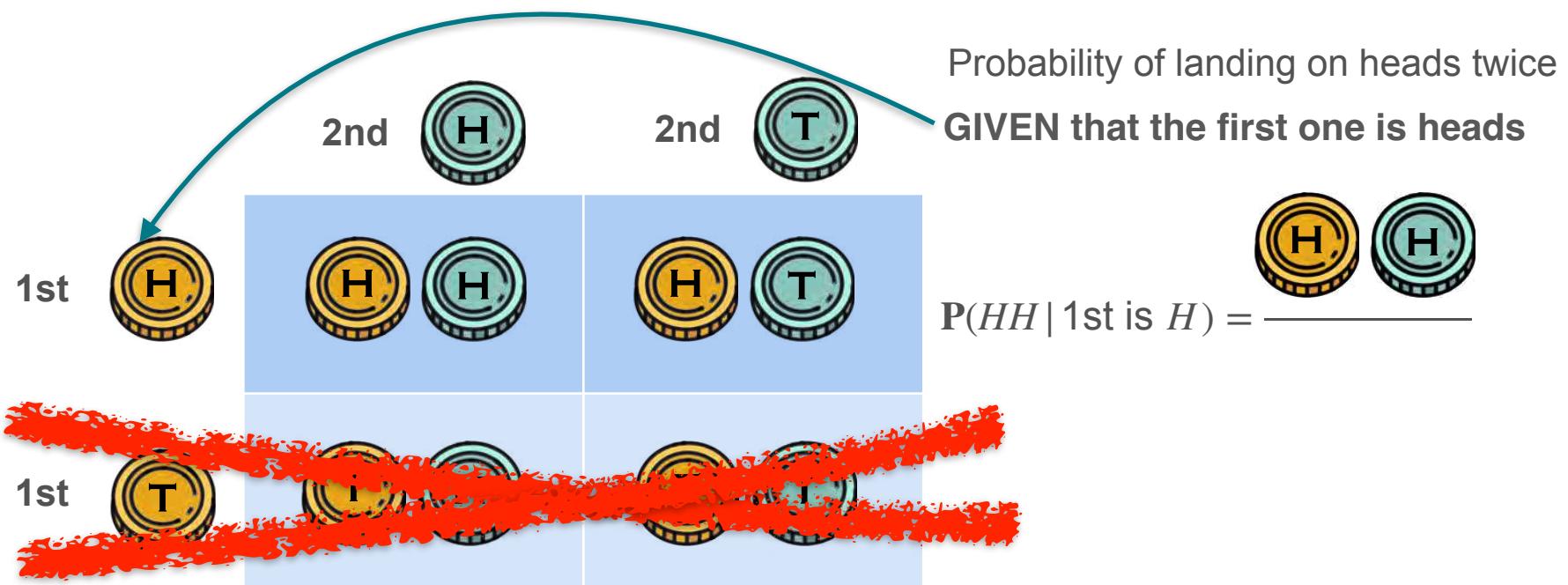
Conditional Probability: Coin Example 1



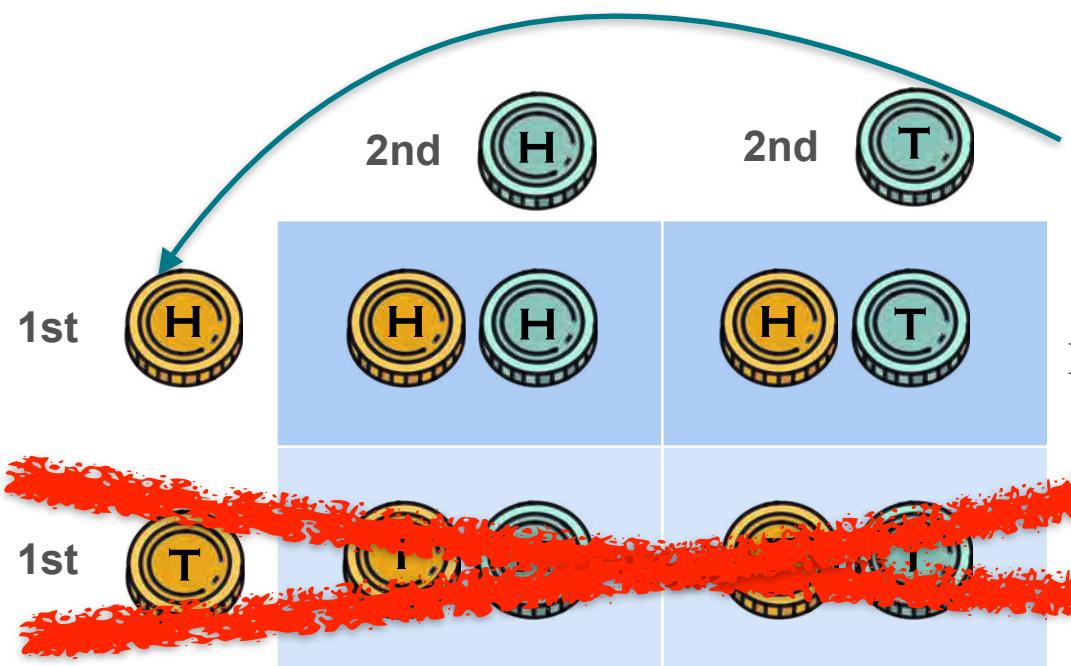
Conditional Probability: Coin Example 1



Conditional Probability: Coin Example 1



Conditional Probability: Coin Example 1



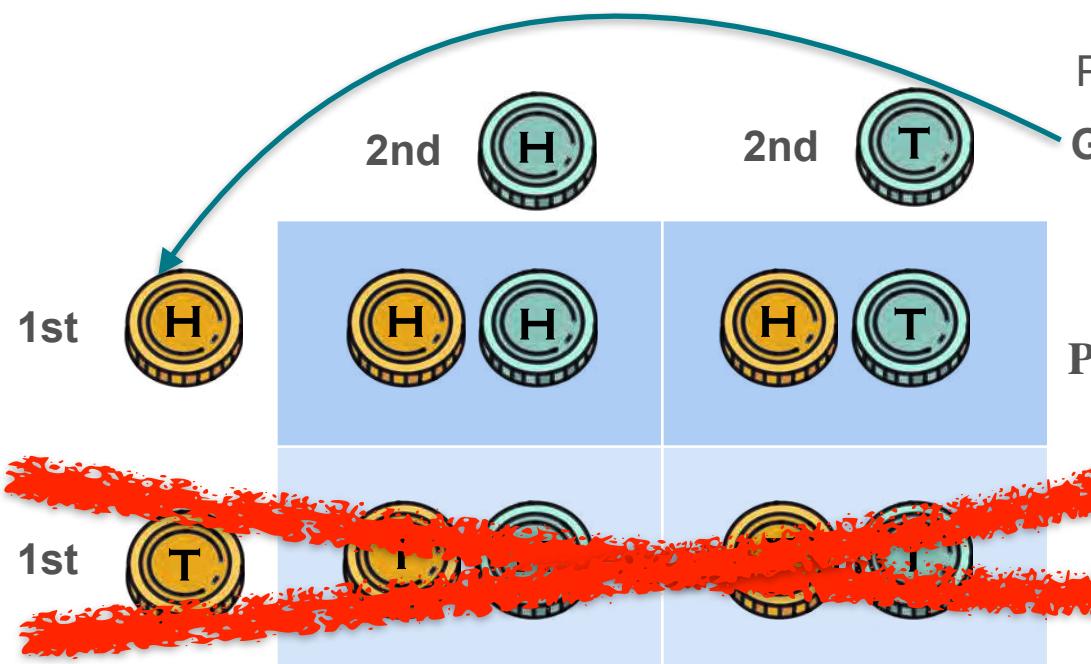
Probability of landing on heads twice
GIVEN that the first one is heads

$$P(HH \mid \text{1st is } H) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

The diagram shows two rows of coins. The top row has two heads (H). The bottom row has one head (H) and one tail (T). A red brush stroke highlights the top row.

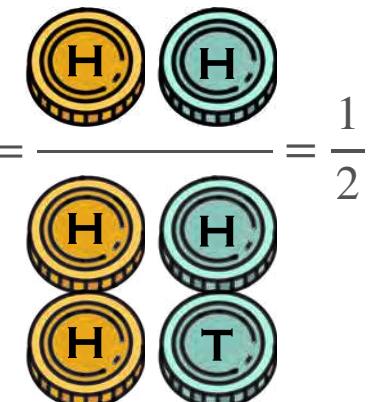
	2nd	2nd
1st	H	H
1st	H	T

Conditional Probability: Coin Example 1



Probability of landing on heads twice
GIVEN that the first one is heads

$$P(HH \mid \text{1st is } H) = \frac{1}{2}$$



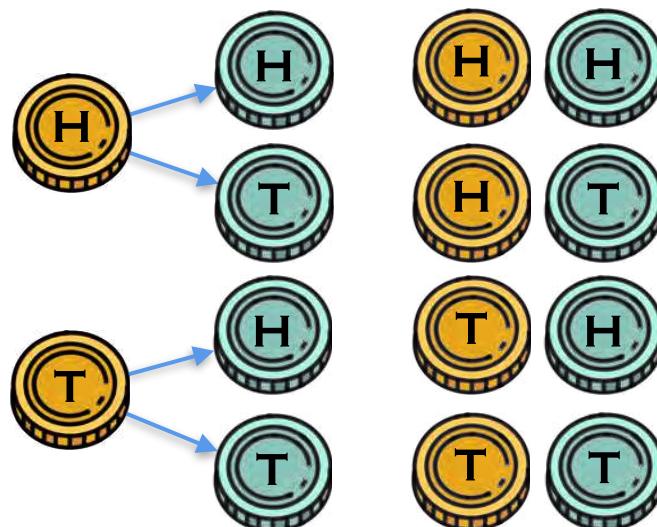
Conditional Probability: Coin Example 2



50% 50%

What is the probability of landing on heads twice?

1st 2nd



$$P(HH) = \frac{1}{4}$$

The equation shows the probability of getting heads on both the 1st and 2nd flip. The numerator is 1, represented by the two yellow coins labeled 'H'. The denominator is 4, represented by the four possible outcomes shown in the grid below the fraction line.

A vertical stack of four pairs of coins. The top pair is yellow with 'H' (1st) and light blue with 'H' (2nd). The middle pair is yellow with 'H' (1st) and light blue with 'T' (2nd). The third pair is yellow with 'T' (1st) and light blue with 'H' (2nd). The bottom pair is yellow with 'T' (1st) and light blue with 'T' (2nd).

Conditional Probability: Coin Example 2

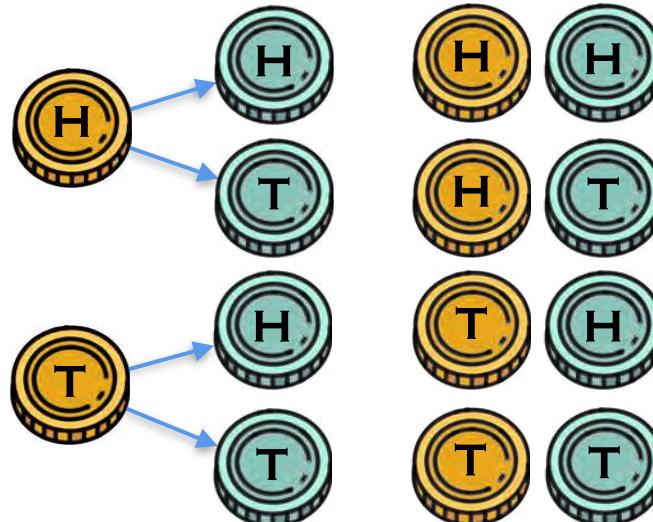


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

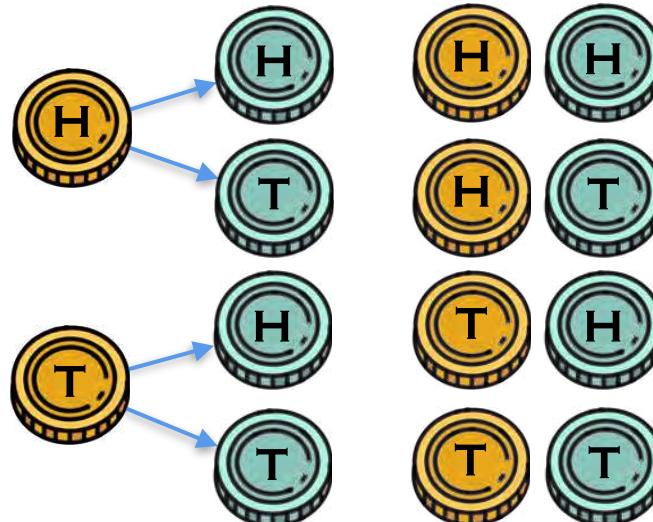


50% 50%

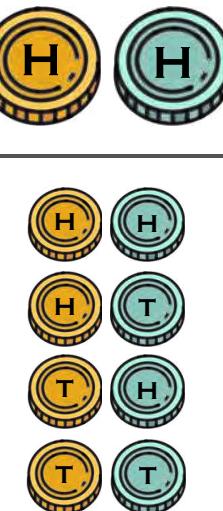
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

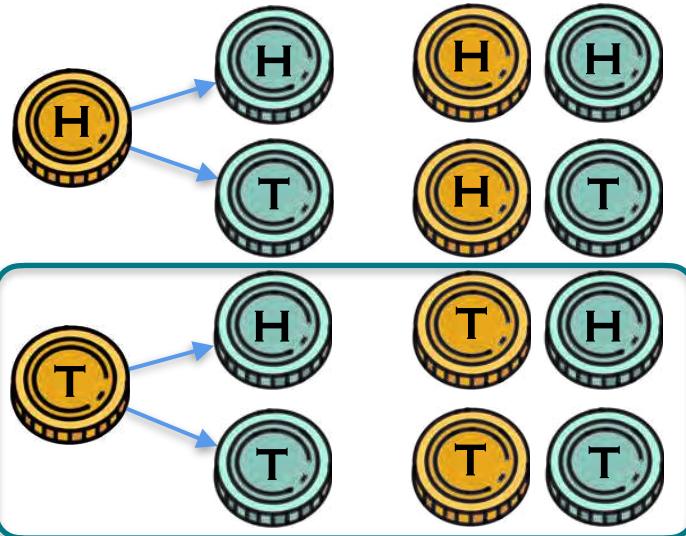


50% 50%

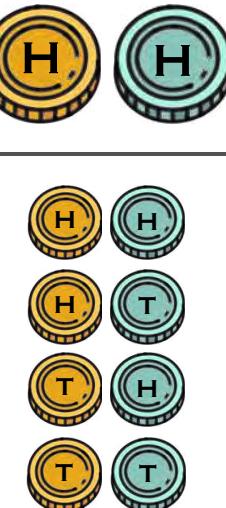
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

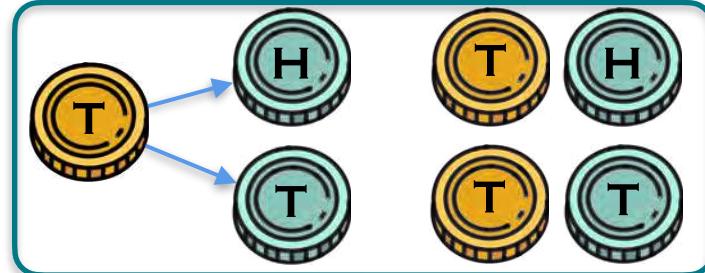


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{8} = \frac{1}{4}$$



Conditional Probability: Coin Example 2



50% 50%

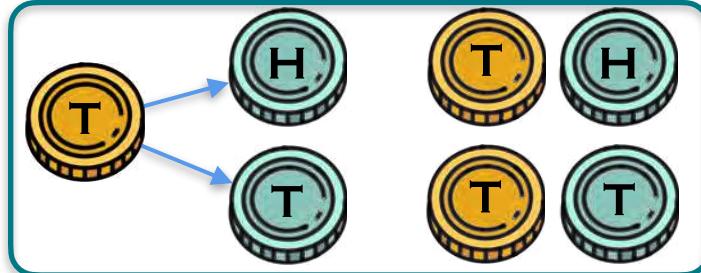
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2



What is the probability of landing on heads twice?

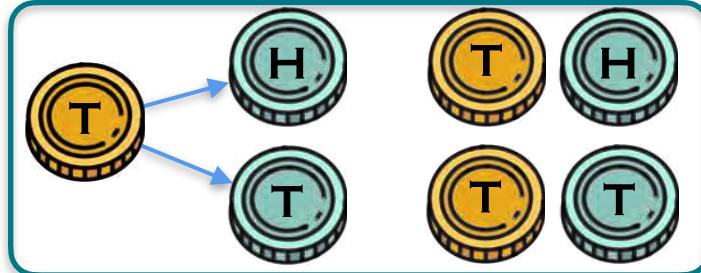
1st 2nd

GIVEN that the first one is tails

50% 50%



$$P(HH) = \text{_____} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

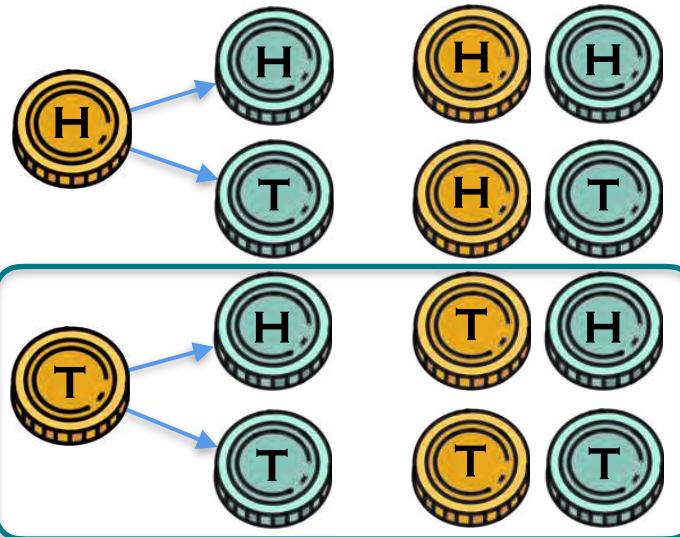


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH \mid \text{1st is } T) = \frac{0}{4} = 0$$

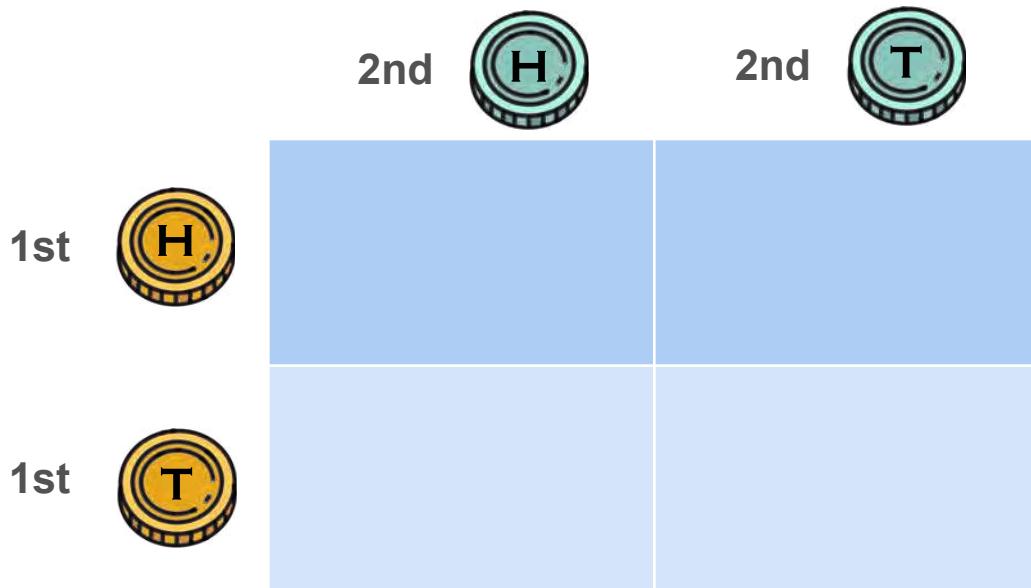


Conditional Probability: Coin Example 2

Conditional Probability: Coin Example 2



Conditional Probability: Coin Example 2



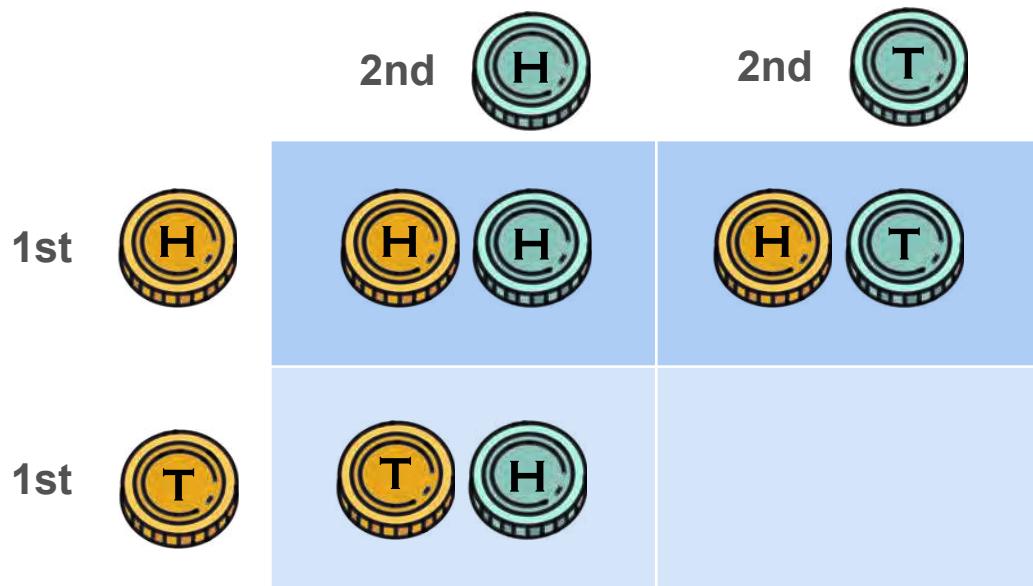
Conditional Probability: Coin Example 2



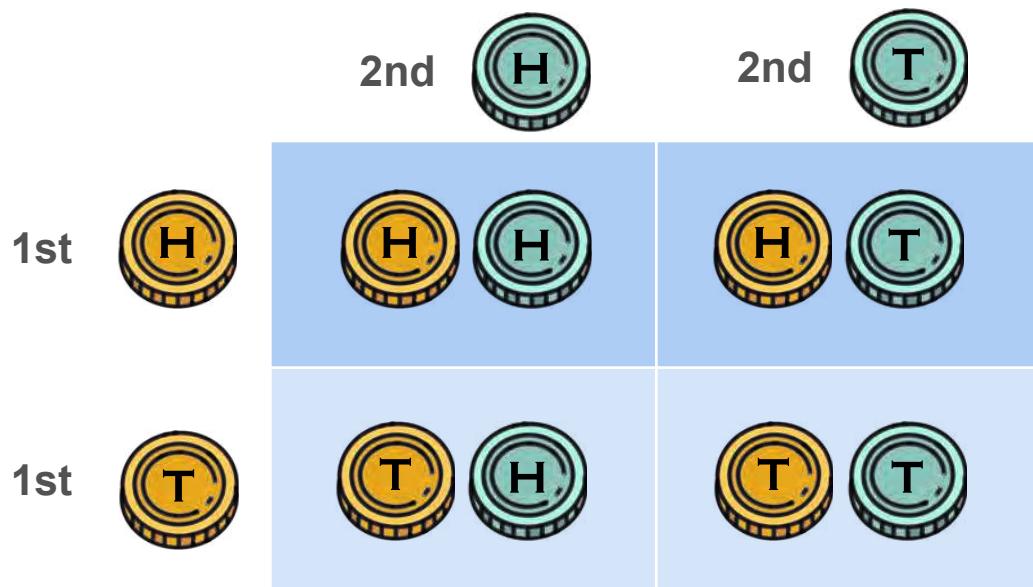
Conditional Probability: Coin Example 2



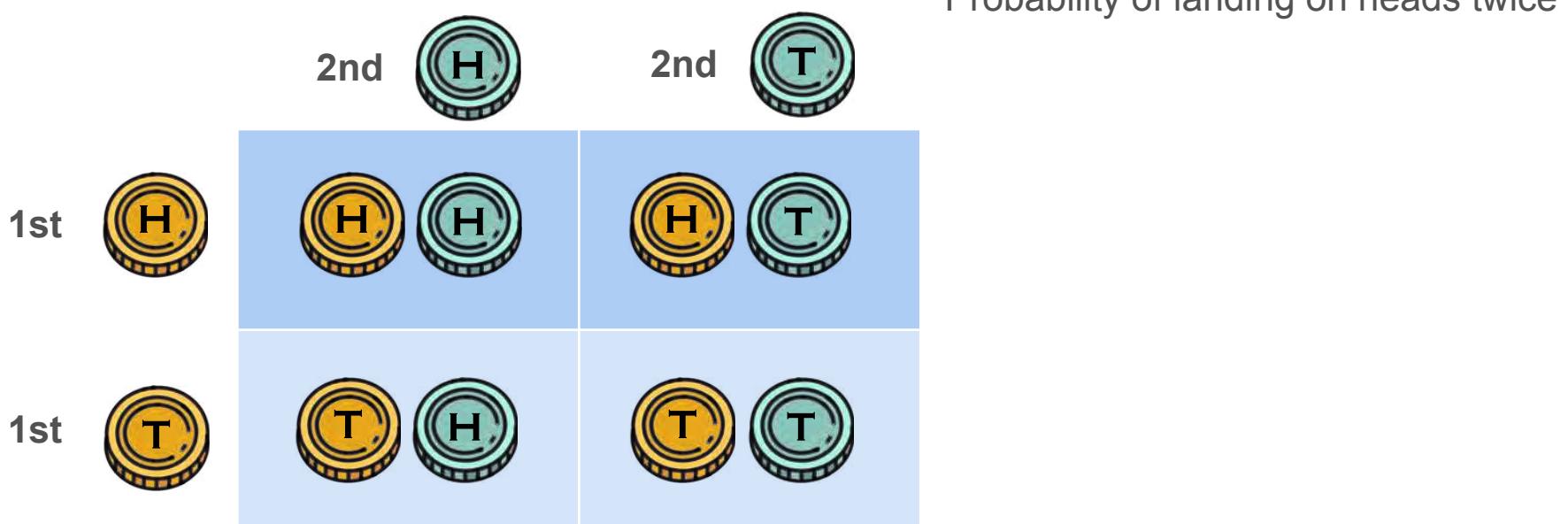
Conditional Probability: Coin Example 2



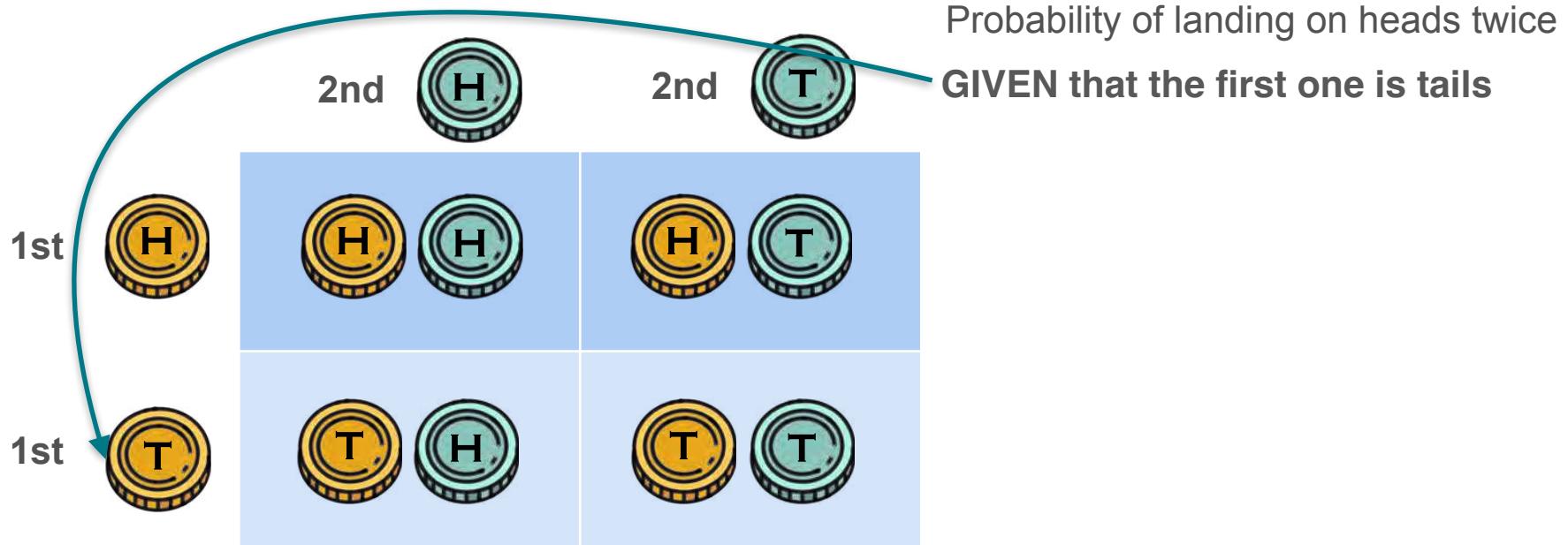
Conditional Probability: Coin Example 2



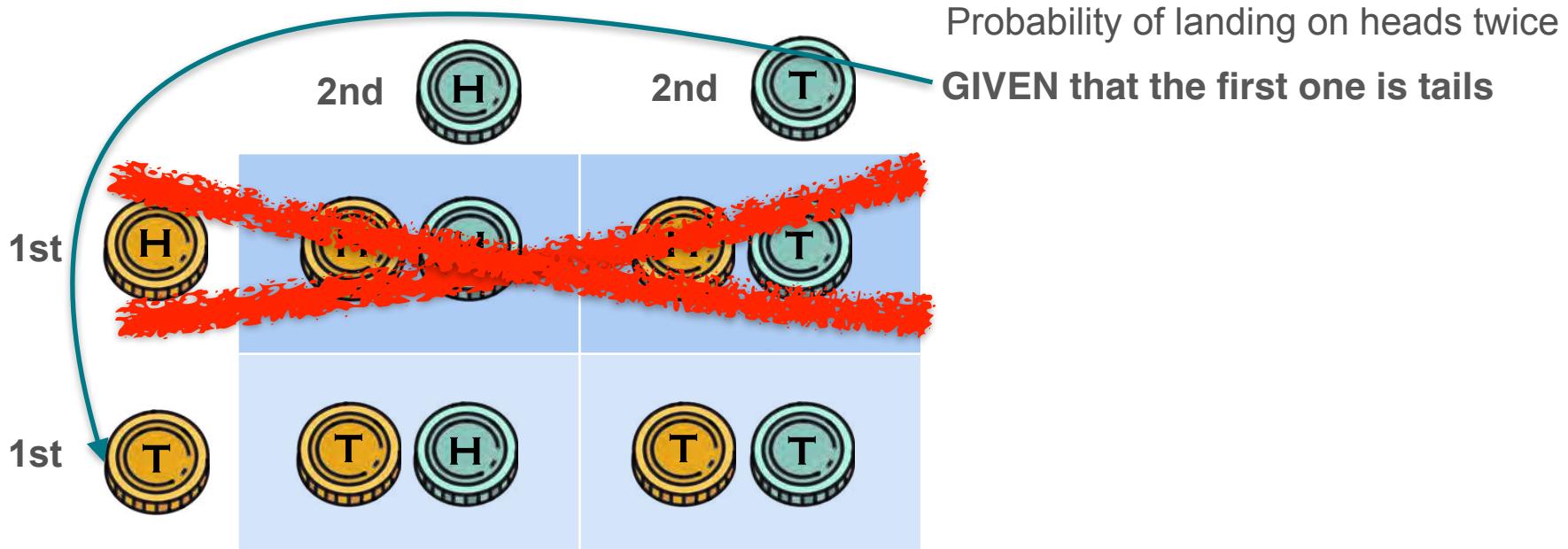
Conditional Probability: Coin Example 2



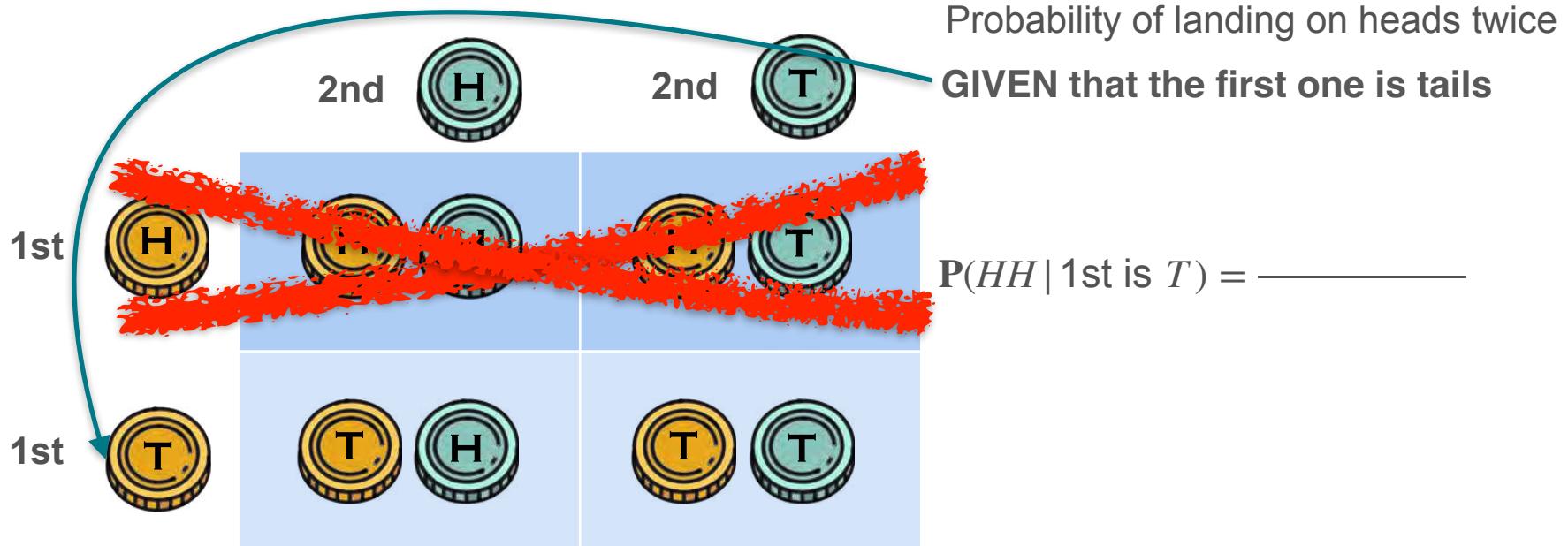
Conditional Probability: Coin Example 2



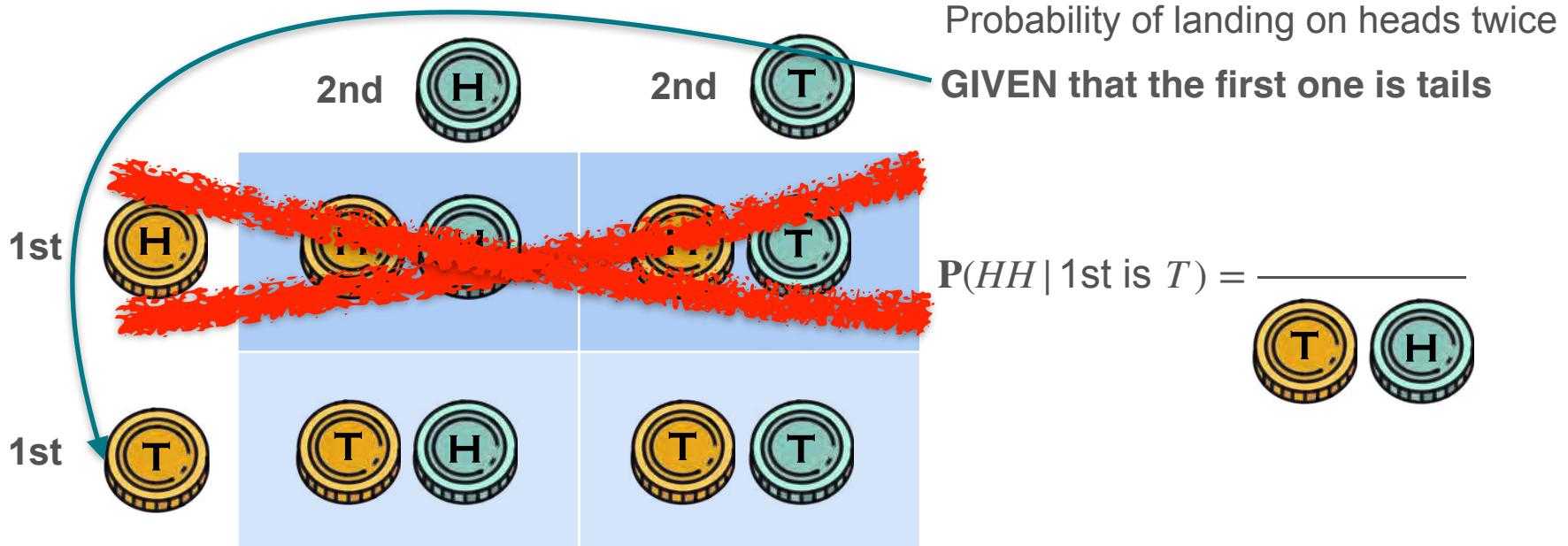
Conditional Probability: Coin Example 2



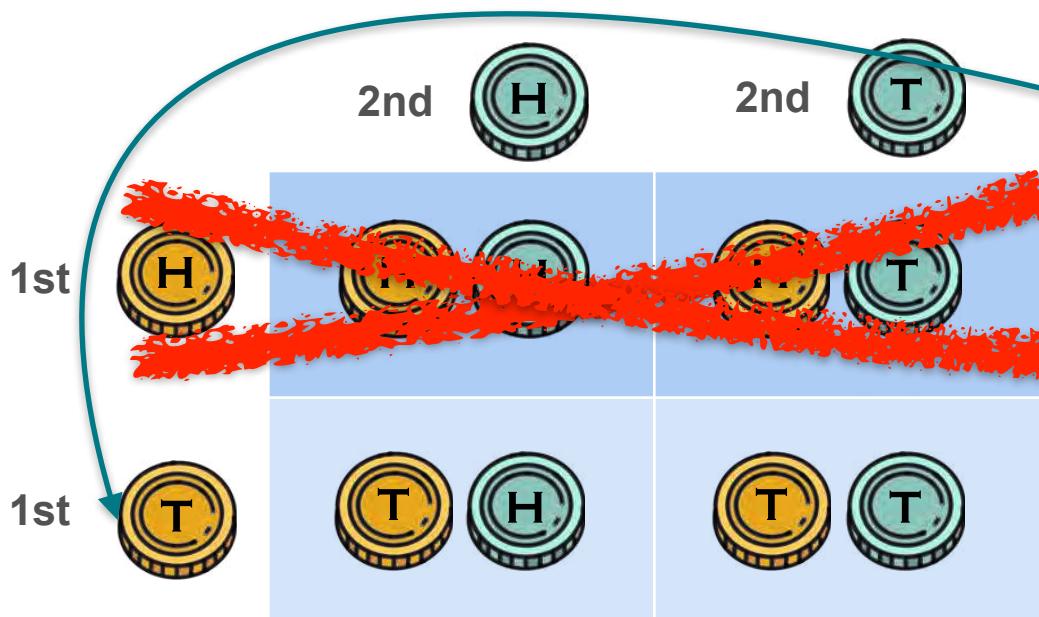
Conditional Probability: Coin Example 2



Conditional Probability: Coin Example 2

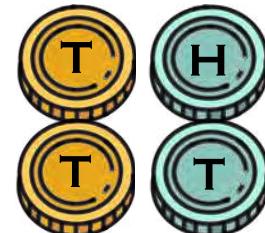


Conditional Probability: Coin Example 2

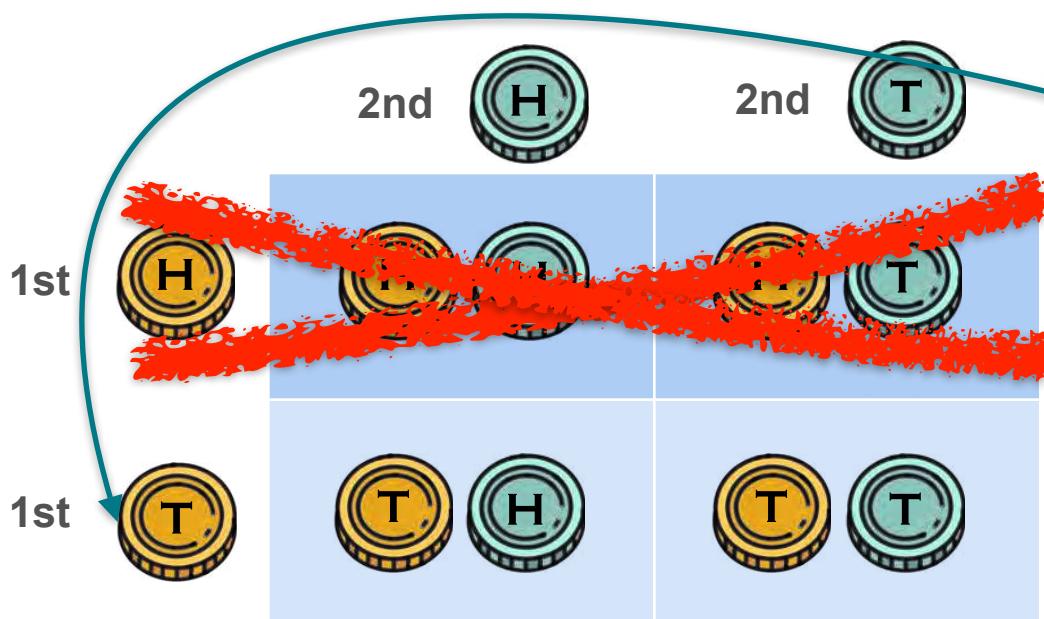


Probability of landing on heads twice
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



Conditional Probability: Coin Example 2

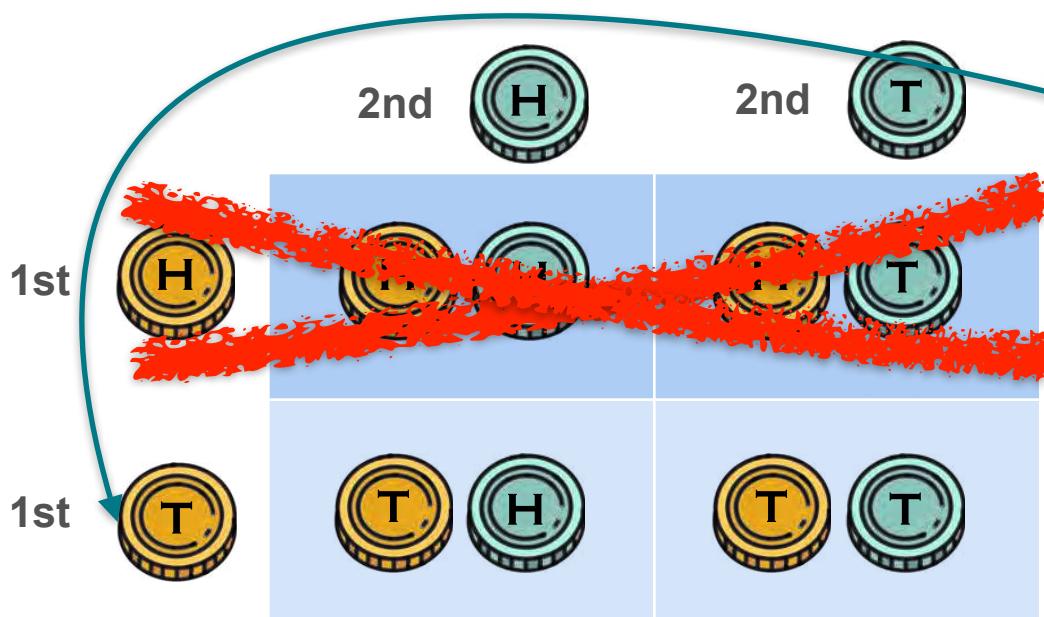


Probability of landing on heads twice
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



Conditional Probability: Coin Example 2

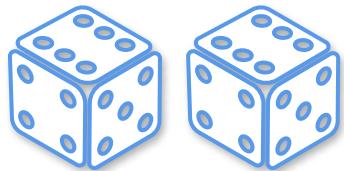


Probability of landing on heads twice
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = 0$$

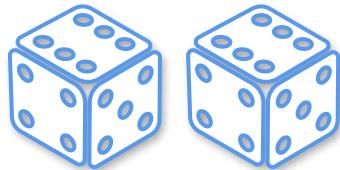


Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

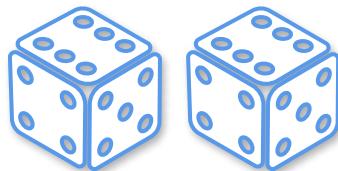
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

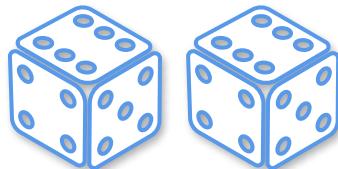
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1

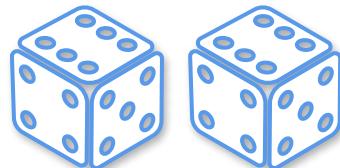


What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \underline{\hspace{10em}}$$

Conditional Probability: Dice Example 1



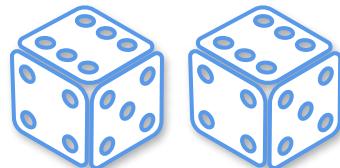
What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. Outcomes where the sum is 10 are highlighted with red boxes: (4,6), (5,5), (6,4), and (6,5). The outcome (5,5) is also highlighted with a red box.

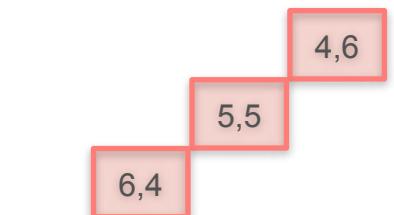
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

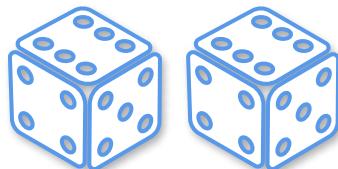
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

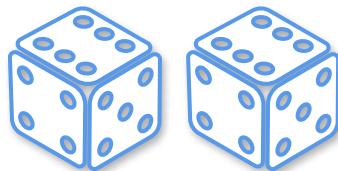
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. The outcomes are represented by small dice icons above each cell. The cells are colored based on their sum: light blue for sums 2 through 9, pink for sum 10, and light green for sums 11 and 12. The three pink cells representing a sum of 10 are highlighted with red boxes. The total number of outcomes is 36.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

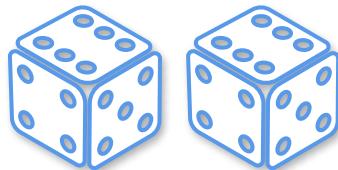
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{12} = \frac{1}{4}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. The outcomes are represented by small dice icons above each cell. The cells are colored based on their sum: light blue for sums 2-5, pink for sum 6, red for sum 7, and light green for sums 8-12. The three outcomes where the sum is 10 (4,6), (5,5), and (6,4) are highlighted with red boxes.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

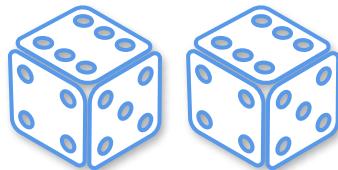
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1

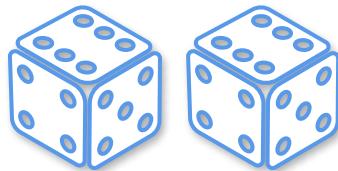


What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



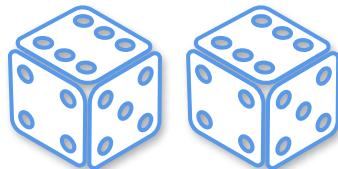
What is the probability that the sum is 10?

GIVEN that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 1



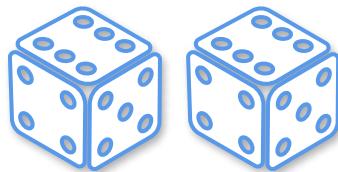
What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 1



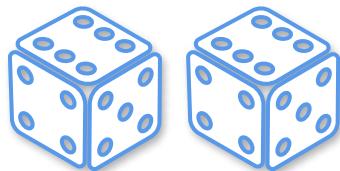
What is the probability that the sum is 10?

GIVEN that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6	
3,1	3,2	3,3	3,4	3,5	3,6	
4,1	4,2	4,3	4,4	4,5	4,6	
5,1	5,2	5,3	5,4	5,5	5,6	
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

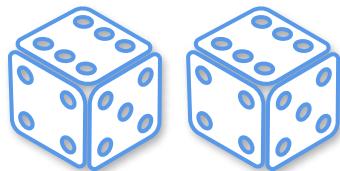
GIVEN that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,1	6,2	6,3	6,4	6,5	6,6
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Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

GIVEN that the first one is 6

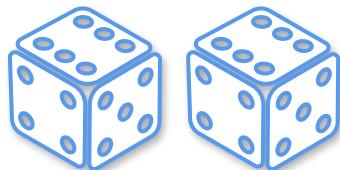
	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,4

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

GIVEN that the first one is 6

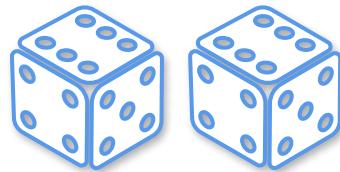
	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,4 6,1 6,2 6,3 6,4 6,5 6,6

$$= \frac{1}{6}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

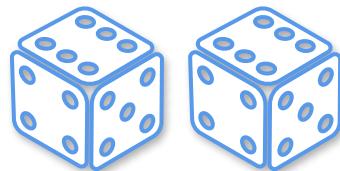
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

A diagram showing a staircase path through a 6x6 grid of dice rolls. The steps are red-bordered boxes. The path starts at (1,1), goes up-right to (2,2), up to (3,3), right to (4,4), up to (5,5), and right to (6,6). The final outcome (6,6) is also highlighted with a red border.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

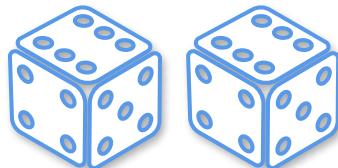
Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 2

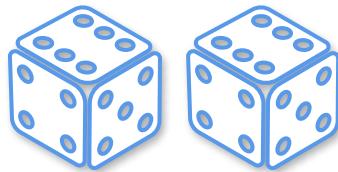


What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 2



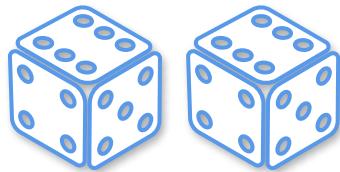
What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

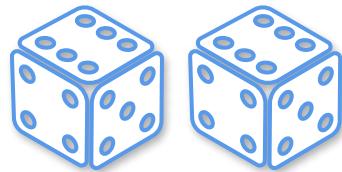
GIVEN that the first one is 1

A 6x6 grid representing all possible outcomes of two dice rolls. The columns are labeled by the outcome of the first die (1, 2, 3, 4, 5, 6) and the rows by the outcome of the second die (1, 2, 3, 4, 5, 6). The first column is highlighted in blue, and the last row is highlighted in red. The cell at the intersection of the first column and the last row (1,6) is also highlighted in red, indicating it is the target event for the conditional probability calculation.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 2



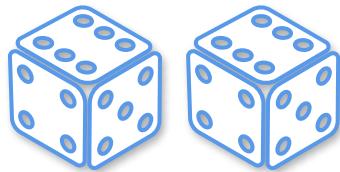
What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

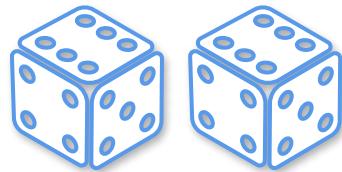
GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

1,1	1,2	1,3	1,4	1,5	1,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

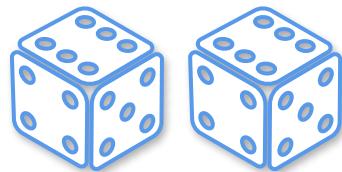
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,2	1,3	1,4	1,5	1,6

1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,2	1,3	1,4	1,5	1,6

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

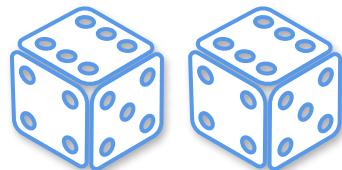
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

1,1	1,2	1,3	1,4	1,5	1,6
0	1,2	1,3	1,4	1,5	1,6

1,1	1,2	1,3	1,4	1,5	1,6
0	1,2	1,3	1,4	1,5	1,6

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The total number of outcomes is 6 (the first die can be 1, 2, 3, 4, 5, or 6).

The number of favorable outcomes is 0 (there are no outcomes where the sum is 10 given that the first die is 1).

$$= 0$$

Product Rule (for Independent Events)

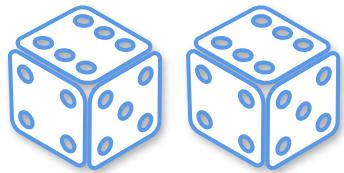
$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Product Rule (for Independent Events)

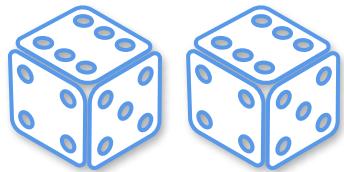
When A and B independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Conditional Probability: Dice Example 3

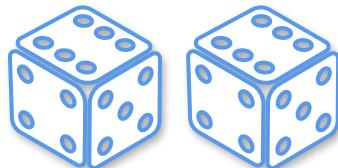


Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

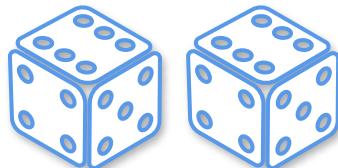
Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

Conditional Probability: Dice Example 3

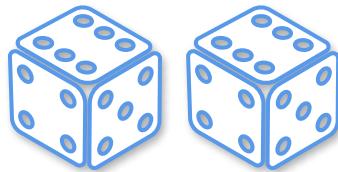


What is the probability that
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 3



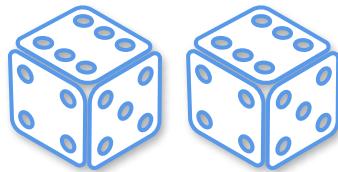
What is the probability that
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) =$$

6,4

Conditional Probability: Dice Example 3



What is the probability that the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) =$$

6,4

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{1}{36}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	2,1	2,2	2,3	2,4	2,5	2,6
2,2						
2,3						
2,4						
2,5						
2,6						
3,1	3,1	3,2	3,3	3,4	3,5	3,6
3,2						
3,3						
3,4						
3,5						
3,6						
4,1	4,1	4,2	4,3	4,4	4,5	4,6
4,2						
4,3						
4,4						
4,5						
4,6						
5,1	5,1	5,2	5,3	5,4	5,5	5,6
5,2						
5,3						
5,4						
5,5						
5,6						
6,1	6,1	6,2	6,3	6,4	6,5	6,6
6,2						
6,3						
6,4						
6,5						
6,6						

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{6}{36}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 \mid \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4
6,1
6,2
6,3
6,4
6,5
6,6

$$\frac{1}{6}$$

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(1\text{st is } 6 \cap \text{sum} = 10) =$$

$$P(1\text{st is } 6)$$

$$\bullet P(\text{sum} = 10 | 1\text{st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4
6,1 6,2 6,3 6,4 6,5 6,6

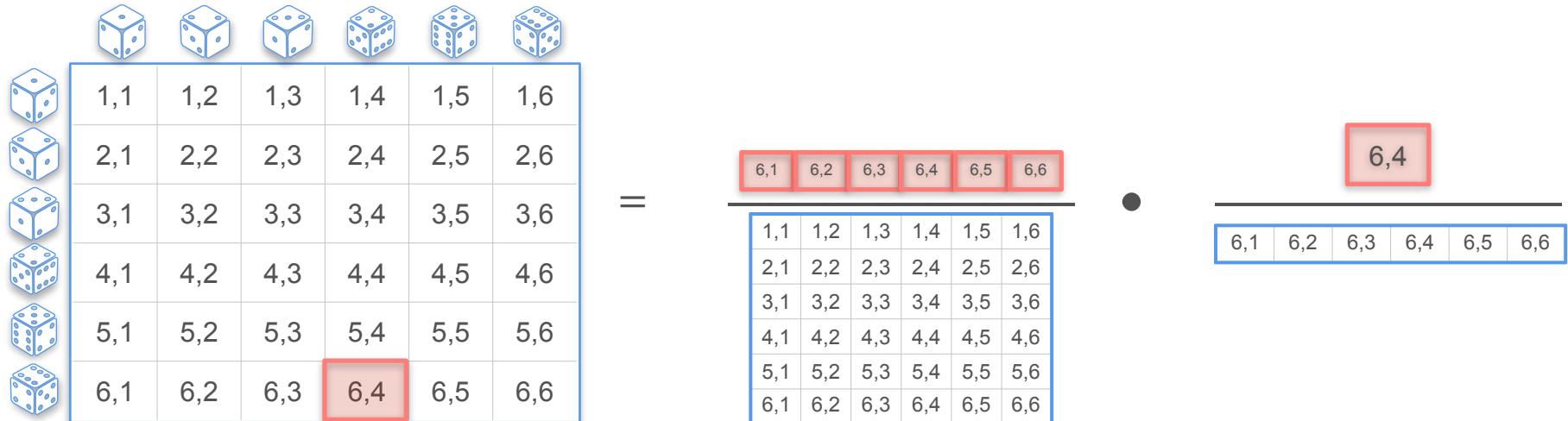
$$\frac{1}{6}$$

•

$$\frac{1}{6}$$

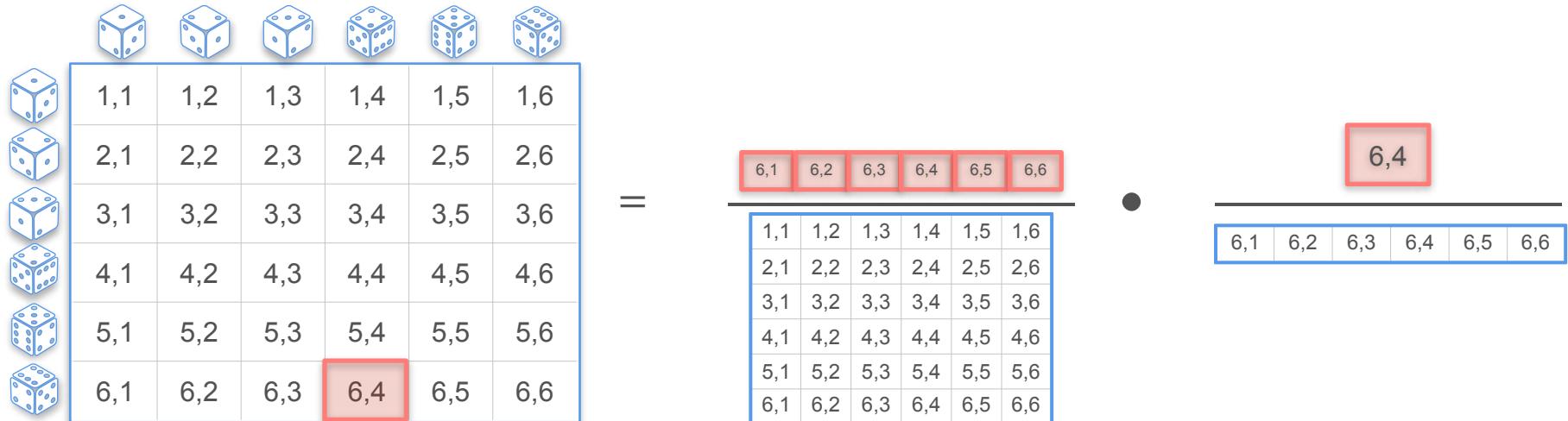
$$= \frac{1}{36}$$

Conditional Probability: Dice Example 3



$$P(1\text{st is } 6 \cap \text{sum} = 10) = P(1\text{st is } 6) \bullet P(\text{sum} = 10 | 1\text{st } 6)$$

Conditional Probability: Dice Example 3



$$P(A \cap B) = P(A) \cdot P(B | A)$$

The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B | A)$$

The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B | A)$$

When independent

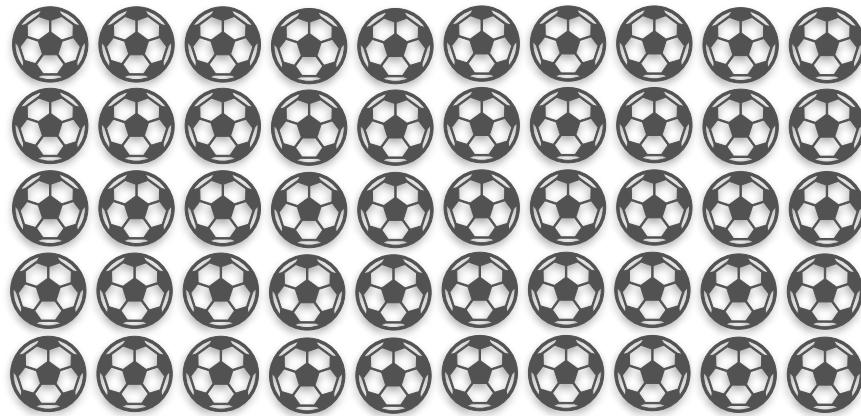
$$\mathbf{P}(B | A) = \mathbf{P}(B)$$

Quiz 1

Quiz 1

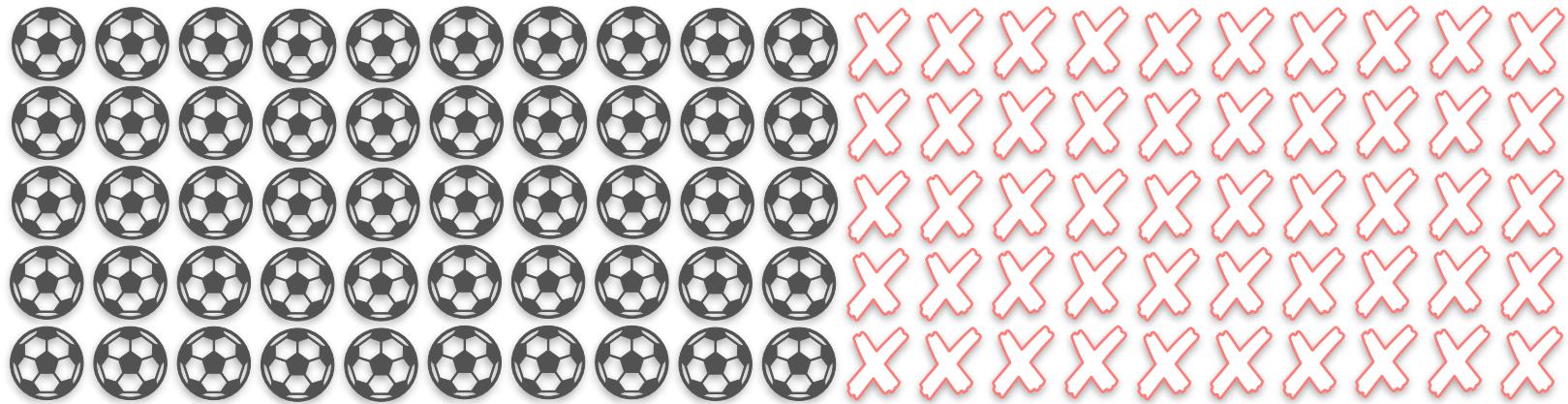
100 kids

Quiz 1



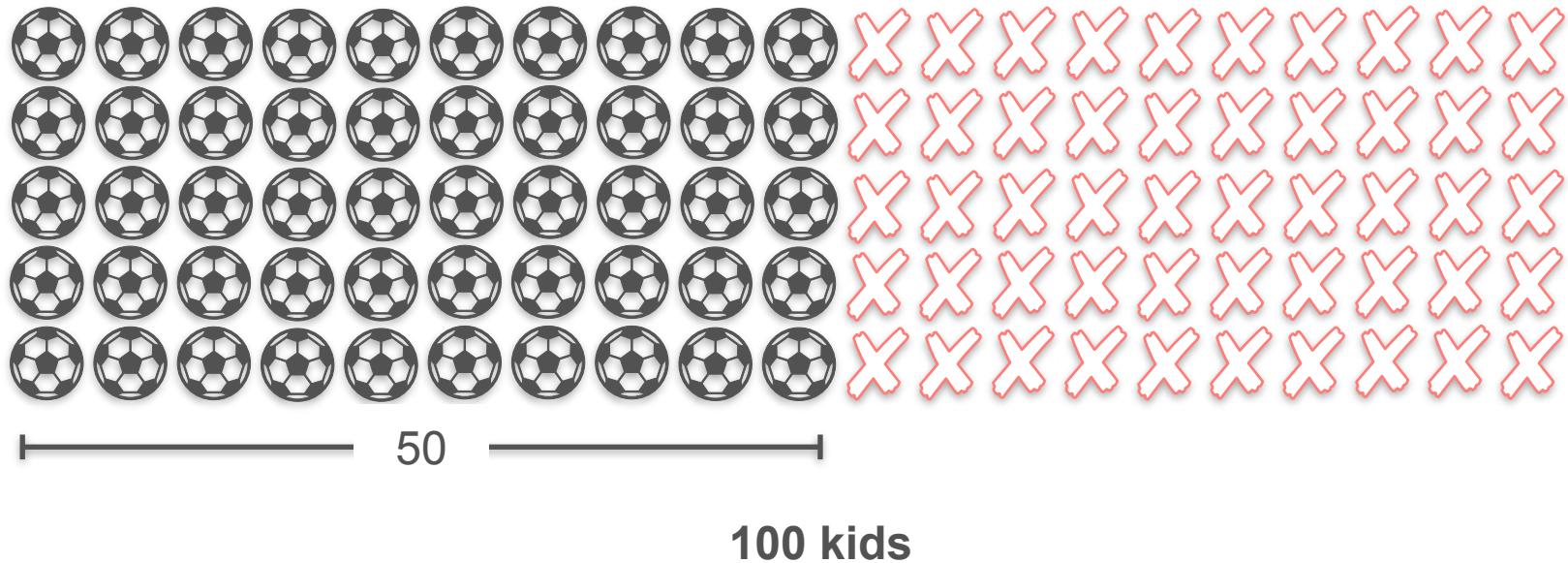
100 kids

Quiz 1

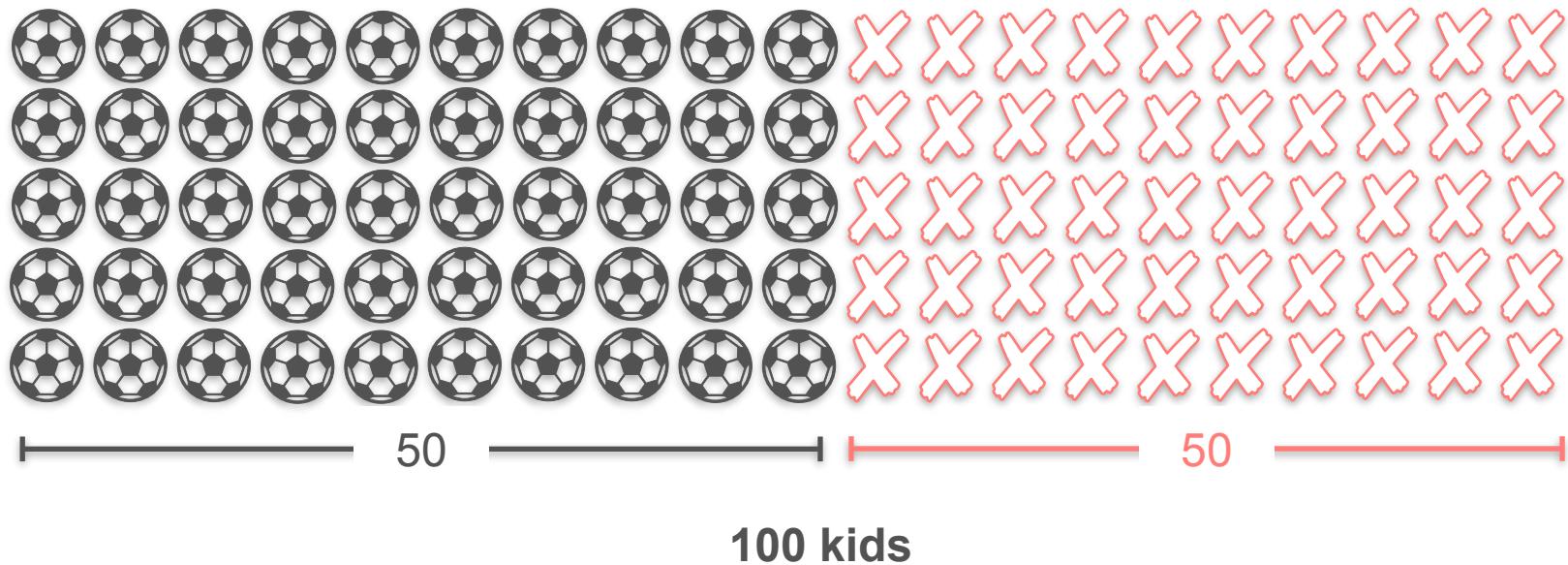


100 kids

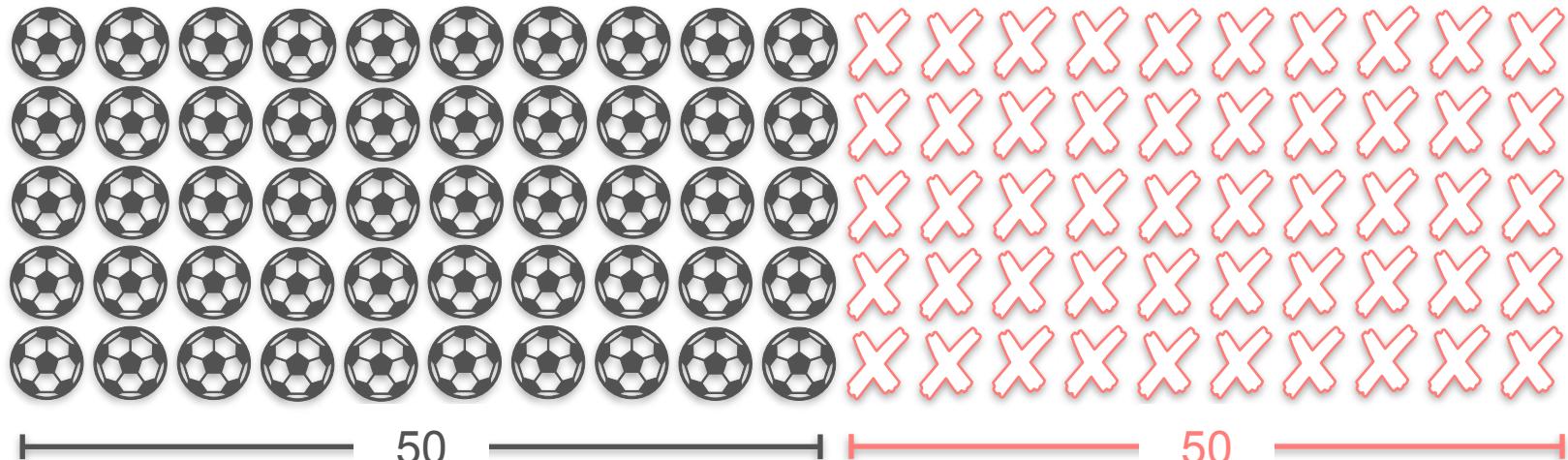
Quiz 1



Quiz 1

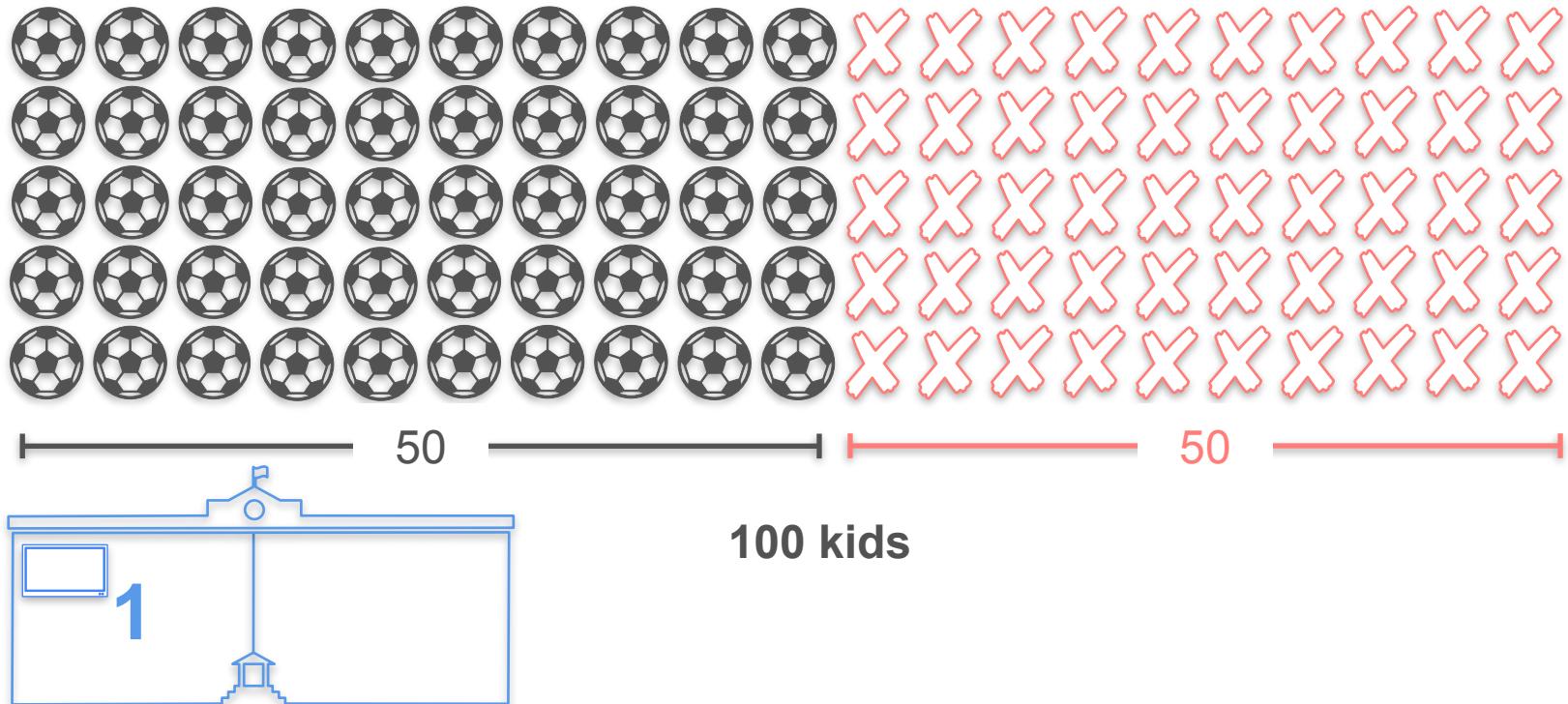


Quiz 1

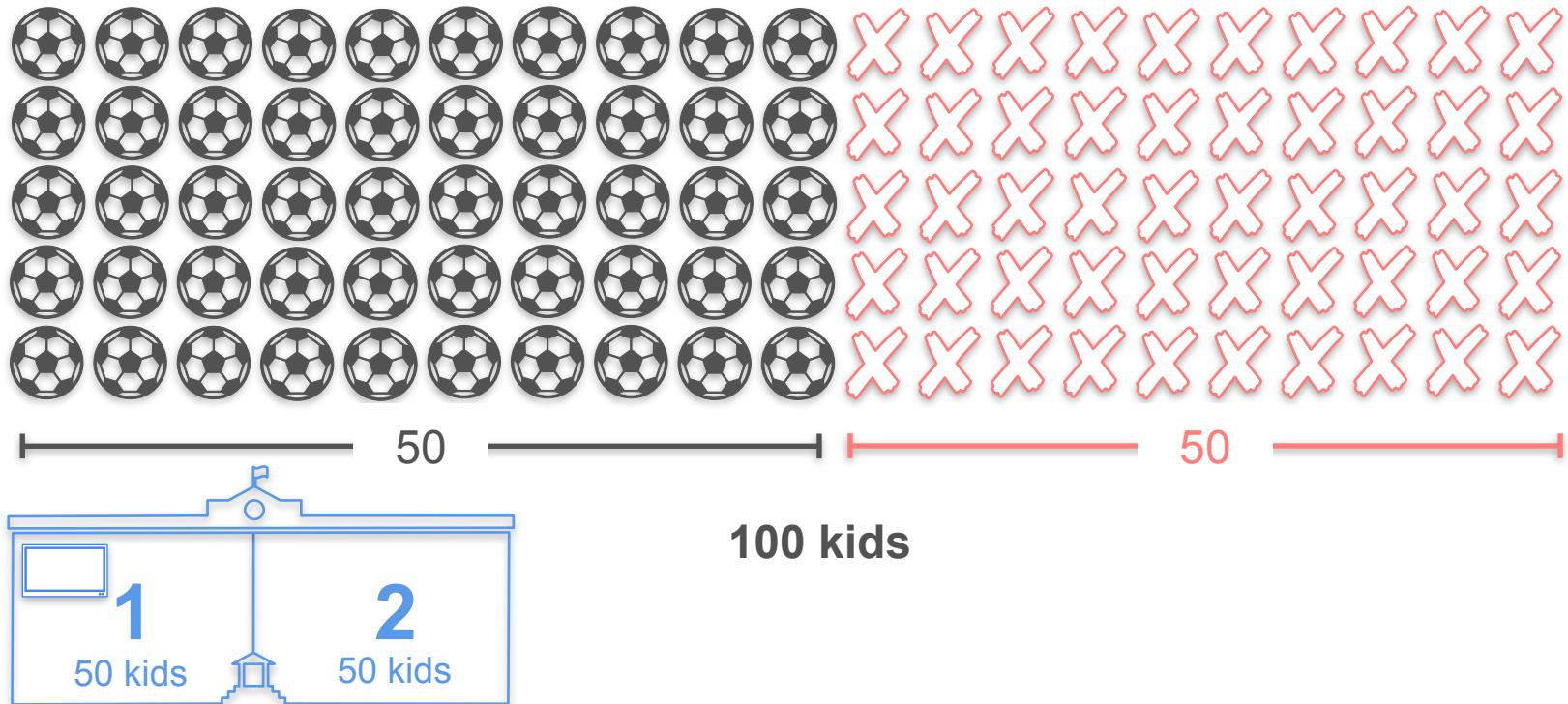


100 kids

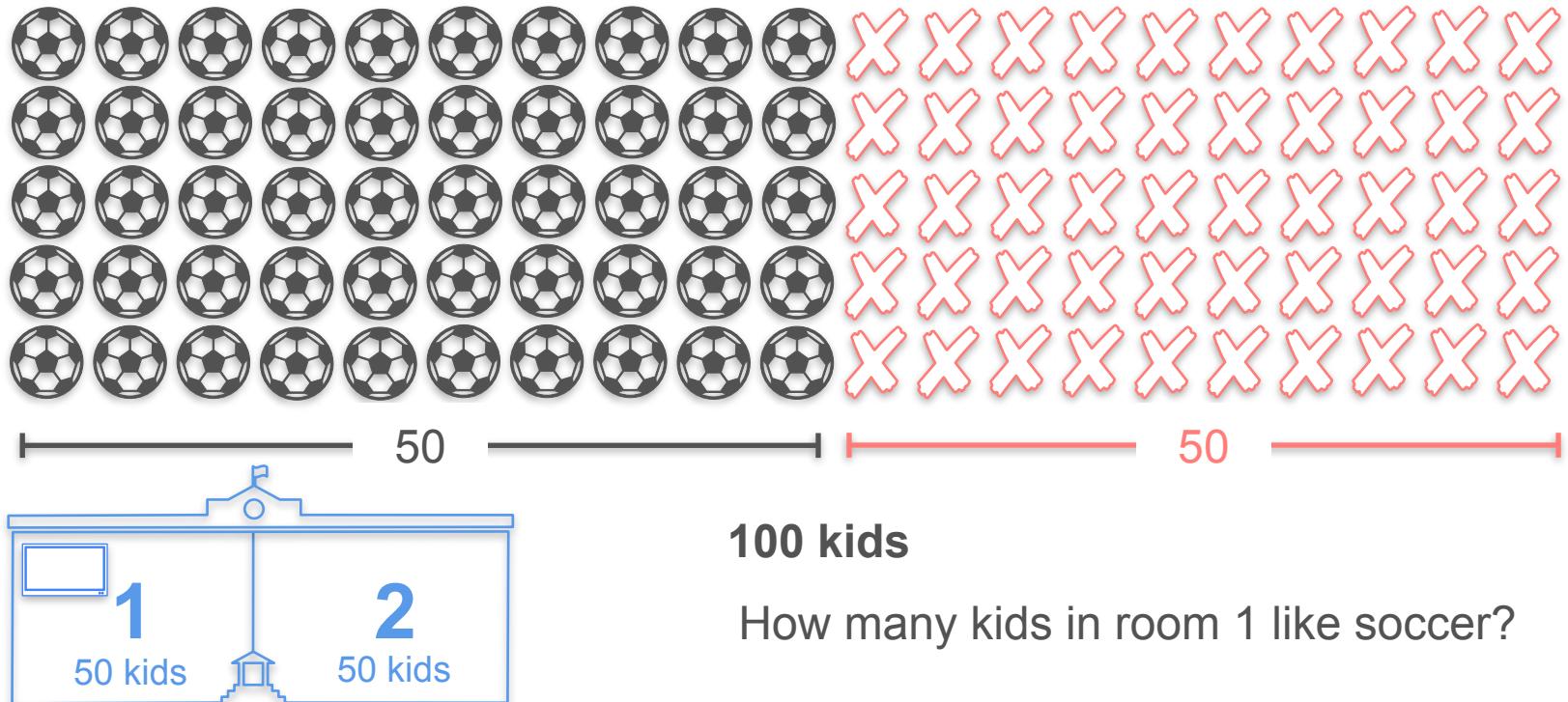
Quiz 1



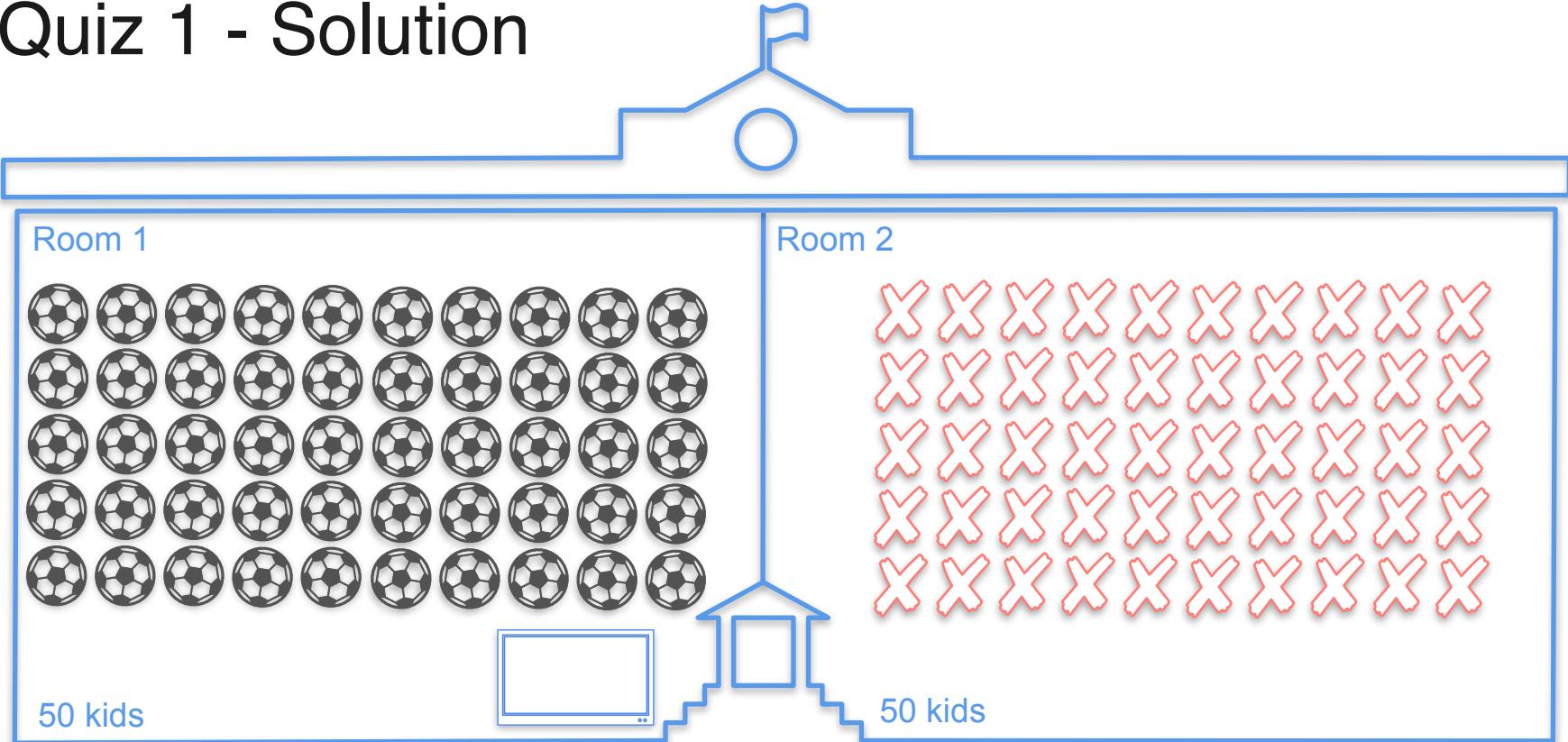
Quiz 1



Quiz 1

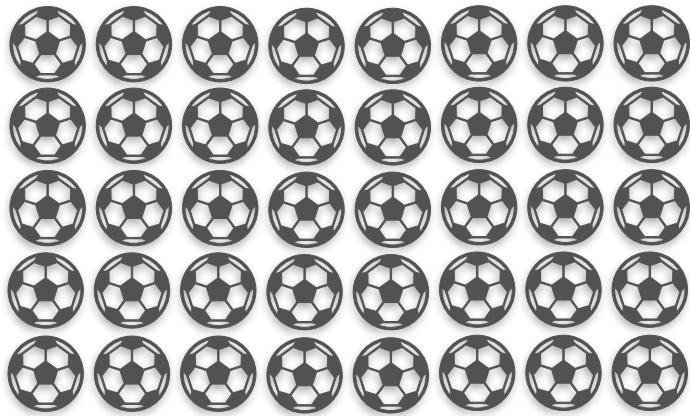


Quiz 1 - Solution

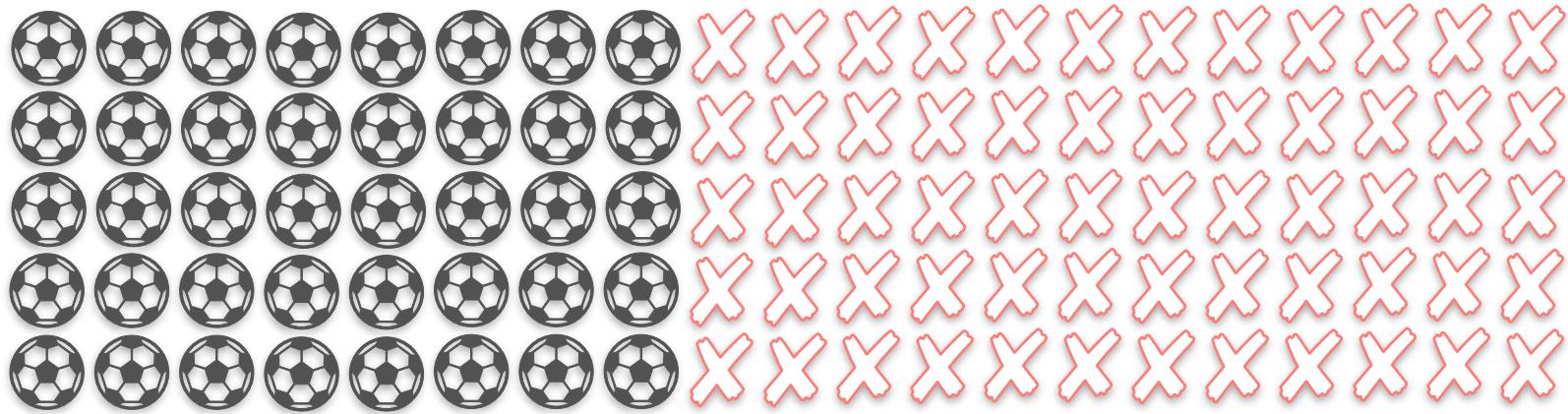


Quiz 2

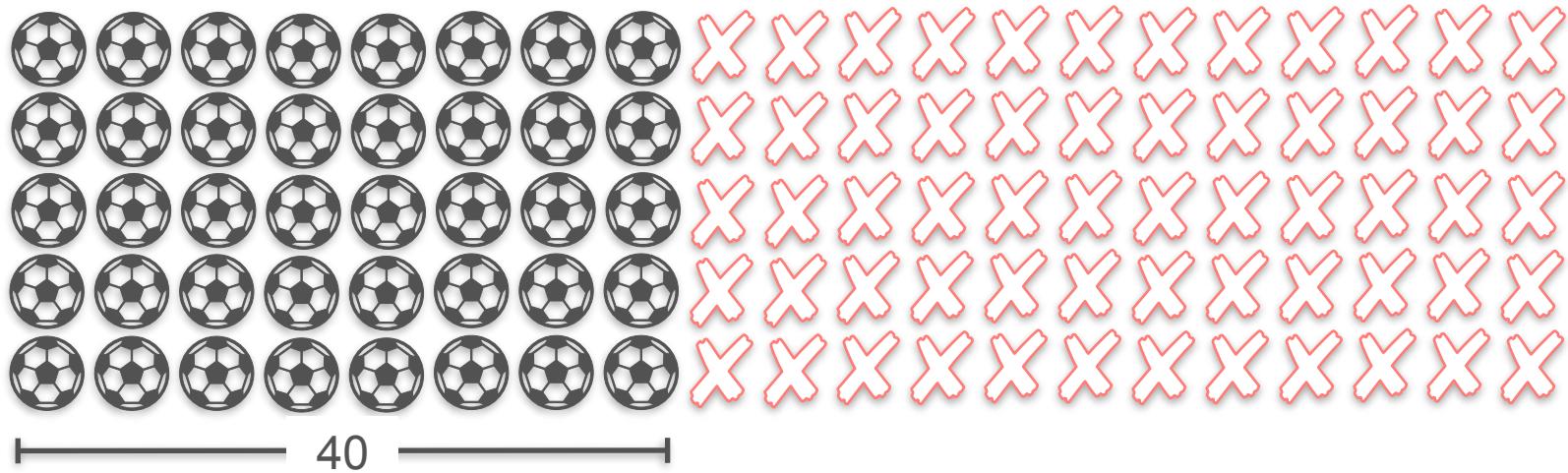
Quiz 2



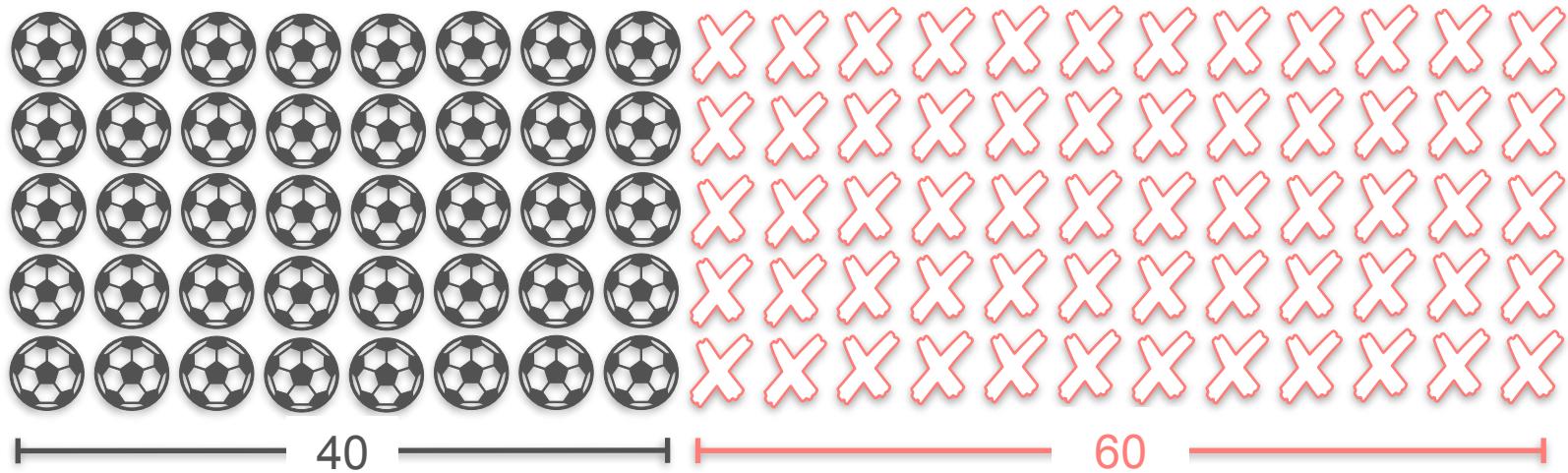
Quiz 2



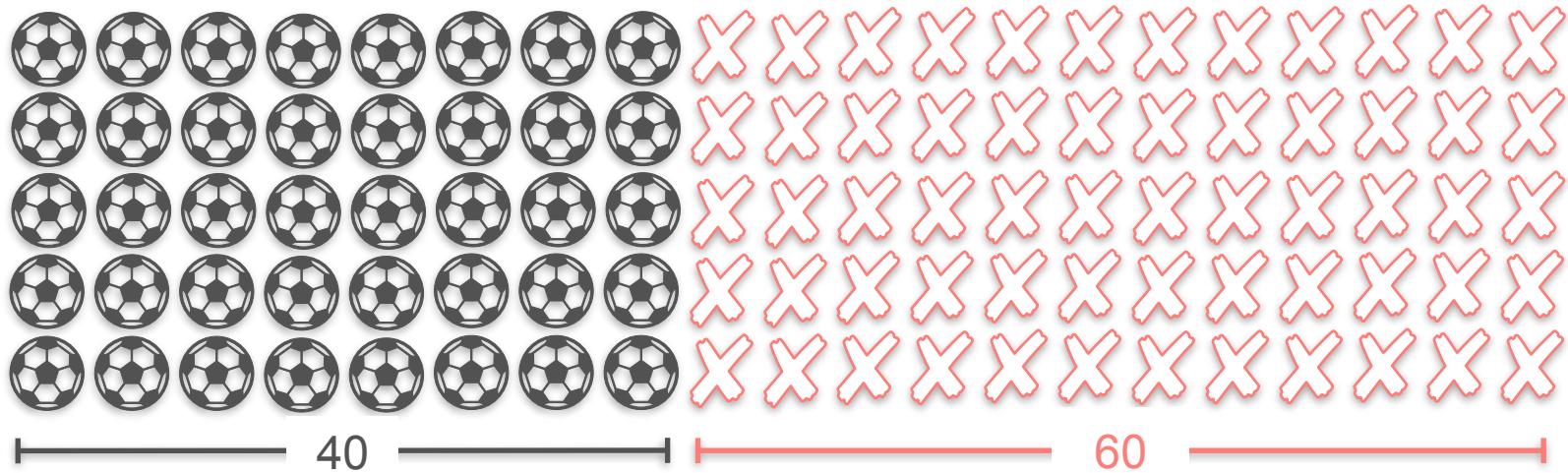
Quiz 2



Quiz 2

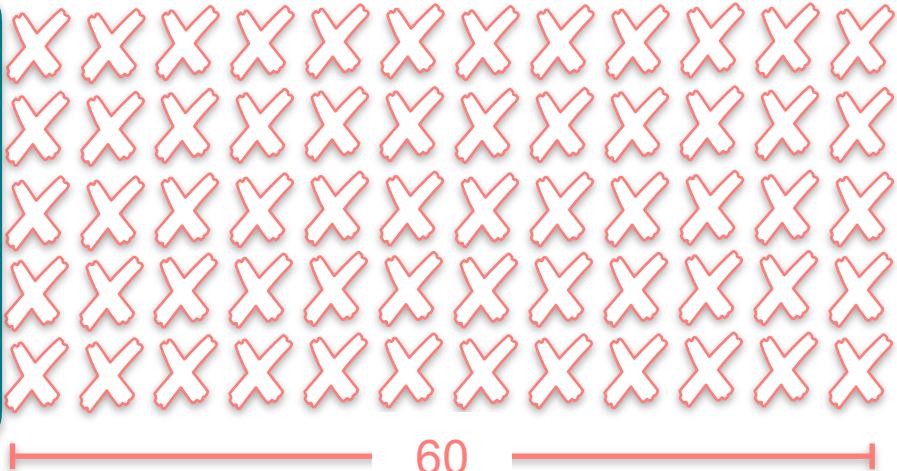
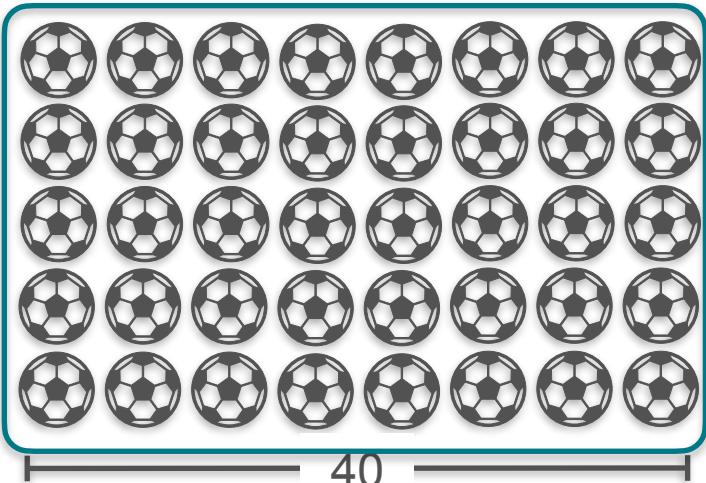


Quiz 2



100 kids

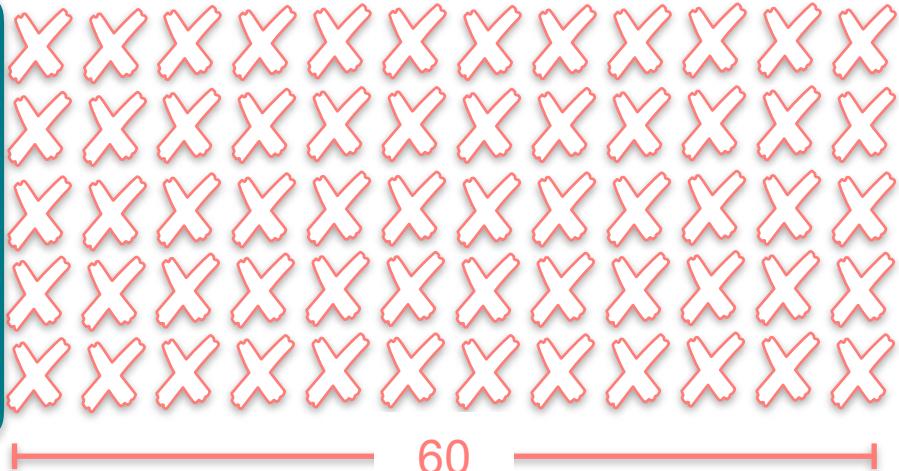
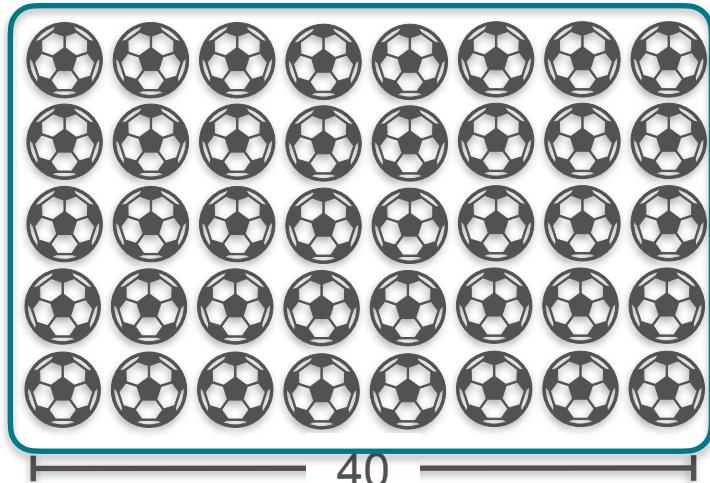
Quiz 2



80%

100 kids

Quiz 2

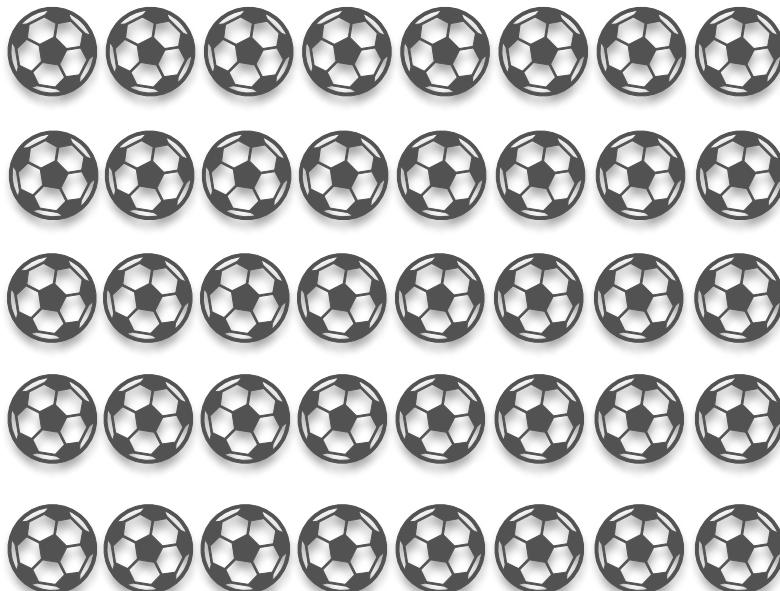


80%

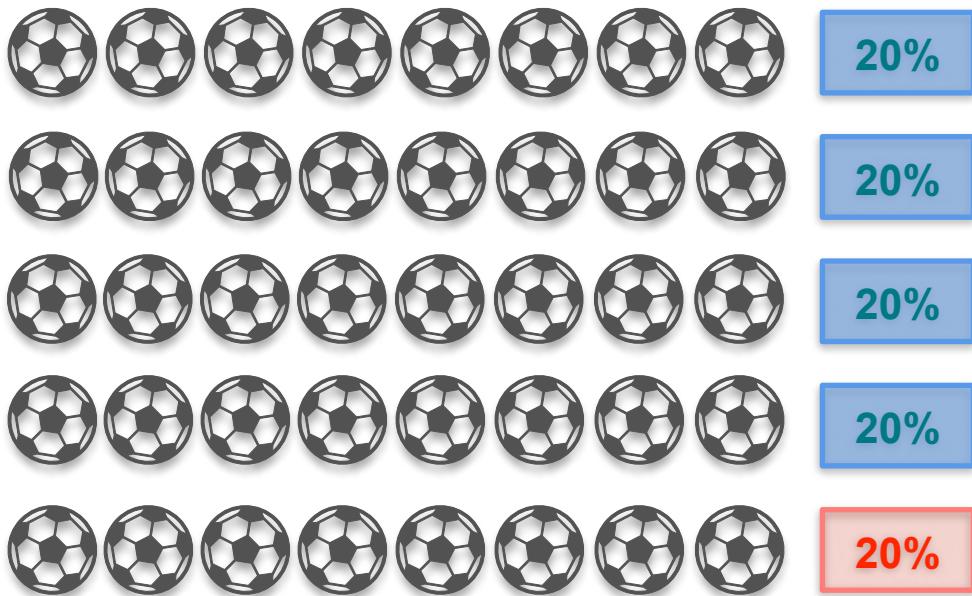
100 kids

How many kids play soccer and wear running shoes

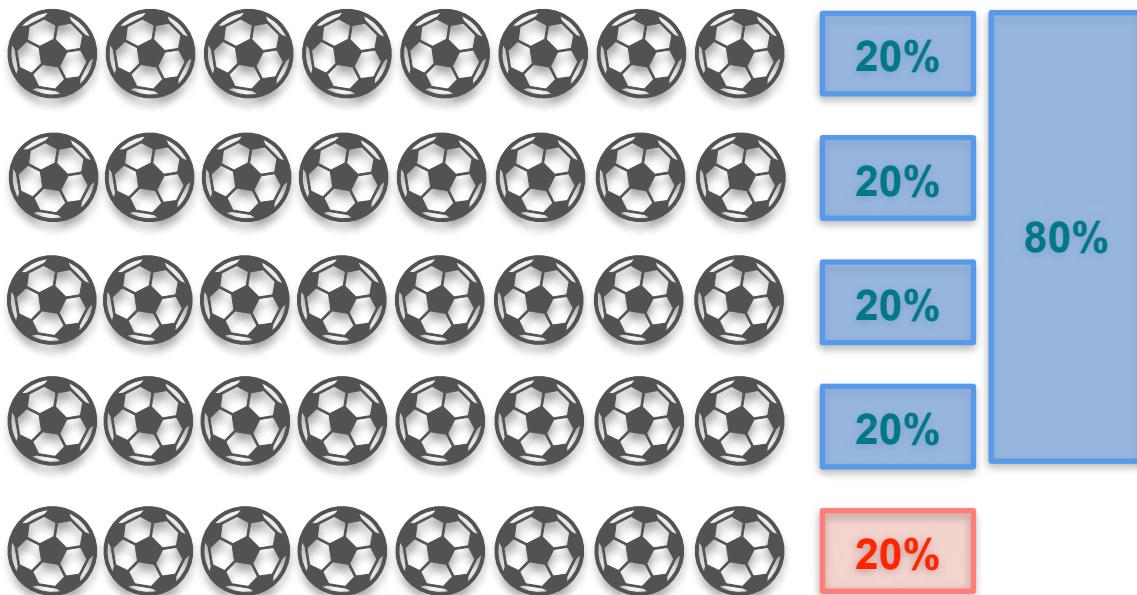
Quiz 2 - Solution



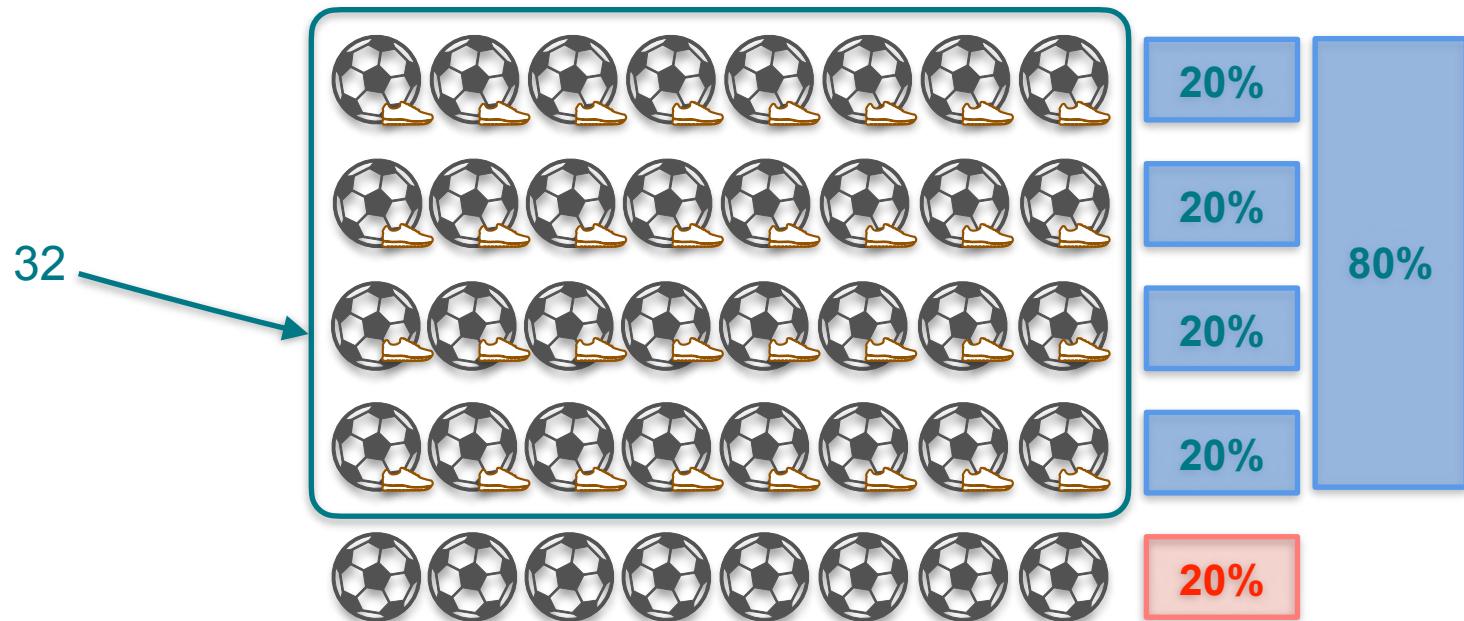
Quiz 2 - Solution



Quiz 2 - Solution

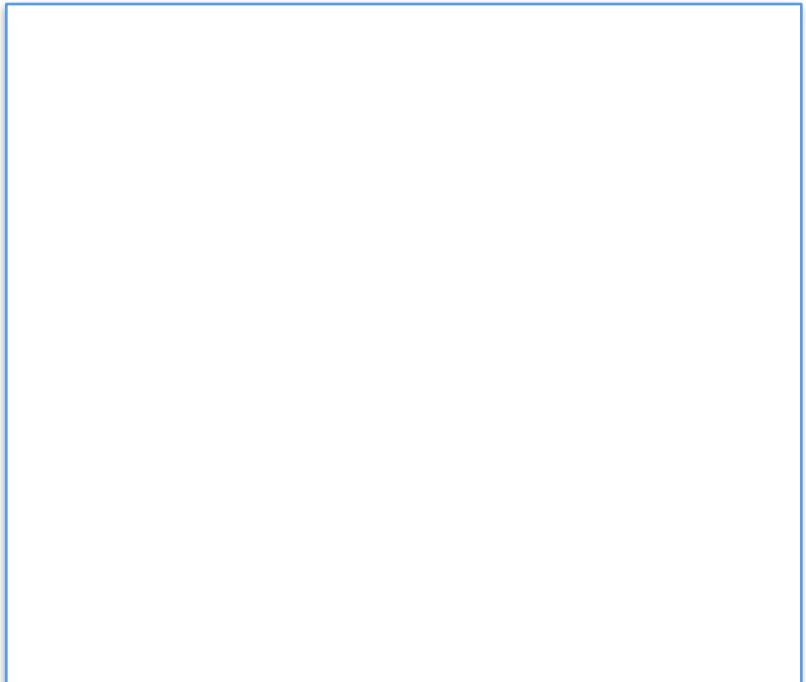


Quiz 2 - Solution



Conditional Probability

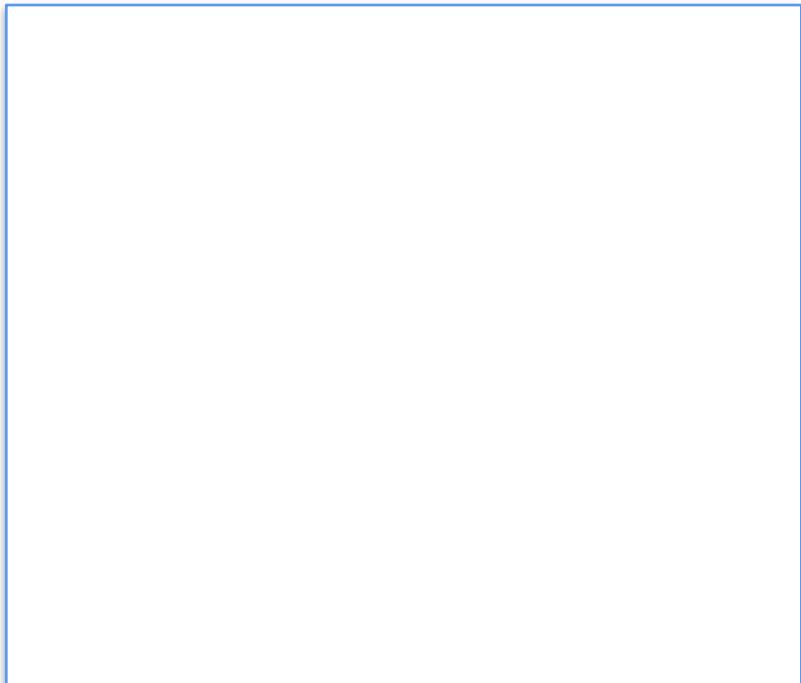
Conditional Probability



Conditional Probability



$$P(S) = 0.4$$



Conditional Probability



$$P(S) = 0.4$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○

Conditional Probability



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○

Conditional Probability



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

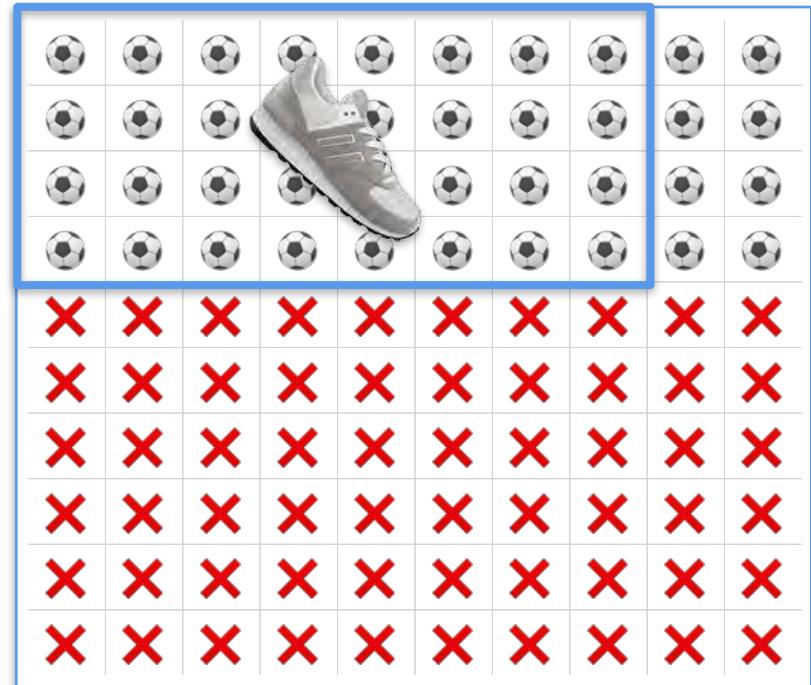
Conditional Probability



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

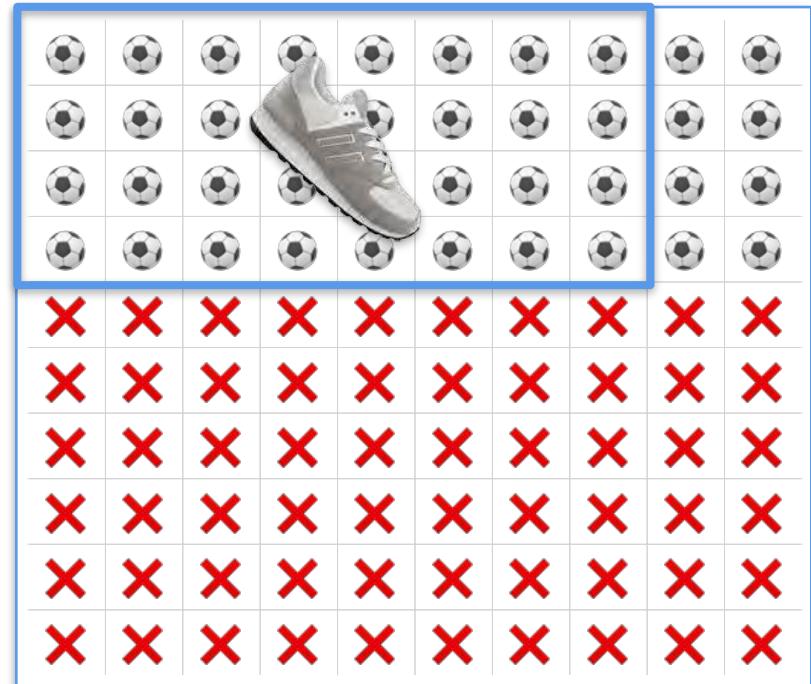


$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

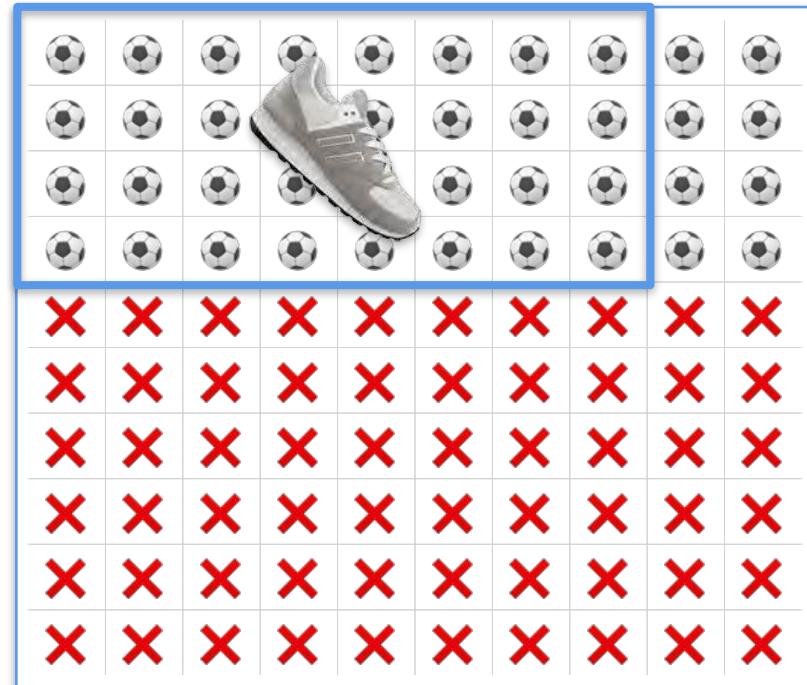


$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

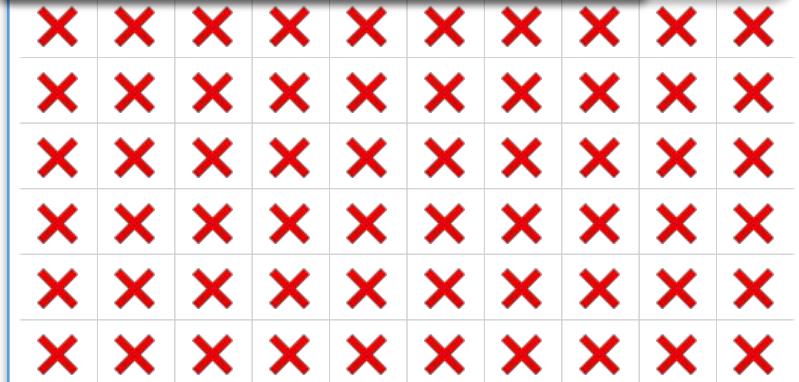
$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

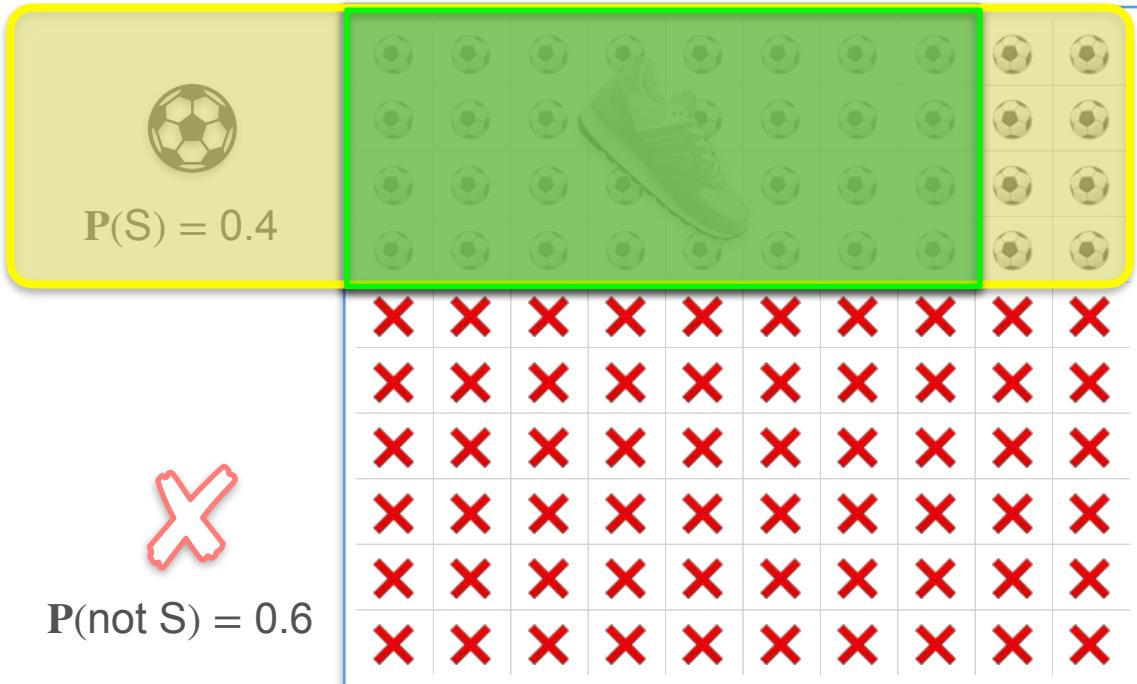
$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

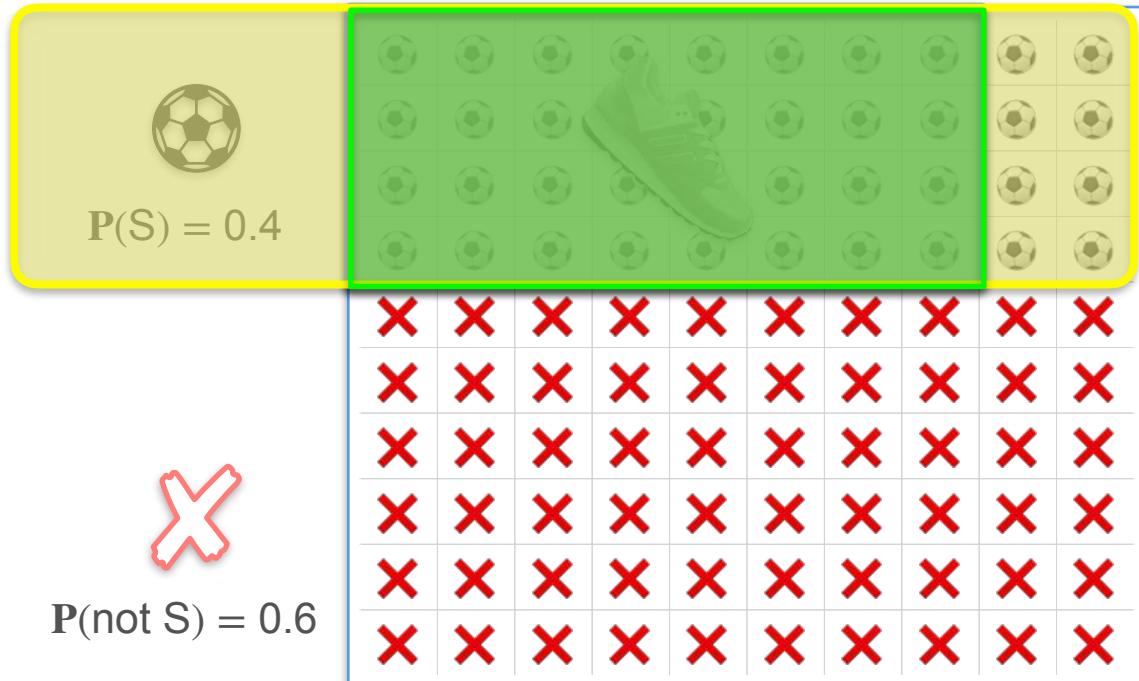
$$P(S \cap R) =$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

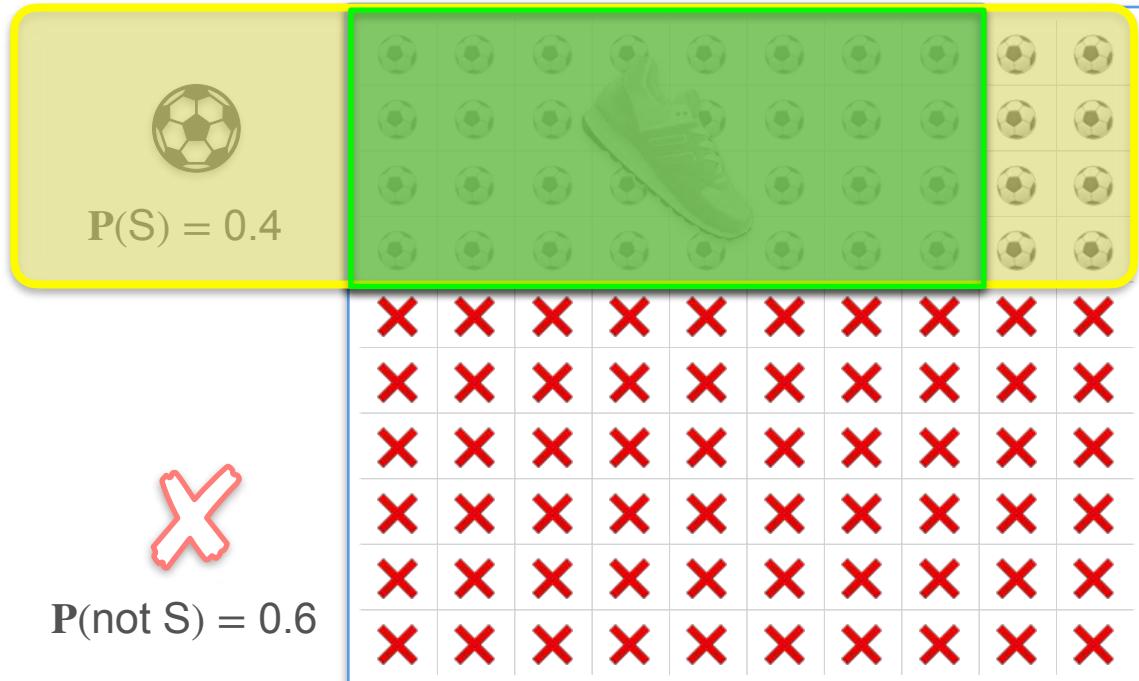
$$P(S \cap R) = P(S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

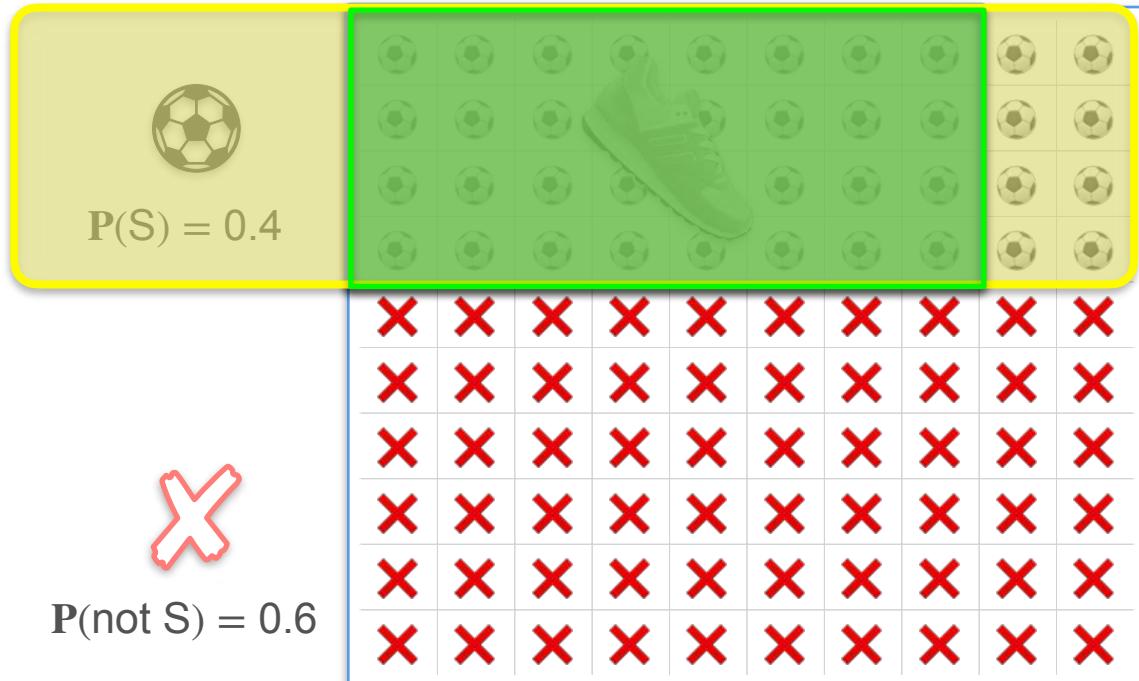
$$P(S \cap R) = P(S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

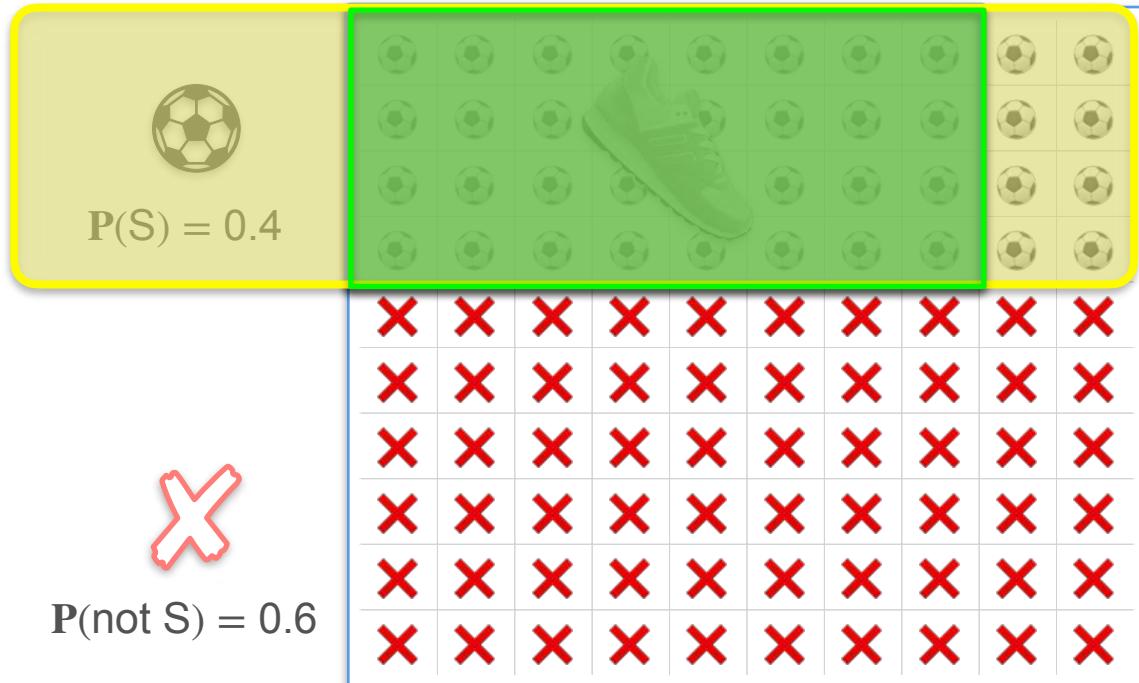
$$P(S \cap R) = P(S) \cdot$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

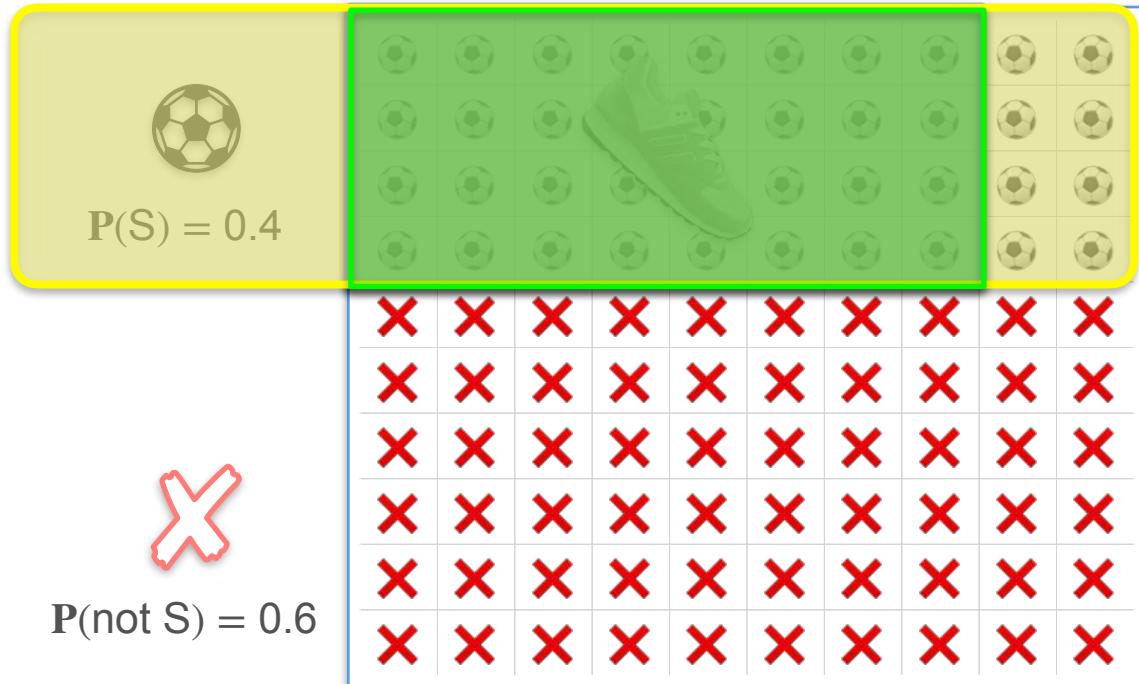
$$P(S \cap R) = P(S) \bullet P(R|S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

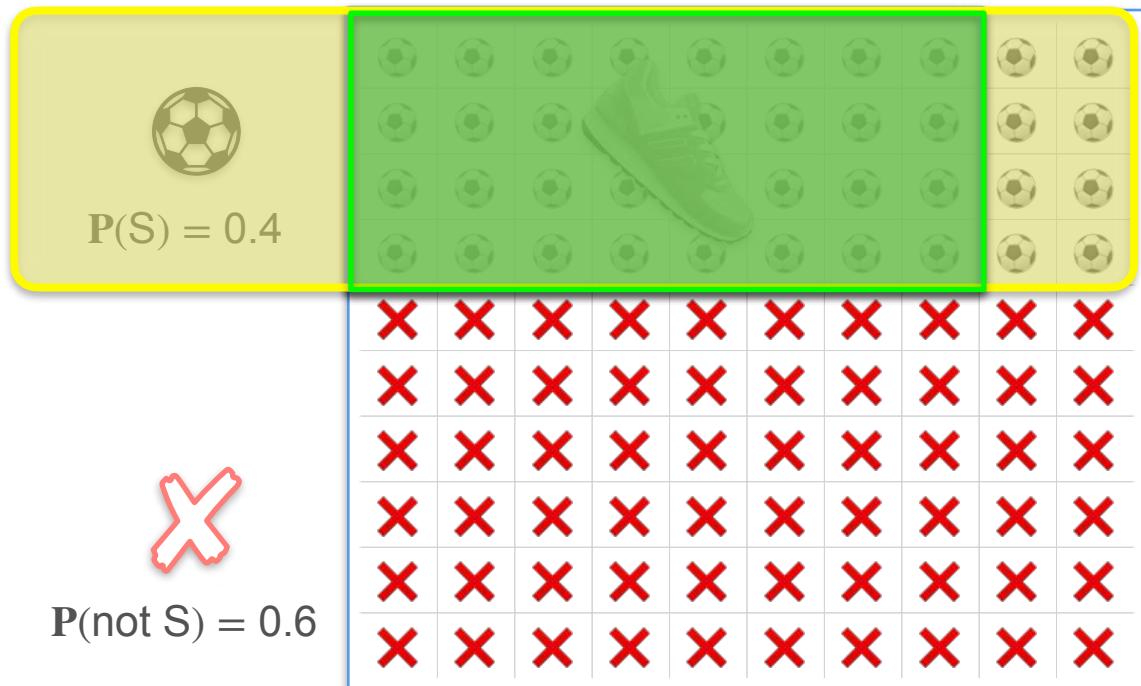


Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$
$$= 0.4$$

$$P(R | S) = 0.8$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

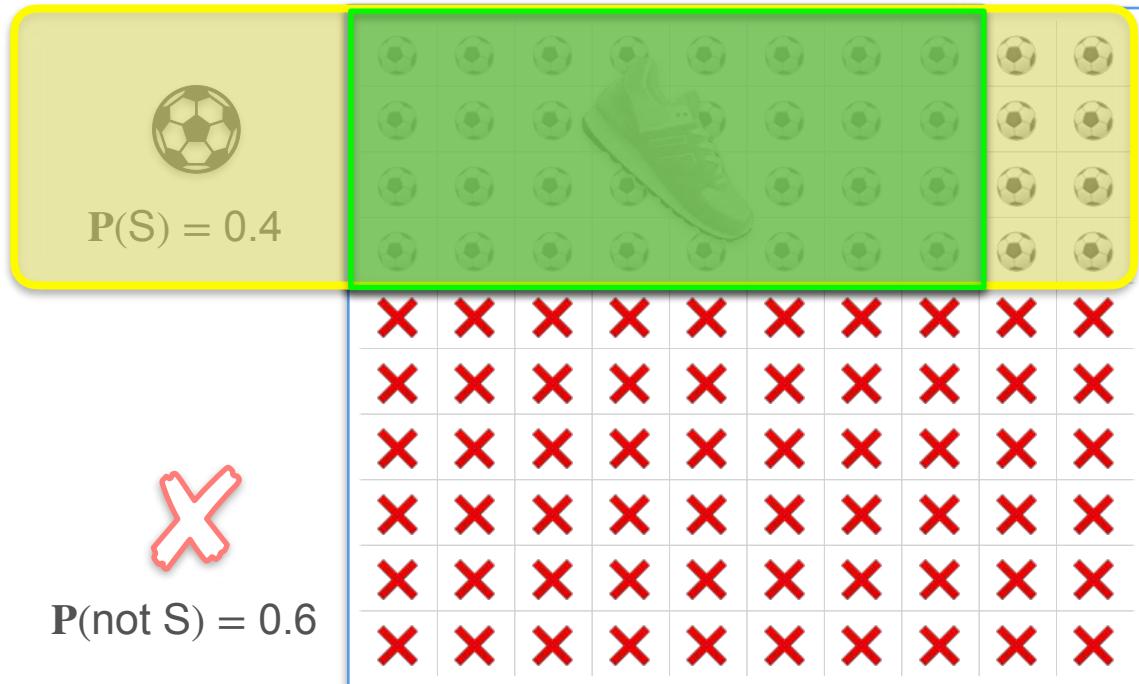
$$= 0.4 \bullet$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$$\begin{aligned} P(S \cap R) &= P(S) \bullet P(R|S) \\ &= 0.4 \bullet 0.8 \end{aligned}$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$P(S \cap R) = 0.32$



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
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○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

Conditional Probability

$P(\text{Soccer and Running shoes})$

$P(S \cap R) = 0.32$



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



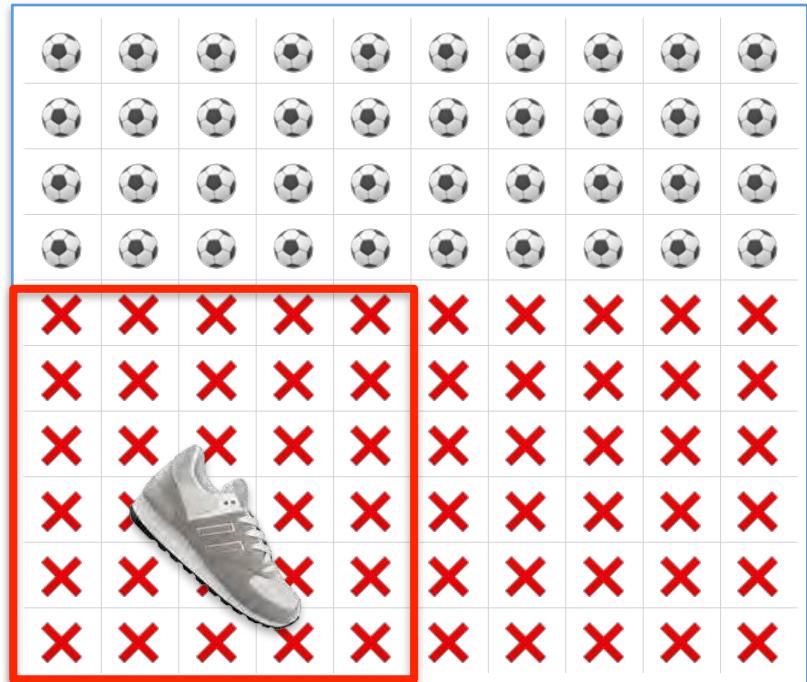
$P(S \cap R) = 0.32$

$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$P(R | S) = 0.8$



$P(R | \text{not } S) = 0.5$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

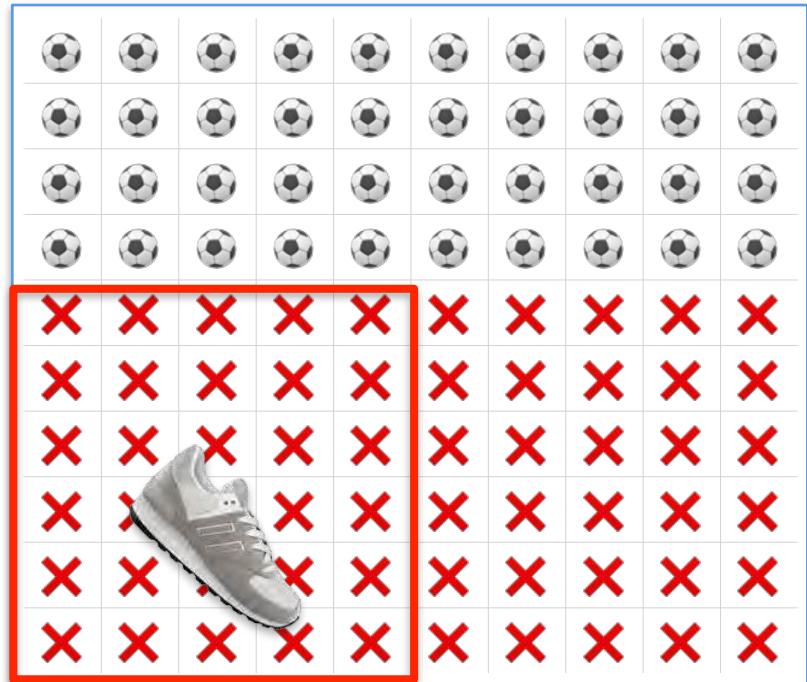
$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

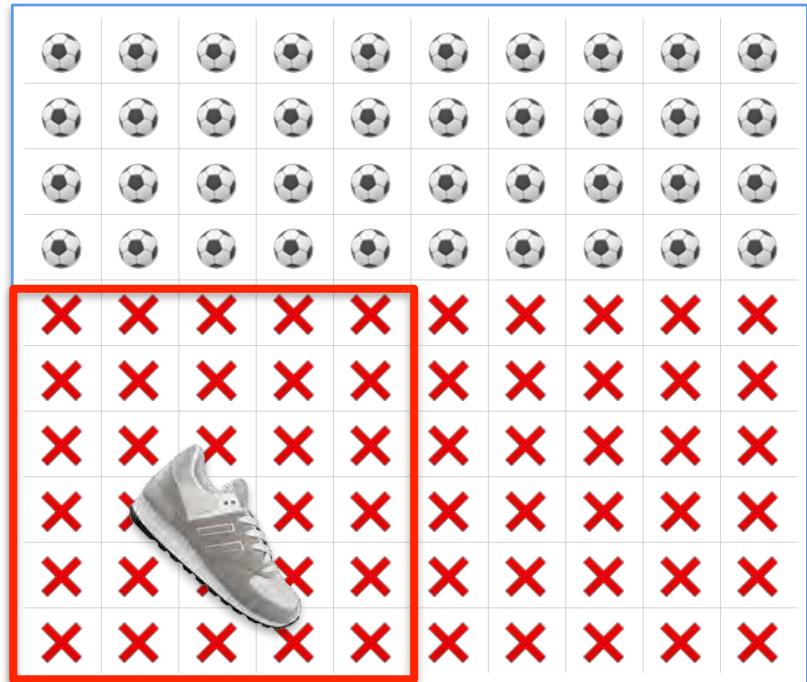
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) =$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

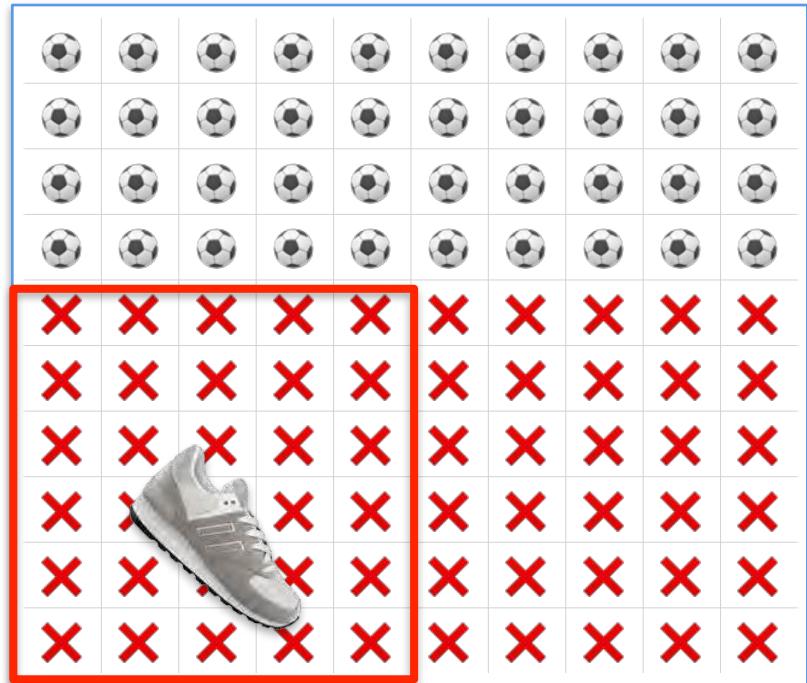
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

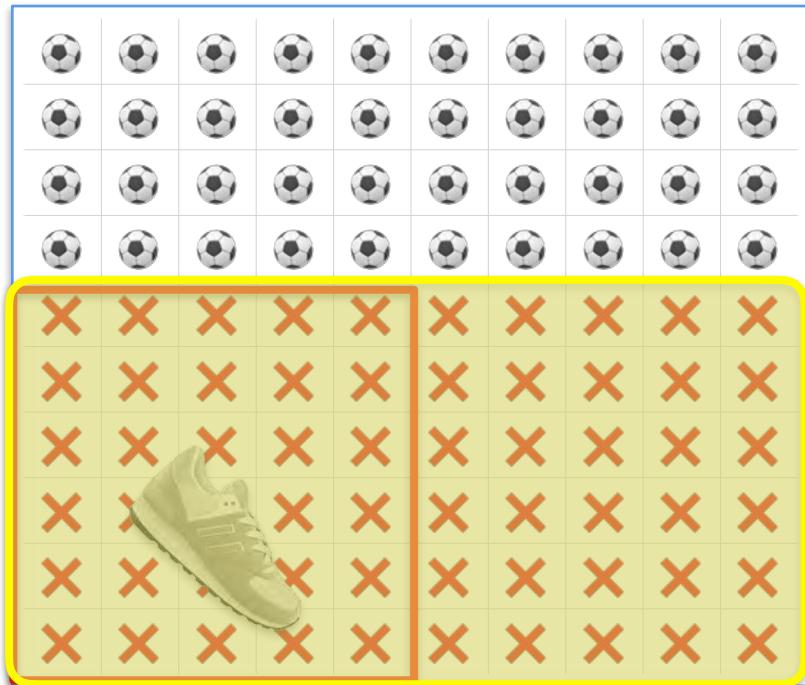
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

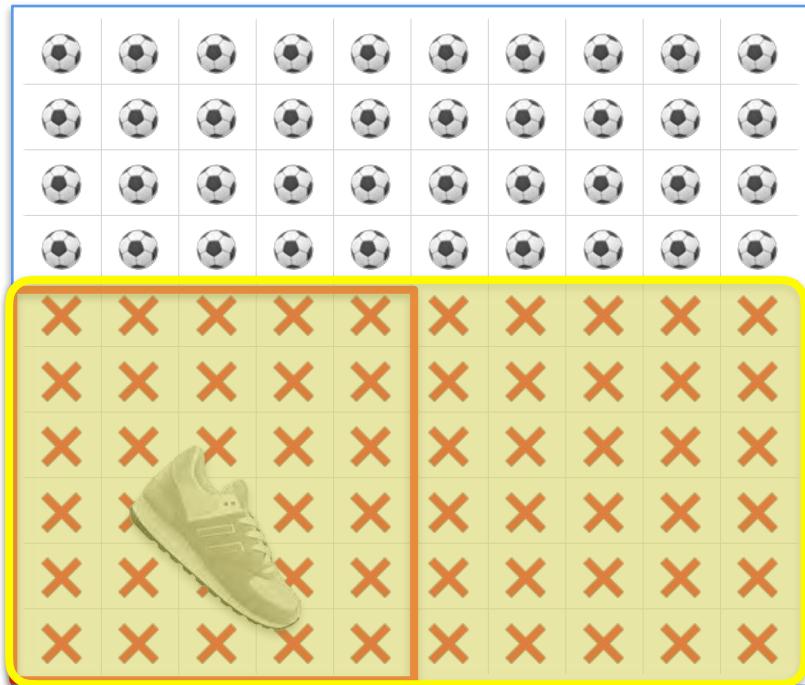
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

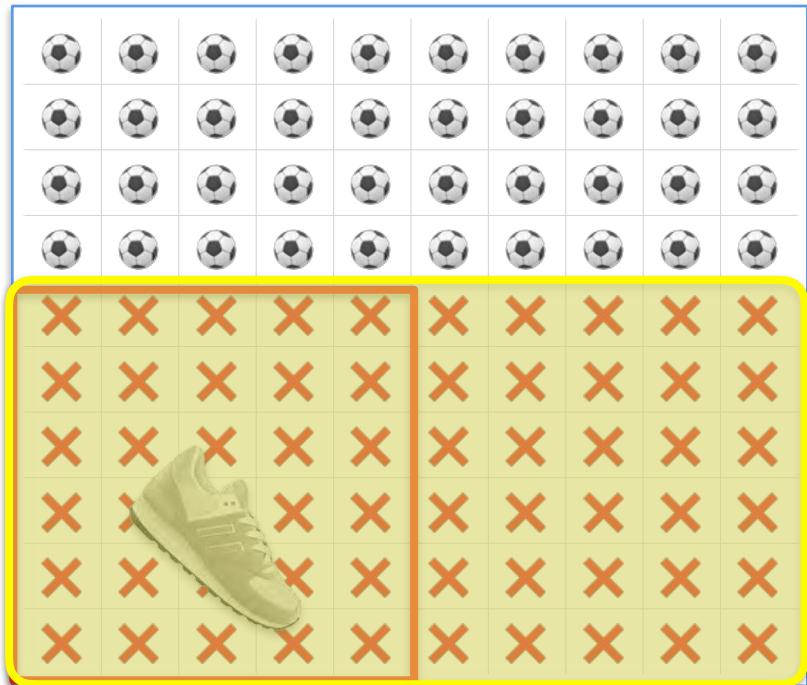
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

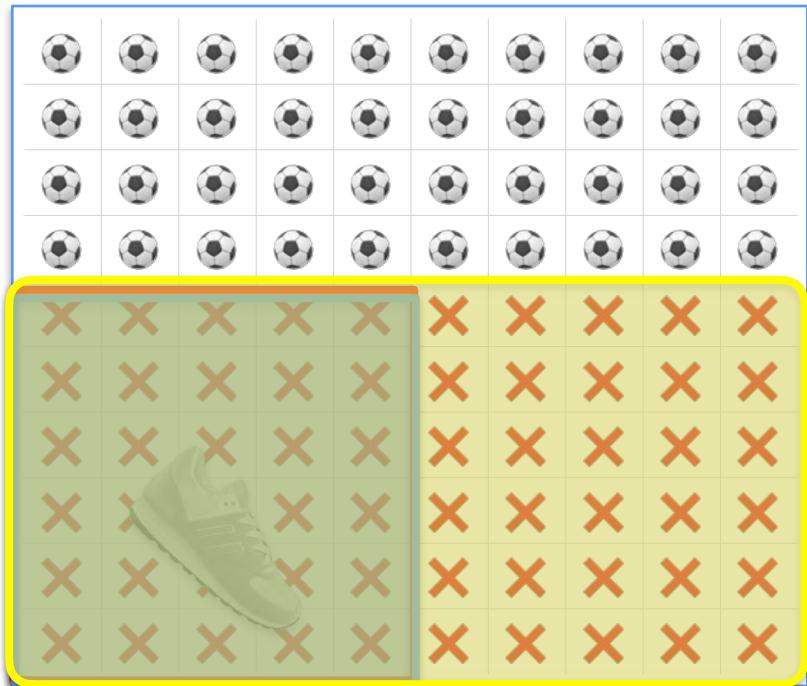
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

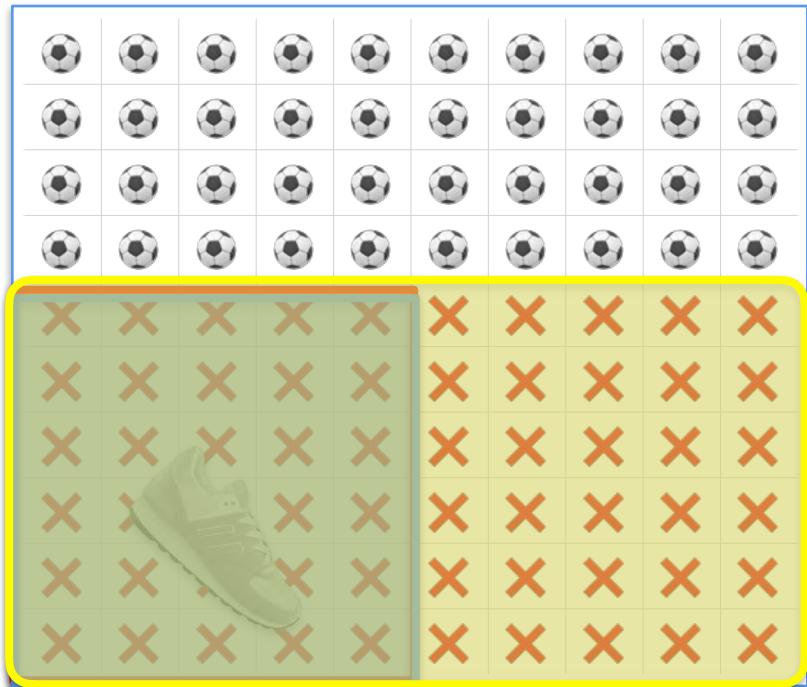
$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

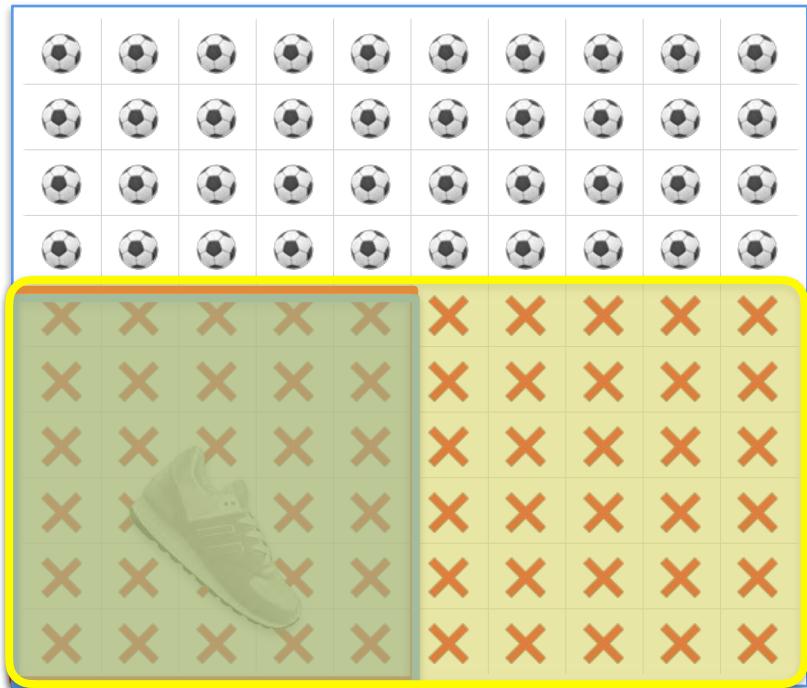
$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6 \bullet$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

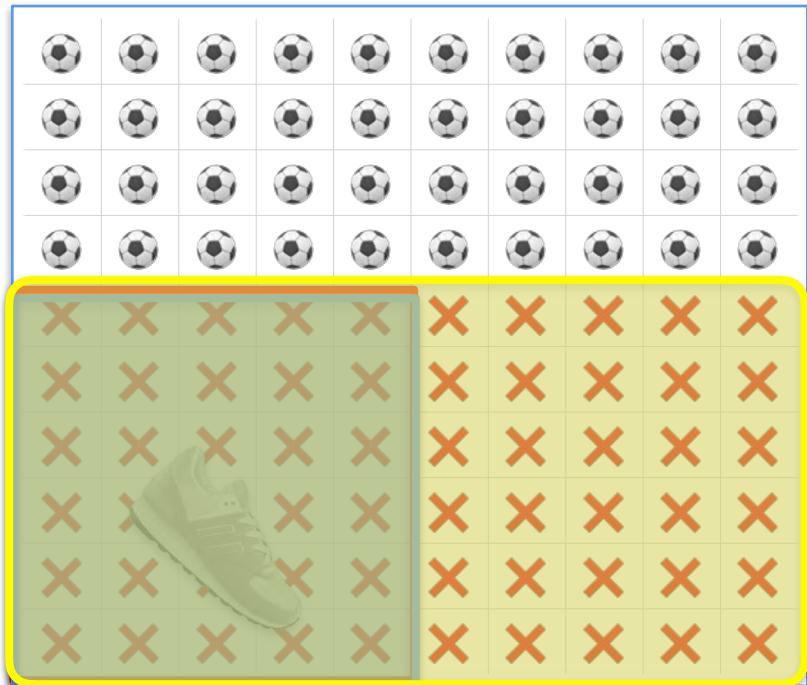
$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6 \bullet 0.5$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

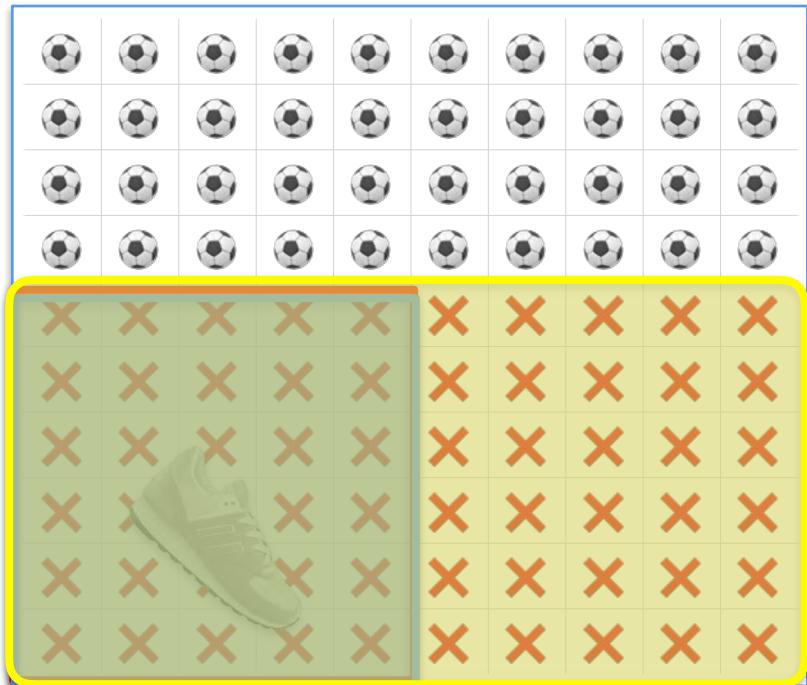
$$= 0.6 \bullet 0.5$$

$$= 0.3$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6 \bullet 0.5$$

$$= 0.3$$

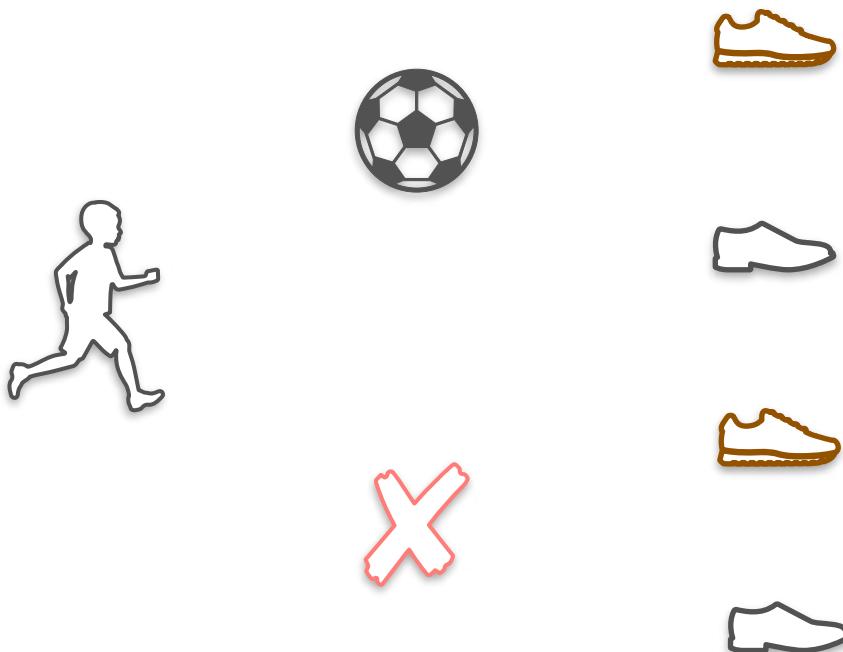


$$P(\text{not } S) = 0.6$$

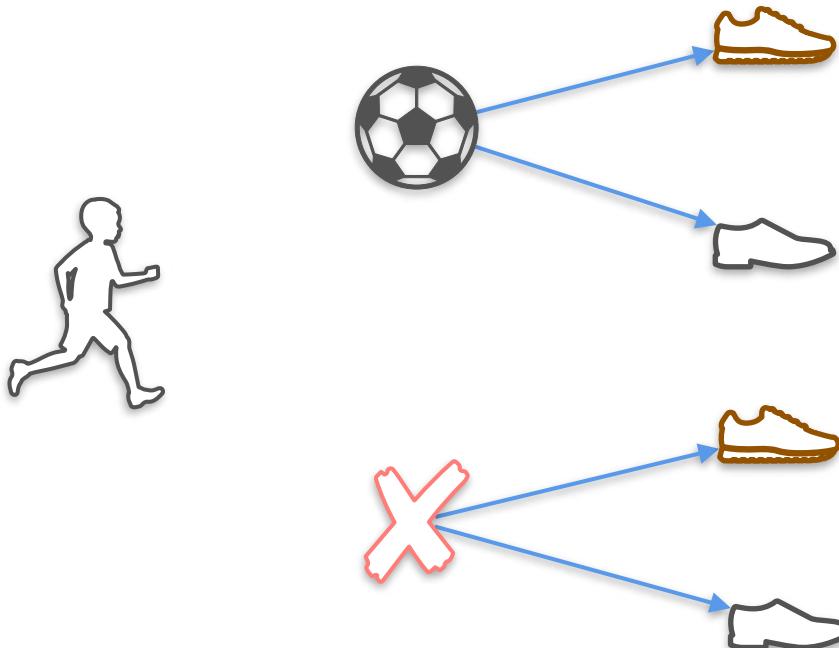
$$P(R | S) = 0.8$$



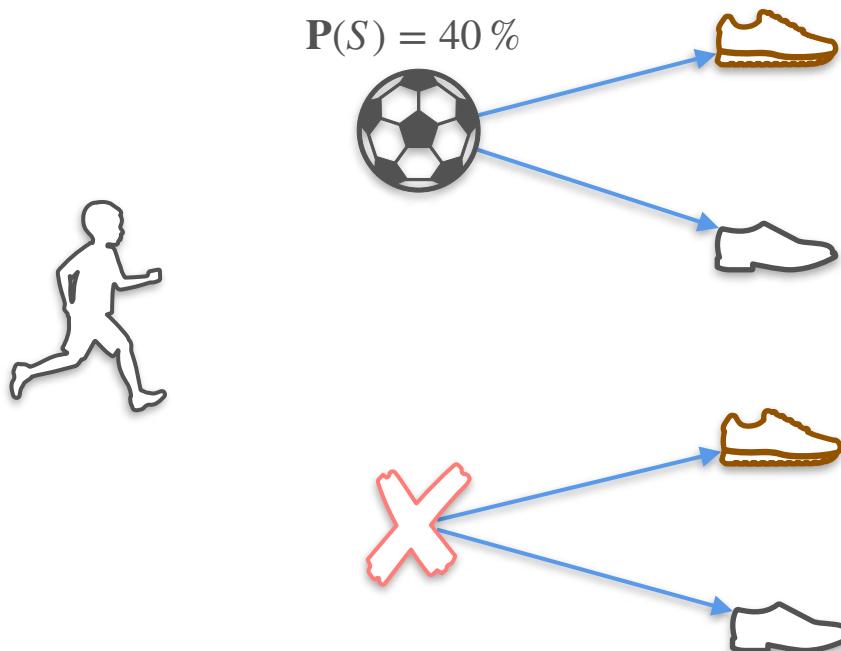
Conditional Probability



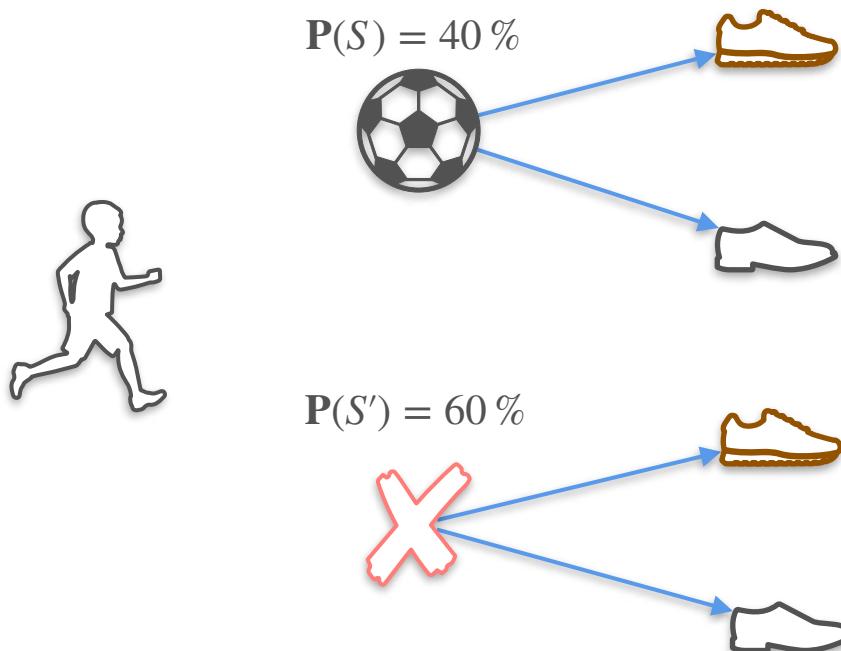
Conditional Probability



Conditional Probability

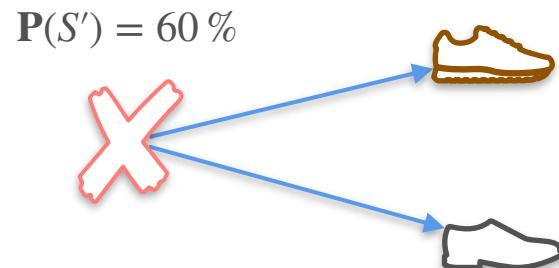
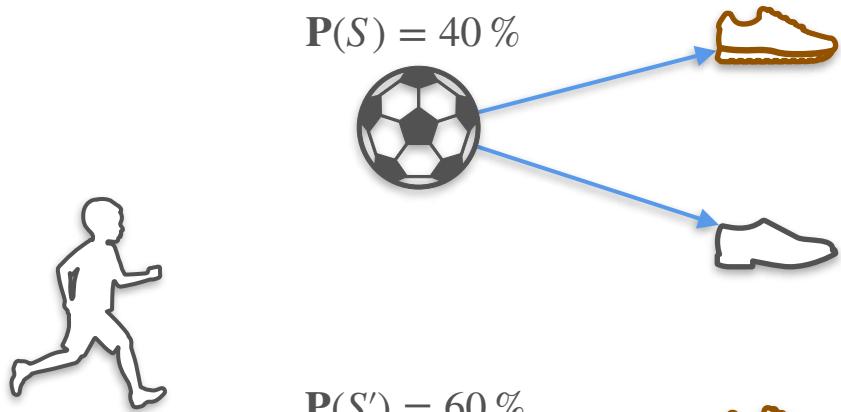


Conditional Probability

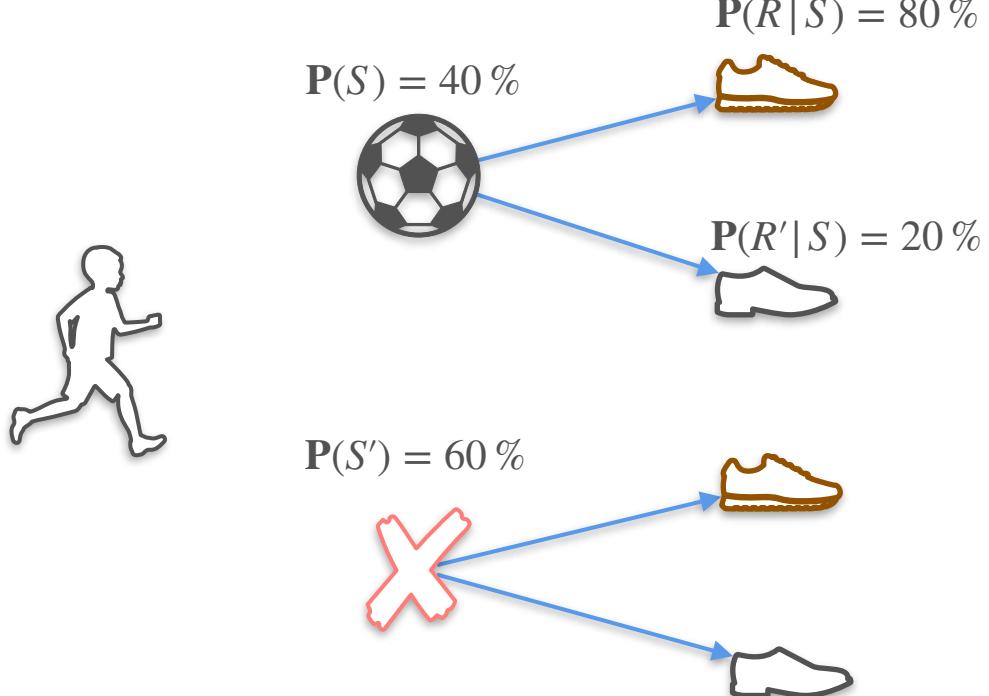


Conditional Probability

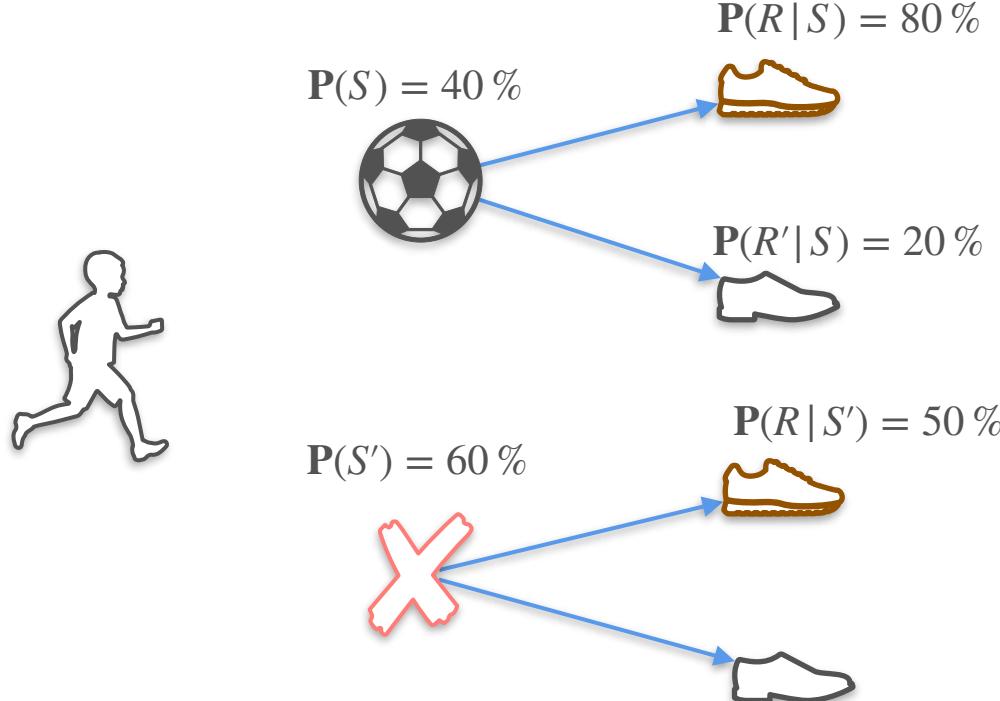
$$P(R | S) = 80 \%$$



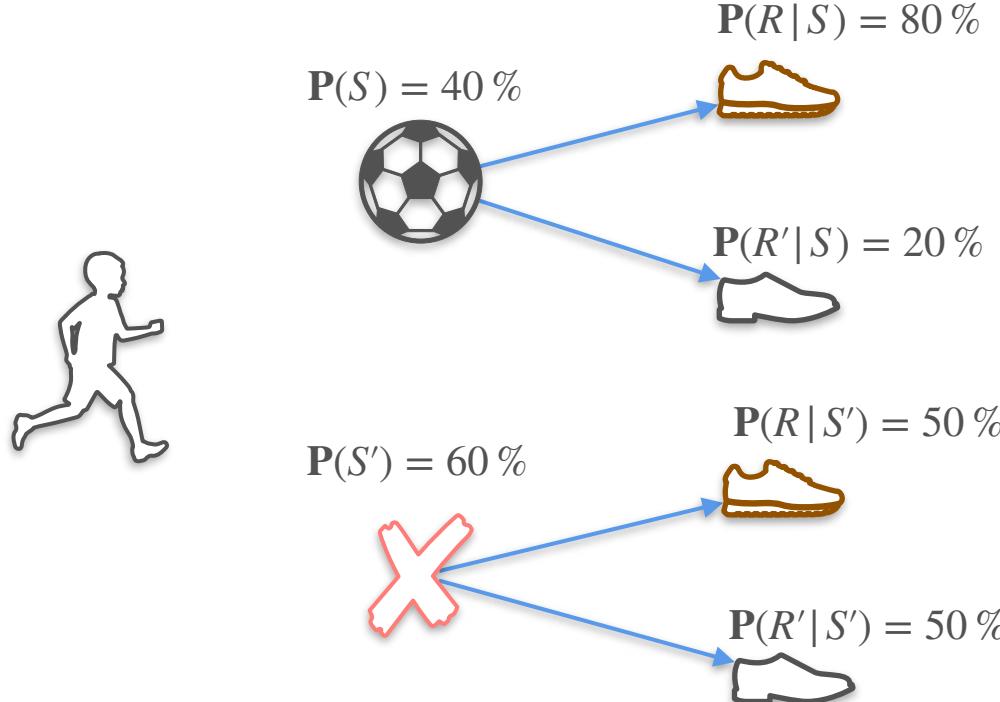
Conditional Probability



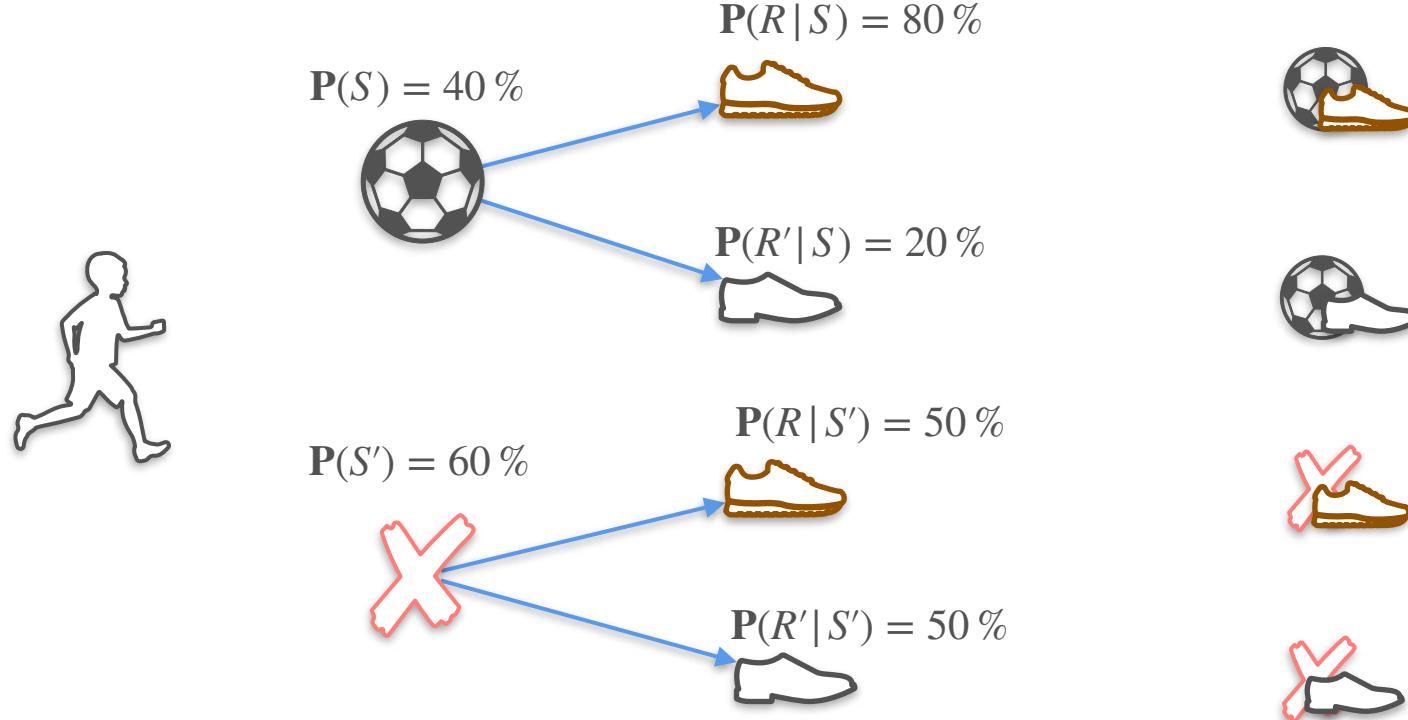
Conditional Probability



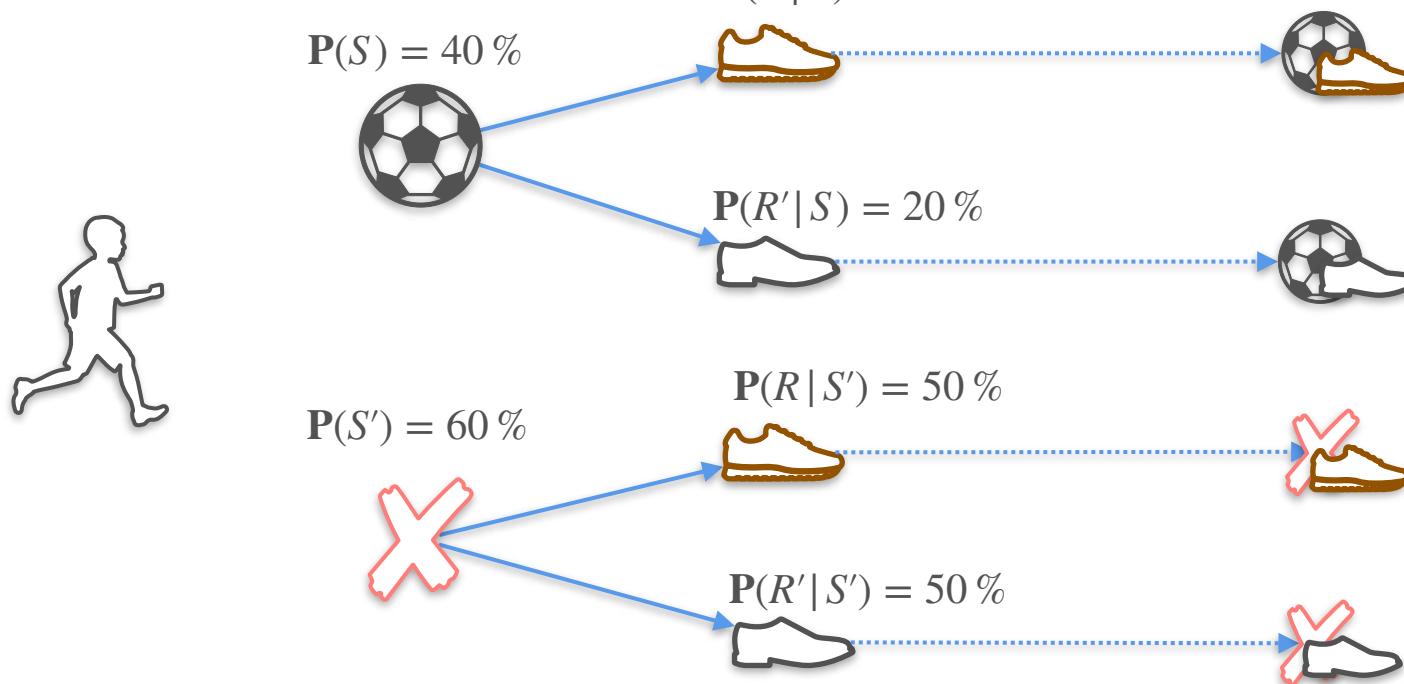
Conditional Probability



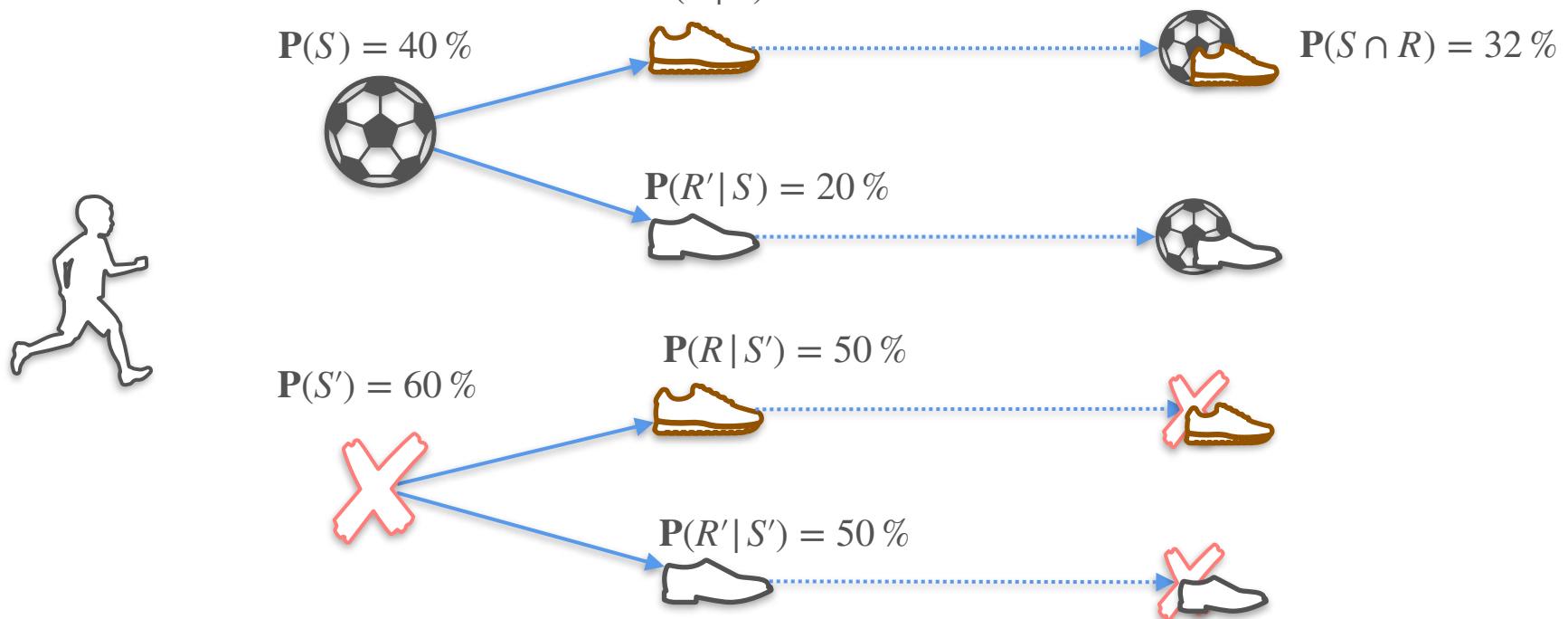
Conditional Probability



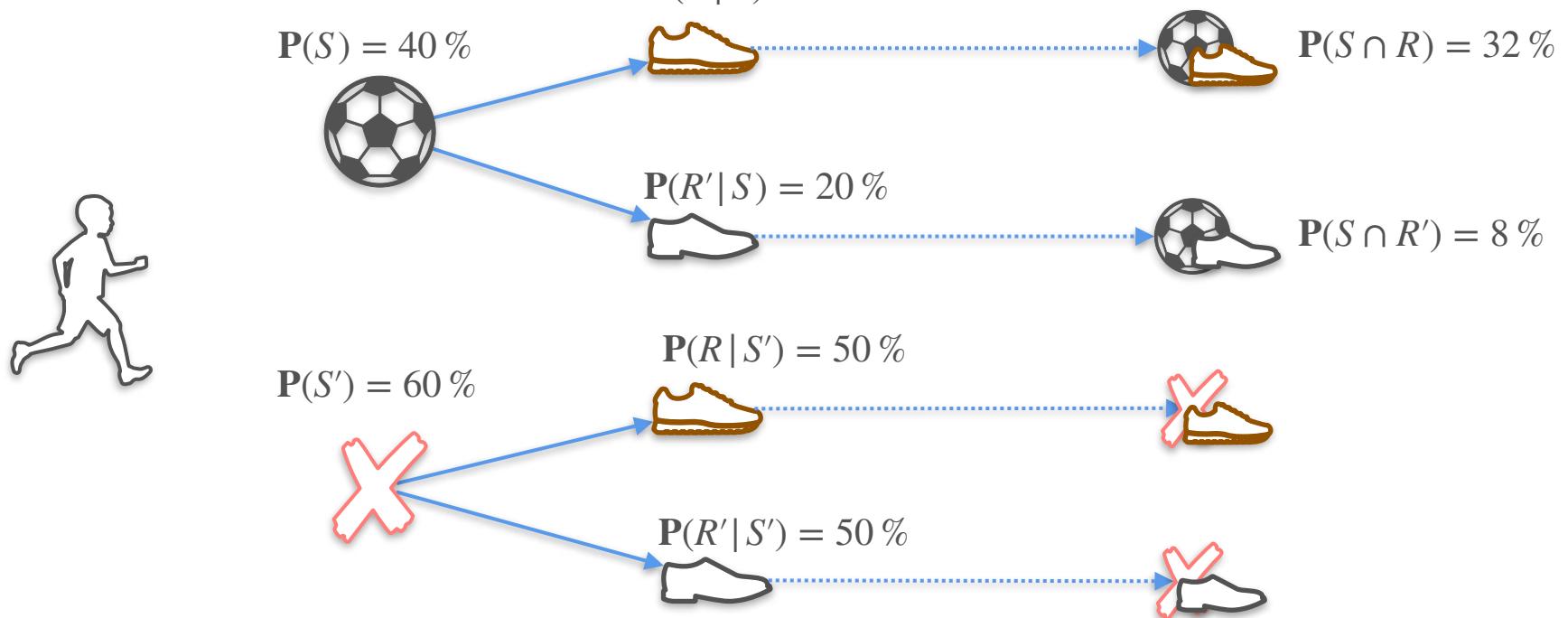
Conditional Probability



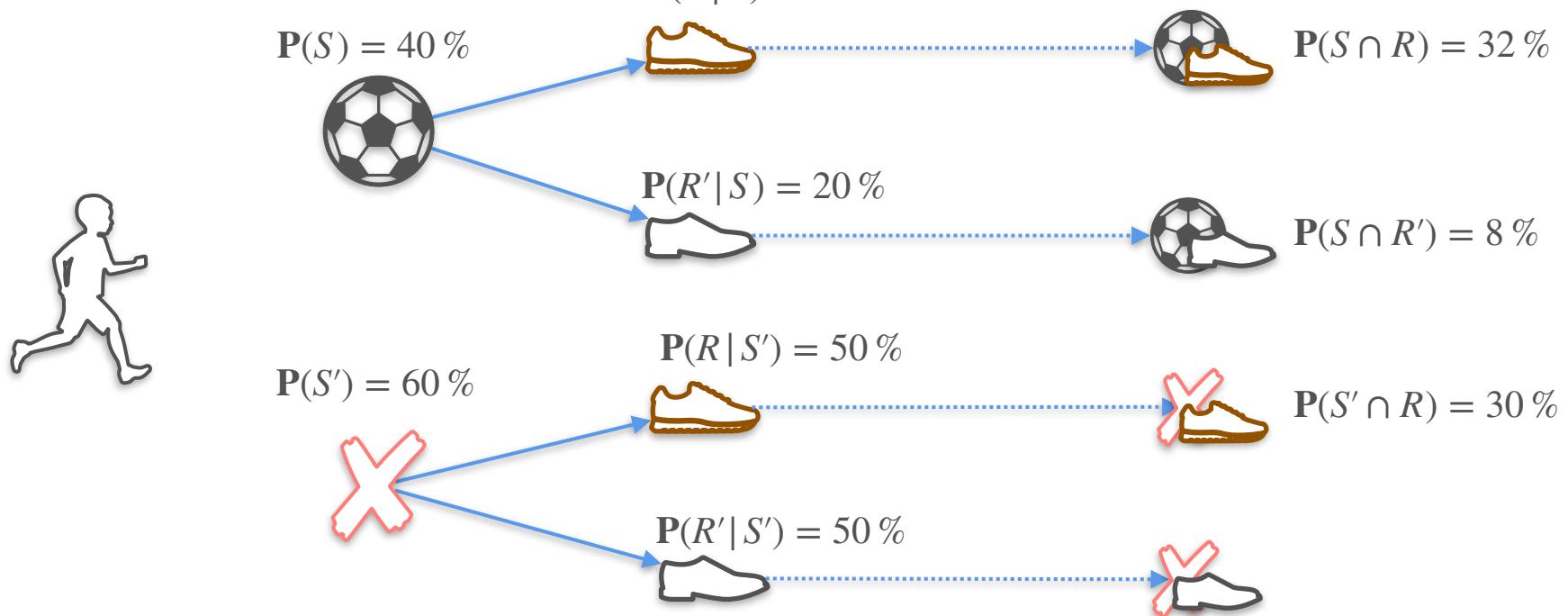
Conditional Probability



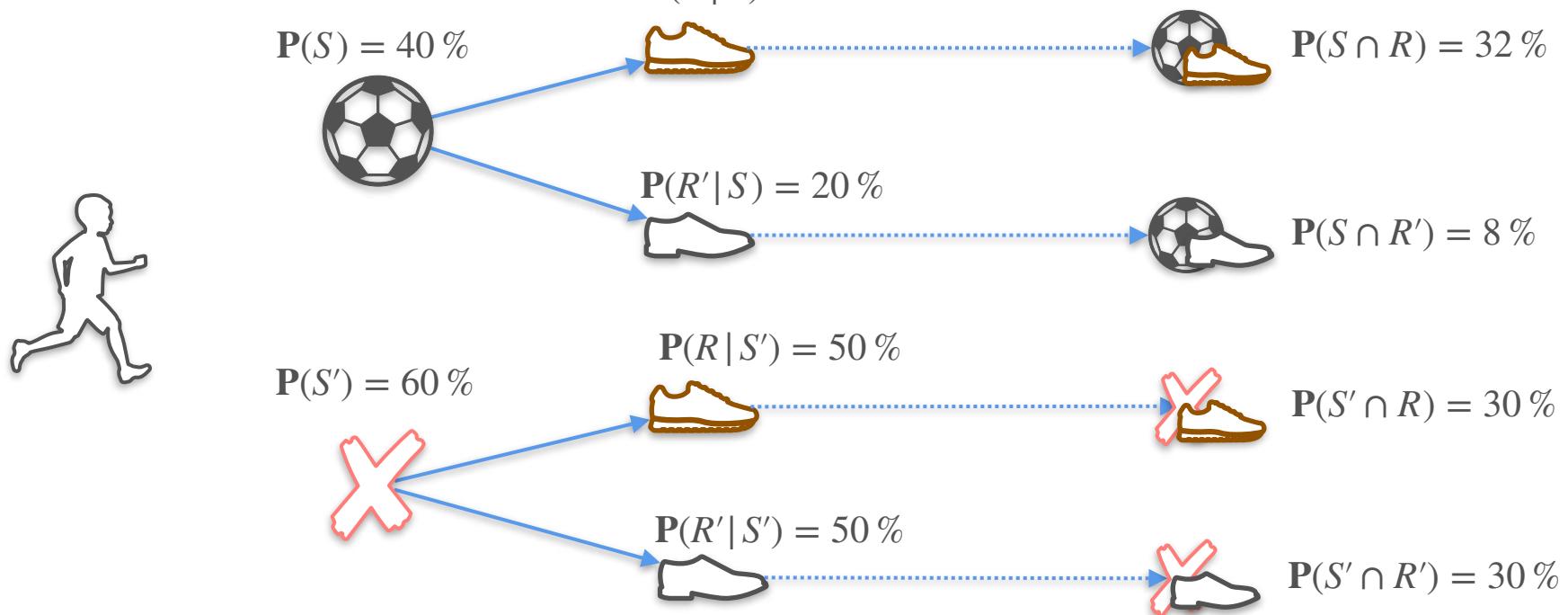
Conditional Probability



Conditional Probability

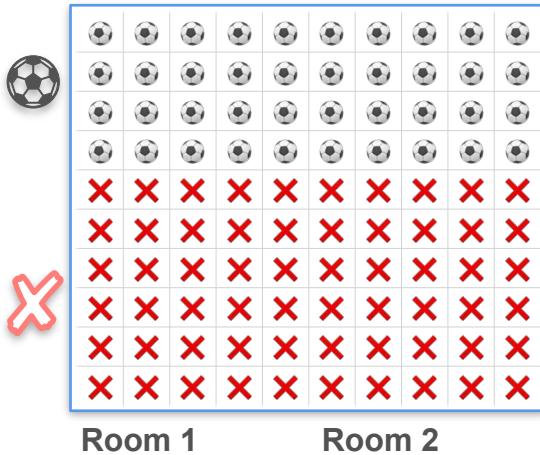


Conditional Probability



Independent vs Dependent Events

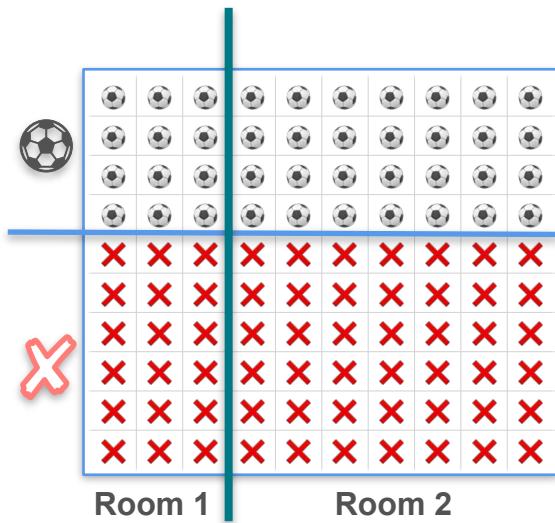
Independent vs Dependent Events



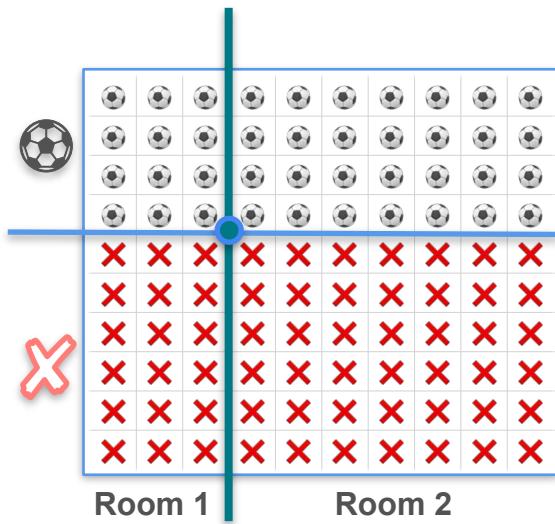
Independent vs Dependent Events



Independent vs Dependent Events



Independent vs Dependent Events



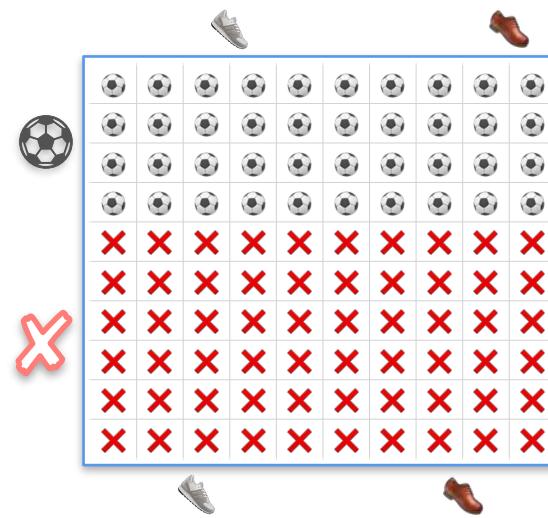
Independent vs Dependent Events

Independent



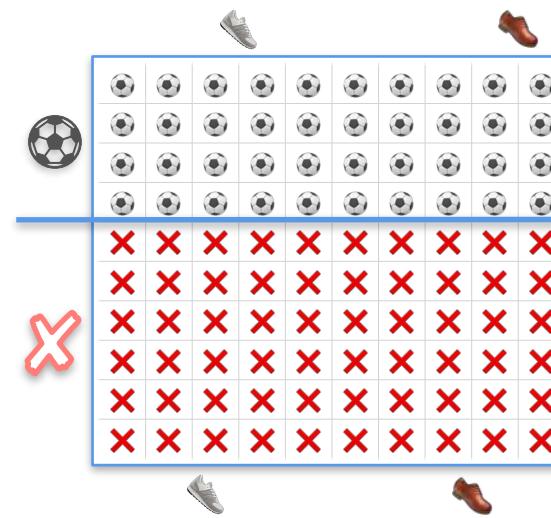
Independent vs Dependent Events

Independent



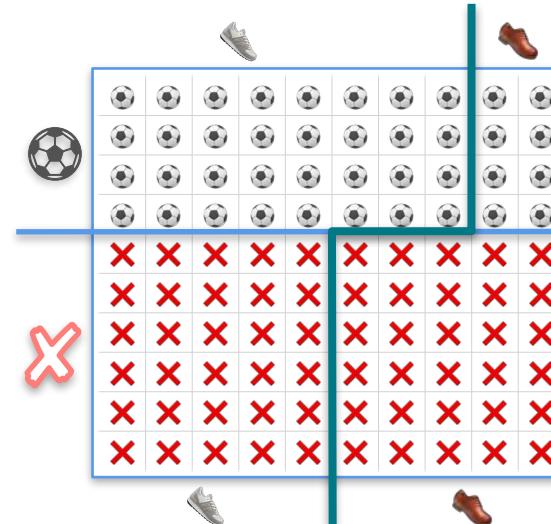
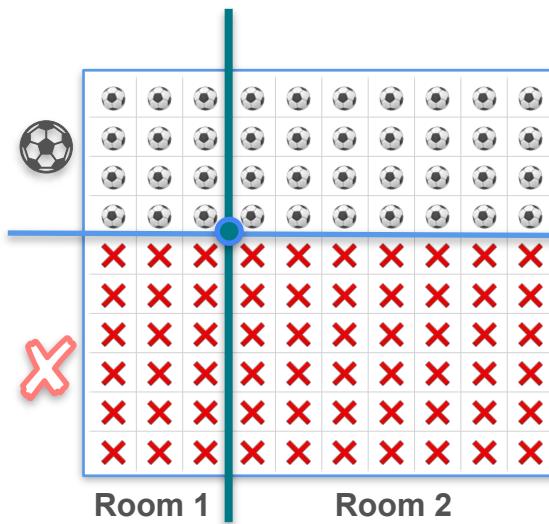
Independent vs Dependent Events

Independent



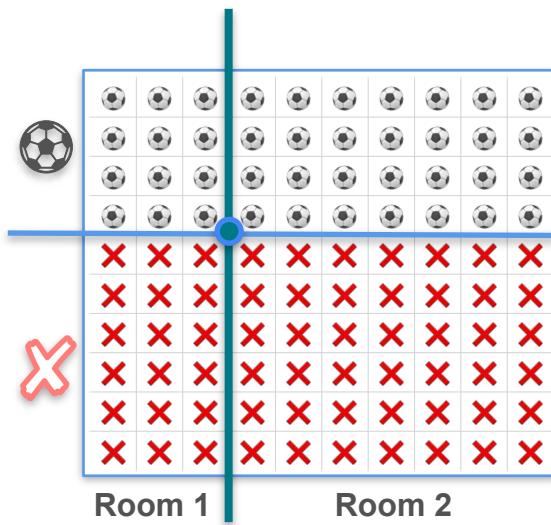
Independent vs Dependent Events

Independent

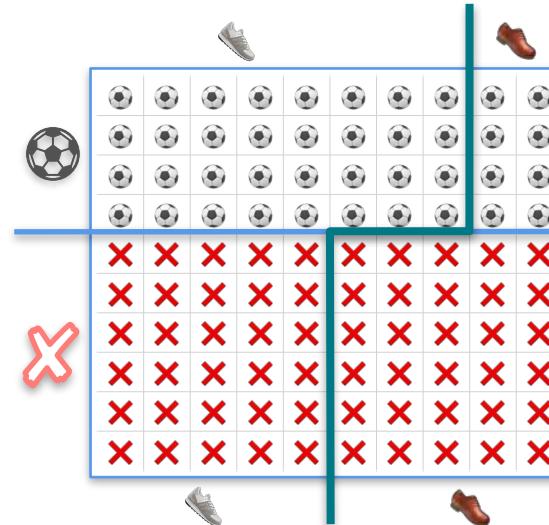


Independent vs Dependent Events

Independent



Dependent

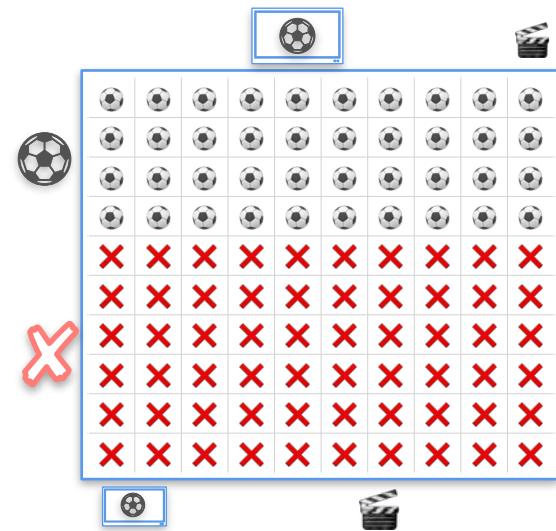
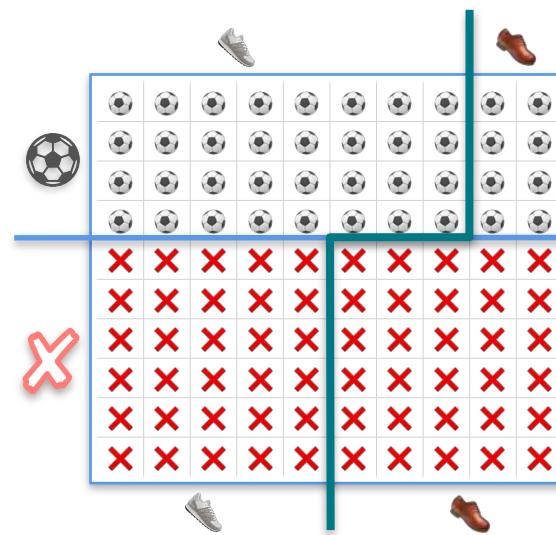


Independent vs Dependent Events

Independent



Dependent

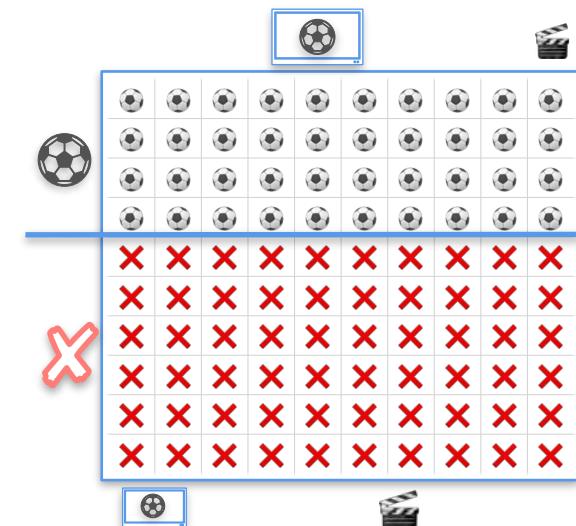
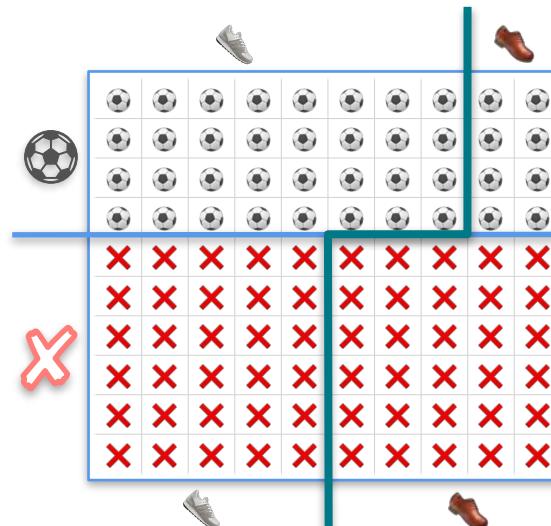


Independent vs Dependent Events

Independent



Dependent

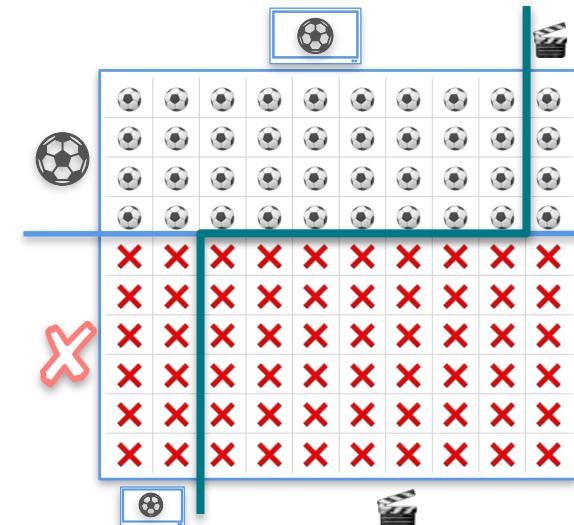
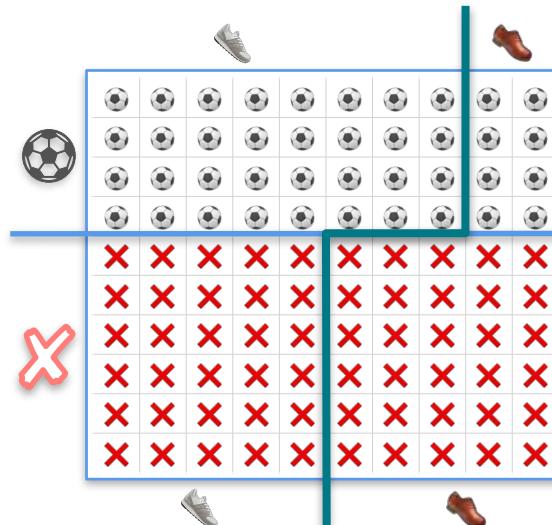


Independent vs Dependent Events

Independent



Dependent

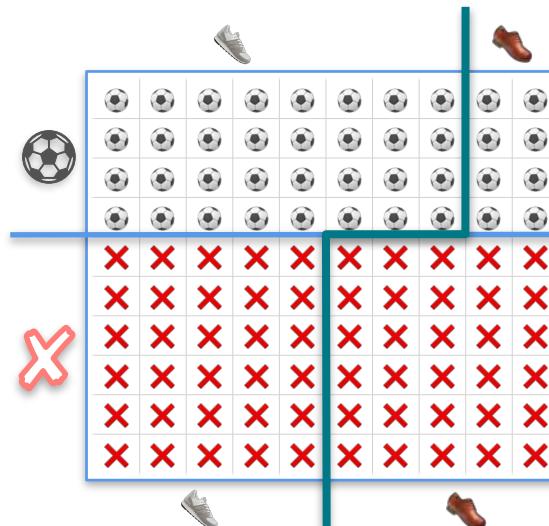


Independent vs Dependent Events

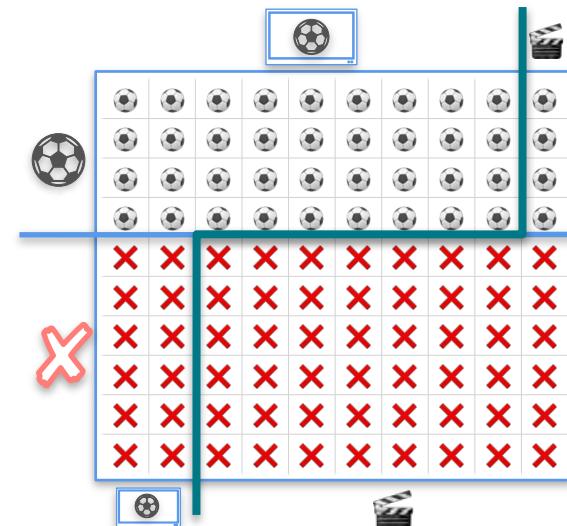
Independent



Dependent



Dependent



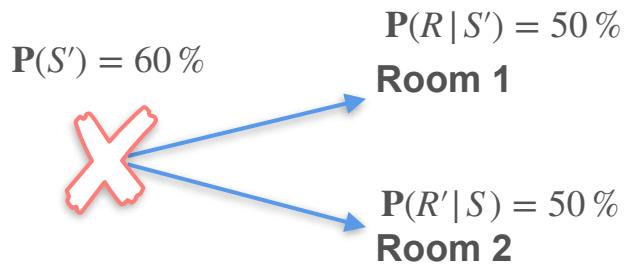
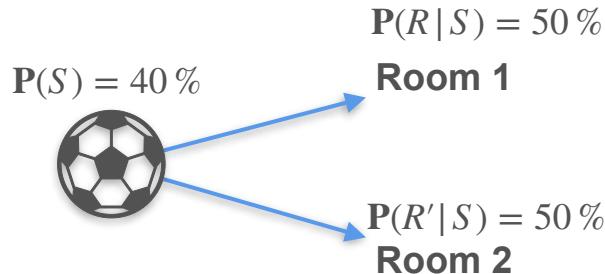
Conditional Probability

Conditional Probability

Independent

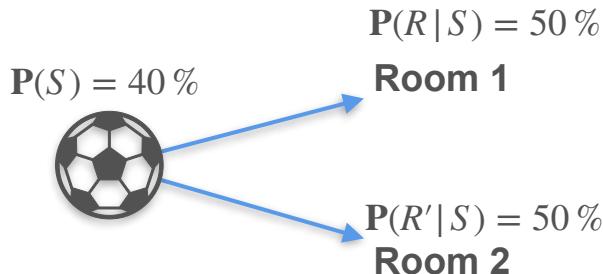
Conditional Probability

Independent

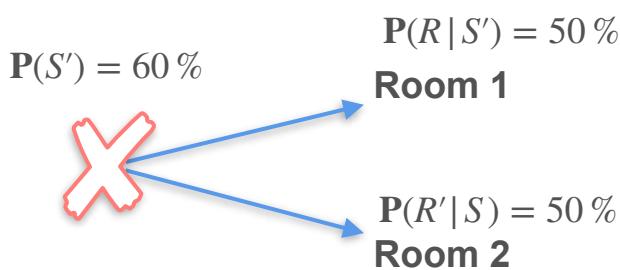


Conditional Probability

Independent

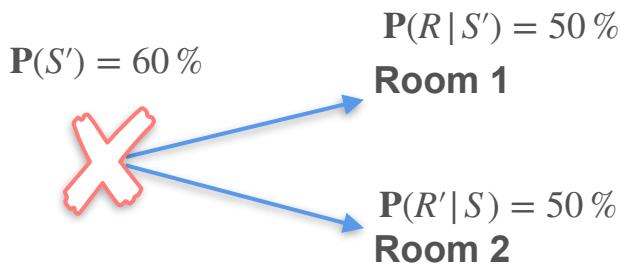
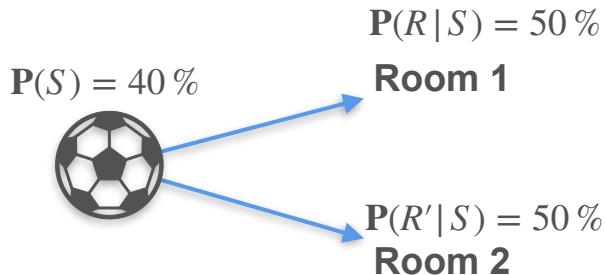


Dependent

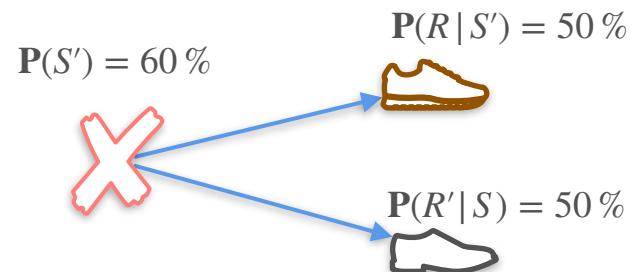
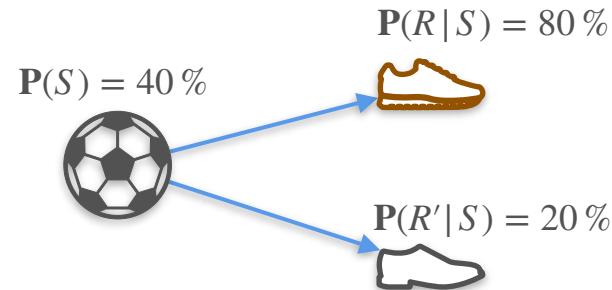


Conditional Probability

Independent

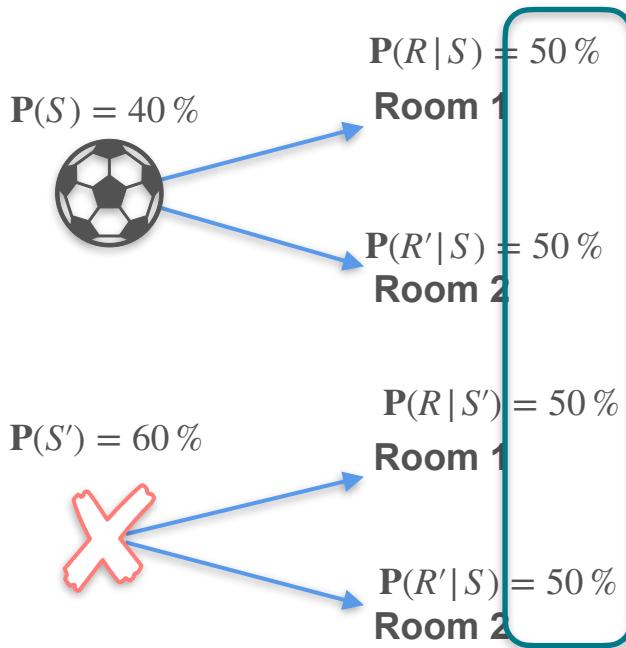


Dependent

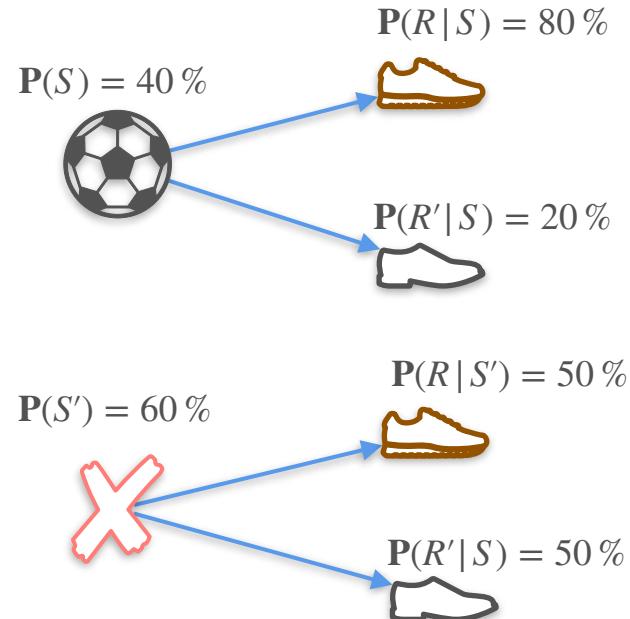


Conditional Probability

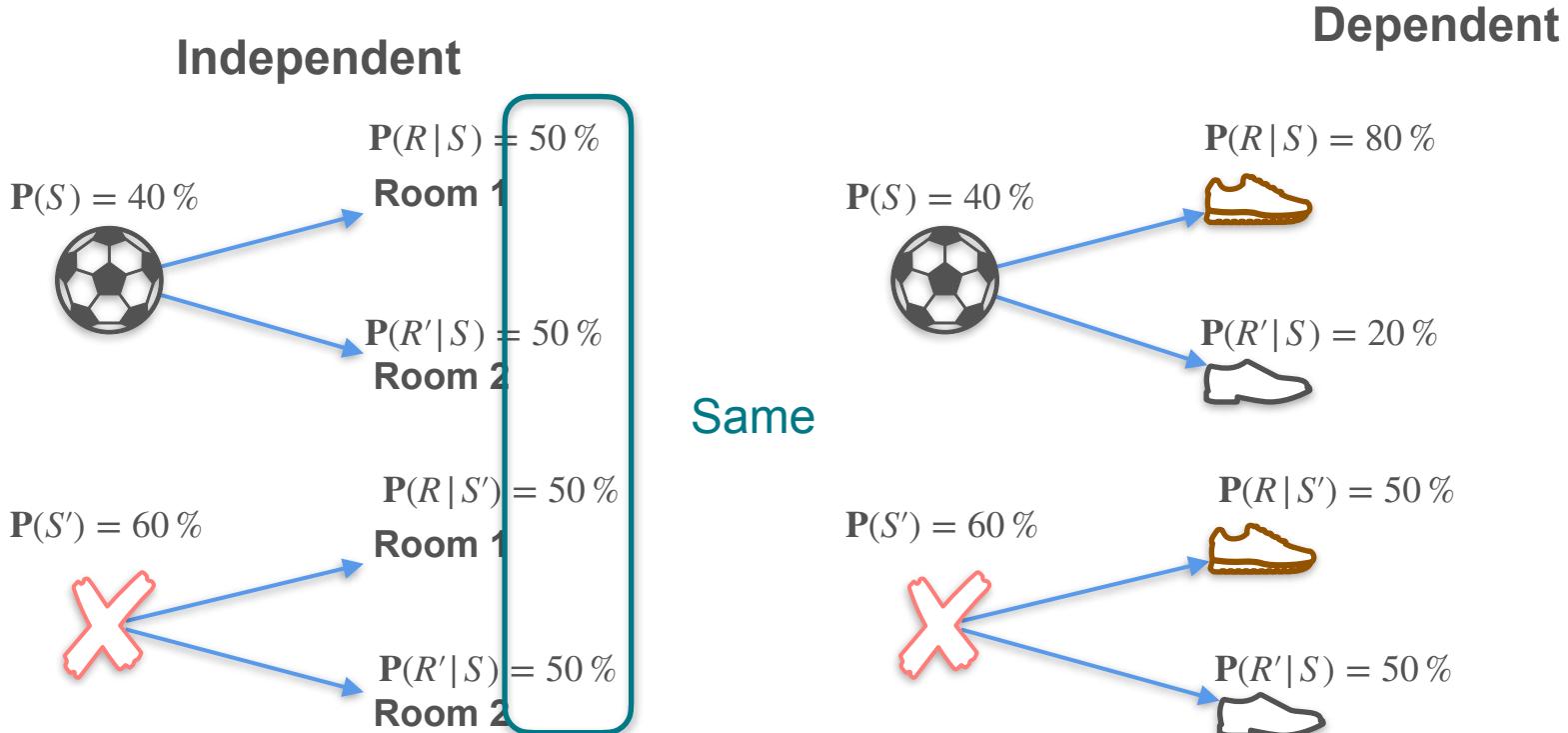
Independent



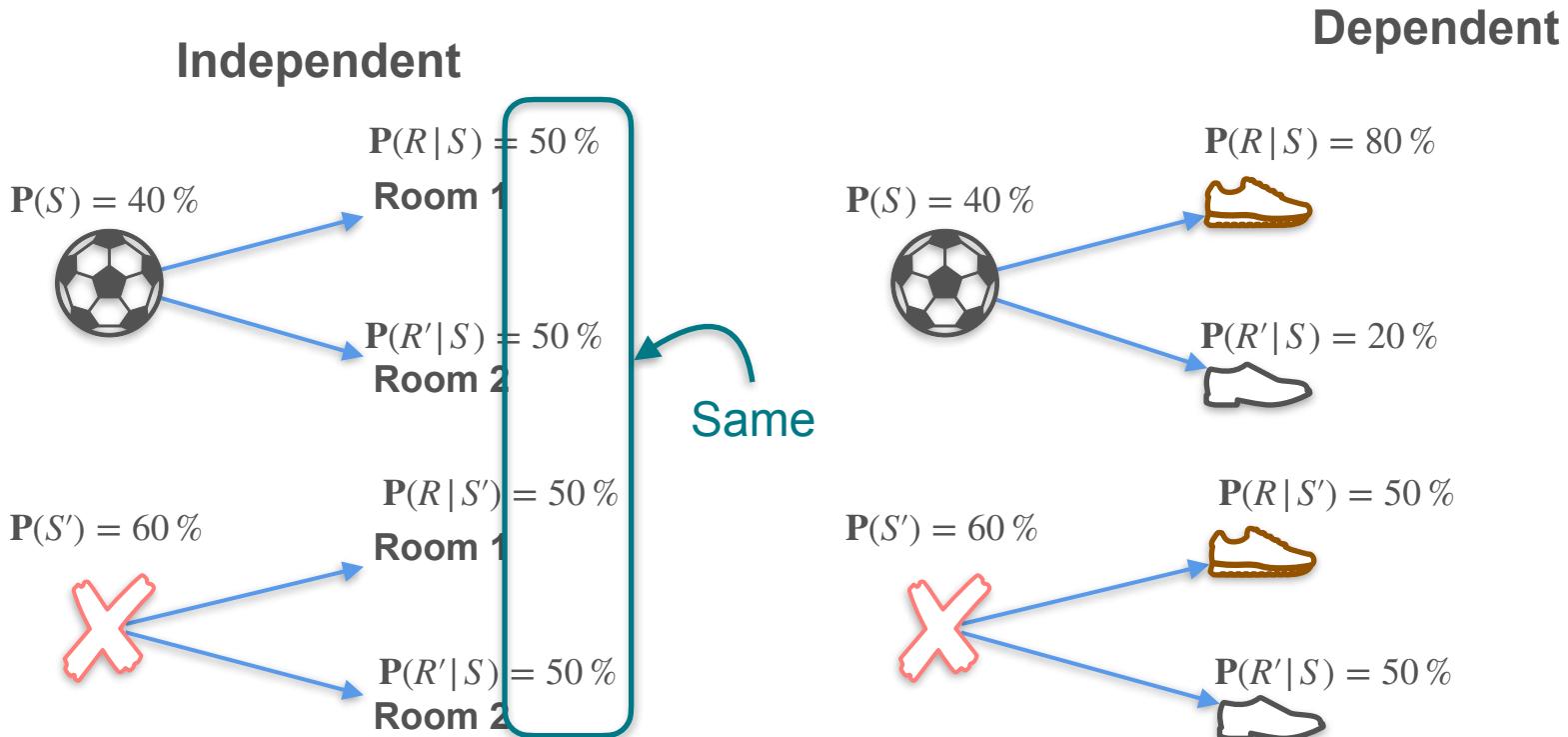
Dependent



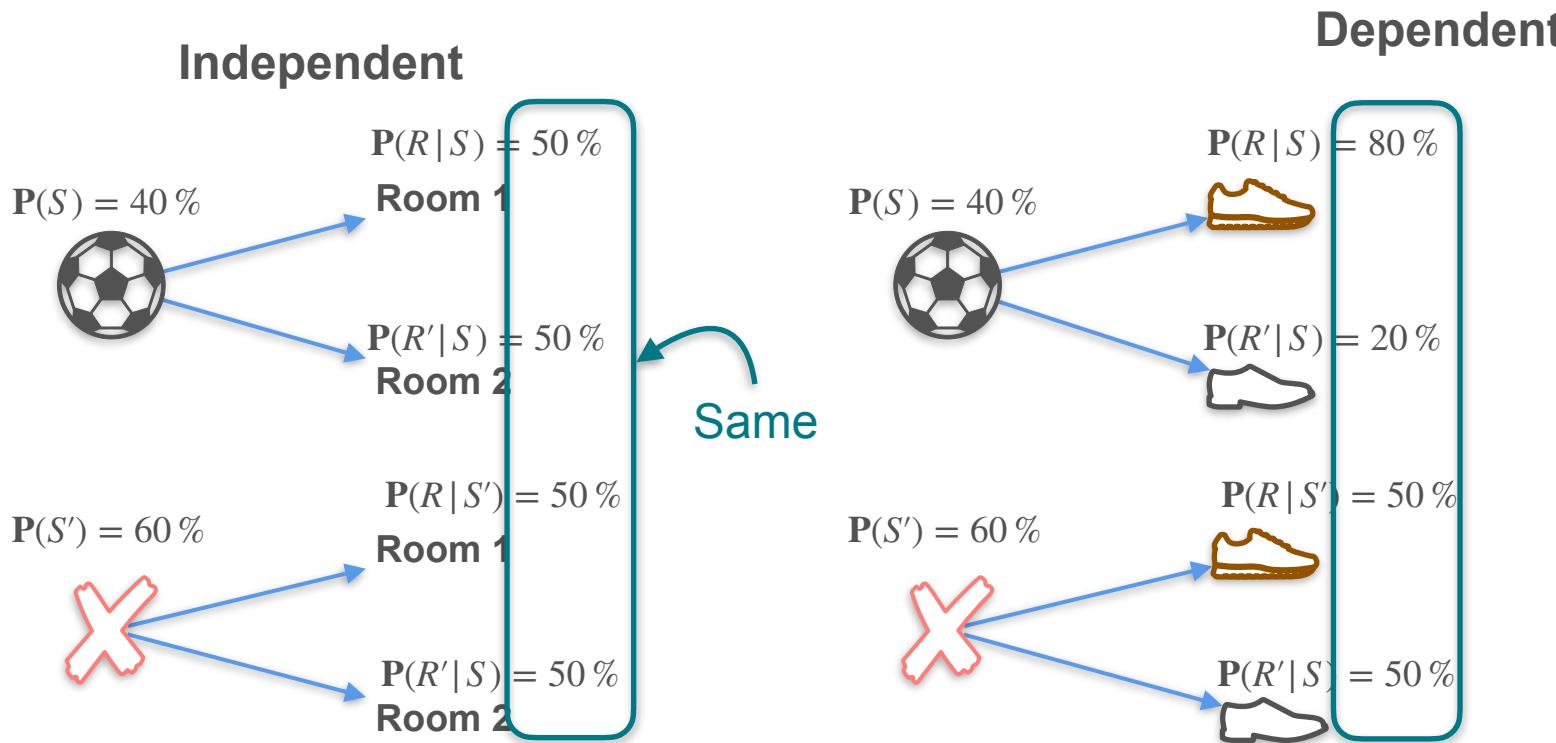
Conditional Probability



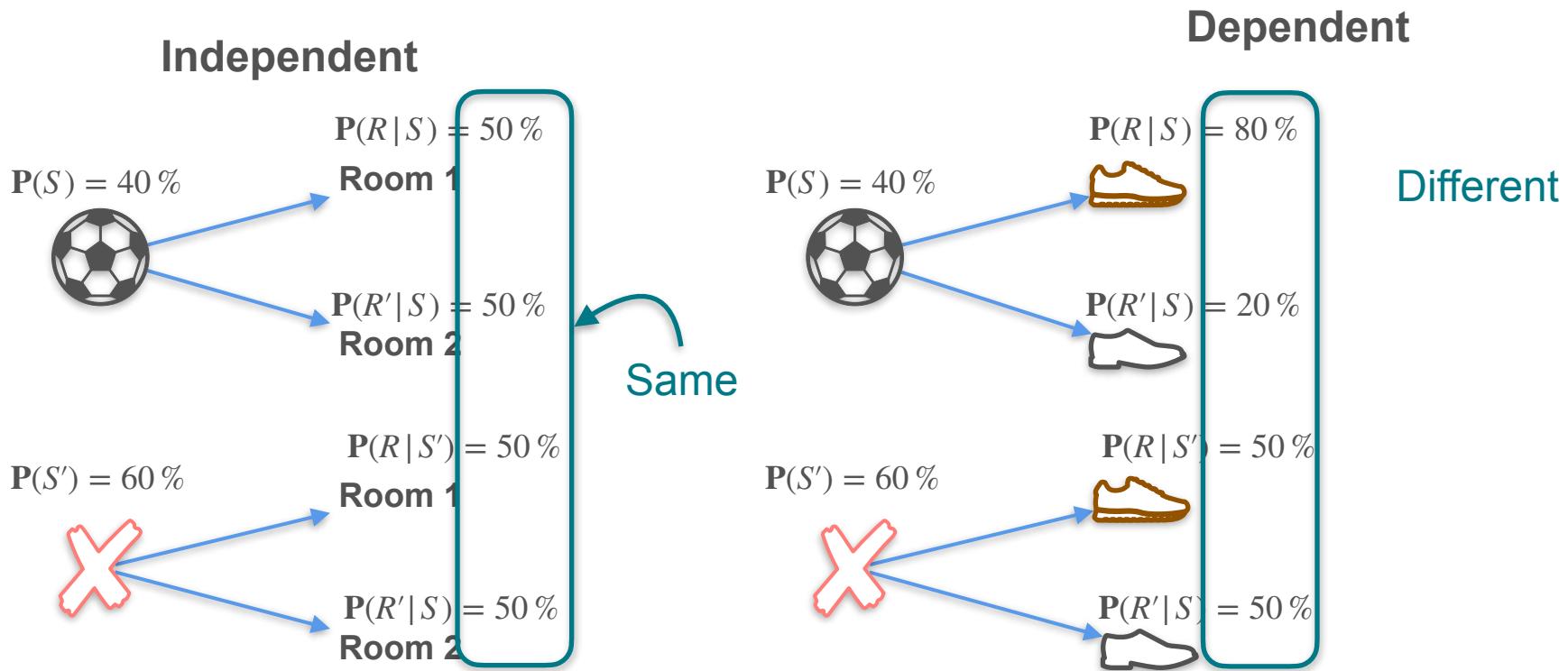
Conditional Probability



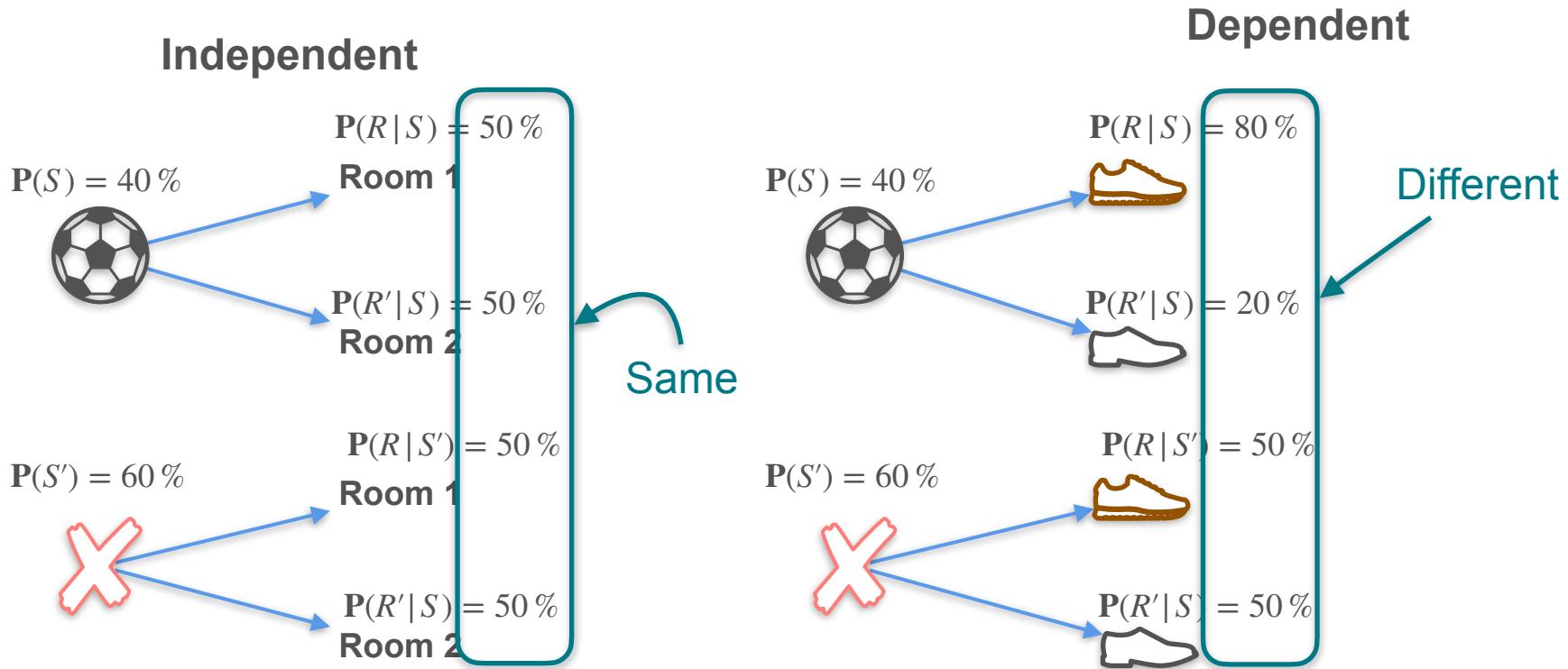
Conditional Probability



Conditional Probability



Conditional Probability





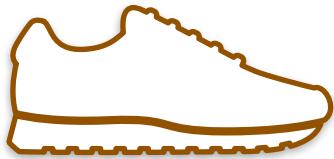
DeepLearning.AI

Introduction to probability

Bayes theorem

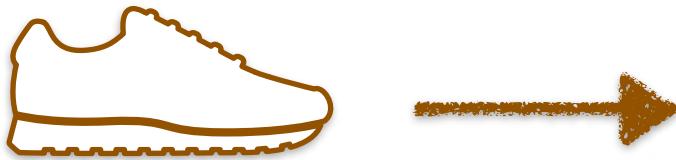
Bayes Theorem - Motivation

Bayes Theorem - Motivation



60%

Bayes Theorem - Motivation

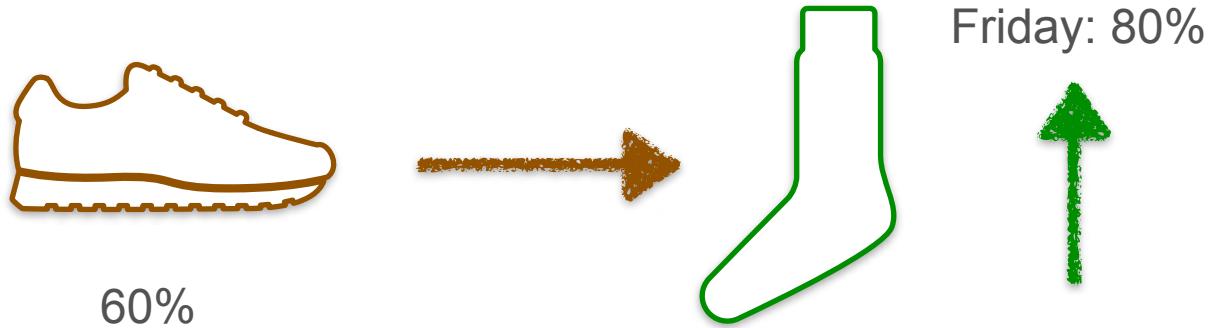


60%

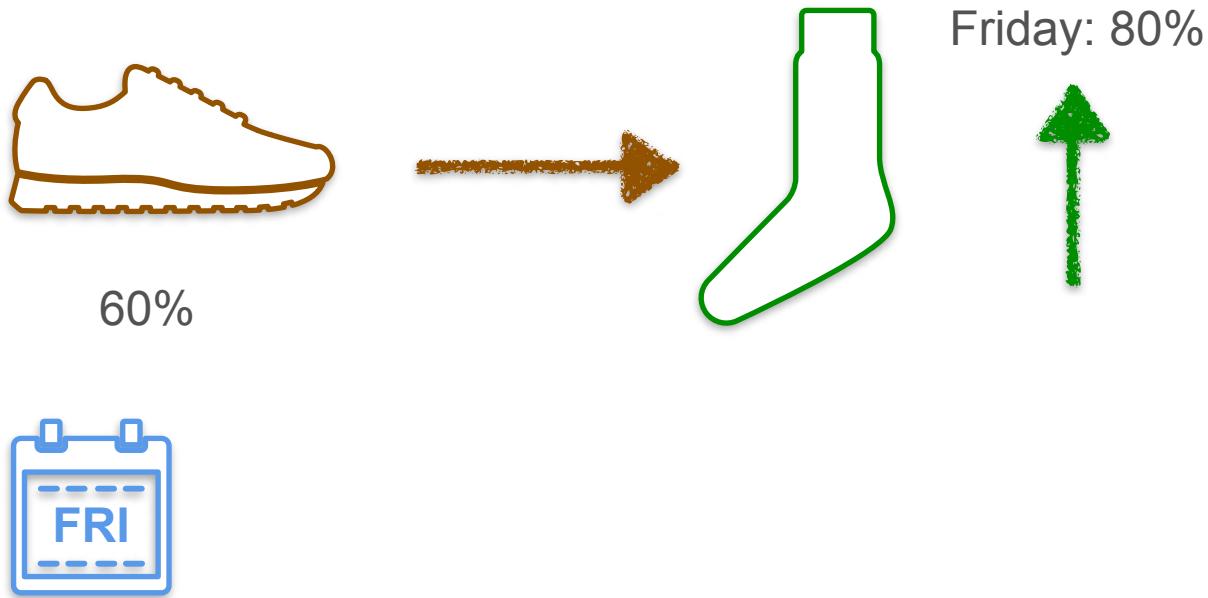
Bayes Theorem - Motivation



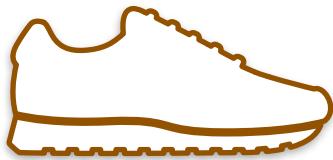
Bayes Theorem - Motivation



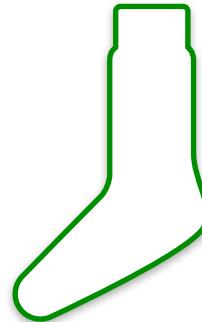
Bayes Theorem - Motivation



Bayes Theorem - Motivation



60%



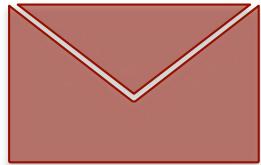
Friday: 80%



What is the probability that a customer will purchase a pair of socks given that they purchased shoes?

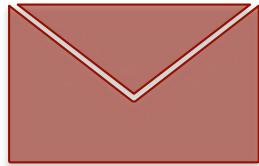
Bayes Theorem: Motivation

Bayes Theorem: Motivation



spam

Bayes Theorem: Motivation

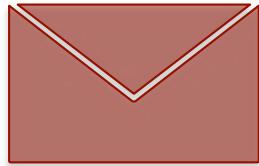


spam



contains
“lottery”

Bayes Theorem: Motivation



spam

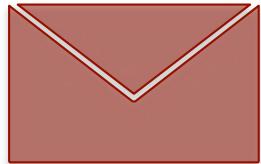


contains
“lottery”

so



Bayes Theorem: Motivation

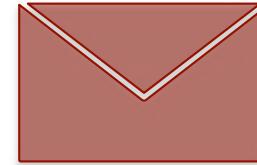


spam



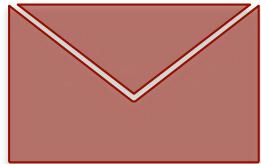
contains
“lottery”

so



spam

Bayes Theorem: Motivation

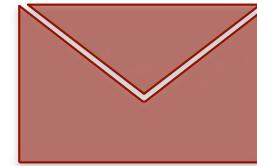


spam

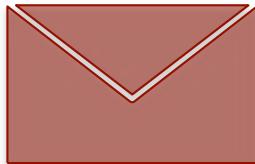


contains
“lottery”

so

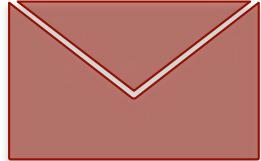


spam



spam

Bayes Theorem: Motivation

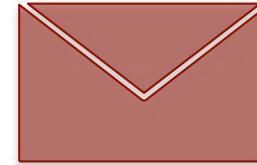


spam

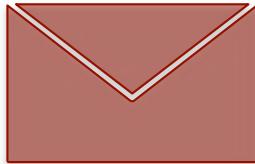


contains
“lottery”

so



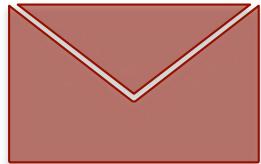
spam



spam

so

Bayes Theorem: Motivation

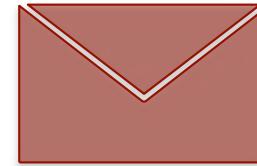


spam

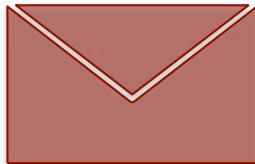


contains
“lottery”

so



spam



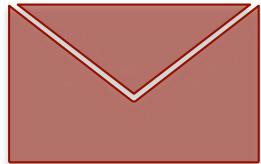
spam

so



contains
lottery

Bayes Theorem: Motivation

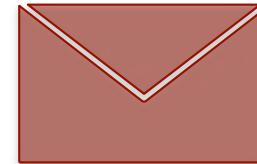


spam

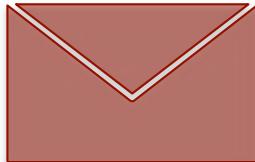


contains
“lottery”

so



spam



spam

?



contains
lottery

Bayes Theorem: Intuition

Bayes Theorem: Intuition



1,000,000 people

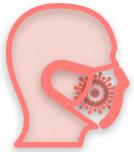


1 / 10,000 people

Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



99% Effective

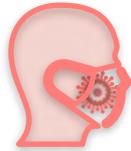
Bayes Theorem: Intuition



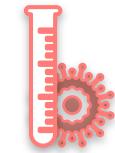
1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective

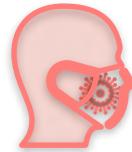
Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people

Diagnosed Sick



99

Diagnosed Healthy



1



99% Effective



100 people



1

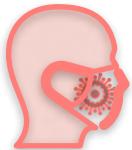


99

Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



100 people



1

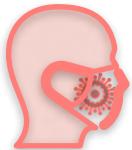


99

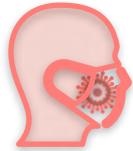
Bayes Theorem: Intuition



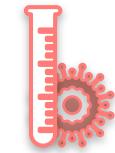
1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



100 people



1



99

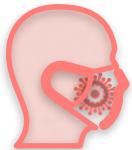


What's the probability that **you are sick**

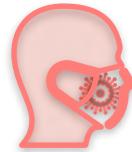
Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



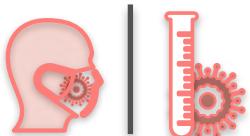
100 people



1



99

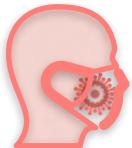


What's the probability that **you are sick**
GIVEN that you tested sick?

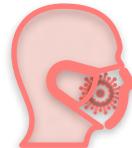
Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people

Diagnosed Sick



99

Diagnosed Healthy



1



99% Effective



Tested Sick



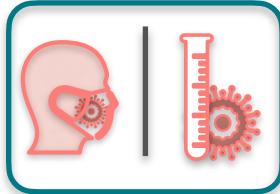
100 people



1

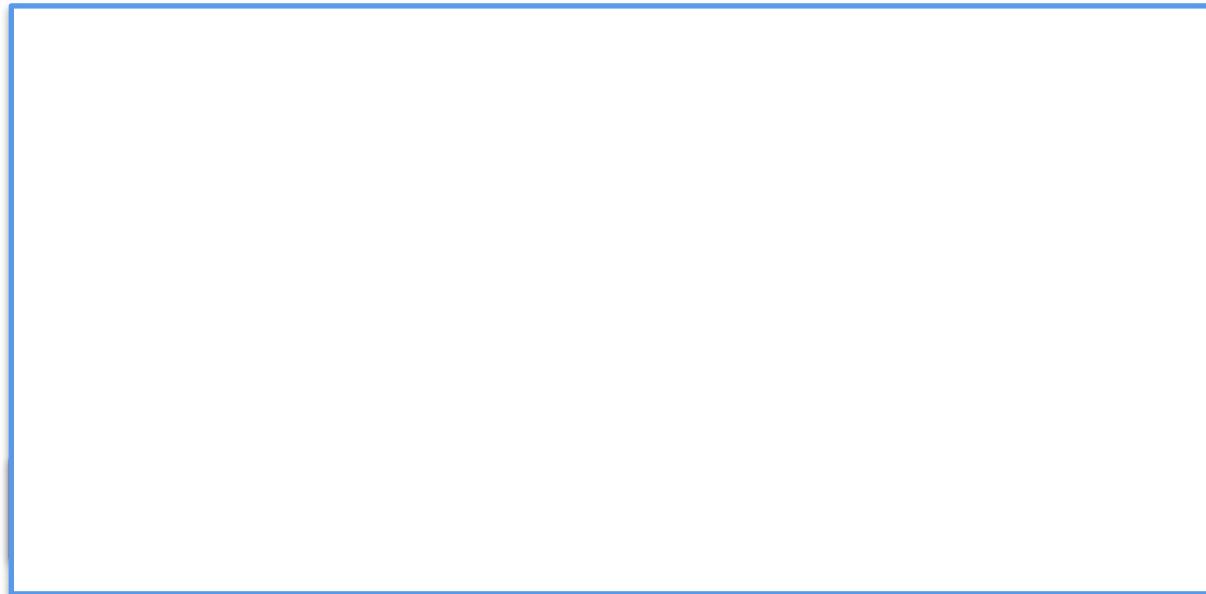


99



What's the probability that **you are sick**
GIVEN that you tested sick?

Bayes Theorem: Intuition



1,000,000 people

Bayes Theorem: Intuition



Healthy

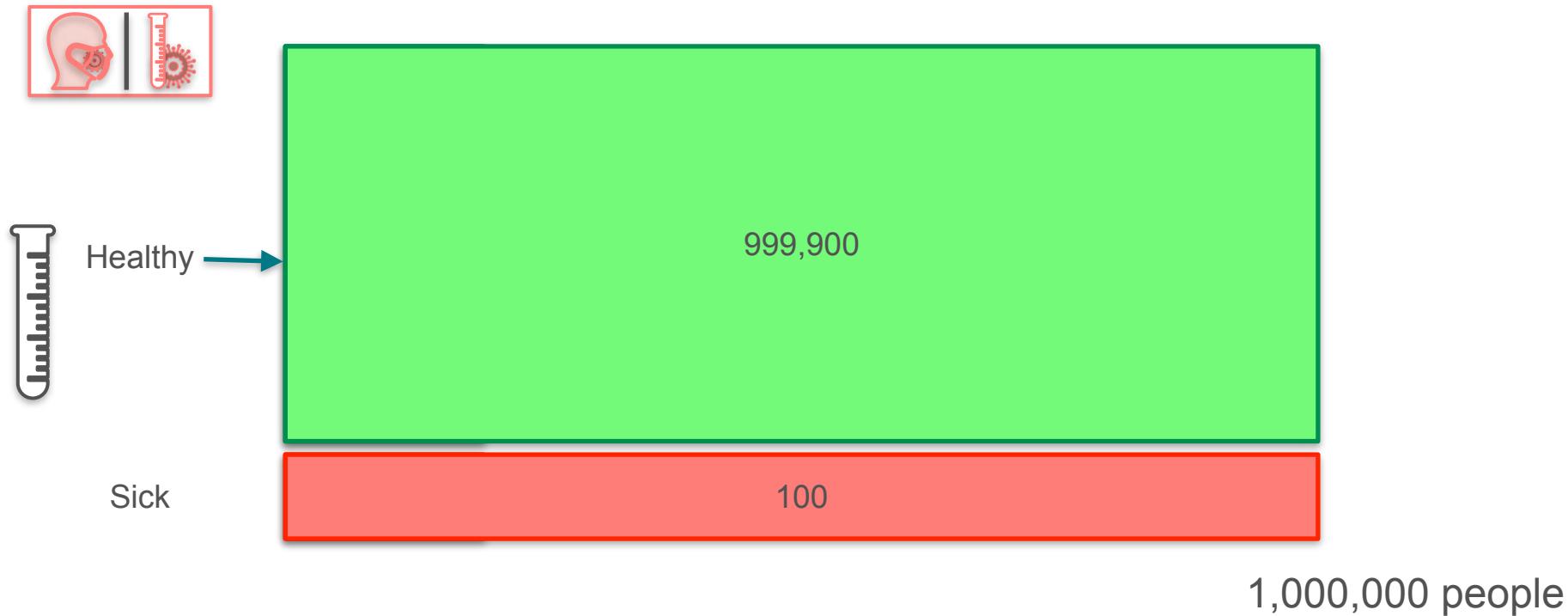
999,900

Sick

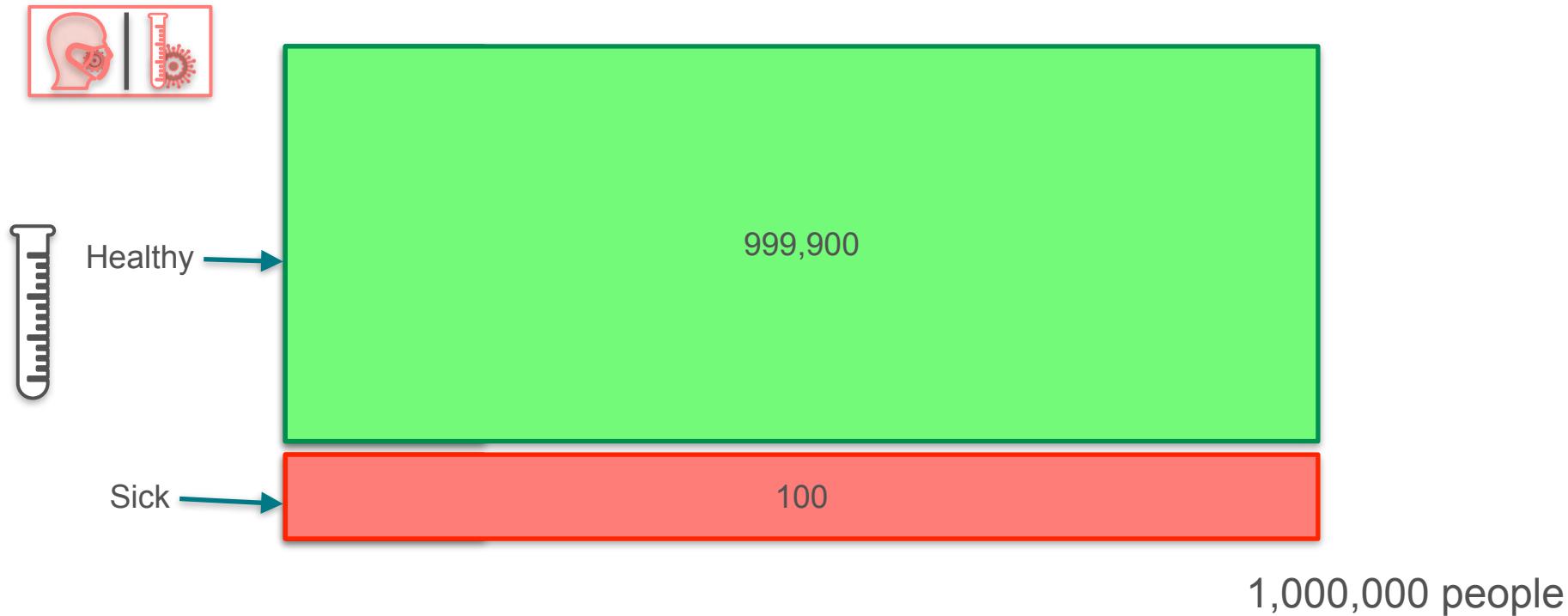
100

1,000,000 people

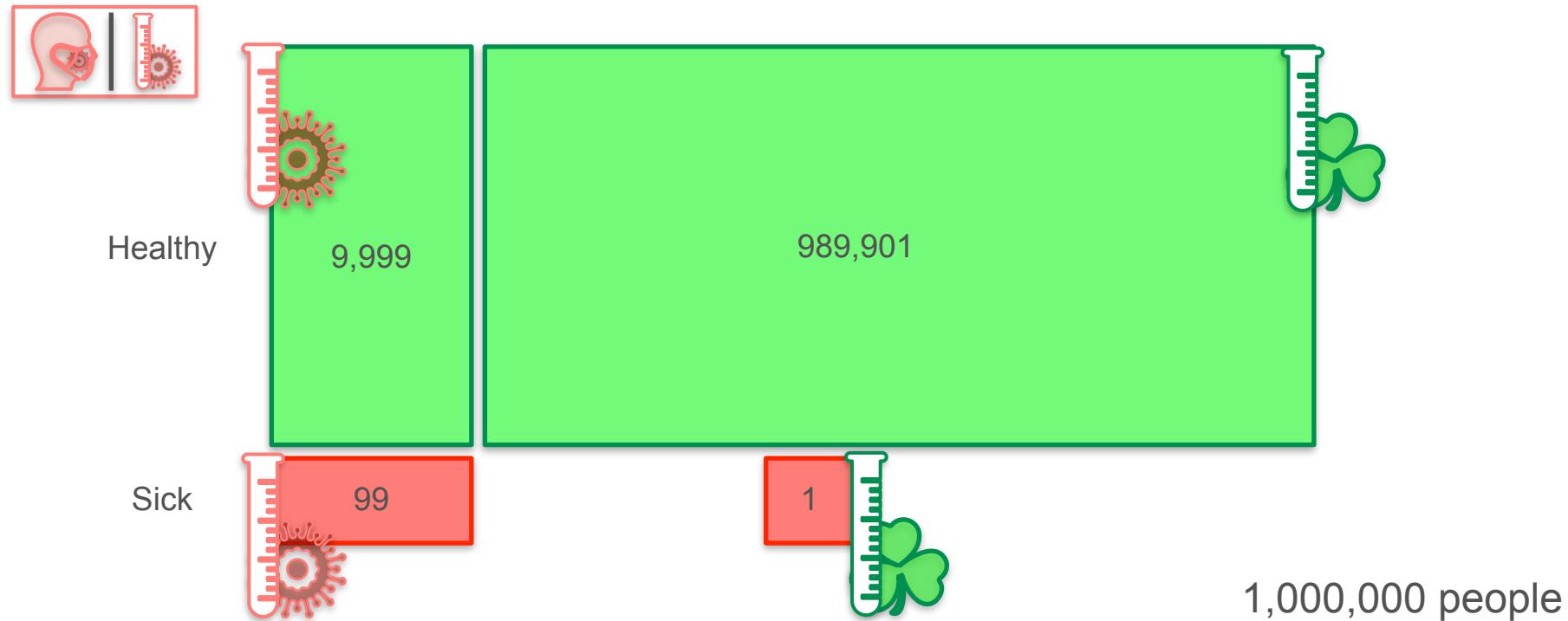
Bayes Theorem: Intuition



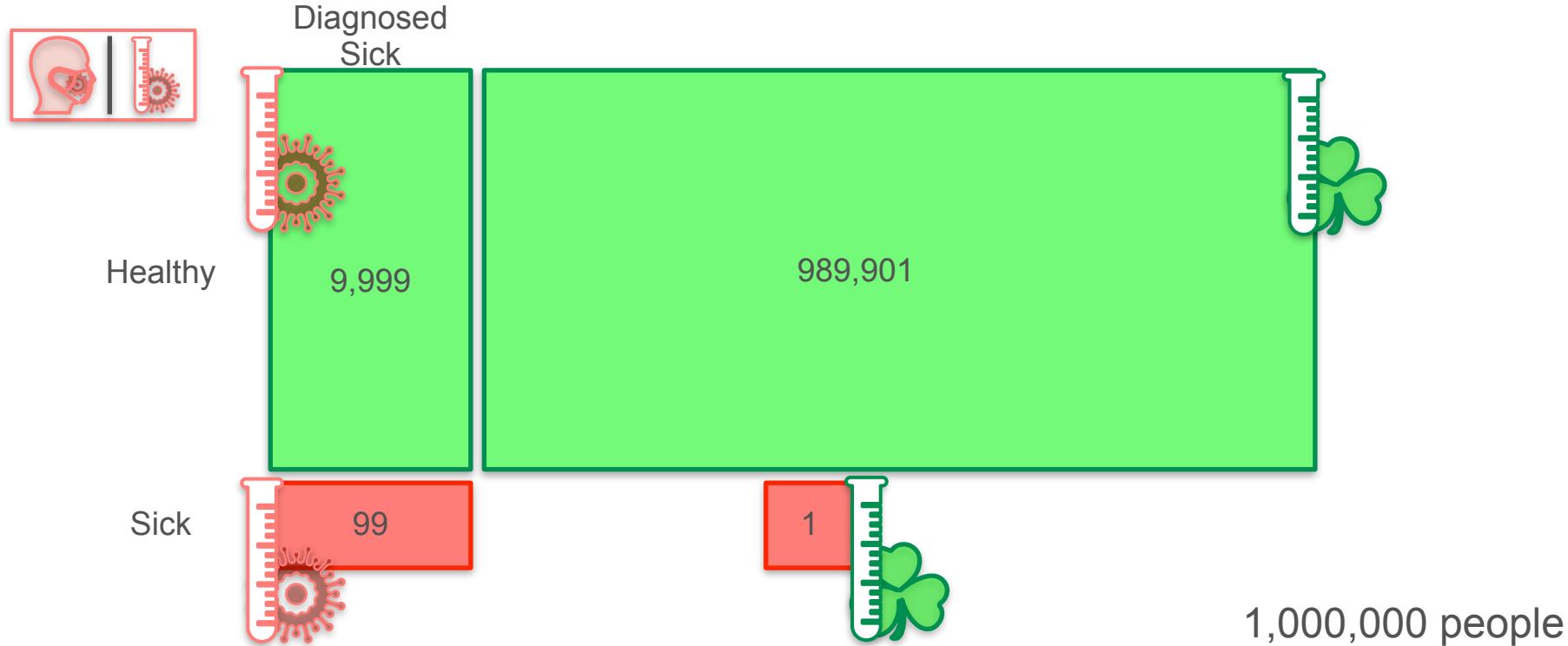
Bayes Theorem: Intuition



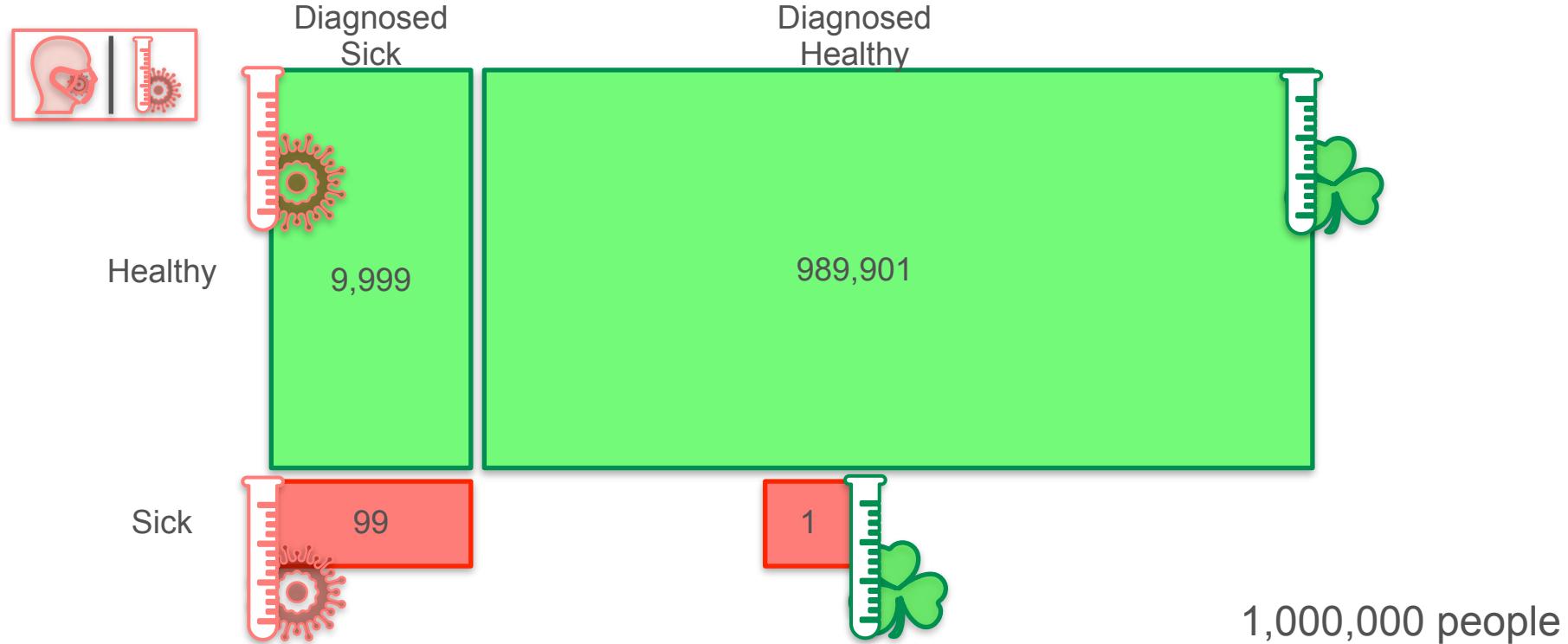
Bayes Theorem: Intuition



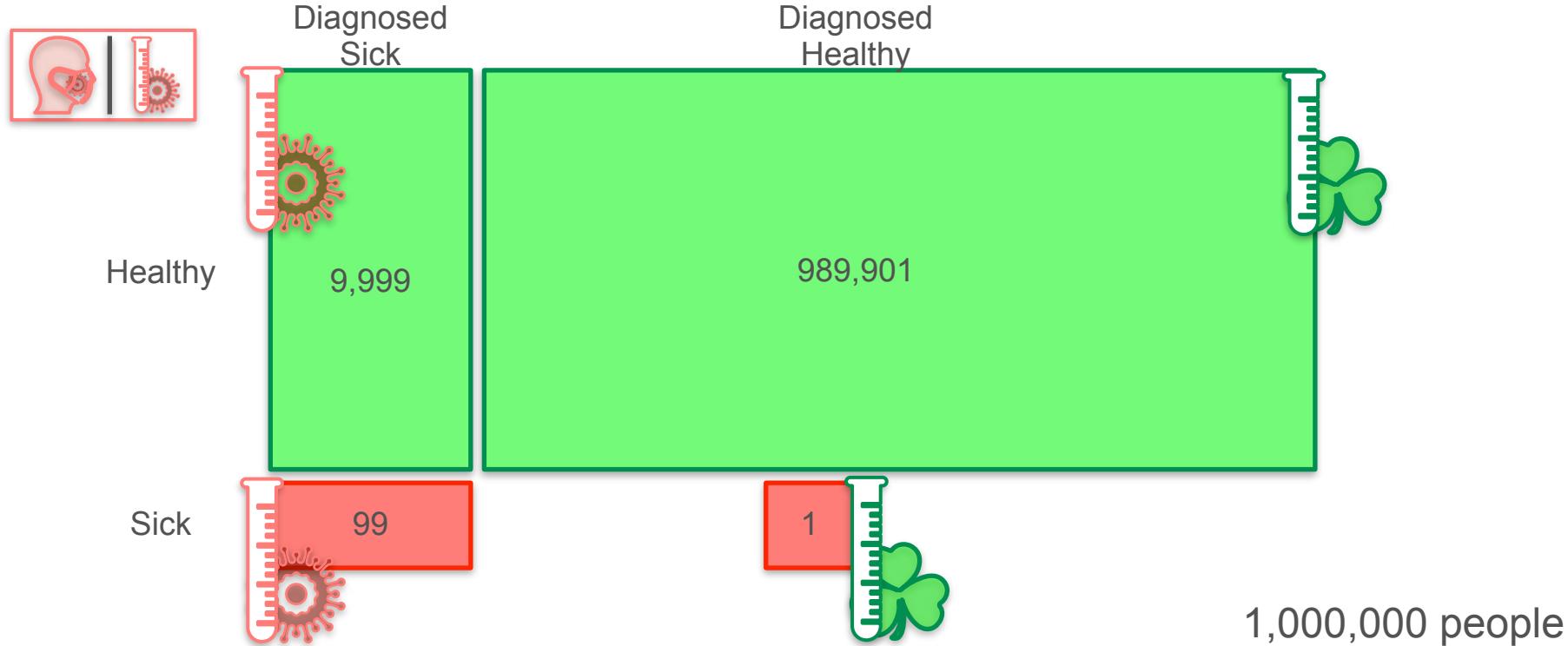
Bayes Theorem: Intuition



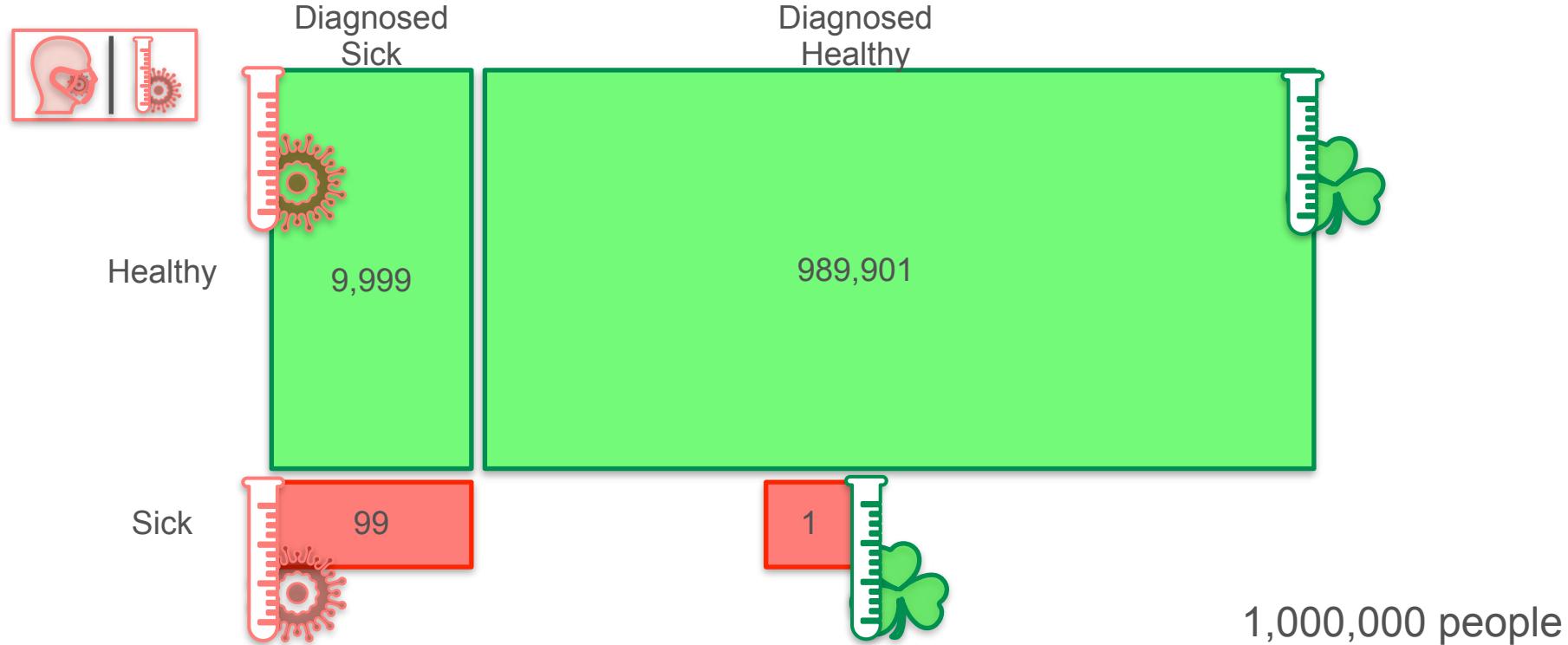
Bayes Theorem: Intuition



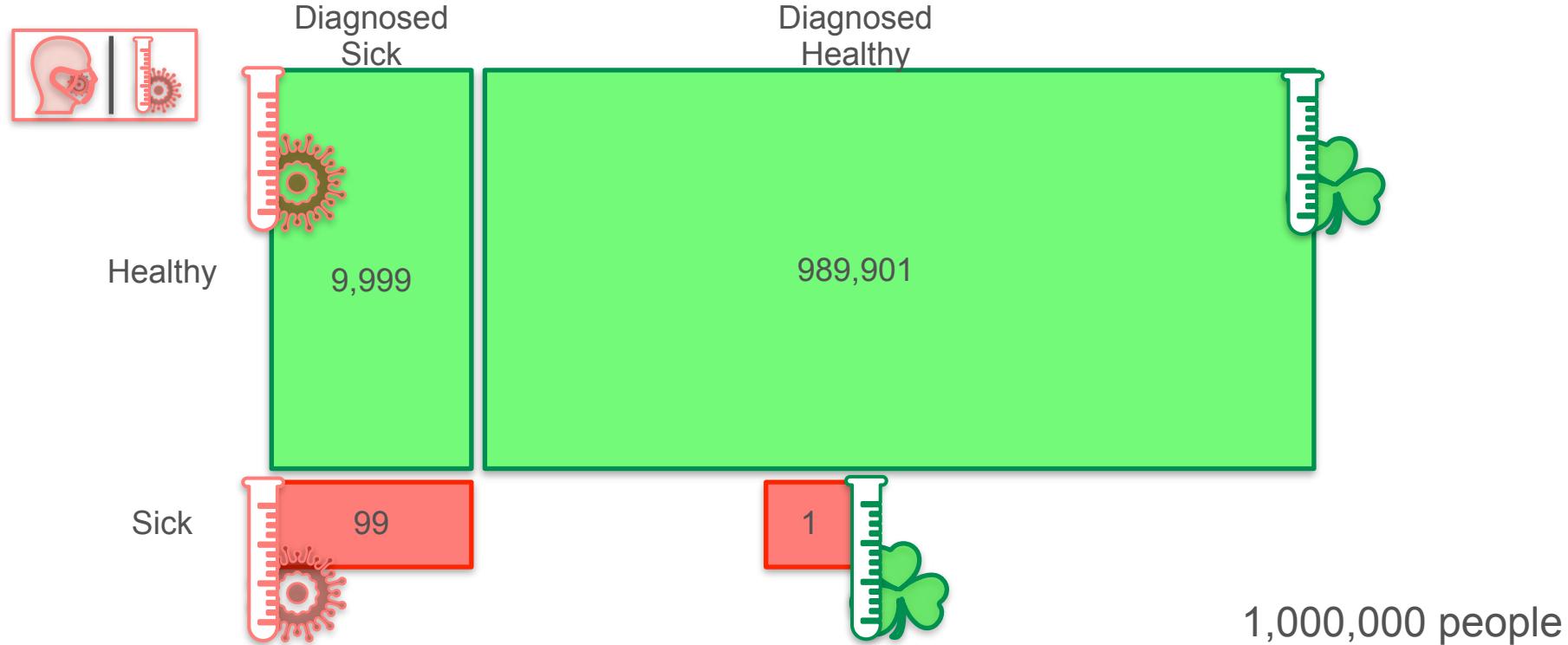
Bayes Theorem: Intuition



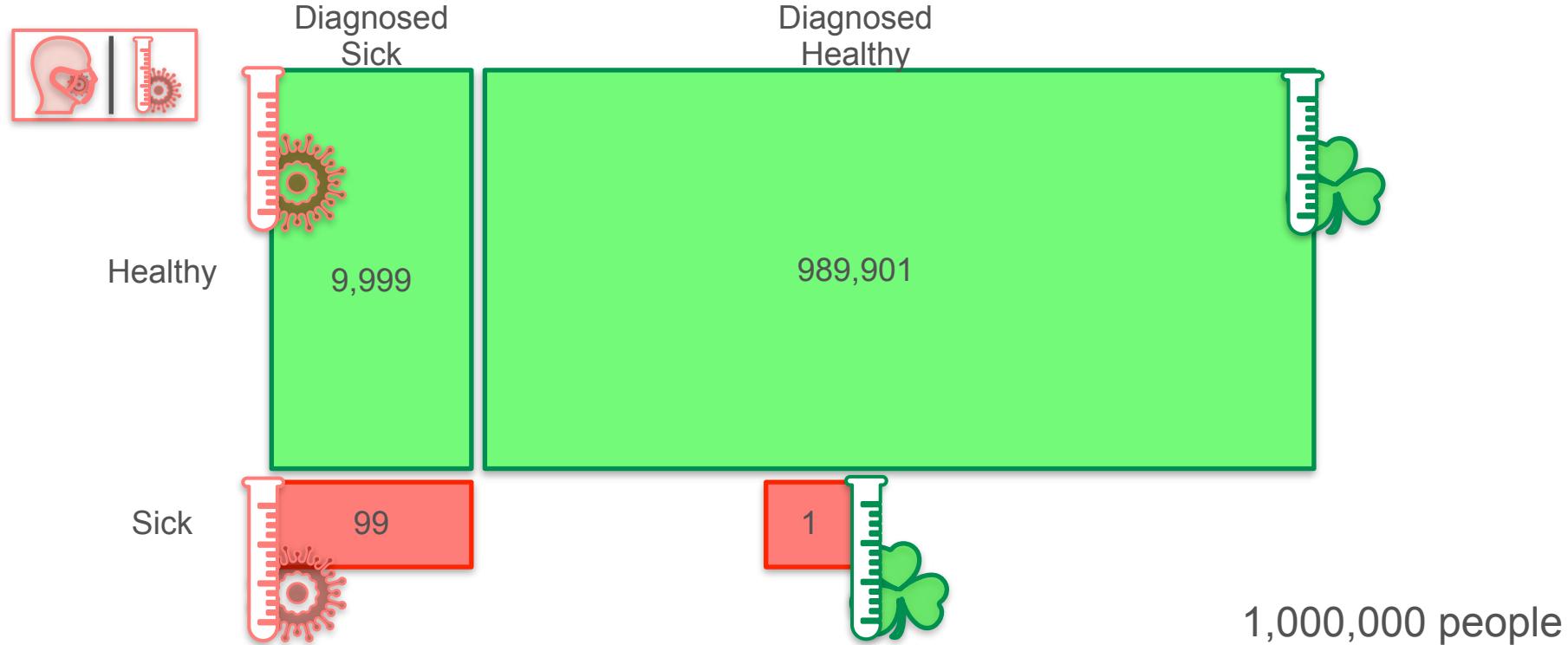
Bayes Theorem: Intuition



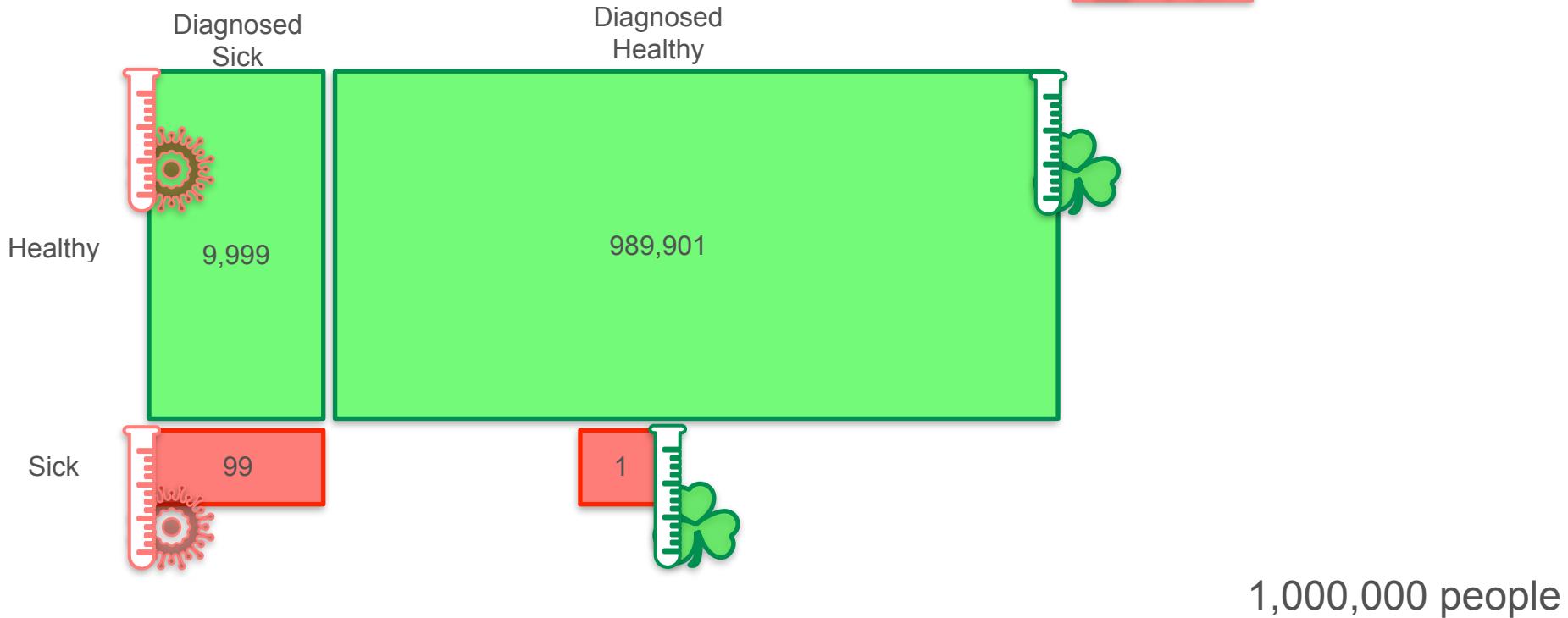
Bayes Theorem: Intuition



Bayes Theorem: Intuition



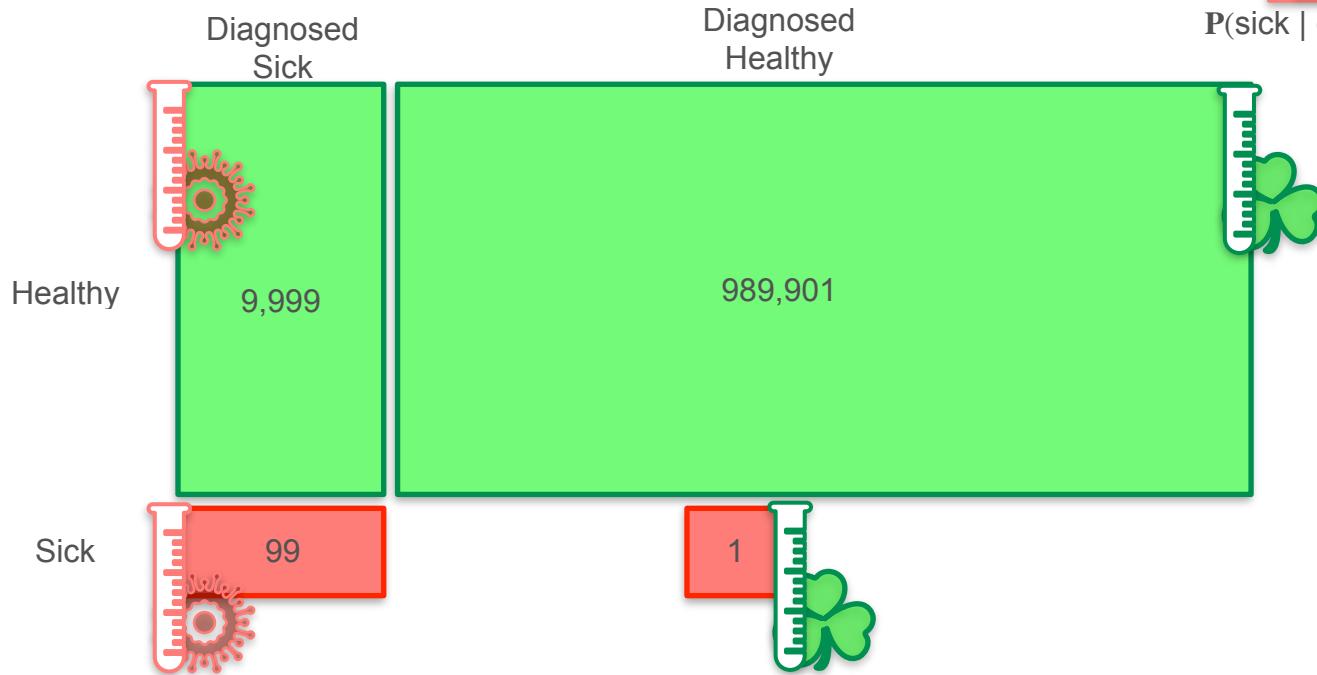
Bayes Theorem: Intuition



Bayes Theorem: Intuition



$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$

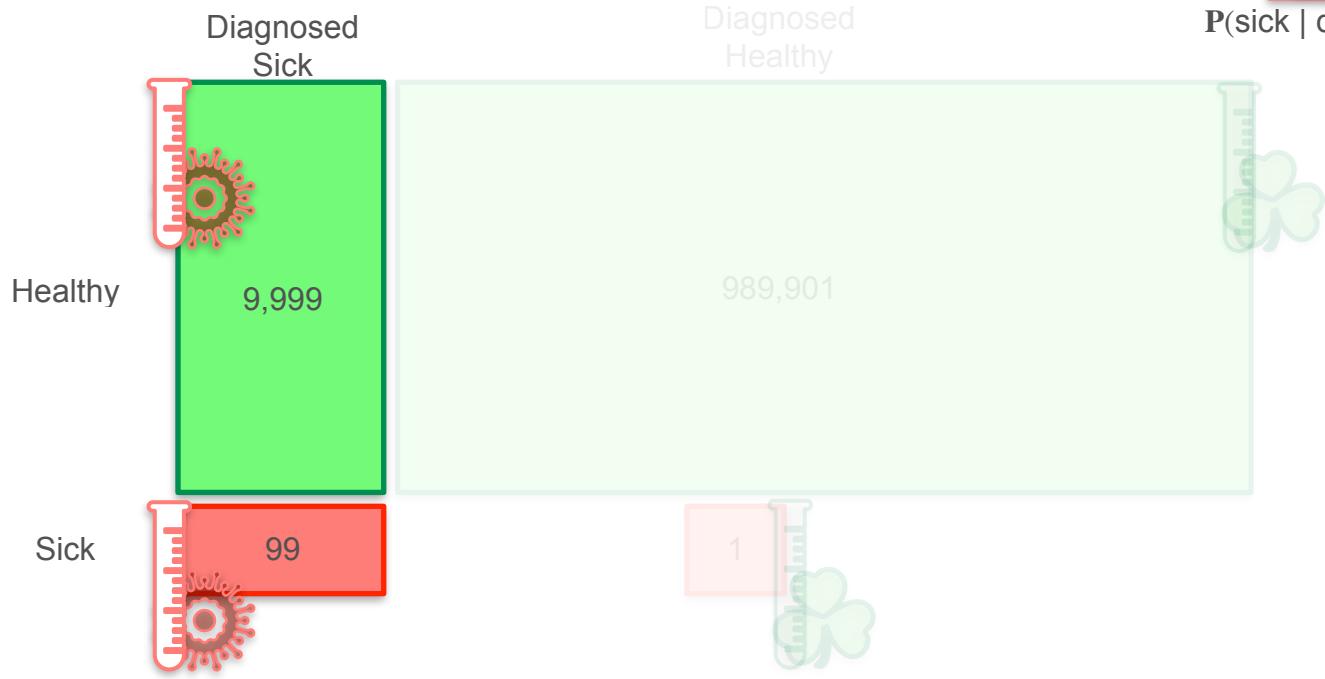


1,000,000 people

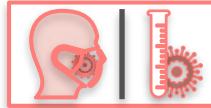
Bayes Theorem: Intuition



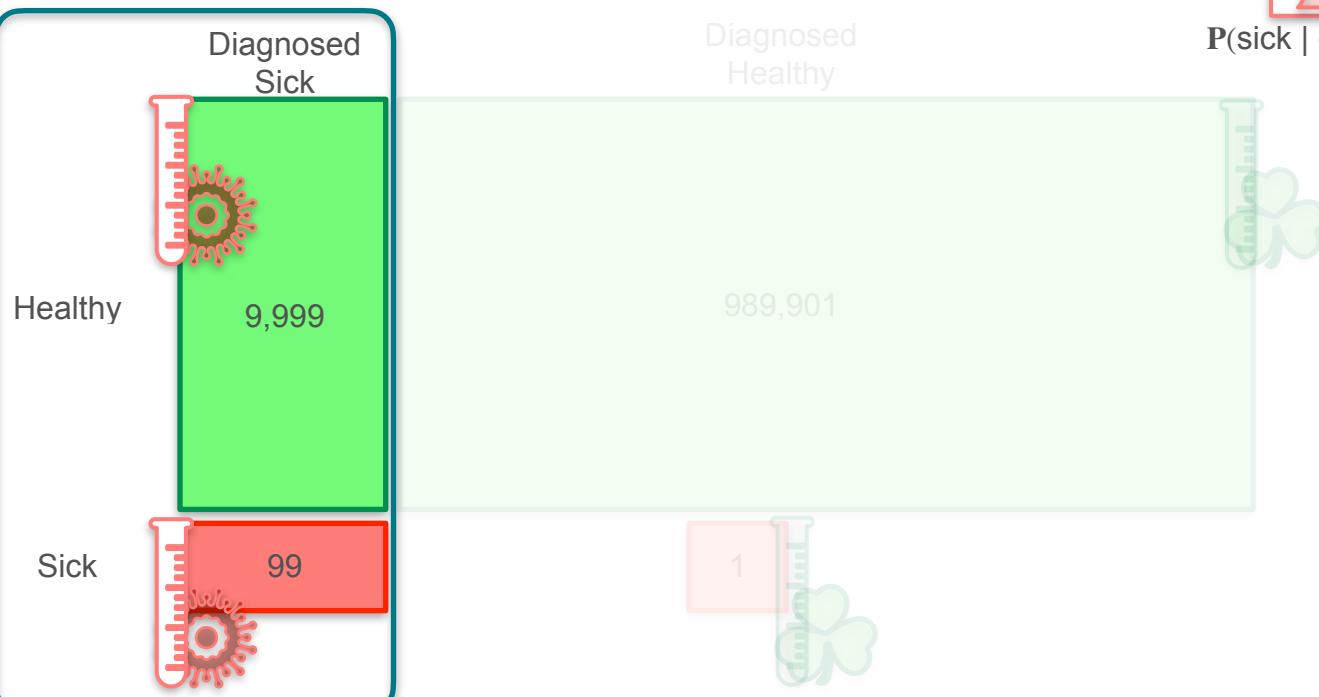
$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$



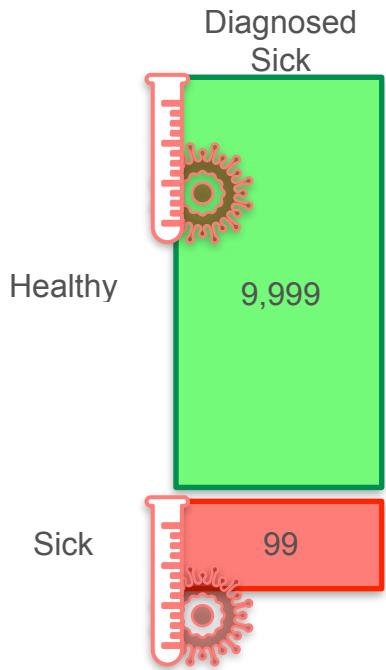
Bayes Theorem: Intuition



$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$

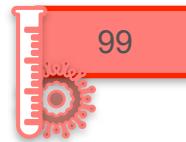
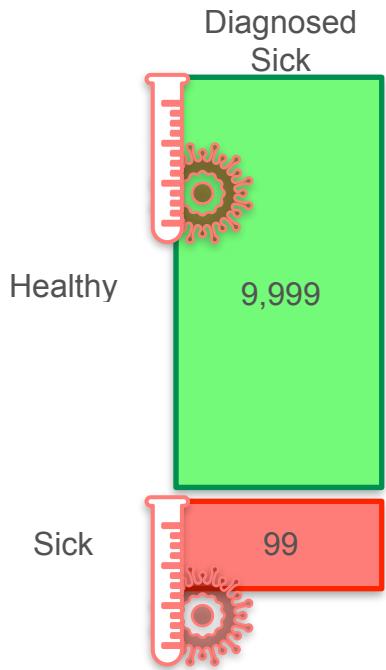


Bayes Theorem: Intuition



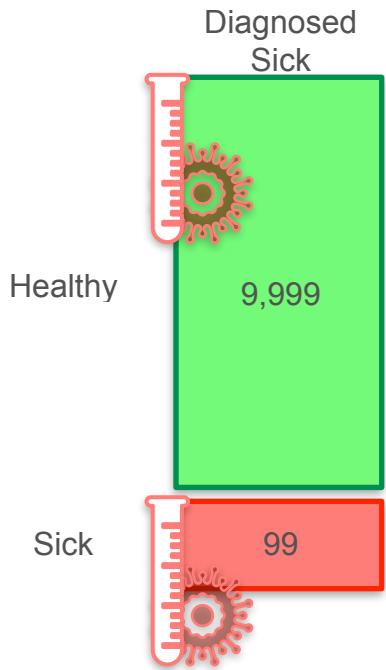
$$P(\text{sick} \mid \text{diagnosed sick}) = \underline{\hspace{2cm}}$$

Bayes Theorem: Intuition

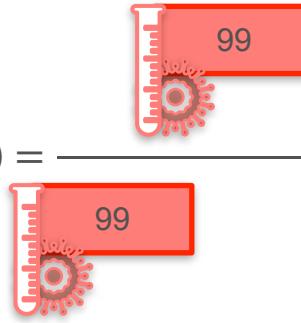


$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$

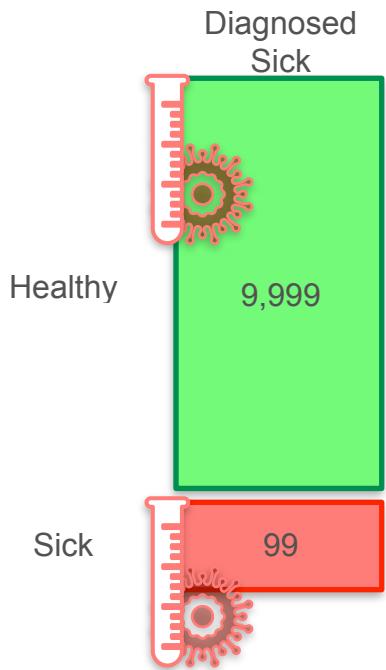
Bayes Theorem: Intuition



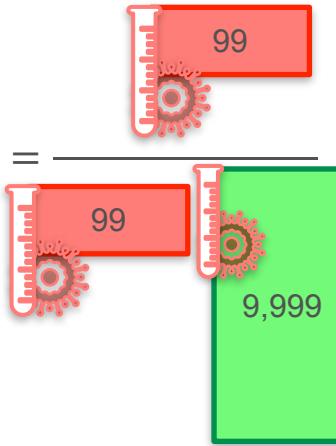
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$



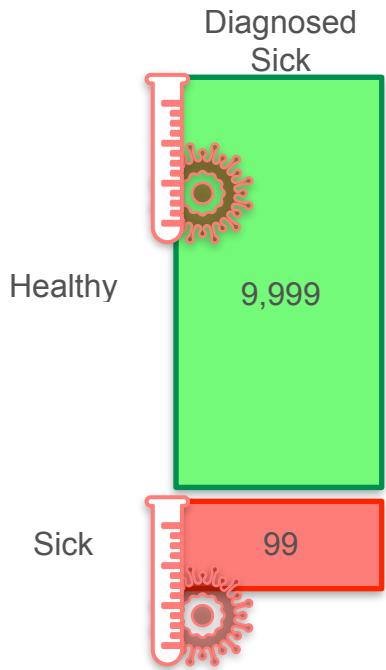
Bayes Theorem: Intuition



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{Probability of true positive}}{\text{Probability of true positive} + \text{Probability of false positive}}$$

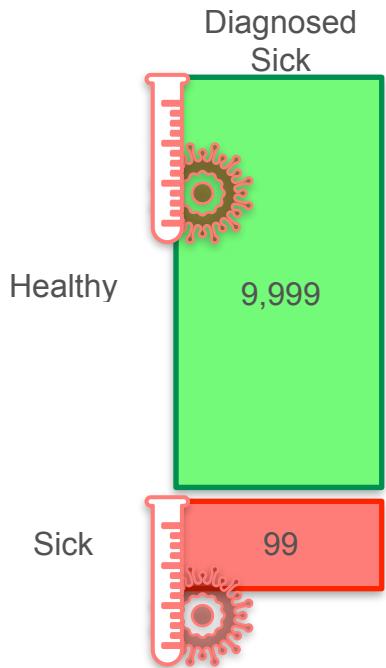


Bayes Theorem: Intuition

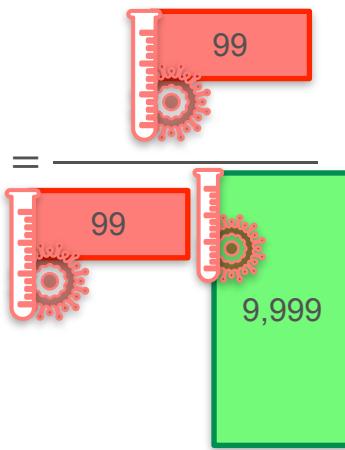


$$\begin{aligned} P(\text{sick} | \text{diagnosed sick}) &= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}} \\ &= \frac{99}{99 + 9,999} \end{aligned}$$

Bayes Theorem: Intuition

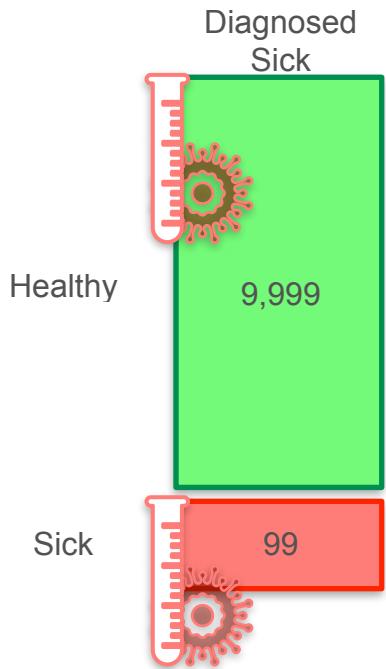


$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}}$$



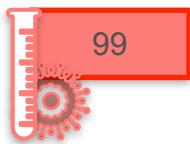
$$= \frac{99}{99 + 9999}$$

Bayes Theorem: Intuition

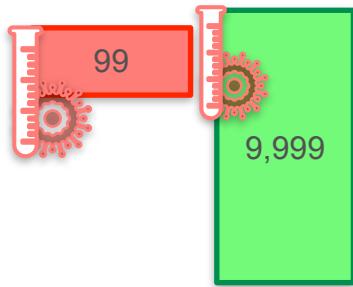


$$\begin{aligned} P(\text{sick} | \text{diagnosed sick}) &= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}} \\ &= \frac{99}{99 + 9999} \\ &= \frac{99}{10098} = 0.0098 \end{aligned}$$

Bayes Theorem: Intuition



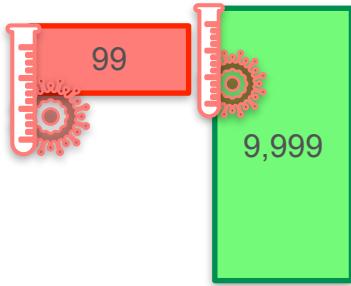
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$



Bayes Theorem: Intuition

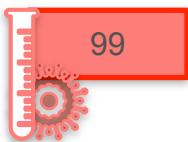


$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$

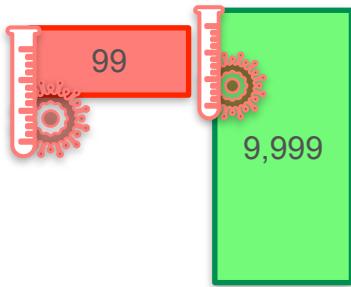


= $\frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$

Bayes Theorem: Intuition



$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

Bayes Theorem: Intuition



1,000,000
people

Bayes Theorem: Intuition

sick = 100



1,000,000
people

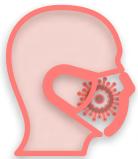


Bayes Theorem: Intuition

sick = 100



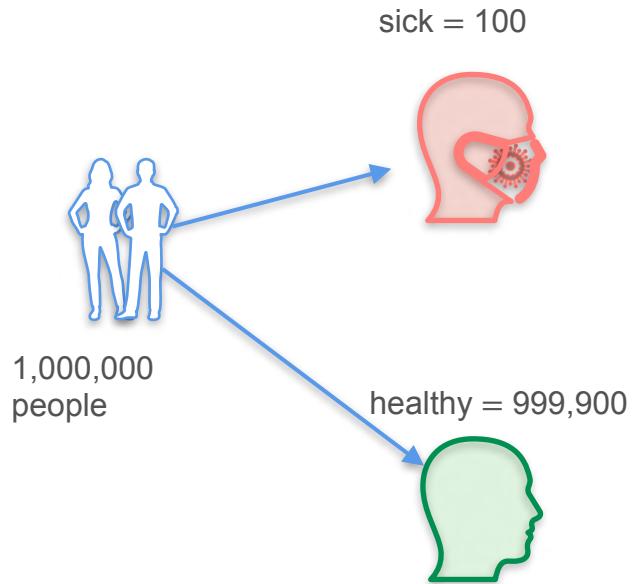
1,000,000
people



healthy = 999,900

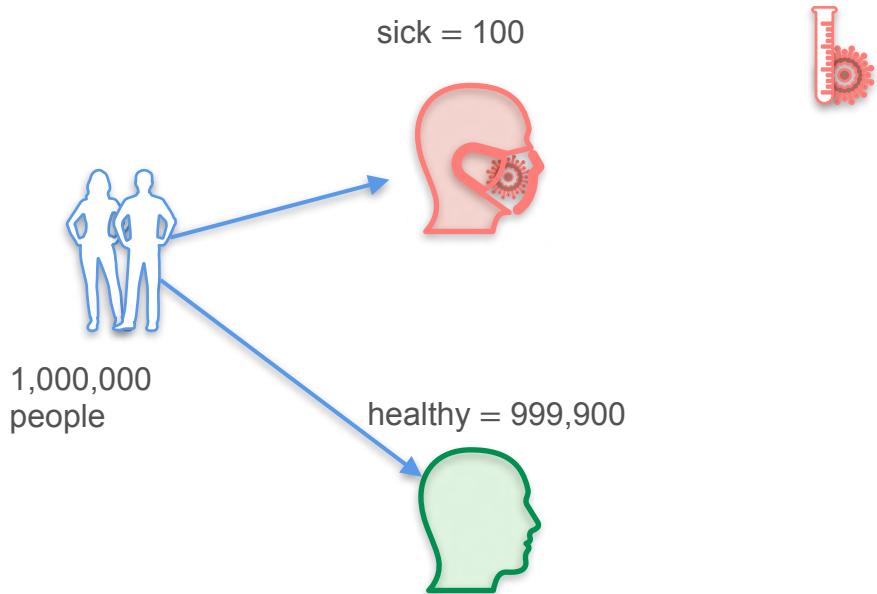


Bayes Theorem: Intuition

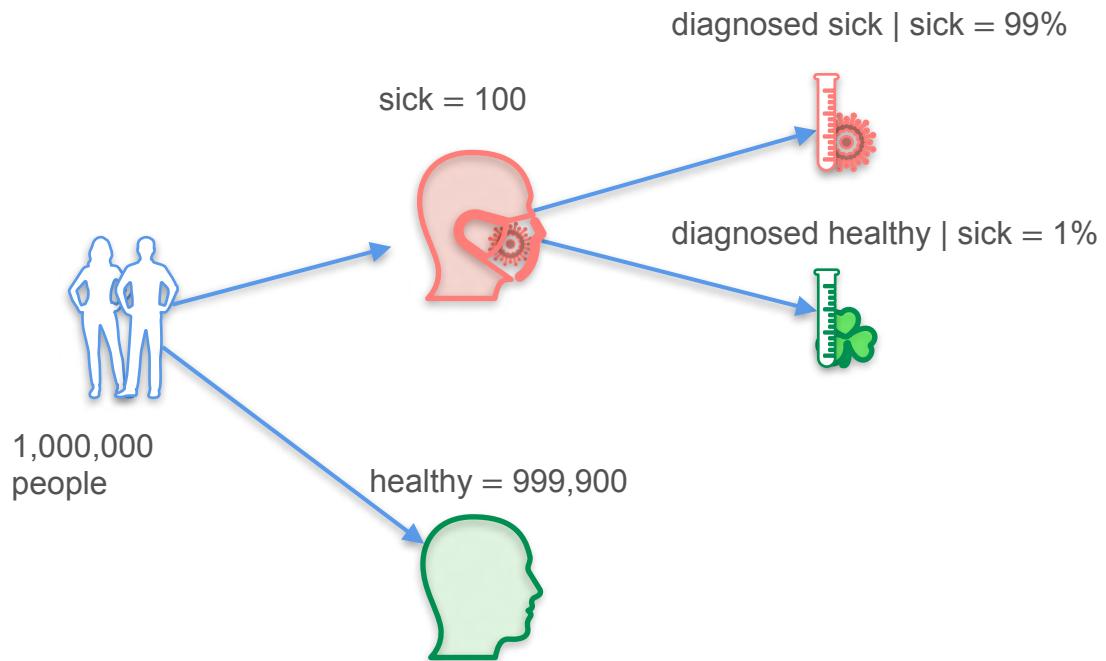


Bayes Theorem: Intuition

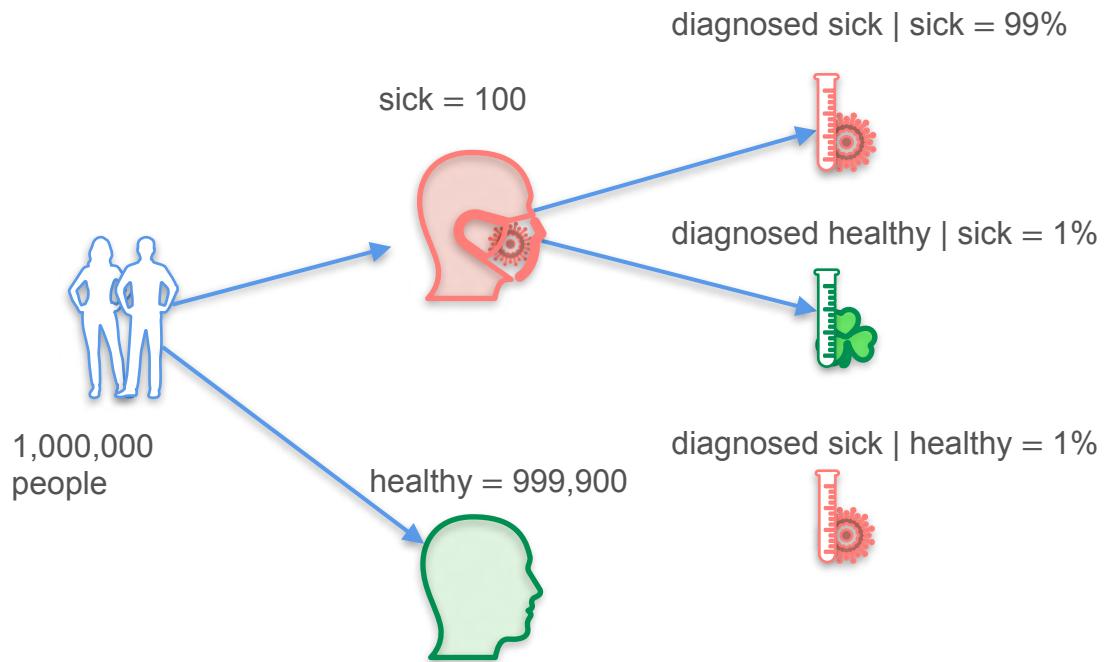
diagnosed sick | sick = 99%



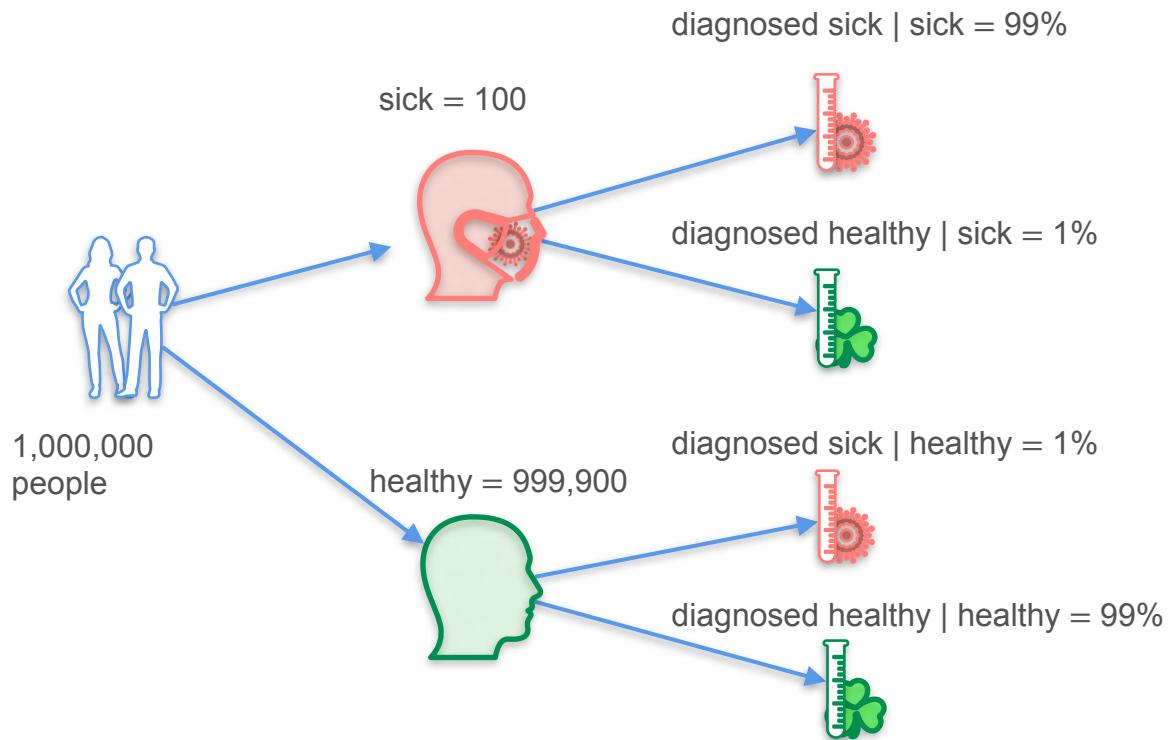
Bayes Theorem: Intuition



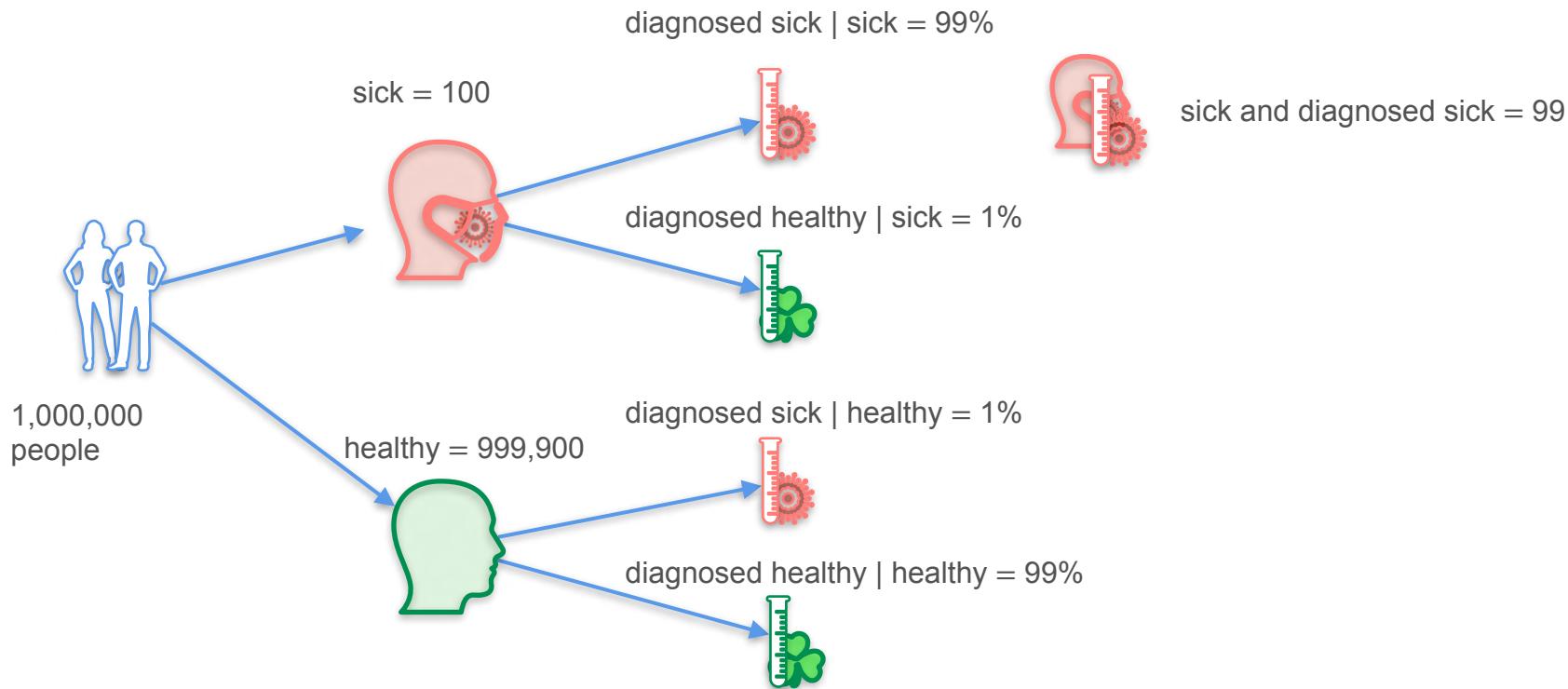
Bayes Theorem: Intuition



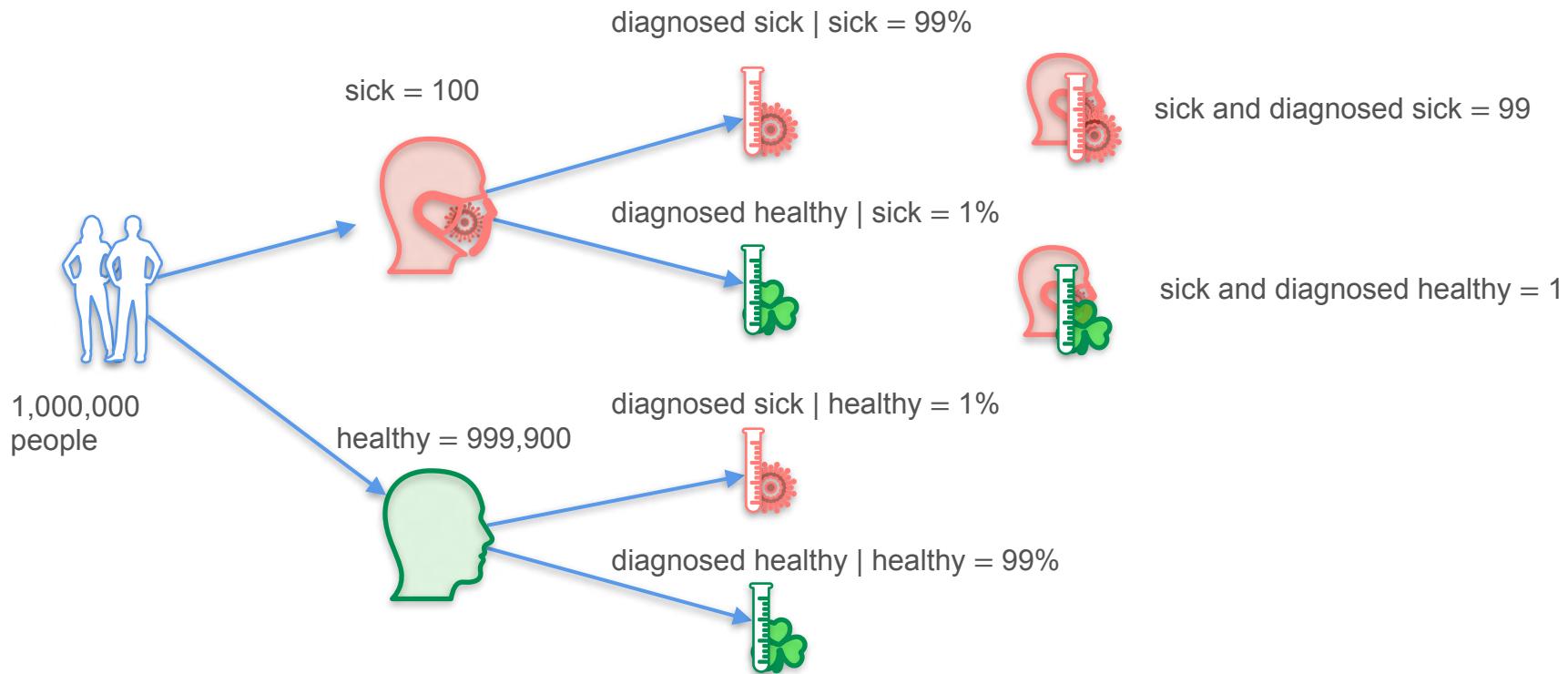
Bayes Theorem: Intuition



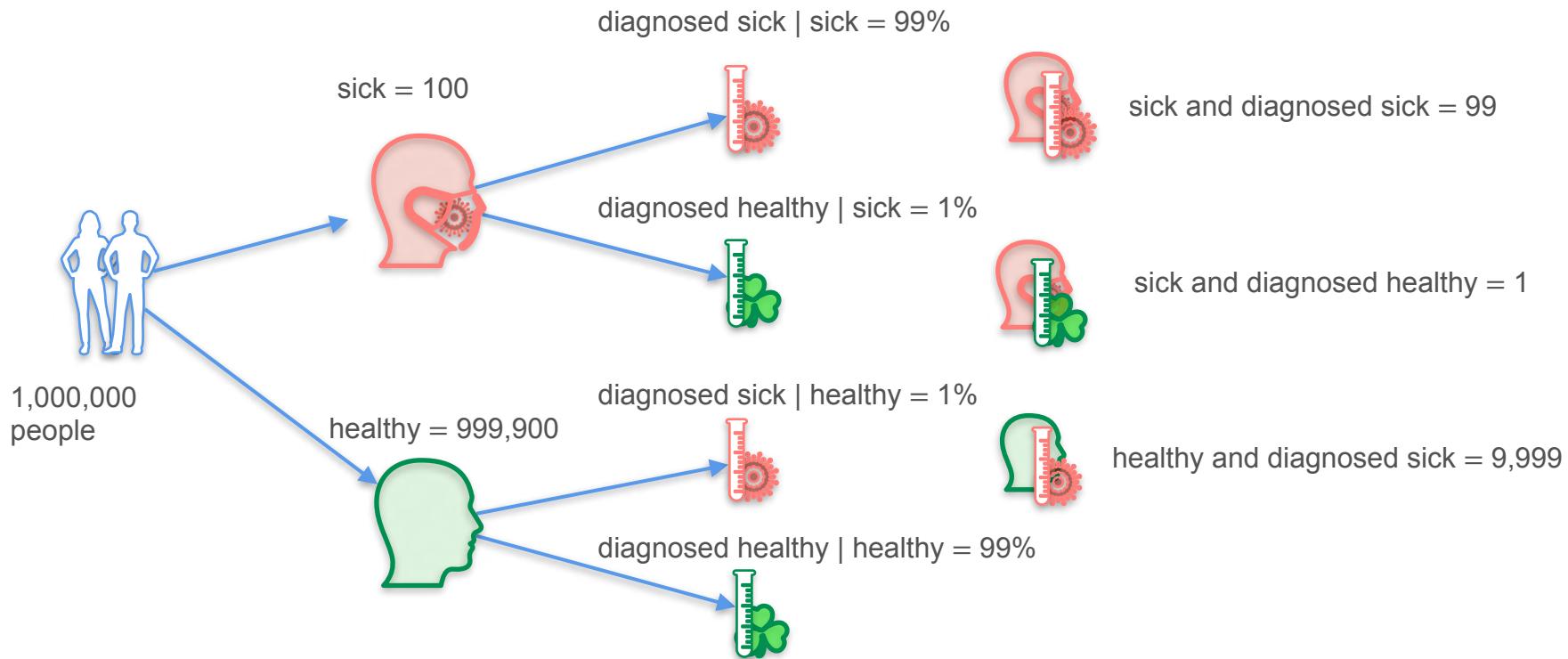
Bayes Theorem: Intuition



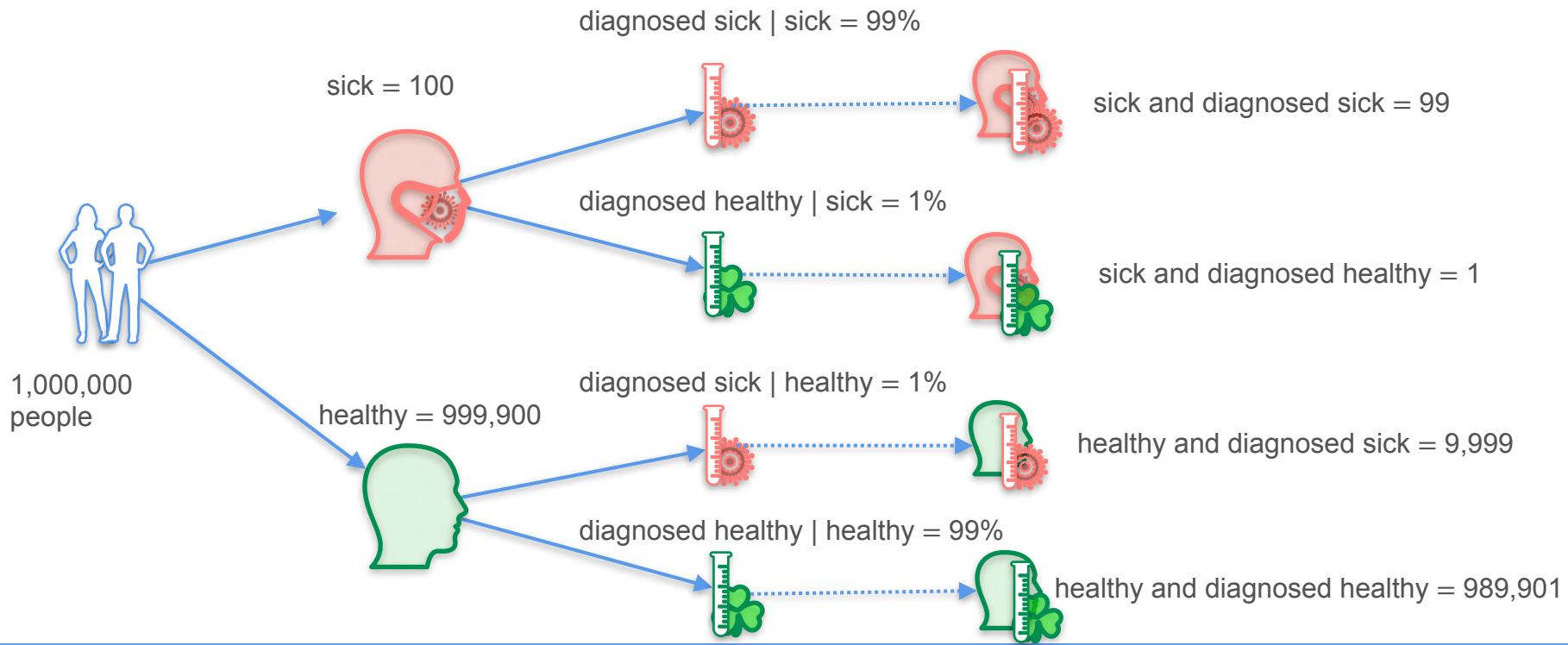
Bayes Theorem: Intuition



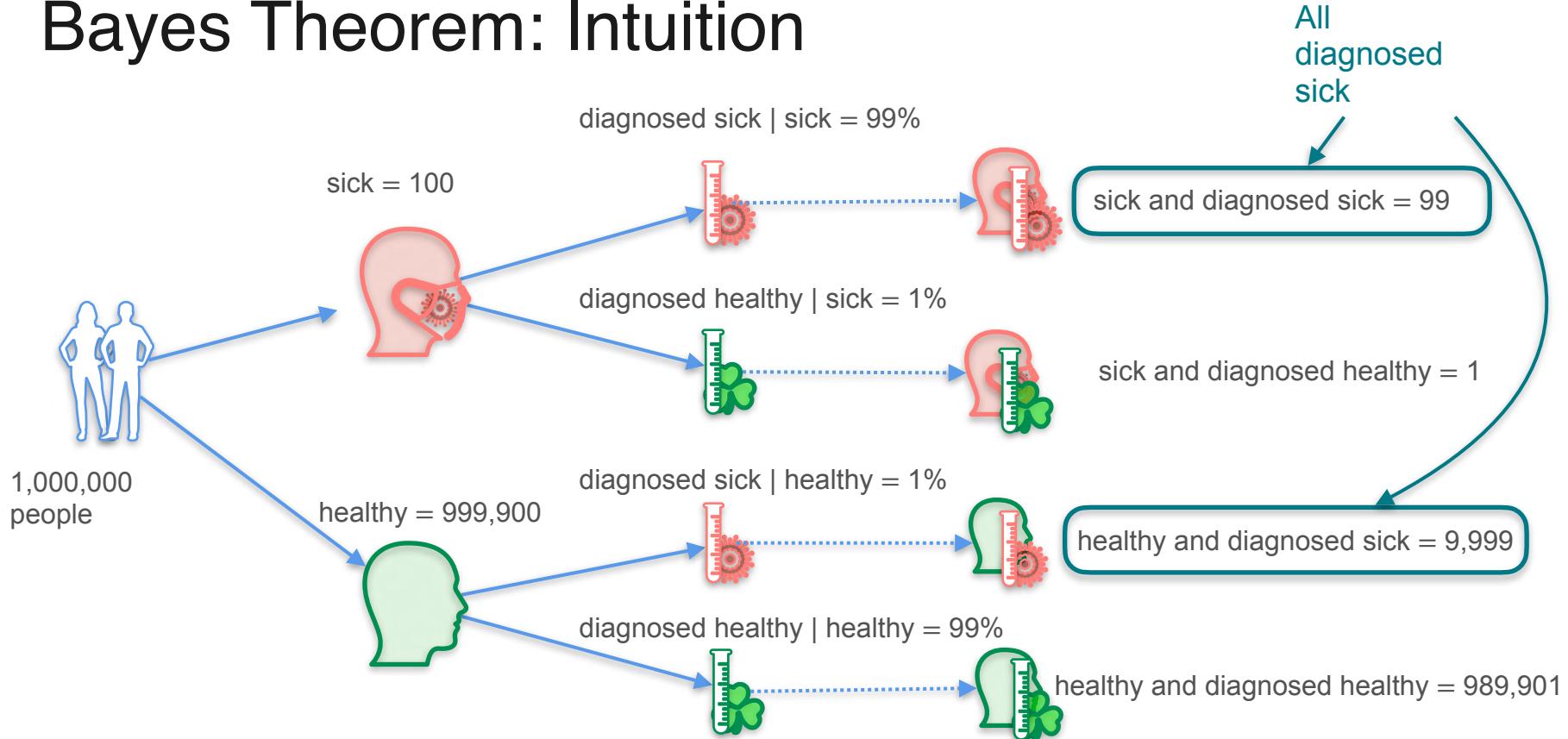
Bayes Theorem: Intuition



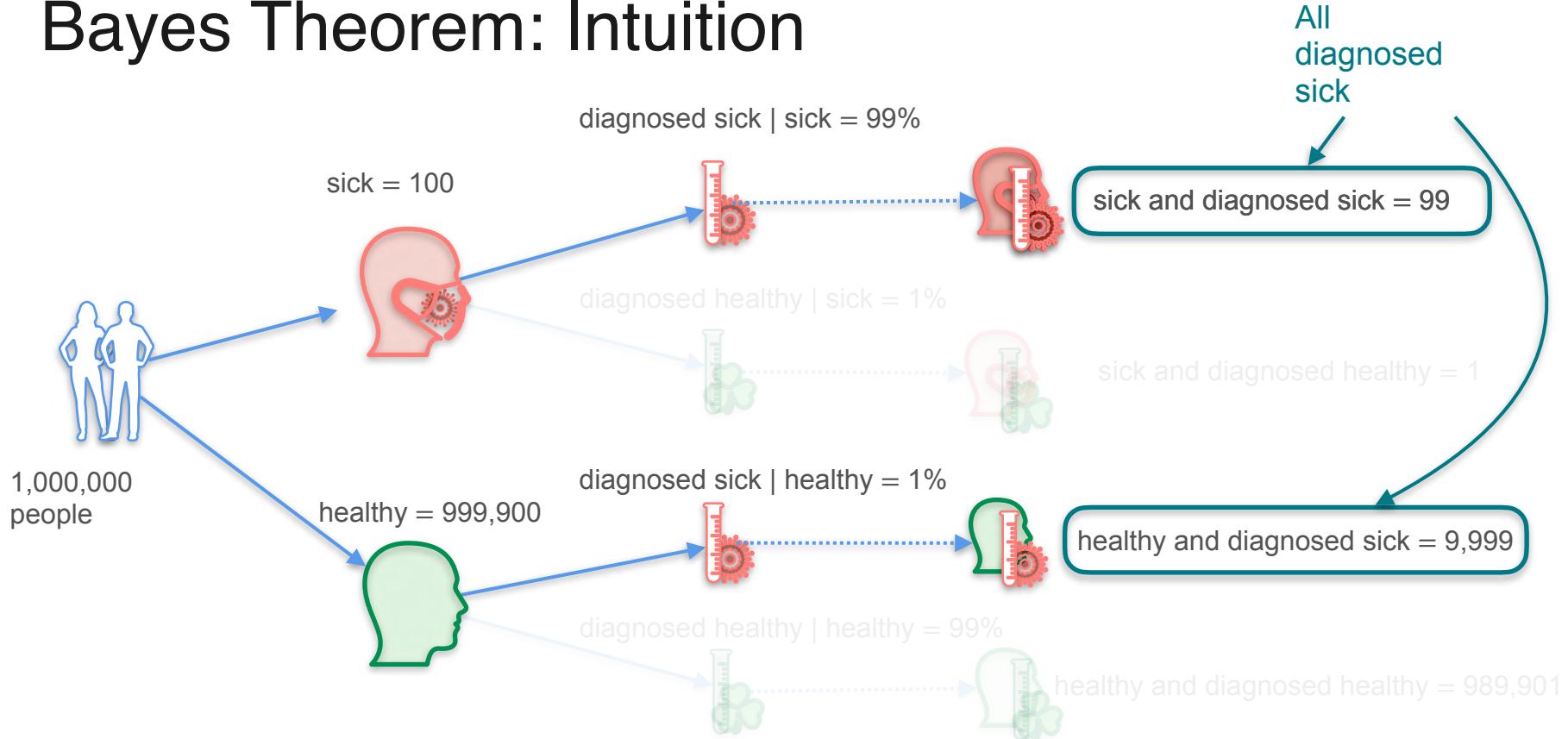
Bayes Theorem: Intuition



Bayes Theorem: Intuition

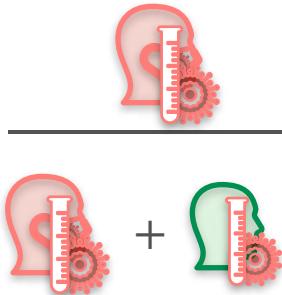


Bayes Theorem: Intuition



Bayes Theorem: Intuition

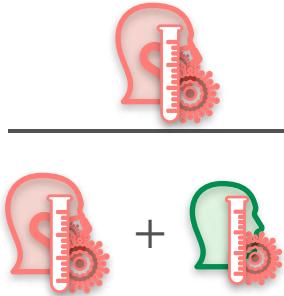
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

Bayes Theorem: Intuition

$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$

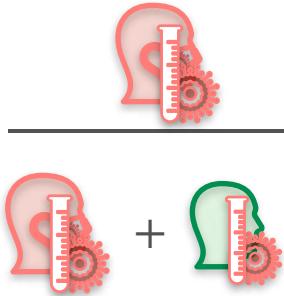


$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

$$P(\text{sick} | \text{diagnosed sick}) = \frac{99}{10098}$$

Bayes Theorem: Intuition

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

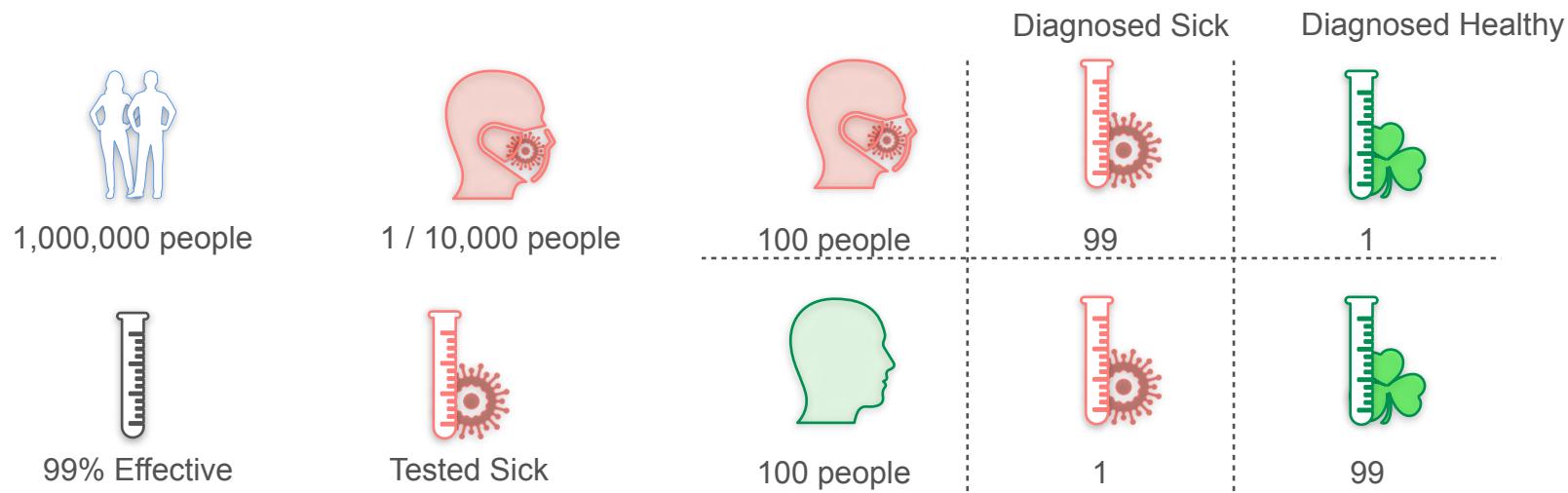
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098} = 0.0098$$

Bayes Theorem: Formula

Bayes Theorem: Formula

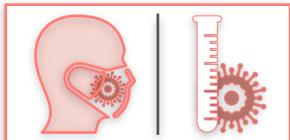
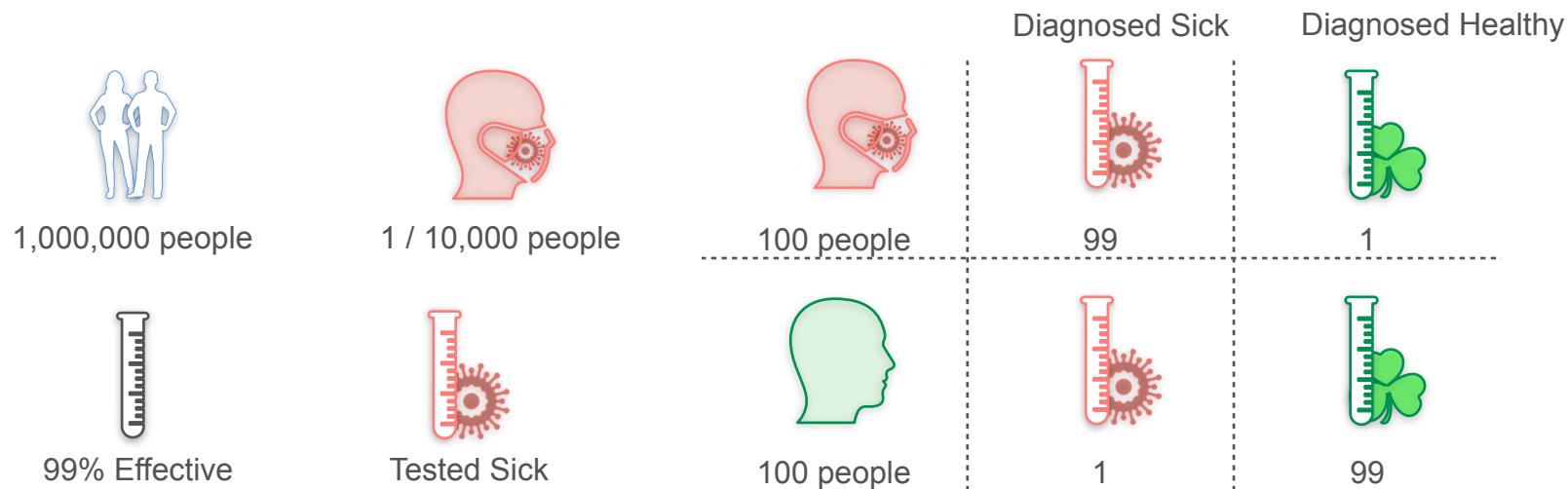
The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula



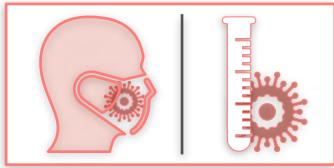
The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula



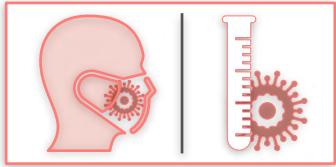
The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula



The probability that **you are sick**
GIVEN that you tested sick

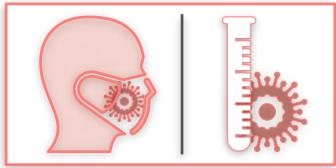
Bayes Theorem: Formula



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula

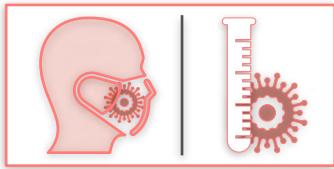


$P(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick**
GIVEN that you tested sick



Bayes Theorem: Formula

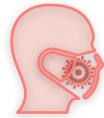


$P(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick**
GIVEN that you tested sick

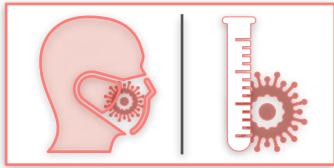


1,000,000



1 / 10,000

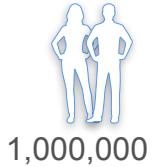
Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

The probability that **you are sick**
GIVEN that you tested sick

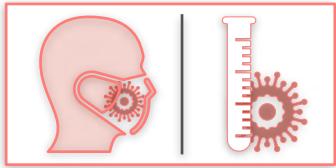


1,000,000



1 / 10,000

Bayes Theorem: Formula

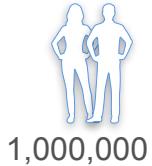


$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

The probability that **you are sick**
GIVEN that you tested sick

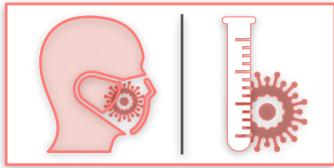


1,000,000



1 / 10,000

Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

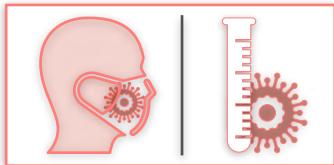


1 / 10,000



99% Effective

Bayes Theorem: Formula



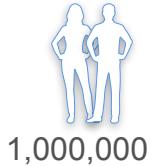
$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

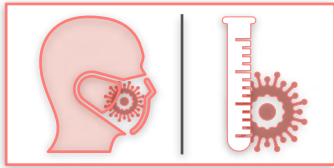


1 / 10,000



99% Effective

Bayes Theorem: Formula



$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

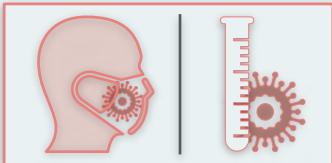


1 / 10,000



99% Effective

Bayes Theorem: Formula



$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

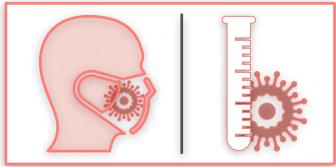


1 / 10,000



99% Effective

Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

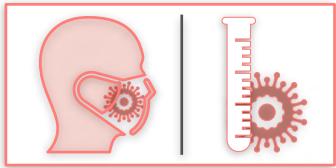
$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} | \text{sick}) = 99\%$

$P(\text{diagnosed sick} | \text{not sick}) = 1\%$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

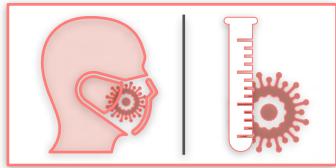
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

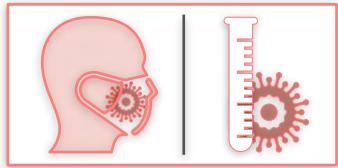
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

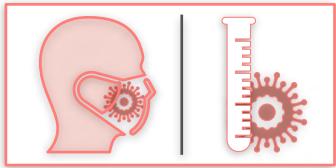
From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

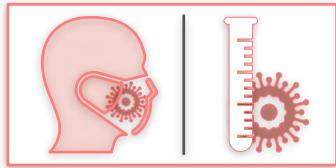
$$P(\text{not sick}) = 99.99\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

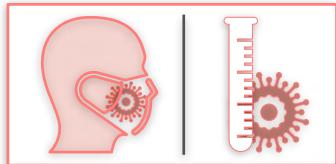
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \underline{\hspace{10em}}$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

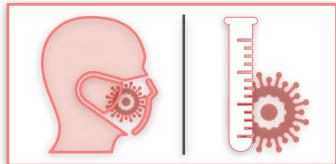
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{}$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

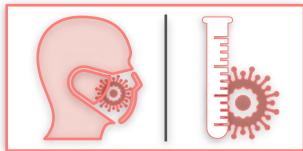
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick} | \text{diagnosed sick}) = ?$$

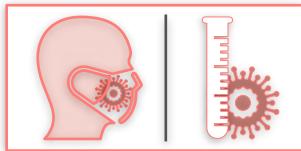
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

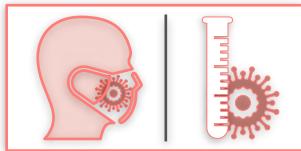
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

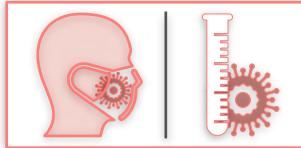
$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$P(\text{sick and diagnosed sick}) = ?$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

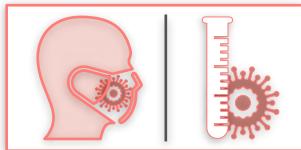
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick and diagnosed sick}) = ?$$

$$P(\text{diagnosed sick}) = ?$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

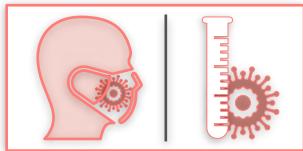
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$P(\text{sick and diagnosed sick}) = ?$

$P(\text{diagnosed sick}) = ?$

BAYES THEOREM FORMULA CAN HELP

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

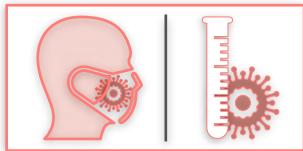
$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

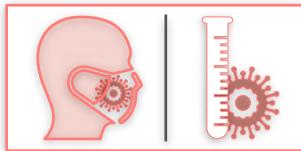
$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

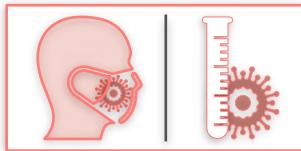
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

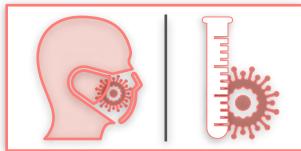
$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$P(\text{sick and diagnosed sick}) =$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

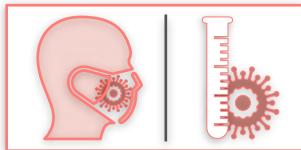
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick})$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

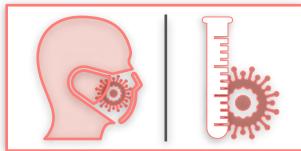
$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick}) \cdot$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

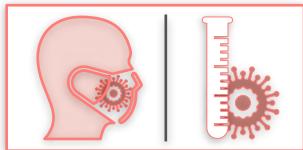
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

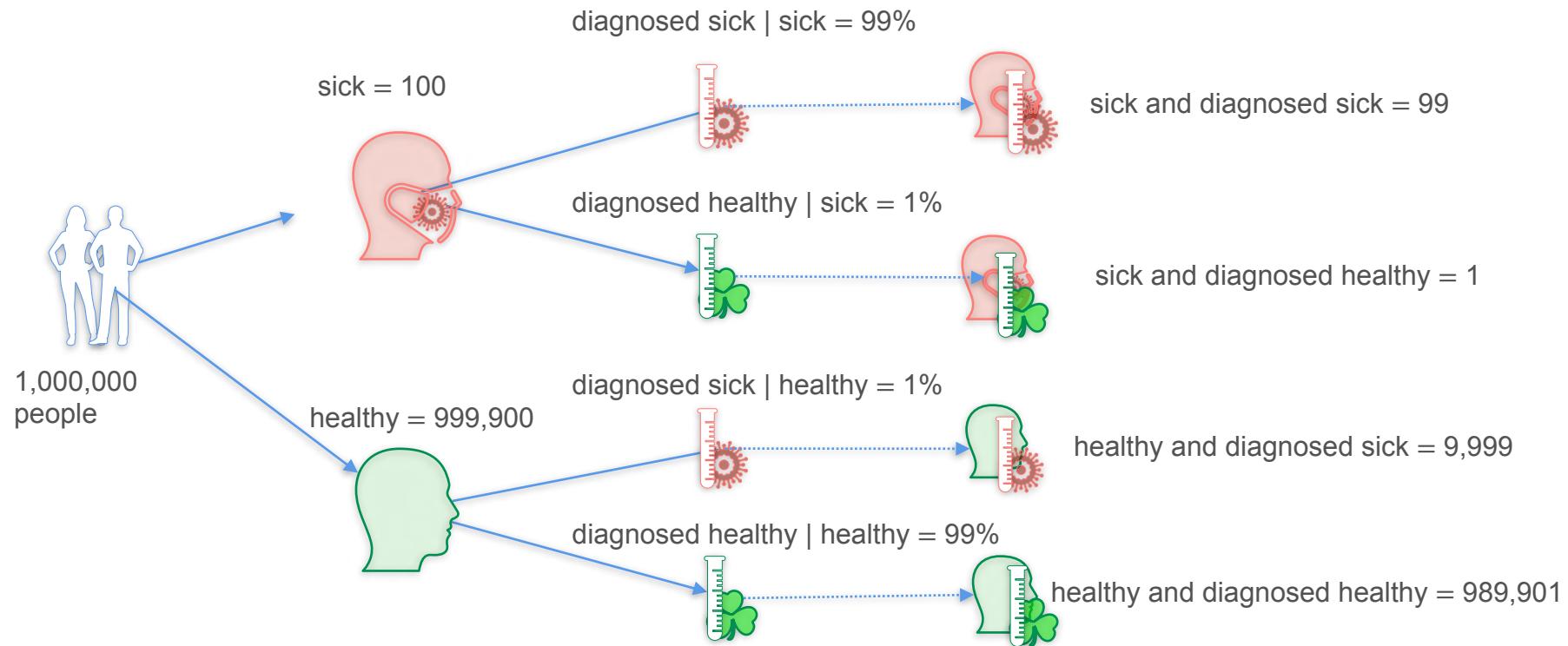
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

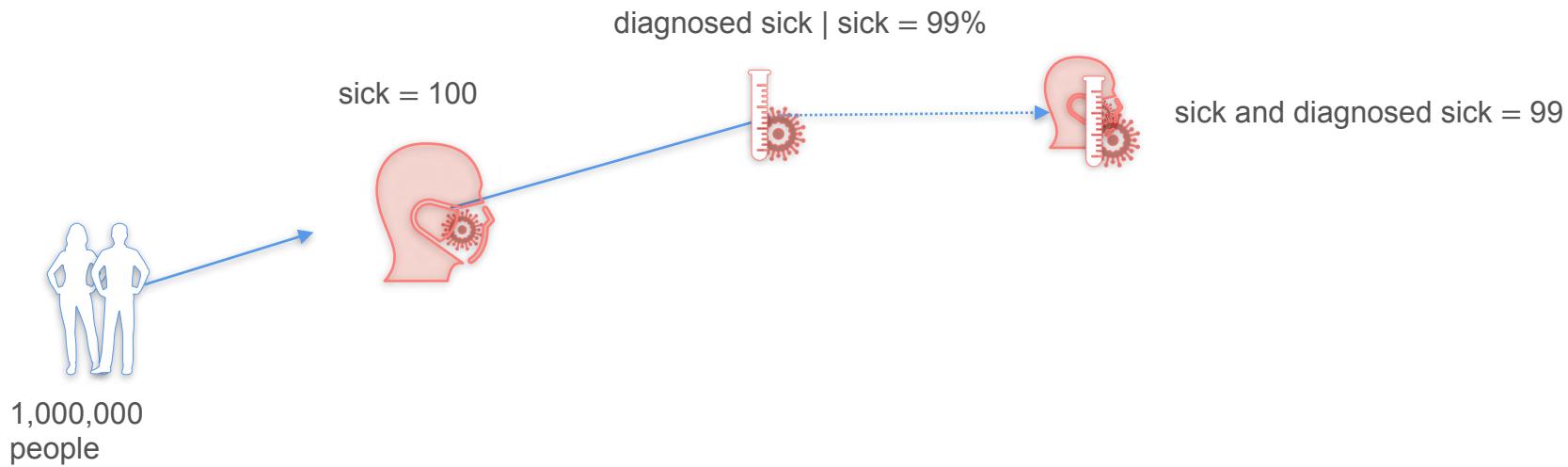
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

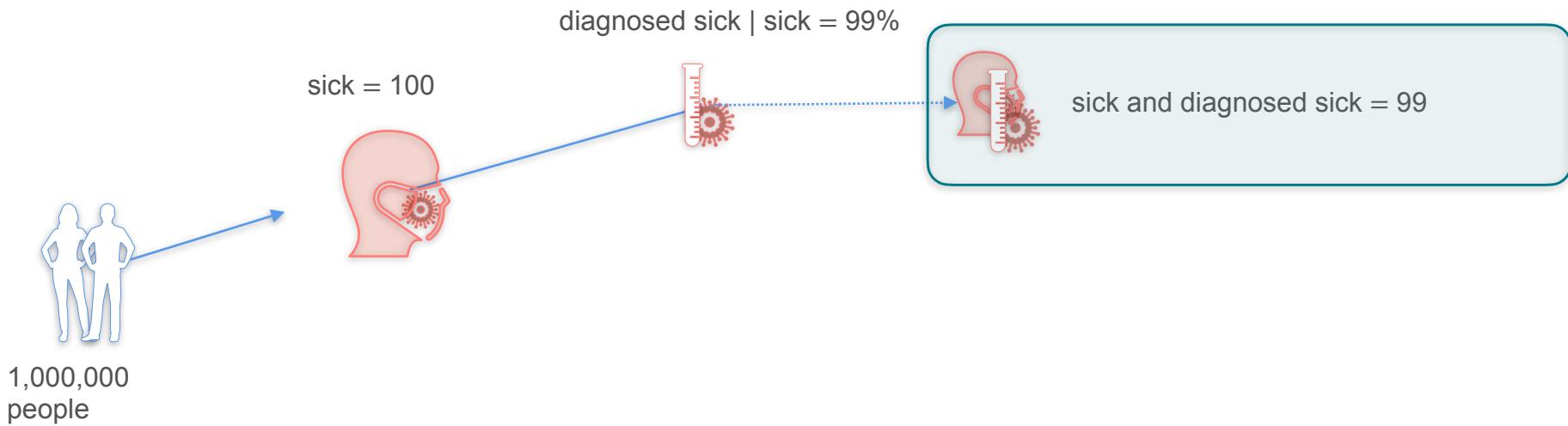
Bayes Theorem: Formula



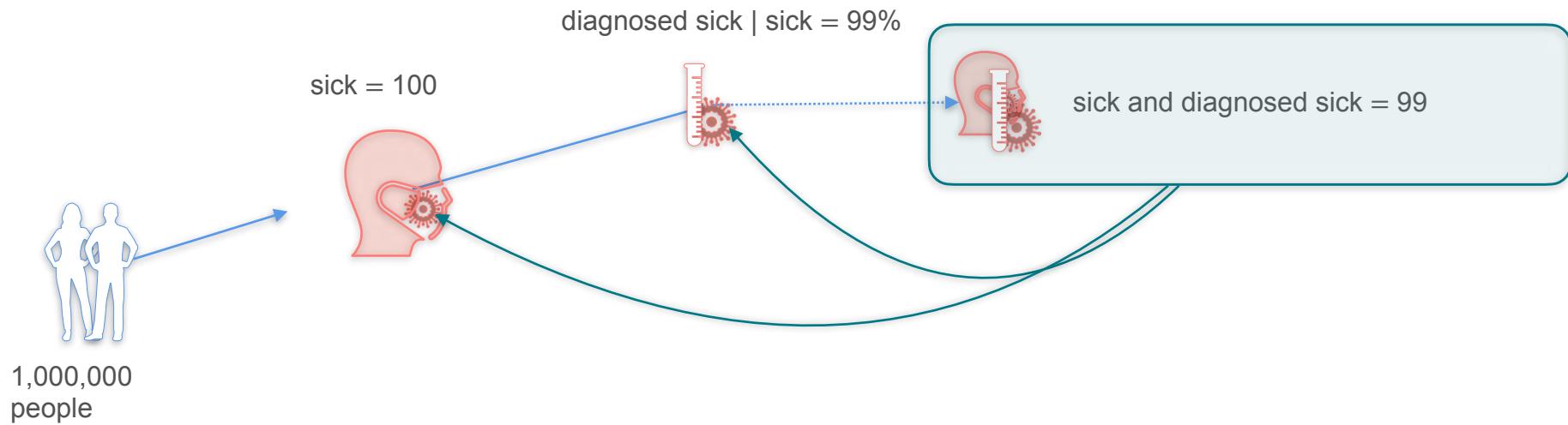
Bayes Theorem: Formula



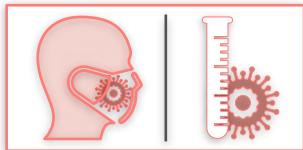
Bayes Theorem: Formula



Bayes Theorem: Formula



Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

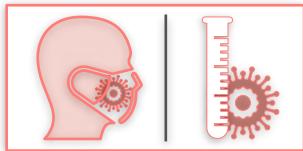
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

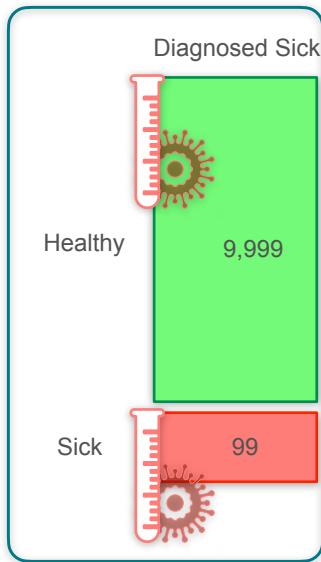
$$P(\text{diagnosed sick}) =$$

Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$

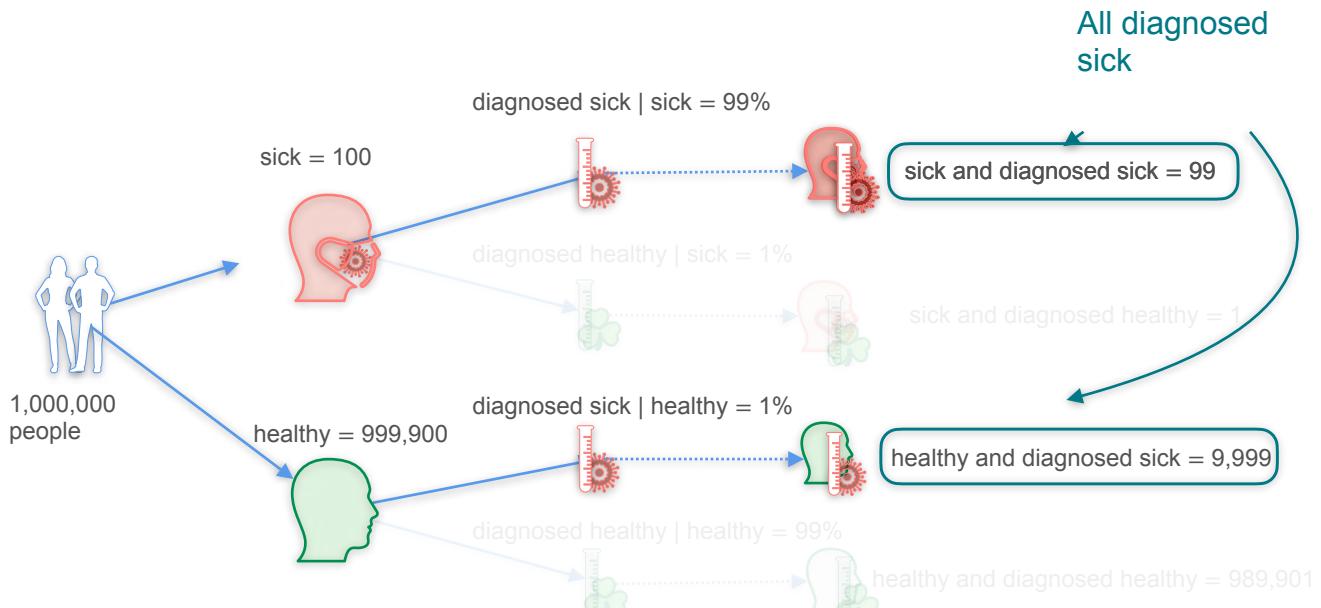
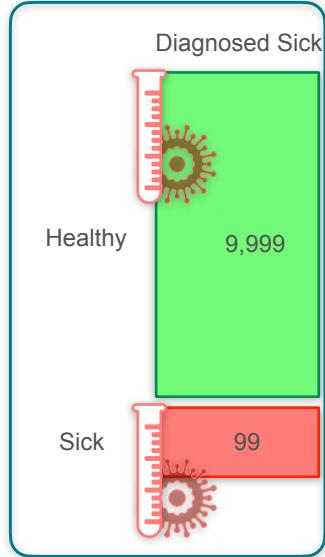
Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$



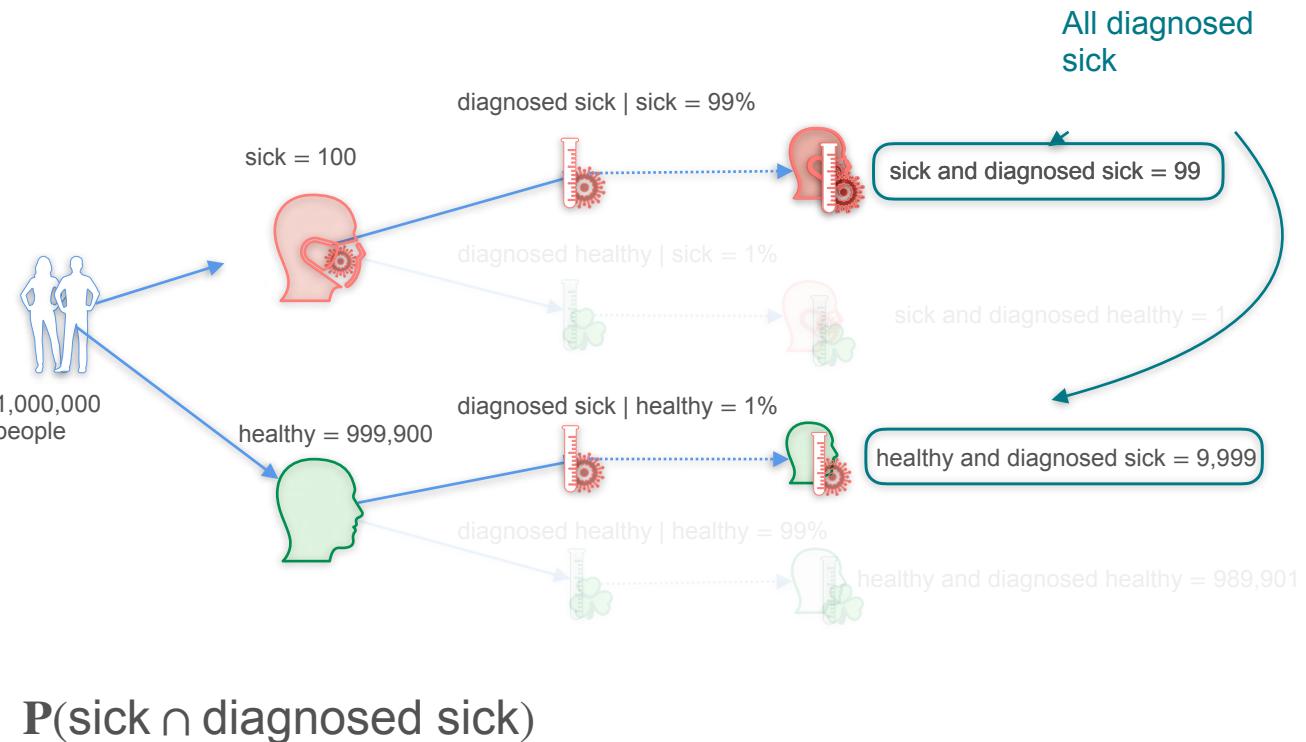
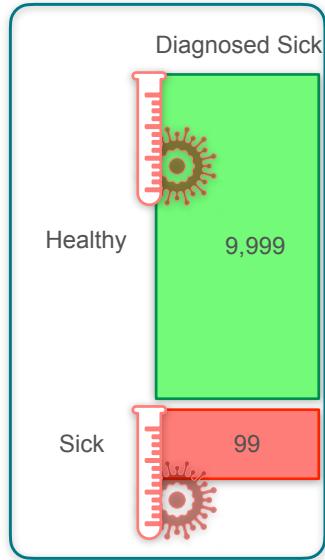
Bayes Theorem: Formula

$$P(\text{diagnosed sick}) =$$



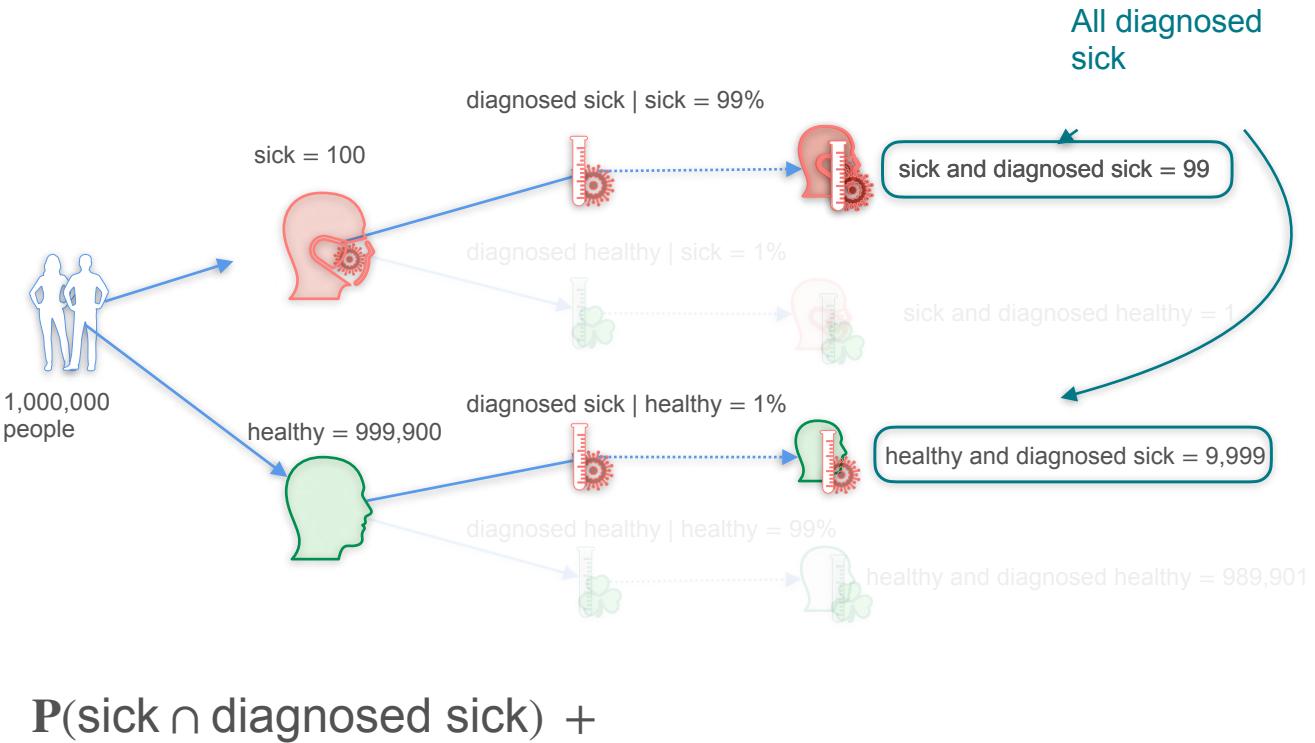
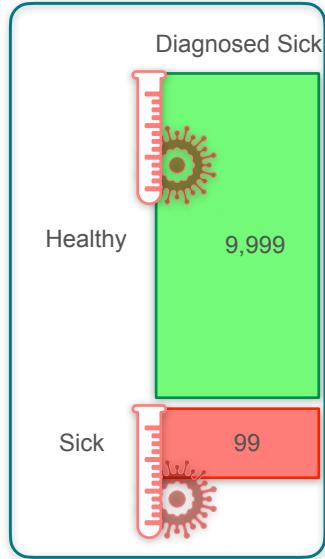
Bayes Theorem: Formula

$$P(\text{diagnosed sick}) =$$



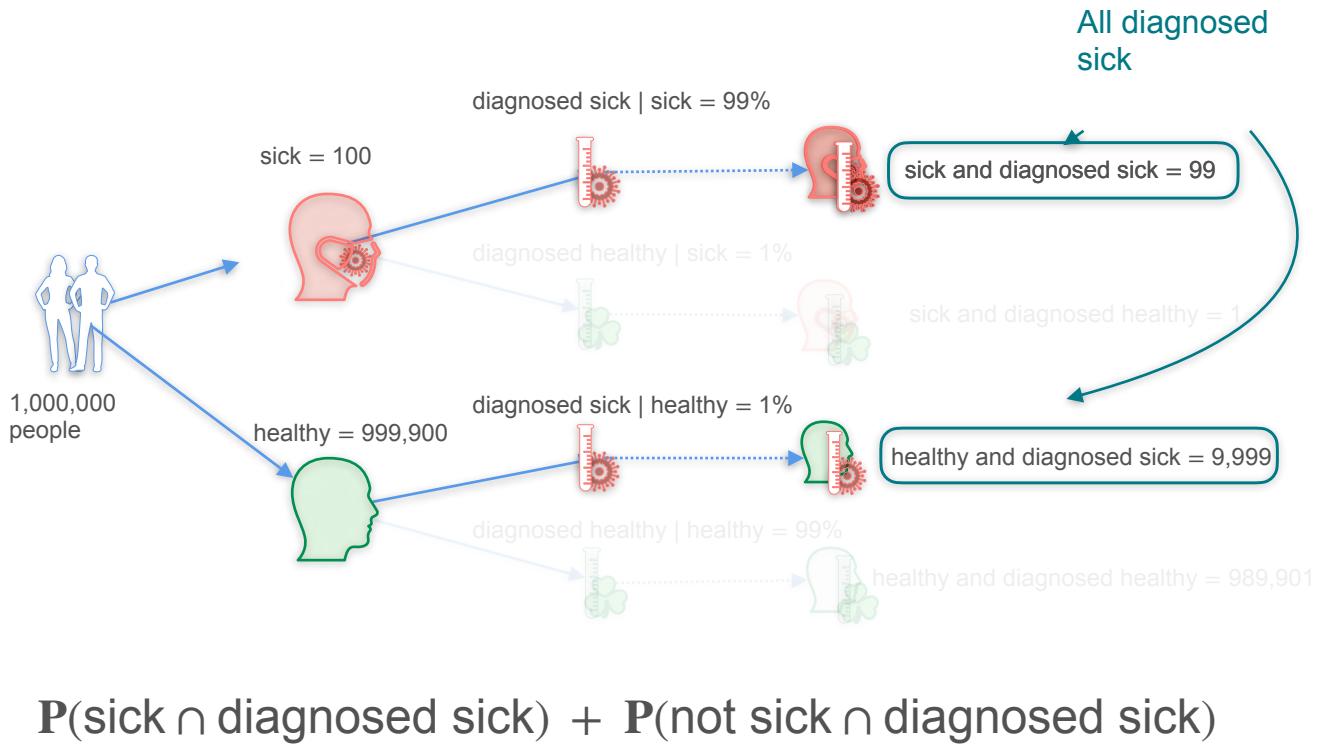
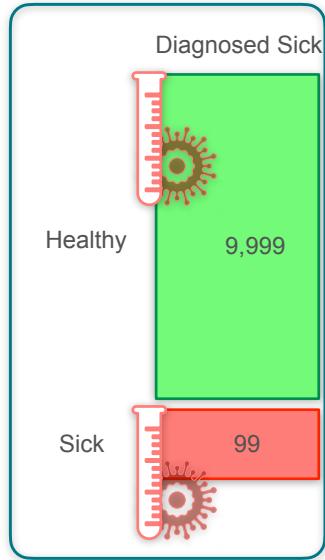
Bayes Theorem: Formula

$$P(\text{diagnosed sick}) =$$



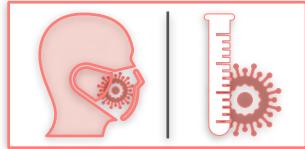
Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$



$$P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



A : sick

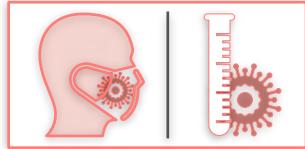
B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{diagnosed sick}) = ?}$$

$$P(\text{diagnosed sick}) = P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



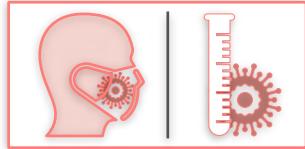
A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

Bayes Theorem: Formula



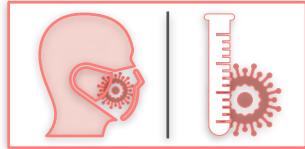
A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

Bayes Theorem: Formula



A : sick

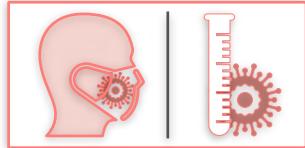
B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



A : sick

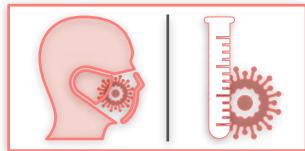
B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick}) = P(A \cap B)$$

Bayes Theorem: Formula



A: sick

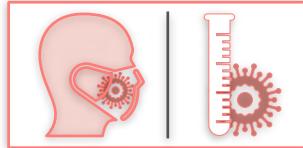
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick}) = P(A \cap B)$$

Bayes Theorem: Formula



A: sick

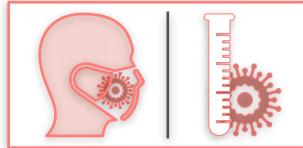
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick}) = P(A \cap B)$$

Bayes Theorem: Formula



A: sick

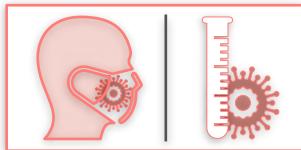
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

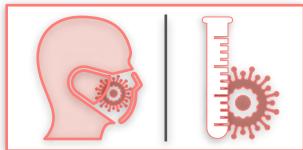
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

$$P(\text{not sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

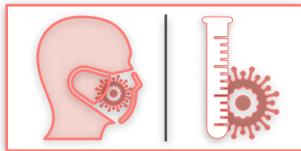
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

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$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

$$P(\text{not sick} \cap \text{diagnosed sick}) = P(A' \cap B)$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

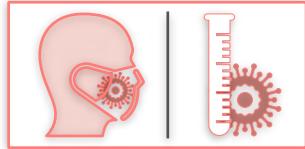
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

$$\begin{aligned} P(\text{not sick} \cap \text{diagnosed sick}) &= P(A' \cap B) \\ &= P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick}) \end{aligned}$$

Bayes Theorem: Formula



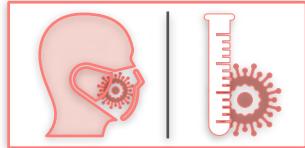
A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

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Bayes Theorem: Formula



A: sick

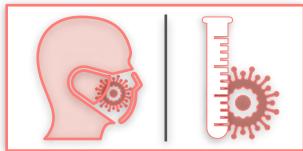
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

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$$P(\text{sick}) = 0.01\%$$

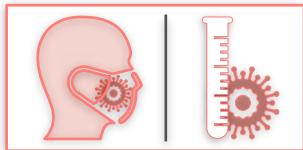
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

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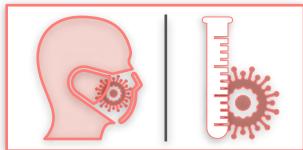
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**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

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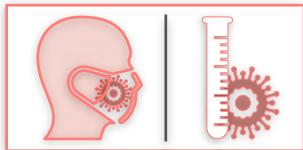
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**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

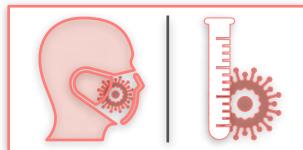
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

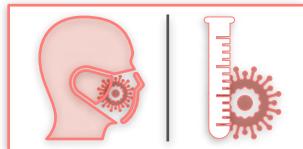
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$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

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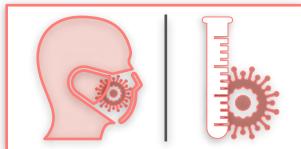
$P(\text{not sick}) = 99.99\%$

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$P(\text{diagnosed sick} | \text{not sick}) = 1\%$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

?

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

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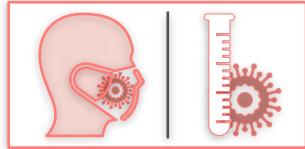
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$P(\text{diagnosed sick} | \text{not sick}) = 1\%$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



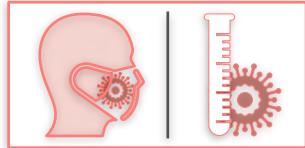
A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

Bayes Theorem: Formula



A: sick

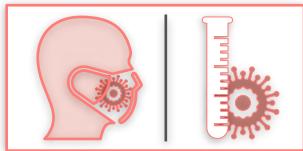
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 1\%$$

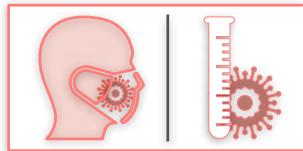
$$P(\text{not sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

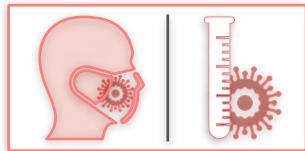
$$P(\text{not sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

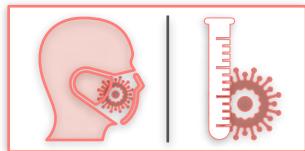
$$P(A') = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

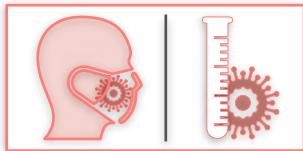
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

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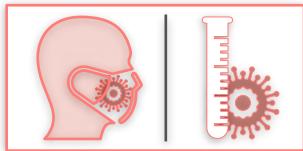
$$P(A') = 99.99\%$$

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$$P(B | A') = 1\%$$

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$$P(A) = 0.01\%$$

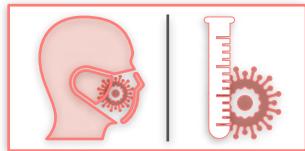
$$P(A') = 99.99\%$$

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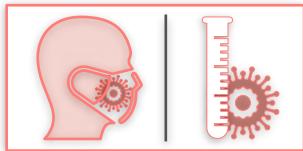
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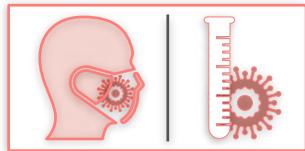
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$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

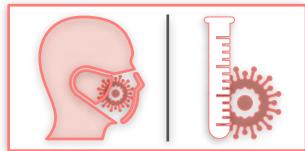
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$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

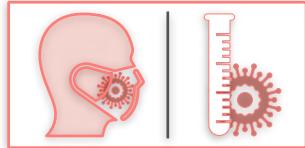
$$P(A') = 99.99\%$$

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Bayes Theorem: Formula



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$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

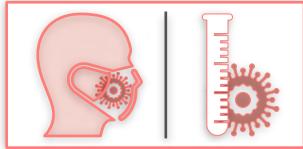
$$P(A') = 99.99\%$$

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$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$\mathbf{P}(A | B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

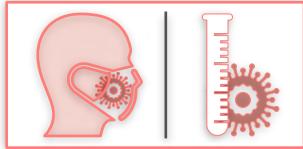
$$\mathbf{P}(A) = 0.01\%$$

$$\mathbf{P}(A') = 99.99\%$$

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$$\mathbf{P}(B | A') = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = 0.01\%$$

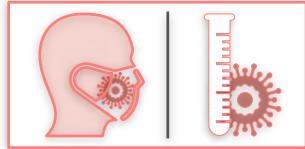
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FORMULA**

Bayes Theorem: Formula



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$$P(A) = 0.01\%$$

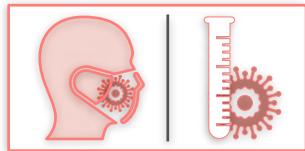
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FORMULA**

Bayes Theorem: Formula



A : sick

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$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

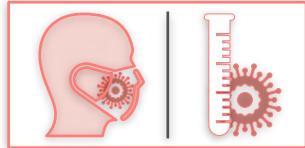
$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

$$P(A | B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

$$P(A | B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

$$P(A | B) = 0.0098$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Spam Example

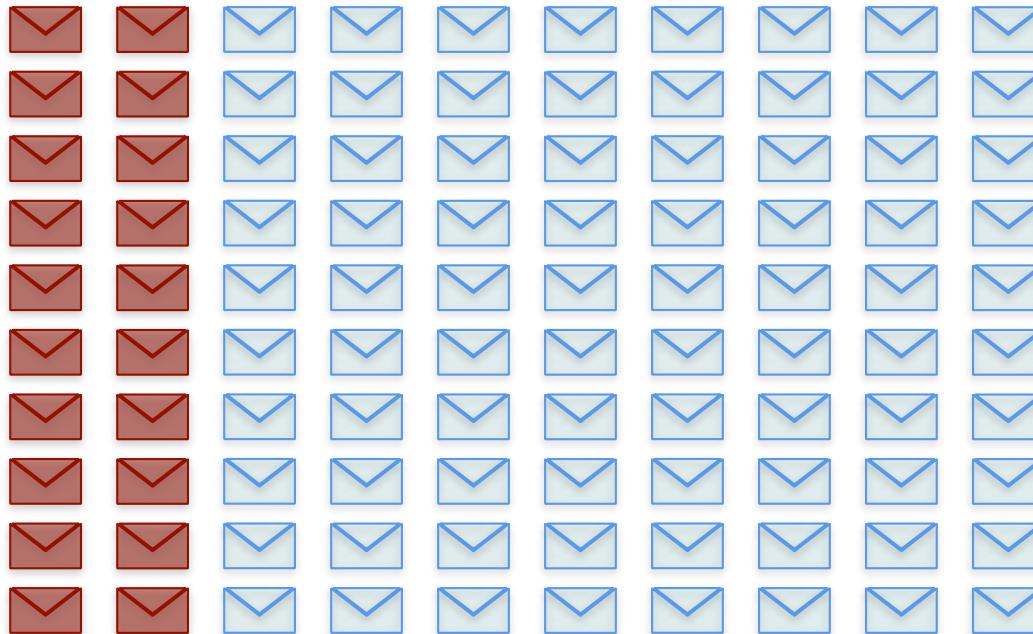
Bayes Theorem: Spam Example



Bayes Theorem: Spam Example



Bayes Theorem: Spam Example



20 spam

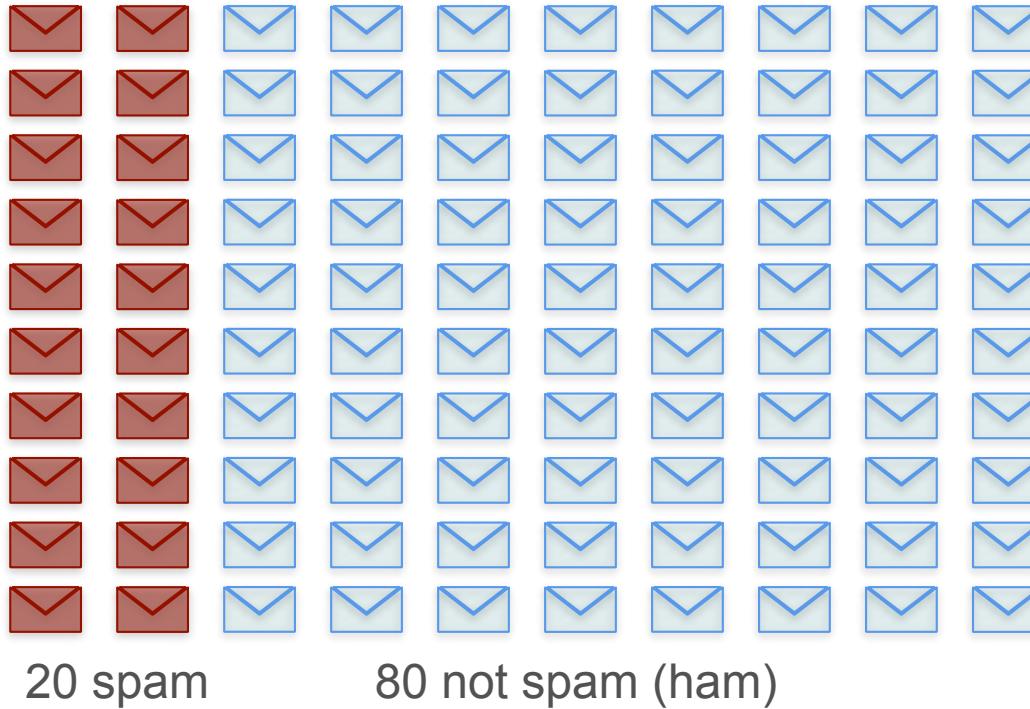
Bayes Theorem: Spam Example



20 spam

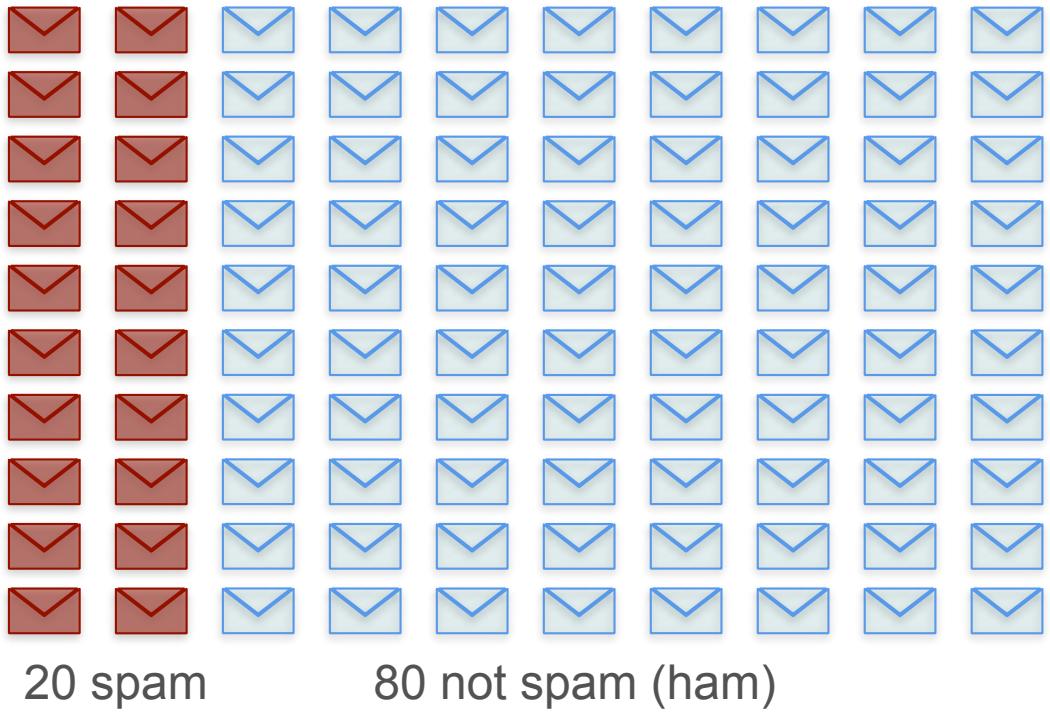
80 not spam (ham)

Bayes Theorem: Spam Example

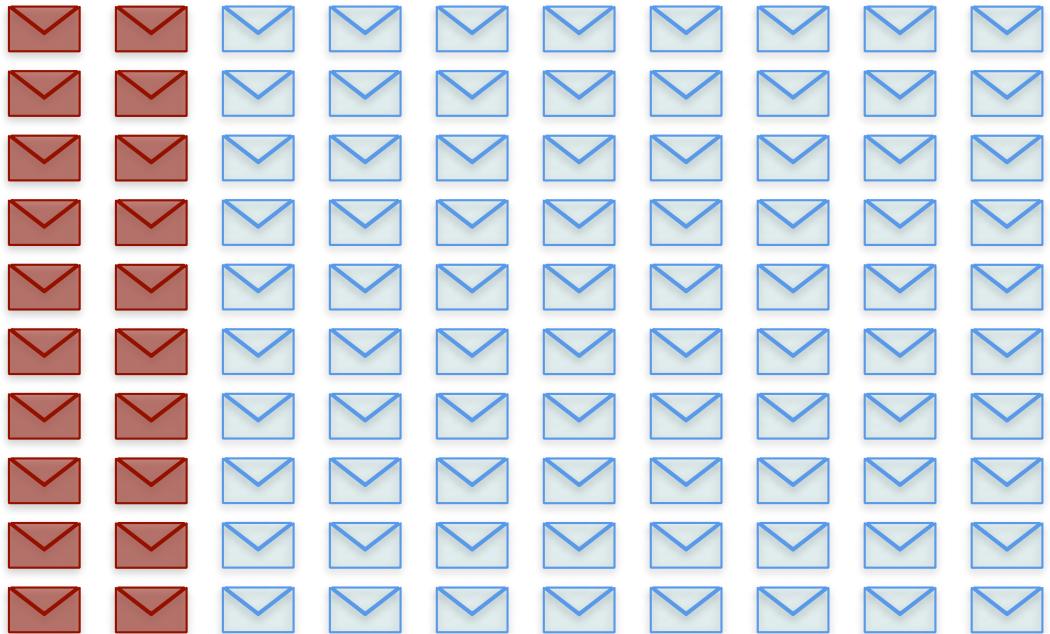
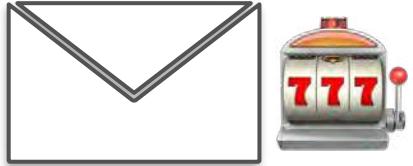


“lottery”

Bayes Theorem: Spam Example



Bayes Theorem: Spam Example



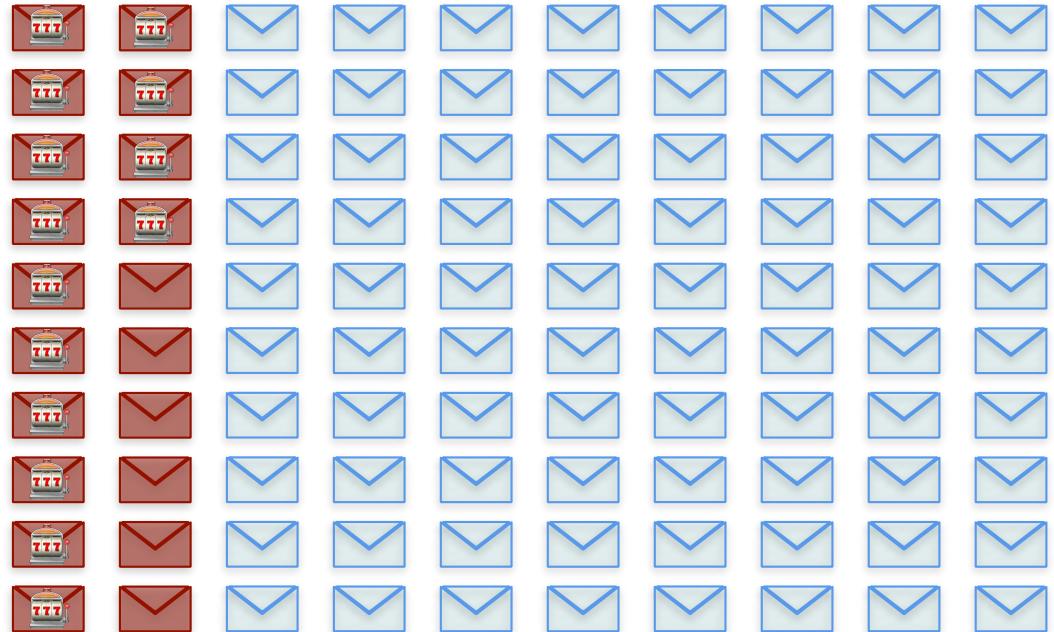
20 spam

80 not spam (ham)

Bayes Theorem: Spam Example



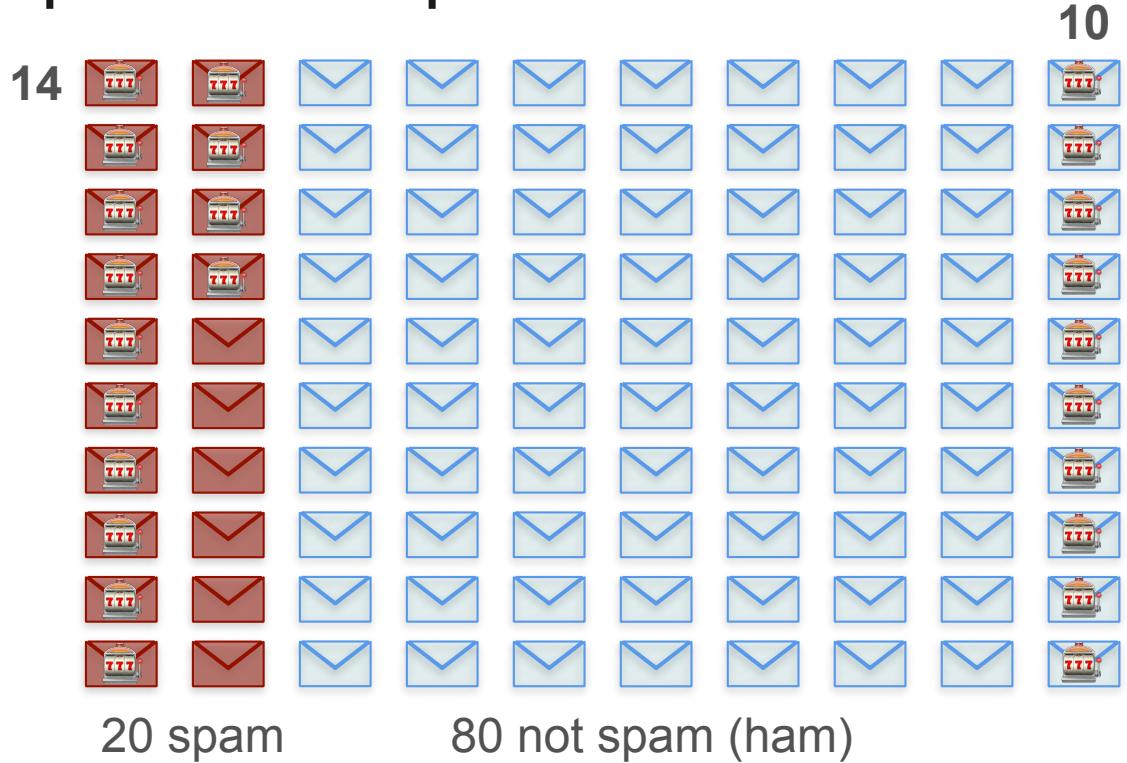
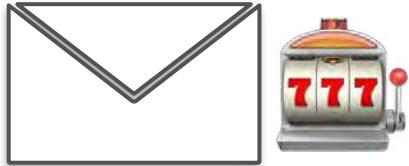
14



20 spam

80 not spam (ham)

Bayes Theorem: Spam Example



Bayes Theorem: Spam Example

10



What is the probability that an email containing lottery is a spam?

14

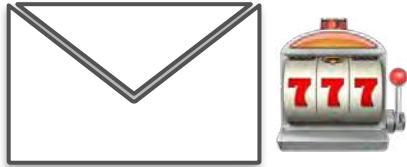


20 spam

80 not spam (ham)

Bayes Theorem: Spam Example

10



What is the probability that an email containing lottery is a spam?

$P(\text{spam} \mid \text{lottery})$

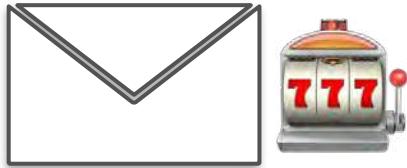
14



20 spam

80 not spam (ham)

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

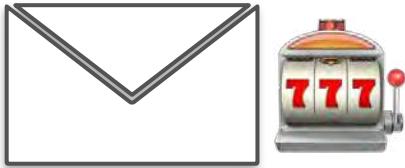


14



10

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$



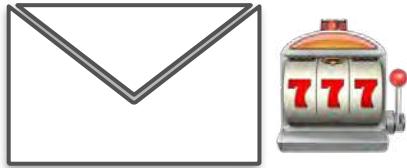
14

24 emails
containing lottery



10

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

24 emails
containing lottery



10

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



24 emails
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

14

10

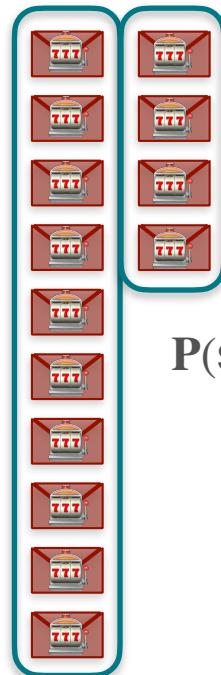


Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



24 emails
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$



Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

24 emails
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

10

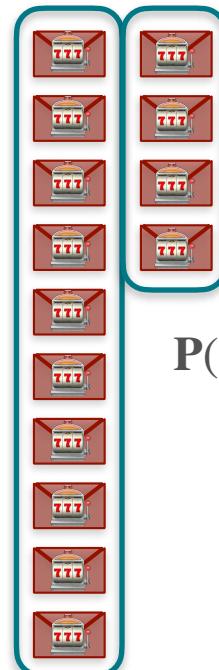


Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

$$= \frac{7}{12} = 0.583$$

10



Bayes Theorem: Spam Example (Formula Solution)

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam} \mid \text{lottery})$$

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

A: Email is spam

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

A : Email is spam B : Email contains lottery

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

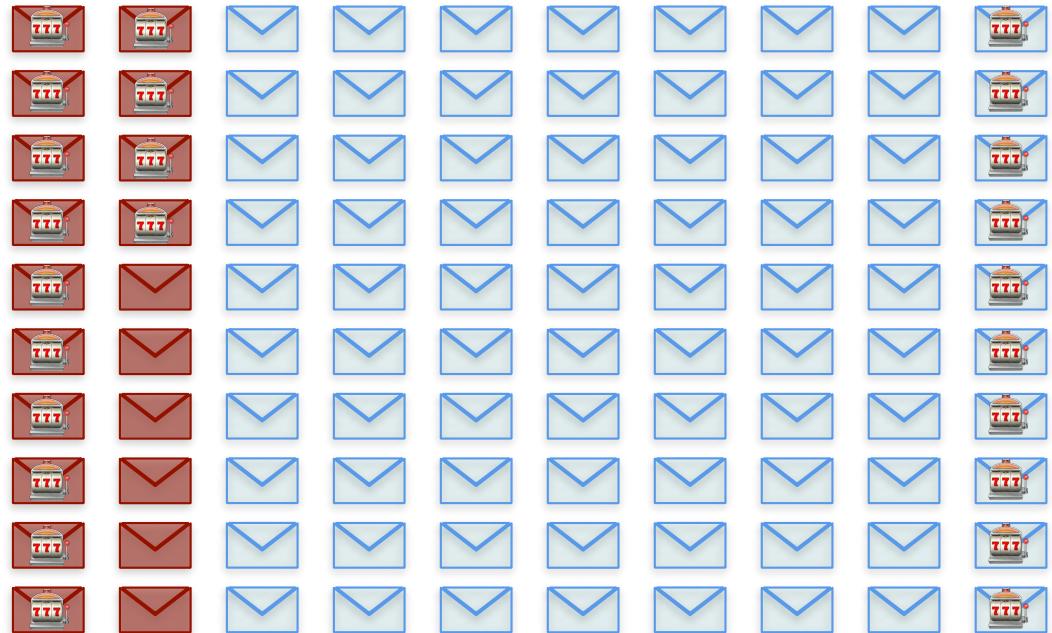
A : Email is spam B : Email contains lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

Bayes Theorem: Spam Example (Formula Solution)

10

14



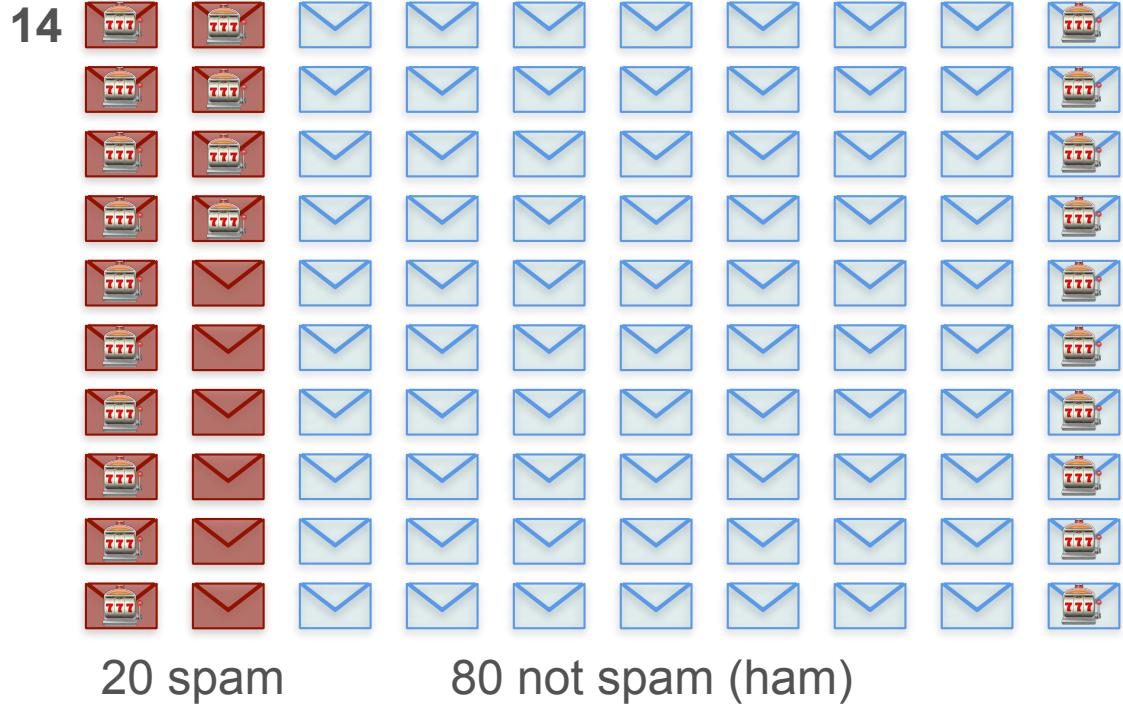
20 spam

80 not spam (ham)

Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

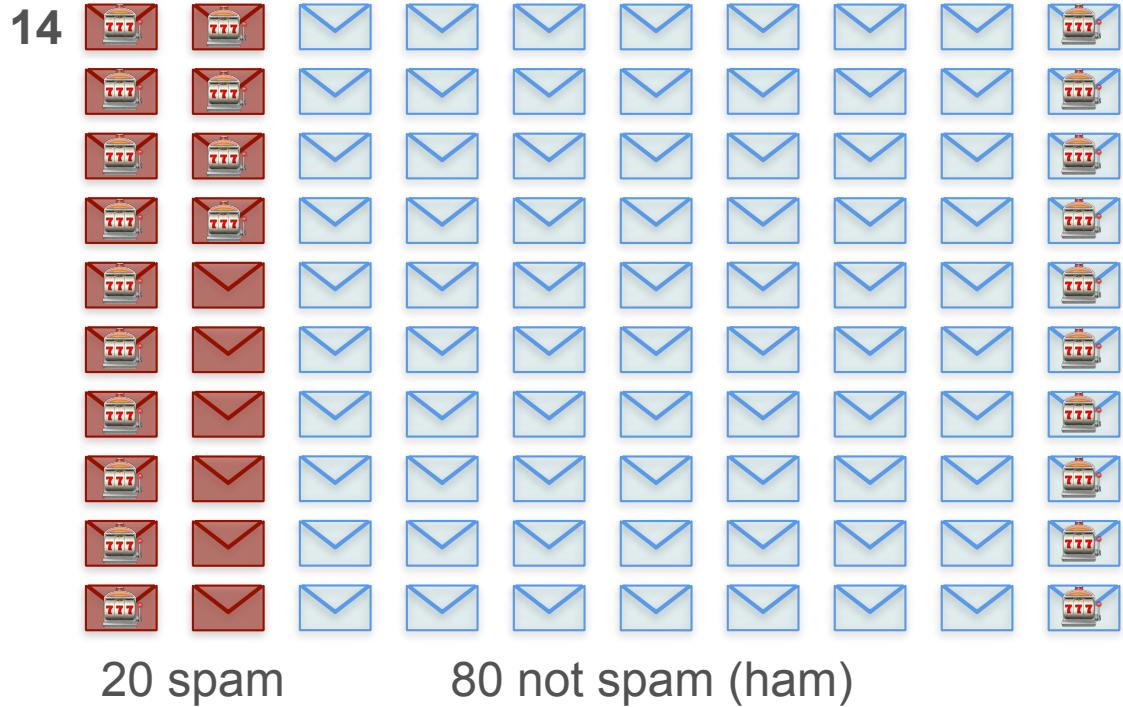


Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$



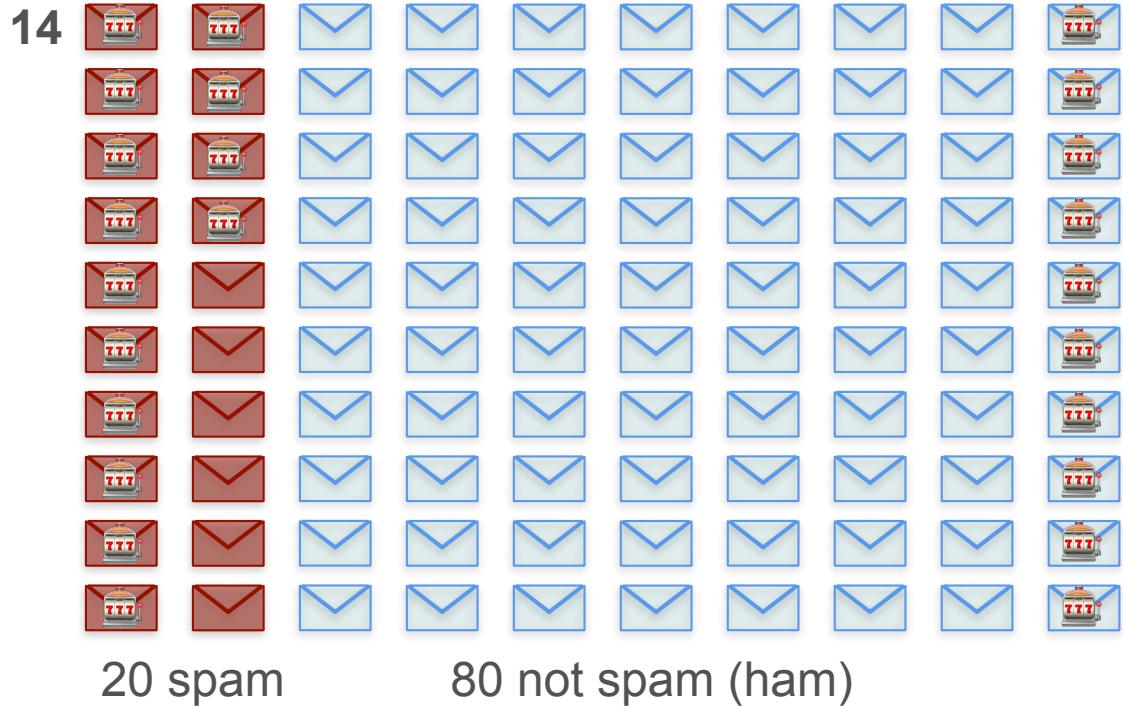
Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



Bayes Theorem: Spam Example (Formula Solution)

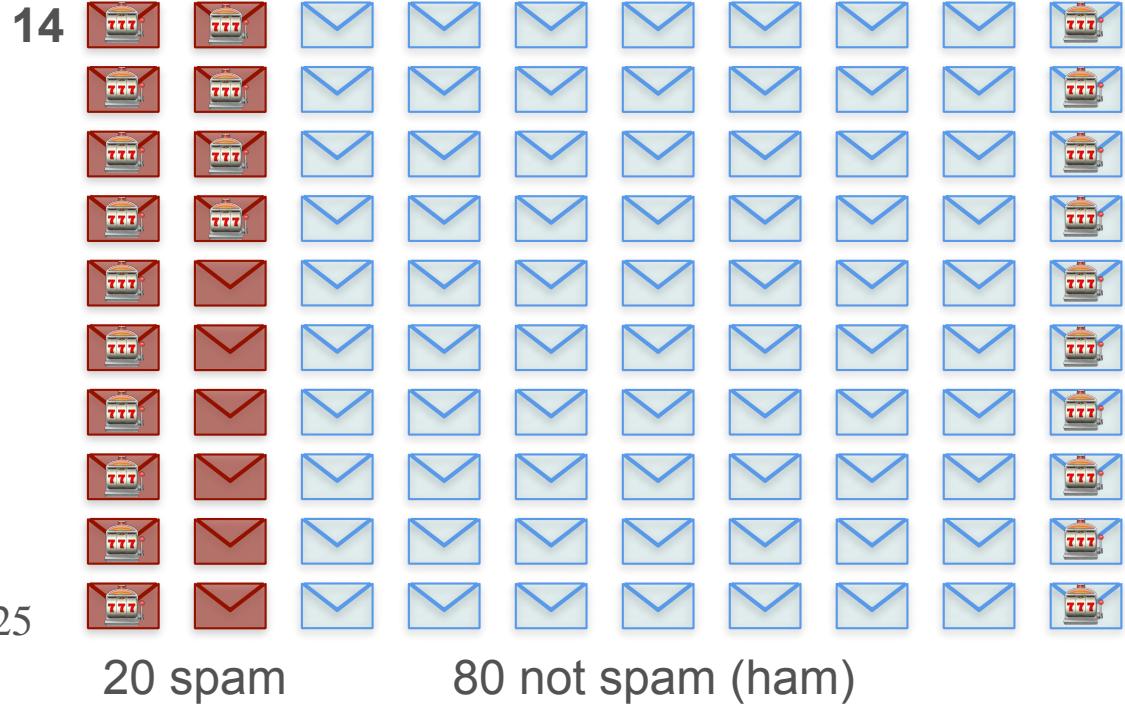
10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} | \text{spam}) = \frac{14}{20} = 0.7$$

$$P(\text{lottery} | \text{not spam}) = \frac{10}{80} = 0.125$$



Bayes Theorem: Spam Example (Formula Solution)

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

$$P(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)}$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

$$P(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)} = 0.583$$

Bayes Theorem

Bayes Theorem

PRIOR

Bayes Theorem

PRIOR

EVENT

Bayes Theorem

PRIOR

EVENT

POSTERIOR

Bayes Theorem

PRIOR

EVENT

POSTERIOR

$$\mathbf{P}(A)$$

Bayes Theorem

PRIOR

$\mathbf{P}(A)$

EVENT

E

POSTERIOR

Bayes Theorem

PRIOR

$$\mathbf{P}(A)$$

EVENT

$$E$$

POSTERIOR

$$\mathbf{P}(A | E)$$

Prior and Posterior

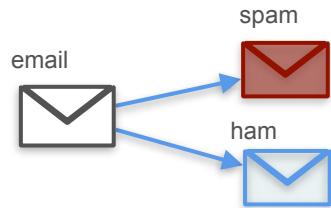
PRIOR

EVENT

POSTERIOR

Prior and Posterior

PRIOR

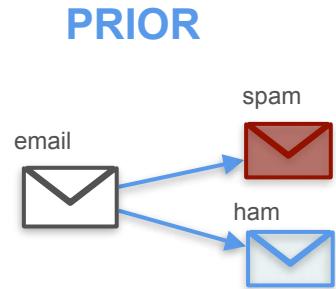


EVENT

POSTERIOR

$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Prior and Posterior



EVENT

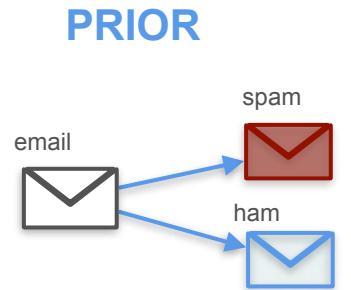


Email contains lottery

POSTERIOR

$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

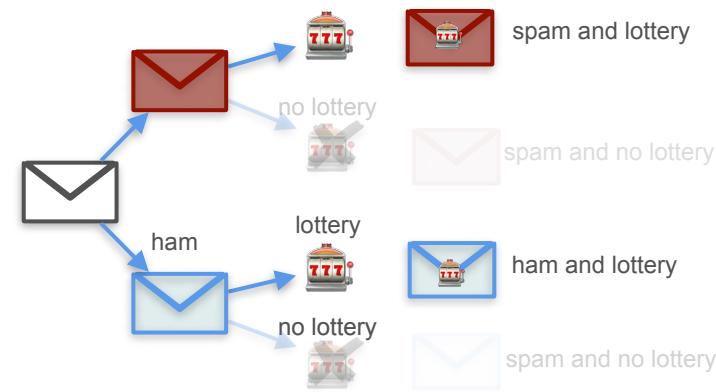
Prior and Posterior



EVENT



Email contains lottery



$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

$$P(\text{spam} | \text{lottery}) = \frac{\text{spam and lottery}}{\text{spam and lottery} + \text{ham and lottery}}$$

Prior and Posterior

PRIOR

EVENT

POSTERIOR

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

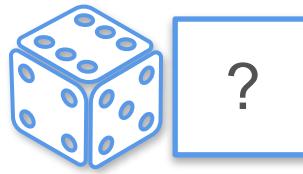
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



POSTERIOR

1st dice is 6

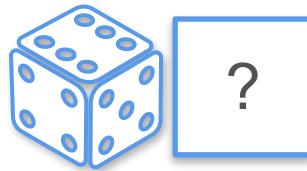
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

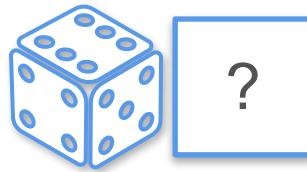
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

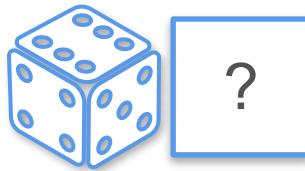
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{3}{36}$$

$$P(\text{sum} = 10 | \text{1st is } 6) = \frac{1}{6}$$

Prior and Posterior

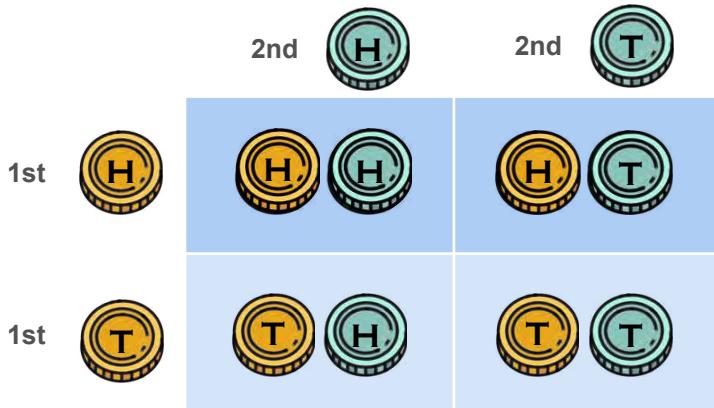
PRIOR

EVENT

POSTERIOR

Prior and Posterior

PRIOR

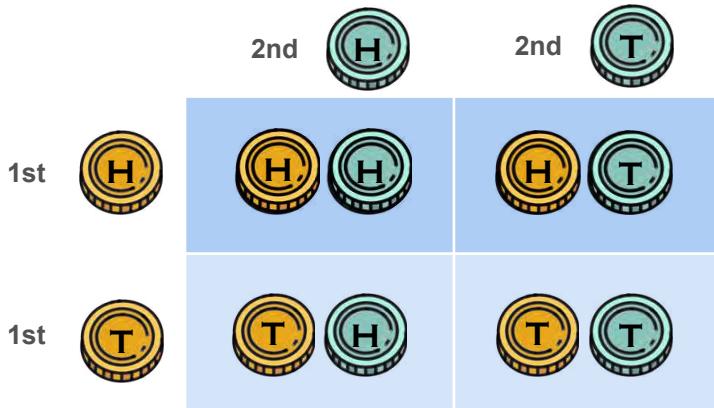


EVENT

POSTERIOR

Prior and Posterior

PRIOR



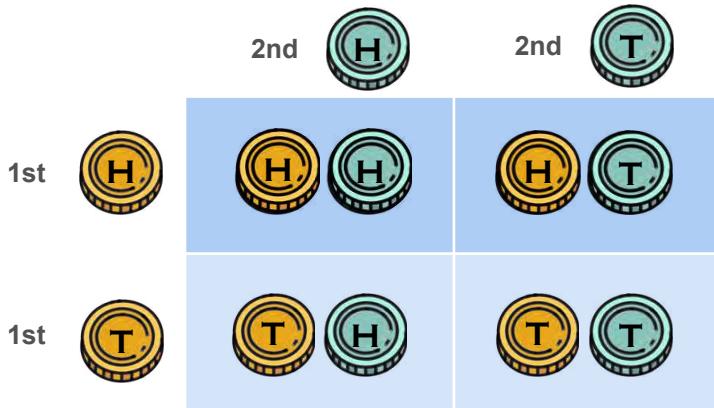
EVENT

POSTERIOR

$$P(HH) = \frac{1}{4}$$

Prior and Posterior

PRIOR



EVENT



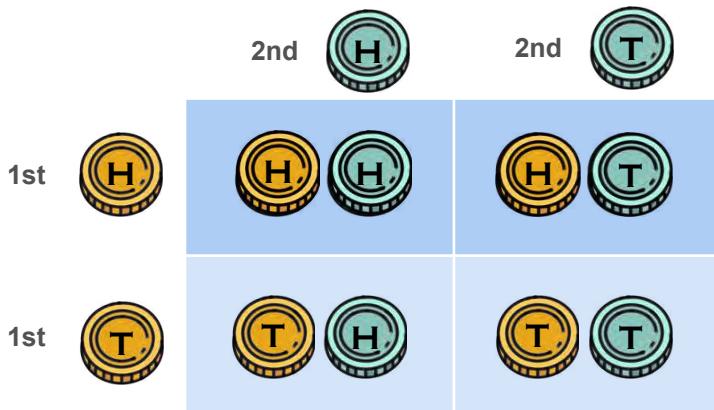
1st coin is H

POSTERIOR

$$P(HH) = \frac{1}{4}$$

Prior and Posterior

PRIOR



$$P(HH) = \frac{1}{4}$$

EVENT



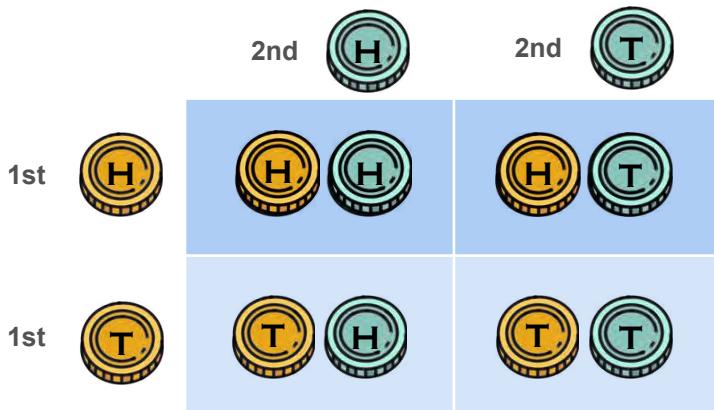
1st coin is H

POSTERIOR



Prior and Posterior

PRIOR



EVENT



POSTERIOR



$$P(HH | \text{1st is } H) = \frac{1}{2}$$

Video 8e: the Naive Bayes Model

What About 2 Events?

PRIOR

EVENT

POSTERIOR

What About 2 Events?

PRIOR

EVENT

POSTERIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'

POSTERIOR

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$



Email contains 'lottery' and 'winning'

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$

?

What About 2 Events?

EVENT



POSTERIOR

Email contains 'lottery' and 'winning'

What About 2 Events?

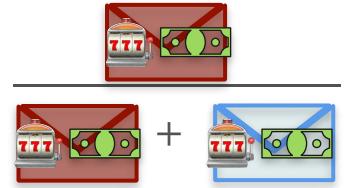
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$



What About 2 Events?

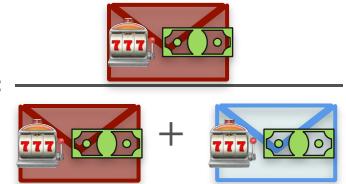
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

What About 2 Events?

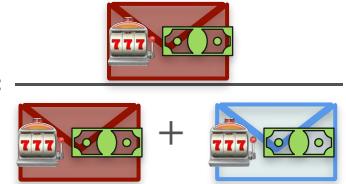
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{?}}{\text{?}}$$



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

What About 2 Events?

EVENT



Email contains 'lottery' and 'winning'

POSTERIOR



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{# Spam emails with 'lottery' and 'winning'}}{\text{# Emails with 'lottery' and 'winning'}}$$

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

What About More Than 2 Events?

EVENT

POSTERIOR

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{P(\text{spam}) P(w_1, \dots, w_{100} | \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

?

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

A red arrow points from the fraction above to the term $P(w_1, \dots, w_{100} | \text{spam})$, which is highlighted with a red oval and followed by a question mark.

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

The fraction $\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}$ is highlighted with a red underline and a red arrow points from it to a question mark $?$. The term $P(w_1, \dots, w_{100} | \text{spam})$ is also highlighted with a red circle.

Is There a Quicker Way To Estimate the Probability?

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

$P(A \cap B)$

↓

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

$P(A \cap B) = P(A) \cdot P(B)$

↓

The terms $P(\text{lottery \& winning} \mid \text{spam})$ and $P(\text{lottery \& winning} \mid \text{ham})$ are circled in red.

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery} \mid \text{ham}) \cdot \mathbf{P}(\text{winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_n \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of the words w_1, w_2, \dots, w_n are independent

$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_n \mid \text{ham})}$$

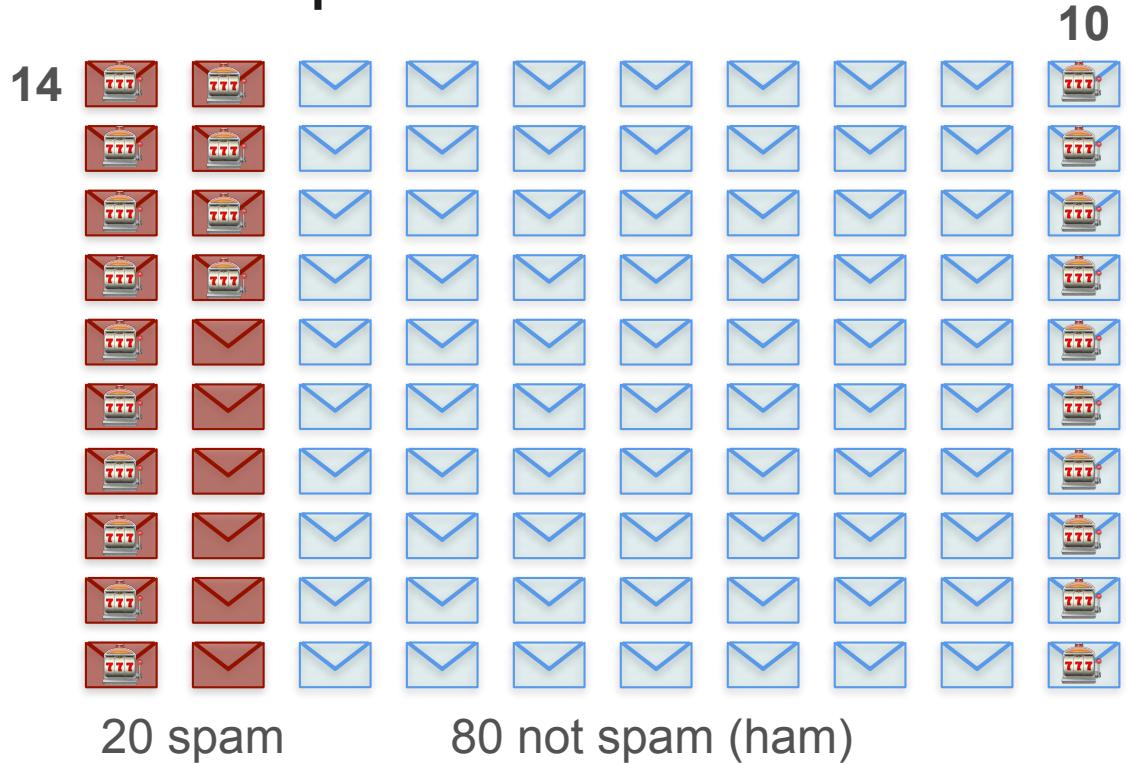
Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of the words w_1, w_2, \dots, w_n are independent

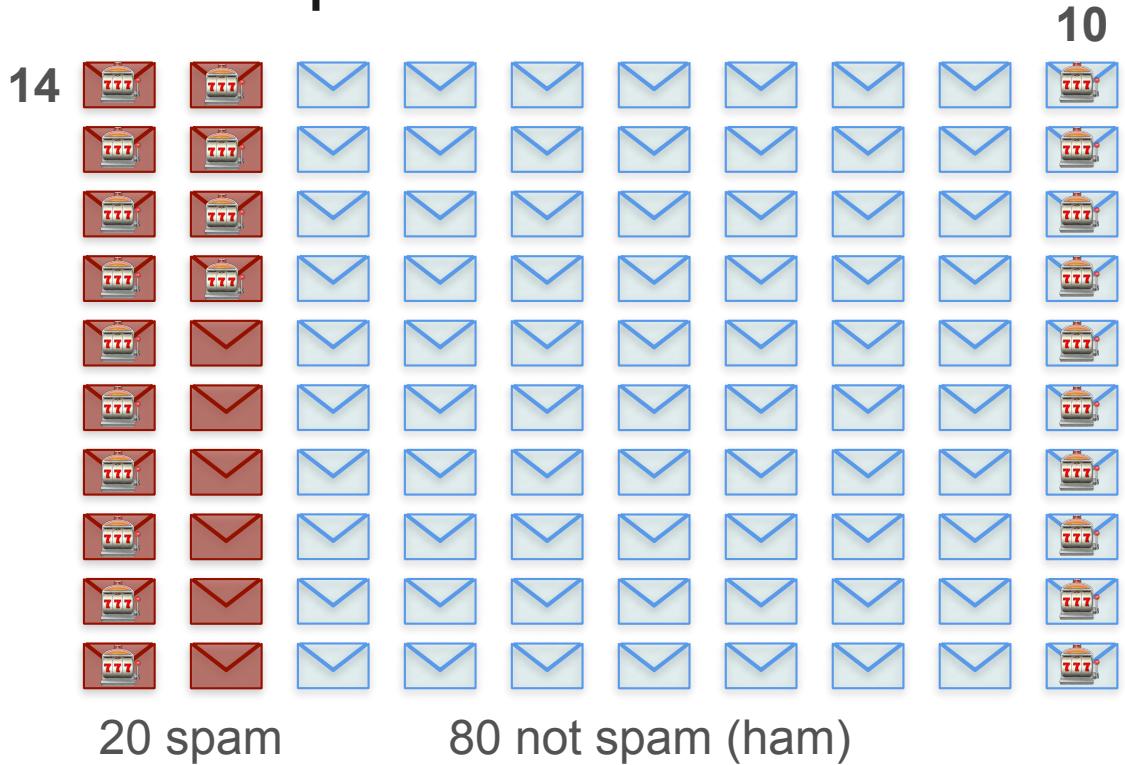
$$P(\text{spam} | w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1 | \text{spam}) \cdots P(w_n | \text{spam})}{P(\text{spam}) \cdot P(w_1 | \text{spam}) \cdots P(w_n | \text{spam}) + P(\text{ham}) \cdot P(w_1 | \text{ham}) \cdots P(w_n | \text{ham})}$$

Naive Bayes: Spam Example



Naive Bayes: Spam Example

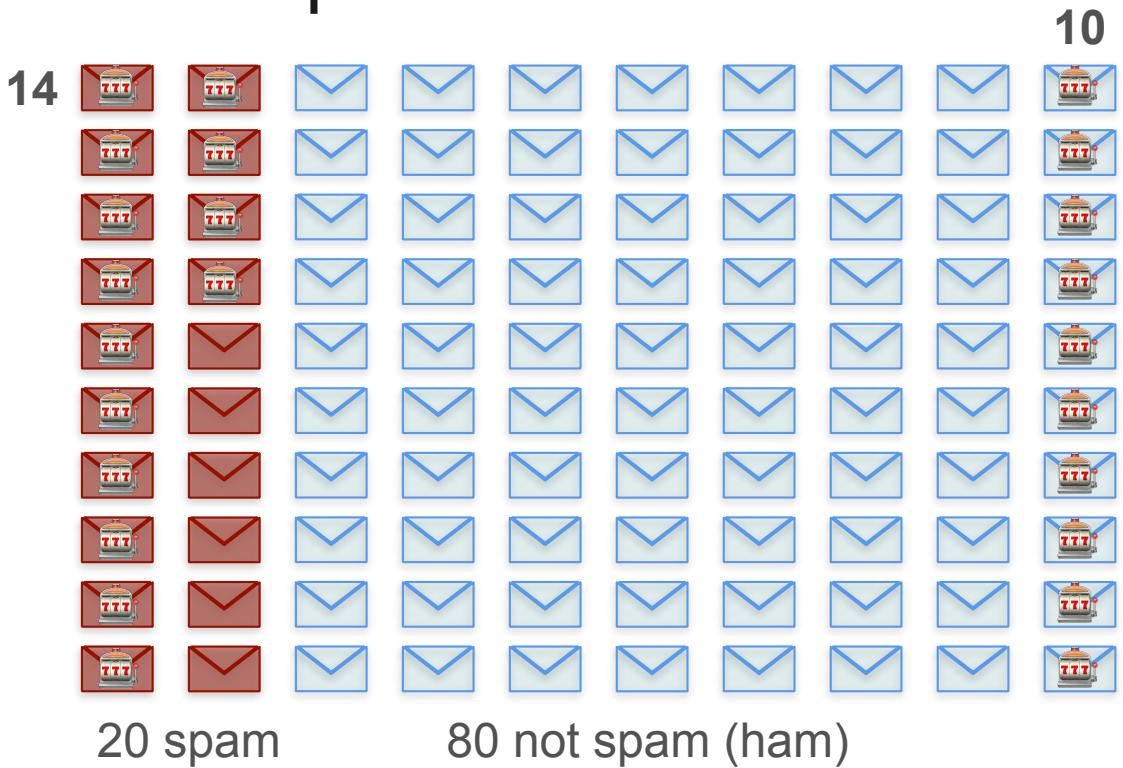
$$P(\text{spam}) = \frac{20}{100} = 0.2$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

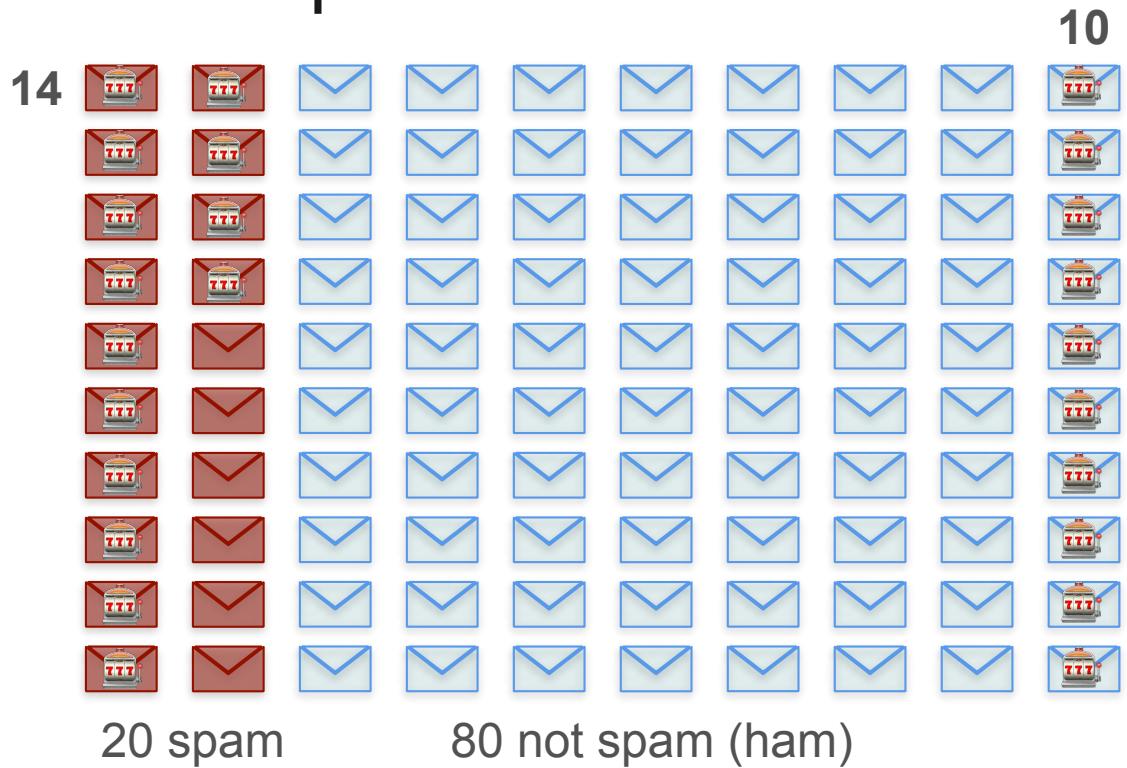


Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



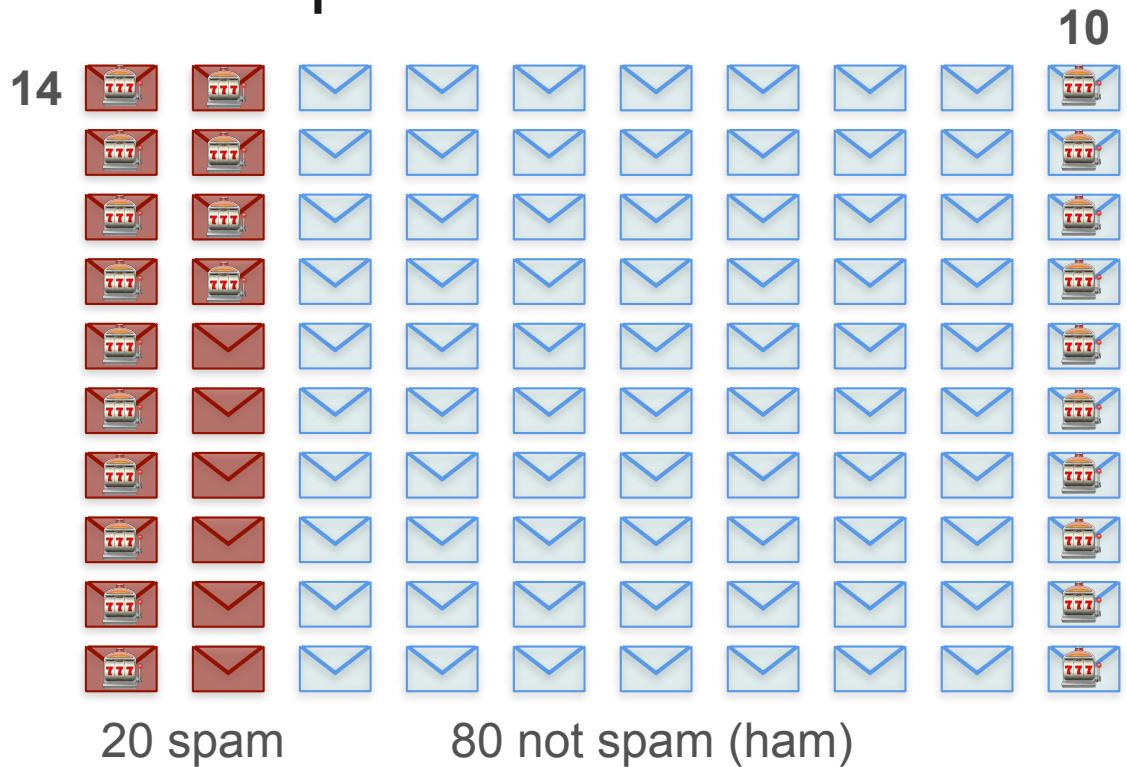
Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} | \text{spam}) = \frac{14}{20} = 0.7$$

$$P(\text{lottery} | \text{ham}) = \frac{10}{80} = 0.125$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$



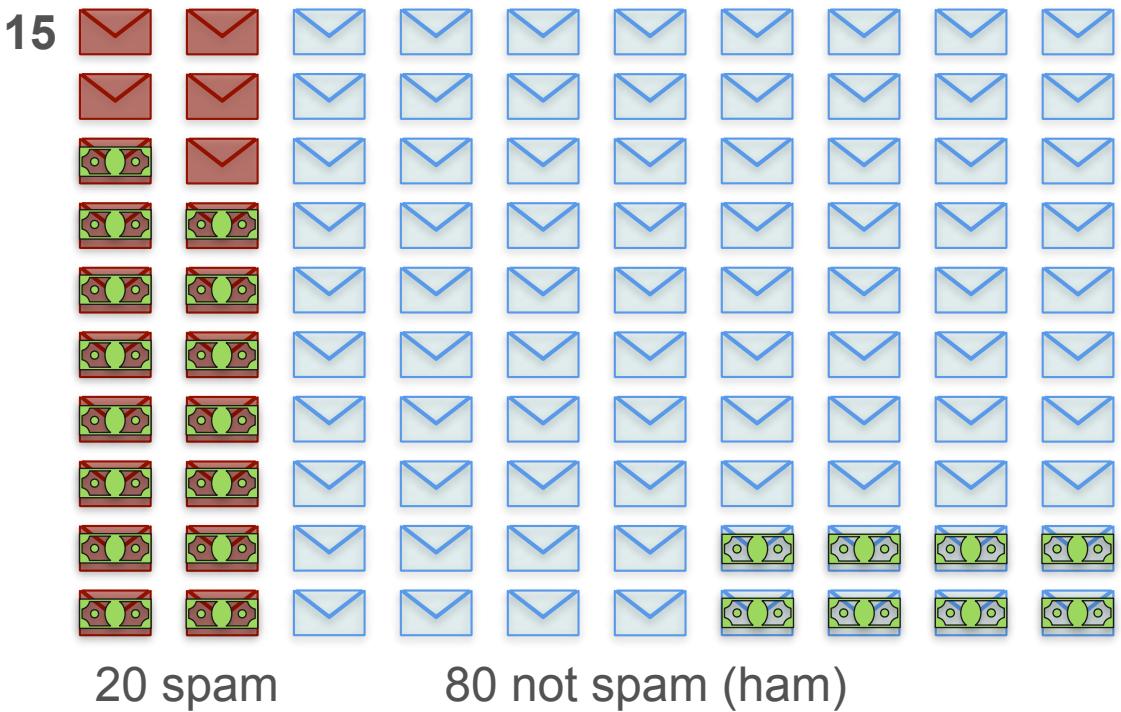
20 spam

80 not spam (ham)

Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

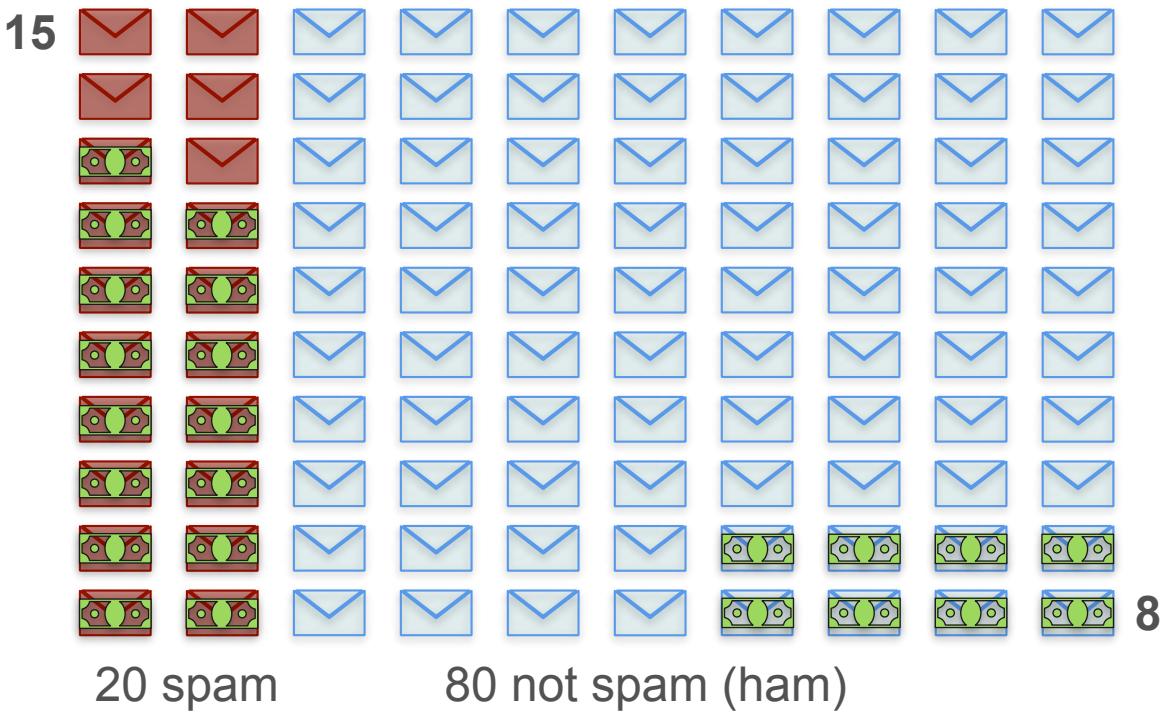
$$P(\text{ham}) = \frac{80}{100} = 0.8$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$



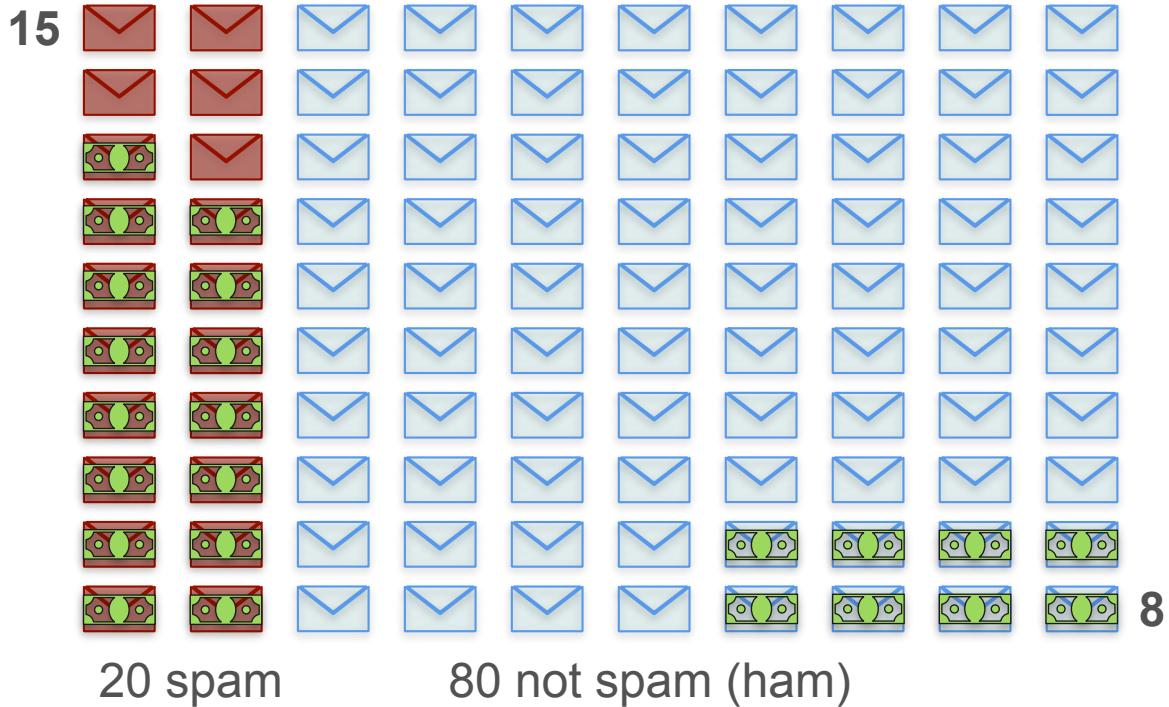
Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$

$$P(\text{winning} \mid \text{ham}) = \frac{8}{80} = 0.1$$



Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

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$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)}$$

Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

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$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)} = 0.913$$



DeepLearning.AI

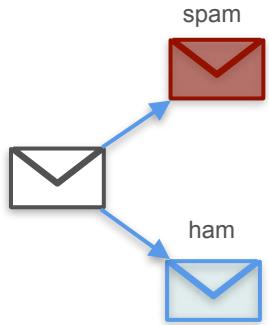
Introduction to probability

Probability in Machine Learning

Bayes Theorem

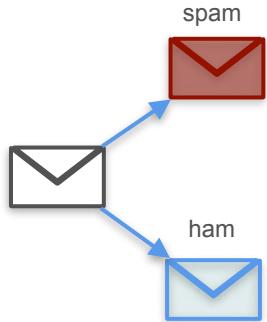
Bayes Theorem

PRIOR



Bayes Theorem

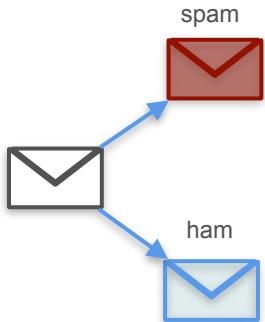
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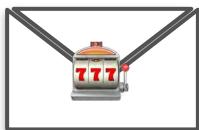
$$P(\text{spam}) = \frac{\text{spam icon}}{\text{spam icon} + \text{ham icon}}$$

Bayes Theorem

PRIOR



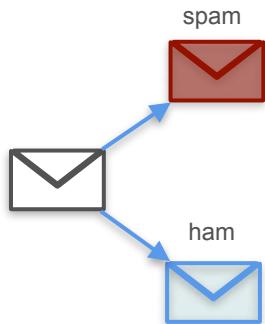
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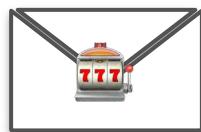
$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

Bayes Theorem

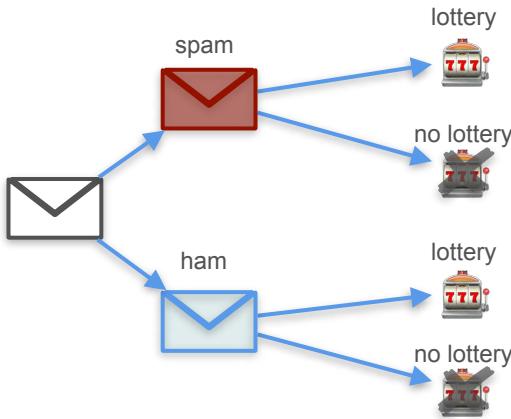
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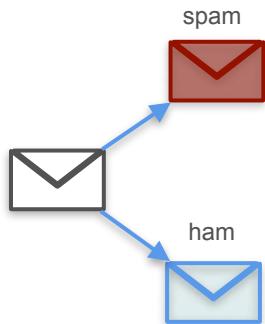
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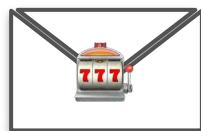
$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

Bayes Theorem

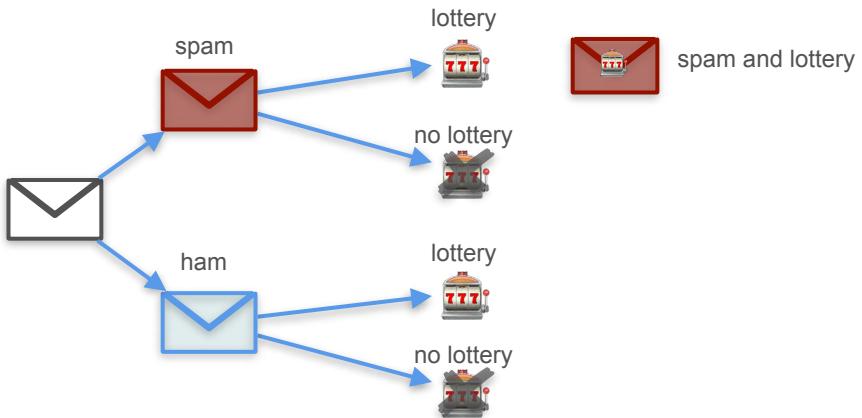
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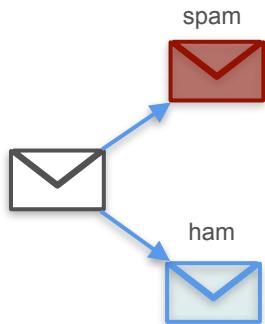
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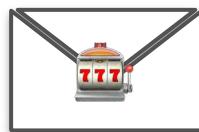
$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Bayes Theorem

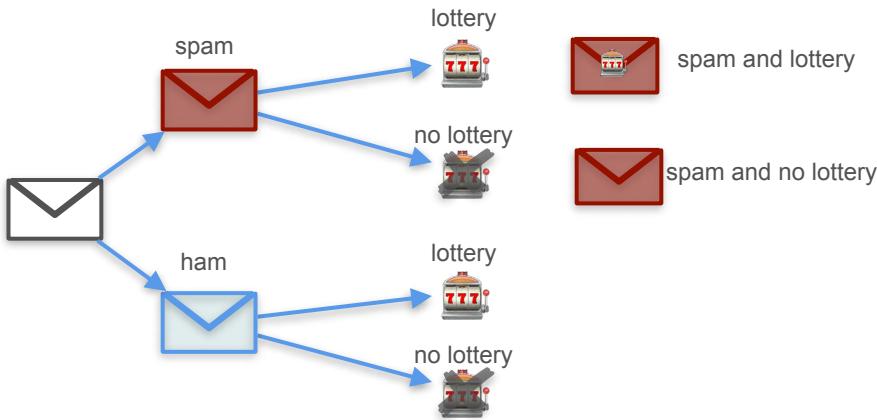
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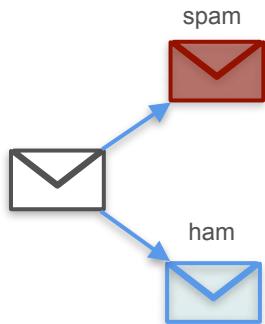
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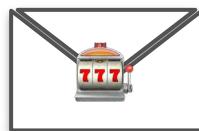
$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

Bayes Theorem

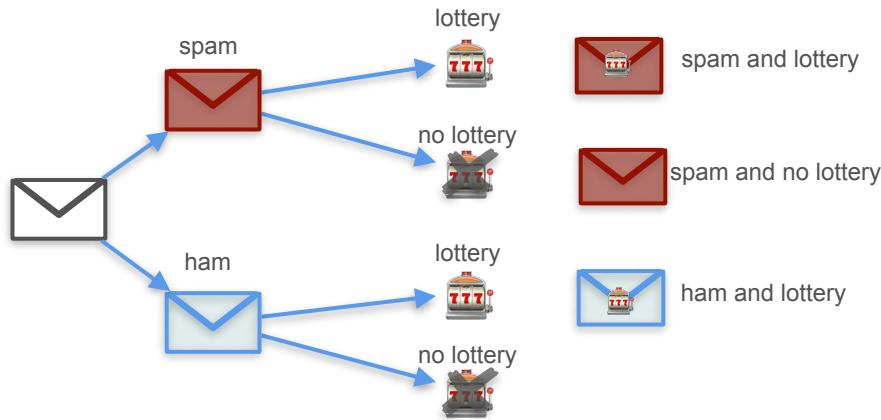
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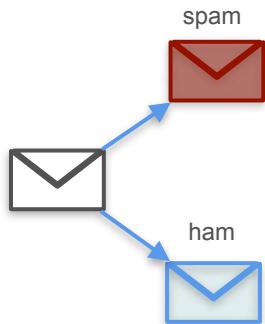
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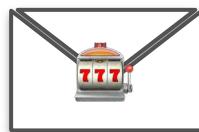
$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Bayes Theorem

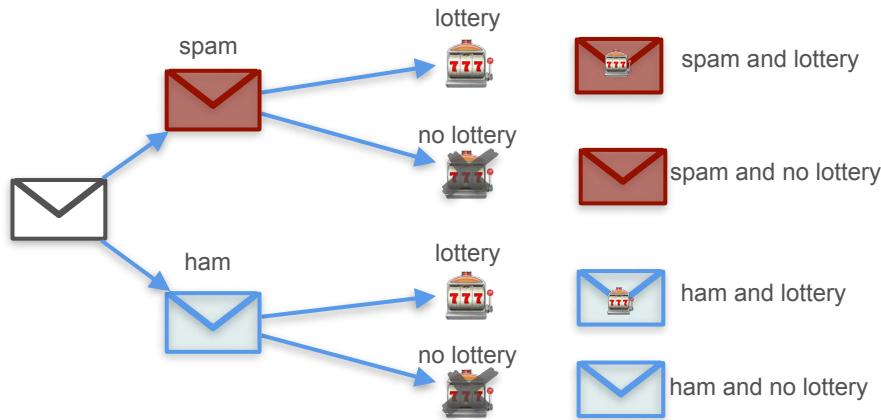
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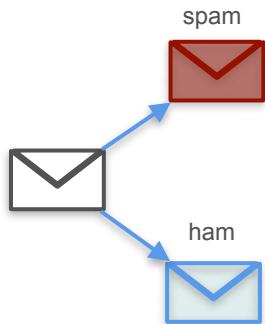
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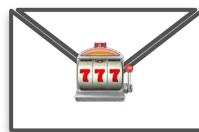
$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Bayes Theorem

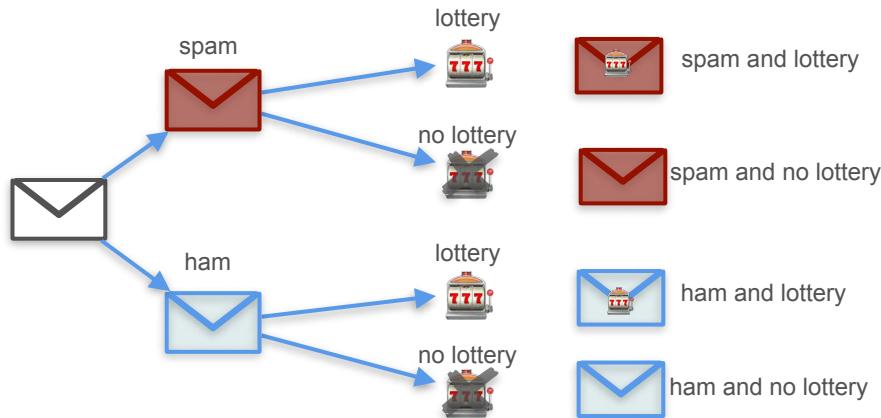
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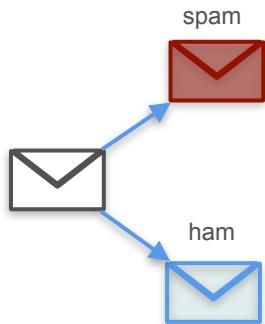


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

$$P(\text{spam} | \text{lottery}) =$$

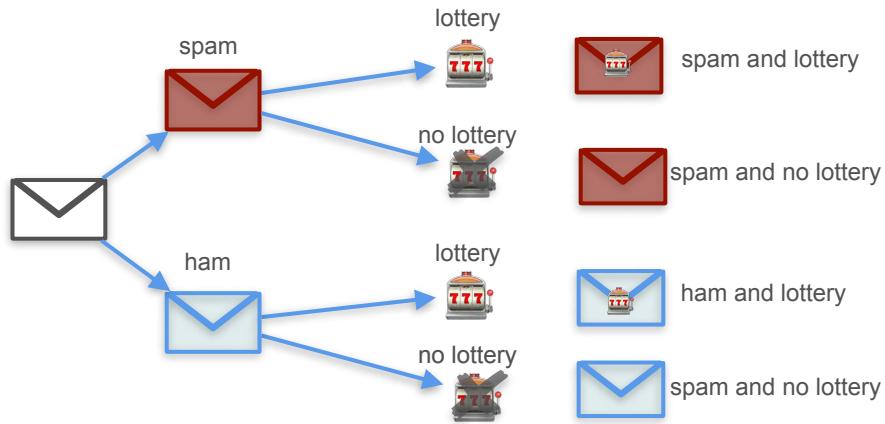
Bayes Theorem

PRIOR



$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

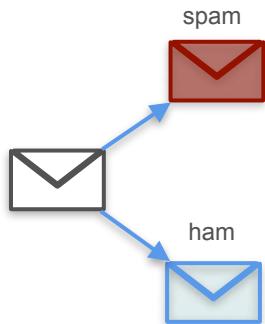
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$$P(\text{spam} | \text{lottery}) =$$

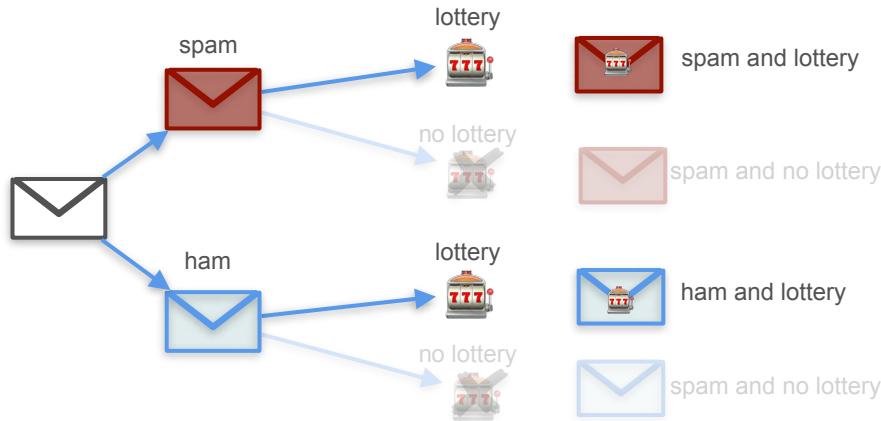
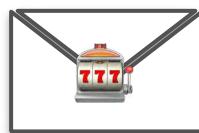
Bayes Theorem

PRIOR



$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

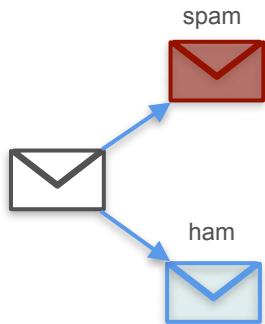
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$$P(\text{spam} | \text{lottery}) =$$

Bayes Theorem

PRIOR

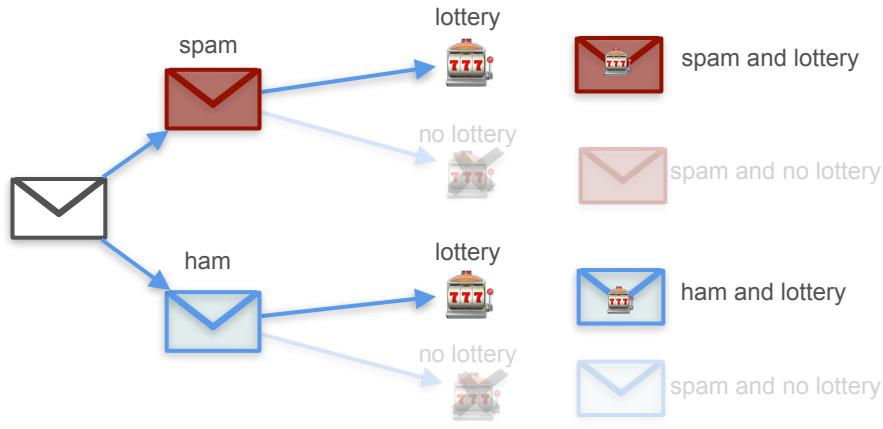


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

EVENT



POSTERIOR



$$P(\text{spam} | \text{lottery}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Example Problem

Example Problem

Image recognition

Example Problem

Image recognition

- What is the probability that there is a cat in the image



Example Problem

Image recognition

- What is the probability that there is a cat in the image
- $P(\text{cat} \mid \text{image}) = P(\text{cat} \mid \text{pixel}_1, \text{pixel}_2, \dots, \text{pixel}_n)$

